Ultimate Strength of Cylindrical Shells with Cutouts

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Cutouts – perforations that are often made in wind turbine towers to allow access or passage – can also reduce the towers' ultimate strength. Thus, cutouts may need to be included in the ultimate strength formulations for wind turbine towers as an influential parameter, where significant. The aims of this study are to examine the effects of cutouts on the ultimate-strength characteristics of wind turbine towers and to propose some practical design formulae to predict the reduced ultimate strength. The structural features of cutouts and towers in real wind turbines are investigated, and the effects of different design variables – such as shape, location, aspect ratio, column slenderness ratio, and column aspect ratio – on the ultimate-strength behaviour are described. The ultimate strengths of the towers are computed using elastic–plastic large-deflection finite element analyses. Practical design formulae accommodating a whole range of actual dimensional characteristics of cutouts and towers are derived and proposed. The findings of this research and the proposed design formulae have the potential to enhance the structural design and safety assessment of wind turbine towers.

Keywords: Cutouts, ultimate strength, wind turbine tower, parameters of influence, nonlinear finite element method

Nomenclature

A — Area of the cutout

b — Breadth of the cutout

D (D_{max} / D_{min}) Diameter (maximum/minimum) of the wind turbine tower

E — Elastic modulus of the material

F_R — Reference force of the wind turbine tower

F_{Tower} — Force acting on the wind turbine tower

F_u — Ultimate force of the wind turbine tower

F_{wind} — Thrust induced by blades

F_z — Load in z-axis

h — Height of the cutout

h_o — Distance from the lower end to the centre of the cutout

H — Height of the wind turbine tower

H_S — Height of 1st section

M — Pure bending moment

M_u — Ultimate bending moment

M_P — Plastic bending moment

M_y — Pure bending moment in y-axis

r — Radius

R — Corner radius of the cutout

R^2 — Adjusted R-square

t (t_{max} / t_{min}) Thickness (maximum/minimum) of the wind turbine tower

t_c — Thickness of the cutout

T Torque moment

u_x, u_y, u_z Translational restraints in the x-, y- and z-axis

w_o Initial imperfection

W Weight

α Aspect ratio (height to breadth) of the cutout

β Slenderness ratio (breadth to thickness) of the cutout

δ_{max} Maximum deformation

γ Column aspect ratio (height to diameter) of the wind turbine tower

$\gamma_{max} / \gamma_{min}$ Maximum/minimum column aspect ratio (height to maximum/minimum diameter) of the wind turbine tower

λ Column slenderness ratio (diameter to thickness) of the wind turbine tower

$\lambda_{max} / \lambda_{min}$ Maximum/minimum column slenderness ratio (maximum/minimum diameter to maximum/minimum thickness) of the wind turbine tower

ν Poisson's ratio

θ Angle of the cutout in the circumferential direction

$\theta_x, \theta_y, \theta_z$ Rotational restraint in the x-, y- and z-axis

σ Coefficient of correlation

σ_Y Yield stress of the material

$\xi_D, \xi_t, \xi_h, \xi_b, \xi_C$ Coefficients of design formula for axial compression

$\zeta_D, \zeta_t, \zeta_h, \zeta_b, \zeta_C$ Coefficients of design formula for pure bending

1. Introduction

Cutouts are widely used in steel structures to provide access to or lighten the structure. It is no wonder, then, that these perforations reduce both the buckling and ultimate strength of such structures. In particular, wind turbines, which have relatively large doors, can be exposed to considerable strength reduction that could result in significant structural failure in their towers. It is thus of great importance to develop advanced technologies that can predict a tower's reduced strength using the size of the cutout.

The regulations for designing reliable and safe wind turbines have been developed and recommended by various authorities (ECCS 1980, DIN 18800-4 1990, EN1993-1.6 2006, DNVGL 2013a, DNVGL 2013b). However, there are no detailed guidelines for predicting the reduced strength of towers.

