Procedural complexity underlies the efficiency advantage in abacus-based arithmetic development.

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Abstract

The abacus is a counting frame designed to facilitate the process of arithmetic calculation.This paper focusses particularly on the Japanese 'soroban' abacus, examining the role of complementary numbers in its operation. We propose that the combination of quinary (base five) and decimal (base ten) systems, and the consequent use of complementary numbers (CN) in calculation, are transferred from physical to mental procedures. The more CN steps involved in a calculation, the more time it should take for mental abacus users to complete. This hypothesis was tested in Experiment 1a: 146 Taiwanese abacus users aged 3-15, identified as either abacus learners (n=126) or abacus experts (n=20), were given parallel sets of serial addition and subtraction problems, matched on conventional metrics of difficulty level, but differing in the number of CN steps entailed in abacus calculation. An effect of CN was found whereby response times were significantly greater for the condition involving double CN steps. Experts were significantly faster than learners, but the effect of CN did not differ between groups. In Experiment 1b, in order to test the specificity of the CN effect, a group of British children (n=20), with no experience of the abacus, was given the same mental calculation tasks. They achieved accuracy levels equivalent to the abacus experts, but their calculation speed was very much slower, and they showed no evidence of the CN effect.

Our findings demonstrate the importance of complementary numbers in children's mental abacus calculation. We found the CN effect to be equally strong in expert and non-expert users, and confirmed the specificity of the effect by showing it to be absent in participants not exposed to abacus use. We argue that the use of complementary numbers in mental abacus calculation provides learners, from the outset, with exposure to a dual-base system which, while procedurally complex, affords an efficiency advantage not enjoyed by learners working within the exclusively decimal system of Arabic numerals.

Key Words: Abacus, Complementary Number, Calculation

Introduction

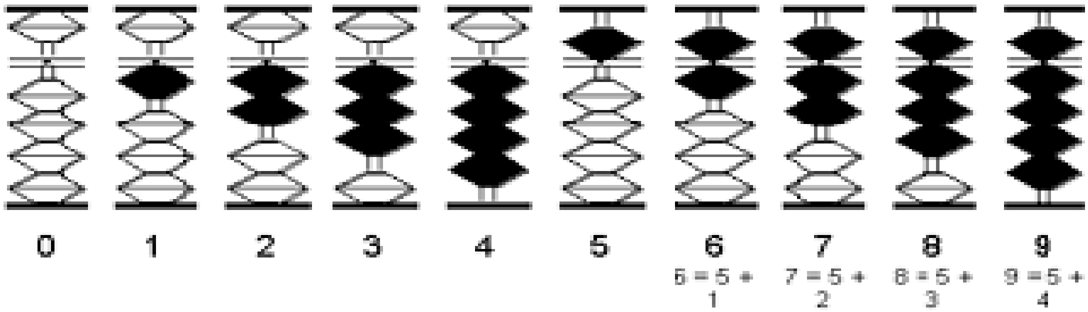
Design and operation of the Soroban Abacus

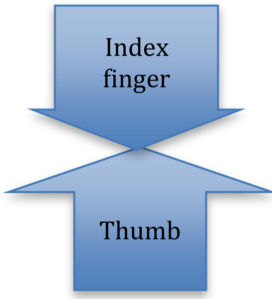
The abacus, of which there are many types across the world, is a counting frame containing sets of moveable beads, mounted on rods. The particular model discussed throughout this paper is the Japanese soroban-abacus (see Figure 1, below). The abacus is made of two parts (the upper part, which contains the “heaven beads” and the lower part, which contains the “earth beads”) separated by a beam. The heaven bead is composite unit (five), the earth bead is a simple unit. A single heaven bead and four earth beads are combined on each rod. Each rod represents a digit within a base-ten (decimal) numeral (Kojima, 1954). Pushing one earth bead up towards the beam represents the value one, the same logic applying from one to four. Pushing the heaven bead downward to the beam represents the value five. Abacus representations 1-9 are shown within a nine digit numeral in Figure 1, below. Particularly noteworthy is the complex nature of abacus representation. At once it offers (a) direct perceptual representation of the numerosity of units, (b) composites (heaven beads) providing a quinary base, and (c) the frame of rods providing decimal place value. The latter, and only the latter, is shared with the Arabic numeral system.

