

1 **Operation stability analysis of district heating substation from the** 2 **control perspective**

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15 **Highlights**

- 16 ✓ Oscillatory of flow rate observed in district heating substation
17 ✓ Mathematical model describing thermal dynamics of heating substation
18 ✓ Control theory based criterion for operation stability of heating substation
19 ✓ Conditions that leads to operation instability of district heating substation
20 ✓ Controller tuning of the plate heat exchanger to ensure robust stability

22 **Keywords**

23 District heating substation; Mathematical model; Operation instability; Stability
24 analysis; Feedback control; Plate heat exchanger

26 **Abstract**

27 Since the heating substation plays a key role in transferring the thermal energy

28 from the primary network to the secondary network and controlling the heat output of
29 district heating system to meet the thermal load, high operation performance of
30 heating substation is essential for energy conservation, cost saving and emission
31 reduction. The dynamic operation stability of heating substation is a very important
32 dynamic characteristic of heating substation and largely affects the operation
33 efficiency of district heating system. The operation instability of heating substation
34 mainly manifest as flow rate and pressure oscillations, which will deteriorate the
35 network hydraulic condition, break the network thermal balance, reduce the consumer
36 comfort and increase the energy cost of the pumping system. Since heating
37 substations will easily operate unstably under some conditions, this paper presents a
38 theoretical method to analyze the stability and retune the feedback controller for
39 operation stability of heating substation. Mathematical model of the plate heat
40 exchanger was established and the feedback control theory was adopted to study the
41 operation stability of heating substation. Based on the mathematical model and
42 feedback control theory, a stability criterion was proposed for analyzing the operation
43 stability of district heating substation effectively. The dynamic model of plate heat
44 exchanger was validated with measured data. Simulation results show that controller
45 tuned at certain operating condition can't ensure operation stability of heating
46 substation, when operating condition varies in large range. The stability analysis
47 method proposed in this paper can be applied to analyzing the operation stability and
48 tuning the controller of heating substation to enhance the operation stability.

49

Nomenclature	
A	matrix of state space model
b	width of the plate heat exchanger flow channel (m)
B_1, B_2, B_3, B_4	matrices of linearized state space model
C	matrix of linearized state space model
c_p	specific heat capacity of water (W/kg · K)
C_v	flow capacity of control valve
C_{Nu}	the empirical parameter
d	distance between neighboring plates (m)
D	hydraulic diameter (m)
$G(s), G_{d,1}(s), G_{d,2}(s), G_{d,3}(s), G_{d,4}(s)$	transfer function of linearized plate heat exchanger model
i	$\sqrt{-1}$
k	overall heat transfer coefficient (W/m ² · K)
K	transfer function of controller
k_c	controller gain
k_v	valve gain
l	length of flow channel
$L(i\omega)$	loop transfer function
M	number of flow channels in each side
N	number of the control volumes in a flow channel
n_1, n_2	the empirical parameter
Nu	the Nusselt number
Pr	the Prantl number
q	volume flow rate (m ³ /s)
R	the rangeability of the valve
Re	the Reynolds number
s	the Laplace variable
t	time (s)
T	temperature (°C)
x	coordinate along the flow channel (m)
z	controller zero
λ	heat conductivity coefficient (W/m ² · K)
μ	the dynamic viscosity
ρ	density of the water (kg/m ³)
τ	time delay of temperature sensor (s)
δ	small deviation & increment symbol of variable
ω	frequency (rad/s)

Nomenclature	
Δx	length of a control volume (m)
Subscripts	
h	high temperature side
in	inlet
l	low temperature side
out	outlet

50

51 **1. Introduction**

52 In China, The total energy consumption of district heating systems in northern
53 areas covers 24% of the total energy cost of building energy systems [1]. Therefore,
54 improving the operation efficiency of district heating system is important to reducing
55 energy cost and enhancing room comfort. In large scale district heating systems, the
56 heating substations are the terminals, which control the heat outputs to the secondary
57 networks. Efficient regulation of the district heating network relies on effective
58 operation and control of the heating substation.

59 There have been numerous researches on improvement of heating substation
60 efficiency by applying new control strategy. Gustafsson et al. [2] developed a new
61 control approach for indirectly connected district heating substations based on a
62 physical model, which maximizes the ΔT of the district heating network. They also
63 verified the control method experimentally through implementation of the control
64 method in a real district heating substation; the results confirms that it is possible to
65 control the radiator system based on the primary supply temperature while
66 maintaining comfort; however, conclusions regarding improvements in ΔT were hard
67 to distinguish [3]. Since high return temperature will lead to large amount of overall

68 distribution energy cost, the temperature difference faults can be detected and
69 eliminated by using fault detection approaches. Gadd and Werner [4, 5] presented a
70 fault detection based method to achieve low return temperatures in district heating
71 substations.

72 Modeling the heating substation is important to analyzing and evaluating the
73 operation performance of heating substation. Brand et al. [6] developed a numerical
74 model for heating substation, which takes into consideration the effect of service
75 pipes. With this model, they studied the effects of service pipe on waiting time for
76 DHW, heat loss, and overall cost. Brand et al. [7] also used the commercial software
77 IDA-ICE and Termis to model and analyze various solutions for controlling the
78 redirected bypass flow and evaluated their performance and the effect on the DH
79 network in heating substation. Kuosa [8] developed a numerical model for a district
80 heating system with ring network, with which the variations of flow rates, pressure
81 losses and overall heat transfer coefficients of plate heat exchanger in heating
82 substation were simulated and analyzed on selected days. Dobos and Abonyi [9]
83 developed the nonlinear dynamic model of the district heating network including the
84 heat exchanger in heating substations, heat production units and pipelines to study the
85 nonlinear model predictive control of district heating network.

86 Since plate heat exchanger is the core component of heating substation,
87 mathematical modeling and control performance analysis of plate heat exchanger is
88 important to improve the operation efficiency of district heating substation. Feedback
89 control analysis and design of plate heat exchangers have been paid attentions.

90 Al-Dawery [10] established a first order model with time delay to suggest the
91 transient responses of a plate heat exchanger, and a fuzzy logic controller of the plate
92 heat exchanger was designed to achieve less settling time and oscillatory behavior.
93 Michel and Kugi [11] developed a control strategy without knowledge of the heat
94 transfer of plate heat exchanger based on controlling the total thermal energy stored in
95 the heat exchanger and a Kalman Filter to estimate the states.

