Appendix: Market Power with Tradable Performance-Based CO_2 Emissions Standards in the Electricity Sector

Yihsu Chen* Makoto Tanaka† Afzal S. Siddiqui‡

Appendix A Proofs of Propositions

Proof of Proposition 3.1

Noting the assumption that $E_i < F < E_j$, we have $a' = -\frac{(F-E_1)}{F-E_2} > 0$, and $b' = \frac{E_1-E_2}{F-E_2} > 0$. Recall also the assumption that $p' < 0, p'' \le 0, c_i' > 0$, and $c_i'' \ge 0$. Let $m^*(g_1)$ denote the left-hand side of Eq. (6). We first calculate the derivative of $m^*(g_1)$:

$$\begin{aligned} m^{*\prime}(g_1) &= p'b' - c_1'' - (E_1 - F)f' \\ &= p'b' - c_1'' - \frac{(E_1 - F)}{E_2 - F}\Big(p'b' - c_2''a'\Big) \\ &< 0. \end{aligned} \tag{A-1}$$

Thus, $m^*(g_1)$ is strictly decreasing, and g_1^*, which is a solution to $m^*(g_1) = 0$ (or Eq. (6)), is unique if an interior solution exists. Next, let $m^c(g_1)$ denote the left-hand-side of Eq. (10) and calculate the derivative as follows:

$$\begin{aligned} m^{c\prime}(g_1) &= p'b' + p' + g_1p''b' - c_1'' - (E_1 - F)h' \\ &= p'b' + p' + g_1p''b' - c_1'' - \frac{(E_1 - F)}{E_2 - F}\Big(p'b' + ap''b' + p'a' - c_2''a'\Big) \\ &< 0. \end{aligned} \tag{A-2}$$

Hence, $m^c(g_1)$ is strictly decreasing, and g_1^c, which is a solution for $m^c(g_1) = 0$ (or Eq.(10)), is unique if an interior solution exists. We now compare g_1^* and g_1^c by calculating the following:

$$\begin{aligned} m^c(g_1) - m^*(g_1) &= p'g_1 - (h - f)(E_1 - F) \\ &= p'g_1 - \frac{(E_1 - F)}{E_2 - F}ap' \\ &< 0. \end{aligned} \tag{A-3}$$

*Corresponding author. Associate Professor, Department of Technology Management, Baskin School of Engineering, University of California Santa Cruz, Santa Cruz, CA, United States. Email: yihsuchen@ucsc.edu.

†Professor, National Graduate Institute for Policy Studies (GRIPS), 7-22-1 Roppongi, Minato-ku, Tokyo 106-8677, Japan. Email: mtanaka@grips.ac.jp.

‡Professor, Department of Statistical Science, University College London, United Kingdom, Professor, Department of Computer and Systems Sciences, Stockholm University, Sweden, and Affiliated Professor, Department of Decision Sciences, HEC Montréal, Canada. Email: afzal.siddiqui@ucl.ac.uk.

Since $m^c(g_1) < m^*(g_1)$, we obtain $g_1^c < g_1^*$. We then compare g_1^c and g_1^s with the assumption of interior solutions by calculating the following:

$$\begin{aligned} m^s(g_1) - m^c(g_1) &= g_1(b' - 1)p' - (E_1 - F)g_1 h' \\ &= -\frac{(E_1 - F)}{E_2 - F}\Big(p' + p'b' + ap''b' + p'a' - c_2''a'\Big)g_1 \\ &< 0. \end{aligned} \tag{A-4}$$

It follows from $m^s(g_1) < m^c(g_1)$ that $g_1^s < g_1^c$ holds for any interior solutions. We, thus, obtain $g_1^s < g_1^c < g_1^*$. Since $a' > 0$, $g_2 = a(g_1)$ is strictly increasing. We, thus, have $g_2^s < g_2^c < g_2^*$. □

Proof of Proposition 3.2

It is straightforward from Proposition 3.1 that $g^s < g^c < g^*$. Since $p' < 0$, $p(g)$ is strictly decreasing. Hence, $p^s > p^c > p^*$ holds. □

Proof of Proposition 3.3

From Eq. (2), $e = E_1 g_1 + E_2 g_2 = Fg$. Hence, e, which is a function of g, is strictly increasing since $e' = F > 0$. It follows from this and Proposition 3.2 that $e^s < e^c < e^*$. □