It is noted that there have been far fewer useful research attempts to investigate the effects of cutouts on the structural capacity of circular cylindrical shells, than those investigating plates (Sabir and Chow 1983, Brown and Yettram 1986, Azizian and Roberts 1983, Shangmugam et al. 1999, Durban and Zuckerman 1999, Betten and Shin 2000, El-Sawy et al. 2004, Paik 2007, Kim et al. 2009, Wang et al. 2009). For a couple of decades, a number of studies were conducted to provide buckling analyses of circular cylindrical shells (Brazier 1927, Reissner 1961, Seide and Weingarten 1961, Fabian 1977, Gellin 1980) with cutouts under axial compression (Schenk and Schuëller 2007, Shariati and Rokhi 2010, Ghazijahani et al. 2015) and pure bending (Yeh et al. 1999, Dimopoulos and Gantes 2012, 2013, 2015, Guo et al. 2013, Dimopoulos et al. 2015).

The aims of this study are to use nonlinear finite element analysis to examine the effect of cutouts on the ultimate-strength characteristics and to propose simple formulae to estimate the reduced ultimate strength of wind turbine towers under axial

compression and pure bending. The structural features of wind turbines are investigated

using data collected from 102 wind turbines in service. Finite element modelling

techniques are developed to calculate the ultimate-strength behaviour of the towers with

a variety of design variables, such as cutout shapes and locations, aspect ratios, column

slenderness ratios and column aspect ratios. The developed nonlinear finite element

method modelling is then validated. For the parametric series analyses, a design of

experiment (DOE) method, such as central composite design (CCD), is applied.

Numerical computations are then used to derive a plausible design formula that predicts

the ultimate strength of the towers.

2. Literature Review

In the early days, buckling analyses of circular cylindrical shells subjected to

pure bending and axial compression were first conducted by analytical and experimental

manners. In particular, Brazier (1927) noted that the ultimate strength is directly related

to the ovalisation of the tube cross-section under bending, and thus derived an

expression for the strain energy per unit tube length in terms of the change in axial

curvature. Reissner (1961) further developed a more general formulation for thin-walled

cylindrical shells of arbitrary cross-sections. Seide and Weingarten (1961) used a

modified Donnell equation and the Galerkin method and found that the maximum

elastic buckling stress under bending is equal to the critical compressive stress under

axial compression. Sherman (1976) experimentally identified that shells with a column

slenderness ratio where λ was greater than about 50 did not have sufficient plastic

hinge rotation capacity to develop the classical ultimate strength. Fabian (1977)

observed two modes of failure of infinitely long cylindrical elastic shells subjected to

bending, pressure and axial loads, and found that the circumferential flattening that

constituted an ultimate load and compression wrinkles generated bifurcation buckling axially. Gellin (1980) demonstrated how extending the results of Brazier (1927) into the plastic range confirmed the results of limit states observed by Fabian (1977).

Traditionally, experimental tests have been regarded as the most efficient way of obtaining technical solutions, despite their high costs. Over the past 50 years, computing speeds and the capabilities of numerical tools have improved significantly. Hence, the contributions made by numerical simulations to engineering applications are increasing. The same trend has been observed in the wind turbine industries, and extensive experimental tests and numerical simulations have been conducted to examine the load-carrying capacity of circular cylindrical shells with cutouts under axial compression and pure bending.

For axial compression, Tennyson (1968) experimentally observed membrane stress distributions and isoclinic patterns around the edges of cutouts using photoelastic shells. Jullien and Limamto (1998) found that the buckling strength is sensitive to a cutout's angle or circumferential size, based on parametric studies of its shape (square, rectangular, circular) and dimensions (axial and circumferential sizes, diameter). Furthermore, they pointed out the importance of initial imperfections for numerical simulations. Schenk and Schuëller (2007) studied the effects of random geometric imperfections on the critical load of thin-walled cylindrical shells under axial compression with rectangular cutouts. They found that the coefficient of variation of the critical load did not decrease with the imperfections' magnitude. Han et al. (2006) observed that the location and size of a cutout significantly affect the buckling load; specifically, cutouts located near the fixed boundary could effectively absorb energy and redistribute the load more efficiently. Shariati and Rokhi (2008 2010) reported that longer shells exhibit much more sensitivity to the positions of the cutouts. Moreover,

they observed that the buckling strength decreases as height increases with a constant

cutout width. Ghanbari Ghazijahani et al. (2015) experimentally found a symmetric

ring-shaped bulging wave in an intact specimen after initiating buckling. They noted

that the effects of the cutouts' heights on the capacity were less than 5% under axial

compression.

