Using pattern to inspire rich mathematical discourse in mixed attainment groups

Sue Gifford and Helen Thouless describe how mathematical discussion about a staircase pattern motivates children to think mathematically.

Teacher: Can you find the tower of thirteen?

Lucasz: [pointing to 13, then making a sweeping gesture across the towers for 11, 12 and 13, then across towers 1, 2 and 3] Because after two, three. Because it's like ten, one, two, three, four.

Magda: It's a bit like counting up stairs. Like counting one, two, three, but eleven, twelve, thirteen.

What struck us about this exchange was the way the 'staircase' stimulated the children to try to describe the 'teens' repeating pattern in their own different ways: identifying a related units pattern and an arithmetic sequence with intervals of one. (This particular exchange happened between two children who both spoke English as an Additional Language (EAL), Lukasz speaking Polish at home and Magda, a more fluent English speaker, speaking Czech.) We found that young children were fascinated by the staircase pattern. As they rushed by us on their way to lunch, two boys stopped dead in their tracks to peer at the pattern, while Latoya sat down to study it more closely.

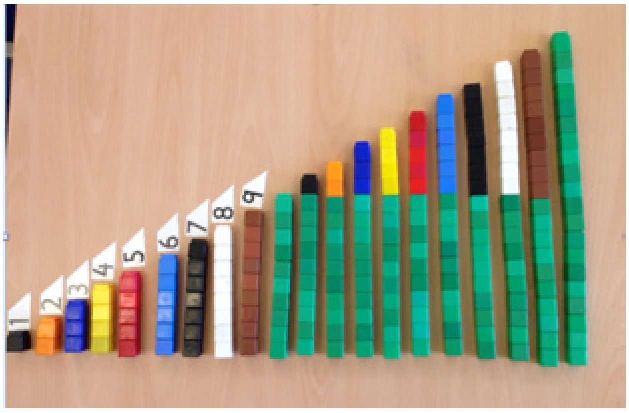

Figure 1: A staircase of towers.

The vignette above occurred while we were investigating 5- and 6- year-olds' understanding of the teen numbers: Helen was the class teacher and Sue was trialling activities for teachers. We were working with small mixed attainment groups in a Year 1 class and were challenging the children to find towers of different numbers from a staircase of 1-20 towers, which was numbered only as far as 9 (see Fig. 1). This was a rich activity which engaged all the children. They used a range of strategies, some of which revealed their difficulties with the teen numbers.

Some children seemed to pick a tower at random and then counted all the cubes by ones, not always accurately:

Clara: [touching every cube in the tower as she counted] One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, fourteen, fifteen.

Clara's omission of 'thirteen' revealed that she was not yet proficient with rote counting to 20. Other children in the same class could remember the sequence, but had difficulties with saying one number for each object. For example, Shopna counted the tower of 12 and concluded it was 'eleven', because she had missed counting one of the cubes. Although she counted accurately with towers smaller than ten, the added cognitive load of remembering the tricky 'teen' number sequence made it difficult for her to maintain one-to-one correspondence. This was a salutary reminder that some Year 1 children may still be developing confidence with skills they learned in the reception class.

Other children were able to use their knowledge strategically to find the target number:

Magda: [counted in ones along the 20 tower and then along the 18 tower] I knew one less would be nineteen and one less would be eighteen.

Magda was using counting to check her reasoning. Some children spotted the repeating number patterns, as Lukasz did. However, we expected more children to use the tens structure and to count on from the green stick of tens. Latoya was rare in using this strategy:

Helen: [pointed to the 15 tower and asked Latoya] How many is this one?

Latoya: Ten, eleven, twelve, thirteen, fourteen, fifteen.

Latoya had worked with the staircase a couple of weeks earlier. Then she had counted by ones, but another girl in her group had counted on from ten. Although Latoya did not take up this practice at the time of the previous session, her new practice emerged in this impromptu session as she was passing on the way to lunch.

What particularly interested us in the exchange between Lucasz and Magda was how Lucasz was eager to articulate the patterns he had found, despite being at an early stage in learning English and how Magda was able to take on his idea and develop it using her own ideas.

Both children were so excited that they had created their own ways of expressing what was, for them, a novel idea, “Because it’s like ten, one, two, three, four” and “It’s a bit like counting up stairs.”

Pattern spotting

It seemed that the visual pattern motivated the children to learn and think mathematically. Lucasz and Magda had implicitly noticed two types of patterns: the repeating units pattern and the stair pattern of equal intervals of one, which is the most basic arithmetic sequence. These patterns can lead to an understanding of the composition of the teen numbers and of place value. These also lead to early algebraic ideas about growing patterns, using general rules, such as ‘one more than’ and the implicit inverse relationship of ‘one less than’, recognised by Magda. Lucasz may also have recognised the repeated units of ten, although this is not clear from his comment.

Mulligan and Michelmore (2009) have proposed that children vary in their awareness of mathematical pattern and structure. They have found that young children who can analyse and recreate spatial patterns, such as staircases, triangular numbers or arrays, also achieve better mathematically. In this case, it certainly seemed that the children’s awareness of the visual pattern helped them to discover the number patterns. Papic, Mulligan and Michelmore (2011) found that pattern awareness can be taught. They argue that recognising the spatial organisation of patterns is linked to abstracting and generalising, with key aspects being the understanding of the unit of repeat as well as the spatial structuring. The Early Years Foundation Stage (Department for Education, 2014, p.11) provides encouragement for developing pattern awareness, with the Early Learning Goal for *Shape, space and measures* requiring children to “recognise, create and describe patterns”. However, this association with shape and space may not alert reception teachers to the potential for pattern to help number and pre-algebraic understanding. While the national curriculum advocates that Year 1 children “recognise and create repeating patterns with objects and with shapes” in the non-statutory guidance for *Number* (DfE 2013 p.6), in Year 2 it requires that children are taught to “order and arrange combinations of mathematical objects in patterns and sequences” as part of *Geometry* (DfE 2013 p.16). These references seem to reflect some uncertainty about the relationship between spatial patterns and mathematical structures, particularly their potential to develop early algebraic reasoning.