96 However there were few studies concerning the dynamic operation stability of
97 heating substation, which is a very important dynamic characteristic for heating
98 substation and largely affects the operation efficiency of district heating system. The
99 operation instability of heating substation mainly manifest as flow rate and pressure
100 oscillations, which will deteriorate the network hydraulic condition, break the network
101 thermal balance, reduce the consumer comfort and increase the energy cost of the
102 pumping system. Since heating substations will easily operate unstably under some
103 conditions, this paper presents a theoretical method to analyze the stability and retune
104 the feedback controller for operation stability of heating substation. The theoretical
105 method presented in this paper mainly utilized the techniques developed in feedback
106 control theory [12]. The feedback control theory has been effectively applied to
107 analyze the operation stability and elimination of oscillations in a central heating
108 system using pump control [13]. Tahersima et al. utilized the feedback control theory
109 to study the stability performance and developed a gain scheduling controller of
110 radiator heating system in a room [14]. In our previous work, the control oriented
111 approaches were adopted to establish an accurate low order model of room heating

112 system and propose a two-degrees-of-freedom H_∞ loop-shaping controller [15].
113 Research on the operation stability of district heating system focuses on dynamic
114 variation and fluctuation of flow rates, pressures and temperatures in the district
115 heating network, which is very important and applicable in improving the operating
116 efficiency of the district heating network. In this paper, the operation stability of
117 district heating substation was studied from the control perspective. An analytical tool
118 was developed to analyzing the operation stability of district heating substation.

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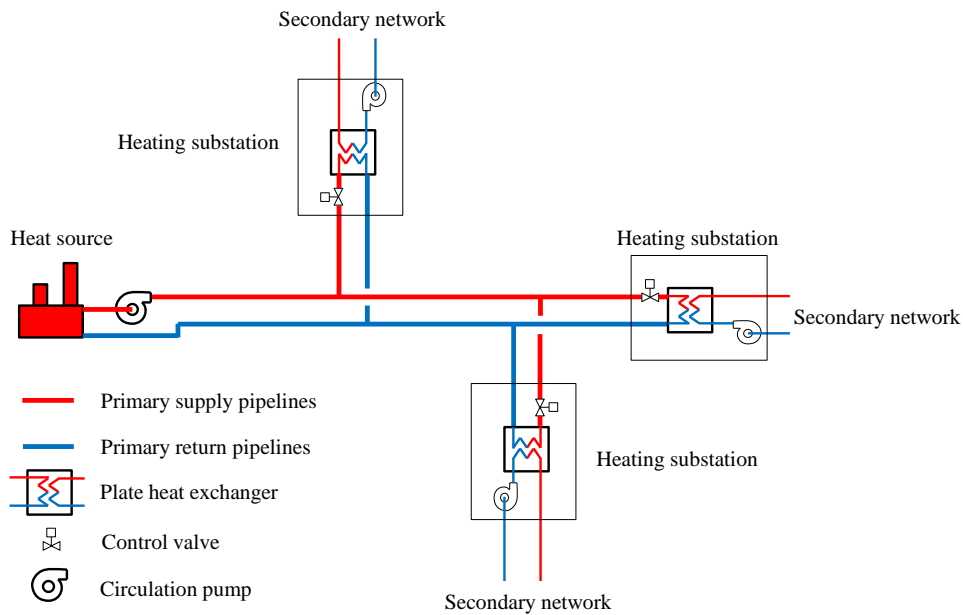
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122 **2. Control levels of district heating system**

123 Fig. 1 shows the schematic of district heating system. The hot water is generated
124 from the heat source and delivered to the heating substations by the primary
125 circulation pump along the primary pipelines. The heating substations are usually
126 located near the center of load regions. The main components of a heating substation
127 are the plate heat exchanger, primary control valve, secondary circulation pump and
128 the control system.

129 In order to provide sufficient thermal energy effectively, the district heating
130 system is usually regulated under three control levels. The first level is named the
131 centralized control; this level functions at the heat sources, which controls the primary
132 supply temperature and the pump speed to meet the total heating load variations of the
133 network. The second level is called the local control; this level functions at each

134 heating substation, which controls the secondary supply temperature and secondary
 135 pump speed to satisfy the variable heating load of the heat consumers. The secondary
 136 supply temperature is controlled by adjusting the control valve of the plate heat
 137 exchanger at the primary side. The third level is personal control; this level functions
 138 at each radiator, which controls the flow rate of radiator according to the room
 139 temperature difference between the desired and value to maintain the room air
 140 temperature around the desired value. In district heating system operation, the three
 141 control levels work simultaneously to allocate heat to each consumer. Fig. 1 shows the
 142 control structure of district heating network. As is shown, local control plays a key
 143 role in controlling the heat output of the primary network.



144

145 Fig. 1. Schematic of district heating system and heating substation

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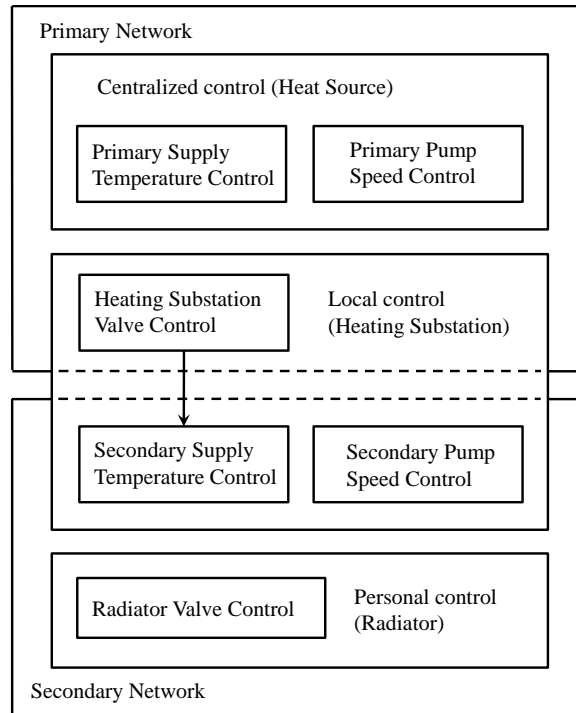


Fig. 2. Control levels of the district heating network.

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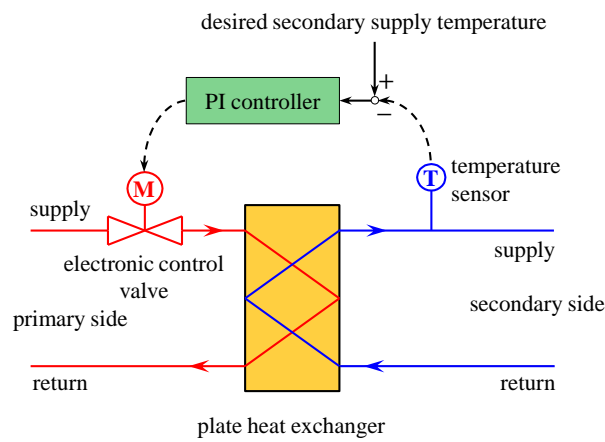
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150 Fig.3 shows the schematic of the feedback control structure in heating substation,
 151 which composes the main part of the local control. Efficient operation of the heating
 152 substation requires the feedback control loop to be stable. However, numerous heating
 153 substations are working in unstable conditions, and oscillations of flow rate and
 154 pressure always occur. The instability is resulted from the nonlinearities of valve and
 155 plate heat exchanger, sensor delay and improperly-tuned controller.