The primary purpose of the abacus is to perform calculation tasks mechanically, thereby minimizing mental workload, by a process that Kojima (1954) called ‘mechanicalization’. This is achieved by users familiarizing themselves with sets of complementary numbers (CN), particularly those number pairs which sum to 5 (the value of the heaven bead) and 10 (the base unit on which the abacus framework is constructed).

Figure 1

A soroban abacus representing "one hundred and twenty three million, four hundred and fifty-six thousand, seven hundred and eighty-nine"

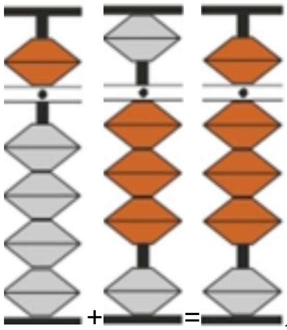
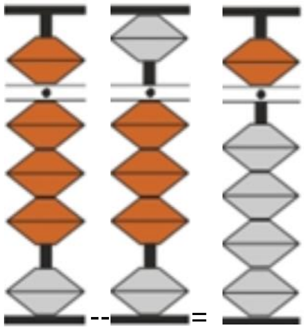




Calculation using the soroban abacus requires knowledge of two sets of CN, based on the heaven bead (value 5) and the base system on which the abacus frame is constructed (value 10). The user must know that 1+4=5, 2+3=5, 3+2=5, 4+1=5, and that 1+9=10, 2+8=10, 3+7=10, 4+6=10, 5+5=10, 6+4=10, 7+3=10, 8+2=10 and 9+1=10. The CN are taught to young children as sets of "friends", e.g. , referring to the heaven bead, 3 has the 'friend' 2, and vice versa. Before a calculation can be made, all rods must be set to zero. Some single rod additions and subtractions can be made without using CN. Figure 2 (below) shows step-by-step examples.

Figure 2

Calculations without use of complementary numbers (No CN)

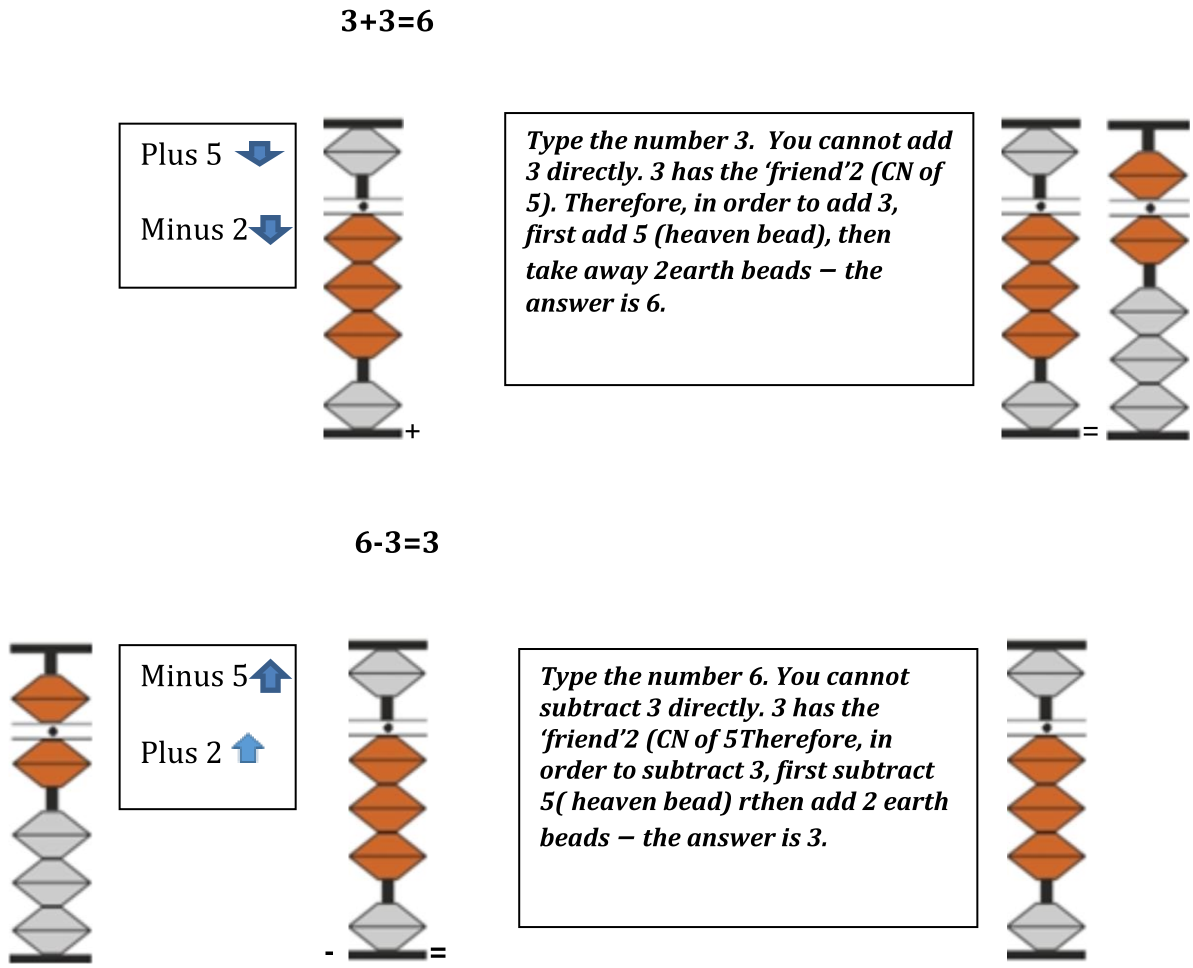
 *Addition 5+3=8*	 *Subtraction 8-3=5*
Push one heaven bead down to type the number 5 on the abacus; then, on the same rod, add 3 by pushing three earth beads up (5+3=8)	Type the number 8 on the abacus by pushing one heaven bead down and three earth beads up, then take away five by pushing the heaven bead up. The rod is left with one heaven bead (8-3=5)

