Power Minimization for Cooperative Wireless Powered Mobile Edge Computing Systems

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Abstract—This paper studies the power-efficient joint radio and computational resource allocation for two near-far mobile devices in a wireless powered mobile edge computing system. To overcome the double-near-far effect for the farther device, cooperative communications in the form of relaying via the nearer device is considered for offloading. The AP's total transmit power minimization problem is formulated under the constraints of the computation tasks, which is equivalent to a min-max problem and can be optimally solved by a two-phase method. Numerical results not only shows the significant performance improvement of the proposed scheme, but also demonstrates its effectiveness in handling computation-intensive latency-critical tasks and resisting the double-near-far effect.

I. INTRODUCTION

With the rapid developments of IoT and 5G communication technologies, a wide range of emerging mobile applications and mass data processing from mobile social networks [1], [2], have driven the increasing computing demands for mobile devices. In recent years, mobile edge computing (MEC) has emerged as a promising concept, which promotes to use cloud-computing facilities at the edge of mobile networks, and is motivated by ultralow latency and high bandwidth [3]. The cross-discipline nature of MEC lays the important role of joint radio-and-computational resource management in achieving energy-efficient or delay-optimal MEC [4]–[8]. Nonetheless, insufficient power supply is a major limitation for mobile devices to make full use of powerful resources at the edges. Wireless power transfer (WPT) particularly in the form of wireless powered communication networks (WPCNs) [9] has recently been considered as an important paradigm to provide sustainability for mobile communications. Many works have seen the possible synergy integrating MEC with WPT [4], [8]. However, WPCNs are known to suffer from the "double-near-far" effect, which occurs because a farther device harvests less energy and is also required to communicate in longer distances [9]. In fact, user cooperation has been extensively investigated in wireless communications to enhance data rate [9]–[11]. Most recently, user cooperation was also considered in MEC [12] to make the most use of the AP's computational resources.

In this paper, we study the wireless powered MEC system to complete the computation-intensive latency-critical (CILC) tasks of two near-far users exploiting cooperative communication. Our objective is to minimize the total transmission power of the AP with joint-optimal power and time allocation [13]. We first formulate AP's transmit power minimization (APTP-

M) problem, and then transform it into an equivalent min-max optimization problem, which is optimally solved by a proposed two-phase method. Simulation results shows that the proposed scheme is very capable of handling CILC tasks and resisting the double-near-far effect in WPCNs.

II. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a wireless powered MEC system that consists of a single-antenna AP (with an integrated MEC server), and two single-antenna mobile devices, $D_i, i \in \{1, 2\}$, both have a CILC task, characterized by $\langle I_i, C_i, T_i \rangle$, where I_i denotes the input data size (in bits), C_i is the amount of required computing resource for 1-bit of input data (i.e., the number of CPU cycles required), and T_i is the maximum tolerable delay. Assume that the AP has perfect knowledge of the channels and task-related parameters, and it is designed to make resource allocation optimally in a block-based structure where each block has a duration of T seconds. In this paper, we assume that T_i for two users is one block length, i.e., $T_1 = T_2 = T$.

A. User Cooperation Model for Computation Offloading

It is assumed that D_2 is nearer to the AP than D_1, and we denote the distances between AP and D_1, AP and D_2, D_1 and D_2 as d_1, d_2, and d_{12}, respectively, with $d_2 \leq d_1$. We also assume that $d_{12} \leq d_1$ for better cooperation. During the first period t_0 in a block, AP broadcasts energy in the downlink with power P_0, and thus the energy harvested by D_i is $E_i = \nu_i g_i P_0 t_0$ where g_i is the downlink channel power gain from the AP to D_i and $0 < \nu_i \leq 1$ is the corresponding energy conversion efficiency. After the WPT period, D_1 transmits its input data with power p_1 during the subsequent period t_1, and both the AP and D_2 decode their received signals from D_1. To overcome the doubly near-far effect, the nearer user D_2 will then first relay the farther user D_1's information with power p_{21} over period t_{21} and then transmits its own input data to the AP with power p_{22} over period t_{22}, all using its harvested energy. We denote the time and power allocation vectors as $\mathbf{t} = [t_0, t_1, t_{21}, t_{22}]$ and $\mathbf{p} = [p_1, p_{21}, p_{22}]$, respectively. According to the results of [9], [11], the offloaded data size of D_1 for computation at the AP can be expressed as

$$L_1(\mathbf{t}, \mathbf{p}) = \min \{L_{1,1}(\mathbf{t}, \mathbf{p}) + L_{1,2}(\mathbf{t}, \mathbf{p}), L_{1,12}(\mathbf{t}, \mathbf{p})\}, \quad (1)$$

where $L_{1,1}(\mathbf{t}, \mathbf{p})$, $L_{1,2}(\mathbf{t}, \mathbf{p})$ and $L_{1,12}(\mathbf{t}, \mathbf{p})$ denote D_1's offloaded data size from D_1 to the AP, from D_2 to the AP, and

from D_1 to D_2, respectively, which are given by

$$L_{1,1}(\mathbf{t}, \mathbf{p}) = t_1 r_{1,1}(\mathbf{p}) = t_1 B \log_2 (1 + p_1 h_1/N_0), \quad (2)$$

$$L_{1,2}(\mathbf{t}, \mathbf{p}) = t_{21} r_{1,2}(\mathbf{p}) = t_{21} B \log_2 (1 + p_{21} h_2/N_0), \quad (3)$$

$$L_{1,12}(\mathbf{t}, \mathbf{p}) = t_1 r_{1,12}(\mathbf{p}) = t_1 B \log_2 (1 + p_1 h_{12}/N_0), \quad (4)$$

where $r_{1,1}(\mathbf{p})$, $r_{1,2}(\mathbf{p})$, and $r_{1,12}(\mathbf{p})$ are the corresponding transmission rates, and h_1, h_2 and h_{12} are the channel power gains from D_1, D_2 to the AP, and from D_1 to D_2, respectively.[1] Also, B is the channel bandwidth, and N_0 is the receiver noise power at the AP and D_2. Similarly, the offloaded data size of D_2 for computing at the AP is described as

$$L_2(\mathbf{t}, \mathbf{p}) = t_{22} r_2(\mathbf{p}) = t_{22} B \log_2 (1 + p_{22} h_2/N_0). \quad (5)$$