156 Fig.4 shows the measured primary side flow rate, supply temperature and
 157 outdoor temperature of a district heating substation in Tianjin, China. The secondary
 158 side of the heating substation is a commercial building with 22 floors. These
 159 measured data are to illustrate the operation instability of a district heating substation.
 160 As is shown, oscillation of primary side flow rate occurs when supply temperature is

161 high. Such oscillation may deteriorate the hydraulic condition of the district heating
 162 network, increase the energy cost of pumping system and reduce the lifetime of
 163 control valve. In order to investigate the instability analytically and stabilize the
 164 heating substation with properly-tuned controller, mathematical models of the heat
 165 exchanger and the feedback controller are established.

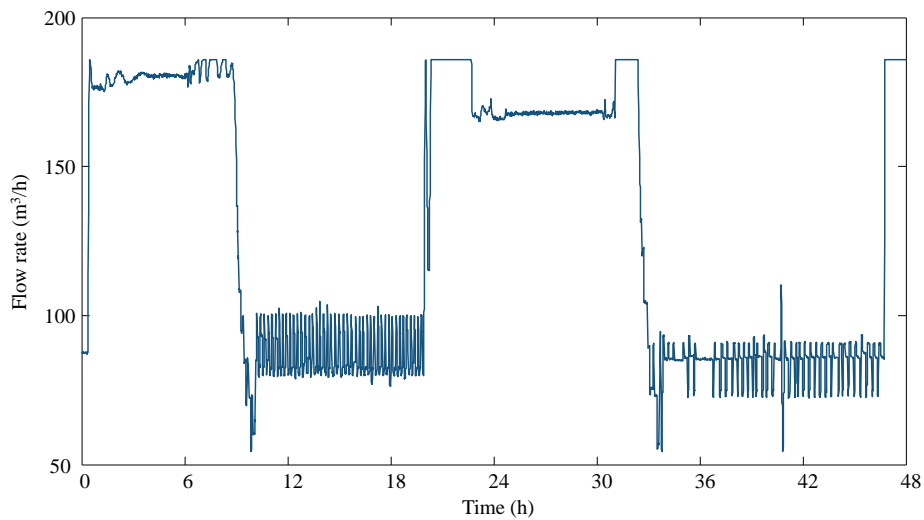
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167

168 Fig. 3. Schematic of heating substation control system

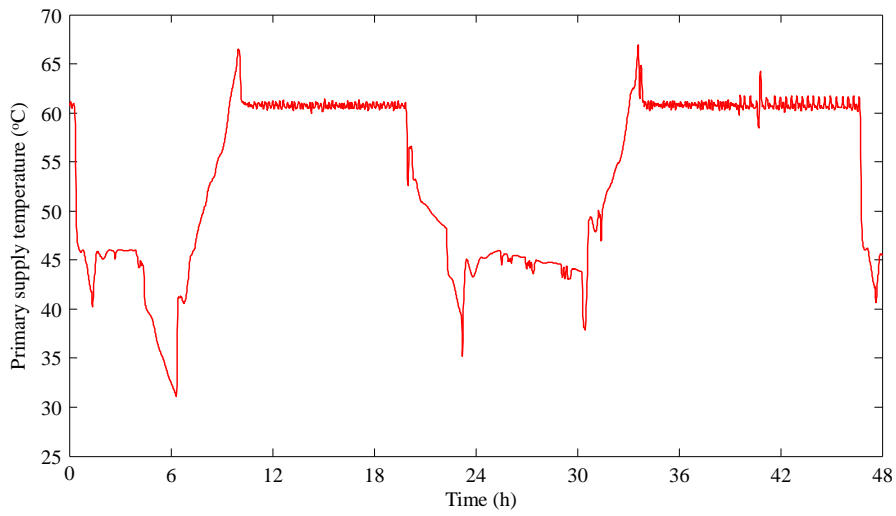
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171 (a)

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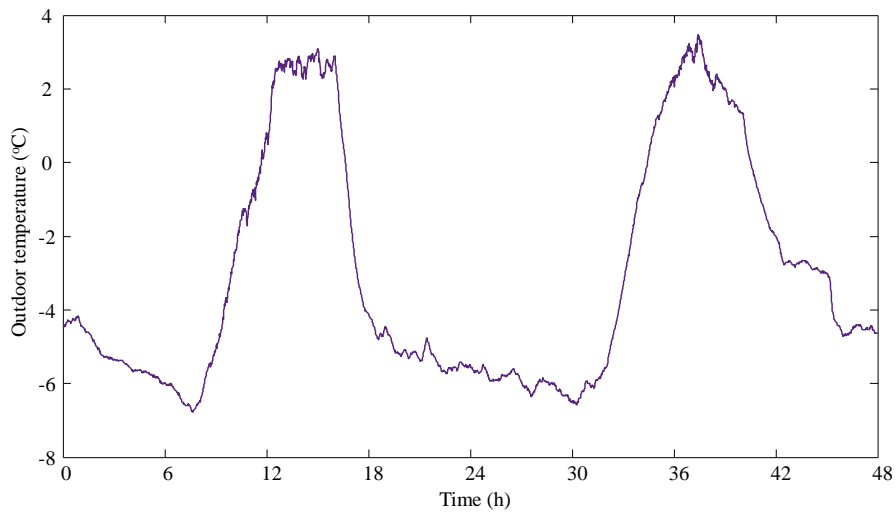


173

174

(b)

175



176

177

(c)

178 Fig.4. Measured primary side data of a heating substation in Tianjin, China. (a) Measured primary

179 side flow rate. (b) Measured primary supply temperature. (c) Measured outdoor temperature.

180

181 **3. Modeling the plate heat exchanger**

182 The thermal dynamics of plate heat exchanger can be described by a pair of
 183 partial differential equations (PDEs) [11]:

$$184 \quad \frac{\partial T_h}{\partial t} = \frac{q_h}{Mbd} \frac{\partial T_h}{\partial x} + \frac{k}{\rho c_p d} (T_l - T_h) \quad (3)$$

$$185 \quad \frac{\partial T_l}{\partial t} = -\frac{q_l}{Mbd} \frac{\partial T_l}{\partial x} + \frac{k}{\rho c_p d} (T_h - T_l) \quad (4)$$

186 where T_h and T_l are the temperature distribution of the high temperature side and
 187 the low temperature side, respectively. M is the number of flow channels in each side.
 188 b is the width of each flow channel. d is the distance between neighboring plates.
 189 q_h and q_l are the flow rates of the high temperature side and low temperature side,
 190 respectively. ρ is the water density. c_p is the specific thermal capacity. k is the
 191 overall heat transfer coefficient of the plate heat exchanger, which is the function of
 192 flow rates of the two sides. Calculation of k is summarized in Appendix A.

