Production Efficiency and Profit Taxation*

Stéphane Gauthier†
PSE and University of Paris 1

Guy Laroque
Sciences-Po, University College London and Institute for Fiscal Studies.

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Abstract

Consider a simple general equilibrium economy with one representative consumer, a single competitive firm and the government. Suppose that the government has to finance public expenditures using linear consumption taxes and/or a lump-sum tax on profits redistributed to the consumer. This note shows that, if the tax rate on profits cannot exceed 100 percent, one cannot improve upon the second-best optimum of an economy with constant returns to scale by using a less efficient profit-generating decreasing returns to scale technology.

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†PSE, 48 bd Jourdan, 75013 Paris, France; stephane.gauthier@univ-paris1.fr.

1 Introduction

In many countries different firms face different price systems. It may be that industrial policies subsidize firms operating in specific sectors of the economy. Also preferential tax treatments based on age, size, legal status or location are used to provide support to start-up companies, small firms, continued family firms or firms established in particular geographical areas. Such policies typically create production inefficiencies: it would be possible to get more output by reallocating inputs across firms. These inefficiencies are typically justified by market failures, e.g., imperfect competition, externalities, or limited capacity to tax some transactions. Absent these markets failures, they are inconsistent with the classical recommendation of production efficiency of Diamond and Mirrlees (1971) and Dasgupta and Stiglitz (1972), according to which all firms should face the same relative prices.

Under constant returns there are no profits in equilibrium and the production efficiency lemma of Diamond and Mirrlees (1971) shows how to improve upon an allocation in the interior of the production set of the economy by manipulating consumer prices. In the generalization to decreasing returns by Dasgupta and Stiglitz (1972) or Stiglitz and Dasgupta (1971) the existence of positive profits complicates the analysis. Indeed consider a tax on profits that has no distortive impact. Efficiency would recommend to use this tax rather than distortive taxes, e.g., consumption taxes acting as wedges between consumer and producer prices. Therefore, if the existence of taxable profits is accompanied with production inefficiency, the government could give up production efficiency in order to exploit the lump-sum feature of profit taxation.

The early literature on optimal taxation actually provides an example based on a similar idea where production efficiency is not socially desirable. Mirrlees (1972) shows that a government may find it welfare improving to use a decreasing returns to scale technology even though this technology is dominated by an available constant returns to scale technology. Decreasing returns allow the government to distribute a surplus to a single consumer using lump-sum taxable profits, rather than distortive consumption subsidies. In the absence of constraints on the profit tax rate, even an infinitesimal level of profits can be exploited to implement large lump-sum transfers: lump-sum taxation is achievable at the cost of small restrictions on the production set of the economy.[1]

The exact reasons why the results that prone departures from production efficiency are at odds with Dasgupta and Stiglitz (1972) or Stiglitz and Dasgupta (1971) analysis are not entirely clear in the existing literature.[2] Whether productive inefficiency is only valid in a few pathological situations or has a more fundamental validity is not clear either.

[1]The literature provides us with many examples where production efficiency is not desirable in the presence of heterogeneity across agents. Sadka (1977) illustrates the desirability of production inefficiency when consumers own different shares of the firms (see also Dasgupta and Stiglitz (1972), page 96): it may be socially desirable to let a firm use a less efficient technology to favor the consumers who own this firm. The case of a single consumer in Mirrlees (1972) makes the property striking.

[2]See Stiglitz and Dasgupta (1971), Hahn (1973), Sadka (1977), Munk (1978), or Blackorby and Brett (2004) for a discussion.