According to the task model, the constraint $L_i(\mathbf{t}, \mathbf{p}) \leq I_i, i \in \{1, 2\}$ should be satisfied. In this paper, we mainly consider the WPT time and the offloading time as the total latency of the WPT-MEC system, and thus we obtain a latency constraint given by $t_0 + t_1 + t_{21} + t_{22} \leq T$. Besides, the energy consumption of D_1 and D_2 for computation offloading equals to the energy consumed for wireless transmissions, given by

$$E_{\text{off},1}(\mathbf{t}, \mathbf{p}) = p_1 t_1, \quad E_{\text{off},2}(\mathbf{t}, \mathbf{p}) = p_{21} t_{21} + p_{22} t_{22}. \quad (6)$$

B. Local Computing Model

Given a pair of $(\mathbf{t}, \mathbf{p})$, the offloaded data sizes $\{L_i(\mathbf{t}, \mathbf{p})\}$ will be known, and hence the remaining input data of each user, i.e., $I_i - L_i(\mathbf{t}, \mathbf{p})$, should be computed locally at D_i, $i \in \{1, 2\}$. For local computing, we assume that the CPU frequency is fixed as f_i for D_i. In order to satisfy the latency constraint, i.e., $(I_i - L_i(\mathbf{t}, \mathbf{p})) C_i/f_i \leq T$, the offloaded data for D_i should have a minimum size of $L_i(\mathbf{t}, \mathbf{p}) \geq M_i^+ = \max\{I_i - f_i T/C_i, 0\}$. The energy consumption per CPU cycle is denoted as $Q_i = \kappa_i f_i^2$, where κ_i is the effective capacitance coefficient, and thus the energy consumption of D_i for local computing can be expressed as

$$E_{\text{loc},i}(\mathbf{t}, \mathbf{p}) = (I_i - L_i(\mathbf{t}, \mathbf{p})) C_i Q_i, \ i \in \{1, 2\}. \quad (7)$$

C. Problem Formulation

Based on the model, the energy saving for $D_i, i \in \{1, 2\}$ is

$$E_{\text{s},i}(P_0, \mathbf{t}, \mathbf{p}) = \nu_i g_i P_0 t_0 - E_{\text{off},i}(\mathbf{t}, \mathbf{p}) - E_{\text{loc},i}(\mathbf{t}, \mathbf{p}). \quad (8)$$

Furthermore, the APTPM problem for minimizing AP's transmit power can be formulated as problem (P1) below

$$\min_{P_0 > 0, \mathbf{t}, \mathbf{p}} \quad P_0 \quad (9a)$$

$$\text{s.t.} \quad E_{\text{s},i}(P_0, \mathbf{t}, \mathbf{p}) \geq 0, \ i \in \{1, 2\}, \quad (9b)$$

$$M_i^+ \leq L_i(\mathbf{t}, \mathbf{p}) \leq I_i, \ i \in \{1, 2\}, \quad (9c)$$

$$T - (t_0 + t_1 + t_{21} + t_{22}) \geq 0, \quad (9d)$$

$$t_0 \geq 0, t_1 \geq 0, t_{21} \geq 0, t_{22} \geq 0, \quad (9e)$$

$$p_1 \geq 0, p_{21} \geq 0, p_{22} \geq 0. \quad (9f)$$

Note that problem (P1) is nonconvex because of the nonconvex expressions of $L_i(\mathbf{t}, \mathbf{p})$, and the product of $P_0 t_0$ in $E_{\text{s},i}(P_0, \mathbf{t}, \mathbf{p}), i \in \{1, 2\}$. In the sequel, we will focus on removing these two obstacles to make problem (P1) solvable.

[1] In order to investigate the effect of user cooperation in resisting the double-near-far problem, we mainly consider the case of $h_1 < h_{12}$.

III. EFFORTS TO MAKE PROBLEM (P1) SOLVABLE

A. Transforming $L_1(\mathbf{t}, \mathbf{p})$ and $L_2(\mathbf{t}, \mathbf{p})$ into Convex

By introducing the variables $q_1 = \frac{p_1 t_1}{\nu_1 g_1 P_0}$, $q_{21} = \frac{p_{21} t_{21}}{\nu_2 g_2 P_0}$ and denoting $\mathbf{q} = [q_1, q_{21}]$, $L_{1,1}(\mathbf{t}, \mathbf{p})$, $L_{1,2}(\mathbf{t}, \mathbf{p})$ and $L_{1,12}(\mathbf{t}, \mathbf{p})$ in (2)–(4) can then be re-expressed as functions of $\mathbf{t}$ and $\mathbf{q}$ as

$$L_{1,1}(\mathbf{t}, \mathbf{q}) = t_1 B \log_2 (1 + \beta_1 P_0 q_1/t_1), \quad (10)$$

$$L_{1,2}(\mathbf{t}, \mathbf{q}) = t_{21} B \log_2 (1 + \beta_2 P_0 q_{21}/t_{21}), \quad (11)$$

$$L_{1,12}(\mathbf{t}, \mathbf{q}) = t_1 B \log_2 (1 + \beta_{12} P_0 q_1/t_1), \quad (12)$$