For pure bending, Kyriakides and Ju (1992) and Ju and Kyriakides (1992)

observed that thinner shells developed short wavelength periodic ripples on the

compressed sides of the shells, and the shells buckled locally and collapsed soon after

the ripples appeared. In contrast, thicker shells were found to exhibit limited load

instability as a direct consequence of the ovalisation of the shell cross-section caused by

pure bending. Yeh et al. (1999) observed that for a shell with a circular cutout, the

ultimate strength decreased as the diameter of the cutout increased, whereas for a shell

with a rectangular cutout, the ultimate strength decreased as the size of the cutout

increased. It was also found that the ultimate strength of a shell with a cutout on the

compression side was less than for a cutout on the tension side, and the ultimate

strength increased when a cutout was close to the end of the clamped shell. Guo et al.

(2013) found that an increase in the D/t ratio resulted in a more pronounced local

buckling phenomenon, with the stiffeners increasing the load carrying capacity while

improving the ductility.

The most distinguished numerical and experimental works were Dimopoulous's

series of studies (Dimopoulos and Gantes 2012, Dimopoulos and Gantes 2013,

Dimopoulos and Gantes 2015, Dimopoulos et al. 2015) on circular cylindrical shell

structures. Experimental and numerical studies of the buckling behaviour of

cantilevered circular cylindrical shells with cutouts and stiffening were conducted, and

it was confirmed that the presence of the cutouts led to a reduction in strength. The

lowest collapse load appeared when a cutout was situated on the compression side (Dimopoulos and Gantes 2012). It was also shown that simple stiffening types, consisting of either a peripheral frame or two longitudinal stiffeners with a ring, were particularly efficient and could be used instead of more complex examples (Dimopoulos and Gantes 2013). The importance of geometrical and material nonlinearities, including initial imperfections, was also noted (Dimopoulos and Gantes 2015). Finally, the stiffening effects of cutouts on circular cylindrical shells under dynamic wind loading were assessed using aeroelastic code. It was concluded that the dynamic effect led to a small decrease in tower strength compared to the effect obtained via static analysis, but this reduction was less than 10% in all investigated cases (Dimopoulos et al. 2015).

3. Structural Features

3.1 Definition of geometrical parameters

Wind turbines typically consist of circular cylindrical shell sections connected by bolted flanges, as shown in Fig. 1. The geometrical attributes of a typical wind turbine tower with a cutout are defined here. The following four parameters for wind turbine towers are considered: minimum column aspect ratio ($\gamma_{min} = H / D_{min}$); maximum column aspect ratio ($\gamma_{max} = H / D_{max}$); minimum column slenderness ratio ($\lambda_{min} = D_{min} / t_{min}$); and maximum column slenderness ratio ($\lambda_{max} = D_{max} / t_{max}$). The other two cutout parameters considered are the aspect ratio ($\alpha = h / b$) and the slenderness ratio ($\beta = b / t_c$).

3.2 Geometrical features

Data on 102 wind turbines and their cutouts were collected where the capacities ranged from 0.5 to 5.0 MW. The principal features are displayed in Appendix Table A1,

and the geometrical characteristics of each parameter predefined in Section 3.1 are analysed. The statistical distribution of the parameters is shown in Figs 2 and 3. Table 1 summarises the range and most probable values of each parameter. These findings are then used to identify the geometrical parameters of the standard wind turbine tower and the cutout: $H = 65,000$ mm; $D_{max} = 3,750$ mm; $t_{max} = 30$ mm; $h = 1,900$ mm; $b = 700$ mm; and $t_c = 30$ mm ($\alpha = 2.875$, $\beta = 25.0$). Based on those parameters, the dimensions of the flanges are designed and illustrated in Table 2 (Sahin 2015). It was assumed that the standard wind turbine tower is composed of 3 sections, as shown in Fig. 4, and that the height of the first section used in the present study was $H_S = 12,655$ mm. Hereafter, because the tower section applied in the present study had a uniform thickness, t, the thickness of the cutout (t_c) and the maximum thickness of the wind turbine tower (t_{max}) are represented as the thickness of the wind turbine tower − t ($= t_c = t_{max}$) − and the diameter of the tower section (D) is the same as the maximum diameter of the wind tower (D_{max}).

