



## Fractal dimension of large aggregates under different flocculation conditions



Rodrigo B. Moruzzi<sup>a,\*</sup>, André L. de Oliveira<sup>a</sup>, Fabiano T. da Conceição<sup>a</sup>, John Gregory<sup>b</sup>, Luiza C. Campos<sup>b</sup>

<sup>a</sup> Universidade Estadual Paulista (UNESP), Instituto de Geociências e Ciências Exatas, Brazil

<sup>b</sup> Department of Civil, Environmental and Geomatic Engineering, University College London, United Kingdom

1

## 2 Fractal dimension of large aggregates under different flocculation conditions

3

4

5

6 Rodrigo B. Moruzzi<sup>a\*</sup>, André L. de Oliveira<sup>a</sup>, Fabiano T. da Conceição<sup>a</sup>, John Gregory<sup>b</sup>, Luiza  
7 C. Campos<sup>b</sup>

8

9

10

11 a – Universidade Estadual Paulista (UNESP), Instituto de Geociências e Ciências Exatas,  
12 Brazil

13 b – Department of Civil, Environmental and Geomatic Engineering, University College  
14 London, United Kingdom

15

16

17

18 **Address:**

19 \* Corresponding author: 1 – Avenida 24-A, nº 1515, C. P. 178, CEP 13506-900, Bela Vista, Rio  
20 Claro, São Paulo, Brazil. Phone: +55 19 3526-9339. E-mail address: [rmoruzzi@rc.unesp.br](mailto:rmoruzzi@rc.unesp.br)

## ABSTRACT

The two-dimensional fractal dimension ( $D_f$ ) of large aggregates of kaolin ( $> 540 \mu\text{m}$ ) during the shear flocculation process for kaolin solution was investigated using non-intrusive *in situ* image-based acquisition system. Separate experiments were also carried out for three different sized sub-ranges of large aggregates (0.540 – 1.125 mm; 1.125 – 1.750 mm; 1.750 – 2.375 mm). Digital images were taken at a frequency of 10 Hz for 10 s for each different pairs of gradients of velocity ( $G_f$ ) of 20 and 60  $\text{s}^{-1}$  and flocculation times of 2; 3; 4; 5; 10; 20; 30; 60; 120 and 180 min. For the same conditions, particle size distribution (PSD) was also determined. Under the investigated conditions, the lowest  $G_f$  produced the greatest  $D_f$  (1.69) at a flocculation time of 30 min for the whole range of aggregates. Also, the evolution of the longest length of aggregate ( $l$ ) and  $D_f$  with time, showed that the dynamic steady-state was reached at different times for each shear rate and  $l$  ranges. However,  $D_f$  varied for each size sub-range (ca. 1.1 to 1.8). Finally, the behavior of the aggregate structure may be understood by the predominance of different aggregation mechanisms such as cluster-cluster for  $G_f$  of 60  $\text{s}^{-1}$  and particle-cluster for  $G_f$  of 20  $\text{s}^{-1}$ .

**Keywords:** Flocculation; Fractal dimension; Shear rate; Aggregates

## 46 1. INTRODUCTION

47 Solid and liquid separation is an essential step in drinking water treatment which  
48 primarily depends on particle characteristics of water. These particles exist in all size  
49 increments; no matter how many sub-ranges one divides the entire size range (Lawler, 1997),  
50 and they usually cannot be removed before coagulation and flocculation processes.  
51 Coagulation destabilizes particles while flocculation is responsible for particle aggregation  
52 which occurs with the majority of particles by means of shear-induced collision and  
53 orthokinetic aggregation. During this step, it is desirable that destabilized colloids ( $5 \cdot 10^{-3}$  to 1  
54  $\mu\text{m}$ ) are in collision to form larger aggregates (flocs), thus altering mass, surface area, number  
55 and morphology as a function of shear rate and time. Large aggregates are defined by Becker  
56 *et al.* (2009) as aggregates formed from more than 305 heterogeneous primary particles. So,  
57 the aggregate size can indicate the stage of flocculation, once it is expected that aggregate  
58 moves from small to large size ranges.

59 The dynamic steady-state is expected during flocculation for a given shear rate, as  
60 aggregation and breakage rates make particle size distribution (PSD) and particle structure  
61 stable over time (Jarvis *et al.*, 2005). In this dynamic stage of equilibrium, aggregate size does  
62 not change significantly but fluctuates within a certain range, which can be either narrow or  
63 broad (He *et al.*, 2012), depending on the applied shear rate and floc strength. Considering the  
64 same primary particles (i.e. colloids) under a fixed coagulation condition (e.g. pH and  
65 coagulant dosage), the time at which the dynamic steady-state is reached is influenced by both  
66 shear rate and aggregate characteristics. This is due to the fact the migration (displacement) of  
67 particles in size sub-ranges depends on the dominant flocculation mechanism upon each  
68 aggregate size responsible for aggregation and breakage (erosion or fragmentation, depending  
69 on either viscous or inertial energy dissipation sub-range). However, aggregates may have the

70 same size but different structures due to different arrangements of particles during  
71 aggregation (Vahedi and Gorczyca, 2012).

72 Thus, the structural characteristic of aggregates should be also relevant for the  
73 comprehension of the flocculation process. Gregory (2009) pointed out that aggregates are  
74 recognized as fractal objects and that fractal structure has important practical implications,  
75 e.g. as aggregate density. Other studies (He *et al.*, 2012; Spicer and Pratsinis, 1996) have  
76 demonstrated how flocculation is influenced by the aggregate structure through the  
77 representation of fractal dimension. Yang *et al.* (2013) proposed a combination of the  
78 Smoluchowski model of flocculation with fractal theory in replacement of Euclidean  
79 geometry to represent particle size. Also, it is suggested that the formation of large aggregates  
80 is not enough to guarantee an improvement of the terminal sedimentation velocity, once it can  
81 vary with fractal dimension (Chakraborti *et al.*, 2000; Gregory, 1997; Johnson *et al.*, 1996;  
82 Vahedi and Gorczyca, 2012).

83 Several factors may affect fractal structures such as mixing (Logan and Kilps, 1995),  
84 pH and coagulant dosage (Xu *et al.*, 2010 and 2011), once floc strength is highly dependent  
85 on the floc formation process (He *et al.*, 2012). Gregory (2009) also indicated that aggregates  
86 formed by perikinetic mechanisms have fractal dimensions lower than those formed during  
87 orthokinetic flocculation. It is known that larger aggregates, formed during sweep-  
88 coagulation, have higher size and fractal dimensions (Kim *et al.*, 2001; Li *et al.*, 2006)  
89 compared to the ones formed during charge neutralization. It is expected that these aggregates  
90 have better performance during sedimentation since the terminal velocity is dependent on  
91 aggregate size and porosity (Johnson *et al.*, 1996). Also, since the settling velocity varies with  
92 the fractal dimension, size is not enough to explain terminal velocity and particle removal by  
93 sedimentation (Johnson *et al.*, 1996; Vahedi and Gorczyca, 2012).