The disturbing Mirrlees (1972) example has been recently reexamined by Reinhorn (2013), Blackorby and Murty (2009) and Murty (2013). Reinhorn (2013) emphasizes possible (dis)continuity properties of dividend payments in response to changes in consumption taxes that may prevent reaching situations where production is efficient. In a rich general equilibrium framework Blackorby and Murty (2009) and Murty (2013) highlight the role played by the government ability to set firm specific, possibly non proportional, profit taxation.[3] Our paper reexamines the Mirrlees (1972) example keeping most of his setup: a single consumer supplies labor and consumes a final good, owning a firm that transforms labor into good. The contribution of our paper is to emphasize that the policies that involve production prices varying across firms cannot be justified by an argument relying on the merit of lump-sum profit taxes compared to consumption taxes. We depart from Mirrlees setup in two crucial dimensions. First the government has to collect taxes to finance positive public expenditures, while in Mirrlees (1972) it had to distribute pre-existing resources. Second we require that profits not be taxed at a rate higher than 100 percent, as in Dasgupta and Stiglitz (1972). That is, for institutional reasons, we require the after-tax income from profits to be non-negative.

We find that profits generated by a dominated decreasing returns to scale technology should always be fully confiscated at the second best optimum: it is always locally advantageous to finance positive public expenditures by a tax on profits to benefit from the lump-sum aspect of this tax. However we also find that substituting a lump-sum tax on profits to distortive consumption taxes cannot improve the general equilibrium welfare upon the level obtained at the second-best allocation of constant returns economy (where there are no taxable profits). The institutional constraint of non-negative after-tax profits binds and leads to zero after-tax income from profits. Then welfare coincides with its second-best level in the constant returns economy. It therefore appears that the Mirrlees (1972) example relies on two features which are uncommon in practice: the government has surpluses to distribute and may subsidize profits. Our analysis therefore reinforces the group of economists that advocate to stick to productive efficiency.

Our paper is organized as follows. Section 2 describes the setup. Then, in Section 3, we specialize to the constant returns to scale linear case. Finally Section 4 considers decreasing returns to scale piecewise linear technologies, following Mirrlees (1972). We provide a simple argument for the desirability of production efficiency, which is extended to the class of all decreasing returns to scale technologies.

[3]In another strand of the literature production inefficiency may be a way to alleviate informational asymmetries, by playing on factor prices as in Wilson (1982), Naito (1999) and Blackorby and Brett (2004) or on observable occupational choices as in Saez (2004) and Gomes, Lozachmeur, and Pavan (2017). Production inefficiency also allows to deter tax evasion in Best, Brockmeyer, Kleven, Spinnewijn, and Waseem (2015).

2 A simple economy

There are three agents in the economy: one representative consumer, one private firm and the government. Using labor supplied by the consumer, the firm produces a final good which may serve for private or public consumption. Labor is chosen as the numeraire. The production price of the good is p and its consumption price is q.

The production set Y describes the couples (y, ℓ) of non negative production and labor that are feasible. It exhibits non-increasing returns to scale and allows inaction, i.e., $(0, 0)$ belongs to Y. The firm is competitive and thus considers the production price of the good as given. It chooses a production level y and a labor input ℓ that maximize its profit $\pi = py - \ell$ on $(y, \ell) \in Y$.

The consumer is the owner of the firm and as such receives after-tax profits m equal to $(1 - \tau)\pi$, where τ is a proportional profit tax rate. The consumer also collects wages and uses all of her incomes to buy final good. Her budget constraint is

$$qc = \ell + m. \tag{1}$$

The consumer choices between consumption and labor are represented with a twice continuously differentiable strictly quasiconcave utility function $u(c, -\ell)$, defined on $\mathcal{R}_+ \times \mathcal{R}_-$, increasing in its two arguments. The pair (c, ℓ) maximizing utility on the budget set is given by (1) and the first-order conditions in consumption and leisure yielding the standard equality between the consumer price and the marginal rate of substitution,

$$q = \frac{u'_c}{u'_\ell}(c, -\ell). \tag{2}$$

The government has to finance public expenditures $p\hat{y} > 0$. It only has two tax instruments at its disposal. First it can use a commodity tax $q - p$, a wedge between producer and consumer prices, yielding tax receipts of $(q-p)c$. Second it can also use a proportional tax τ on private profits, yielding revenues equal to $\tau\pi$. We require

$$\tau \leq 1. \tag{3}$$

This constraint is satisfied in actual tax systems and may be linked to the absence of lower bounds on other incomes. The public budget constraint is

$$(q - p)c + \tau\pi = p\hat{y}.$$

3 Linear technology

Following Mirrlees (1972) our starting point is the economy where the production set is $Y = \{(y, -\ell) \mid y - a\ell \leq 0\}$ for some positive a. Labor supply must be at least as large as $\hat{y}/a$ to produce the public good. The first-best optimum supply of labor is

$$\ell_{\text{FB}} = \arg\max_{\ell} \{u(a\ell - \hat{y}, -\ell) \mid a\ell \geq \hat{y}\}. \tag{4}$$

