Distributed Fault Detection and Isolation for Interconnected Systems: a Non-Asymptotic Kernel-Based Approach

Peng Li * **Francesca Boem** * **Gilberto Pin** ** **Thomas Parisini** ***

* *Dept. of Electrical and Electronic Engineering, Imperial College London, UK (peng.li13, f.boem@imperial.ac.uk)*
** *Electrolux Professional S.p.A., Italy (gilberto.pin@electrolux.it).*
*** *Dept. of Electrical and Electronic Engineering, Imperial College London, UK and Dept. of Engineering and Architecture, University of Trieste, Italy (t.parisini@gmail.com)*

Abstract: In this paper, a novel framework is proposed for deadbeat distributed Fault Detection and Isolation (FDI) of large-scale continuous-time LTI dynamic systems. The monitored system is composed of several subsystems which are linearly interconnected with unknown parameterization. Each subsystem is monitored by a local diagnoser based on the measured local output, local inputs and the interconnection variables from the neighboring subsystems. The local FDI decision is based on two non-asymptotic state-parameter estimators using Volterra integral operators which eliminate the effect of the unknown initial conditions so that the estimates converge to the true value in a deadbeat manner and therefore the fault diagnosis can be achieved in finite time. Moreover, the unknown interconnection parameters and the unknown fault parameters are simultaneously estimated. Numerical examples are included to show the effectiveness of the proposed FDI architecture.

1. INTRODUCTION

Among the fault diagnosis literature, major efforts have been devoted to centralized FDI algorithms (see Patton et al. (1989) and Blanke et al. (2006) for preliminaries and typical FDI techniques). However, with the rapid development of the modern technology, the complexity of the systems in terms of order and scale is continuously increasing. Moreover, the development of the communication networks gives rise to the realization of distributed and networked systems, where centralized approaches are not always feasible nor reliable. In the recent years, distributed model-based FDI schemes have been proposed for continuous and discrete-time systems (e.g. Hassan et al. (1992); Shames et al. (2011); Stankovic et al. (2010); Zhang and Zhang (2012); Keliris et al. (2015); Reppa et al. (2015); Boem et al. (2017)). When considering interconnected systems, the effect of the neighboring subsystems on the local model could be unknown or modeled as a function with unknown parameters. Some FDI methodologies consider this issue using adaptive approximation methods (see Boem et al. (2011)); some works consider bounded interconnections (Riverso et al. (2016)). However, faster and more accurate FDI schemes are the goal to pursue in order to guarantee a prompt fault accommodation.

In this paper, a novel non-asymptotic distributed FDI architecture is proposed for LTI linearly interconnected subsystems with unknown interconnection parameters, by developing two deadbeat joint parameters-state estimators. In fact, non-asymptotic estimation methodologies have been proposed in the recent literature (Haskara (1998); Mboup (2009); Fedele and Coluccio (2010)), where the convergence can be obtained in arbitrarily short time. As demonstrated in Pin et al. (2016), deadbeat parametric estimation can be achieved for continuous-time LTI dynamic systems with a class of Bivariate Causal Non-asymptotic Kernels, therein defined. In Pin et al. (2013), a non-asymptotic state observer for continuous-time SISO linear systems with a newly defined Bivariate Feed-through Non-asymptotic Kernels (BF-NK) is proposed and shown to have very fast convergence rate. The kernel-based deadbeat approaches, being superior in convergence rate and accuracy, have found applications for practical problems, e.g. localization (Li et al. (2016a)), parameter estimation for sinusoidal signals(Li et al. (2016b)).

In this paper, the deadbeat observer introduced in Pin et al. (2013) is extended to jointly estimate the local state of the interconnected subsystems and the parameters of the unknown interconnection, in a distributed scenario. Similarly, a second estimator is designed, able to estimate after fault detection both the state of the faulty subsystem and the parameters of the unknown fault function. Remarkably, with a set of BFNK functions, the estimators enjoy a non-asymptotic feature, thus converging to the true value in finite time, significantly reducing the fault diagnosis time with respect to asymptotic observer-based methods. To the authors' knowledge, this represents the first contribution in the literature dealing with the fault detection and isolation problem using the non-asymptotic kernel-based estimation. A specific PE condition is introduced providing guidance to the system decomposition. The proposed FDI architecture has been tested in a numerical example where its effectiveness is evaluated.

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider a large-scale system $\mathcal{S}$, composed of N MISO LTI subsystems, in which the I-th subsystem $\mathcal{S}_I$ is modeled as

$$\begin{cases} x_I^{(1)}(t) = A_{II}x_I(t) + B_I u_I(t) + \sum\limits_{J\in\mathcal{N}_I} A_{IJ}y_J(t) + f_I(t, y_I, u_I, w_I), \\ y_I(t) = c_I^\top x_I(t), \end{cases} \tag{1}$$

with $x_I(t) \in \mathbb{R}^{n_I}$, $y_I(t) \in \mathbb{R}$, $u_I(t) \in \mathbb{R}^{m_I}$ denoting the state vector, the output and the input vector of the i-th subsystem respectively. $A_{II} \in \mathbb{R}^{n_I \times n_I}$, $B_i \in \mathbb{R}^{n_I \times m_I}$ and $c_I \in \mathbb{R}^{n_I}$ are assumed to be known. The term $\sum_{J \in \mathcal{N}_I} A_{IJ} y_J(t)$ represents the interconnection with the neighboring subsystems, where $\mathcal{N}_I$ denotes the index set of p_I interconnected subsystems affecting the dynamics of I-th subsystem and the interconnection parameter vector $A_{IJ} \in \mathbb{R}^{n_I}$ is unknown. The local fault function vector $f_I(t, y_I, u_I, w_I) \in \mathbb{R}^{n_I}$ is modeled as

$$f_I(t, y_I, u_I, w_I) = \mathcal{B}(t - T_{0,I})\phi_I(y_I, u_I, w_I),$$

where $\mathcal{B}(t - T_{0,I})$ defines the time profile of the local fault that occurs at an unknown time instant $T_{0,I}$:

$$\mathcal{B}(t - T_{0,I}) \triangleq \begin{cases} 0 & \text{if } t < T_{0,I} \\ 1 & \text{if } t \geq T_{0,I} \end{cases}. \qquad (2)$$