94           Theoretically, self-similar and scale invariants are the most important characteristics  
95 of fractal objects (Gregory, 1997; Johnson, 1996). However, aggregates in natural systems do  
96 not generally follow the theoretical scaling laws, but it is expected this concept is valid for  
97 large aggregates with size much bigger than primary particles (Chakraborti *et al.*, 2003).  
98 Furthermore, large aggregates are likely to have a different structures and size changes during  
99 flocculation as the factors that control their formation are more complex (Vahedi and  
100 Gorczyca, 2012).

101           Becker *et al.* (2009) used simulation to investigate aggregate behavior based on inter-  
102 particle forces and bending moments. The simulations pointed out that for small aggregates,  
103 formed by 55 primary particles (names as Aggregate I), stress forces are not sufficient to  
104 overcome both bond and bending resistance, so that aggregates remain unchanged and  
105 rotating as a rigid body. On the other hand, for very large aggregates, formed by 1000 primary  
106 particles, (named as Aggregate III) they will break if they are exposed to shear flows. For  
107 aggregates of intermediate size, formed by 305 primary particles (named as Aggregate II), the  
108 structure of aggregates change as consequence of primary particles rearrangements. For this  
109 intermediate case, shear forces are high enough to promote aggregate restructuring but not  
110 high enough to break primary particles in small fragments.

111           There is no literature reporting temporal characteristic of aggregates, restricted to large  
112 size domain, in terms of fractal dimension evolution. So, the main question is what are the  
113 main characteristics of these large aggregates in different shear rates, flocculation times and  
114 size sub-ranges? The aim of this study was to investigate the temporal evolution of two-  
115 dimensional fractal dimensions of large aggregates in different size ranges from a series of  
116 flocculation tests (shear rates of 20 and 60 s<sup>-1</sup> and mixing times ranging from 2 to 180 min).  
117 The evolution of PSD and the two-dimensional fractal dimensions were all measured by a  
118 non-intrusive image analysis. The results may provide new insight into solid and liquid

119 separation processes where the domain of large aggregates is the deciding factor in the  
120 sedimentation performance.

121

122

## 2. MATERIALS AND METHODS

123

### 124 2.1. Kaolin suspension

125 Water was prepared from a stock solution of kaolin suspension based on the studies by  
126 Pádua (1994), and Yukselen and Gregory (2004). A commercial kaolin (Sigma-Aldrich) was  
127 used as primary particles. The structure composition of dry kaolin was identified by X-ray  
128 diffractometry (XRD – Siemens D5000), using a wide angle X-ray diffractometer, operating  
129 at 40 kV and 40 mA, with  $\text{CuK}\alpha$  radiation. In addition, the surface characterization of the  
130 kaolin particles was observed under a JEOL JSM-6010LA scanning electron microscope  
131 (SEM), equipped with integrated Energy Dispersive Spectroscopy (EDS) Dry SD Hyper (EX-  
132 94410T1L11). A MALVERN Mastersizer 2000 was used to measure the size distribution of  
133 the kaolin particles. Two-dimensional fractal dimension of dry kaolin particles was  
134 determined through the same methodology used for large aggregates described in Section 2.2,  
135 and the images taken by SEM. In total, 138 primary particles were analysed.

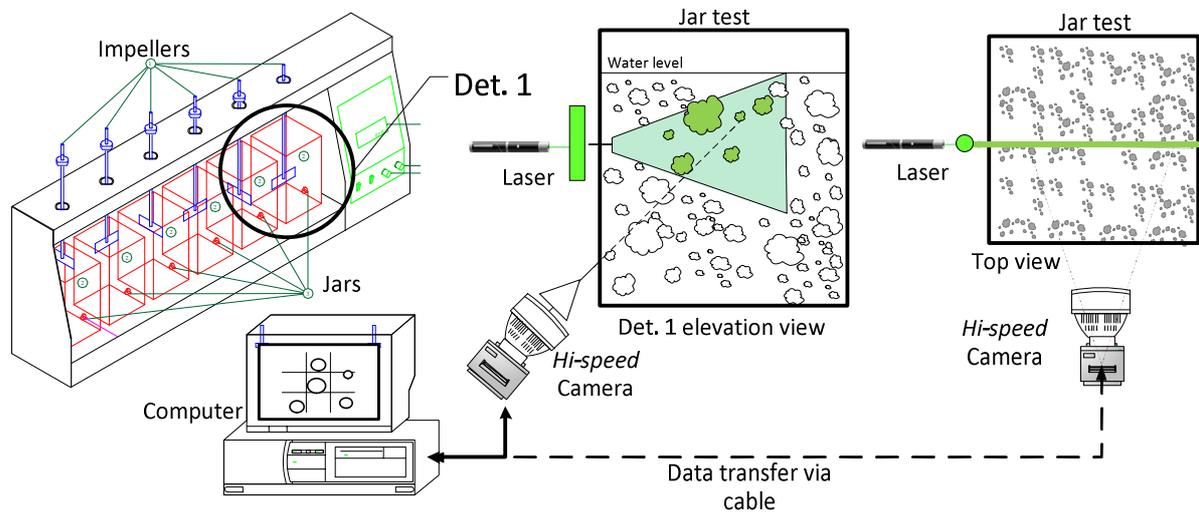
136 Kaolin solution was used as representative of clay suspension, differently of uniform-  
137 sized spheres reported by Chakraborti et al. (2003). The stock solution turbidity was around  
138  $5000 \pm 200$  NTU. Stock solution (10 mL) was diluted in deionized water to produce 2 L of  
139 water with a turbidity of  $25 \pm 2$  NTU. Analytic alum ( $\text{Al}_2(\text{SO}_4)_3 \cdot 14\text{H}_2\text{O}$ ) from Sigma was  
140 used as coagulant and dosages were presented as  $\text{Al}^{3+}$ . Analytic sodium bicarbonate  
141 ( $\text{NaHCO}_3$ ) was used as the buffer during coagulation tests.

142

### 143 2.2. Jar-Tests and Image Capture

144 Jar-tests were performed as recommended by Yukselen and Gregory (2004) and the jar  
145 was coupled to a non-intrusive image capturing system (Figure 1). All tests were carried using  
146 the same jar (Ethik technology Model 218/6 LDB) and at room temperature at  $20 \pm 2$  °C.

147



148  
149

150 Figure 1 – Schematic representation of the experimental device. Tests were carried out using  
151 the same jar. Adapted from Oliveira *et al.* (2015).

152

153 Optimum conditions for coagulation-flocculation were taken from Oliveira *et al.*  
154 (2015) for image acquisition tests. Therefore, coagulant dosage and solution pH after  
155 coagulation were kept constant at 2 mg Al<sup>3+</sup>/L and 7.5, respectively, and coagulation velocity  
156 gradient ( $G_{rm}$ ) and mixing time were 800 s<sup>-1</sup> and 10 s, respectively. Velocity gradients ( $G_f$ ) for  
157 flocculation were varied from 20 to 60 s<sup>-1</sup> with time ( $T_f$ ) of 1, 2, 3, 4, 5, 10, 20, 30, 60, 120  
158 and 180 min. However, the 100 samples collected at 1 min were not used once image  
159 resolutions were out of the range recommended by Chackraborti *et al.* (2003). For this reason,  
160 only samples collected from 2 min onwards were used. Velocity gradients were previously  
161 calibrated by a torque gauge, thus permitting quoting of velocity gradients by means of the  
162 equipment's rotation (rpm). Alum was used because it is the most widely used coagulant

163 (Sahu and Chaudhari, 2013) and velocity gradients and flocculation time (Table 1) were  
164 chosen based on usual range used in treatment process (Chackraborti et al., 2003).