3.3 Selection of parameters

CCD is an experimental design for building a second-order model without needing to use a complete 3-level factorial experiment. The CCD with the fractional factorial design was used in this study as a deterministic method for parameter selection. The locations of the design points were determined based on the number of input parameters, in accordance with the DOE method. The example points of the CCD with three input parameters are shown in Fig. 5.

3.4 Reference capacity of circular cylindrical shells without the cutout

In the section on wind turbine structures with cutouts, the first yield occurred near the cutout where the highest compression developed and rapidly expanded around the cutout with further loading. The entire load-carrying capacity of the wind turbine structure with the cutouts depended on the geometrical dimensions and material properties. In the present study, the reference buckling loads of the shells without cutouts subjected to axial compression (Shariati and Rokhi 2008) and pure bending moments (Dimopoulous and Gantes 2013) were defined as:

$$F_R = \pi D t \times \sigma_Y \tag{1}$$

$$M_P = \frac{4}{3}\pi\left(\left(r+\frac{t}{2}\right)^3 - \left(r-\frac{t}{2}\right)^3\right)\times\sigma_Y \tag{2}$$

where F_R is the reference load of a wind turbine, M_P is the plastic bending moment of a wind turbine, D is the diameter of the wind turbine tower, r is the radius of the wind turbine tower, t is the thickness of the wind turbine tower and σ_Y is the yield stress of the material.

4. Nonlinear Finite Element Modelling

4.1 Finite-element model

In this study, nonlinear finite element analysis is performed using ANSYS-Workbench (2015) to accommodate both geometrical and material nonlinearities. To solve highly nonlinear problems, including the very unstable snap-through response of shell structures, nonlinear stabilisation – which uses the line search method together with auto time stepping – is used.

The SHELL181 element, which has four nodes with 6 degrees of freedom at each node, is used to model circular cylinder shells and the SOLID185 element, which has 8 nodes with 3 degrees of freedom at each node, is used to model the ring frame located at both ends of the circular cylinder shell section. The wind turbine is modelled on the result of the quasi-static material test of mild steel grade A, as shown in Fig. 6.

As in previous studies (Jullien and Limamto 1998, Schenk and Schuëller 2007, Dimopoulos and Gantes 2015), the effect of initial imperfection is properly applied. The maximum magnitude of initial deflection w_o is assumed to be 30% of the thickness of the wind turbine tower; that is, $w_o = 0.3t$. The eigenvalue buckling mode is used to determine the shape of the initial deflection. Fig. 7 provides examples of the smallest buckling mode near the cutout, obtained from the eigenvalue buckling analyses for intact (no opening) and cutout under axial compression and pure bending. In order to maintain the simplicity of finite element method computations, the residual stress caused by welding is not considered in the present study.

4.2 Loading conditions

The loading regimes of wind turbines during operation are extremely complex. A proper understanding of loads on wind turbines and their structural responses are crucial to avoiding their catastrophic failure. In general, the types of loads acting on wind turbines in service can be classified into five categories: static, cyclic, stochastic, aerodynamic and mechanical. As shown in Fig. 8, the schematic free-body diagram of a wind turbine structure can be represented as three loads: a torque due to the blades, an axial force due to gravity and a bending moment due to the thrust of the blades and the transverse force on a tower.

In the present study, it is assumed that an axial force and bending moment are closely related to the wind turbine tower's failure or collapse. They are applied in an

isolated manner rather than in combination to investigate the effect of each load on the ultimate strength of the wind turbine tower with a cutout. When it is essential to predict the ultimate strength accurately, the combined loads should be considered.

4.3 Boundary conditions

The boundary conditions investigated in this study are described in Fig. 9. The coordinate system used for their measurement is shown in Fig. 9(a). The restraints are described in detail below.

