Note: This is a pre-print version of the article. It may not exactly replicate the authoritative document to be published in Cognitive Processing. The publisher version of the article can be found at https://link.springer.com/article/10.1007/s10339-018-0867-4

Number line estimation and complex mental calculation: is there a shared cognitive process driving the two tasks?

Maria Montefinese[a,b,c,*] and Carlo Semenza[b,c]

[a] Department of Experimental Psychology, University College London, 26 Bedford Way, WC1H 0AP London, United Kingdom

[b] Department of Neuroscience, University of Padua, Via Giustiniani 5, 35128 Padua, Italy

[c] IRCCS San Camillo Hospital Foundation, Via Alberoni 70, 30126 Venice, Italy

*Corresponding author:

Maria Montefinese

Department of Experimental Psychology,

University College London,

London, United Kingdom

Email address: m.montefinese@ucl.ac.uk, maria.montefinese@gmail.com

Abstract

It is widely accepted that different number-related tasks, including solving simple addition and subtraction, may induce attentional shifts on the so-called mental number line, which represents larger numbers on the right and smaller numbers on the left. Recently, it has been shown that different number-related tasks also employ spatial attention shifts along with general cognitive processes. Here we investigated for the first time whether number line estimation and complex mental arithmetic recruit a common mechanism in healthy adults. Participants' performance in two-digit mental additions and subtractions using visual stimuli was compared with their performance in a mental bisection task using auditory numerical intervals. Results showed significant correlations between participants' performance in number line bisection and that in two-digit mental arithmetic operations, especially in additions, providing a first proof of a shared cognitive mechanism (or multiple shared cognitive mechanisms) between auditory number bisection and complex mental calculation.

Keywords: two-digit mental operations; number bisection **task**; mental number line; verbal working memory

Introduction

Space-number association has been represented by the metaphor of the mental number line (MNL), a horizontal representation of numerical magnitude in which larger numbers are associated with the right side of the line and smaller numbers with the left side (see recent review by Winter, Matlock, Shaki, & Fischer, 2015). A first demonstration of this association is the so-called Spatial-Numerical Association of Response Codes (SNARC) phenomenon. In magnitude comparison (e.g., indicating the numerically larger number of two presented numbers) or parity judgment (e.g., indicating whether a given number is odd or even) tasks, participants are faster at giving left hand responses to small numbers and right hand responses to large numbers (Dehaene, Bossini, & Giraux, 1993). Moreover, numerous eye-tracking studies have documented the impact of this horizontal spatial-numerical mapping on eye movements, which typically accompany shifts of visual attention (Fischer, Warlop, Hill, & Fias, 2004; Moeller, Fischer, Nuerk, & Willmes, 2009; Myachykov, Cangelosi, Ellis, & Fischer, 2015; Myachykov, Ellis, Cangelosi, & Fischer, 2016; Schwarz & Keus, 2004).

An important part of the research on space-number association has started with neuropsychological studies on patients with unilateral neglect, a deficit in attending and reporting stimuli on the controlesional side of space despite apparently normal visual perception, following brain lesions in the contralateral posterior parietal cortex (for a review see Corbetta & Shulman, 2011). For example, when asked to indicate the midpoint of a visual line, left neglect patients exhibit a rightward misplacement (e.g., Binder, Marshall, Lazar, Benjamin, & Mohr, 1992; Pizzamiglio, Committeri, Galati, & Patria, 2000). A pioneering study by Zorzi and colleagues (2002) revealed that left neglect patients showed a rightward shift in bisecting orally presented numerical intervals (e.g., saying that 8 is halfway between 5 and 9), with an error pattern that closely resembles that of the visual line bisection task (i.e., a greater rightward displacement of the midpoint with increasing interval lengths), suggesting the spatial nature of the MNL and its functional isomorphism to the visual physical line.

Importantly, a leftward bias in healthy adults has been found in the number interval bisection task analogous to that found in the line bisection task (see review by Jewell & McCourt, 2000), suggesting that participants employ the same spatial representation to solve both the number and line bisection tasks (Longo & Lourenco, 2007). In particular, the numerical bias increased with the magnitude of the numbers to be bisected (Longo & Lourenco, 2007) and changed significantly with the size of the numerical interval similarly to the line bisection task (Göbel et al., 2006). However, Rotondaro et al. (2015) failed to replicate the correlation between the two tasks when the numerical intervals were presented auditorily in the number bisection task, suggesting that a possible correlation between these two tasks could be induced only by the visual presentation of numerical stimuli.

Recently, this spatial-numerical congruence effect has been extended to arithmetic processes, by testing whether solving addition and subtraction may imply attentional shifts to the right and left of the space, respectively. For example, psychophysical studies of approximate mental calculation, where participants were asked to simply estimate (rather than actually calculate) the results of addition and subtraction operations, observed an over- and under-estimation for the results of additions and subtractions respectively (so-called operational momentum). This has been shown with operations presented mainly with sets of objects (e.g., Knops et al., 2009; McCrink et al., 2007), and also with problems without a carry operation or with zero as second operand (Lindemann & Tira, 2015). Furthermore, Pinhas et al. (2014) suggested that arithmetical operation signs might induce spatial associations (i.e., an association between the minus sign and leftward space; and between the plus sign and rightward space) that may in turn bias arithmetic performance. Crucially, in line with the idea of a space-number association, recent studies showed how left- and rightward body movements experienced by participants while carrying out subtractions and additions, respectively, influenced arithmetic calculation (Anelli, Lugli, Baroni, Borghi, & Nicoletti, 2014; Wiemers, Bekkering, & Lindemann, 2014).

To date, to the best of our knowledge, only a few studies have investigated the spatial nature of complex exact mental calculation, reporting attentional displacements induced by solving simple and complex operations (Masson & Pesenti, 2014, 2016). A recent rTMS study also showed that two-digit mental addition and subtraction causally involve cerebral areas of posterior parietal cortex to some degree, which subserve spatial attention as well (Montefinese, Turco, Piccione & Semenza, 2017). Taken together, these results suggest that complex mental calculation and number line estimation processes employ common brain areas underlying spatial attentional processes. For this reason, is it plausible to think that these processes are mediated by either a common spatial representation, akin to a mental number line or by more general cognitive processes?