NB All operations represented here take place on a single rod.

Where a particular calculation cannot be directly represented by simple movement of beads, the user must now think in terms of CN. The learning opportunity afforded by the use of CN is demonstrated in the operations of simple addition and subtraction, which are taught together. The inverse relation between these operations (if a + b = c, then c − b = a) , often regarded as a late-acquired concept (Dowker 2014), is procedurally represented in the abacus calculation.

Figure 3

Calculation with one set of complementary numbers (1CN)



An example is given in Figure 3, above. In each case the same set of CN of 5 is employed in both addition and subtraction. Thus the use of complementary numbers and the principle of inversion are represented procedurally from the outset of arithmetic instruction (as early as age 3 or 4). These procedures are complex compared to those typically employed in early arithmetic learning using Arabic numerals. Here instruction is largely based on counting (Siegler 1987; Donlan et al. 2007), often implemented using a simple linear representation or 'number line' (UK Department for Education, 2014). Systematic exposure to the inversion principle may occur at a much later stage for Arabic numeral learners e.g. 7-8 years (Canobi 2009).

Calculation with two sets of complementary numbers (2 CN).

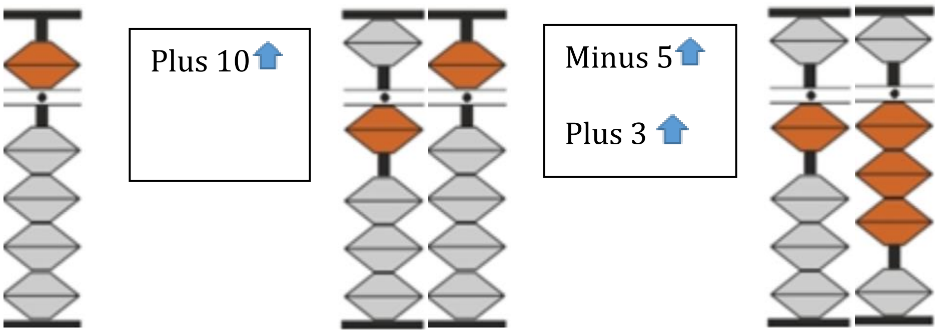
Some calculations, including some single digit additions, require the operation of two sets of CNs simultaneously. Figure 4 gives an example. Here the requirement

Figure 4

Calculation with two sets of complementary numbers (2CN)

5+8 = 13

Type the number 5. You cannot add 8 beads directly. 8 has the 'friend' 2 (CN of 10). Therefore, add an earth bead on the next rod (+10) and subtract 2. You cannot subtract 2 directly. 2 has the 'friend' 3 (CN of 5). Therefore subtract the heaven bead and add three earth beads – the answer is 13.