The structural function of the local fault is linearly parameterized as

$$\phi_I(y_I, u_I, w_I) \triangleq \theta_I^\top g_I(y_I, u_I, w_I),$$

where $w_I \in \mathbb{R}^{p_I} \triangleq col\{y_J : J \in \mathcal{N}_I\}$ [1] collects the interconnection variables and the parameter vector $\theta_I \in \mathbb{R}^{n_I}$ is assumed to be unknown. $g_I(\cdot,\cdot,\cdot) : \mathbb{R}^{n_I} \times \mathbb{R}^{m_I} \times \mathbb{R}^{p_I} \to \mathbb{R}$ is a scalar function representing the functional structure of the effect of the fault on the I-th subsystem. For fault isolation purposes, for each subsystem a set $\mathcal{F}_I$ is defined containing all the $N_{\mathcal{F}_I}$ possible fault functions, i.e. $g_{Ip}(y_I, u_I, w_I)$, $p \in \{0, \ldots, N_{\mathcal{F}_I} - 1\}$, affecting $\mathcal{S}_I$.

Remark 2.1. For the sake of notational simplicity, here we consider the scalar fault function case. It is anyway trivial to extend the algorithm for vector fault functions.

The following assumption is needed:

Assumption 1. The pair $(A_{II}, c_I^\top)$ is completely observable.

The proposed Distributed Fault Detection and Isolation (DFDI) architecture consists of N Local Fault Diagnosing Agents (LFDAs), monitoring the health status of the corresponding subsystem. Each LFDA is equipped with two estimators. The first is called Fault Detection and Parameter Estimator (FDPE). It is based on the local nominal model (1) and, making use of a modified version of the kernel-based deadbeat observer inspired by Pin et al. (2013), it estimates both the local state x_I and the interconnection parameters A_{IJ}. The difference between the local output and the estimated output allows the detection of the fault. The second estimator is called fault isolation estimator (FIE); it is activated after fault detection and allows to identify the type of fault occurring in the subsystem by estimating the fault parameters θ_I and in the mean time providing the state estimates in the faulty scenario. Both estimators make use of the algebra of the linear integral Volterra operator.

3. DISTRIBUTED FAULT DETECTION AND ISOLATION

3.1 Fault detection and parameter estimation scheme

By exploiting the deadbeat SISO observer proposed in Pin et al. (2013), in this subsection, we develop a deadbeat state-parameter joint estimator for MISO systems. Based on the nominal model (1) and on the available data, i.e. $y_I(t)$, $u_I(t)$ and $y_J(t)$, $J \in \mathcal{N}_I$, and treating the interconnection function term as an additional input of the subsystem, the FDPE

- estimates the state variables of the MISO subsystem;
- estimates the unknown interconnection A_{IJ}.

Thanks to the observability of $(A_{II}, c_I^\top)$, the state vector $x_I(t)$ admits a linear coordinates transformation $z_I(t) = Tx_I(t)$ such that the equivalent LTI system with respect to $z_I(t)$ can be rewritten in the observer canonical form:

$$\begin{cases} z_I^{(1)}(t) = \bar{A}_{II} z_I(t) + TB_I u_I(t) + \sum_{J \in \mathcal{N}_I} TA_{IJ} y_J(t), \\ y_I(t) = \bar{c}_I^\top z_I(t), \end{cases} \qquad (3)$$

where $\bar{c}_I^\top = c_I^\top T^{-1} = [\,1\ 0\ \ldots\ 0\,]$,

$$\bar{A}_{II} = TA_{II}T^{-1} = \begin{bmatrix} a_{n_I-1} & 1 & 0 & \cdots & 0 \\ a_{n_I-2} & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ a_1 & 0 & \ldots & 0 & 1 \\ a_0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

and where $(-a_0, -a_1, \ldots, -a_{n_I-1})$ are the coefficients of the characteristic polynomial of the subsystems determined by the eigenvalues of matrix A_{II}. We define

$$\begin{aligned} TB_I &= [b_{n_I-1}, b_{n_I-2}, \ldots, b_0]^\top \\ TA_{IJ} &= [\alpha_{J,n_I-1}, \alpha_{J,n_I-2}, \ldots, \alpha_{J,0}]^\top,\ J \in \mathcal{N}_I. \end{aligned}$$

With the above parameter definitions, (3) admits the following I/O model

$$y^{(n)}(t) = \sum_{i=0}^{n_I-1} a_i y^{(i)}(t) + \sum_{i=0}^{n_I-1} b_i u^{(i)}(t) + \sum_{J \in \mathcal{N}_I} \sum_{i=0}^{n_I-1} \alpha_{J,i} y_J^{(i)}(t). \qquad (4)$$

For $r \in \{0, \ldots, n_I - 1\}$ the r-th element of the state of the realization (3) can be expressed as

$$\begin{aligned} z_{I,r}(t) = y_I^{(r)}(t) &- \sum_{j=0}^{r-1} a_{n_I-r+j} y_I^{(j)}(t) - \sum_{j=0}^{r-1} b_{n_I-r+j} u_I^{(j)}(t) \\ &- \sum_{J \in \mathcal{N}_I} \sum_{j=0}^{r-1} \alpha_{J,n_I-r+j} y_J^{(j)}(t), \end{aligned}$$

where $\sum_{j=0}^{k}\{\cdot\} = 0$, for $k < 0$.