where $\beta_1 = \frac{\nu_1 g_1 h_1}{N_0}$, $\beta_2 = \frac{\nu_2 g_2 h_2}{N_0}$, and $\beta_{12} = \frac{\nu_1 g_1 h_{12}}{N_0}$. Note that the above three functions equal to 0 when $t_1 = 0$, $t_{21} = 0$ and $t_1 = 0$, respectively. Using the property of perspective function [14], it is easily verified that $L_{1,1}(\mathbf{t}, \mathbf{q})$, $L_{1,2}(\mathbf{t}, \mathbf{q})$ and $L_{1,12}(\mathbf{t}, \mathbf{q})$ are all joint concave functions of $\mathbf{t}$ and $\mathbf{q}$. Next, we introduce a new variable

$$L_1 = \min\{L_{1,1}(\mathbf{t}, \mathbf{q}) + L_{1,2}(\mathbf{t}, \mathbf{q}), L_{1,12}(\mathbf{t}, \mathbf{q})\} \quad (13)$$

to replace $L_1(\mathbf{t}, \mathbf{p})$ in (P1) with two additional convex constraints, $L_{1,1}(\mathbf{t}, \mathbf{q}) + L_{1,2}(\mathbf{t}, \mathbf{q}) \geq L_1$ and $L_{1,12}(\mathbf{t}, \mathbf{q}) \geq L_1$. Besides, we redefine the offloaded data size of D_2 as an independent variable L_2, and then by defining a function $g(x) = N_0(2^{\frac{x}{B}} - 1)$, $x \geq 0$, the offloading power p_{22} can be described as $p_{22} = \frac{1}{h_2} g\left(\frac{L_2}{t_{22}}\right)$ according to (5). Hence, the energy savings for D_1 and D_2 can be rewritten as

$$E_{\text{s},1}(P_0, \mathbf{t}, \mathbf{q}, L_1) = \nu_1 g_1 P_0 (t_0 - q_1) - (I_1 - L_1) G_1, \quad (14)$$

$$E_{\text{s},2}(P_0, \mathbf{t}, \mathbf{q}, L_2) = \nu_2 g_2 P_0 (t_0 - q_{21})$$
$$- \frac{t_{22}}{h_2} g\left(\frac{L_2}{t_{22}}\right) - (I_2 - L_2) G_2, \quad (15)$$

where $G_i = C_i Q_i, i \in \{1, 2\}$. As $g(x)$ is a convex function, its perspective function $t_{22} g(\frac{L_2}{t_{22}})$ is a joint convex function of t_{22} and L_2 considering both the cases of $t_{22} > 0$ and $t_{22} = 0$ [14]. Hence, problem (P1) can be equivalent transformed into another APTPM problem (P2),

$$\min_{P_0 > 0, \mathbf{t}, \mathbf{q}, L_1, L_2} \quad P_0 \quad (16a)$$

$$\text{s.t.} \quad E_{\text{s},i}(P_0, \mathbf{t}, \mathbf{q}, L_i) \geq 0, i \in \{1, 2\}, \quad (16b)$$

$$L_{1,1}(\mathbf{t}, \mathbf{q}) + L_{1,2}(\mathbf{t}, \mathbf{q}) \geq L_1, \quad (16c)$$

$$L_{1,12}(\mathbf{t}, \mathbf{q}) \geq L_1, \quad (16d)$$

$$M_i^+ \leq L_i \leq I_i, \ i \in \{1, 2\}, \quad (16e)$$

$$T - (t_0 + t_1 + t_{21} + t_{22}) \geq 0, \quad (16f)$$

$$t_0 \geq 0, t_1 \geq 0, t_{21} \geq 0, t_{22} \geq 0, \quad (16g)$$

$$q_1 \geq 0, q_{21} \geq 0. \quad (16h)$$

In problem (P2), only constraints in (16b) remain non-convex since P_0 is coupled with t_0, q_1 and q_{21}. Next, we will separate this coupling to facilitate the problem solving.

B. Separation for the Coupled Parameters

In order to further solve the APTPM problem (P2), we first introduce an equivalent min-max optimization problem (P3):

$$\min_{P_0 > 0} \max_{\mathbf{t}, \mathbf{q}, L_1, L_2} \quad \sum_{i=1}^{2} E_{\text{s},i}(P_0, \mathbf{t}, \mathbf{q}, L_i)$$
$$\text{s.t.} \quad (16b)–(16h), \quad (17)$$

which is still nonconvex in the above form, but can be optimally solved by a two-phase method. In the first phase, we solve the inner sub-problem with a given P_0 where the sum-energy-saving (SES) is maximized under the constraints in (P3), referred to as the SESM problem (P4):

$$\max_{\mathbf{t},\mathbf{q},L_1,L_2} \quad E_{\mathrm{s},1}(\mathbf{t},\mathbf{q},L_1) + E_{\mathrm{s},2}(\mathbf{t},\mathbf{q},L_2)$$
$$\text{s.t.} \quad (16b)\text{–}(16h), \tag{18}$$

where is a convex optimization problem, through which the optimal $\mathbf{t}^*$, $\mathbf{q}^*$ (or $\mathbf{p}^*$), L_1^* and L_2^* corresponding to the given P_0 can be obtained. It is easy to understand that if we assume the given P_0 is the optimal minimum, then the obtained $(\mathbf{t}^*, \mathbf{p}^*, L_1^*, L_2^*)$ is actually the optimal solution to problem (P2). If we find the minimum given $P_0^\star$ that maximizes the SES, then the obtained $(P_0^\star, \mathbf{t}^\star, \mathbf{p}^\star, L_1^\star, L_2^\star)$, i.e., the optimal solution of (P3), is actually the joint-optimal solution of (P2). In the second phase of the method, we will find the minimum P_0 by a bi-section search method. In the following section, we will demonstrate the problem-solving process of problem (P3) with the proposed two-phase method.

