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Mathematics education and technology

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Abstract
Mathematics education has relied on tools for over 2,000 years in the form of physical instruments that both embody and represent mathematical concepts - such as a pair of compasses for the construction of distances and angles. The arrival of mechanical tools (i.e. comptometers) and subsequently digital mathematical tools was met with both enthusiasm and great scepticism. Policy makers grappled hard to identify which traditional (paper and pencil) skills to remove from the taught curriculum to be replaced with which digital alternatives. For example, the removal of printed tables of logarithms, trigonometric ratios and exponentials from most secondary curricular to be replaced with electronic calculators (that pre-stored this data) is a transition that most (but not all) countries in the world have now made. This article has used an international survey methodology to provide a glimpse of the state of the adoption of educational technology within school mathematics education in sixteen countries, as viewed through each country's curriculum, formal assessment and teacher education policies and practices.

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Introduction

Recent international surveys such as the Organisation for Economic Co-operation and Development (OECD) Report *Students, Computers and Learning* (2015) highlight the wide gap in students' access to, and use of, technology in secondary mathematics in the participating countries. The OECD 'snapshot' methodology in which 15-year-old students were asked if they (or their teachers) had performed a range of mathematical tasks using computers in the month preceding their completion of the Programme for International Student Assessment (PISA) survey revealed low levels of technology use (see Figure 1).

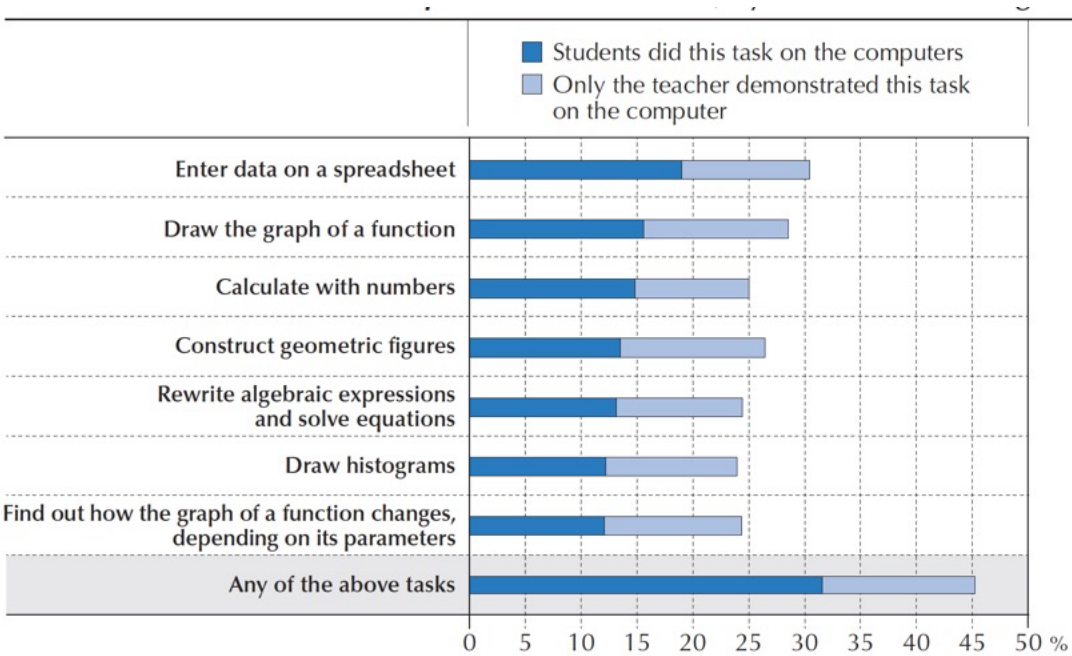

Figure 1 OECD average use of computers during mathematics lessons (OECD 2015, p. 56)

However, these data revealed big differences between countries. For example, in Norway, 73.1% of students reported at least one of these uses of technology, whereas for Ireland, the figure was 17.6%.