Let us consider a n_I-th order non-asymptotic kernel $K(t,\tau)$ verifying the condition $K^{(i)}(t, 0) = 0, \forall i \in \{0, \ldots, n_I - 1\}$ [Pin et al. (2013)]. Applying the Volterra operator [2] V_K induced by $K(t,\tau)$ to the output signal $y_I(t)$ and its derivatives, we have

$$[V_K y_I^{(i)}](t) \triangleq \int_0^t K(t,\tau) y_I^{(i)}(\tau) d\tau, \forall i \in \{0, \ldots, n_I - 1\}, \qquad (5)$$

which, thanks to the integral by parts and the non-asymptotic condition, can be rearranged as

$$\begin{aligned} \left[V_K y_I^{(i)}\right](t) = &\sum_{j=0}^{i-1} (-1)^{i-j-1} y_I^{(j)}(t) K^{(i-j-1)}(t,t) \\ &+ (-1)^i \left[V_{K^{(i)}}\, y_I\right](t) \end{aligned} \qquad (6)$$

and similarly for $u_I^{(i)}(t)$ and $y_J^{(i)}(t)$. Let us consider the $i = 1$ case; we have

$$\left[V_{K^{(1)}}\, y_I\right](t) = y_I(t) K(t,t) - \left[V_K y_I^{(1)}\right](t). \qquad (7)$$

Replacing $y_I(t)$ with $y_I^{(n_I-1)}(t)$, (7) becomes

[1] The notations $col\{\}$ and $row\{\}$ are defined as $col\{v_1, v_2\} = [v_1^\top\ v_2^\top]^\top$ and $row\{v_1, v_2\} = [v_1, v_2]$ for vectors , and as $col\{A, B, C\} = [A^\top\ B^\top\ C^\top]^\top$ and $row\{A, B, C\} = [A\ B\ C]$ for matrices with compatible dimension.

[2] For details about Volterra algebra, see Pin et al. (2016) and the reference therein.

$$\left[V_{K^{(1)}}\, y_I{}^{(n_I-1)}\right](t) = y_I^{(n_I-1)}(t)K(t,t) - \left[V_K y_I^{(n_I)}\right](t),$$

which is equivalent to

$$\begin{aligned}&(-1)^{n_I-1}\left[V_{K^{(n_I)}}\, y_I\right](t) = \\ &-\sum_{j=0}^{n_I-2}(-1)^{n_I-2-j}y^{(j)}(t)K^{(n_I-j-1)}(t,t) + y^{(n_I-1)}(t)K(t,t) \\ &-\sum_{i=0}^{n_I-1}a_i\left[V_K y_I^{(i)}\right](t)-\sum_{i=0}^{n_I-1}b_{I,i}\left[V_K u_I^{(i)}\right](t)-\sum_{J\in\mathcal{N}_I}\sum_{i=0}^{n_I-1}\alpha_{J,i}\left[V_K y_J^{(i)}\right](t)\end{aligned} \tag{8}$$

thanks to (4). By substituting (6) and its counterpart for $\left[V_K u_I^{(i)}\right](t)$ into (8), after some algebra, we have

$$\begin{aligned}&(-1)^{n_I-1}\left[V_{K^{(n_I)}}\, y_I\right](t) + \sum_{i=0}^{n_I-1} a_i(-1)^i\left[V_{K^{(i)}}y_I\right](t) \\ &+\sum_{i=0}^{n_I-1}(-1)^i b_i\left[V_{K^{(i)}}u_I\right](t) = -\sum_{J\in\mathcal{N}_I}\sum_{i=0}^{n_I-1}(-1)^i\alpha_{J,i}\left[V_{K^{(i)}}y_J\right](t) \\ &+\sum_{r=0}^{n_I-1}(-1)^{n_I-r-1}K^{(n_I-r-1)}(t,t)z_r(t),\end{aligned} \tag{9}$$

where the state variables $z_r(t)$ and the unknown interconnection parameters $\alpha_{J,i}$, $J \in \mathcal{N}_I, i \in \{0,\ldots,n_I-1\}$ appear linearly parameterized on the right hand side. Define

$$\begin{aligned}\mu_I(t) \triangleq\, &(-1)^{n_I-1}\left[V_{K^{(n_I)}}\, y_I\right](t) + \sum_{i=0}^{n_I-1} a_i(-1)^i\left[V_{K^{(i)}}y_I\right](t) \\ &+\sum_{i=0}^{n_I-1}(-1)^i b_i\left[V_{K^{(i)}}u_I\right](t)\end{aligned} \tag{10}$$

$$\gamma_I(t) \triangleq \Big[-\eta_I^\top(t), (-1)^{n_I-1}K^{(n_I-1)}(t,t),\ldots,K(t,t)\Big] \tag{11}$$

where

$$\eta_I(t) \triangleq col\left\{\Big[(-1)^{n_I-1}\big[V_K^{(n_I-1)}y_J\big](t),\ldots,\big[V_K y_J\big](t)\Big]^\top, J\in\mathcal{N}_I\right\}.$$

Therefore, (9) can be rewritten as

$$\mu_I(t) = \gamma_I(t)\begin{bmatrix}\lambda_I\\ z_I(t)\end{bmatrix}, \tag{12}$$

where $\lambda_I \in \mathbb{R}^{n_I p_I} \triangleq col\{TA_{IJ},\, J\in\mathcal{N}_I\}$.