5+(10-2) = 5+(10-5+3) = 8

Likewise, the subtraction 13-8 =13-10+2 = 13- 10+(5-3) = 5

Soon after the introduction of calculation using the physical abacus, learners are encouraged to implement the procedures mentally. This typically occurs around 5-7 years of age, and may be accompanied the use of a ‘paper abacus’ (an image of the abacus, set at zero), as an intermediate form of presentation.

As has been made clear above, the dual-base nature of the soroban abacus, and multiple steps involved in abacus calculation (e.g. add 5 and subtract 2 in order to add 3) make procedural demands on the user which are substantially more complex than count-based systems of instruction. Nonetheless the abacus frame provides a concrete and precise tactile and visual representation of these procedures which is well adapted to the needs of the young learner.

Cognitive underpinnings of the mental abacus

A pioneering small-n study by Stigler (1984) explored the extent to which the 'mental abacus' has psychological reality, taking particular account of the short term memory load entailed in multi-digit operations. Restrictions on mental vs. physical abacus addition increased with the number of digits per addend. This effect was attributed to capacity limitation in visual short-term memory. Observing the process of abacus addition in detail, Stigler (1984) found a dominant error pattern in which responses missed the target by 5 (indicating mental manipulation of heaven beads), was found only amongst participants with abacus experience, providing suggestive evidence to confirm his proposal that the expert abacus user calculates by 'mentally computing complements to 5 and to 10, plotting the necessary bead movements, and planning the finger movements that will realize the bead movements '(Stigler 1984, p. 105).

Further studies have explored the nature of memory function in abacus users. Hatano *et al.* (1987) recruited groups of abacus users ranging from novices to experts and gave them spoken digit span tasks with verbal or spatial distraction. More skilful learners were less vulnerable to verbal distractors, and more vulnerable to visual-spatial distractors, suggesting enhanced use of abacus-based mental representations.

Theses studies provide evidence for the reliance of mental abacus calculation on visual coding in memory, and suggest a developmental specialization whereby cognitive systems may be specifically shaped (even enhanced) by abacus training. Participants in the study by Hatano et al. (1987) covered an age range from 9 to 23, but were grouped according to their level of abacus skill. Significant group differences in digit span, favouring higher skill groups, remained after the effect of age was removed, suggesting an effect of training beyond simple maturation. Frank and Barner (2011) further proposed that the particular application of visual-memory systems observed in mental abacus users provides them with a specialized cognitive system for mental arithmetic, a system which is resistant to verbal interference, and is not generally exploited in individuals without abacus experience. Under Frank and Barner's (2011) account, the representation of exact numerosity afforded by the abacus is inconsistent with the involvement of the approximate number system (ANS) proposed by Feigenson et al. (2004) as a core system for representation of number, and widely held to underlie mathematical development through the lifespan (Gilmore et al. 2010; Halberda et al. 2012), but consistent with the involvement of Feigenson's 'Core system 2' for exact representation of small sets of individual items (Feigenson et al. 2004, p.310)). This proposal carries with it a particular developmental implication: basic mental calculation may be served by different and broadly independent cognitive systems according to cultural inputs.

Central to Frank and Barner's (2011) account is the configuration of the soroban abacus, composed of elements (rods) comprising just five physical entities (one heaven bead and four earth beads). The heaven bead is essential to this configuration, largely defining not only the physical structure of the abacus, but also the distinctive use of CN in calculation. However, no study to our knowledge has examined the extent to which this distinctive use of CN operates in mental as well as physical abacus use, nor explored the possibility that developmental changes may occur in these procedures. Stigler (1984) gave a detailed account of intermediate states of the abacus during the performance of multi-digit addition (Stigler, 1984, p. 152). The intermediate states identified included single rod bead manipulations, but Stigler's study does not make explicit the ways in which CN operate within the calculation process. The present study tests the transfer of this key component of physical abacus calculation to mental calculation. We constructed parallel sets of combined addition and subtraction problems, matched on conventional metrics of difficulty level, but differing in the number of CN steps entailed in abacus calculation. The more CN steps involved in a calculation, the more time it should take participants to respond, if indeed they are using mental representation of the abacus to perform the calculation. In Experiment 1a we test mental calculation speed in abacus users at different levels of expertise, in order to examine the possibility that CN use in mental calculation is dependent on extended practice. In Experiment 1b we compare performance of abacus users with that of participants with no abacus training, in order to test the specificity of the effect of CN use.