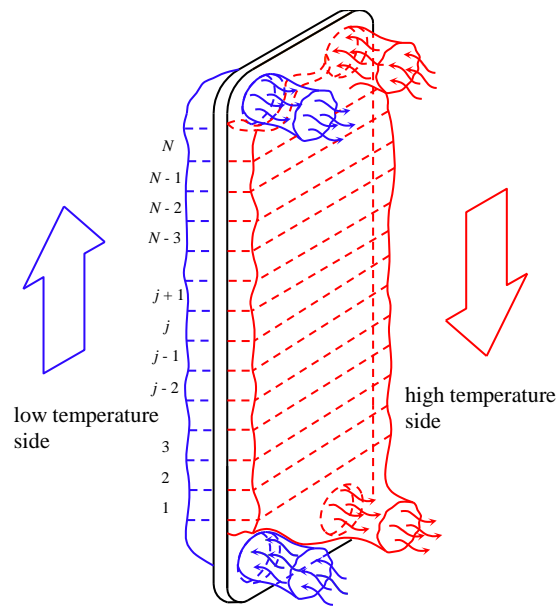
193 The PDE model can be reduced to an ordinary differential equation (ODE)
 194 model with the finite volume/difference method. Fig. 4 illustrates the finite volume
 195 division of the plate heat exchanger. The ODE model of plate heat exchanger can be
 196 derived as:

$$197 \quad \frac{dT_{h,j}}{dt} = \frac{q_h}{Mbd\Delta x} (T_{h,j+1} - T_{h,j}) + \frac{k}{\rho c_p d} (T_{l,j} - T_{h,j}) \quad (5)$$

$$198 \quad \frac{dT_{l,j}}{dt} = -\frac{q_l}{Mbd\Delta x} (T_{l,j} - T_{l,j-1}) + \frac{k}{\rho c_p d} (T_{h,j} - T_{l,j}) \quad (6)$$

199 where $\Delta x = l/N$; $j = 1, \dots, N$; l is the channel length; N is the number of volumes
 200 divided in each channel; $T_{h,N+1} = T_{h,in}$ and $T_{l,0} = T_{l,in}$ are the inlet temperatures of
 201 the high temperature and low temperature sides, respectively; $T_{h,1} = T_{h,out}$ and
 202 $T_{l,N} = T_{l,out}$ are the outlet temperatures of the high temperature and low temperature
 203 sides, respectively.

204



205

206

Fig. 4. Finite volume division of the plate heat exchanger flow channel.

207

208 The feedback control theory based method adopted in this paper generally
209 includes three steps. The first step is to develop the dynamic model of plate heat
210 exchanger, control valve and controller. The dynamic model of plate heat exchanger is
211 a pair of nonlinear ordinary differential equations describing the thermal dynamics of
212 the heat transfer process between the high temperature and low temperature sides.
213 Since the models of plate heat exchanger and control valve are nonlinear, they should
214 be linearized at an equilibrium point for stability analysis of the heating substation
215 control loop with the feedback control theory. The second step is to do Laplace
216 transform for the linearized models and derive the loop transfer function of the whole
217 system $L(s)$ (transfer function from primary flow rate to secondary supply
218 temperature) [12]. The third step is to draw the curve of $L(s)$ on the complex plane
219 for s varying along the imaginary axis from 0 to $i\infty$. This curve is called the

220 Nyquist curve [12]. The operation stability of heating substation can be judged with
221 the relation between the Nyquist curve and the point $(0, -1)$ on the complex plane.
222 The Nyquist curve method for analyzing stability of dynamic systems named the
223 Nyquist criterion was developed in 1930s, which has become a core concept and
224 technique of classical control theory [12]. The Nyquist criterion is very applicable to
225 practical problems and has been extended to more modern control technique [17].

226 The rest of this paper is organized as follows. The next section illustrates the
227 control levels of the whole district heating system. The primary flow rate oscillation
228 was observed from the measured data of a heating substation in Tianjin, China. Then
229 the nonlinear ordinary differential equation model of plate heat exchanger was derived.
230 The stability analysis method for heating substation was developed with the linearized
231 model of heating substation and the Nyquist criterion. The nonlinear plate heat
232 exchanger model was validated with measured data from the literature. The dynamic
233 responses and operation stability of a heating substation were studied for application
234 of the proposed stability analysis method.

235

236 **4. Operation stability analysis method**

237 *4.1. Model linearization*

238 Dynamic model of plate heat exchanger Eq. (5) and (6) is nonlinear. In order to
239 analyze the operation stability with the frequency domain method, model linearization
240 is required [17]. The nonlinear model described by Eq. (5) and (6) can be linearized
241 into the following linear state space form:

242
$$\frac{dT}{dt} = AT + B_1q_h + B_2q_l + B_3T_{h,in} + B_4T_{l,in} \quad (7)$$

243
$$T_{l,out} = CT \quad (8)$$

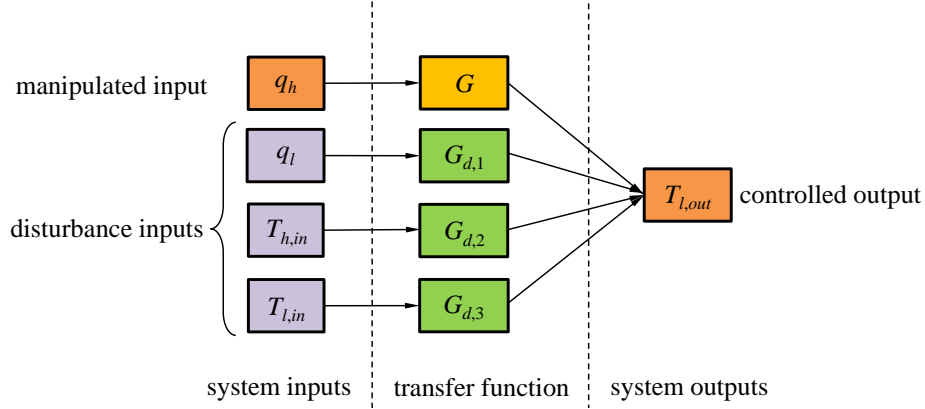
244 where $T = [T_{h,1} \ T_{h,2} \ \dots \ T_{h,N} \ T_{l,1} \ T_{l,2} \ \dots \ T_{l,N}]^T$. A , B_1 , B_2 , B_3 , B_4
 245 and C are constant matrices (details for derivation of the matrices are listed in
 246 **Appendix A**). Doing Laplace transform to Eq. (7) and (8), the input-output model of
 247 plate heat exchanger can be written in the following form:

248
$$T_{l,out} = G(s)q_h + G_{d,1}(s)q_l + G_{d,2}(s)T_{h,in} + G_{d,3}(s)T_{l,in} \quad (9)$$

249 where $G(s)$, $G_{d,1}(s)$, $G_{d,2}(s)$ and $G_{d,3}(s)$ are transfer functions, that can be
 250 calculated with the matrices A , B_1 , B_2 , B_3 , B_4 and C (see **Appendix A**). When
 251 linearizing a nonlinear system like Eq. (5) and (6), an equilibrium point should be
 252 specified. The equilibrium point of Eq. (5) and (6) is the solution of the steady state
 253 Eq. (5) and (6) (of which the time derivatives are made zero), with specified steady
 254 state inputs: q_h , q_l , $T_{h,in}$ and $T_{l,in}$. Therefore, with different steady state inputs,
 255 different equilibrium points can be derived. Since the matrices: A , B_1 , B_2 , B_3 and
 256 B_4 of the linearized model Eq. (7) and (8) are dependent on the equilibrium point.
 257 Different selection of equilibrium points will lead to different linearized models.
 258 However, all of the possible equilibrium points of the nonlinear system (Eq. (5) and
 259 (6)) lead to a set of linearized models, with which it is sufficient to study the robust
 260 stability of the nonlinear system [17].

261 The input-output structure of plate heat exchanger is illustrated in Fig. 5. The
 262 inputs can be divided into two categories: manipulated inputs and disturbance inputs.
 263 In district heating substation, the secondary supply temperature $T_{l,o}$ is controlled by

264 adjusting the primary flow rate q_h . Therefore q_h is the manipulated input, and the
 265 primary supply temperature $T_{h,in}$, secondary flow rate q_l and secondary return
 266 temperature $T_{l,in}$ are disturbance inputs .



267

268

Fig. 5. Input-output structure of the plate heat exchanger.

269

270 4.2. Models of the controller and valve

271 The PI control law is usually adopted as feedback controller K in district
 272 heating substation, and K can be represented as the Laplace transform form:

$$273 \quad K = k_c \frac{s+z}{s} \quad (10)$$

274 where $k_c > 0$ is the controller gain and $z > 0$ is controller zero.

275 The equal percentage valve is usually used in heating substation. The
 276 characteristic for equal percentage valve is nonlinear. The relation between flow rate
 277 and valve opening of equal percentage valve can be characterized by the following
 278 formula [13, 18]:

$$279 \quad q = C_v \sqrt{\Delta p} R^{x-1} \quad (11)$$

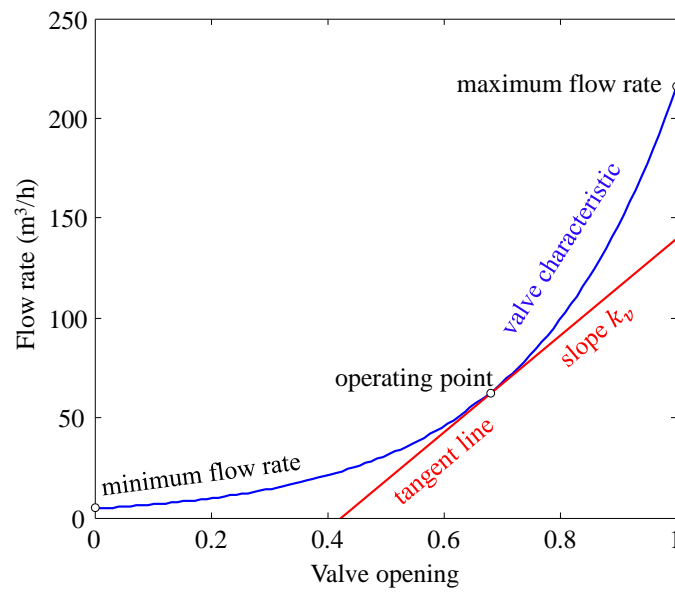
280 where C_v is the flow capacity of the valve; R is the rangeability of the valve; Δp
 281 is the pressure difference of the valve. Fig. 6 shows the characteristic of an equal

282 percentage valve with $C_v = 450$, $\Delta p = 0.23\text{atm}$ and $R = 50$. Fig. 6 also shows the
 283 tangent line of valve characteristic line. The slope of the tangent line can be derived
 284 by:

$$285 \quad k_v = \left(\frac{\partial q}{\partial x} \right)_{x=x_0} \quad (12)$$

286 x_0 is the operating point.

287



288

289 Fig. 6. Characteristic of an equal percentage valve.

290

291 The block diagram of heating substation feedback control system can be
 292 illustrated as Fig. 7. Since the valve characteristic is nonlinear, the valve gain k_v is
 293 varying with the operating point x_0 changing. The varying range of valve gain
 294 $k_{v,min} \leq k_v \leq k_{v,max}$ can be derived from the valve characteristic Eq. (11) and (12)
 295 as follows.

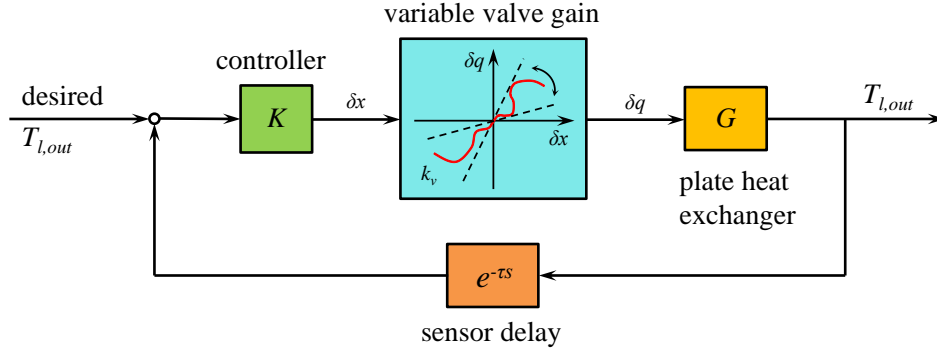
$$296 \quad k_{v,min} = \left(\frac{\partial q}{\partial x} \right)_{x=0} = C_v \sqrt{\Delta p} R^{-1} \ln R$$

297 $k_{v,max} = \left(\frac{\partial q}{\partial x}\right)_{x=1} = C_v\sqrt{\Delta p}\ln R$

298 Therefore, the varying valve gain satisfies:

299 $C_v\sqrt{\Delta p}R^{-1}\ln R < k_v \leq C_v\sqrt{\Delta p}\ln R$ (13)