165 Images were taken at a frequency of 10 Hz for 10 s using the camera coupled to a set  
166 of lenses that allowed for an 840 pixel x 640 pixel resolution with a 30  $\mu\text{m}$  pixel size. The  
167 *Image-Pro Plus*® software was used to develop the images, i.e. conversion from  $2^8$  to  $2^1$  bits,  
168 enhancement and measurement. A laser light sheet of 2000 mW with a 532 nm wavelength  
169 and 2 mm thickness was used as an illumination source, in order to obtain good contrast and  
170 to define a spatial position for image acquisition. The laser light was accurately adjusted by  
171 the focus of the image capture system to allow acquisition control at the middle height of the  
172 jar. The main advantage of this non-intrusive method is that the samples can be analyzed  
173 without risk of the deformation of flocs due to breakage.

174 For the PSD analyses, 20 jar-test samples were evaluated. The longest length ( $l$ ) of  
175 aggregates was used to determine the relative proportions of the images. So, a continuous  
176 function of PSD (Lawler, 1997) was obtained by adjusting Equation 1 to experimental results,  
177 using particle ranges from 0.540 to 2.375 mm.

$$\frac{dN}{d(d_p)} = k(d_p)^{-\beta} \quad (1)$$

178 Where:

179  $k$  = power law density coefficient (dimensionless);

180  $d_p$  = geometric average of the sub-range where aggregates were classified (mm);

181  $\beta$  = power law slope coefficient (dimensionless).

182

183 The same image acquisition system was used to determine the two-dimensional fractal  
184 dimension ( $D_f$ ), based on the power law relationship given by Equation 2. For each condition,  
185  $D_f$  values were determined from the slope of regression line and longest length ( $l$ ) on a *log-log*  
186 plot (Johnson *et al.*, 1996). However,  $D_f$  was calculated using aggregates larger than 0.540

187 mm for the entire range (0.540 to 2.375 mm) and for three size sub-ranges (sub-range 1: 0.540  
188 to 1.125 mm; sub-range 2: 1.125 to 1.750 mm; sub-range 3: 1.750 to 2.375 mm). For these  
189 size sub-ranges, image resolution ( $l/\text{pixel}$  ratio) was between 5.2 and 7.5 thus resulting in an  
190 average area ratio around 1.3-1.1 (Chackraborti *et al.*, 2003). More information on image  
191 acquisition and processing procedures can be found in Moruzzi and Reali (2007 and 2010).

$$A \sim l^{D_f} \quad (2)$$

192 Where:

193  $A$  = aggregate area ( $\text{mm}^2$ );

194  $l$  = characteristic length of the aggregate, here the longest length was used (mm);

195  $D_f$  = two-dimensional fractal dimension (dimensionless).

196 At this point, it is important to make clear the difference between the characteristic  
197 dimensions  $d_p$  and  $l$ . Here,  $d_p$  of Equation 1 is the geometric mean of the classification range,  
198 *i.e* it represents the geometric average for each classification interval, using the average length  
199 of the aggregates. These average length measures were taken at  $2^\circ$  intervals around the  
200 centroid of each one of the aggregates. The  $l$  used in Equation 2 is the longest length for each  
201 floc, and its variant  $l_m$  is the average of the longest length. So,  $d_p$  comes from the discrete  
202 classification of data while  $l$  comes from the longest length and  $l_m$  is the average of all the raw  
203 data.

204 For each shear rate, the Kolmogorov microscale was calculated using Equation 3  
205 (Spicer and Pratsinis, 1996).

$$\eta = \left(\frac{\nu}{G}\right)^{\frac{1}{2}} \quad (3)$$

206

207 Where:

208  $\eta$  = Kolgomorov microscale (m);

209  $\nu$  = kinematic viscosity of water ( $\text{m}^2 \text{s}^{-1}$ );

210  $G$  = gradient of velocity ( $\text{s}^{-1}$ ).

211

### 212 **3. RESULTS AND DISCUSSION**

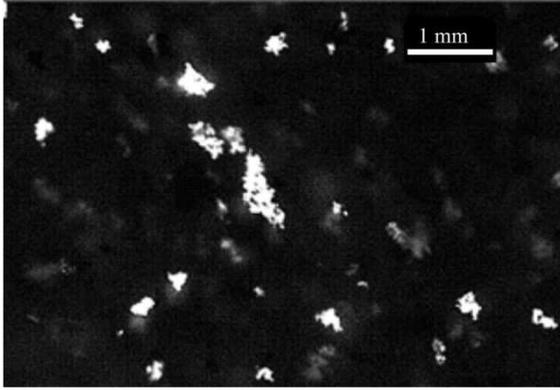
213

#### 214 **3.1 Kaolin characterisation**

215

216 X-Ray diffraction (XRD) patterns of dry kaolin used in this study is shown in Figure  
217 2a. It can be seen that this material is composed mainly of kaolinite ( $\text{Al}_2\text{Si}_2\text{O}_5(\text{OH})_4$ ), illite  
218 [ $\text{K}_{0.9}\text{Al}_2\text{Si}_4\text{O}_{10}(\text{OH})_2 \cdot (\text{H}_2\text{O})$ ] and microcline ( $\text{KAlSi}_3\text{O}_8$ ). It also consists of kaolinite plates  
219 and particles of different size, shape and texture (Figure 2b). The particle size distribution  
220 (Figure 2c) of this raw material indicates a heterogeneous size range, varying from lower than  
221  $1 \mu\text{m}$  up to higher than  $100 \mu\text{m}$  of diameter, with a median of  $7.5 \mu\text{m}$ . The fractal dimension  
222 ( $D_f$ ) of kaolin was around 1.54, indicating the natural flat surface of this material (Malekani et  
223 al., 1996).



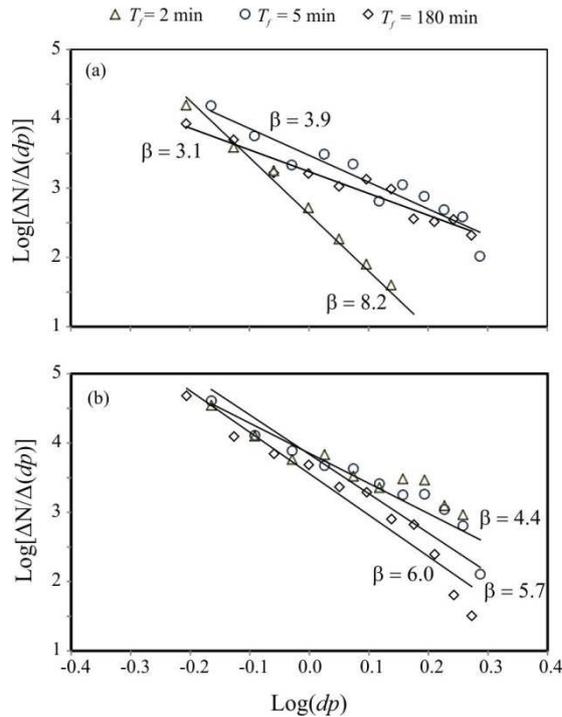


234

235 Figure 3 – Example of raw image (as captured) of aggregates taken at monochromatic mode  
236 with  $2^8$  bits, pixel of  $30\ \mu\text{m}$ .