To estimate the unknown elements $[\lambda_I\ z_I(t)]^\top \in \mathbb{R}^{n_I(p_I+1)}$, (12) is augmented into $n_I(p_I+1)$ linearly independent equations by applying $n_I(p_I+1)$ different non-asymptotic kernel functions. Let us consider a set of Bivariate Feedthrough Non-asymptotic Kernel functions (BF-NK) introduced in Pin et al. (2013)

$$K_h(t,\tau) = e^{-\omega_h(t-\tau)}(1-e^{-\bar{\omega}t})^\delta, h\in\{0,\ldots,n_\xi-1\}, \tag{13}$$

containing $n_\xi = n_I(p_I+1)$ kernels parameterized by the same $\bar{\omega}$ but distinct $\omega_0,\ldots\omega_{n-1}$. It is necessary that $\delta \geq n_I$ to guarantee the at least n_I-th order of non-asymptoticity of the kernel functions. For any $h \in \{0,\ldots,n_I(p_I+1)-1\}$, the kernel function verifies

$$\frac{\partial K_h^{(i)}(t,\tau)}{\partial t} = -\omega_h K_h^{(i)}(t,\tau). \tag{14}$$

Eq. (12) can thus be rewritten as

$$\nu_I(t) = \Gamma_I(t)\begin{bmatrix}\lambda\\ z_I(t)\end{bmatrix}, \tag{15}$$

where $\nu_I(t) = \big[\mu_0(t),\mu_1(t),\ldots,\mu_{n_\xi-1}(t)\big]^\top$ and $\Gamma_I(t) = \big[\gamma_0(t)^\top,\gamma_1(t)^\top,\ldots,\gamma_{n_\xi-1}(t)^\top\big]^\top$ with $\mu_h(t)$ and $\gamma_h(t)$ defined in (10) and (11) and induced with the h-th kernel function $h\in\{0,\ldots,n_\xi\}$. For signals y_I, u_I and y_J, the vector of auxiliary signals is defined as

$$\xi_\star(t) \triangleq \big[\xi_{\star,0}(t),\xi_{\star,1}(t),\ldots,\xi_{\star,n_\xi-1}(t)\big]^\top, \tag{16}$$

where $\star$ represents signals y_I, u_I and y_J respectively, with

$$\begin{aligned}\xi_{y_I,h}(t) &\triangleq \Big[[V_{K_h}y_I](t),[V_{K_h^{(1)}}y_I](t),\ldots,[V_{K_h^{(n_I)}}y_I](t)\Big]\\ \xi_{u_I,h}(t) &\triangleq \Big[[V_{K_h}u_I](t),[V_{K_h^{(1)}}u_I](t),[V_{K_h^{(n_I-1)}}u_I](t)\Big]\\ \xi_{y_J,h}(t) &\triangleq \Big[[V_{K_h}y_J](t),[V_{K_h^{(1)}}y_J](t),[V_{K_h^{(n_I-1)}}y_J](t)\Big].\end{aligned}$$

Thanks to the kernel feature (14), the auxiliary signals (16) can be calculated by

$$\begin{cases}\xi_\star^{(1)}(t) &= G_\star\xi_\star(t) + E\star(t)\\ \xi_\star(0) &= 0,\end{cases} \tag{17}$$

where

$$\begin{aligned}G_{y_I} &= blockdiag[G_{1,h},\, h=0,\ldots,n_\xi-1]\\ G_{1,h} &= diag(-\omega_h)\in\mathbb{R}^{(n_I+1)\times(n_I+1)}\\ G_{u_I} &= G_{y_J} = blockdiag[G_{2,h},\, h=0,\ldots,n_\xi-1]\\ G_{2,h} &= diag(-\omega_h)\in\mathbb{R}^{n_I\times n_I}\end{aligned}$$

$$E_{y_I}(t) = \begin{bmatrix}E_{1,0}(t)\\E_{1,1}(t)\\ \vdots\\ E_{1,n_\xi-1}(t)\end{bmatrix}, E_{1,h}(t,t) = \begin{bmatrix}K_h(t,t)\\ \vdots\\ K_h^{(n_I)}(t,t)\end{bmatrix}$$

$$E_{u_I}(t) = E_{y_J}(t) = \begin{bmatrix}E_{2,0}(t)\\E_{2,1}(t)\\ \vdots\\ E_{2,n_\xi-1}(t)\end{bmatrix}, E_{2,h}(t,t) = \begin{bmatrix}K_h(t,t)\\ \vdots\\ K_h^{(n_I-1)}(t,t)\end{bmatrix}$$

In order to guarantee the persistent invertibility of the square matrix $\Gamma_I(t)$ of (15), so to allow the local state-parameters estimation and the detection of possible faults, the following definitions and assumptions are needed.

Definition 3.1. (Sufficient richness (SR) of order n [Kreisselmeier and Rietze-Augst (1990)]) The signal $r(t)$ is said to be sufficiently rich of order n in an arbitrary finite time interval $[t_1,t_2]$ if there does not exist a non-zero vector $q = [q_0,\ldots,q_{n-1}]$ verifying $\sum_{i=0}^{n-1} q_i r^{(i)}(t) = 0, \forall t\in(t_1,t_2)$.