300



301

302 Fig. 7. Block diagram of heating substation control system.

303

304 *4.3. Stability criterion of heating substation*

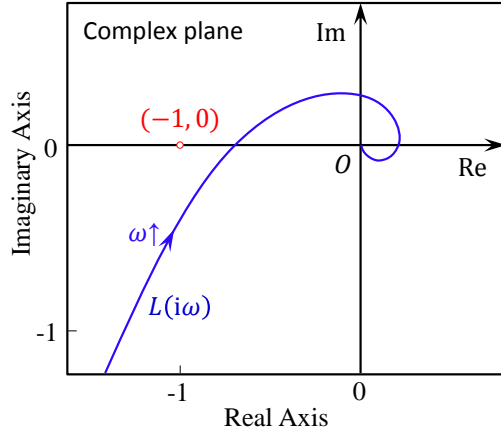
305 According to the feedback control theory [17], the operation stability of feedback
 306 control system can be judged by the Nyquist stability criterion. If introducing this
 307 criterion to heating substation system, the operation stability of district heating
 308 substation can be judged by the following criterion:

309 If the curve of $L(s) = k_v K(s)G(s)e^{-\tau s}$ (s is varying along the imaginary axis
 310 from 0 to $i\infty$) encircles or crosses the point $(-1, 0)$ on the complex plane, the heating
 311 substation control system will be unstable.

312 Here the complex variable s can be replaced by the pure imaginary variable $i\omega$
 313 with $0 \leq \omega < +\infty$, where $i = \sqrt{-1}$. Fig. 8 shows three cases of the relation between
 314 the curve of $L(i\omega)$ and the point $(-1, 0)$. For cases (b) and (c), the heating substation

315 will be unstable.

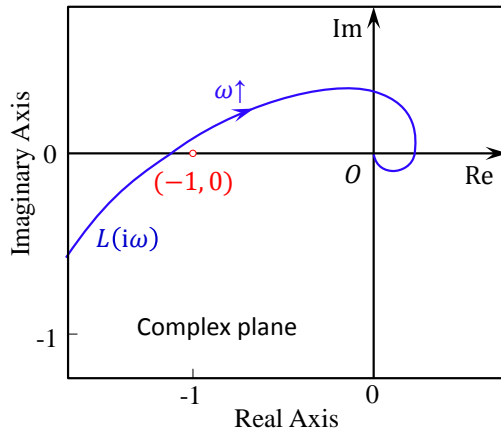
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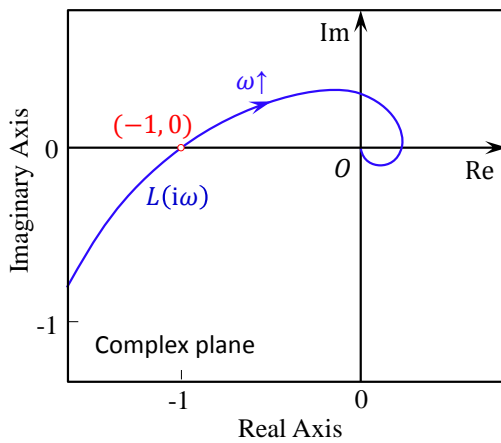
(a)



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(b)



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(c)

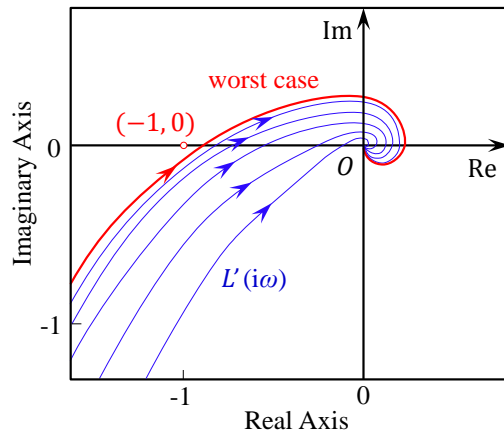
323 Fig. 8. Nyquist plots for operation stability analysis of heating substation. (a) The curve of $L(i\omega)$
324 doesn't encircle point $(-1, 0)$. (b) The curve of $L(i\omega)$ encircles point $(-1, 0)$. (c) The curve of
325 $L(i\omega)$ crosses the point $(-1, 0)$.

326

327 Since the dynamics of plate heat exchanger is nonlinear, the linearized model G
328 will be perturbed to G' ($G' \neq G$), if the operating condition (equilibrium point) is
329 changed. And the perturbation may lead to instability of the heating substation control
330 system. According to the proceeding criterion and Fig.8, operation stability of heating
331 substation will be damaged, if the perturbed system G' causes the curve of $L'(i\omega) =$
332 $k_v'K(i\omega)G'(i\omega)e^{-i\omega\tau}$ to encircle or cross the point $(-1, 0)$. The conversion from
333 stability to instability may happen, when operating condition of plate heat exchanger
334 in heating substation changes largely. As is observed in Fig. 3, with the primary
335 supply temperature increasing to a high level, the operation stability of heating
336 substation will be damaged, and the oscillatory will occur.

337 Therefore, case (a) for a certain operating condition doesn't mean that the
338 heating substation will be stable for all operating conditions. To ensure robust stability
339 of heating substation at all conditions, the case (a) should be held for all perturbed
340 models G' at any operating conditions. This criterion also indicates that robust
341 stability for all operating conditions can be ensured with a small loop gain: $|L(i\omega)|$,
342 which means that if the absolute value of $L(i\omega)$ is small enough, operation stability
343 can always be satisfied. This can be intuitively observed from Fig. (8). Fig. 9 shows

344 the Nyquist plots of all possible operating conditions (equilibrium points). As is
 345 shown if all of these curves do not encircle or cross point $(-1, 0)$, the heating
 346 substation will be stable at all operating conditions. This is equivalent to that the
 347 worst case Nyquist curve doesn't encircle or cross point $(-1, 0)$.
 348



349

350 Fig. 9. Nyquist plots of all possible operating conditions (equilibrium points).

351

352 With this criterion, the operation stability of heating substation is predictable.
 353 Besides, for an unstable operation condition of heating substation, this criterion can
 354 also be used to analyze the key factor which leads to the instability of the heating
 355 substation and tune the controller to ensure robust stability.

