

# SU(6) Grand Unification of 3-3-1 Model

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**Abstract.** We discuss a sequential variant of the  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  model which can fit within a minimal SU(6) grand unification. Interestingly, this minimal SU(6) embedding can allow a  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  symmetry breaking scale within the reach of LHC and with seesaw-type neutrino masses.

**Keywords:** grand unification, 3-3-1 model, SU(6)

**Introduction:** The  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  [1] gauge extension of the SM provides a strong promise of new physics that can be observed at the LHC or the next generation accelerators [2, 3] and also can provide novel ways to understand neutrino masses. The  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  model proposed by Singer, Valle and Schechter (SVS) [2] is not anomaly free for each generation of fermions, however, when all the three generations of fermions are included the theory becomes anomaly free. Consequently, different multiplets of the  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  group appear with different multiplicity and it becomes difficult to unify the model within usual grand unified theories. In Ref. [4] we have studied how such a theory can be unified in a SU(6) gauge theory that can emerge from a E(6) Grand Unified Theory (GUT). Interestingly, the SVS 3-3-1 model can readily be refurbished into an anomaly free multiplet structure which can be right away embedded in a minimal anomaly free combination of representations of SU(6) as an E(6) subgroup. We refer to this new 3-3-1 model as the sequential 3-3-1 model. This scheme is particularly intriguing since this SU(6) embedding does not require any bulk exotics to account for the chiral families; and in that sense it provides a truly minimal unification scenario in the same spirit akin to the minimal SU(5) construction [5].

**The sequential  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  Model:** In this model we assign the fields in such a way that the anomalies are cancelled for each generation separately. The multiplet structure is given by

$$\begin{aligned} Q_{aL} &= (u_{aL}, d_{aL}, D_{aL})^T \equiv [3, 3, 0], \quad u_{aR} \equiv [3, 1, 2/3], \quad d_{aR} \equiv [3, 1, -1/3], \\ D_{aR} &\equiv [3, 1, -1/3], \quad \psi_{aL} = (e_{aL}^-, \nu_{aL}, N_{aL}^1)^T \equiv [1, 3^*, -1/3], \\ \xi_{aL} &= (E_{aL}^-, N_{aL}^2, N_{aL}^3)^T \equiv [1, 3^*, -1/3], \quad \chi_{aL} = (N_{aL}^4, E_{aL}^+, e_{aL}^+) \equiv [1, 3^*, 2/3]. \end{aligned} \quad (1)$$

In order to drive symmetry breaking and generate the charged fermion masses, we assume a Higgs sector similar to the SVS 3-3-1 model. The Yukawa Lagrangian for the quark sector is given by

$$\mathcal{L}_{\text{quarks}} = y_{u_a} \overline{Q_{aL}} u_{aR} \phi_0^* + y_{d_a}^i \overline{Q_{aL}} d_{aR} \phi_i^* + y_{D_a}^i \overline{Q_{aL}} D_{aR} \phi_i^* + \text{h.c.} \quad , \quad (2)$$

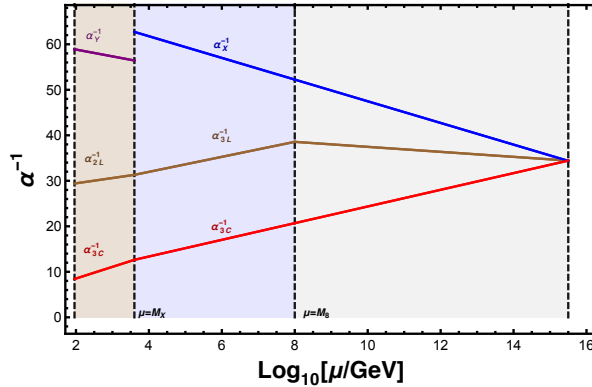
where  $a = 1, 2, 3$ ,  $i = 1, 2$  (neglecting any flavour mixing). In the leptonic sector the relevant Yukawa interactions are given by

$$\mathcal{L}_{\text{leptons}} = \epsilon_{\alpha\beta\gamma} [\psi_{\alpha L}^T C^{-1} (y_1 \xi_{\beta L} \phi_{0\gamma} + y_2^i \chi_{\beta L} \phi_{i\gamma}) + \xi_{\alpha L}^T C^{-1} y_3^i \chi_{\beta L} \phi_{i\gamma}] + \text{h.c.} \quad (3)$$

where  $\alpha, \beta, \gamma$  are the  $SU(3)_L$  tensor indices giving antisymmetric Dirac mass terms,  $C$  is the charge conjugation matrix, and  $i = 1, 2$ . After the symmetry breaking, the  $5 \times 5$  neutrino mass matrix for each generation can be diagonalized to obtain two quasi-Dirac heavy neutrinos with mass around  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  symmetry breaking scale, two Dirac states with mass of the order of the electroweak symmetry breaking scale and a light seesaw Majorana neutrino.