IV. THE TWO-PHASE METHOD FOR COOPERATIVE MEC

A. Problem-Solving with Lagrange Method

To gain more insights of the solution to problem (P4), we next solve it optimally using the Lagrange method [14]. The partial Lagrange function of (P4) is defined as

$$\begin{aligned}
&\mathcal{L}(\mathbf{t},\mathbf{q},L_1,L_2,\eta,\boldsymbol{\lambda}) \\
&\triangleq (1+\lambda_1)E_{\mathrm{s},1}(\mathbf{t},\mathbf{q},L_1) + (1+\lambda_2)E_{\mathrm{s},2}(\mathbf{t},\mathbf{q},L_2) \\
&\quad + \lambda_3(L_{1,1}(\mathbf{t},\mathbf{q}) + L_{1,2}(\mathbf{t},\mathbf{q}) - L_1) \\
&\quad + \lambda_4(L_{1,12}(\mathbf{t},\mathbf{q}) - L_1) \\
&\quad + \eta(T - (t_0 + t_1 + t_{21} + t_{22})),
\end{aligned} \tag{19}$$

where $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_4] \succeq \mathbf{0}$ ($\succeq$ denotes the componentwise inequality) and $\eta \geq 0$ consist of the Lagrange multipliers associated with the constraints (16b)-(16d) and (16f) in problem (P4), respectively. In order to facilitate the analysis in the sequel, we define another two functions

$$f(x) = \ln(1+x) + \frac{1}{1+x} - 1, \quad h(x) = g(x) - xg'(x). \tag{20}$$

Hence, the following lemma is established, which can be easily proved using the property of Lambert function.

Lemma 1. $f(x)$ $(g(x))$ is a monotonic increasing (decreasing) function of $x \geq 0$ with $f(0) = 0$ $(g(0) = 0)$. Given $C > 0$ $(G < 0)$, there exists a unique positive solution for equation $f(x) = C$ $(g(x) = G)$, given by $x^* = -\left(1 + (W_0(-e^{(-(C+1))}))^{-1}\right)$ $(x^* = \frac{B}{\ln 2}\left[W_0\left(\frac{G/N_0 + 1}{-e}\right) + 1\right])$, where $W_0(z)$ is the principal branch of the lambert W function [15], and e is the base of the natural logarithm.

We first assume that (P4) is feasible and let $\boldsymbol{\lambda}^*$, η^* denote the optimal Lagrange multipliers under a given P_0. Then applying the Karush-Kuhn-Tucker (KKT) conditions [14] leads

to the following necessary and sufficient conditions:

$$\frac{\partial \mathcal{L}}{\partial t_0^*} = (1+\lambda_1^*)\nu_1 g_1 P_0 + (1+\lambda_2^*)\nu_2 g_2 P_0 - \eta^* = 0, \tag{21}$$

$$\frac{\partial \mathcal{L}}{\partial t_1^*} = \frac{B\lambda_3^*}{\ln 2}f\left(\beta_1 P_0 \frac{q_1^*}{t_1^*}\right) + \frac{B\lambda_4^*}{\ln 2}f\left(\beta_{12}P_0 \frac{q_1^*}{t_1^*}\right) = \eta^*, \tag{22}$$

$$\frac{\partial \mathcal{L}}{\partial t_{21}^*} = \frac{B\lambda_3^*}{\ln 2}f\left(\beta_2 P_0 \frac{q_{21}^*}{t_{21}^*}\right) - \eta^* = 0, \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial t_{22}^*} = -(1+\lambda_2^*)\frac{1}{h_2}h\left(\frac{L_2^*}{t_{22}^*}\right) - \eta^* = 0, \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial q_1^*} = -(1+\lambda_1^*)\nu_1 g_1 P_0 + \frac{B}{\ln 2}F(t_1^*, q_1^*, \lambda_3^*, \lambda_4^*) = 0, \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial q_{21}^*} = \frac{B}{\ln 2}\left(\frac{\lambda_3^* \beta_2 P_0}{1 + \beta_2 P_0 \frac{q_{21}^*}{t_{21}^*}}\right) - (1+\lambda_2^*)\nu_2 g_2 P_0 = 0, \tag{26}$$

$$\frac{\partial \mathcal{L}}{\partial L_1^*} = (1+\lambda_1^*)G_1 - \lambda_3^* - \lambda_4^* \begin{cases} < 0, & L_1^* = M_1^+, \\ = 0, & L_1^* \in (M_1^+, I_1), \\ > 0, & L_1^* = I_1, \end{cases} \tag{27}$$

$$\frac{\partial \mathcal{L}}{\partial L_2^*} = (1+\lambda_2^*)H\left(\frac{L_2^*}{t_{22}^*}\right) \begin{cases} < 0, & L_2^* = M_2^+, \\ = 0, & L_2^* \in (M_2^+, I_2), \\ > 0, & L_2^* = I_2, \end{cases} \tag{28}$$

$$\lambda_1^* E_{\mathrm{s},1}(\mathbf{t}^*,\mathbf{q}^*,L_1^*) = 0, \quad \lambda_2^* E_{\mathrm{s},2}(\mathbf{t}^*,\mathbf{q}^*,L_2^*) = 0 \tag{29}$$

$$\lambda_3^*(L_{1,1}(\mathbf{t}^*,\mathbf{q}^*) + L_{1,2}(\mathbf{t}^*,\mathbf{q}^*) - L_1^*) = 0, \tag{30}$$

$$\lambda_4^*(L_{1,12}(\mathbf{t}^*,\mathbf{q}^*) - L_1^*) = 0, \tag{31}$$

$$\eta^*(T - (t_0^* + t_1^* + t_{21}^* + t_{22}^*)) = 0. \tag{32}$$

where $F(t_1^*, q_1^*, \lambda_3^*, \lambda_4^*) = \frac{\lambda_3^* \beta_1 P_0}{1 + \beta_1 P_0 \frac{q_1^*}{t_1^*}} + \frac{\lambda_4^* \beta_{12} P_0}{1 + \beta_{12} P_0 \frac{q_1^*}{t_1^*}}$, $H(\frac{L_2^*}{t_{22}^*}) = G_2 - \frac{1}{h_2}g'(\frac{L_2^*}{t_{22}^*})$. Note that $t_0^* + t_1^* + t_{21}^* + t_{22}^* = T$ must hold; otherwise, we can always allocate the remaining time to t_0^* to further increase the energy saving of both two users, and thus $\eta^* > 0$ holds for sure. Furthermore, the following lemma describes an important result concerning $\mathbf{t}^*$, $\mathbf{q}^*$ and L_1^*.