Whist the OECD reports teacher and student usage, it gives no insight into the context in which this usage is taking place. A country's (or region's) policy for technology use in its mathematics curriculum (and the associated assessment regime) has long been cited as a potential catalyst or barrier to technology use (Hoyles & Lagrange, 2009). For example, mandating technology use in examinations is a known factor that increases classroom usage by teachers and students. This article aims to reveal similarities and differences between the international policies and practices as a means to shed more light on the OECD data. There are two important caveats:

- national curriculum and assessment policies are dynamic, with varying timescales for review and implementation. As this study reflects the current picture, it cannot be assumed that the 15-year-olds that took part in the PISA study experienced the curriculum and assessment regime as reported.
- in some countries such as the United States of America, Australia and Germany there is wide regional variation due to locally mandated curriculum and assessment policies.

The categories used in the OECD survey reflect the international evolution of how technology is now being used within mathematics education. The following entries of the Springer Encyclopaedia of Mathematics Education expand on the evolution of these uses with respect to: technology types and design considerations (Freiman, 2014a, 2014b); the affordances of

technologies for mathematics education (Hegedus & Moreno-Armella, 2014); and the implications for mathematics curricula (Sutherland & Rojano, 2014).
Technology is interpreted as a mathematical 'tool' that offers a mathematical environment within which to explore, express and communicate mathematical ideas. (For more on mathematical digital tools, see Monaghan, Trouche and Borwein (2016)). Hence, it has many uses within the design, teaching and assessment of mathematical activity, which forms the main focus of this text.

As an example, consider Figures 2 and 3, which show a digital resource for lower secondary students that aims to provide them opportunities to "Find out how the graph of a function changes, depending on its parameters", one of the seven task types that features in the OECD survey. In Figure 2, the character 'Shakey the Robot' has been animated to travel along the number line in real-time. Simultaneously, the character's position-time graph is plotted and the corresponding data-line highlighted in the table. The linear function that describes the motion algebraically is also given in the form of an equation. Figure 2 is a 'snapshot' of this motion when the lapsed journey time is equal to 2 seconds.