Recently the space-number association has been questioned by dissociations in neglect patients' and healthy participants' performances in number and line bisection tasks (Rotondaro et al., 2015; Storer and Demeyere, 2014), supporting the idea that a common spatial representation is insufficient to capture the variety of number–space interactions across different tasks (van Dijck et al., 2012). Since both number bisection and complex mental calculation recruit several cognitive processes, a common spatial representation might be insufficient to capture the interaction between number bisection task and complex mental calculation.

Here, we sought to tackle this issue at the behavioural level, by directly comparing healthy young participants' performance in two-digit mental addition and subtraction with their performance in a mental number line bisection task. The aim is to find supporting evidence for the existence of a common cognitive process driving the two tasks. We will first test some of the classical effects described in the literature (e.g., worse performance for subtraction compared to addition and participants' bias in the number bisection task). We will then assess the association between the participants' performance in two-digit calculation and in mental number line bisection in order to understand whether a common cognitive mechanism is employed.

Method

Participants

Sixteen native Italian speakers (four males; mean age = 24.75 years, *SD* = 4.14 years) from the local university participated in this study. For the two-digit mental calculation tasks, the sample size was chosen based on an a-priori (sensitivity) power analysis (G*Power 3 software; Faul, Erdfelder, Buchner, & Lang, 2009) for *t* tests. The power analysis revealed that our sample size was large enough to detect a significant (α = .05) difference corresponding to an effect size (Cohen's *d* = .965) equivalent to a partial eta squared (η^2_p) of .49, as estimated based on a previous report (Masson & Pesenti, 2016), with a statistical power ($1 - \beta$) of .95. Moreover, for the number bisection task, an a-priori sensitivity analysis for *F* tests (Montefinese, Sulpizio, Galati, & Committeri, 2015; Montefinese, Zannino, & Ambrosini, 2015) revealed that our sample size was large enough to detect the significant highest interaction (α = .05), corresponding to an effect size (η^2_p) as small as .1 with a statistical power ($1 - \beta$) of .80. The mean years of education of the participants was 15.38 (*SD* = 1.75). Participants had normal or corrected-to-normal vision and were right-handed. The procedure was approved by the local Ethics Committee (IRCCS San Camillo Hospital Foundation, Venice, Italy) and performed in accordance with the ethical standards of the Declaration of Helsinki for human studies (World Medical Association, 2013). All participants gave written informed consent before participating in the study.

Apparatus and stimuli

Participants sat comfortably in a sound- and light-attenuated room, facing an LCD 16-in. laptop monitor (resolution: 640×480 pixels) at a distance of 57 cm, with the centre of the monitor aligned with their eyes. The presentation of stimuli was controlled by the E-prime software (Schneider, Eschman, & Zuccolotto, 2002), and participants' vocal responses were recorded by the internal microphone of the computer.

Mental two-digit calculations

In order to minimize the occurrence of confounders, several criteria for stimuli selection were adopted. In particular, the stimuli set consisted of two-digit additions and subtractions, in order to limit the use of automatic retrieval processes of arithmetic facts for one-digit calculations. The problems were presented in column format and were well within foveal dimensions (horizontal visual angle < 3°). This experimental manipulation prevented participants from making left-to-right eye movements that could have primed large numbers or addition solving (right side) or vice versa. The operations were presented in white font (24-point monospace Courier New font) on a black background (see Figure 1). The stimuli set was derived from a pilot study: an independent sample of fifteen participants (mean age: 23.65, *SD* = 3.26 years) mentally solved two-digit additions and subtractions (102 distinct problems for each of the two orders of operands) presented at the centre of the screen in separate blocks and provided the result verbally; the modality of presentation of the stimuli and the procedure were equal to that of the mental two-digit calculation task (see Figure 1 and the Procedure section for further details). For addition, both operands ranged from 23 to 75; the result (i.e., the sum) ranged from 47 to 98. The stimuli for subtraction were created by reversing those for addition (see below): the minuend was the addition result, the subtrahend was the second addition operand, and the result (i.e., the difference) was the first addition operand. To select stimuli as similar as possible in terms of performance across the operations, we performed a regression analysis on participants' log-transformed vocal response times (vRTs) for both operations. This analysis showed a positive linear correlation between addition and subtraction ($r = .191$, $p < .0087$, $R^2 = .037$). We chose to select the stimuli with at most one error in both operations and those with smaller residuals. The final set of stimuli was constituted by 80 total operations (40 distinct operations for addition and subtraction) for which the same relation between the operations was observed ($r = .823$, $p < .0001$, $R^2 = .677$).

Moreover, to minimize the occurrence of confounds, several criteria for stimuli selection were adopted. In particular, we included only two-digit additions and subtractions without carrying/borrowing, in order to eliminate confounds due to their greater difficulty. To match low-level stimulus properties, subtraction stimuli were created by reversing the operands/result of the addition stimuli. In line with previous studies (Avancini et al., 2014; Muluh et al., 2011; Zhou et al., 2006), we discarded operations with repeated operands (e.g., 23+23=46) and with one repeated digit between operands/result (e.g., 23+53=76; 76-53=23), since they have a privileged memory access compared to other operations (Ashcraft and Battaglia, 1978; Campbell and Gunter, 2002). Furthermore, operations containing “0” and “1” digits in the operands or result (e.g., 57–31=26, 42+20=62, 23+37=60) were not included in the stimulus set, because they involve rule-based operations (Jost et al., 2004; Lefevre et al., 1988; McCloskey et al., 1991). The first operand for addition (and result for subtraction) ranged from 23 to 74, the second operand from 23 to 75, and the result for addition (and first operand for subtraction) ranged from 57 to 98.