Assumption 2. There exists a time interval (t_1,t_2), with $t_2 > t_1$ such that $\forall J\in\mathcal{N}_I$ the interconnection variables $y_J(t)$ are at least sufficiently rich of order n_I in (t_1,t_2).

Proposition 3.1. (Implication) For LTI autonomous systems $\mathcal{S}_J$, $J\in\mathcal{N}_I$, it is sufficient that the subsystems are at least of order n_I to satisfy Assumption 2 in (t_1,∞). For LTI subsystems with inputs, if the Laplace transform of the output $Y_J(s)$ of the interconnected subsystems $J\in\mathcal{N}_I$ is at least n_I-th order on the denominator, after canceling the poles and the zeros of the transfer function $F_J(s)$ with $U_J(s)$, then Assumption 2 is satisfied in (t_1,∞).

The proof is omitted due to length constraints.

Assumption 3. (Independence of the interconnection variables) There exists a time interval (t_1, t_2), with $t_2 > t_1$ such that the interconnection variables $y_J(t)$, $J \in \mathcal{N}_I$ satisfy the following property: there does not exist a nonzero vector $q = col\{q_{i,J}, i \in \{0, \ldots, n_I - 1\}, J \in \mathcal{N}_I\}$ such that

$$\sum_{J\in\mathcal{N}_I} \sum_{i=0}^{n_I-1} q_{iJ} y_J^{(i)}(t) = 0, \, \forall t \in (t_1, t_2). \tag{18}$$

Remark 3.1. It is worth noting Assumption 3 is not so restrictive. In the case that an interconnection variable $y_J(t)$, $J \in \mathcal{N}_I$ is a linear combination of one (or more) interconnection variables $y_L(t)$, $L \in \mathcal{N}_I$ so that Assumption 3 is not satisfied, but Assumption 2 is, it is anyway possible to estimate the state of the local I-th subsystem, and perform the monitoring activity, by reducing the order of the augmented system (15) and defining a novel interconnection variable which is the sum of the two (or more) not independent original interconnection variables.

Theorem 3.1. (Invertibility and persistency of excitation) Given the output $y_I(t)$, the input $u_I(t)$ and the interconnection variables $y_J(t)$, $J \in \mathcal{N}_I$ verifying Assumptions 2 and 3 in (t_1, t_2), with the designed set of kernel functions (13), the square matrix $\Gamma_I(t)$ is invertible in (t_1, t_2).

Remark 3.2. Proposition 3.1 presents some conditions to guarantee SR in (t_1, ∞). In the case that either Ass. 2 or 3 are not satisfied at some time t, the proposed scheme will detect the singularity of matrix $\Gamma_I(t)$ and freeze the fault detection action.

Having guaranteed the invertibility of $\Gamma_I(t)$ in a time interval (t_1, t_2), the unknown vector can be estimated by

$$\begin{bmatrix} \hat{\lambda}_I \\ \hat{z}_I \end{bmatrix}(t) = \Gamma_I(t)^{-1}\nu_I(t). \tag{19}$$

Therefore, the interconnection parameter vectors A_{IJ}, $J \in \mathcal{N}_I$ and the state variables $z_I(t)$ can be estimated as

$$col\{\hat{A}_{IJ}, \, J \in \mathcal{N}_I\} = T^{-1}\lambda, \tag{20}$$

$$\hat{x}_I(t) = T^{-1}\hat{z}_I(t), \tag{21}$$

and the local estimated output $\hat{y}_I(t)$ can be computed as $\hat{y}_I(t) = c_I^\top \hat{x}_I(t)$.

For fault detection purposes the local output estimation error is then considered: $e_I(t) = |\hat{y}_I(t) - y_I(t)|$. To ensure that the fault can be detected by the FDPE scheme, the following assumption is needed.

Assumption 4. (Detectability) The fault f_I occurs at an unknown time $T_{0,I}$ in the time interval (t_1, t_2), in which we have invertibility of the matrix $\Gamma_I(t)$ and persistency of excitation.

Fault detection decision Under assumption 4, a fault occurring in the I-th subsystem is immediately detected by the proposed FDPE scheme at time $t = T_{d,I}$ (the local fault detection time) when the output estimation error is nonzero, i.e. $e_I(T_{d,I}) \neq 0$.

3.2 Fault isolation scheme

As stated in Section 2, a set $\mathcal{F}_I$ is defined for each subsystem collecting all the fault functions $g_{Ip}(\cdot,\cdot,\cdot)$. Then the main logic of the FIE is to define a linear combination of all the fault functions in $\mathcal{F}_I$ as

$$F_I(y_I, u_I, w_I) \triangleq \sum_{p=0}^{N_{\mathcal{F}_I}-1} \theta_{Ip} g_{Ip}(y_I, u_I, w_I). \tag{22}$$

and estimate the parameter vectors θ_{Ip}, $p \in \mathcal{F}_I$. Once at least one component of the p-th vector θ_{Ip} is not zero, the p-th fault can be isolated.