**SU(6) Grand Unification:** It is easy to verify that each generation of the fermionic multiplets of the sequential 3-3-1 model written in Eq. (1) can be embedded perfectly in the anomaly free combination of  $SU(6)$  representations:  $\bar{6} + \bar{6}' + 15$ , where  $\bar{6}$  contains  $d_L^c \equiv [3, 1, -1/3]$  and  $\psi_L \equiv [1, 3^*, -1/3]$ ;  $\bar{6}'$  contains  $D_L^c \equiv [3, 1, -1/3]$  and  $\xi_L \equiv [1, 3^*, -1/3]$ ; and 15 contains  $u_L^c \equiv [3^*, 1, -2/3]$ ,  $\chi_L \equiv [1, 3^*, 2/3]$  and  $Q_L \equiv [3, 3, 0]$ . Now the  $E(6)$  fundamental representation 27 decomposes under the maximal  $SU(2) \otimes SU(6)$  subgroup as  $27 = [2, \bar{6}] + [1, 15]$ . Three 27s of  $E(6)$  containing three sets of  $\bar{6} + \bar{6}' + 15$  can accommodate three generations of the fermionic multiplets of the sequential 3-3-1 model. However, the minimal content of the sequential 3-3-1 model does not lead to a successful unification scenario. Interestingly, by adding three generations of the fermionic octets leads to a successful gauge coupling unification.

The one-loop beta-coefficients for the phase between the electroweak symmetry breaking and the  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  symmetry breaking ( $M_Z$  to  $M_X$ ) are given by  $b_{2L} = -19/6$ ,  $b_Y = 41/10$ ,  $b_{3C} = -7$ . The one-loop beta-coefficients for the phase between the  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  symmetry breaking scale and the octet mass scale ( $M_X$  to  $M_8$ ) are given by  $b_{3L} = -9/2$ ,  $b_X = 13/2$ ,  $b_{3C}^{331} = -5$ . Finally, the one-loop beta-coefficients for the phase between the octet mass scale to the unification scale ( $M_8$  to  $M_U$ ) are given by  $b_{3L}^8 = 2n - 9/2$ , where  $n$  is the number of generations of the fermionic octets ( $\Omega \equiv [1, 8^*, 0]$ ),



**Fig. 1.** The running of the gauge couplings in the sequential 3-3-1 Model with three generations of fermionic octets with  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  symmetry breaking scale  $M_X = 4000$  GeV and octet mass scale  $M_8 = 10^8$  GeV, demonstrating successful gauge unification at the scale  $M_U = 10^{15.5}$  GeV with  $n_Y = \sqrt{5/3}$  and  $n_X = 2/\sqrt{3}$ .

$b_X^8 = 13/2$ ,  $b_{3C}^8 = -5$ . Fig. 1 shows the gauge coupling running of the sequential  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  model and three generations of fermionic octets with  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  symmetry breaking scale  $M_X = 4000$  GeV and octet mass scale  $M_8 = 10^8$  GeV, demonstrating successful gauge unification at the scale  $M_U = 10^{15.5}$  GeV with  $n_Y = \sqrt{5/3}$  and  $n_X = 2/\sqrt{3}$ . Taking  $M_U = 10^{15.5}$  GeV and  $\alpha_{\text{GUT}}^{-1} \sim 35$  in sequential 3-3-1 model, the lifetime of the proton decay mode  $p \rightarrow e^+ \pi^0$  is roughly  $\sim 10^{34}$  yrs, which is consistent with the current experimental limit [6].

**Concluding Remarks:** We have discussed a minimal SU(6) grand unification of the sequential variant of the  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  model which allows for a TeV scale  $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$  model as well as seesaw-induced neutrino masses. The gauge coupling unification can be associated to the presence of sequential leptonic octets at some intermediate scale between the 3-3-1 scale and the unification scale. The presence of the octet scale can have interesting implications for radiative origin of neutrino masses [7].

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