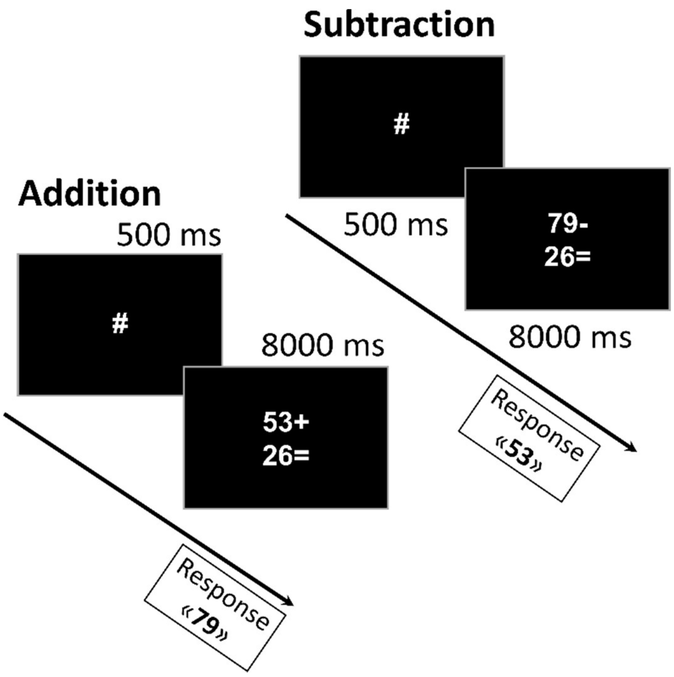

Figure 1. Time-course of a trial for two-digit mental additions and subtractions.

Mental bisection task of number intervals

The stimuli were the same as those used in previous studies (e.g., Priftis, Pitteri, Meneghello, Umiltà, & Zorzi, 2012; Priftis et al., 2006; Zorzi et al., 2002). Each trial involved the auditory presentation of a pair of number words defining a number interval (e.g., one – nine, nineteen – twelve). We decided to use auditory stimuli to avoid attentional bias related to visuospatial coding of visually presented stimuli. Moreover, the different presentation modalities between the calculation task and the number bisection task ensured that a common mechanism underlying performance in the two tasks does not depend on the similarity of the presentation modalities in the two tasks (visual and auditory, respectively for the calculation and number line estimation tasks). The length of the number intervals could be three (e.g., 21-23), five (e.g., 21-25), seven (e.g., 21-27), or nine (21-29), such that the midpoint was always an integer. Each number interval was presented within the units, teens, and twenties. The total set of stimuli included 48 forward number intervals (e.g., 23-27) divided in subsets of 16 units, teens, and twenties (see Figure 2). The same partition was performed for the 48 backward number intervals (e.g., 27-23).

The auditory stimuli were created with the DSpeech software (http://dimio.altervista.org/ita/), a text-to-speech synthesizer. It converted the written number intervals into the auditory ones generated as an Italian female voice with 32-bit resolution and sampling rate of 16 KHz. The average duration of the auditory stimuli was 2367 ms (*SD* = 324 ms). They were presented at a constant (but comfortable) intensity using stereo headphones.

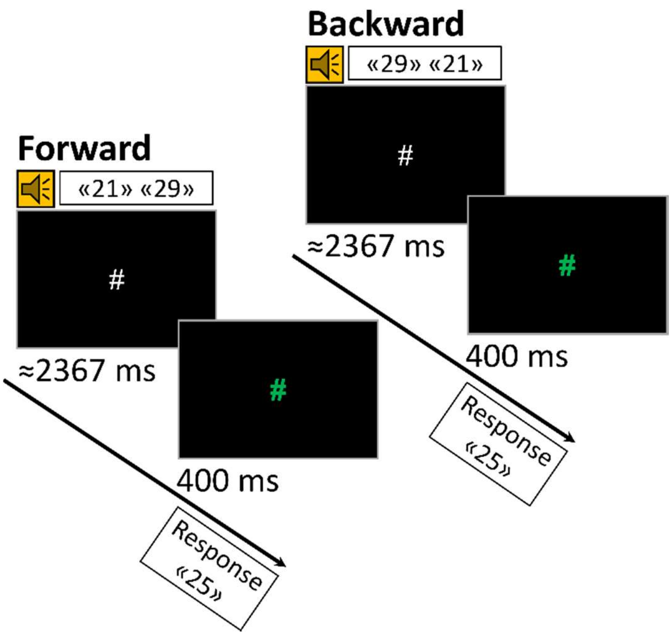

Figure 2. Time-course of a trial for both the orders (forward and backward) of number line estimation.

Procedure

Mental two-digit calculations

Participants had to mentally solve two-digit additions and subtractions and then verbally provide the result. To ensure that participants computed the result with the same calculation procedure, we asked them to explicitly calculate the result according to the procedure taught at school, that is, by first subtracting/adding the units and then the tens of each operation. Moreover, we chose to use a vocal response to avoid the use of a motor response which could affect the participant's performance.

The problems were presented one at a time at the centre of the screen for 8000 ms with an inter-trial interval (a white hash symbol in 24-point Courier New font on a black background) of 500 ms. Additions and subtractions were presented in separate blocks and the trial order within each block was randomized across participants. After twelve practice trials for each operation, participants performed one experimental block for each operation (presented in random order across participants). Each block comprised 40 two-digit operations (additions or subtractions) without carrying/borrowing.

Mental bisection task of number intervals

Participants were asked to name the midpoint number for each number interval (e.g., the midpoint number for the 22-28 interval is 25). To limit calculation strategies, we explicitly asked participants to estimate the midpoint number and not to calculate it. The auditory number intervals were presented while participants fixated a white hash symbol (24-point Courier New font) on a black background for the duration of the auditory stimulus. Next, a green hash symbol (24-point Courier New font) was presented for 400 ms, indicating participants to provide the result verbally.

After ten practice trials (5 for each order), participants performed two experimental blocks of 48 trials. In order to avoid cueing responses for participants, each number pair occurred only once in each block. The order of forward and backward trials as well as the length of number intervals were randomized within and across participants.

The high pressure to respond quickly (a 400 ms post-stimulus response window) prevented participants from adopting calculation strategies (i.e., to calculate the mean of the two numbers: add them and divide them by two). We also asked participants to explain the strategy that they used to solve the task after completing the experimental session; they all reported that they estimated the midpoint of the number interval rather than calculating it.

Data analysis

Mental two-digit calculations

Response latencies were measured from the onset of the stimulus to the beginning of the vocal response (audio captured via E-Prime's SoundIn function). Responses were recorded as WAV files, which were later analyzed using CheckVocal (Protopapas, 2007).