Experiment 1.

In order to test the hypothesis that CN are employed in mental abacus calculation in a similar way to their operation on the physical abacus, we constructed to two sets of serial mixed addition and subtraction tasks matched for number of additions and subtractions, problem size (Ashcraft, Guillaume & Michelle, 2009) and number of carries (Frank & Barner 2011), but differing in the number of CN operations required in physical abacus calculation. If mental abacus calculation imitates physical abacus calculation, then the set of tasks entailing more CNs should be more demanding and time consuming. We test two groups of abacus learners differing in expertise, in order to examine the extent to which the use of CNs in mental abacus depends on skill level (Experiment 1a). In order to confirm the specificity of any effect of CNs on performance, we go on to compare the performance of abacus users with a group of individuals with no exposure to the abacus (Experiment 1b).

Experiment 1a

Participants

The participants in this experiment were 200 Taiwanese children between the ages of four and twelve years old. The participants were drawn from three year groups: pre-P1 (pre-primary school) 3-6 years olds; P1-P3 7-10 year olds, and P4-P6 11 to 15 year olds. Mean age and gender by year group are shown in Table 1. The participants were recruited through local abacus organizations and attended ten different afterschool abacus programmes in the Greater Taipei area.

Table 1
Gender and age of participants by year group.

	Year Groups	N	Male	Female	Mean Age	Age SD
	Before P1	21	9	12	5.62	0.49
	P1-P3	142	77	65	8.23	1.02
	P4-P6	37	15	22	11.70	0.74
Total		200	100	100	8.60	1.92

Each child was graded by the abacus programme according to the level of expertise. For the purpose of the present study, children with abacus knowledge between Grades (Ji) 12~1 were designated ‘abacus users’; children at Level (Duan) 1 and above were designated ‘abacus experts’. Abacus users were drawn from all year groups. Abacus experts were evenly distributed between P1-P3 – 7-10 year olds, and P5-P6 – 15 year olds. No experts came from the pre-primary school group (see Table 2, below)..

Table 2
Skill level of participants by year group.

	Pre P1	P1-P3	P4-P6	Total
Abacus Users	21	133	26	180
Abacus Experts	0	9	11	20
Total	21	142	37	200

Materials

The materials consisted of two matched sets of ten serial single-digit mixed addition and subtraction problems (trials). Each trial entailed four addition or subtraction calculations, giving a total of forty calculations per set. Each set included seven subtractions (see Appendix A for full details). Sets were balanced on conventional metrics of difficulty as follows: Problem size defined as sum of solutions was 191 for Set A and 194 for Set B; Absolute problem size (sum of addends and subtrahends) was 242 for Set A and 252 for Set B; Total of place value conversions (carrying in addition and borrowing in subtraction) was 19 for Set A and 21 for Set B. Differentiation of sets according to number of CN operations was achieved as follows: Set A contained no operations with 2C, 24 operations with 1CN, 16 operations with no CN; Set B contained 4 operations of 2CN, 24 operations of 1CN, 12 operations with no CN. Problems were printed on A4 paper in two columns (Set A to the right and Set B to the left for half the participants, vice versa for the remaining participants). See Appendix B for full layout.

Procedure

Participants were tested in groups of up to 8 in their own classrooms. No abacus frames were available to the participants. Individual A4 test sheets were placed face down on their tables. When the experimenter said 'start', they turned the test sheet over and completed all 10 questions in the right hand column. Half the group received test sheets with Set A as the right-hand, column, and the other half of the class received test sheets with Set B as the right-hand, column. When the participants finished the right-hand, column, they were instructed to put their hands up straight away so that the experimenter could record their finishing time accurately, using a lap-timer. Each participant was given an order number that corresponded to their 'lap' time. After the group had completed the column, individual completion times were recorded. The entire class then moved on to repeat the steps to complete the left-hand column.