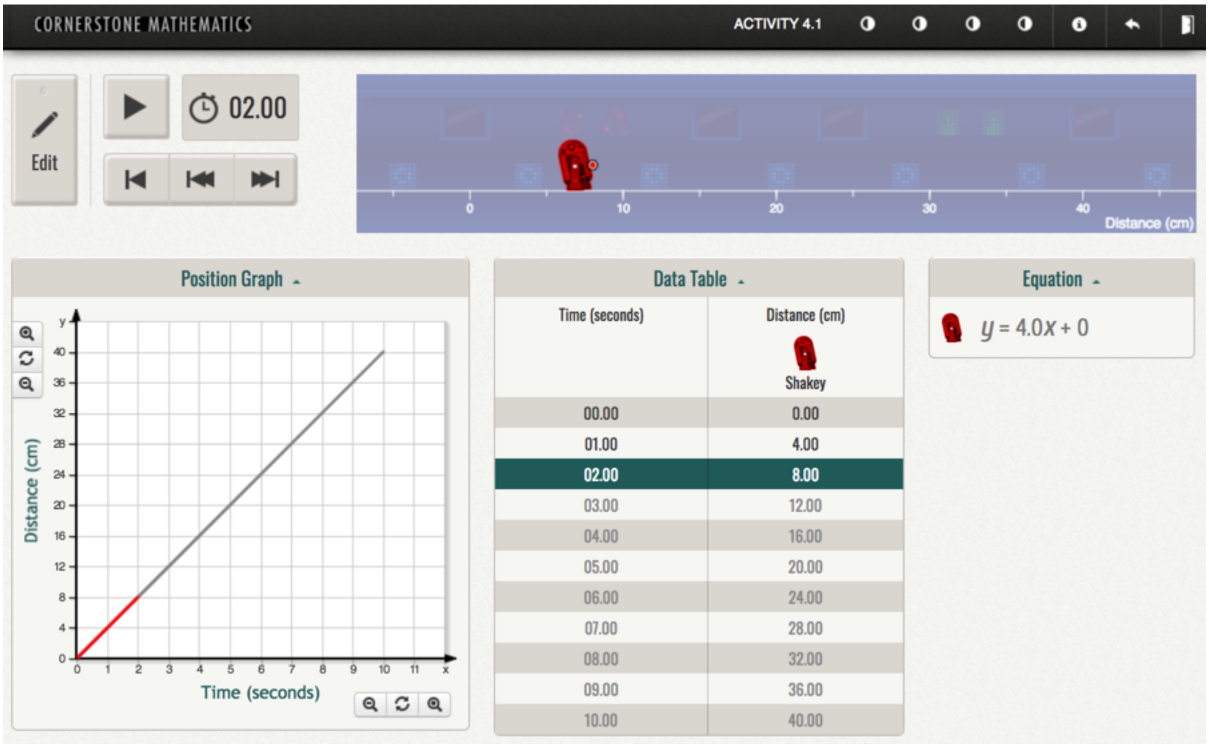

Figure 2 'Shakey the robot' – a snapshot of an animated journey

In Figure 3, the simulation is shown in edit mode, which affords the student the choice to vary a number of parameters (start position, finish position and speed) by dragging hotspots or changing numerical values.

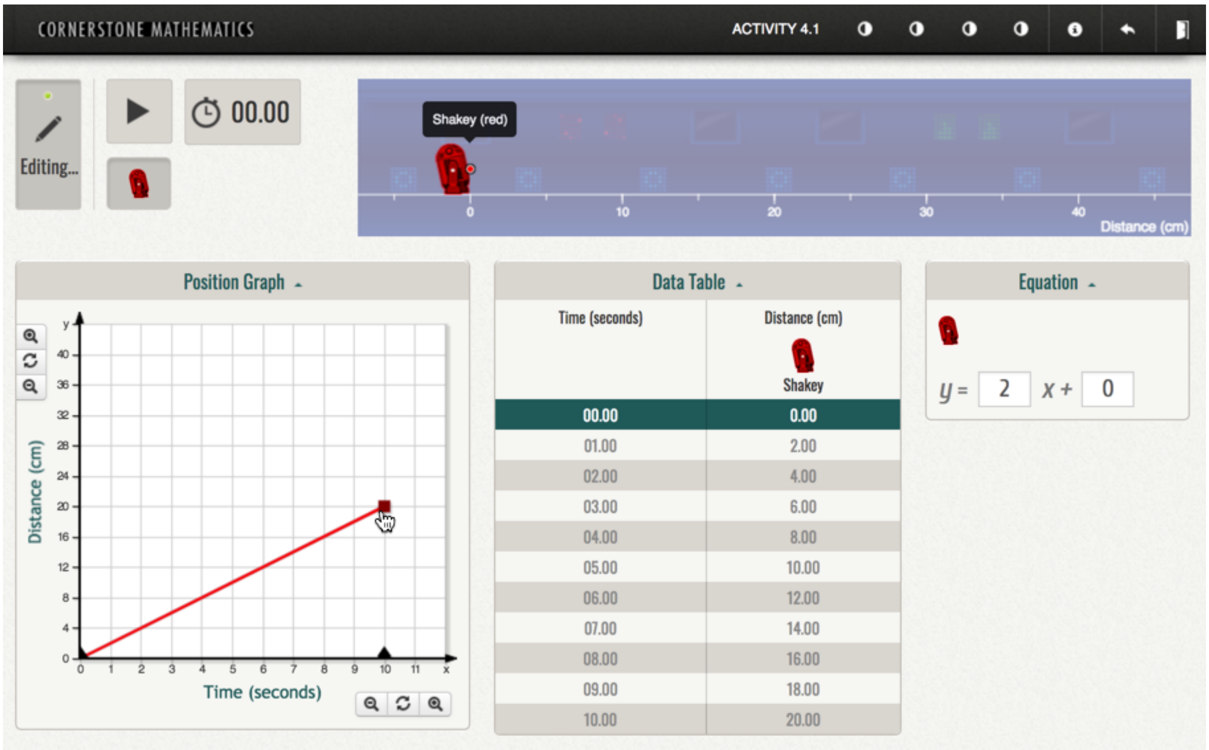

Figure 3 Shakey the robot – editing the animation.

This resource, which is taken from the Cornerstone Maths linear functions curriculum (www.ucl.ac.uk/cornerstone-maths), has its antecedents in the work of Jim Kaput in the United States of America and has been extensively evaluated with positive impacts on students' mathematical outcomes in a number of studies in the USA and England (Hegedus & Roschelle, 2013; Hoyles, Noss, Vahey, & Roschelle, 2013; Kaput, 1989; Kaput & Schorr, 2008).