Appendix B Nomenclature

Indices and Sets

Γ: upper-level decision variables
Ξ: lower-level primal decision variables
Ψ: lower-level dual variables
Φ: decision variables for MILP
$i \in \mathcal{I}$: power producers
s: strategic producer index
$j \in \mathcal{J}$: non-strategic producers[1]
$k \in \mathcal{K}$: discrete generation level
$\ell \in \mathcal{L}$: transmission lines
$n', n \in \mathcal{N}$: power network nodes
$u', u \in \mathcal{U}_{n,i}$: generation units of producer $i \in \mathcal{I}$ at network node $n \in \mathcal{N}$

Parameters

$B_{n,n'}$: element (n, n') of node susceptance matrix, where $n, n' \in \mathcal{N}$ (1/Ω)
$C_{n,i,u}$: generation cost of unit $u \in \mathcal{U}_{n,i}$ from producer $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (\$/MW)
D_n^{int}: intercept of linear inverse demand function at node $n \in \mathcal{N}$ (\$/MW)
D_n^{slp}: slope of linear inverse demand function at node $n \in \mathcal{N}$ (\$/MW^2)
$E_{n,i,u}$: CO_2 emission rate of unit $u \in \mathcal{U}_{n,i}$ from producer $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (t/MWh)
F: regulated CO_2 emissions rate under performance (rate)-based policy (t/MW)
$\overline{F}$: regulated CO_2 emissions cap under mass-based policy (t)
$G_{n,i,u}$: maximum generation capacity of unit $u \in \mathcal{U}_{n,i}$ from producer $i \in \mathcal{I}$ at node $n \in \mathcal{N}$ (MW)
$H_{\ell,n}$: element (ℓ, n) of network transfer matrix, where $\ell \in \mathcal{L}$ and $n \in \mathcal{N}$ (1/Ω)

[1] $\mathcal{J} \cap \{s\} = \emptyset$, $\mathcal{J} \cup \{s\} = \mathcal{I}$

K_ℓ: maximum capacity of power line $\ell \in \mathcal{L}$ (MW)
$\overline{G}_{n,s,u,k}$: discrete generation level $k \in \mathcal{K}$ of strategic producer's unit $u \in \mathcal{U}_{n,s}$ located at node $n \in \mathcal{N}$ (MW)
$\overline{E}_{n,s,u,k}$: discrete CO_2 emissions associated with discrete generation level $k \in \mathcal{K}$ of strategic producer's unit $u \in \mathcal{U}_{n,i}$ located at node $n \in \mathcal{N}$ (t)
$M^\lambda, M^p, M^y, M, \overline{M}, \check{M}, \hat{M}, \tilde{M}, \underline{M}$: large constants used in disjunctive constraints and binary expansion

Primal Variables

$g_{n,i,u}$: generation at node $n \in \mathcal{N}$ by producer $i \in \mathcal{I}$ using unit $u \in \mathcal{U}_{n,i}$ (MW)
d_n: consumption at node $n \in \mathcal{N}$ (MW)
v_n: voltage angle at node $n \in \mathcal{N}$ (rad)
$y_{n,s,u,k}$: strategic generator's electricity sales revenue at node $n \in \mathcal{N}$ using unit $u \in \mathcal{U}_{n,s}$ at generation level $k \in \mathcal{K}$ (\$)
$z_{n,s,u,k}$: strategic generator's CO_2 permit revenue (or cost) at node $n \in \mathcal{N}$ using unit $u \in \mathcal{U}_{n,s}$ at generation level $k \in \mathcal{K}$ (\$)
$q^y_{n,s,u,k}$: auxiliary variable to linearize the strategic generator's objective function with respect to electricity sales at node $n \in \mathcal{N}$ using unit $u \in \mathcal{U}_{n,s}$ at generation level $k \in \mathcal{K}$
$p_{n,s,u,k}$: auxiliary variable used to associate CO_2 permit price for the output level of producer at node $n \in \mathcal{N}$ using unit $u \in \mathcal{U}_{n,s}$ at generation level $k \in \mathcal{K}$ (\$/t)