We logarithmically transformed the vRTs of correct responses to satisfy the assumption of normality for the analyses and calculated a robust estimation of central tendency for each condition of interest (Ambrosini & Vallesi, 2016; Rousseeuw & Verboven, 2002), as implemented by the mloclogist and madc functions in the LIBRA Matlab library (Verboven & Hubert, 2005, 2010). To assess possible differences between additions and subtractions, we performed two-tailed by-subjects ($t1$) and by-items ($t2$) t-tests on the vRTs and accuracy.

Mental bisection task of number intervals

In contrast to the mental calculation task, response latencies were measured from the offset of the stimulus to the beginning of the vocal response. We discarded all trials with incorrect or missing responses (12.63% of the trials). We logarithmically transformed the vRTs of the correct responses and calculated a robust estimation of central tendency (e.g., Verboven & Hubert, 2010). We carried out by-subjects repeated measures ANOVAs on the vRTs and accuracy data with order (forward, backward) and interval length (3, 5, 7, 9) as within-subjects factors, as well as the corresponding by-items repeated measures ANOVAs with order as within-items factor and the interval length factor as between-items factor. Post-hoc Newman–Keuls's tests were used when necessary. To assess the likely presence of an attentional bias in participants (Loftus, Nicholls, Mattingley, Chapman, & Bradshaw, 2009; Longo & Lourenco, 2007, 2010) we performed two-tailed by-subjects and by-items *t*-tests comparing whether the difference between the actual midpoint number and that provided by participants was affected by order (forward vs. backward). Positive and negative values indicate, respectively, shifts to the right and the left of the true midpoint. We also performed a series of regression analyses (Method 3 of Lorch & Myers, 1990) to test whether the participants' bisection errors were modulated either by the number interval length (3, 5, 7, 9) or by each of the numbers composing the number interval. One participant was excluded from this analysis because they did not make bisection errors.

Finally, for the main goal of this study, we performed correlational analyses on the participants' accuracy between the calculation and bisection tasks (it was not possible to perform correlations between the error biases observed in additions and subtractions and those observed in the forward and backward bisection of the MNL because of the paucity of errors). To compare the derived correlation coefficients, we used Pearson and Filon's *z* test (1898) as implemented in the *cocor* R package (Diedenhofen & Musch, 2015).

Results

Mental two-digit calculations

T-tests showed that participants were significantly slower in solving two-digit subtractions ($M = 7.94$, $SD = .46$) compared to additions ($M = 7.50$, $SD = .40$; $t1(15) = 10.089$, $p < .0001$, $d = 2.523$; $t2(39) = 69.587$, $p < .0001$, $d = 11.003$). These results were confirmed by the analysis on the participants' mean accuracy: Participants showed a significantly worse performance for subtractions ($M = .90$, $SD = .11$) as compared to additions ($M = .97$, $SD = .03$; $t1(15) = 3.163$, $p = .0064$, $d = .790$; $t2(39) = 5.637$, $p < .0001$, $d = .891$).

Mental bisection task of number intervals

The ANOVAs on the participants' vRTs revealed a significant main effect of order ($F1_{(1,15)} = 7.074$, $p = .018$, $\eta^2_p = .320$; $F2_{(1,44)} = 28.505$, $p < .0001$, $\eta^2_p = .393$), with slower vRTs for the backward order compared to the forward one. In addition, interval length was also significant ($F1_{(1,15)} = 40.822$, $p < .0001$, $\eta^2_p = .731$; $F2_{(3,44)} = 1507.752$, $p < .0001$, $\eta^2_p = .990$). The post-hoc analysis showed that participants' vRTs significantly increased ($ps < .0002$) as the interval length increased from 3 to 7 (i.e., 3 < 5 < 7). No difference emerged between the intervals 5 and 9 ($ps > .2058$) due to the decrease of the vRTs in the interval 9. The interaction between order and interval length was significant by-items ($F2_{(3, 44)} = 6.636$, $p = .0008$, $\eta^2_p = .312$) (see Figure 3) but not by-subjects ($F1(_{3, 45)} = .908$, $p = .4445$, $\eta^2_p = .057$). This two-way interaction was due to the fact that the significant effect of order reported above was modulated by the interval length value. Indeed, the post-hoc analysis revealed that this interaction was driven by the fact that the participants' vRTs were significantly different for the forward and backward orders in interval 3 ($p = .0001$), 5 ($p = .0130$) and 7 ($p = .0001$). In contrast, no difference emerged between forward and backward orders in interval 9 ($p = .1242$).

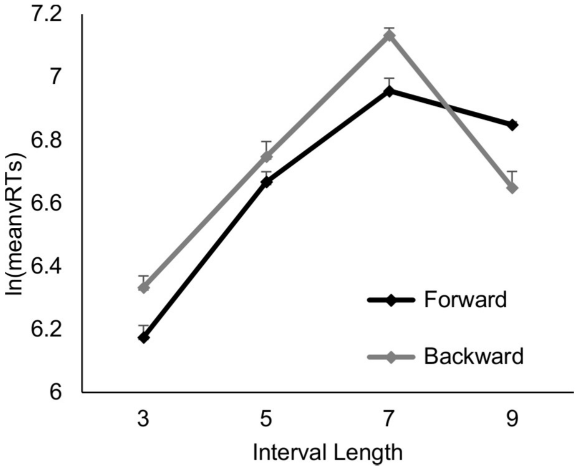

Figure 3. Log-trasformed vocal response times before the calculation of the robust estimation of central tendency are shown as a function of order (backward, forward) and interval length (3, 5, 7, 9) factors. Error bars indicate within-subjects standard error of the mean (Morey, 2008). Note that some are so small that the corresponding error bars are hidden behind the symbol on the graph.

The results on vRTs were partially confirmed by the ANOVAs on accuracy. As in the vRT analysis, we found a significant main effect of interval length ($F1_{(3,45)} = 29.564$, $p < .0001$, $\eta^2_p = .663$; $F2_{(3,44)} = 52.595$, $p < .0001$, $\eta^2_p = .782$). Post-hoc analyses showed that accuracy tended to decrease from interval 3 to interval 7 (3 vs. 5: ps $< .0972$; 5 vs. 7: $p < .0002$). No difference emerged between intervals 5 and 9 ($ps > .1407$), due to the increase of accuracy in interval 9 (see Table 1 for the descriptive statistics). Neither the main effect of order ($F1_{(1,15)} = .111$, $p = .7435$, $\eta^2_p = .007$; $F2_{(1,44)} = .072$, $p = .7900$, $\eta^2_p = .002$) nor the two-way interaction ($F1_{(3,45)} = 1.101$, $p = .3586$, $\eta^2_p = .068$; $F2_{(3,44)} = 1.204$, $p = .3195$, $\eta^2_p = .076$) were significant.