Results

200 Taiwanese abacus learners completed the task, with mean accuracy 85.30% (S.D. 0.25) for Set A, and 85.05 (S.D. 0.22) for Set B. Individual trial response times were not recorded. In order that analysis of response time per set of problems should be meaningful, a criterion of 80% accuracy was established. One hundred and forty six participants met this criterion. Mean accuracy and response times for Experts and Users meeting accuracy criterion are shown in Table 3.

Table 3
Accuracy and response times by problem set and group for participants meeting accuracy criterion.

	Experts (n=20)		Users (n=126)	
	Set A	**Set B**	**Set A**	**Set B**
Mean Accuracy %	96.50	95.50	95.08	95.32
(SD)		(6.8)	(7.0)	(6.8)

	(6.7)			
Mean RT secs.	26.29	37.32	64.39	71.21
(SD)	(9.72)	(16.16)	(39.37)	(40.83)

A two-way mixed ANOVA was conducted in order to examine the effects of problem set (A vs B, within subjects) and group (Experts vs. Users, between subjects) on time taken to complete the task. The effect of problem set was significant with Set B taking longer than Set A ($F(1,144)=14.14$, $p<0.01$). The effect of group was significant, with experts performing faster than users ($F(1, 144)= 16.80$, $p<0.01$). The interaction was not significant ($F(1, 144)= 0.66$, $p=0.42$).

Experiment 1b

In order to test the specificity of the effect of problem set found in Experiment 1a, data were gathered from a group of children educated in the English school system who had no exposure to the abacus, and whose training in arithmetic was based on Arabic symbols. We made no attempt to control differences in culture and experience between the Taiwanese and English groups. The English sample was broadly matched in years of compulsory education with the Taiwanese Expert group. The year groups from which the English sample were drawn ensured that they had experience of performing the sort of mental arithmetic required by our calculation task. By recruiting only volunteers, and explaining the nature of the task in advance, we attempted to ensure that the English participants were confident in their performance of the tasks.

Participants

Twenty children, similar in years of compulsory education to the Taiwanese abacus experts, were recruited as volunteers at the homework club of a Central London school. None had any exposure to abacus calculation. This group was designated 'non-users'. All performed the calculation task as described above (Experiment 1a), and all reached the 80% accuracy criterion.

Results

Gender and mean age, accuracy levels and response times for each set of problems for each group are shown in Table 4, below.

Table 4
Gender and mean age, accuracy levels and response times for each set of problems for experts and non-users.

	Expert (n=20)		Non-Users (n=20)	
Gender	M:10 F:10		M:11 F:9	
Mean Age (years)	10.65		12.18	
	Set A	**Set B**	**Set A**	**Set B**
Mean Accuracy	96.50%	95.50%	92.73%	91.36%
Mean Response Time	26.29 sec	37.32 sec	143.72 sec	144.00 sec

A two-way mixed ANOVA was conducted in order to examine the effects of problem set (A vs B, within subjects) and group (experts vs. users, between subjects) on time taken to complete the task. The effect of problem set was significant with Set B taking longer (F(1,40)=6.23, p=0.017). The effect of group was significant with experts performing faster than the non-users (F(1, 40)=87.66, p<0.001,). The interaction was significant (F(1, 40)= 5.63, *p*= 0.023). The experts showed a time advantage for Set B compared with Set A (t(19)=-3.43, p=0.003); no such effect was found for the non-users (t(21)=-0.87, p=0.93).

Discussion

A number of previous studies have produced convergent evidence supporting the view that the remarkable calculation skills of abacus users might be attributable to the creation of mental images of the abacus (Stigler 1984), supported by visual memory systems (Hatano et al 1987), and conforming to the particular configuration of the soroban abacus (Frank and Barner 2011). However, no study to our knowledge had examined the possible effects of the unique combination of quinary (heaven bead) and decimal (rod) bases which define the operation of the soroban abacus. Through classroom observation we examined the workings of the dual base system as taught to young children, and the key role played by sets of complementary numbers. We noted striking differences from the practices of basic calculation in classrooms where Arabic numerals and counting provide the basis for instruction and learning.