An alternative technology might be to use an online graphing tool such as Desmos (https://www.desmos.com/calculator/56envakzbp), a graphing software package or a graphical calculator. There is consistent research evidence to suggest that such technologies support students' understanding of the function concept (Kieran, 2014), although it is important to highlight the key role of the teacher in the successful integration of such technologies in classrooms (Clark-Wilson, Robutti, & Sinclair, 2014). Consequently, this text also sheds light on the policy and practice concerning initial and in-service teacher education for technology use in mathematics in the different countries.

Although the main tenet of the article is focused on upper secondary education (14–19 years), it begins with a brief commentary on the policy differences regarding primary (4–9 years) and lower secondary education (9–13 years) as indicators of the varying trajectories of students' experiences through their schooling.

Technology use in primary mathematics education

The majority of the sixteen countries (or regions) do not mandate for the use of mathematical technologies in the primary mathematics curriculum, although the use of general, non-specified technologies is sometimes required. The exceptions are France, Portugal, Norway, Wales and the state of Victoria in Australia.

Table 1 shows the distribution of countries and their technology use.

Table 1 Technology use in primary mathematics education (5-8 years)

	Technology use is mandated in the national/regional curriculum	**Technology use is optional in the mathematics curriculum**
No standardized high-stakes testing	Australia (Victoria) France Norway	Germany Netherlands
Students can choose to use technology in some or all high stakes mathematics assessments		Israel
Students are not allowed to use technology in any high stakes mathematics assessment	Portugal Slovakia Wales	England Hong Kong Ireland Italy Scotland Taiwan Turkey

The French national curriculum includes a particularly forward-looking vision statement, which translates to:

> ‘The School contributes to the project of a society of information and communication for all by initiating, in partnership with communities and different actors, actions to generalise uses and develop digital resources for education. It trains students to master these digital tools and prepares the future citizen to live in a society whose technological environment is constantly evolving.’ (http://www.education.gouv.fr/pid24307/les-programmes-de-l-ecole-elementaire.html).

This vision is supported by a government department, the ‘Digital Education Directorate’, which provides links and resources for teachers to realise this aim. Furthermore, all trainee teachers in France are required to pass a Master’s level course on the use of technology for teaching and learning, although for primary phase teachers this might not necessarily have a mathematical focus.

In Norway, a similar broad vision for mathematics exists that encompasses the role of digital skills:

> *Digital skills* in Mathematics involves using digital tools to learn through play, exploration, visualisation and presentation. It also involves learning how to use and assess digital aids and tools for calculating, problem solving, simulation and modelling. It also means it is important to find information, analyse, process and present data using appropriate tools, and being critical of sources, analyses and results. The development of digital skills involves working with complex digital texts with an increasing degree of complexity. It also involves developing an increasing awareness of the new digital tools that exist for learning in the subject of Mathematics.
> https://www.udir.no/kl06/MAT1-04?lplang=http://data.udir.no/kl06/eng

Within the primary curriculum, this vision is exemplified within one of the content statements for geometry, where pupils are enabled to 'make and explore geometric patterns, with and without using digital tools, and describe them orally' (ibid).

The approach to the assessment of primary mathematics varies internationally. Some countries require classroom-based teacher assessment (e.g. France, Norway) whereas other countries conduct high-stakes examinations at a national or regional level (i.e. England, Netherlands, Hong Kong, Singapore, Slovakia, Taiwan, Turkey, Wales). For those countries with high stakes testing, none permit the use of technology by primary students.

Technology use in lower secondary mathematics education

In the lower secondary phase (9-14 years), there is a shift to more mandated use of technology in the mathematics curriculum, with several countries typically specifying the use of spreadsheet, function plotting and dynamic geometry software applications (i.e. all states in Germany, Portugal, France).
Table 2 shows the distribution of countries and their technology use in this education phase.