Lemma 2. The optimal time and power allocation $(\mathbf{t}^*, \mathbf{q}^*)$ ensures the following property of D_1's offloaded data size, L_1^*.

$$L_1^* = L_{1,1}(\mathbf{t}^*,\mathbf{q}^*) + L_{1,2}(\mathbf{t}^*,\mathbf{q}^*) \leq L_{1,12}(\mathbf{t}^*,\mathbf{q}^*). \tag{33}$$

Proof. We know that $\frac{\partial(t_{22}g(\frac{L_2}{t_{22}}))}{\partial t_{22}} = h\left(\frac{L_2}{t_{22}}\right) < 0$ for $t_{22} > 0$, which indicates that $t_{22}g(\frac{L_2}{t_{22}})$ is a monotonically decreasing function of t_{22}. It is easy to prove that the inequality $L_{1,1}(\mathbf{t}^*,\mathbf{q}^*) < L_{1,12}(\mathbf{t}^*,\mathbf{q}^*)$ always holds for the considered case of $h_1 < h_{12}$. If $L_{1,1}(\mathbf{t}^*,\mathbf{q}^*) + L_{1,2}(\mathbf{t}^*,\mathbf{q}^*) > L_{1,12}(\mathbf{t}^*,\mathbf{q}^*)$ holds, we can always allocate part of t_{21}^* to t_{22}^* while maintaining the same L_1^*, L_2^*, $\mathbf{q}^*$, t_0^*, t_1^* and the sum of t_{21}^*, t_{22}^*, which will decrease $L_{1,2}(\mathbf{t}^*,\mathbf{q}^*)$ until the equality holds. This operation will result in an increased $E_{\mathrm{s},2}(\mathbf{t}^*,\mathbf{q}^*,L_2^*)$ by decreasing $t_{22}^*g(\frac{L_2^*}{t_{22}^*})$ without reducing $E_{\mathrm{s},1}(\mathbf{t}^*,\mathbf{q}^*,L_1^*)$, and thus will increase the objective function of problem (P4). Hence, expression (33) always holds with the optimal solution of problem (P4). $\square$

Based on the result of Lemma 2, we can derive that $\lambda_3^* > 0$ and $\lambda_4^* = 0$. Furthermore, for $\mathbf{t}^* \succ \mathbf{0}$ and $\mathbf{q}^* \succ \mathbf{0}$, it can be derived from (22), (23) and Lemma 1 that

$$\beta_1 \frac{q_1^*}{t_1^*} = \beta_2 \frac{q_{21}^*}{t_{21}^*} = -\frac{1}{P_0}\left(1 + \left(W_0\left(-e^{-\left(\frac{\eta^* \ln 2}{\lambda_3^* B}+1\right)}\right)\right)^{-1}\right). \tag{34}$$

Moreover, through the KKT conditions (25) and (26), we can respectively derive that

$$\beta_1 \frac{q_1^*}{t_1^*} = \frac{\lambda_3^* B \beta_1}{(1+\lambda_1^*)\nu_1 g_1 P_0 \ln 2} - \frac{1}{P_0}, \tag{35}$$

$$\beta_2 \frac{q_{21}^*}{t_{21}^*} = \frac{\lambda_3^* B \beta_2}{(1+\lambda_2^*)\nu_2 g_2 P_0 \ln 2} - \frac{1}{P_0}. \tag{36}$$

Based on (34)-(36), we obtain that $(1 + \lambda_1^*)\nu_1 g_1 P_0 = \frac{\beta_1}{\beta_2}(1+\lambda_2^*)\nu_2 g_2 P_0$. Combining the condition (21), the optimal Lagrange multipliers have the following property:

$$(1+\lambda_i^*)\nu_i g_i P_0 = \frac{\beta_i \eta^*}{\beta_1 + \beta_2}, \ i \in \{1,2\}. \tag{37}$$

Hence, by substituting (37) into (35) and (36), we obtain

$$\beta_1 \frac{q_1^*}{t_1^*} = \beta_2 \frac{q_{21}^*}{t_{21}^*} = \frac{B\lambda_3^*(\beta_1+\beta_2)}{\eta^* \ln 2} - \frac{1}{P_0}. \tag{38}$$

Based on these results, the optimal resource allocation of (P4) for a given P_0 is characterized in the following subsections.

B. Optimal Offloading Decisions with Power Allocation

First, we define an offloading indicator for D_i as $\mu_i \triangleq Bh_iC_iQ_i/(N_0 \ln 2), i \in \{1,2\}$. Note that μ_i depends on the variables quantifying uplink channel (h_i), local computing (C_iQ_i), and it is a monotonically increasing function of h_i, C_i and Q_i. The following theorem shows the relationship between the optimal solution with μ_i.