Dual Variables

$\beta_{n,i,u}$: shadow price on generation capacity at node $n \in \mathcal{N}$ for generation unit $u \in \mathcal{U}_{n,i}$ of producer $i \in \mathcal{I}$ (\$/MW)
$\overline{\mu}_\ell, \underline{\mu}_\ell$: shadow prices on transmission capacity for transmission line $\ell \in \mathcal{L}$ (\$/MW)
λ_n: market-clearing price at node $n \in \mathcal{N}$ (\$/MW)
ν: hub price (\$/MW)
ρ: shadow price on emissions rate (\$/t)

Integer Variables

q^λ_n: auxiliary variable used to indicate whether market-clearing price at node $n \in \mathcal{N}$ is positive
$q_{n,s,u,k}$: auxiliary variable used to discretize the strategic generator's electricity generation at node $n \in \mathcal{N}$ using unit $u \in \mathcal{U}_{n,s}$ at generation level $k \in \mathcal{K}$
$\overline{r}_{n,j,u}$: auxiliary variable used to handle the Karush-Kuhn-Tucker (KKT) condition with respect to non-strategic producer $j \in \mathcal{J}$'s generation at node $n \in \mathcal{N}$ using unit $u \in \mathcal{U}_{n,j}$ and $g_{n,j,u}$
r_n: auxiliary variable used to handle the KKT condition with respect to consumption at node $n \in \mathcal{N}$ and d_n
$\check{r}_{n,j,u}$: auxiliary variable used to handle complementarity condition between generation constraint of non-strategic producer $j \in \mathcal{J}$'s unit $u \in \mathcal{U}_{n,j}$ located at node $n \in \mathcal{N}$ and shadow price of generation capacity
$\hat{r}_\ell$: auxiliary variable used to handle the complementarity condition between transmission line ℓ's capacity constraint and the shadow price in positive direction
$\tilde{r}_\ell$: auxiliary variable used to handle the complementarity condition between transmission line ℓ's capacity constraint and the shadow price in negative direction

$\underline{r}$: auxiliary variable used to handle the complementarity condition between the emissions constraint and the CO_2 price

Appendix C KKT Conditions for Lower-Level Equilibrium Problem

$$0 \leq g_{n,j,u} \perp D_n^{\text{slp}} \sum_{u' \in \mathcal{U}_{n,j}} g_{n,j,u'} + C_{n,j,u} + \beta_{n,j,u} - \lambda_n + \rho\left(E_{n,j,u} - F\right) \geq 0, \forall n, \forall j, \forall u \in \mathcal{U}_{n,j} \tag{C-5}$$

$$0 \leq d_n \perp -D_n^{\text{int}} + D_n^{\text{slp}} d_n + \lambda_n \geq 0, \forall n \tag{C-6}$$

$$\sum_{\ell \in \mathcal{L}} \overline{\mu}_\ell H_{\ell,n} - \sum_{\ell \in \mathcal{L}} \underline{\mu}_\ell H_{\ell,n} - \sum_{n' \in \mathcal{N}} (\lambda_{n'} - \nu) B_{n',n} = 0 \text{ with } v_n \text{ u.r.s.}, \forall n \tag{C-7}$$

$$0 \leq \beta_{n,j,u} \perp G_{n,j,u} - g_{n,j,u} \geq 0,\ \forall n, \forall j, \forall u \in \mathcal{U}_{n,j} \tag{C-8}$$

$$0 \leq \overline{\mu}_\ell \perp K_\ell - \sum_{n \in \mathcal{N}} H_{\ell,n} v_n \geq 0\ ,\ \forall \ell \tag{C-9}$$

$$0 \leq \underline{\mu}_\ell \perp K_\ell + \sum_{n \in \mathcal{N}} H_{\ell,n} v_n \geq 0\ ,\ \forall \ell \tag{C-10}$$

$$d_n - \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{n,i}} g_{n,i,u} + \sum_{n' \in \mathcal{N}} B_{n,n'} v_{n'} = 0 \text{ with } \lambda_n \text{ u.r.s.}, \forall n \tag{C-11}$$

$$\sum_{n \in \mathcal{N}} \sum_{n' \in \mathcal{N}} B_{n,n'} v_{n'} = 0 \text{ with } \nu \text{ u.r.s.} \tag{C-12}$$

$$0 \leq \rho \perp \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{I}} \sum_{u \in \mathcal{U}_{n,i}} \left(F - E_{n,i,u}\right) g_{n,i,u} \geq 0 \tag{C-13}$$