Table 2 Technology use in lower secondary mathematics education (9-14 years)

	Technology use is mandated in the national/regional curriculum	**Technology use is optional in the mathematics curriculum**
No standardized high-stakes testing	Australia (Victoria)	Netherlands
Students are mandated to use technology in some or all high stakes mathematics assessments	Norway Portugal	
Students can choose to use technology in some or all high stakes mathematics assessments	France	Hong Kong Ireland Israel
Students are not allowed to use technology in any high stakes mathematics assessment	Slovakia Wales	England Germany Italy Scotland Taiwan Turkey

Again, some countries have offered a more visionary stance, such as Portugal:

> All Secondary Schools must equip themselves as soon as possible with Mathematical Laboratories. The didactics for Mathematics in secondary education presupposes the possibility of using diversified materials and equipment: graphical calculators with the possibility of using programs; computers; data collection sensors for both calculators and computers. It is considered indispensable to use: graphing calculators (for regular classroom work or for demonstrations with all students, using a "view-screen" calculator); a computer room with software suitable for work as regular as possible; a computer connected to a data-show or video projector (for demonstrations, simulations or

> classroom work with all students at the same time). (Ministério da Educação e Ciência, 2001, pp. 14-15)

In this phase of education, the shift is towards optional use of technology within mathematics assessments, which seems to accompany more explicit curriculum guidance for its use. The trend is to permit the use of scientific and/or graphical calculators within high-stakes testing.

Some countries are in the process of reviewing the role and use of technology in this educational phase. For example, in Ireland, the forthcoming introduction of an element of classroom-based assessment in mathematics during the 'junior cycle' is expected to stipulate the use of digital technology - especially for statistical investigations. The influence of 'big data' in wider society seems to be influencing attitudes to technology use in mathematics education as the need to develop students' mathematical and statistical thinking through more open exploration and modelling of large data sets gains importance.

Technology use in upper secondary mathematics education

Unsurprisingly, most policy guidance on technology use with respect to the curriculum and its assessment is visible in the upper secondary phase, which coincides with the age range of the students who participated in the OECD survey.
The distribution of countries is shown in Table 3.

Table 3 Technology use in pre-university mathematics education (14-19 years)

	Technology use is mandated in the national/regional curriculum	**Technology use is optional in the mathematics curriculum**
Students are mandated to use technology in some or all high stakes mathematics assessments	Victoria, Australia Germany Netherlands Norway Portugal	
Students can choose to use technology in some or all high stakes mathematics assessments	England France Slovakia Wales	Hong Kong Ireland Israel Italy Scotland Taiwan
Technology use is not permitted in assessments		Turkey

At this educational phase, several countries' curricula refer explicitly to the (mathematical) affordances of the technologies that students should use. For example, the Welsh curriculum states:

> At Advanced level, calculators must include the following features:
> - an iterative function;
> - the ability to compute summary statistics and access probabilities from standard statistical distributions.
> - and they must not, for example, be designed or adapted to offer

- symbolic algebra manipulation;
- symbolic differentiation or integration

(Welsh Joint Education Committee, 2017, p. 39)

Alternatively, other countries' curricula offer a set of mathematical-technical competencies that students should acquire. For example in Norway, students are expected to be enabled to: "perform, describe and provide rationale for geometric constructions using a compass and ruler and dynamic geometry programs" in geometry; and "prepare functions that describe numerical relationships and practical situations, on paper and digitally, describe and interpret them and convert between various representations of functions, such as graphs, tables, formulas and text".
https://www.udir.no/kl06/MAT1-04/Hele/Kompetansemaal/competence-aims-after-year-10?lplang=http://data.udir.no/kl06/eng

In Victoria, Australia, where technology use in pre-university mathematics education has been mandatory since 2009, high stakes assessments only occur in upper secondary mathematics, where the scores on examinations contribute to a score used for university entrance for many courses. The mathematical methods course incorporates the use of computer algebra system (CAS) technology, which is also required in the examination.

This contrasts with the consistent approach in Germany states where digital tools are mandated in the curriculum, with most schools choosing to use graphics calculators (with and without CAS functionality). The 'Abitur' forms the final qualification, which involves a project element that can include the use of (CAS or non-CAS) technology within tasks that have been approved by the German 'Institute for Educational Quality Improvement'. The German and Australian experiences have influenced the trajectory of curriculum and assessment in England (Button, 2013).

Teacher preparation and support for teaching mathematics with technology

There is wide variance in the countries' policy and practice concerning the preparation of teachers to use technology within mathematics teaching. For example, in Australia, there is an expectation that graduate teachers can use ICT in their teaching (as prescribed in the teaching standards: https://www.aitsl.edu.au/teach/standard). The implication is that initial teacher training would prepare pre-service teachers to meet these standards. However, although there is no national policy about the technologies used in mathematics specifically, taking the standards and curriculum together suggest that pre-service teachers do learn about mathematics specific technologies, as well as general technologies.