- Fixed boundary condition, as shown in Fig. 9(b)

- Bottom surface: translational restraints in the *x*-, *y*- and *z*-directions,

    $u_x = u_y = u_z = 0$; rotational restraint in the *x*-, *y*- and *z*-direction, $\theta_x = \theta_y = \theta_z = 0$.

    As mentioned, a wind turbine with a cutout – as detailed in Section 3.2 – is regarded as being subject to axial compression in the z-axis and pure bending in the y-axis, as shown in Fig. 9(c).

4.4 Mesh-convergence study

This section presents the results of the mesh convergence study for 6 types of element sizes under pure bending when $\sigma_Y = 299$ MPa and $w_o = 0.3t$. In the mesh convergence study, 6 element sizes are tested under pure bending. The ultimate strength is summarised in Fig. 10, and it is found that approximately 35,000 elements (F5, size = 40 mm) are sufficient to estimate the ultimate bending moment of the wind turbine. The authors assume that the mesh convergence for axial compression may agree with the results for pure bending.

4.5 Validation

The finite element modelling techniques developed in the present study are

validated by the experimental results under axial compression (Ghanbari Ghazijahani et al. 2015) and pure bending (Dimopoulos and Gantes 2012). Fig. 12 shows the results of a validation study for the models shown in Fig. 11. It confirms that the developed finite element modelling technique is generally effective for simulating the ultimate strength of the wind turbine towers under axial compression and pure bending.

5. Effects of Variables

In this section, 3 sets of parametric studies of the results are presented. First, to investigate the effect of a cutout shape on the ultimate strength, 3 shapes – rectangular, elliptical and half-elliptical-rectangular – are considered. Second, to examine the effect that a cutout location has on the ultimate strength in the vertical and circumferential directions, 5 locations in the vertical direction and 9 in the circumferential direction are considered. Third, the cutout shape, aspect ratio, column slenderness (diameter to thickness) ratio and column aspect (height to diameter) ratio are taken as the design variables and their effects on the ultimate strength are widely calculated. To identify the combined effects of these variables on the ultimate strength, a DOE with the CCD method is applied for the selection of design points for a given range of each parameter from Section 3.2.

5.1 Effects of cutout shape

As stated earlier, previous researchers (Julien and Limam 1998, Yeh et al. 1999) attempted to examine the effects of cutout shape on the load-carrying capacities of circular cylindrical shells. They concluded that the existence of cutouts alters the nature of the moment-end-rotation response under pure bending. However, the effects of cutout shape on the load-carrying capacities were weak, and sometimes negligible.

The authors attempted to improve their understanding of the effects of cutout

shape on the ultimate strength using the standard model predefined in Section 3.2. 3 types of shapes – rectangular, elliptical and half-rectangular-elliptical – were considered. To perform an accurate comparison, the area of each cutout, A, was kept the same. The parameters considered in this study are as follows:

- Shape: intact (no-opening), rectangular, elliptical, half-rectangular-elliptical
- Loading condition: axial compression, pure bending

The applied geometries are illustrated in Fig. 13, and their dimensions for the cutouts are summarised in Table 2. Fig. 14 describes a comparison of the load-carrying capacity with the no-opening model. The reduction rate of the ultimate strength of each shape for both loading conditions appeared to be around 80% intact. The present results confirm the previous findings (Julien and Limam 1998, Yeh et al. 1999) that the effect of shape is negligible.

Figs. 15 and 16 illustrate von Mises stress contours at various loading stages for the standard wind turbines with the rectangular cutout under axial compression and pure bending as an example. It was found that the stress concentration is initiated regions around the cutout and it increases with the further load. As it increases, the level of stress rapidly rises so that these regions yield before the tower reaches the buckling state. Finally, tower shell around the cutout buckles. It was surmised that the local buckling occurs at first around the cutout and then the tower buckling follows.