By convexity the first-best labor supply ℓ_{FB} is unique, when it exists. The first-best optimum can be decentralized by equalizing consumer and producer prices to the social value of the good $1/a$ (measured in units of labor), $q_{\mathrm{FB}} = p_{\mathrm{FB}} = 1/a$. The public expenditures $p_{\mathrm{FB}}\hat{y}$ can then be financed using a lump-sum transfer

$$m_{\mathrm{FB}} = -p_{\mathrm{FB}}\hat{y} = -\hat{y}/a < 0.$$

The requirement (3) and the fact that profits are non-negative imply $m \geq 0$, a condition that is violated at $m = m_{\mathrm{FB}}$. Thus the government cannot implement the first-best optimum with the available fiscal tools. Under constant returns to scale, the firm makes zero profits, implying $m = 0$. The only way to finance public expenditures is therefore to introduce a positive wedge between the consumer and producer prices, q and p.

The aggregate demand for the good is the sum of the private consumption c satisfying (1) and (2) and public consumption $\hat{y}$. Facing the producer price $p = 1/a$ the firm produces y and uses y/a units of labor. The market clearing conditions in the markets for the good and labor are

$$y = c + \hat{y},$$

and

$$\ell = \frac{y}{a}.$$

We are now in a position to define an equilibrium of the economy with linear technology as follows:

Definition 1. An equilibrium of the economy $(a, \hat{y})$ with constant returns to scale is a triple (m, q, y) such that

$$q = \frac{u'_c}{u'_\ell}\left(y - \hat{y}, -\frac{y}{a}\right),$$

$$q\,(y - \hat{y}) = \frac{y}{a} + m,$$

$$m = 0.$$

The first two conditions are (1) and (2) evaluated at a market clearing allocation. Zero after-tax income from profits comes from the use of a constant returns to scale technology.

In the sequel our benchmark will be the equilibrium which yields the highest level of welfare, i.e., the highest consumer utility. We refer to it as the second-best equilibrium. Since $m = 0$ in equilibrium, the second-best equilibrium is $(0, q_{\mathrm{SB}}, y_{\mathrm{SB}})$ where q_{SB} is the smallest (non-negative) equilibrium consumption price. The corresponding consumer's utility level is denoted v_{SB}.

4 Piecewise linear technology

Profit taxation cannot be used in the linear economy since profits are 0 in equilibrium. We study whether there exists a less efficient technology than that of the previous section, which would generate profits and therefore provide a new tax instrument to the government allowing to achieve a larger welfare than the second-best equilibrium.

We first restrict our attention to a special form of piecewise linear production functions such that one unit of labor still produces a units of the final good, but only up to some production level $\tilde{y}$, $\tilde{y} > 0$. For production levels larger than $\tilde{y}$, one unit of labor now produces 0 unit of the good. The supply of good is

$$\begin{cases} 0 & \text{if } p < 1/a \\ [0, \tilde{y}] & \text{if } p = 1/a \\ \tilde{y} & \text{if } p > 1/a. \end{cases}$$

By playing on $\tilde{y}$ this technology may yield positive profits at no real cost in terms of feasible production compared to the linear technology case. For $p > 1/a$ the firm indeed produces $\tilde{y}$ units of the good, uses $\tilde{y}/a$ units of labor and thus gets positive profits $(p - 1/a)\,\tilde{y} > 0$. Therefore any equilibrium production level y in the linear technology case can be obtained in the piecewise linear case by making the threshold $\tilde{y}$ equal to y.