Theorem 1. *(Optimal Cooperative Computation Offloading Decisions with Power Allocation).*
1) If $M_1^+ > 0$ or $\mu_1 \geq (\beta_1 + \beta_2)P_0/z^*$, the optimal L_1^*, p_1^* and p_{21}^* (all in semi-closed from) can be expressed as

$$L_1^* \begin{cases} = M_1^+, & \mu_1 < (\beta_1+\beta_2)P_0/z^*, \\ \in (M_1^+, I_1), & \mu_1 = (\beta_1+\beta_2)P_0/z^*, \\ = I_1, & \mu_1 > (\beta_1+\beta_2)P_0/z^*, \end{cases} \tag{39}$$

$$p_1^* = \frac{N_0}{h_1}\left((\beta_1+\beta_2)P_0/z^* - 1\right) > 0, \tag{40}$$

$$p_{21}^* = \frac{N_0}{h_2}\left((\beta_1+\beta_2)P_0/z^* - 1\right) > 0, \tag{41}$$

in which z^* is the unique solution of the equation given by $e^{\left(\frac{1}{(\beta_1+\beta_2)P_0}-1\right)z} - \frac{e}{(\beta_1+\beta_2)P_0}z = 0$ on the specific range of $z \in (0, (\beta_1+\beta_2)P_0)$. If $M_1^+ = 0$ and $\mu_1 < (\beta_1+\beta_2)P_0/z^*$, it is optimal to set $L_1^* = 0$, $p_1^* = 0$, and $p_{21}^* = 0$.
2) If $M_2^+ > 0$ or $\rho(\mu_2) \geq (\beta_1+\beta_2)P_0$, the optimal L_2^* and p_{22}^* (all in closed form) are given by

$$L_2^* \begin{cases} = M_2^+, & \rho(\mu_2) < (\beta_1+\beta_2)P_0, \\ \in (M_2^+, I_2), & \rho(\mu_2) = (\beta_1+\beta_2)P_0, \\ = I_2, & \rho(\mu_2) > (\beta_1+\beta_2)P_0, \end{cases} \tag{42}$$

$$p_{22}^* = \frac{1}{h_2}g\left(\frac{B}{\ln 2}\left[W_0\left(\frac{(\beta_1+\beta_2)P_0 - 1}{e}\right) + 1\right]\right), \tag{43}$$

where $\rho(\mu_2) \triangleq \mu_2 \ln \mu_2 - \mu_2 + 1$. If $M_2^+ = 0$ and $\rho(\mu_2) < (\beta_1+\beta_2)P_0$, it is optimal to set $L_2^* = 0$ and $p_{22}^* = 0$.

Proof. See Appendix A. $\square$

Theorem 1 shows that L_i, $i \in \{1,2\}$ have a threshold-based structure. Since the exact cases of $\mu_1 = (\beta_1+\beta_2)P_0/z^*$ in (39) and $\rho(\mu_2) = (\beta_1+\beta_2)P_0$ in (42) rarely occur in practice, the optimal policy makes binary offloading decisions for both cooperative users. Besides, L_i grows with increasing μ_i, which is consistent with the intuition that more resources should be scheduled to computation offloading when users have good channels (i.e., large h_i) or consume high local computing energy (i.e., large C_i and Q_i). Moreover, the same item in the thresholds of the offloading decisions for two users, i.e., $(\beta_1+\beta_2) = (\nu_1 g_1 h_1 + \nu_2 g_2 h_2)/N_0$ reflects the energy harvesting potential of two users (i.e., $\nu_1 g_1$ and $\nu_2 g_2$) and the quality of uplink offloading channels (i.e., h_1 and h_2), which demonstrates the effect of user cooperation that either user's offloading decision is affected by the other user's energy-harvesting ability and offloading-channel quality. Based on the results in Theorem 1, it is easy to obtain the following lemma.

Lemma 3. *For $L_1^* > 0$, the optimal transmit rates of D_1 and D_2 for offloading D_1's input data are same, i.e.,*

$$r_{1,1}(\mathbf{p}^*) = r_{1,2}(\mathbf{p}^*) = B\log_2\left((\beta_1+\beta_2)P_0/z^*\right). \tag{44}$$

C. Optimal Power-Efficient Time Allocation

The corresponding optimal time allocation, i.e., $\mathbf{t}^* = (t_0^*, t_1^*, t_{21}^*, t_{22}^*)$, is summarised in Theorem 2.

Theorem 2. *(Optimal Time Allocation for WPT and Cooperative Computation Offloading).*
1) The optimal time allocation for offloading D_2's input data is given by

$$t_{22}^* = \frac{L_2^* \ln 2}{B}\left[W_0\left(\frac{(\beta_1+\beta_2)P_0 - 1}{e}\right) + 1\right]^{-1}. \tag{45}$$

2) The optimal WPT duration time can be derived as

$$t_0^* = T - t_{22}^* - L_1^*/r_{1,1}(\mathbf{p}^*). \tag{46}$$

3) The optimal time allocation for offloading D_1's input data, i.e., (t_1^*, t_{21}^*) can be expressed as[2]

$$(t_1^*, t_{21}^*) = \left(L_1^*/r_{1,12}(\mathbf{p}^*), L_1^*/r_{1,1}(\mathbf{p}^*) - t_1^*\right), \tag{47}$$

where $(t_1^*, t_{21}^*) = (0,0)$ when $L_1^* = 0$.

Proof. See Appendix B. $\square$

D. Optimal Resource Allocation for obtaining $P_0^\star$

In this section, we will discuss the second phase of solving problem (P2). Note that with a larger P_0, the feasible region of problem (P4) will be lager as well, and thus more energy will be saved, which means that the maximum SES obtained by (P4) is an increasing function of P_0. Hence, the minimum $P_0^\star$ of the original APTPM problem (P1) can be obtained through

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[2]In this paper, we mainly consider the usual encountered case of $h_1 < h_2$. Actually, if the case of $h_1 > h_2$ does happen, we can simply exchange the roles of the two devices to apply the proposed scheme.
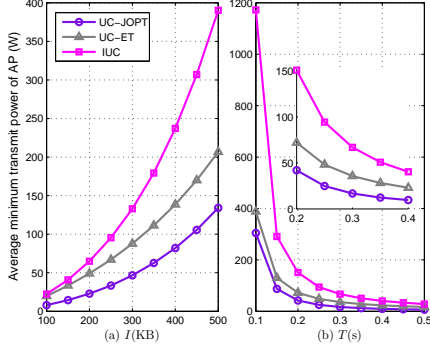
Fig. 1. Average minimum transmit power of AP versus I and T.