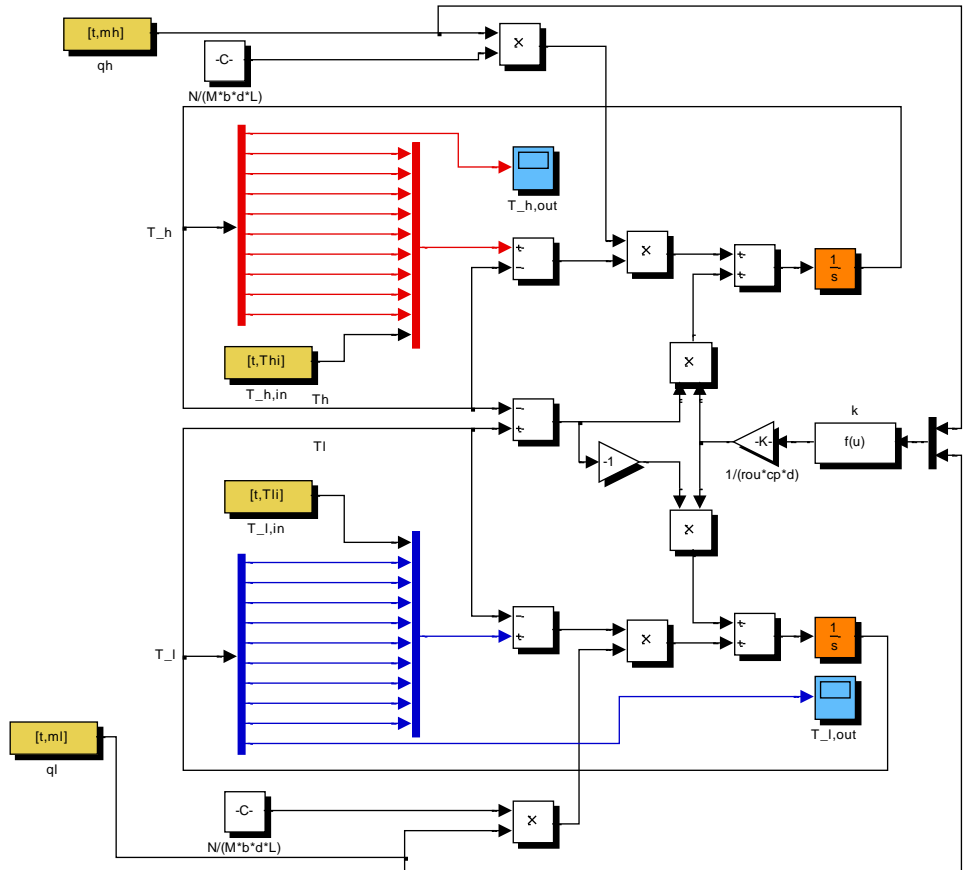
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357 5. Results and discussion

358 5.1. Validation of the nonlinear model

359 The nonlinear model of plate heat exchanger described by Eq. (5) and (6) was
 360 established in Simulink (Fig. 10). In Ref. [16], Michel and Kugi have tested the

361 dynamic operation of plate heat exchanger, and the measured data were used to
362 validate the dynamic model of plate heat exchanger. If the nonlinear model of plate
363 heat exchanger described by Eq. (5) and (6) is effective, the model should be able to
364 predict the dynamic operation in Ref. [16]. The plate heat exchanger parameters and
365 measured data including the boundary conditions of inlet temperatures and flow rates
366 in both sides given in Ref. [16] have been used to validate the proposed nonlinear
367 model described by Eq. (5) and (6). The nonlinear model was validated using the
368 measured data and plate heat exchanger parameters given in Ref. [16]. The simulated
369 and measured outlet temperatures $T_{h,out}$ and $T_{l,out}$ are shown in Fig. 11, and the
370 relative errors of $T_{h,out}$ and $T_{l,out}$ are shown in Fig.12. Relative errors of the
371 simulated $T_{h,out}$ and $T_{l,out}$ are both varying within $\pm 8\%$, which is in a satisfied
372 range. Therefore, the nonlinear model of the plate heat exchanger Eq. (5) and (6) can
373 describe the thermal dynamics of the plate heat exchanger with satisfied accuracy.

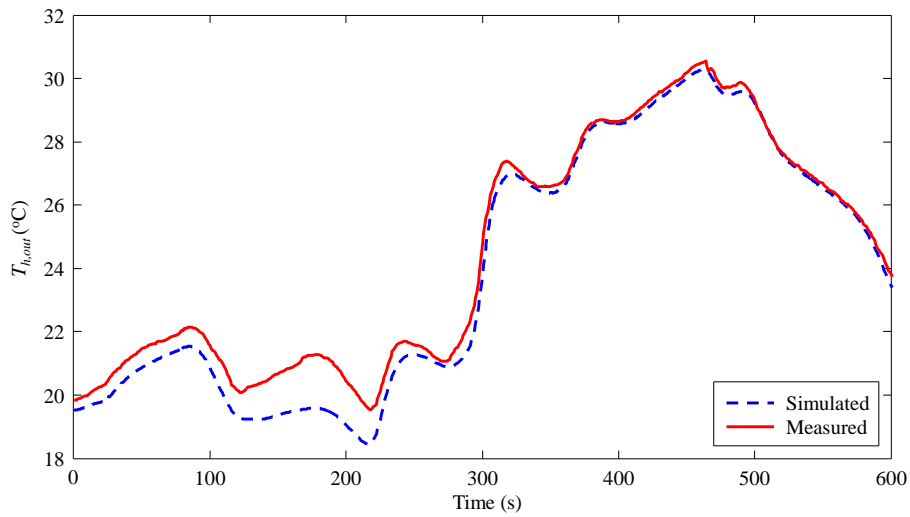


374

375

Fig. 10. Simulink model of the nonlinear plate heat exchanger model Eq. (5) and (6).

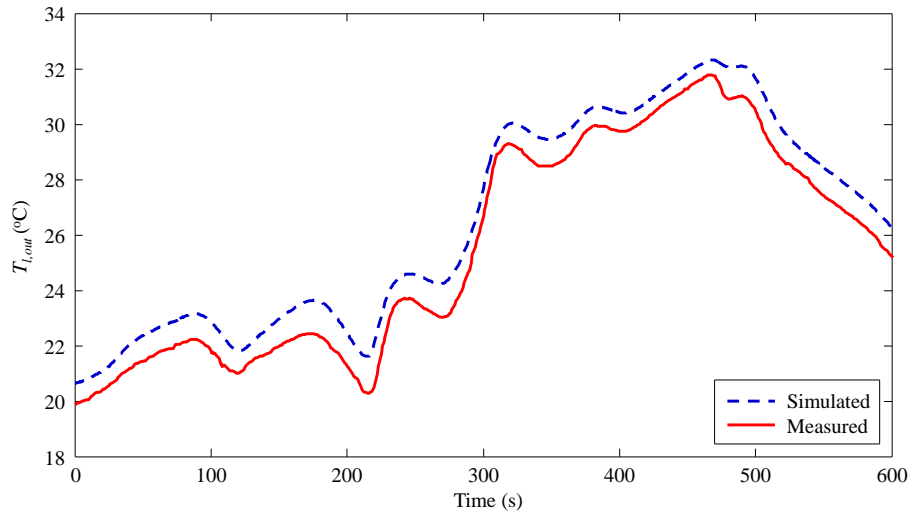
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(a)