If $p \leq 1/a$ profits are zero and so the after-tax profit m is also zero. By definition of the second-best equilibrium, this cannot improve upon the welfare level v_{SB}. Hence in the sequel we focus on the case where $p > 1/a$.

Definition 2. An equilibrium of the economy $(a, \tilde{y}, \hat{y})$ with piecewise linear technology is a triple (m, q, y) with $m \geq 0$, such that

$$q = \frac{u'_c}{u'_\ell}\left(y - \hat{y}, -\frac{y}{a}\right),$$

$$q\,(y - \hat{y}) = \frac{y}{a} + m,$$

$$y = \tilde{y} > \hat{y}.$$

Figure 1 presents the second-best equilibrium in the linear technology E, together with a putative welfare improving equilibrium A.[4] At the second-best, the budget constraint is $q_{\mathrm{SB}} c \leq \ell$ since after tax profits are 0. The solution of the consumer's program is $(c_{\mathrm{SB}}, \ell_{\mathrm{SB}})$, and the corresponding indifference curve is blue long dash-dotted, while feasibility requires $c_{\mathrm{SB}} = a\ell_{\mathrm{SB}} - \hat{y}$. Figure 1 depicts in green thin dash a candidate indifference curve for a feasible allocation A at the kink of the piecewise linear technology where labor input is $\tilde{\ell} = \tilde{y}/a$. Were A to satisfy all the conditions given in Definition 2, which our next

[4] We thank a referee for urging us to use a diagram to clarify the discussion.

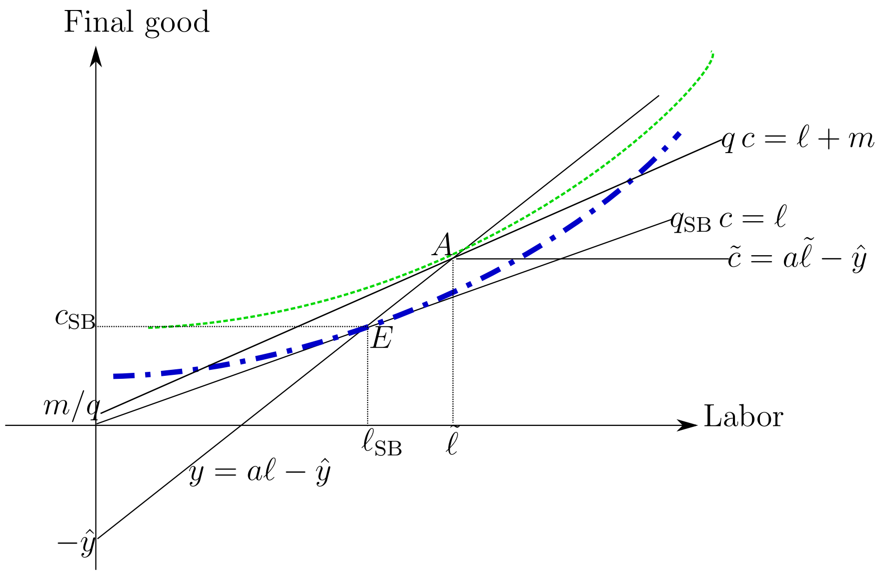

Figure 1: Improving upon the second-best equilibrium

proposition proves to be impossible, it would yield an equilibrium allocation of the piecewise linear technology that improves upon the second-best equilibrium.

Proposition 1. Consider a government that has to finance a positive public consumption $\hat{y}$. When the profit tax rate τ is constrained to be less than 1, there is no equilibrium of the economy $(a, \tilde{y}, \hat{y})$ with piecewise linear technology, which improves upon the second-best equilibrium of the constant returns to scale economy $(a, \hat{y})$.