We sought to examine whether the properties of the physical abacus, and, in particular, the complex exploitation of complementary numbers it requires, may be found to operate in mental as well as physical abacus calculation. We also sought to establish whether such apparently sophisticated procedures might be observable not only in expert users, but also in younger less experienced users. The serial mixed addition/subtraction task we employed was derived from abacus instruction materials, reflecting the practice of simultaneous teaching of addition and subtraction. We were able to compile parallel sets of such problems which were comparable in their levels of difficulty on the conventional metrics of problem size and number of carries, but which differed in the specific requirements of abacus calculation. One set of problems contained four examples of operations entailing repeated application of the complementary number procedure (2CN); the other set contained none. In line with our hypothesis, the set containing 2CN problems took significantly longer for abacus-trained participants to complete. Furthermore, while experts were faster than non-expert users, we found no evidence of change in the CN effect with expertise. Two important implications may be drawn from these findings. First, we have shown that complex CN operations play a role in mental calculation in individuals trained to use mental representations of the soroban abacus. In this way we have operationalized theoretical proposals made by Stigler (1984) and provided experimental evidence of the importance of particular procedures associated with the combined quinary and decimal properties of the abacus. Second, we have shown that such operations are entailed in mental calculation in younger experienced users as well as experts. This is a surprising finding, somewhat opposed to the finding of Hatano et al. (1987) that enhancement of visual memory processes (underlying mental abacus use) is driven by expertise. However, our finding focuses particularly on mental calculation procedure, and is in line with our observation that children as young as three or four are exposed to complex CN-based procedures on the physical abacus.

Some concern might be raised about the specificity of our findings. Had we sufficiently controlled factors in our problem sets which might produce similar findings for reasons other than CN-based complexity? Our observations of abacus instruction and learning led us to propose that the procedures entailed are substantially different from those used in educational approaches based on Arabic numerals and counting. This allowed us an opportunity to test the specificity of our findings by comparing abacus-trained calculators to Arabic-trained calculators. In Experiment 1b we recruited a volunteer sample of participants trained in the English school system, matched them broadly for years of compulsory education with our Taiwanese abacus experts, and compared performance of the two groups on the calculation task. Sampling in this way we were able to compare groups performing at similar levels of accuracy on our tasks. There was a great difference between groups in speed of performance. However, the more important finding from this experiment is the interaction between group and problem set. No trace of an effect of CN complexity was found in the English group, but the effect in the Taiwanese group was significant. This confirmed (a) the matching of problem sets on criteria of problem size and number of carries and (b) the specificity of the CN complexity effect to abacus-trained participants.

Given this confirmation we suggest that the phenomenon of mental abacus calculation can be characterized not only by its particular adaptation to properties of the visual memory system (Frank and Barner 2011), but also by its exploitation of combined quinary and decimal bases. We have observed 'costs' in terms of time taken for mental calculation where exchanges in both bases are required. Noting the early stage at which instruction in the use of CN in addition and subtraction occurs, we suggest that very early exposure to these procedures may facilitate not only efficiency but also conceptual understanding. In a seminal paper Siegler, Rittle-Johnson and Alibali (2001), proposed that procedural and conceptual learning in mathematics proceed in iterative fashion. Their study of fraction learning in 11 year olds in the U.S. found prediction from children's procedural skills to their conceptual knowledge, and vice versa; importantly, prediction was shown to be mediated by the accuracy of children's representations of fractions. Based on these principles it is possible that early learning of the procedural relation between addition and subtraction as represented on the abacus, affords enhanced conceptual understanding. Altogether we propose that the procedures underlying physical abacuscalculation are transferred to mental abacus calculation, and that the high efficiency of the process may be attributable, in part at least, to the particular combination of quinary and decimal bases used.

Conclusion

We have shown that the procedural complexity required in calculation using the dual-base soroban abacus is transferred from physical to mental calculation, in line with the proposal of a 'mental abacus'. We argue that the particular adaptation of the soroban to visual memory systems (Frank and Barner, 2011), is dependent on the conciseness of the soroban's dual base structure, and that early exposure to dual base procedures may contribute to the remarkable efficiency observed in mental abacus calculation. The developmental trajectory underlying our observations would appear to be striking in its divergence from the pathways of conceptual and procedural development widely observed in children whose arithmetic instruction is based on counting and Arabic numerals.