Appendix D MILP Reformulation

The complementarity conditions in Eqs. (C-5)–(C-6), (C-8)–(C-10), and (C-13) can be converted to disjunctive constraints using sufficiently large constants (Fortuny-Amat and McCarl, 1981; Gabriel and Leuthold, 2010). Another computational difficulty is the bilinear terms, $\lambda_n g_{n,s,u}$ and $\rho\left(E_{n,s,u} - F\right) g_{n,s,u}$, in Eq. (17a). We apply binary expansion to linearize those bilinear terms (Barroso et al., 2006; Gabriel and Leuthold, 2010). Taking discrete generation level k of strategic producer's unit $u \in \mathcal{U}_{n,i}$ located at node $n \in \mathcal{N}$, i.e., $\overline{G}_{n,s,u,k}$, we consider the following linearization.

$$y_{n,s,u,k} = \begin{cases} \lambda_n \overline{G}_{n,s,u,k} & \text{if } q_{n,s,u,k} = q_n^\lambda = 1 \\ 0 & \text{otherwise} \end{cases} \tag{D-1}$$

$$z_{n,s,u,k} = \begin{cases} \rho\left(E_{n,s,u} - F\right) \overline{G}_{n,s,u,k} & \text{if } q_{n,s,u,k} = \underline{r} = 1 \\ 0 & \text{otherwise} \end{cases} \tag{D-2}$$

If generation level $\overline{G}_{n,s,u,k}$ is selected and power price λ_n is positive, then we have the strategic generator's electricity sales revenue, $y_{n,s,u,k}$. Moreover, if generation level $\overline{G}_{n,s,u,k}$ is selected and the CO_2 allowance price ρ is positive, then we have strategic generator's

CO_2 permit revenue (or cost), $z_{n,s,u,k}$. Our formulation is an extension of Gabriel and Leuthold (2010) in which one type of bilinear term was considered.

$$\underset{\Phi}{\text{Maximize}} \sum_{n\in\mathcal{N}} \sum_{u\in\mathcal{U}_{n,s}} \left(\sum_{k\in\mathcal{K}} y_{n,s,u,k} - \sum_{k\in\mathcal{K}} z_{n,s,u,k} - C_{n,s,u} g_{n,s,u} \right) \tag{D-3}$$

$$\text{s.t. } (C-7), (C-11), (C-12)$$

$$0 \leq \lambda_n \leq M^{\lambda} q_n^{\lambda}, \ \ \forall n \tag{D-4}$$

$$g_{n,s,u} = \sum_{k\in\mathcal{K}} q_{n,s,u,k} \overline{G}_{n,s,u,k}, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s} \tag{D-5}$$

$$\sum_{k\in\mathcal{K}} q_{n,s,u,k} = 1, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s} \tag{D-6}$$

$$q_{n,s,u,k}^{y} \leq q_n^{\lambda}, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s}, \ \forall k \tag{D-7}$$

$$q_{n,s,u,k}^{y} \leq q_{n,s,u,k}, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s}, \ \forall k \tag{D-8}$$

$$q_{n,s,u,k} + q_n^{\lambda} - 1 \leq q_{n,s,u,k}^{y}, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s}, \ \forall k \tag{D-9}$$

$$y_{n,s,u,k} \leq \lambda_n \overline{G}_{n,s,u,k}, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s}, \ \forall k \tag{D-10}$$

$$0 \leq y_{n,s,u,k} \leq M^{y} q_{n,s,u,k}^{y}, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s}, \ \forall k \tag{D-11}$$

$$0 \leq p_{n,s,u,k} \leq M^{p} q_{n,s,u,k}, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s}, \ \forall k \tag{D-12}$$

$$\sum_{k\in\mathcal{K}} p_{n,s,u,k} = \rho, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s} \tag{D-13}$$

$$-\left(\overline{E}_{n,s,u,k} - F\overline{G}_{n,s,u,k}\right) p_{n,s,u,k} + z_{n,s,u,k} \geq 0, \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s}, \ \forall k \tag{D-14}$$

$$0 \leq -D_n^{\text{int}} + D_n^{\text{slp}} d_n + \lambda_n \leq M r_n, \ \ \forall n \tag{D-15}$$