Proof. To each $\tilde{y} > \hat{y}$ is associated one equilibrium (the third equation gives $y = \tilde{y}$, the first yields q, while the middle one gives m) when the value of m is non-negative, or no equilibrium when it is negative. In an equilibrium,

$$m = \left(q - \frac{1}{a}\right)\tilde{y} - q\hat{y} \geq 0,$$

so that q is larger than $1/a$. The social welfare is $u(\tilde{y} - \hat{y}, -\tilde{y}/a)$. It decreases with $\hat{y}$. Following Definition 2, the change in social welfare with $\tilde{y}$ is

$$u\left(\tilde{y} + \mathrm{d}\tilde{y} - \hat{y}, -\frac{\tilde{y} + \mathrm{d}\tilde{y}}{a}\right) - u\left(\tilde{y} - \hat{y}, -\frac{\tilde{y}}{a}\right) \simeq \left(u'_c - \frac{u'_\ell}{a}\right)\mathrm{d}\tilde{y} = u'_\ell\left(q - \frac{1}{a}\right)\mathrm{d}\tilde{y}.$$

At any equilibrium the social welfare is increasing in $\tilde{y}$. The best equilibrium is therefore the one with the largest $\tilde{y}$. There exists an equilibrium $(m, q, \tilde{y})$ where m equals 0: the second best equilibrium $(0, q_{\mathrm{SB}}, y_{\mathrm{SB}})$ is the equilibrium of the economy with the piecewise linear technology where $\tilde{y} = y_{\mathrm{SB}}$. By (3) at the best equilibrium m is equal to zero: otherwise if m were positive, a small enough increase in $\tilde{y}$ would yield a welfare improving equilibrium. Thus the best equilibrium is the second-best equilibrium: the social welfare in an economy with the piecewise linear technology cannot be greater than v_{SB}. $\square$

At first the result is surprising (see also Dasgupta and Stiglitz (1972), bottom of page 91): even though the piecewise linear technology allows for positive profits that can be taxed in a lump-sum fashion without sacrificing feasible production, profits cannot be exploited to reduce distorsions from consumption taxes and raise social welfare. The proof of Proposition 1 highlights the crucial role played by our two departures from Mirrlees (1972) example, namely a positive amount $\hat{y}$ to be financed and the institutional restriction (3), $\tau \leq 1$, on profit taxation. Since $\hat{y} > 0$, the first-best optimum requires a negative non-labor income $m = -\hat{y}/a$ which corresponds to a tax rate on profits higher than 100 percent, a level prohibited by the restriction (3). The best option for the government is to fully tax profits. The consumer thus receives no income from the firm, so that $m = 0$ as in the case of the linear technology where there are no profits: the best equilibrium coincides with the second-best equilibrium. By (3) the profit-generating piecewise linear technology can never yield a welfare improvement upon the linear technology case.

This property even holds when profits are large enough to finance the public deficit, $\pi > p\hat{y}$. In this configuration, public expenditures could be financed lump-sum by setting $\tau = p\hat{y}/\pi < 1$, using zero distortive consumption taxes, i.e., $q = p$. However, from the condition for positive profits $p > 1/a$, both p and q must be greater than the social value of the final good $1/a$. By Proposition 1, however, it is socially better to set a 100 percent profit tax and to subsidize consumption.

Proposition 1 considers equilibrium allocations. We now investigate whether an allocation in the interior of the production set of the economy $(a, \hat{y})$ with constant returns to scale may yield a welfare improvement upon v_{SB}. We consider a quantity z, $z \geq 0$, of the final good which can be interpreted as a waste: output is $\tilde{y} - z$ when labor input is $\tilde{\ell}$ with $a\tilde{\ell} = \tilde{y}$. The corresponding private consumption is $c = \tilde{y} - z - \hat{y}$. The decentralization of this allocation requires

$$q = \frac{u'_c}{u'_\ell}\left(\tilde{y} - z - \hat{y}, -\frac{\tilde{y}}{a}\right).$$

and

$$m = q\left(\tilde{y} - z - \hat{y}\right) - \frac{\tilde{y}}{a} \geq 0,$$

The issue is whether there exists $z > 0$ such that

$$u\left(\tilde{y} - z - \hat{y}, -\frac{\tilde{y}}{a}\right) > v_{\mathrm{SB}}.$$

The following result shows that this is never the case.