T**able 1.** Descriptive statistics for the number line bisection task.

	vRTs [ln(ms)]		**Accuracy (%)**	
	M	*SD*	*M*	*SD*
Order				
Forward	6.67	0.66	83.85	21.73
Backward	6.77	0.62	82.85	19.49
Interval Length				
3	6.24	0.55	97.17	5.78
5	6.17	0.54	90.21	11.73
7	7.11	0.48	60.76	18.93
9	6.82	0.68	84.38	20.71
Interaction				
Forward-3	6.16	0.51	98.21	4.88
Forward-5	6.67	0.53	89.58	13.55
Forward-7	7.05	0.48	58.33	23.13
Forward-9	6.82	0.78	87.50	16.67
Backward-3	6.32	0.59	96.13	6.56
Backward-5	6.76	0.55	90.83	10.00
Backward-7	7.17	0.48	63.19	13.89
Backward-9	6.82	0.60	81.25	24.25

Participants' estimation bias

We then assessed whether participants showed an over- or under-estimation bias in the bisection of numerical interval. First, participants' errors in bisecting the number interval were significantly different between the two orders (forward: $M = .02$, $SD = .09$ vs. backward: $M = -.05$, $SD = .06$; $t1_{(15)} = 3.493$, $p = .0033$, $d = .873$; $t2_{(98)} = 2.502$, $p = .0141$, $d = .255$). When performing the backward order trials, participants significantly underestimated the midpoint (one-sample $t1_{(15)} = -3.220$, $p = .006$, $d = -.81$; $t2_{(95)} = -2.672$, $p = .0089$, $d = -.27$), while in the forward order trials they did not overestimate the midpoint ($t1_{(15)} = -1.031$, $p = .3190$, $d = .258$; $t2_{(95)} = .668$, $p = .5054$, $d = .068$).

These results seem to support the idea that participants had a bias towards the last number they heard. To test this hypothesis, we coded the participants' errors as the bias towards the last number heard (i.e., by coding the errors toward the first numbers heard by participants as negative numbers and those toward the last numbers heard by participants as positive numbers). Next, we performed *t*-test comparisons on the mean of such re-coded errors. Results confirmed our hypothesis by showing a significant bias towards the last number heard by the participants ($t1_{(15)} = 3.493$, $p = .0033$, $d = .873$; $t2_{(95)} = 2.502$, $p = .0141$, $d = .255$).

Effects of the interval length and numbers composing intervals on the bisection bias

Next, we tested the effect of the interval length and that of the numbers composing the number interval on the participants' bisection bias. Firstly, the participants' performance was not affected by the number interval length (mean $\beta = .18$, $SD = .56$, $t_{(14)} = 1.259$, $p = .2295$, $d = .325$), suggesting that this variable was not a reliable predictor of the participants' bias errors. Nonetheless, for the forward order, participants showed an effect of both the first (mean $\beta = -.08$, *SD* = .13, $t_{(14)}$ = -2.843, *p* = .0123, *d* = -.711) and second number (mean β = -.07, *SD* = .14, $t_{(14)}$ = -2.153, *p* = .0480, *d* = -.539) composing the number interval, highlighting that the bias error shifted leftward as the magnitude of numbers composing the intervals increased. This was not true for the backward order ($ts_{(14)}$ < -.574, *p*s > .5747, *d*s < .143).

Relation between mental calculation and bisection tasks

We carried out correlational analyses between the participants' performance in the calculation task and that in the mental bisection task. The vRTs for both addition and subtraction tasks were significantly correlated to that in the number bisection task, for both orders (all *N*s = 16, *r*s > .645, all *p*s < .0070, R^2s > .416). These correlations were not significantly different from each other (*z* = -1.672, *p* = 0.0945). Instead, accuracy for addition was significantly correlated to that for the number bisection task with both orders (*N* = 16, *r* = .616 and .524, *p* = .0110 and .0359, R^2 = .379 and .275, for the forward and backward order, respectively), whereas the accuracy for subtraction was not significantly correlated to either the forward (*N* = 16, *r* = .472, *p* = .0649, R^2 = .223) or backward (*N* = 16, *r* = .197, *p* = .4646, R^2 =.039) orders. Moreover, this latter correlation was significantly different from that between addition and forward order (*z* = 1.967, *p* = 0.0491).

Discussion

In this study, we tested whether there is a common cognitive mechanism in complex mental calculation and number line estimation: until now, there has been little evidence from which only indirect conclusions could be drawn. Our behavioural results showed a relation between two-digit mental operations and bisection of auditory numerical intervals, suggesting that these two tasks are driven by a shared cognitive mechanism (or multiple shared cognitive mechanisms) employed in both number bisection and complex mental calculation tasks. Because all participants reported that they solved the number line bisection task by estimating the number midline, we can rule out the possibility that the relation we found between the two numerical tasks was due to the use of a common calculation strategy to solve both tasks. We thus propose that our results occur because both two-digit addition and subtraction rely on a visuospatial strategy. Calculating the result of an arithmetic operation implies attention shifting to the left or right side of a spatial representation (for subtractions and additions, respectively), akin to a mental number line (Masson and Pesenti, 2014, 2016). This idea is supported by the fact that brain areas usually involved in visuospatial attention (Dehaene & Cohen, 2007) are involved to some extent in line (Fierro et al., 2000) and number (Göbel, Calabria, Farnè, & Rossetti, 2006) bisection tasks, as well as complex mental arithmetic (Montefinese et al., 2017). Unlike studies on operational momentum (e.g., Lindemann & Tira, 2015; Marghetis et al., 2014), we could not compare the participants' bias (i.e., an over- or under-estimation of the correct result) in both arithmetic and bisection of numerical intervals tasks because of the small number of errors in our calculation task. Although this could limit the impact of our results, it is worth noting that studies on operational momentum (e.g., Lindemann & Tira, 2015; Marghetis et al., 2014) used either one-digit operations, that are believed to be based on automatic retrieval processes, or a result verification task based mostly on a subjective sensation (i.e., "feeling right" the correct results), rather than on actual calculation procedures. We tried to overcome these drawbacks by asking participants to mentally solve two-digit additions and subtractions and to provide the result verbally, without a cued result.