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Appendix A

Stimuli for Calculation Task

Abacus Calculation Task 珠算行不行

	Set A	Ans 答案	Set B	Ans 答案
1 PS 36	9-7+8-4+8= 9-7=2 no cn 2+8=10 1cn 10-4= 6 1cn 6+8= 14 2cn	14 1x2 2x1 1x0	7+5+9+8+1= 7+5= 12 1cn 12+9=21 1cn 21+8=29 no cn 29+1= 30 1cn	30 3x1 1x0 Ps30
2 PS 28	8+1+2+9+8= 8+1=9 no cn 9+2=11 ! cn 11+9= 20 1 cn 20+8= 28 no cn	28 2x1 2x0	5+4+8-4-4= 5+4=9 no cn 9+8=17 1cn 17-4=13 1cn 13-4= 9 1cn	9 3x1 1x0 Ps25
3 PS 22	9-1+4+5-3= 9-1=8 no cn 8+4= 12 ! cn 12+3= 15 1cn 15-3= 12 1 cn	14 3x1 1x0	6+9+7+4+2= 6+9=15 1cn 15+7= 22 1cn 22+4= 26 1cn 26+2=28 no cn	28 3x1 1x0 Ps28
4 PS 25	8+4-5+1+7= 8+4= 12 !cn 12-5= 7 1cn 7+1=8 no cn 8+7= 15 1 cn	15 3x1 1x0	1+2+9+8+3= 1+2=3 no cn 3+9=12 1cn 12+8=20 1cn 20+3= 23 no cn	23 2x1 2x0 Ps23
5 PS 20	9+3+2+3+3= 9+3= 12 1cn 12+2= 14 no cn 14+3= 17 1cn 17+3= 20 1cn	20 3x1 1x0	3+6+3+1+2= 3+6=9 no cn 9+3=12 1cn 12+1=13 no cn 13+2= 15 1cn	15 2x1 2x0 Ps15
6 PS 26	2+7+6+8+3= 2+7=9 no cn 9+6= 15 1cn 15+8= 23 2cn 23+3= 26 1cn	26 1x2 2x1 1x0	8+5+9-8+6= 8+5=13 1cn 13+9=22 1cn 22-8=14 1cn 14+6=20 1cn	20 4x1 Ps36
7 PS 20	6+6+1-4+3= 6+6=12 1cn 12+1= 13 no cn 13-4=9 1cn 9+3=12 1cn	12 3x1 1x0	8+3+7+1+4= 8+3=11 1cn 11+7=18 no cn 18+1=19 no cn 19+4=23 1cn	23 2x1 2x0 Ps23

8 PS 19	3+8+1-4+3= 3+8=11 1cn 11+1=12 no cn 12-4=8 1cn 8+3= 11 1cn	11 3x1 1x0	6+9+4+8-5= 6+9=15 1cn 15+4=19 no cn 19+8= 27 1cn 27-5=22 no cn	22 2x1 2x0 Ps32
9 PS 23	9+2+5+4+3= 9+2=11 2 cn 11+5= 16 no cn 16+4= 20 1cn 20+3= 23 no cn	23 1x2 1x1 2x0	3-2+4-4+8= 3-2=1 no cn 1+4=5 1 cn 5-4=1 1 cn 1+8=9 no cn	9 2x1 2x0 Ps21
10 PS 33	5+8+5+7+8= 5+8=13 2 cn 13+5= 18 no cn 18+7= 25 1 cn 25+8= 33 1cn	33 1x2 2x1 1x0	9+9-5-3-2= 9+9=18 1cn 18-5=13 no cn 13-3=10 no cn 10+2=12 no cn	12 1x1 3x0 Ps28

CNs Set A 4x2, 24x1, 12x0 Set B 24x1, 16x0

Absolute Problem Size Set A 252 Set B 242

Some of solutions Problem Size Set A 194 Set B 191

Total carries/borrows A=21 B=19

Appendix B

Calculation Task Question Sheet

	2C	Ans 答案	1C	Ans 答案
1	9-7+8-4+8=		7+5+9+8+1=	
2	8+1+2+9+8=		5+4+8-4-4=	
3	9-1+4+3-3=		6+9+7+4+2=	
4	8+4-5+1+7=		1+2+9+8+3=	
5	9+3+2+3+3=		3+6+3+1+2=	
6	2+7+6+8+3=		8+5+9-8+6=	
7	6+6+1-4+3=		8+3+7+1+4=	
8	3+8+1-4+3=		6+9+4+8-5=	
9	9+2+5+4+3=		3-2+4-4+8=	
10	5+8+5+7+8=		9+9-5-3+2=	
Time 時間				