237

238 The adjustment of the *log-log* plot of Equation 1 can be observed in Figure 4 for  $G_f$  of  
239  $20$  and  $60\ \text{s}^{-1}$  in flocculation time ( $T_f$ ) of  $2$ ,  $5$  and  $180$  min. Considering the initial range of size  
240 of kaolin (particles ranging from lower than  $1\ \mu\text{m}$  up to higher than  $100\ \mu\text{m}$  of diameter, with  
241 a median of  $7.5\ \mu\text{m}$ ), there was clearly a particle displacement from the lowest to the highest  
242 sub-ranges of size caused by flocculation for both  $G_f$  values here exemplified. However, for  
243  $G_f$  of  $20\ \text{s}^{-1}$  (Figure 4a), the transition of small particles from low to high sub-ranges took  
244 more time. This can be observed by means of the slope of the adjusted curves for  $T_f$  of  $2$  min  
245 (Figures 4a and b). For  $G_f$  of  $60\ \text{s}^{-1}$ , there were fewer large flocs (high slope). These behaviors  
246 were expected due to the flocculation kinetics where aggregation rate and aggregate size are  
247 dependent of  $G_f$  (Jarvis *et al.* 2005). For all tests, the power law slope coefficient  $\beta$  values  
248 started from around  $8$  for  $G_f$  of  $20\ \text{s}^{-1}$  and  $6$  for  $G_f$  of  $60\ \text{s}^{-1}$ , reaching a minimum value for a  $T_f$   
249 of  $5$  min and then increasing again for a  $T_f$  of  $180$  min.



250

251 Figure 4 – Example of large aggregate size distribution written as *log-log* plot of Equation 1

252 for three flocculation times. The slope of the trend line gives the  $\beta$  value. (a)  $G_f = 20 \text{ s}^{-1}$  and  $T_f$

253 = 2, 5 and 180 min. (b)  $G_f = 60 \text{ s}^{-1}$  and  $T_f = 2, 5$  and 180 min.

254

### 255 3.2. Characterization of the whole population of large aggregates

256 The images of flocs formed for shear rates of 20 and 60  $\text{s}^{-1}$  in different flocculation

257 times were analyzed and the two-dimensional fractal dimension ( $D_f$ ) for the whole population

258 of large aggregates was derived as shown in Table 1. According to Waite (1999),  $D_f$  is usually

259 an integer number for Euclidean objects, but  $D_f$  values do not follow Euclidean geometry for

260 fractal objects. More circular aggregates have a greater fractal dimension (closer to 2), while

261 aggregates with a looser structure have a smaller fractal dimension (closer to 1). The median

262 of the longest dimension for aggregate size for the whole distribution ( $l_m$ ), Kolmogorov

263 microscale ( $\eta$ ) and the  $R^2$  value for *log-log* slope can be seen in Table 1. Once the  $l_m$

264 represents the median of the longest length of aggregates, it is influenced by the relative

265 frequency in each size range. This measure is influenced by asymmetry as frequency of low

266 sub ranges is more pronounced.  $D_f$  values were calculated based on the whole large aggregate  
 267 sample ( $> 0.540$  mm), which is formed by more than 305 primary particles. Standard  
 268 deviations for each  $D_f$  value are given in parentheses. A statistical analysis of the all the  
 269 experimental data indicated that  $D_f$  values were statistically different ( $p < 0.05$ ) after 10 min  
 270 of flocculation time.

271

272 Table 1 – Characterization of large aggregates for the whole sample (0.540 to 2.375 mm) for  
 273 ten flocculation times ( $T_f$ ) and two shear rates ( $G_f$ ).  $\eta$  is the Kolmogorov microscale.  $l_m$  is the  
 274 median size for the longest dimension of the whole distribution of large aggregates (mm).  $D_f$   
 275 is the two-dimensional fractal dimension. Numbers within parentheses are standard deviation  
 276 values of  $D_f$  and  $l_m$ .

277

$G_f$ (s <sup>-1</sup> )	$\eta$ (μm)		$T_f$ (min)									
			2	3	4	5	10	20	30	60	120	180
20	239±13	$l_m$	0.67 (0.12)	0.70 (0.20)	0.70 (0.22)	0.78 (0.29)	0.97 (0.45)	0.94 (0.44)	0.90 (0.39)	0.85 (0.34)	0.81 (0.27)	0.80 (0.31)
		$D_f$	1.17 (0.22)	1.29 (0.10)	1.16 (0.20)	1.54 (0.07)	1.68 (0.18)	1.68 (0.11)	1.69 (0.10)	1.48 (0.28)	1.52 (0.18)	1.56 (0.12)
		R <sup>2</sup>	0.90	0.86	0.84	0.87	0.90	0.89	0.89	0.88	0.86	0.87
60	138±07	$l_m$	0.83 (0.38)	0.74 (0.27)	0.74 (0.26)	0.76 (0.30)	0.68 (0.24)	0.74 (0.25)	0.68 (0.22)	0.70 (0.23)	0.68 (0.23)	0.68 (0.21)
		$D_f$	1.41 (0.05)	1.36 (0.05)	1.40 (0.10)	1.43 (0.04)	1.38 (0.11)	1.35 (0.06)	1.32 (0.09)	1.31 (0.06)	1.30 (0.08)	1.28 (0.08)
		R <sup>2</sup>	0.86	0.82	0.81	0.82	0.79	0.81	0.8	0.83	0.82	0.82

278

279

280 For both shear rates studied, aggregate size ( $l_m$ ) values are above the Kolmogorov  
 281 microscale ( $\eta < 540$  μm), suggesting that breakage mostly occurs by fragmentation (He *et al.*  
 282 2012; Thomas *et al.*, 1999). For 20 s<sup>-1</sup>, the median aggregate size increased from 0.67 to 0.97  
 283 mm during the first stage of flocculation (2 to 10 min), after which decreasing until a value of  
 284 0.80 mm was reached at 180 min. On the other hand,  $l_m$  values rapidly reached the maximum  
 285 value of 0.83 mm at 2 min for  $G_f$  of 60 s<sup>-1</sup> and then decreases over time to the final value of  
 286 0.68 mm at 180 min.