This relation between two-digit operations and number bisection task was more evident for additions, supporting the view of a greater use of a spatial strategy in this kind of operation than with subtraction. It has already been shown that gestures and spatial mapping can support arithmetic learning (Goldin-Meadow, Cook, & Mitchell, 2009; Wiemers et al., 2014), and Anelli and colleagues (2014) found a congruence effect between the type of motion performed by the participants and the type of calculation made (i.e., leftward motion – subtraction; rightward motions– additions), especially for the addition. This result might be due to the fact that the addition is taught at school to a greater extent (Barrouillet et al., 2008) and that participants with greater mathematical expertise use a mostly visuospatial strategy (Marghetis et al., 2014). Moreover, brain areas of posterior parietal cortex involved in visuospatial processes have been found to play a greater role in solving two-digit mental additions compared to two-digit mental subtractions (Montefinese et al., 2017). However, the larger effect we found for addition compared to subtraction might be due to fact that we used relatively large numbers. Consistent with this explanation, Masson and colleagues (2014) observed that attentional shifts influenced the speed of detecting a target presented on the left or right of the screen specifically when participants solved one-digit subtractions and two-digit additions, respectively (see Masson & Pesenti, 2014; 2016). Along the same line of reasoning, it is possible that no attentional shift to the left occurred when solving our subtraction problems because all numbers involved in our problems were large. A non-spatial explanation of our results suggests the role of other common cognitive processes between complex calculation and number line estimation (such as, e.g., verbal working memory; van Dijck et al., 2012), although our task does not distinguish the specific contribution of attentional shifts on the MNL from that of verbal working memory.

We also observed that participants under- and over-estimated the midpoint of the numerical interval when performing the backward and forward order trials, respectively. A non-spatial account of this bias might invoke easier memory access to last heard numbers (i.e., a sort of recency memory effect) (Cattaneo et al., 2011; Ranzini et al., 2017), suggesting that the participants' errors in bisecting the number interval might be tied to the shift of attention on the last number heard and consequently, to the sensory modality of presentation. This result merits further investigation to assess whether misplacement in the auditory mental number bisection task may have been driven by recency memory effect in previous reports.

We also investigated the effect of the interval length and number size on the bisection bias on the MNL. On the one hand, we found that the participants' performance was not affected by number interval length, suggesting that this variable was not a reliable predictor of the participants' bias errors. Consistent with these results, Longo and Lourenco (2007) found no relation between line length and pseudoneglect in the physical line bisection task. Together, these results tell a convergent story: bias error does not increase with the increase of difficulty. On the other hand, we found an effect of both the first and second numbers composing the numerical intervals, highlighting that the bias error shifted leftward as the magnitude of numbers composing the intervals increased, but only for the forward order. A spatial interpretation surmises an attentional advantage for the processing of small numbers compared to large ones (e.g., Loetscher & Brugger, 2007; Loetscher et al., 2008), which translates to a leftward bias along the MNL in healthy participants (e.g., Longo & Lourenco, 2007) as that on the physical line (Jewell & McCourt, 2000). Our results are also consistent with the idea of a logarithmic compression of the MNL, suggesting that the attentional bias follows Weber's law: the discriminability of two numbers decreases as the magnitude of numbers increases (i.e., the numerical size effect), since the greater numbers are closer along the MNL (Dehaene & Mehler, 1992; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Izard & Dehaene, 2008; Knops et al., 2009; Nieder & Miller, 2003; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004).

Finally, we confirmed some classical effects in literature on the calculation and MNL. In particular, we found better performance in solving two-digit mental additions compared to subtractions, as well as in estimating the midpoint of numerical intervals presented in the forward order compared to those presented in the backward order. This could be because that addition is used more frequently in daily life compared to subtraction (Kong et al., 2005). Similarly, facilitation for the forward order may occur because it follows the traditional spatial representation of numbers in Western populations.

Interestingly, we also observed a significant interaction between order and interval length. This interaction was due to the greater difficulty related to the increase of number interval size (exacerbated in the backward order) which disappeared for interval 9, given by the better mapping of this number interval in the MNL, which also facilitates the transposition of the number interval from backward to forward order. Indeed, as pointed by Pinhas and Fischer (2008; see note 1), the edges or midpoint of interval 9 might be much easier to attain compared to the midpoint of interval 7 because the numbers composing this interval represent the start point and endpoint of each ten, and might be better mapped onto the MNL. This result is analogous to those of Rotondaro and colleagues (2015), who observed a similar effect of length interval in large samples of participants, strengthening the validity of our result (see Fig. 2B, pag. 568 by Rotondaro et al., 2015).

Conclusion

In the present study, we provided evidence for the relation between two-digit mental addition and subtraction, and a mental number line estimation task, suggesting that a shared cognitive mechanism (or multiple shared cognitive mechanisms) underlies these two tasks. More importantly, we observed a stronger relation between the performance in two-digit mental addition and that in the number bisection task compared to that involving subtraction, highlighting that addition is more related to the use of a common spatial representation. This interpretation is in line with the idea that number processing is formed into cultural spatial experience.

Acknowledgements

This work was supported by the University of Padua (CPDA131328 and NEURAT STPD11B8HM_004).

Conflict of Interest: The authors declare that they have no conflict of interest.

References

Ambrosini, E., & Vallesi, A. (2016). Asymmetry in prefrontal resting-state EEG spectral power underlies individual differences in phasic and sustained cognitive control. *NeuroImage, 124,* 843-857.

Anelli, F., Lugli, L., Baroni, G., Borghi, A. M., & Nicoletti, R. (2014). Walking boosts your performance in making additions and subtractions. *Frontiers in psychology, 5.*

Ashcraft, M. H., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. *Journal of Experimental Psychology: Human Learning and Memory, 4*(5), 527.