Similarly as in (3), after the fault time $T_{0,I}$ the model (1) can be rearranged into the observer canonical form

$$\begin{cases} z_I^{(1)}(t) \, = \, \bar{A}_{II} z_I(t) + TB_I u_I(t) + \sum\limits_{J\in\mathcal{N}_I} TA_{IJ} y_J(t) \\ \qquad\qquad + \sum\limits_{p=0}^{N_{\mathcal{F}_I}-1} T\theta_{Ip} f_{Ip}(y_I, u_I, w_I), \\ y_I(t) \quad = \bar{c}_I^\top z_I(t), \end{cases} \tag{23}$$

where variables are defined as in (3) and $T\theta_{Ip} \triangleq [\sigma_{p,0}, \sigma_{p,1}, \ldots, \sigma_{p,n-1}]^\top$. In the following we will omit the dependence on $y_I(t)$, $u_I(t)$ and $w_I(t)$ of $g_{Ip}(y_I, u_I, w_I)$. Similarly as what done for eqs. (3)-(9), we obtain:

$$\begin{aligned} &(-1)^{n_I-1}\big[V_{K^{(n_I)}} y_I\big](t) + \sum_{i=0}^{n_I-1} a_i(-1)^i \big[V_{K^{(i)}} y_I\big](t) \\ &+\sum_{i=0}^{n_I-1}(-1)^i b_i\big[V_{K^{(i)}} u_I\big](t) + \sum_{J\in\mathcal{N}_I}\sum_{i=0}^{n_I-1}(-1)^i \alpha_{J,i}\big[V_{K^{(i)}} y_J\big](t) \\ &=-\sum_{p=0}^{N_{\mathcal{F}_I}-1}\sum_{i=0}^{n_I-1}(-1)^i\sigma_{p,i}\big[V_{K^{(i)}} g_{Ip}\big](t) \\ &\quad+\sum_{r=0}^{n_I-1}(-1)^{n_I-r-1}K^{(n_I-r-1)}(t,t) z_r(t), \end{aligned} \tag{24}$$

where $(N_{\mathcal{F}_I} + 1)n_I$ unknown terms are on the right hand side. Therefore, in the proposed FIE, $n_{\xi_2} \triangleq (N_{\mathcal{F}_I} + 1)n_I$ different kernel functions of the shape of (13) are needed, denoted as $\{K_h(t, \tau), h \in \{0, \ldots, n_{\xi_2} - 1\}\}$, so that (24) can be augmented, yielding to the square algebraic system

$$\varphi_I(t) = \Upsilon_I(t) \begin{bmatrix} \varrho_I \\ z_I(t) \end{bmatrix} \tag{25}$$

where $\varphi_I(t) = [\phi_0(t), \phi_1(t), \ldots, \phi_{n_{\xi_2}-1}]^\top$

$$\begin{aligned} \phi_k(t) \, &= (-1)^{n_I-1}\big[V_{K_k{}^{(n_I)}} y_I\big](t) + \sum_{i=0}^{n_I-1} a_i(-1)^i\big[V_{K_k^{(i)}} y_I\big](t) \\ &+\sum_{i=0}^{n_I-1}(-1)^i b_i\big[V_{K_k^{(i)}} u_I\big](t) + \sum_{J\in\mathcal{N}_I}\sum_{i=0}^{n_I-1}\hat{\alpha}_{J,i}\big[V_{K^{(i)}} y_J\big](t) \\ \varrho_I \, &= col\{T\theta_{Ip}, p \in \{0, \ldots, N_{\mathcal{F}_I} - 1\}\} \\ \Upsilon_I(t) \, &= [\, \Xi_I(t) \;\; \Gamma_3(t) \,] \\ \Xi_I(t) \, &= \big[-\chi_0(t) \;\; -\chi(t) \;\; \ldots \;\; -\chi_{n_{\xi_2}-1}(t)\big]^\top \\ \chi_k(t) \, &= col\Big\{\Big[(-1)^{n_I-1}\big[V_{K_k{}^{(n_I-1)}} g_{Ip}\big](t), \ldots, \\ &\quad -\big[V_{K_k{}^{(1)}} g_{Ip}\big](t), \big[V_{K_k} g_{Ip}\big](t)\Big], p\in\{0, \ldots, N_{\mathcal{F}_I} - 1\}\Big\}, \\ k \, &\in \{0, \ldots, n_{\xi_2} - 1\} \end{aligned}$$

$$\Gamma_3(t) = \begin{bmatrix} (-1)^{n_I-1}K_0^{(n_I-1)}(t,t) & \ldots & K_0(t,t) \\ (-1)^{n_I-1}K_1^{(n_I-1)}(t,t) & \ldots & K_1(t,t) \\ \vdots & & \vdots \\ (-1)^{n_I-1}K_{n_{\xi_2}-1}^{(n_I-1)}(t,t) & \ldots & K_{n_{\xi_2}-1}(t,t) \end{bmatrix}$$

and the unknown parameters $(\alpha_0, \alpha_1, \ldots, \alpha_{n-1})$ are replaced by the estimates $(\hat{\alpha}_0, \hat{\alpha}_1, \ldots, \hat{\alpha}_{n-1})$ obtained by FDPE. The filtered signals $[V_{K_k^{(i)}} \star](t)$ can be calculated by the LTI system (17), where $\star$ represents y_I, u_I, y_J

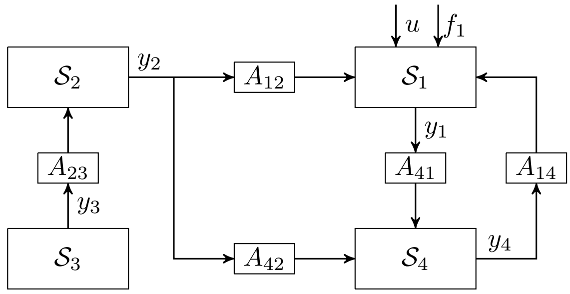

Fig. 1. System block diagram.

and $g_{Ip}, \forall J \in \mathcal{N}_I, p \in \{0,\ldots,N_{\mathcal{F}_I}\}$ with $E_{y_I}(t) = col\{E_{1,h}, h = 0,\ldots,n_{\xi_2}-1\}\}$, $E_{g_{Ip}}(t) = E_{u_I}(t) = E_{y_J}(t) = col\{E_{2,h}, h = 0,\ldots,n_{\xi_2}-1\}$, $G_{y_I} = blockdiag[G_{1,h}, h = 0,\ldots,n_{\xi_2}-1]$ and $G_{g_{Ip}} = G_{u_I} = G_{y_J} = blockdiag[G_{2,h}, h = 0,\ldots,n_{\xi_2}-1]$.