Fig. 2. Average minimum transmit power of AP versus ξ.

a bisection search of P_0. Note that t_{22}^* in (45) monotonically decreases with P_0, thus a lower bound of P_0, denoted as P_0^L, can be obtained by solving the equation $t_{22}^*(P_0) = T$ with $L_2^* = I_2$. Based on this P_0^L, we can further obtain a proper upper bound of P_0, denoted as P_0^U, which should make (P4) feasible and lead to positive energy savings for both of the users, and the optimal $P_0^\star$ must be in the range of (P_0^L, P_0^U). With the contradiction method, the following lemma showing a property of $P_0^\star$ can be easily obtained.

Lemma 4. *When the minimum feasible $P_0^\star$ is used in problem (P4), at least one of the two users should use up all its harvested energy, i.e., $E_{s,1}^*(P_0^\star) = 0$ or $E_{s,2}^*(P_0^\star) = 0$.*

V. SIMULATION RESULTS

In this section, the performance of the proposed scheme (UC-JOPT) is investigated by simulations. Also, we include the results of two baselines: UC-ET and IUC, which represent the schemes with equal time allocation for offloading D_1's input data, i.e., $t_{21} = t_1 = L_1^*/r_{1,12}(\mathbf{p}^*)$, and with inactive user cooperation by letting $t_{21} = 0$ and $t_1 = \overline{L}_1^*/r_{1,1}(\mathbf{p}^*)$, respectively. The other variables of these two baselines are obtained from Theorem 1 and Theorem 2. Comparisons with these two baselines can further show the capability of the proposed UC-JOPT scheme in handling CILC tasks and resisting the double-near-far effect in WPCNs.

The simulation parameters are set as follows unless specified otherwise. We set $B = 10$MHz, $T = 0.2$s, $N_0 = 10^{-9}$W, $\nu_i = 0.8$ and $\kappa_i = 10^{-28}$, respectively. It is assumed that the channel reciprocity holds for the downlink and uplink, and thus $g_1 = h_1$, $g_2 = h_2$. The channel power gain is modeled as $h_j = 10^{-3}d_j^{-\alpha}\phi_j$, $j \in \{1, 2, 12\}$, where ϕ_j represents the short-term Rayleigh fading and α is the path-loss exponent. We assume that $d_1 = 10$m, $d_2 = 6$m, $d_{12} = 6$m and $\alpha = 2$. For each user D_i, $i \in \{1, 2\}$, f_i is uniformly selected from the set of $\{0.1, 0.2, \ldots, 1.0\}$GHz. Besides, I_i and C_i follow the uniform distribution with $I_i \in [100, 500]$ KB and $C_i \in [1000, 2000]$ cycles/bit, respectively. The simulated figures in this section are based on 1000 realizations.

Fig. 1 depict the average minimum transmit power (AMTP) versus the same input data size $I = I_1 = I_2$ and the block
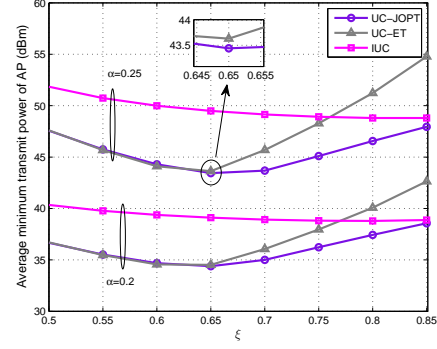
length T in (a) and (b), respectively. The AMTP of all the schemes increase with I but decrease with T, as expected. We can see that the UC-JOPT scheme obviously outperforms the baselines, indicating the significance of the optimization and user cooperation in UC-JOPT. Also, it is noted that the gaps of AMTP between different schemes become more significant for a larger I or a shorter T, demonstrating superiority of the proposed UC-JOPT scheme in handling the CILC tasks.

Fig. 2 shows the AMTP with respect to ξ for $\alpha = 2, 2.5$ where $d_2 = \xi d_1$, $d_{12} = (1 - \xi)d_1$, and $\phi_1 = \phi_2$. We can see that the proposed UC-JOPT scheme is superior to the benchmarks, and the improvements are even more pronounced with a larger α, indicating that UC-JOPT is highly effective in resisting the attenuation caused by path loss. It is also noticed that the curves of UC-JOPT and UC-ET, first decrease then increase with ξ, achieving the minimum AMTP at a saddle point of ξ. This is because for the cooperative computation offloading schemes, the performance depends not only on h_2 but also on h_{12}, and there exists a tradeoff between the two values. It is interesting to note that the performance of UC-JOPT converges to IUC as ξ gradually tends to 1 since both D_1 and D_2 suffer from severe signal attenuation. However, the performance of UC-ET is even worse than IUC when ξ becomes larger approaching to 1, which shows the importance and effect of optimizing the offloading time fraction.

VI. CONCLUSIONS

In this paper, we investigated the use of cooperative communications in computation offloading for a WPT-MEC system. Joint power and time allocation for cooperative computation offloading has been considered with the aim to minimize the transmit power of the AP for completing the computation tasks of the two near-far users. A two-phase method was proposed to find the optimal solution. Simulation results revealed that the proposed scheme greatly outperforms the baselines in handling CILC tasks and resisting double-near-far effect in WPCNs.

APPENDIX A

1) The following lemma is useful to prove the first result, which can be easily proved using the property of derivative.