Association, W. M. (2013). World Medical Association Declaration of Helsinki: ethical principles for medical research involving human subjects. *Jama, 310*(20), 2191.

Avancini, C., Galfano, G., & Szűcs, D. (2014). Dissociation between arithmetic relatedness and distance effects is modulated by task properties: An ERP study comparing explicit vs. implicit arithmetic processing. *Biological psychology, 103,* 305-316.

Barrouillet, P., Mignon, M., & Thevenot, C. (2008). Strategies in subtraction problem solving in children. *Journal of Experimental Child Psychology, 99*(4), 233-251.

Binder, J., Marshall, R., Lazar, R., Benjamin, J., & Mohr, J. (1992). Distinct syndromes of hemineglect. *Archives of Neurology, 49*(11), 1187.

Campbell, J. I., & Gunter, R. (2002). Calculation, culture, and the repeated operand effect. *Cognition, 86*(1), 71-96.

Cattaneo, Z., Fantino, M., Silvanto, J., Vallar, G., & Vecchi, T. (2011). Tapping effects on numerical bisection. *Experimental brain research, 208*(1), 21-28.

Corbetta, M., & Shulman, G. L. (2011). Spatial neglect and attention networks. *Annual review of neuroscience, 34,* 569.

Dehaene, S., & Cohen, L. (2007). Cultural recycling of cortical maps. *Neuron, 56*(2), 384-398.

Dehaene, S., & Mehler, J. (1992). Cross-linguistic regularities in the frequency of number words. *Cognition, 43*(1), 1-29.

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*(3), 371.

Diedenhofen, B., & Musch, J. (2015). Cocor: A comprehensive solution for the statistical comparison of correlations. *PloS one, 10*(4), e0121945.

Faul, F., Erdfelder, E., Buchner, A., & Lang, A.-G. (2009). Statistical power analyses using G* Power 3.1: Tests for correlation and regression analyses. *Behavior research methods, 41*(4), 1149-1160.

Fierro, B., Brighina, F., Oliveri, M., Piazza, A., La Bua, V., Buffa, D., & Bisiach, E. (2000). Contralateral neglect induced by right posterior parietal rTMS in healthy subjects. *Neuroreport, 11*(7), 1519-1521.

Fischer, M. H., Warlop, N., Hill, R. L., & Fias, W. (2004). Oculomotor bias induced by number perception. *Experimental psychology, 51*(2), 91-97.

Göbel, S. M., Calabria, M., Farnè, A., & Rossetti, Y. (2006). Parietal rTMS distorts the mental number line: simulating 'spatial'neglect in healthy subjects. *Neuropsychologia, 44*(6), 860-868.

Goldin-Meadow, S., Cook, S. W., & Mitchell, Z. A. (2009). Gesturing gives children new ideas about math. *Psychological Science, 20*(3), 267-272.

Halberda, J., Ly, R., Wilmer, J. B., Naiman, D. Q., & Germine, L. (2012). Number sense across the lifespan as revealed by a massive Internet-based sample. *Proceedings of the National Academy of Sciences, 109*(28), 11116-11120.

Hoffman, J. E., & Subramaniam, B. (1995). The role of visual attention in saccadic eye movements. *Perception & psychophysics, 57*(6), 787-795.

Izard, V., & Dehaene, S. (2008). Calibrating the mental number line. *Cognition, 106*(3), 1221-1247.

Jewell, G., & McCourt, M. E. (2000). Pseudoneglect: a review and meta-analysis of performance factors in line bisection tasks. *Neuropsychologia, 38*(1), 93-110.

Jost, K., Beinhoff, U., Hennighausen, E., & Rösler, F. (2004). Facts, rules, and strategies in single-digit multiplication: evidence from event-related brain potentials. *Cognitive Brain Research, 20*(2), 183-193.

Knops, A., Viarouge, A., & Dehaene, S. (2009). Dynamic representations underlying symbolic and nonsymbolic calculation: Evidence from the operational momentum effect. *Attention, Perception, & Psychophysics, 71*(4), 803-821.

Kong, J., Wang, C., Kwong, K., Vangel, M., Chua, E., & Gollub, R. (2005). The neural substrate of arithmetic operations and procedure complexity. *Cognitive Brain Research, 22*(3), 397-405.

Lefevre, J.-A., Bisanz, J., & Mrkonjic, L. (1988). Cognitive arithmetic: Evidence for obligatory activation of arithmetic facts. *Memory & Cognition, 16*(1), 45-53.

Lindemann, O., & Tira, M. D. (2015). Operational momentum in numerosity production judgments of multi-digit number problems. *Zeitschrift für Psychologie.*

Loetscher, T., & Brugger, P. (2007). Exploring number space by random digit generation. *Experimental Brain Research, 180*(4), 655-665.

Loetscher, T., Schwarz, U., Schubiger, M., & Brugger, P. (2008). Head turns bias the brain's internal random generator. *Current Biology, 18*(2), R60-R62.

Longo, M. R., & Lourenco, S. F. (2007). Spatial attention and the mental number line: Evidence for characteristic biases and compression. *Neuropsychologia, 45*(7), 1400-1407.

Longo, M. R., & Lourenco, S. F. (2010). Bisecting the mental number line in near and far space. *Brain and cognition, 72*(3), 362-367.

Lorch, R. F., & Myers, J. L. (1990). Regression analyses of repeated measures data in cognitive research. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 16*(1), 149.

Marghetis, T., Núñez, R., & Bergen, B. K. (2014). Doing arithmetic by hand: Hand movements during exact arithmetic reveal systematic, dynamic spatial processing. *The Quarterly Journal of Experimental Psychology, 67*(8), 1579-1596.

Masson, N., & Pesenti, M. (2014). Attentional bias induced by solving simple and complex addition and subtraction problems. *The Quarterly Journal of Experimental Psychology, 67*(8), 1514-1526.

Masson, N., & Pesenti, M. (2016). Interference of lateralized distractors on arithmetic problem solving: a functional role for attention shifts in mental calculation. *Psychological Research, 80*(4), 640-651.

McCloskey, M., Aliminosa, D., & Sokol, S. M. (1991). Facts, rules and procedures in normal calculation: Evidence from multiple single-patient studies of impaired arithmetic fact retrieval. *Brain and cognition, 17*(2), 154-203.