Mathematical discourse

The children’s pattern awareness was also fostered by the mathematical discussion. As they bounced ideas off each other, an apparently half-formed idea got fleshed out. This could lead later to more generalised explanations of the patterns. We would argue that the discussion was richer because the children were in a mixed attainment group. If they had been grouped by attainment, Lukasz and Magda may never have had the opportunity to have this discussion. If Lucasz had been placed in a ‘lower’ group he would not have had the opportunity to hear his idea articulated differently and Magda might not have noticed the patterns if she had not heard Lukasz pointing them out. This exchange seems to demonstrate the positive impact of collaborative learning and peer tutoring in mixed attainment classes.

We were also intrigued by the way that Lucasz’s use of gesture supported both his mathematical thinking and his developing command of English. His gestures were an important part of the discussion, as they helped him to describe the pattern by indicating the repetition and so expand on a partly articulated idea. This may have been an example of a child using gesture to express emergent mathematical learning, as described by Garber, Alibali and Goldin-Meadow (1998). On the other hand, Lukasz may have already been familiar with this pattern and he might have been able to express it fluently in Polish, we do not know. Either way, the use of gesture allowed Lucasz to communicate his idea so that Magda could then extend their mathematical thinking. It also helped us to see his competence and confidence, providing important clues about his mathematical thinking. As Moschkovich (2007) found, gestures are an important form of support for children with EAL, allowing them to participate in mathematical discussions, thus countering the assumption that students with EAL will not be active participants in mathematics discussions because they will find the language too difficult. It also suggests that teachers should encourage the use of gesture to support mathematics learning and engagement with patterns.

Implications for teaching

These children were at the beginning stages of spotting patterns in the teen numbers. A later step would be to ask them to explain the repetition of the number sequence. However, most did not yet understand the significance of the ‘teen’ in the teen numbers. While the children showed their knowledge, some confidently recognising two digit symbols and even mentally adding ten to a number, they did not explain the numbers as made up of ten and some ones. Our sessions impressed on us the complexity of the ‘teen’ numbers and the importance of engaging children with them through a variety of manipulatives. We then went on to explore the numbers with other resources such as beadstrings, Numicon and place value cards.

We think that this brief episode has several important implications for our practice:

- a simple pattern activity can engage children mathematically across the attainment range.
- we should not assume that children are fluent with the previous year's curriculum.
- mixed attainment groups that include children with EAL support learning for all children.
- mathematical discussions become more powerful in mixed attainment groups.
- teachers need to observe and encourage the use of gesture.
- we need to encourage children to look for, describe and recreate patterns from an early age.

This activity, of identifying individual towers in a staircase made of manipulatives, showed us how a simple pattern can provoke rich responses from children in a mixed attainment group and provide important information about learning. The staircase stimulated children to see patterns, to express relationships and to reason mathematically. Other 'staircases' can be equally intriguing (see Fig. 2):

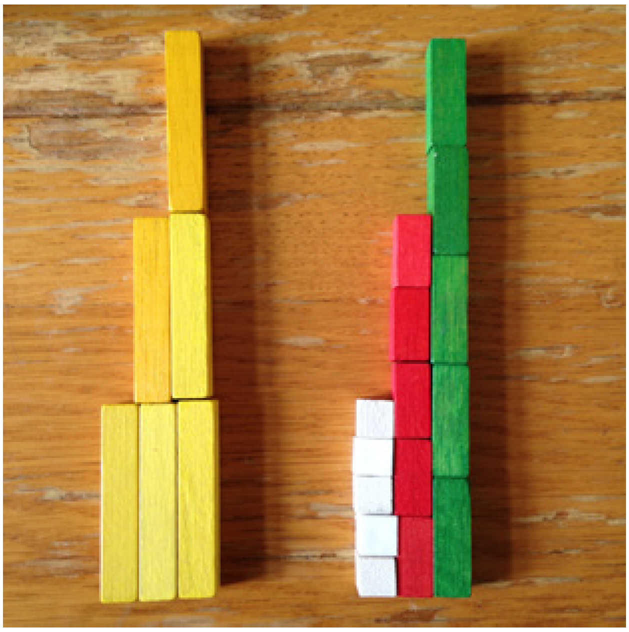

Figure 2: Staircases for the five-times table.

We think these activities provide stimulating, rich and productive experiences, building on children's strengths with visual memory and multisensory learning to prompt discussion and foster greater understanding of mathematical relationships. Patterns like these also meet national curriculum aims of providing a foundation for "an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject" (DfE 2013 p.3). It seems that teaching pattern awareness may not only be important for higher -level mathematical reasoning and understanding but also for those rarely voiced national curriculum aims, enjoyment and curiosity about mathematics.

References

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For more early pattern spotting activities see the Australian Association of Mathematics Teachers website: http://topdrawer.aamt.edu.au/Patterns.

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