5.2 Effects of cutout location

The effects of cutout location in the vertical or circumferential directions were previously investigated by a number of researchers (Kyriakides and Ju 1992, Ju and Kyriakides 1992, Yeh et al. 1999, Han et al. 2006, Dimopoulos and Gantes 2012), who noted that as the cutout location neared the boundaries, the ultimate strength increased under axial compression. Moreover, the ultimate strength of a circular cylindrical shell

with a cutout on the compression side was smaller than that of one with a cutout on the tension side under pure bending (Kyriakides and Ju 1992, Ju and Kyriakides 1992, Yeh et al. 1999, Dimopoulos and Gantes 2012). In order to assess the ultimate strength of circular cylindrical shells with cutouts, a series of nonlinear finite element method computations are performed for various cutout locations in the vertical and circumferential directions.

5.2.1 Vertical direction

To investigate the effects of cutout location in the vertical direction on the ultimate strength, the thickness, t, is kept the same at 30 mm. 50 cases of series analyses were performed, and the parameters considered are as follows:

- Shape: elliptical

- Location in vertical direction, h_o / H_s: 0.1, 0.2, 0.3, 0.4, 0.5

- Column slenderness ratio, $\lambda = D / t$: 90, 110, 125, 130, 150

- Loading condition: axial compression, pure bending

Fig. 17 displays an example of applied geometries with varying cutout locations in the vertical direction, $h_o / H_s = 0.1{\sim}0.5$.

Figs 18 and 19 illustrate the force-displacement and moment-rotation histories for various column slenderness ratios. It was found that the closer the cutout is to the fixed boundary, the more the ultimate strength increases in both loading conditions. However, while numerical results of cutouts located near the fixed boundaries measured the higher ultimate strength than others, a sudden drop in strength appears after the ultimate state for all cases in axial compression and some cases in pure bending. Although the closest cutout reveals the highest ultimate strength, it can not be regarded as the safest wind turbine in terms of structural integrity. It indicates that current

locations of the cutout in the vertical axis may not be the optimum location in terms of safety and structural integrity. Therefore, it is recommended that when structural engineers determine the location of the cutout in the vertical direction, they must carefully consider it as one of crucial design aspects and have a design solution for enhancing structural capacity.

Fig. 20 summarises the non-dimensionalised load-carrying capacity, which varies with the cutout location in the vertical direction. The ultimate strength increases almost linearly as a function of the cutout location, and as the column slenderness ratio decreases, the reduction rate increases under both loading conditions. Moreover, the ultimate strength under pure bending is not sensitive to the variations in the column slenderness ratio; namely, increasing diameter except for $\lambda = 150$.

5.2.2 Circumferential direction

The thickness, t, is kept the same at 30 mm under pure bending to identify the effects that cutout location in the circumferential direction has on the ultimate strength because the structural response under axial compression is symmetrical. 90 cases of series analyses are performed in total, and the parameters considered are as follows:

- Shape: half-rectangular-elliptical
- Column slenderness ratio, $\lambda = D/t$: 125
- Cutout angle, θ: 0, 15, 30, 45, 60, 90, 120, 150, 180 degree
- Loading condition: pure bending

Fig. 21 illustrates an example of applied geometries with varying cutout angles in the circumferential direction, $\theta = 0 \sim 180$ degrees.

Fig. 22 presents an example of moment-rotation histories with varying cutout angles and the non-dimensionalised load carrying strength. It confirms that when a cutout is located on the compression side ($\theta = 0$ degrees), the ultimate strength is

minimal, and as the cutout angle increases, so do the ultimate strength. It is observed that the ultimate strength increases almost linearly until $\theta = 90$ degrees, whereas it appears almost constant over 90 degrees ($\theta = 120$, 150, 180 degrees).

5.3 Combined effects of aspect, column slenderness, and column aspect ratios

To examine the effects of the aspect, column slenderness and column aspect ratios on the ultimate strength, the dimensions of the cutouts vary based on the statistical distribution boundaries, $1800 \leq h \leq 2900$ and $600 \leq b \leq 1100$, as illustrated in Section 3.2. For the selection of parameters, DOE is applied, using the CCD method. Four cases of the maximum diameter varying from 2750 mm to 4250 mm are noted with the locations of the cutouts in the vertical and circumferential directions, $h_o / H_s = 0.1$ and $\theta = 0$ degree. A total of 1,080 cases of series analyses are performed, and the parameters considered are as follows:

- Shape: rectangular, elliptical, half-rectangular-elliptical
- Height of the cutout, h : 1800~2900 mm
- Width of the cutout, b : 600~1100 mm
- Diameter, D : 2750, 3250, 3750, 4250 mm
- Column slenderness ratio, $\lambda = D / t$: 90, 110, 125, 130, 150
- Loading condition: axial compression and pure bending

The volume of the cutout is defined as:

$$V = At \qquad\qquad (3)$$

where V is the volume of the cutout, A is its area and t is the thickness of the wind turbine tower. It is assumed that the change in the breadth of the cutout due to curvature

is ignored to simplify the calculation of the cutout area.

Figs. 23 and 24 show an example of the effect of column slenderness and aspect ratios on the non-dimensionalised ultimate strength ($D = 3750$ mm) for 3 shapes under axial compression and pure bending. It is found that as the height of the cutout, h increases for the same breadth, b, the ultimate strength clearly decreases. It is observed that as the volume of the cutout increases for the same aspect ratio, the strength decreases due to increasing thickness, yet the reduced ultimate strength appears nearly the same regardless of the cutout shape.

Figs 25 and 26 illustrate the non-dimensionalised ultimate strength of selected design points for the 3 cutout shapes under axial compression and pure bending. It is found that the ultimate strength reduction appears within a range of 50-80% of the reference strengths for both loading conditions.

6. Empirical Formulation of the Ultimate Strength

The results of the parametric analysis described in Section 5.3 are used to derive empirical formulations that can predict the ultimate strength of a circular cylindrical shell with the cutout. The linear regression approach with the least-squares method is used, as follows:

$$F_u / F_R = \xi_D D_{\max} + \xi_t t + \xi_h h + \xi_b b + \xi_C \qquad (3)$$

$$M_u / M_P = \zeta_D D_{\max} + \zeta_t t + \zeta_h h + \zeta_b b + \zeta_C \qquad (4)$$

where F_u / F_R and M_u / M_P are the non-dimensionalised ultimate strengths under axial compression and pure bending, respectively. The coefficients for axial compression and pure bending are indicated in Table 4. The regression statistics, including the correlation coefficients and the adjusted R-square (R^2), are illustrated in Table 5.

The correlation between the numerical results and the empirical estimations of the ultimate strength is illustrated in Fig. 27. It is found that the estimations made by the proposed empirical equations well agree with the numerical calculations. This implies that the proposed empirical equations can be an effective way to estimate the reduced ultimate strength of circular cylindrical shells with cutouts.

The design formulae developed in the present study cover an extensive range of possible geometrical variations in circular cylindrical shells with cutouts, and thus they would provide guidance for wind turbine tower design. It should be cautioned that they may need to be validated further by comparison with more accurate computations and experiments before being used in special cases with geometric and boundary conditions.

7. Conclusion

The aims of this study were to develop a numerical modelling technique that accurately predicts structural responses by considering nonlinearities, and to numerically examine the effects of various variables on the ultimate-strength characteristics of wind turbine towers with cutouts. A series of nonlinear finite element computations were undertaken to achieve these objectives, and several conclusions can be drawn from the results, as outlined below.

(1) First, the wind turbine structures in service were investigated, and their actual dimensional characteristics and those of their cutouts were identified from the data collected and analysed.

(2) A nonlinear finite element modelling technique was developed based on a mesh convergence study and validation studies for wind turbine towers with cutouts.

(3) It was confirmed that the effect of the cutout shape is negligible, whereas placing the cutout on the compression side produced the minimum ultimate

strength. Further, under pure bending, the ultimate strength appeared to be uniform when the cutout angle was over 90 degrees.

(4) Based on the results of a parametric analysis, a design formula accommodating a whole range of actual dimensional characteristics is proposed. This formula has the potential to improve the design and safety assessment of circular cylindrical shells with cutouts.

(5) Given the numerous uncertainties involved due to geometrical and boundary conditions, among others, further studies are recommended to examine models used to examine the collapse mechanism more realistically.

Appendix A.
A.1. Tables

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