In Ireland, where the initial teacher education curriculum is not prescribed at a national level, there is an expectation (rather than a policy) that pre-service teachers will be introduced to appropriate mathematical technology. This picture is replicated in several other countries (i.e. England, Israel, Scotland). A number of countries report compulsory or elective courses within initial teacher education programmes that are specifically focused on technology use in mathematics education (France, Portugal, Turkey).

For in-service teachers, again the picture is varied. Few countries report state-funded professional development (PD) initiatives in this respect, with most indicating that it would be the choice of the teacher to focus on mathematical (as opposed to generic) technologies. (i.e. England, France, Germany, Netherlands, Norway, Slovakia). In some cases, the PD opportunity is offered through local or regional networks, resulting in a varied picture of uptake, and therefore impact. Alternatively, government-funded initiatives led by university-based mathematics education teachers/researchers, such as the 'm@t.abel' project in Italy (http://www.scuolavalore.indire.it/superguida/matabel/) have involved large numbers of teachers in focused PD.

Massive open online courses (MOOCs) are an emerging phenomenon for teacher PD, with colleagues in France, Italy (http://www.difima.unito.it/mooc/) and Norway all reporting their development, alongside some early evaluations (Panero, Aldon, Trgalová, & Trouche, 2017; Taranto et al., 2017). A significant international community that has evolved from the GeoGebra software user group has now established formal PD programmes in many countries (Italy, Hong Kong).

A number of studies are identifying the design principles for, and impact of, larger-scale PD programmes for mathematics teachers, exploring the role of PD 'multipliers (Barzel & Biehler, 2017) and online PD models (Clark-Wilson & Hoyles, 2017).

Concluding comments

Very few of the countries reporting for this review indicate any large-scale evaluation of the impacts of classroom use of mathematical technologies on students' educational outcomes. Where evaluations have occurred, these were mostly smaller scale studies of particular technology interventions (Wang, 2011). Norway and Germany seem most advanced in this respect, conducting annual surveys of technology that reveal patterns of use. However, the most recent German report reveals that current teaching in science, technology, engineering and mathematics was not bringing about the desired societal impacts relating to high-level technological skills (https://www.telekom-stiftung.de/sites/default/files/files/media/publications/Schule_Digital_2017__Web.pdf).

Regarding the wider use of technology, a large-scale study in Taiwan adapted a set of teacher technology integration performance standards (developed by the International Society of Technology in Education) to evaluate teachers' technology integration proficiency. This study revealed six scales for teachers' technology integration, including (1) teaching preparation and information gathering, (2) instructional material and hardware problem-solving, (3) management, communication, and sharing, (4) planning, instruction, and evaluation, and (5) professional development and self-study. Factor analysis revealed that teachers who had experienced more professional development scored higher (Hsu, 2010).

As previously mentioned, the societal interest in 'big data' is catalysing curriculum developments with respect to increased use of technology in mathematical tasks that require statistical hypotheses, analyses and modelling. In England and Ireland technology is becoming mandated as part of classroom-based tasks and/or high stakes assessment in this respect.

But a number of key challenges remain. The discussion about digital tools is often driven by the technical aspects of the hardware, rather than the deeper consideration of the (desirable) mathematical affordances to improve students' engagement and participation in mathematics. This is reflected in the presentation of curriculum policy guidance that emphasise the students' attributes in terms of their mathematical activity with the technology. Herein lies the biggest challenge. For teachers who have not been inducted into mathematics with technology, significant PD is required to enable them to rethink their own mathematics, alongside the pedagogies that accompany this.

This final comment sums up this issue and highlights the importance that the curriculum and assessment policy developments are underpinned by clear and wide communications of the vision concerning technology use in mathematics - and accompanying professional development opportunities for teachers:

> Last summer (2017) Grade 13 students (19-years-of-age) in the scientific high schools were suddenly allowed to use graphical calculators in the National examination of grade 13. This new permission was given in spring 2017, unfortunately without any prior indication or support for teachers. This has caused some stressful situations for these students' teachers because they did not expect such a decision by the Ministry. [Questionnaire respondent]

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