$$0 \leq d_n \leq M\left(1 - r_n\right), \ \ \forall n \tag{D-16}$$

$$0 \leq D_n^{\text{slp}} \sum_{u'\in\mathcal{U}_{n,j}} g_{n,j,u'} + C_{n,j,u} - \lambda_n + \beta_{n,j,u} \leq \overline{M}\overline{r}_{n,j,u}, \ \ \forall n, \ j, \ u \in \mathcal{U}_{n,j} \tag{D-17}$$

$$0 \leq g_{n,j,u} \leq \overline{M}\left(1 - \overline{r}_{n,j,u}\right), \ \ \forall n, \ j, \ u \in \mathcal{U}_{n,j} \tag{D-18}$$

$$0 \leq K_{\ell} - \sum_{n} H_{\ell,n} v_n \leq \hat{M}\hat{r}_{\ell}, \ \ \forall \ell \tag{D-19}$$

$$0 \leq \overline{\mu}_{\ell} \leq \hat{M}\left(1 - \hat{r}_{\ell}\right), \ \ \forall \ell \tag{D-20}$$

$$0 \leq K_{\ell} + \sum_{n} H_{\ell,n} v_n \leq \tilde{M}\tilde{r}_{\ell}, \ \ \forall \ell \tag{D-21}$$

$$0 \leq \underline{\mu}_{\ell} \leq \tilde{M}\left(1 - \tilde{r}_{\ell}\right), \ \ \forall \ell \tag{D-22}$$

$$0 \leq -g_{n,j,u} + \overline{G}_{n,j,u} \leq \check{M}\check{r}_{n,j,u}, \ \ \forall n, \ j, \ u \in \mathcal{U}_{n,j} \tag{D-23}$$

$$0 \leq \beta_{n,j,u} \leq \check{M}\left(1 - \check{r}_{n,j,u}\right), \ \ \forall n, \ j, \ u \in \mathcal{U}_{n,j} \tag{D-24}$$

$$0 \leq \sum_{n\in\mathcal{N}} \sum_{i\in\mathcal{I}} \sum_{u\in\mathcal{U}_{n,i}} \left(F - E_{n,i,u}\right) g_{n,i,u} \leq \underline{M}(1 - \underline{r}) \tag{D-25}$$

$$0 \leq \rho \leq \underline{Mr} \tag{D-26}$$

$$\underline{r} \in \{0,1\}; r_n \in \{0,1\}, \ \ \forall n; \overline{r}_{n,j,u} \in \{0,1\}, \check{r}_{n,j,u} \in \{0,1\}, \ \ \forall n, \ j, \ u \in \mathcal{U}_{n,j};$$

$$\hat{r}_{\ell} \in \{0,1\}, \ \ \tilde{r}_{\ell} \in \{0,1\} \ \ \forall \ell \tag{D-27}$$

$$q_n^{\lambda} \in \{0,1\} \ \ \forall n; q_{n,s,u,k} \in \{0,1\}, q_{n,s,u,k}^{y} \in [0,1] \ \ \forall n, \ \forall u \in \mathcal{U}_{n,s}, \ \forall k \tag{D-28}$$

where we define:
$\Phi = \{d_n, g_{n,i,u}, v_n, \lambda_n, \nu, \overline{\mu}_\ell, \underline{\mu}_\ell, \beta_{n,j,u}, \rho, \underline{r}, r_n, \overline{r}_{n,j,u}, \check{r}_{n,j,u}, \hat{r}_\ell, \tilde{r}_\ell, y_{n,s,u,k}, z_{n,s,u,k}, q_{n,s,u,k}, q^\lambda_n,$
$q^y_{n,s,u,k}, p_{n,s,u,k}\}$.

References

Barroso, L.A., R.D. Carneiro, S. Granville, M.V. Pereira, and M.H.C. Fampa (2006). "Nash Equilibrium in Strategic Bidding: a Binary Expansion Approach." *IEEE Transactions on Power Systems*. 21(2): 629–638.

Fortuny-Amat, J. and B. McCarl (1981). "A Representation and Economic Interpretation of a Two-Level Programming Problem." *The Journal of the Operational Research Society*. 32(9): 783–792.

Gabriel, S. A. and F.U. Leuthold (2010). "Solving Discretely-Constrained MPEC Problems with Applications in Electric Power Markets." *Energy Economics*. 32: 3–14.