Lemma 5. *For function* $q(z) = e^{(m-1)z} - emz = 0$ $(m > 0$ *is a constant), there exists a unique root on* $z \in (0, \frac{1}{m})$, *which can be easily obtained by a bisection search method.*

We will first consider the cases of $M_1^+ > 0$ or $\mu_1 \geq (\beta_1 + \beta_2)P_0/z^*$. From (34) and (38), we can get the equation given below

$$W_0\left(-e^{-(\frac{\eta^* \ln 2}{\lambda_3^* B} + 1)}\right) = \frac{-\eta^* \ln 2}{\lambda_3^* B (\beta_1 + \beta_2)P_0}. \quad (48)$$

Denoting $z^* = \frac{\eta^* \ln 2}{\lambda_3^* B} > 0$ and using the definition of the Lambert function, the above equation can be rewritten as

$$e^{\left(\frac{1}{(\beta_1+\beta_2)P_0} - 1\right)z^*} - \frac{e}{(\beta_1+\beta_2)P_0}z^* = 0. \quad (49)$$

Note that $\beta_1 \frac{q_1^*}{t_1^*} = \frac{\lambda_3^* B(\beta_1+\beta_2)}{\eta^* \ln 2} - \frac{1}{P_0} = \frac{(\beta_1+\beta_2)}{z^*} - \frac{1}{P_0} > 0$, which means that equation (49) should have a unique root z^* on $(0, (\beta_1 + \beta_2)P_0)$. According to Lemma 5, solving (49) is equivalent to find the unique root of $q(z) = 0$ on $z \in (0, (\beta_1 + \beta_2)P_0)$ with $m = 1/(\beta_1 + \beta_2)P_0$, and this unique root can be obtained by a bi-section search on $z^* \in (0, (\beta_1 + \beta_2)P_0)$. Therefore, λ_3^* can be expressed as $\lambda_3^* = \frac{\eta^* \ln 2}{Bz^*}$. Substituting λ_3^* and $(1 + \lambda_1^*)$ (in (37)), and β_1 into (27) leads to

$$\frac{\partial \mathcal{L}}{\partial L_1^*} = \frac{\ln 2}{B}\left(\frac{\mu_1}{(\beta_1+\beta_2)P_0} - \frac{1}{z^*}\right)\eta^*, \quad (50)$$

which establishes the result of L_1^* in (39). Similarly, substituting $\lambda_3^* = \frac{\eta^* \ln 2}{Bz^*}$ into (38), we have

$$\left(\frac{q_1^*}{t_1^*}, \frac{q_{21}^*}{t_{21}^*}\right) = \left(\frac{1}{\beta_1}V(z^*), \frac{1}{\beta_2}V(z^*)\right) \succ \mathbf{0}, \quad (51)$$

where $V(z^*) = \frac{\beta_1+\beta_2}{z^*} - \frac{1}{P_0}$. Based on these, we can further obtain p_1^* and p_{21}^* through the variable revivification, i.e., $p_1^* = \nu_1 g_1 P_0 \frac{q_1^*}{t_1^*}$ and $p_{21}^* = \nu_2 g_2 P_0 \frac{q_{21}^*}{t_{21}^*}$, leading to the results in (40) and (41). If $M_1^+ = 0$, $\mu_1 < (\beta_1 + \beta_2)P_0/z^*$, it can be derived that $L_1^* = 0$ according to (27), and thus $p_1^* = 0, p_{21}^* = 0$.

2) Next, we will prove the second result of Theorem 1. Similarly, we also first consider the cases of $M_2^+ > 0$ or $\rho(\mu_2) \geq (\beta_1 + \beta_2)P_0$. According to Lemma 3, the optimal transmission rate for offloading D_2's input data, i.e., $\frac{L_2^*}{t_{22}^*}$ can be obtained through (24) as

$$r_2^* = \frac{B}{\ln 2}\left[W_0\left(\frac{(\beta_1+\beta_2)P_0 - 1}{e}\right) + 1\right] > 0, \quad (52)$$

where the property of λ_2^* in (37) and the definition of β_2 are used. It is known that $g'(x) = \frac{N_0 \ln 2}{B} 2^{\frac{x}{B}}$ is a monotonically increasing function of x. Through (28), we can derive that $\frac{\partial \mathcal{L}}{\partial L_2^*}(<, =, >)0$ hold if and only if $\frac{L_2^*}{t_{22}^*}(>, =, <)\frac{B}{\ln 2}\ln \mu_2$, respectively. Hence, the result of L_2^* in (42) can be obtained by comparing the expression of $\frac{L_2^*}{t_{22}^*}$ in (52) and $\frac{B}{\ln 2}\ln \mu_2$. Hence, the optimal transmit power for offloading D_2's data is $p_{22}^* = \frac{1}{h_2}g\left(\frac{L_2^*}{t_{22}^*}\right)$, giving the result in (43). For the case of $M_2^+ = 0$, $\rho(\mu_2) < (\beta_1 + \beta_2)P_0$, it can be derived that $L_2^* = 0$ according to (28), and thus $p_{22}^* = 0$.

APPENDIX B

Based on the results of Theorem 1, we can easily derive t_{22}^* through $t_{22}^* = \frac{L_2^*}{r_2^*}$ with the expression of r_2^* in (52). For the case of $L_1^* = 0$, we understand that $t_1^* = 0$ and $t_{21}^* = 0$, and thus $t_0^* = T - t_{22}^*$. For the case of $L_1^* > 0$, combining the results of Lemma 2, Lemma 3, and the active time-sharing constraint in (16f), establishes the following equation

$$t_1^* + t_{21}^* = L_1^*/r_{1,1}(\mathbf{p}^*) = T - t_{22}^* - t_0^*, \quad (53)$$

which leads to (46). As for the derivation of (t_1^*, t_{21}^*) when $L_1^* > 0$, we resort to the results of Lemma 2 and Theorem 1, and further derive the following lemma, which can be proved by contradiction and here we omit the proof to save space.

Lemma 6. *The optimal time allocation* (t_1^*, t_{21}^*) *for cooperatively offloading* D_1's *input data satisfies*

$$L_1^* = L_{1,1}(t_1^*) + L_{1,2}(t_{21}^*) = L_{1,12}(t_1^*). \quad (54)$$

The above lemma is equivalent to that $L_1^* = (t_1^* + t_{21}^*)r_{1,1}(\mathbf{p}^*) = t_1^* r_{1,12}(\mathbf{p}^*)$, from which we can deduce the optimal time division parameters (t_1^*, t_{21}^*) as in (47).

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