The following assumption is required.

Assumption 5. There exist a time interval $[t_2, t_3]$ with $t_2 > T_{d,I}$ such that $\forall p \in \{0,\ldots,N_{\mathcal{F}_I}-1\}$ the fault functions $g_{Ip}(t)$ are mutually independent (as in Assumption 3) and are at least sufficient rich of order n_I in $[t_2, t_3]$.

Thanks to the results of Theorem 3.1, under Assumption 5, $\Upsilon_I(t)$ is invertible within the interval (t_2, t_3). Therefore, the unknown terms in (24) can be estimated as

$$\begin{bmatrix} \hat{\varrho}_I \\ \hat{z}_I \end{bmatrix}(t) = \Upsilon_I(t)^{-1}\varphi_I(t), \tag{26}$$

The parameters of each fault function and the state variables can be obtained as

$$\begin{aligned} [\hat{\theta}_{I0}, \ldots, \hat{\theta}_{I(N_{\mathcal{F}_I}+1)n_I}] &= T^{-1}\hat{\varrho}(t), \\ \hat{x}_I(t) &= T^{-1}\hat{z}_I(t). \end{aligned} \tag{27}$$

Fault isolation With the proposed FIE (26), the p-th fault is isolated when at least one element of the corresponding parameter vector $\hat{\theta}_{Ip}$ is different from zero.

4. NUMERICAL EXAMPLE

In this section, a numerical example is presented to show the effectiveness of the proposed deadbeat distributed FDI architecture. The simulation is carried out in the Matlab Simulink environment with sampling time $T_s = 10^{-4}s$. We consider a 9-th order LTI system decomposed in 4 subsystems (see Fig. 1):

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + f(t,y,u) \\ y(t) = Cx(t), \quad t \in \mathbb{R}_{\geq 0} \end{cases} \tag{28}$$

where

$$A = \begin{bmatrix} A_{11} & A_{12}c_2^\top & 0 & A_{14}c_4^\top \\ 0 & A_{22} & A_{23}c_3^\top & 0 \\ 0 & 0 & A_{33} & 0 \\ A_{41}c_1^\top & A_{42}c_2^\top & 0 & A_{44} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} c_1^\top & 0 & 0 & 0 \\ 0 & c_2^\top & 0 & 0 \\ 0 & 0 & c_3^\top & 0 \\ 0 & 0 & 0 & c_4^\top \end{bmatrix}, \quad f(t,y,u) = \begin{bmatrix} f_1(t,y,u) \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

with

$$A_{11} = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix}, A_{22} = \begin{bmatrix} -5 & -1 \\ 0 & -4 \end{bmatrix}, A_{33} = \begin{bmatrix} -3 & 1 & -2 \\ -5 & -2 & 0 \\ -1 & 0 & -7 \end{bmatrix}, A_{44} = \begin{bmatrix} -1 & -0.5 \\ -2 & -1 \end{bmatrix},$$

$$b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.5 \\ 2 \end{bmatrix}, A_{14} = \begin{bmatrix} 1 \\ 0.6 \end{bmatrix}, A_{23} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}, A_{41} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, A_{42} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$c_1^\top = \begin{bmatrix} 0.7 & 0.3 \end{bmatrix}, \quad c_2^\top = \begin{bmatrix} 1 & 0 \end{bmatrix}, c_3^\top = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}, c_4^\top = \begin{bmatrix} 5 & 1 \end{bmatrix},$$

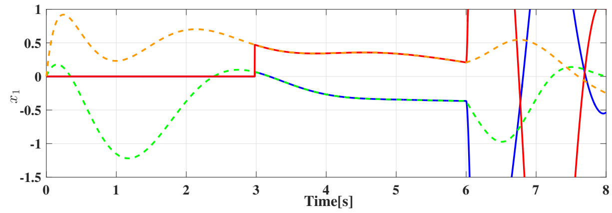

Fig. 2. Local state $x_1(t)$ (dotted lines) and state estimates $\hat{x}_1(t)$ (red and blues lines) by the local FDPE activated at $t = 2.98s$. The estimates are set to zero before the activation time. A fault occurs at $t = 6s$.

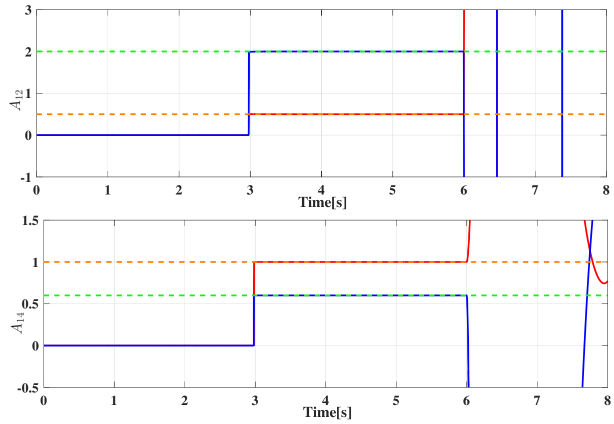

Fig. 3. Interconnection parameters $A_{12}(t)$ and $A_{14}(t)$ (dotted lines) and estimated values $\hat{A}_{12}(t)$ and $\hat{A}_{14}(t)$ (red and blue lines) by the local FDPE activated at $t = 2.98s$. The estimates are set to zero before the activation time. A fault occurs at $t = 6s$.