287 Referring to the evolution of two-dimensional fractal dimension, at a shear rate of 20  
288  $s^{-1}$ ,  $D_f$  values increased during the first 10 min, stabilized over 10 to 30 min, reached the  
289 highest value (*i.e.* 1.69) at 30 min, decreased to 1.48 at 60 min and increased up to 1.56 at 180  
290 min. However, for  $G_f$  of  $60 s^{-1}$   $D_f$  values decreased with time from 1.41 ( $T_f$  of 2 min) to 1.28  
291 ( $T_f$  of 180 min), similar to the observations made by Chakraborti *et al.* (2003).  $D_f$  values were  
292 1.32 ( $G_f$  of  $60 s^{-1}$ ) and 1.69 ( $G_f$  of  $20 s^{-1}$ ) at 30 min, and these are in agreement with those  
293 values reported by Li *et al.* (2007).  $R^2$  values ranged from 0.79 to 0.90, indicating a good fit  
294 for the *log-log* plot of the whole population of aggregates, in terms of area and size ( $l$ ).

295 The steady state was reached around 30 min for both  $l_m$  and  $D_f$  for  $G_f$  equal to  $60 s^{-1}$ .  
296 In contrast, for a  $G_f$  equal to  $20 s^{-1}$  the plateau was reached only after a  $T_f$  of 60 min which  
297 corroborates well with Spicer and Pratsinis (1996), Chakraborti *et al.* (2003) and He *et al.*  
298 (2012). Furthermore, comparing results from Figure 3 and Table 1, it can be observed that  $D_f$   
299 varied while  $\beta$  was stable after  $T_f$  of 5 min. This emphasizes that PSD may not be enough to  
300 evaluate flocculation of large aggregates, since aggregate size and structure are both modified  
301 and the steady state may occur differently depending on size and structure of aggregates  
302 (Vahedi and Gorczyca, 2012). Also, PSD (or even  $l$ ) is influenced by small particles and thus  
303 the identification of changes in large aggregates may not be accurate.

304 The relation between  $l_m$  and  $D_f$  obtained in this study seems to contradict those  
305 reported by Chakraborti *et al.* (2003) and Li *et al.* (2007), who found that higher  $D_f$  were  
306 observed for higher  $G_f$  ( $80 s^{-1}$ ) *i.e.* lower aggregate sizes. In this study, higher  $D_f$  values were  
307 observed for  $G_f$  of  $20 s^{-1}$  after  $T_f$  of 10 min which means higher aggregate sizes. It is important  
308 to highlight here that the results presented in this paper refer strictly to aggregates of large  
309 size and that small aggregates influence the  $D_f$  values when the slope of *log-log* plot is used  
310 for a population of aggregates, as suggested by Chakraborti *et al.* (2003).

311

312 **3.3 Aggregate characteristics within size sub-range**

313 In order to check the influence of  $G_f$  and  $T_f$  for different size sub-ranges,  $D_f$  values for  
 314 each one of the investigated sub-ranges were calculated. However, before presenting  $D_f$   
 315 results, it is important to show the temporal evolution of the absolute and relative frequency  
 316 of each sub-range at the two investigated shear rates, since restructuring, fragmentation and  
 317 reformation may also alter the aggregate size. Ten flocculation times were tested for each  
 318 velocity gradient (from 2 to 180 minutes), and a different number of aggregates (N) was taken  
 319 due to the random sampling procedure (Table 2).

320

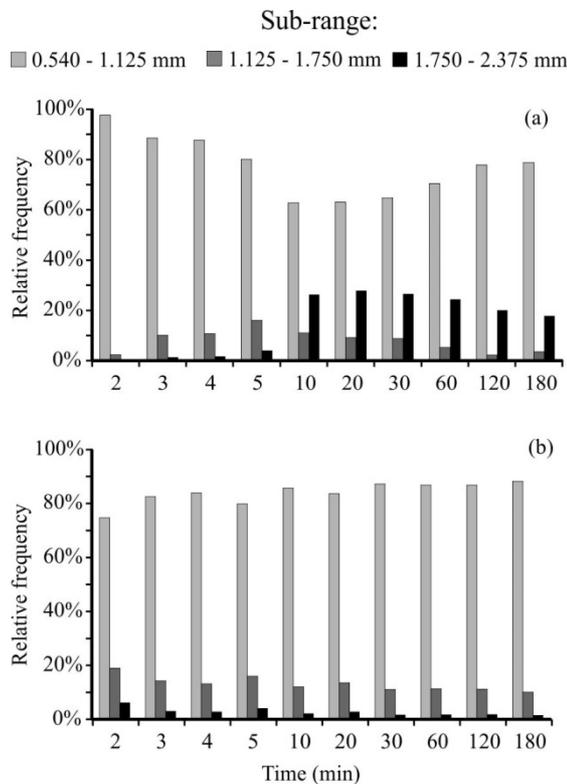
321 Table 2 – Number of aggregates counted (N) at ten sampling times, taken from 2 to 180  
 322 minutes, for each sub-range of size at  $G_f$  of 20 and 60  $s^{-1}$ .

323

Time (min)	Number of aggregates			Total
	0.540 – 1.125 mm	1.125 – 1.750 mm	1.750 – 2.375 mm	
<b><math>G_f</math> of 20 <math>s^{-1}</math></b>				
<b>2</b>	12837	303	14	13154
<b>3</b>	29437	3353	470	33260
<b>4</b>	20077	2440	401	22918
<b>5</b>	12753	2537	646	15936
<b>10</b>	3010	527	1263	4800
<b>20</b>	3363	488	1484	5335
<b>30</b>	3731	504	1534	5769
<b>60</b>	3694	274	1279	5247
<b>120</b>	3805	107	980	4892
<b>180</b>	8466	374	1917	10757
<b><math>G_f</math> of 60 <math>s^{-1}</math></b>				
<b>2</b>	30786	7820	2570	41176
<b>3</b>	30278	5270	1130	36678
<b>4</b>	26204	4120	895	31219
<b>5</b>	27665	5571	1432	34668
<b>10</b>	24776	3490	629	28895
<b>20</b>	28924	4699	973	34596
<b>30</b>	27366	3486	532	31384
<b>60</b>	30466	3978	633	35077
<b>120</b>	31487	4077	687	36251
<b>180</b>	29629	3419	532	33580

324

325 Figure 5 shows the frequency in relative numbers of large aggregates within three size  
 326 sub-ranges at ten sampling times and two shear rates. Aggregates in the sub-range 1 from  
 327 0.540 to 1.125 mm were predominant for both shear rates at all sampling times, thus  
 328 explaining the proximity of the average of  $l$  (*i.e.*,  $l_m$ ) values in Table 1. However, it can be seen  
 329 up to 30 % of aggregates in the sub-range 3 from 1.175 to 2.375 mm at a shear rate of  $20 \text{ s}^{-1}$   
 330 and between 10 and 30 min. Relatively, the amount of aggregates decreased at the sub-range  
 331 3 (1.750 to 2.375 mm) for  $G_f$  of  $60 \text{ s}^{-1}$ .  
 332