Proposition 2. Consider a government that has to finance a positive public consumption $\hat{y}$. When the profit tax rate τ is constrained to be less than 1, the sustainable allocation $c = \tilde{y} - z - \hat{y}$, $\ell = \tilde{y}/a$ of the economy with piecewise linear technology, and waste $z > 0$, never improves upon the second-best equilibrium of the constant returns to scale economy $(a, \hat{y})$.

Proof. Indeed the allocation $c = \tilde{y} - z - \hat{y}$ and $\ell = \tilde{y}/a$ is an equilibrium of the economy $(a, \tilde{y}, \hat{y} + z)$. The welfare at this equilibrium $u[\tilde{y} - (\hat{y} + z), -\tilde{y}/a]$ is decreasing with z. Since

$\hat{y} + z > \hat{y} > 0$ and $\tau \leq 1$, Proposition 1 implies that this level of welfare cannot improve upon the second best v_{SB} of the economy $(a, \hat{y})$. $\square$

Remark 1. The same properties hold in the case of a general decreasing returns to scale technology. Suppose that the set of feasible allocations consists of (y, ℓ) satisfying $y - F(k, \ell) \leq 0$, with k a scaling real parameter, and F is increasing and concave in labor ℓ. Given k an equilibrium is defined as in Definition 2, replacing $\tilde{y}$ with the supply

$$y(p, k) \in \arg\max_{y} \{py - \ell \mid y - F(k, \ell) \leq 0\}.$$

If k can be adjusted to yield a rise in equilibrium output, the best equilibrium is associated with a value of k such that $m = 0$. If $F(k, \ell) \leq a\ell$ for every ℓ, then the second-best equilibrium of the economy $(a, \hat{y})$ with linear technology yields a higher welfare than the best equilibrium of the economy with the decreasing returns to scale technology. Production efficiency is therefore desirable. This extends the result of Dasgupta and Stiglitz (1972) obtained for smooth technologies when $\hat{y}$ is positive and τ is smaller than 1.

Remark 2. Mirrlees (1972) example. The marginal rate of substitution between consumption and leisure is $1/a$ if private consumption is $c = a\ell_{\mathrm{FB}} - \hat{y}$. To decentralize such an allocation as an equilibrium of the economy $(a, \tilde{y}, \hat{y})$ where $\tilde{y} = y_{\mathrm{FB}} = a\ell_{\mathrm{FB}}$, it must consequently be that $q = q_{\mathrm{FB}} = 1/a$. The budget constraint of the consumer then gives $m = m_{\mathrm{FB}} = -\hat{y}/a$. By Walras law the government budget constraint is satisfied for every p: since $\tau\pi = \pi - (1 - \tau)\pi = \pi - m$, we have

$$(q - p)c + \tau\pi = \left(\frac{1}{a} - p\right)(y_{\mathrm{FB}} - \hat{y}) + \left(p - \frac{1}{a}\right) y_{\mathrm{FB}} - \left(-\frac{\hat{y}}{a}\right) = p\hat{y}.$$

One can thus set $p > 1/a$. Then positive profit yields

$$m_{\mathrm{FB}} = (1 - \tau)\pi = -\frac{\hat{y}}{a} \Leftrightarrow 1 - \tau = -\frac{\hat{y}}{a\pi}.$$

We know that there is no equilibrium of the economy $(a, y_{\mathrm{FB}}, \hat{y})$ if $\hat{y} > 0$ since then $m < 0$, or equivalently, $\tau > 1$. Financing public expenditures would require to set a tax on profits greater than the dividends received by the consumer, the supplement being paid from labor income. The first-best allocation can however be implemented as an equilibrium if $\hat{y} < 0$, i.e., a public surplus is to be distributed to the consumer, as in the initial example of Mirrlees (1972). In this last case where $\hat{y} < 0$, the highest level of social welfare that can be obtained as an equilibrium is lower in the economy $(a, \hat{y})$ than in $(a, y_{\mathrm{FB}}, \hat{y})$. Thus production inefficiency is socially desirable if $\hat{y} < 0$.

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