and $f_1(t,y,u) = \mathcal{B}(t-6)\theta_1\big[\sin(0.5u) + \cos([0\ 0\ 0\ 1]y)\big]$ with $\theta_1 = [-2\ 0.5]^\top$. The initial state vector is $x_0 = [0\ 0\ 5\ 2\ 0\ 2\ -1\ 0\ 0]^\top$. The input is $u(t) = 2\sin(0.5t)$. The fault affects the dynamics of $\mathcal{S}_1$ and we consider a fault set $\mathcal{F}_I = \big\{g_1 = \sin(0.5u) + \cos(y_4),\ g_2 = \sin(y2 + y1) + 1\big\}$. The proposed FDPE is implemented for each subsystem with kernel functions characterized by $\delta = 4$, $\bar{\omega} = 2.5$ and $[\omega_0, \ldots, \omega_5] = [1,2,3,4,5,6]$,

In the simulations, in order to avoid huge overshoot in the initial phase of estimation caused by inverting a small matrix $\Gamma_I(t)$, an activation threshold $\epsilon_{a,I}^{FD}$ is defined such that only when $det(\Gamma_I(t))$ exceeds $\epsilon_{a,I}^{FD}$, the inversion of $\Gamma_I(t)$ is performed, denoting $T_{a,I}^{FD}$ the activation time. For $\mathcal{S}_1$, we set the activation threshold $\epsilon_{a,1}^{FD} = 1.5 \times 10^{-10}$. The local FDPE is activated at $t = 2.98s$ estimating the states $x_1(t)$ (see Fig. 2) and the interconnection parameters A_{12} and A_{14} as illustrated in Fig. 3 in a deadbeat manner. After the occurrence of the fault at $t = 6s$, the estimates of the local FDPE are not accurate anymore. In Fig. 4, we can see that the output estimation error is different than zero and we have immediate fault detection. After fault detection by the local FDPE, the local FIE is switched on to identify the fault parameters using the same kernel set used for the FDPE. After the activation time at $t = 8.78s$, the FIE provides accurate estimations of the fault parameters $\hat{\theta}_{11}(t)$ and $\hat{\theta}_{12}(t)$, as shown in Fig. 5), thus immediately isolating the correct fault function g_1 and excluding g_2. Moreover, the local FIE can simultaneously estimate the local states of the faulty subsystem as illustrated in Fig. 6.

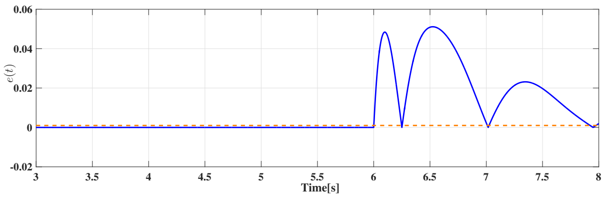

Fig. 4. Output estimation error $e_1(t)$(blue line). A threshold ϵ_1^{FD} (dotted line) is set considering computational accuracy to 0.001. The fault is detected at $t = 6.001s$.

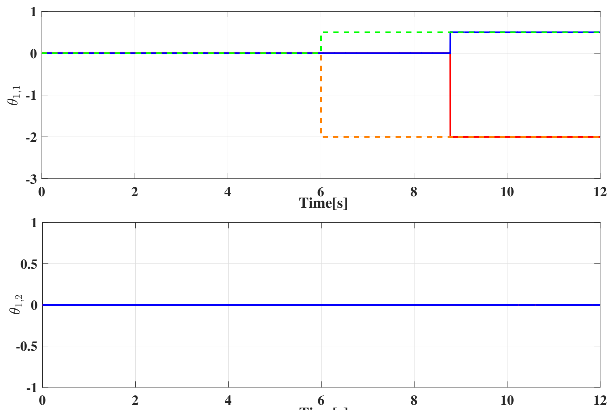

Fig. 5. Estimated fault parameters (red and blues lines) $\hat{\theta}_{11}(t)$ and $\hat{\theta}_{12}(t)$ and the true values (dotted lines). FIE activation time at $t = 8.78s$.

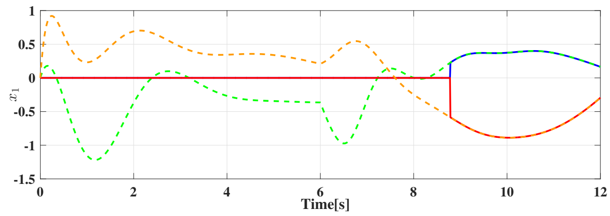

Fig. 6. Local state $x_1(t)$ (dotted lines) and state estimates $\hat{x}_1(t)$ (red and blues lines) computed by the local FIE after activation time $t = 8.78s$. The estimates are set to zero before the activation time.

In the meantime, the local FDPEs of the other subsystems have non-asymptotic convergence on state-parameter estimations. This is not shown due to space constraints.

5. CONCLUDING REMARKS

In this paper, a non-asymptotic distributed FDI scheme for interconnected LTI subsystems with unknown linear parameterization is designed. The unknown interconnection parameters and the states are locally estimated and the fault can be identified within finite time, thus effectively cutting down the diagnosis time and enhancing the reliability of the proposed FDI scheme. As a future work, we will characterize the robustness of the proposed FDI scheme with respect to noise and uncertainty.

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