333

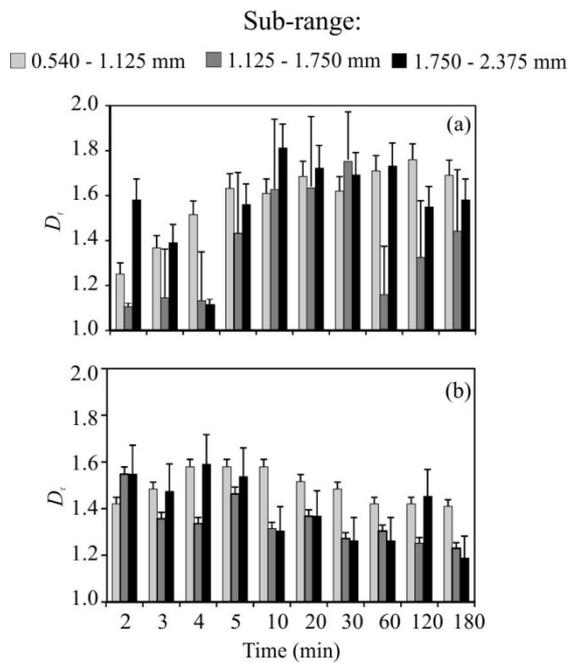
334

335 Figure 5 - Relative frequency of analyzed aggregates at ten flocculation times for three size  
 336 sub-ranges for  $G_f$ :  $20 \text{ s}^{-1}$  (a) and  $60 \text{ s}^{-1}$  (b).  
 337

337

338 Figure 6 shows the  $D_f$  values at three size sub-ranges within the large domain of  
 339 aggregate ( $> 0.540 \text{ mm}$ ) for ten sampling times and two shear rates. For all sampling times,  $D_f$   
 340 varied from  $\sim 1.1$  to  $1.8$  for all size sub-ranges, confirming that  $D_f$  for large aggregates varies

341 widely (Vahedi and Gorczyca, 2012). Similar to the results presented in Table 1, Figure 6  
 342 shows that  $D_f$  values varied with time, and the time at which dynamic steady state was  
 343 reached was different from those observed when the whole large domain was considered. In  
 344 general, it is important to note that  $D_f$  tended to be greater for lower size sub-ranges and *vice*  
 345 *versa*, confirming the idea that the restructuring of large aggregates may result in more  
 346 compact flocs.



347  
 348 Figure 6 - Evolution of  $D_f$  with flocculation time for three size sub-ranges for  $G_f$  of  $20 \text{ s}^{-1}$  (a)  
 349 and  $60 \text{ s}^{-1}$  (b). Bars indicate standard deviation for each sub-range (5 – 30%). In total 2,000  
 350 images were taken capturing ca. 465,000 aggregates.

351  
 352 It is still possible to see in Figure 6 that for the sub-range 1 (0.540 to 1.125 mm),  
 353 aggregates rapidly reach a plateau at around 20 min, and then values fluctuated in a range of  $\pm$   
 354 8 %. On the other hand, it took about 30 min for the sub-range 2 to reach a plateau and the  
 355 values fluctuated over a wider range ( $\pm 30 \%$ ) than the sub-range 1. In addition,  $D_f$  also varied  
 356 in a wider range for sub-range 3, especially for time less than 60 min. In general, the  $G_f$  of  $20$   
 357  $\text{s}^{-1}$  showed the wider fluctuation for all sub-range samples. This was expected, since for the  $G_f$

358 of  $20 \text{ s}^{-1}$  there were aggregates in all large size sub-ranges, especially in the widest sub-range  
359 (as shown in Figure 4a), where the mechanisms for complexes govern aggregation and  
360 breakage (Vahedi and Gorczyca, 2012).

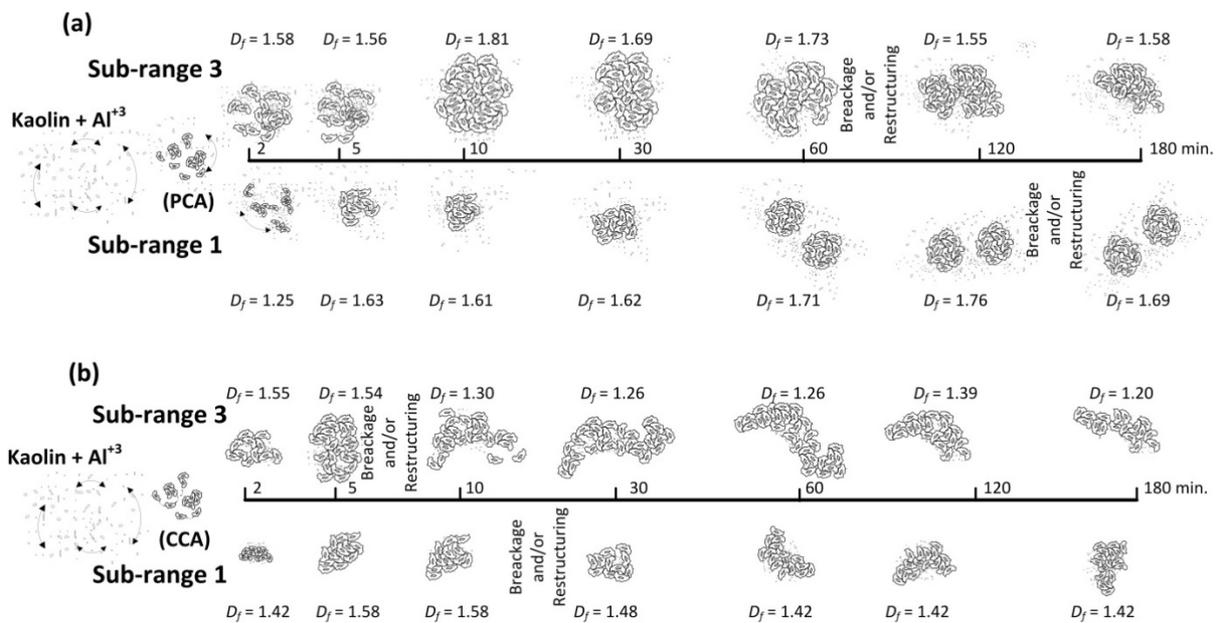
361 Finally, it can be also seen that  $D_f$  values for each sub-range clearly present a peak at  
362 different times, and there is an increase followed by a decrease (presented by a peak) as  
363 shown in Figure 6a or there is a simple decrease with time (Figure 5b), as observed by  
364 Chakraborti *et al.* (2003). However, a huge variation of the  $D_f$  values obtained from the  
365 samples within different size sub-ranges extracted at the same time was observed,  
366 contradicting the results presented by Chakraborti *et al.* (2003) for aggregates ranging from  
367 10 to 45  $\mu\text{m}$ , not much larger than the primary particles ( $9.975 \pm 0.061 \mu\text{m}$ ). Here, only a  
368 large domain of aggregates was considered for the  $D_f$  calculation. Further, here heterodisperse  
369 kaolin particles were used, differently to monodisperse microspheres of latex used by  
370 Chakraborti *et al.* (2003). Moreover, it is important to highlight that it is possible that the  
371 overestimation of the area for the lower sub-ranges may result in smaller  $D_f$  values, when the  
372 fractal dimension is calculated using the whole population of aggregates (Chakraborti *et al.*,  
373 2003).