McCrink, K., Dehaene, S., & Dehaene-Lambertz, G. (2007). Moving along the number line: Operational momentum in nonsymbolic arithmetic. *Perception & psychophysics, 69*(8), 1324-1333.

Moeller, K., Fischer, M., Nuerk, H.-C., & Willmes, K. (2009). Eye fixation behaviour in the number bisection task: evidence for temporal specificity. *Acta psychologica, 131*(3), 209-220.

Montefinese, M., Sulpizio, V., Galati, G., & Committeri, G. (2015). Age-related effects on spatial memory across viewpoint changes relative to different reference frames. *Psychological research, 79*(4), 687-697.

Montefinese, M., Turco, C., Piccione, F., & Semenza, C. (2017). Causal role of the posterior parietal cortex for two-digit mental subtraction and addition: a repetitive TMS study. *NeuroImage, 155,* 72-81.

Montefinese, M., Zannino, G. D., & Ambrosini, E. (2015). Semantic similarity between old and new items produces false alarms in recognition memory. *Psychological research, 79*(5), 785-794.

Morey, R.D. (2008). Confidence intervals from normalized data: a correction to Cousineau (2005). *Tutorial in Quantitative Methods for Psychology 4,* 61–64.

Muluh, E., Vaughan, C., & John, L. (2011). High resolution event-related potentials analysis of the arithmetic-operation effect in mental arithmetic. *Clinical Neurophysiology, 122*(3), 518-529.

Myachykov, A., Cangelosi, A., Ellis, R., & Fischer, M. H. (2015). The oculomotor resonance effect in spatial–numerical mapping. *Acta psychologica, 161,* 162-169.

Myachykov, A., Ellis, R., Cangelosi, A., & Fischer, M. H. (2016). Ocular drift along the mental number line. *Psychological research,* 1-10.

Nieder, A., & Miller, E. K. (2003). Coding of cognitive magnitude: Compressed scaling of numerical information in the primate prefrontal cortex. *Neuron, 37*(1), 149-157.

Pearson, K., & Filon, L. N. G. (1898). Mathematical contributions to the theory of evolution. IV. On the probable errors of frequency constants and on the influence of random selection on variation and correlation. *Philosophical Transactions of the Royal Society of London.* Series A, Containing Papers of a Mathematical or Physical Character, 191, 229-311.

Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron, 44*(3), 547-555.

Pinhas, M., & Fischer, M. H. (2008). Mental movements without magnitude? A study of spatial biases in symbolic arithmetic. *Cognition, 109*(3), 408-415.

Pinhas, M., Shaki, S., & Fischer, M. H. (2014). Heed the signs: Operation signs have spatial associations. *The Quarterly Journal of Experimental Psychology, 67*(8), 1527-1540.

Pizzamiglio, L., Committeri, G., Galati, G., & Patria, F. (2000). Psychophysical properties of line bisection and body midline perception in unilateral neglect. *Cortex, 36*(4), 469-484.

Priftis, K., Pitteri, M., Meneghello, F., Umiltà, C., & Zorzi, M. (2012). Optokinetic stimulation modulates neglect for the number space: evidence from mental number interval bisection. *Frontiers in Human Neuroscience, 6*(23.10), 3389.

Priftis, K., Zorzi, M., Meneghello, F., Marenzi, R., & Umiltà, C. (2006). Explicit versus implicit processing of representational space in neglect: Dissociations in accessing the mental number line. *Journal of Cognitive Neuroscience, 18*(4), 680-688.

Protopapas, A. (2007). Check Vocal: A program to facilitate checking the accuracy and response time of vocal responses from DMDX. *Behavior Research Methods, 39*(4), 859-862.

Ranzini, M., Carbè, K., & Gevers, W. (2017). Contribution of visuospatial attention, short-term memory and executive functions to performance in number interval bisection. *Neuropsychologia, 99,* 225-235.

Rotondaro, F., Merola, S., Aiello, M., Pinto, M., & Doricchi, F. (2015). Dissociation between line bisection and mental-number-line bisection in healthy adults. *Neuropsychologia, 75*, 565-576.

Rousseeuw, P. J., & Verboven, S. (2002). Robust estimation in very small samples. *Computational Statistics & Data Analysis, 40*(4), 741-758.

Schneider, W., Eschman, A., & Zuccolotto, A. (2002). E-Prime: User's guide: Psychology Software Incorporated.

Schwarz, W., & Keus, I. M. (2004). Moving the eyes along the mental number line: Comparing SNARC effects with saccadic and manual responses. *Perception & Psychophysics, 66*(4), 651-664.

van Dijck, J. P., Gevers, W., Lafosse, C., & Fias, W. (2011). The heterogeneous nature of number–space interactions. *Frontiers in human neuroscience, 5,* 182.

Storer, L., Demeyere, N., 2014. Disruptions to number bisection after brain injury: neglecting parts of the mental Number Line or short-term memory impairments?. *Brain & Cognition, 86,* 116–123.

Verboven, S., & Hubert, M. (2005). LIBRA: a MATLAB library for robust analysis. *Chemometrics and intelligent laboratory systems, 75*(2), 127-136.

Verboven, S., & Hubert, M. (2010). Matlab library LIBRA. *Wiley Interdisciplinary Reviews: Computational Statistics, 2*(4), 509-515.

Wiemers, M., Bekkering, H., & Lindemann, O. (2014). Spatial interferences in mental arithmetic: Evidence from the motion–arithmetic compatibility effect. *The Quarterly Journal of Experimental Psychology, 67*(8), 1557-1570.

Winter, B., Matlock, T., Shaki, S., & Fischer, M. H. (2015). Mental number space in three dimensions. *Neuroscience & Biobehavioral Reviews, 57,* 209-219.

Zhou, X., Chen, C., Dong, Q., Zhang, H., Zhou, R., Zhao, H., . . . Guo, Y. (2006). Event-related potentials of single-digit addition, subtraction, and multiplication. *Neuropsychologia, 44*(12), 2500-2507.

Zorzi, M., Priftis, K., & Umiltà, C. (2002). Brain damage: neglect disrupts the mental number line. *Nature, 417*(6885), 138-139.