374

### 375 **3.4 Evolution of $D_f$ within size sub-ranges of large aggregates**

376 Upon evaluation of the results presented, it has been found that large aggregates  
377 behaved differently from the smaller ones reported in the literature (e.g. Chakraborti *et al.*,  
378 2003). Here, large aggregate size ( $l_m$ ) seemed not to change significantly after the dynamic  
379 steady state was reached but their structure, measured by means of  $D_f$ , still changed with time.  
380 Also, different sub-ranges of large aggregates behaved differently with time, resulting in both  
381 different  $D_f$  values and steady state position with time. The lower sub-range of the large  
382 aggregate domain presented a more stable behavior than the larger sub-ranges.

383 In general, a broad range of two-dimensional fractal dimension ( $D_f$ ) was found ~1.1-  
 384 1.8 for different size sub-ranges domains, and these ranges for  $D_f$  are in agreement with  
 385 results reported by Chang *et al.* (2005) and Li *et al.* (2007). Also, clusters formed from  
 386 heterodisperse primary particles of kaolin coagulated with  $Al^{3+}$  reaching both a greater size  
 387 and higher  $D_f$  values for the lower shear rate investigated ( $G_f$  of  $20\text{ s}^{-1}$ ). This contradicts  
 388 results obtained from small aggregates, as presented by Chakraborti *et al.* (2003) and Li *et al.*  
 389 (2007). Figure 7 shows the possible pathways during flocculation of large aggregates  
 390 domains.  
 391



392  
 393 Figure 7 – Schematic of aggregates growth during flocculation for two shear rates: (a)  $20\text{ s}^{-1}$   
 394 and (b)  $60\text{ s}^{-1}$  in two size sub-ranges (sub-range 1 from 0.540 to 1.125 mm and sub-range 3  
 395 from 1.750 to 2.375 mm). The values above the floccs represent  $D_f$  values.

396  
 397 A possible explanation is that a mechanism similar to particle-cluster aggregation  
 398 (PCA) may occur for low  $G_f$ , once there are still significant amounts of primary particles  
 399 remaining (or even relatively small clusters, that may behave as single particles when

400 precipitate of alum is formed), thus favoring particle attachment into large precipitate clusters  
401 of alum precipitate. In this way, primary particles, or small clusters, may adhere into alum  
402 precipitate favouring a large  $D_f$ . Otherwise, the probability of large  $D_f$  values for cluster  
403 approximation is near to zero as aggregate size increases, according to predictions made by  
404 Brasil *et al.* (2001). Further, the coagulant type and dose here investigated may overlap the  
405 isolated effect of particle and cluster mechanisms for large aggregates, once the properties of  
406 flocs are strongly dependent on coagulant dose and type, as described by Yu *et al.* (2015).

407 Results also suggest that after aggregation, fragmentation and restructuring may  
408 change large aggregates, according to their size and compaction. Becker *et al.* (2009) studied  
409 the behavior of small and large aggregates in shear rates, and represented two possible draw  
410 for restructuring of large aggregates (Aggregate II and III). For Aggregate-II, restructuring  
411 starts with stretching followed by compaction, and for Aggregate III break up and reformation  
412 may occur, thus leading to stronger and more compact flocs. Aggregates formed by shear rate  
413 of  $G_f$  of  $20 \text{ s}^{-1}$  might have behaved like Aggregate-II and Aggregate-III, as suggested by  
414 Becker *et al.* (2009). For  $G_f$  of  $20 \text{ s}^{-1}$  an initial stretched phase (lower  $D_f$  values) is followed  
415 by an aggregate compaction (higher  $D_f$ ). Afterward, rupture leading to fragmentation results  
416 in a slightly stretched structure.

417 For  $G_f$  of  $60 \text{ s}^{-1}$  cluster-cluster like aggregation (CCA) may better explain aggregate  
418 structures. In this case, aggregates seems to behave more like in reaction-limited domain  
419 [slow aggregation, as presented by Lin *et al.* (1989)], when an energy barrier has to be  
420 overcome before attaching. Also, formed clusters of large aggregates have shown good  
421 agreement with computer simulations presented by Becker *et al.* (2009). For this shear rate,  
422 Aggregate-I seems more appropriate to explain  $D_f$  behavior, since a fragmented small cluster  
423 may rotate like a rigid body (Becker *et al.*, 2009). The behavior of  $D_f$  values for  $G_f$  of 60 and  
424  $20 \text{ s}^{-1}$  may also be explained using the findings reported by Deng and Davé (2017). According

425 to these authors,  $D_f$  values strongly depend upon the initial velocities of the aggregates, where  
426 higher random velocities lead to more irregular shape, i.e. lower  $D_f$  values. Again, alum  
427 precipitate may determine aggregate characteristics.

428

#### 429 **4. CONCLUSIONS**

430 The results presented here may provide new insight on two-dimensional fractal  
431 dimensions of a large domain of aggregates, which are fundamental to separation processes  
432 since large aggregates hold most of the mass of contaminant and the majority of the energy  
433 applied in the flocculation process by mixers. The main conclusions of this study are:

- 434 - While considering different sub-ranges of large domains,  $D_f$  values varied with the  
435 lowest range of size, resulting in the highest  $D_f$  for each  $G_f$ ;
- 436 - The growth evolution of the longest length of aggregate ( $l_m$ ) and the change of the  $D_f$   
437 with time demonstrated that the dynamic steady-state was reached, but over different  
438 times for each shear rates and size ( $l_m$ ) ranges;
- 439 -  $D_f$  values for different times and shear rate were obtained, and the structure behavior  
440 of aggregates may be explained by aggregation mechanisms (like cluster-cluster for  $G_f$   
441 of  $60 \text{ s}^{-1}$  and like particle-cluster for  $G_f$  of  $20 \text{ s}^{-1}$ ). Also for large aggregates, the  
442 complex mechanism of restructuring probably leads to different structures and  $D_f$ . It is  
443 possible that precipitated alum determine large aggregates' characteristics.

444

#### 445 **ACKNOWLEDGMENTS**

446 The research was supported by the São Paulo Research Foundation (*Fundação de*  
447 *Amparo à Pesquisa do Estado de São Paulo – FAPESP*). Proc. 2010/50694-0; 2013/25641-9;  
448 2013/21355-1; 2013/01634-3. The authors dedicate this work in memory of Dr Marcelo De  
449 Julio who contributed to the advancement of the coagulation-flocculation research in Brazil.

450

451 **REFERENCES**

- 452 Becker, V., Schalauch, E., Behr, M., Briesen, H., 2009. Restructuring of colloidal aggregates  
453 in shear flows and limitations of the free-draining approximation. *Journal of Colloid and*  
454 *Interface Science*. 339 (2), 362-372. doi: <http://dx.doi.org/10.1016/j.jcis.2009.07.022>
- 455 Brasil, A. M., Farias, T. L., Carvalho, M. G., Koylu, U. O., 2001. Numerical characterization  
456 of the morphology of aggregated particles. *Aerosol Science*. 32, 489 -508.
- 457 Chakraborti, R.K., Atkinson, J.F., Van Benschoten, J.E., 2000. Characterization of alum floc  
458 by image analysis, *Environmental Science and Technology* 34, 3969–3976. doi:  
459 <http://dx.doi.org/10.1021/es990818o>
- 460 Chakraborti, R.K., Gardner, K.H., Atkinson, J.F., Van Benschoten., J.E., 2003. Changes in  
461 fractal dimension during aggregation. *Water Res.*, 37, 873–883.
- 462 Deng, X. Davé, R.N., 2017. Breakage of fractal agglomerates. *Chemical Engineering Science*,  
463 161, 117-126. <http://dx.doi.org/10.1016/j.ces.2016.12.018>
- 464 Gregory, J., 1997. The density of particle aggregates, *Water Science and Technology*. 36, 1–  
465 13.
- 466 Gregory, J., 2009. Monitoring particle aggregation processes. *Advances in Colloid and*  
467 *Interface Science* 147–148, 109–123. <http://dx.doi.org/10.1016/j.cis.2008.09.003>
- 468 He, W.P., Nan, J., Li, H.Y., Li, S.N, 2012. Characteristic analysis on temporal evolution of  
469 floc size and structure in low-shear flow. *Water Res.*, 46, 509–520.  
470 <http://dx.doi.org/10.1016/j.watres.2011.11.040>
- 471 Jarvis, P., Jefferson, B., Gregory, J., Parsons, S.A., 2005. A review of floc strength and  
472 breakage. *Water Res.*, 39, 3121–3137. <http://dx.doi.org/10.1016/j.watres.2005.05.022>
- 473 Johnson, C.P., Li, X., Logan, B.E., 1996. Settling velocities of fractal aggregates. *Environ.*  
474 *Sci. Technol.*, 30, 1911-1918.

475 Kim, S-H., Moon, B-H., Lee, H-I., 2001. Effects of pH and dosage on pollutant removal and  
476 floc structure during coagulation. *Microchemical Journal*. 68, 197-203

477 Lawler, D.F., 1997. Particle size distributions in treatment processes: theory and practice.  
478 Proceedings of the Fourth International Conference: The Role of Particle Characteristics  
479 in Separation Processes, IAWQ–IWSA Joint Specialist Group on Particle Separation,  
480 Jerusalem, 28–30 October 1996.

481 Li, T., Zhu, Z., Wang, D., Yao, C., Tang, H., 2006. Characterization of floc size, strength and  
482 structure under various coagulation mechanisms. *Power Technology*, 168, 104-110.  
483 <http://dx.doi.org/10.1016/j.powtec.2006.07.003>

484 Lin, M. Y., Lindsay, H.M., Weitz, D.A., Ball, R.C., Klein, R., Meakin, P., 1989. Universality  
485 in colloid aggregation. *Nature*, 339, 360-362.

486 Logan, B.E., Kilps, J.R., 1995. Fractal dimensions of aggregates formed in different fluid  
487 mechanical environments. *Water Res.* 29, 443-453.

488 Malekani K, Rice JA, Lin JS. 1996. Comparison of techniques for determining the fractal  
489 dimensions of clay minerals. *Clays Clay Miner.* 44, 677-685.

490 Moruzzi, R.B., Reali, M.A.P., 2007. *Método para determinação de distribuição de tamanho*  
491 *de microbolhas (DTMB) em sistemas flotação (FAD) para tratamento de águas*  
492 *utilizando a análise de imagem digital* [Method for measuring bubbles sizes distribution  
493 (BSD) in drinking water treatment flotation (DAF) systems by using digital image  
494 analysis], *Eng. Sanit. Ambient.* 12(3), 273–283. [http://dx.doi.org/10.1590/S1413-](http://dx.doi.org/10.1590/S1413-41522007000300007)  
495 [41522007000300007](http://dx.doi.org/10.1590/S1413-41522007000300007)

496 Moruzzi, R.B., Reali, M.A.P., 2010. Characterization of microbubble size distribution and  
497 flow configuration in DAF contact zone by a non-intrusive image analysis system and  
498 tracer tests, *Water Sci. Technol.* 61(1), 253–262. <http://dx.doi.org/10.2166/wst.2010.784>

499 Oliveira, A.L.de, Moreno, P., Silva, P.A.G. da, Julio, M.D., Moruzzi, R.B., 2015. Effects of  
500 the fractal structure and size distribution of flocs on the removal of particulate matter.  
501 Desalination and Water Treatment. Taylor & Francis, 1-12.  
502 <http://dx.doi.org/10.1080/19443994.2015.1081833>

503 Pádua, V.L., 1994. *Metodologia para determinação dos gradientes de velocidade médios em*  
504 *unidades de floculação de mistura completa com câmaras em série e escoamento*  
505 *contínuo a partir de reatores estáticos* [Methodology to determine mean velocity  
506 gradients in complete mixture flocculation units with series chambers and continuous  
507 flow from static reactors]. Thesis (Masters in Hydraulics and Sanitation), São Carlos,  
508 São Carlos School of Engineering, University of São Paulo (Escola de Engenharia de  
509 São Carlos, Universidade de São Paulo), p. 165.

510 Pedro, G., Sieffermann, G. 1979. Weathering of rocks and formation of soils. In: Siegel, F. R.  
511 (ed.), Review in Modern Problems of Geochemistry. UNESCO, pp. 39-55.

512 Spicer, P.T., Pratsinis, S.E., 1996. Shear-induced flocculation: The evolution of floc structure  
513 and the shape of the size distribution at steady state. *Water Res.*, 30, 1049–1056.

514 Sahu, O.P., Chaudhari, P.K., 2013. Review on Chemical treatment of Industrial Waste Water.  
515 *J. Appl. Sci. Environ. Manage.* 17(2) 241-257.

516 Vahedi, A., Gorczyca, B., 2012. Predicting the settling velocity of flocs formed in water  
517 treatment using multiple fractal dimensions, *Water Res.* 46, 4188 – 4194.  
518 <http://dx.doi.org/10.1016/j.watres.2012.04.031>

519 Waite, T.D., 1999. Measurement and implications of floc structure in water and wastewater  
520 treatment. *Colloid Surface. A* 151, 27–41.

521 Xu, W.Y., Gao, B.Y., Yue, Q.Y., Wang, Y., 2010. Effect of shear force and solution  
522 pH on flocs breakage and re-growth formed by nano-Al13 polymer, *Water Res.*  
523 44, 1893–1899. <http://dx.doi.org/10.1016/j.watres.2009.11.029>

524 Xu, W. , Gao, B., Yue, Q., Bo, X., 2011. Influence of pH on Floccs Formation, Breakage and  
525 Fractal Properties — The Role of Al<sub>13</sub>Polymer. Journal of Water Sustainability, Volume  
526 1, Issue 1. 45–57

527 Yang, Z. , Yang, H. , Jiang, Z., Huang, X. , Li, H. , Li, A., Cheng, R., 2013. A new method  
528 for calculation of flocculation kinetics combining Smoluchowski model with fractal  
529 theory. Colloids and Surfaces A: Physicochem. Eng. Aspects . 423, 11–19.  
530 <http://dx.doi.org/10.1016/j.colsurfa.2013.01.058>

531 Yukselen, M.A., Gregory, J., 2004. The reversibility of floc breakage, Int. J. Mineral Process.  
532 73, 251–259. [http://dx.doi.org/10.1016/S0301-7516\(03\)00077-2](http://dx.doi.org/10.1016/S0301-7516(03)00077-2)

533