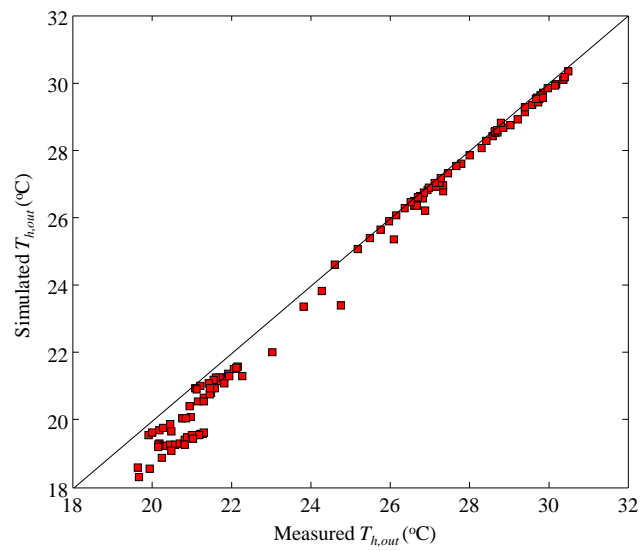


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(b)

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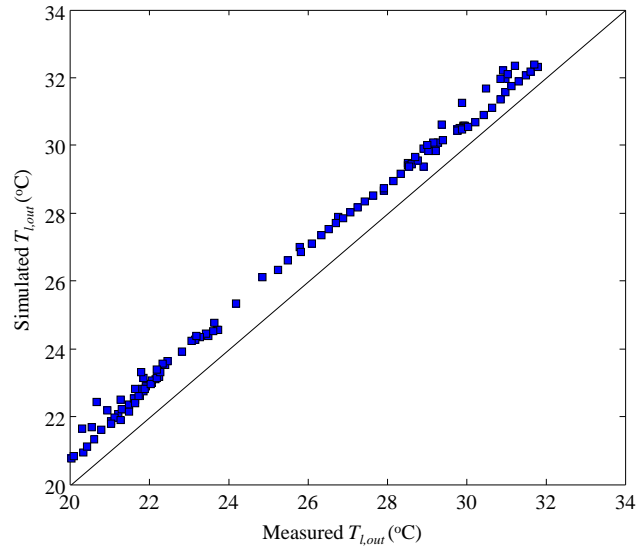


382

383

(c)

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385

386

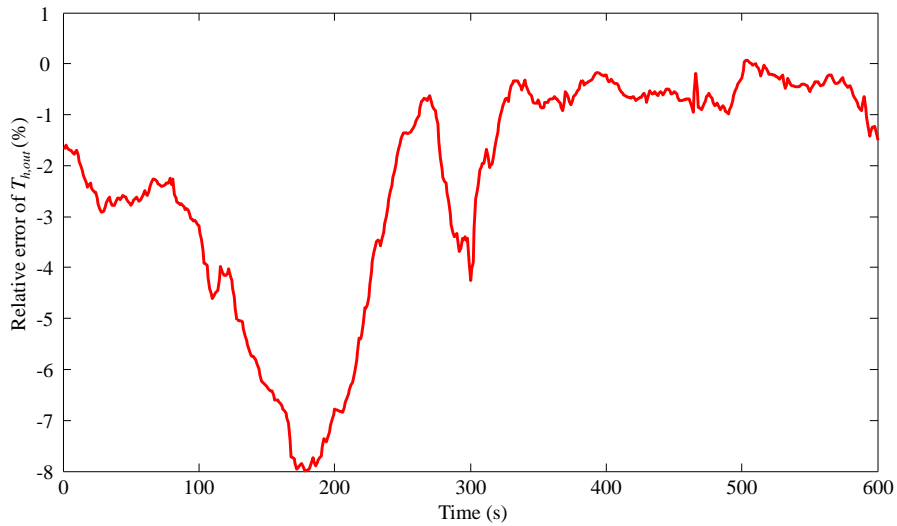
(d)

387 Fig. 11. Comparison of the measured and simulated outlet temperatures using parameters and

388 measured data provided in Ref. [16]. (a) Outlet temperature of high temperature side. (b) Outlet

389 temperature of low temperature side. (c) Validation of $T_{h,out}$. (d) Validation of $T_{l,out}$.

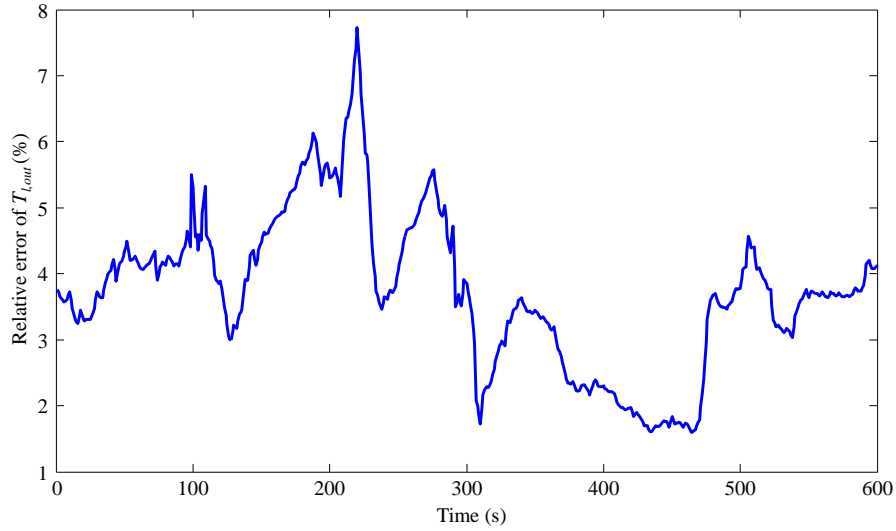
390



391

392

(a)



393

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(b)

395 Fig. 12. Relative errors of the nonlinear model Eq. (5) and (6). (a) Relative error of $T_{h,out}$. (b)

396 Relative error of $T_{l,out}$.

397

398 5.2. Control and stability of heating substation

399 The nonlinear heating substation control system composed of plate heat

400 exchanger, equal percentage valve and PI controller was also established in Simulink

401 with the mathematical models Eq. (5), (6), (10) and (11). The dynamic responses of

402 the heating substation control system were calculated with Simulink. The parameters

403 of plate heat exchanger are listed in Table 1. The valve characteristic is shown in Fig.

404 6. The time delay of temperature sensor is $\tau = 5s$.

405

Description	Symbol	Value	Unit
channel width	b	0.8	m
channel length	l	1.36	m

water specific heat capacity	c_p	4220	kJ/(kg · K)
empirical parameters	C	0.64	/
distance between neighboring plates	d	4.5	mm
plate thickness	d_p	0.5	mm
number of flow channels in each side	M	137	/
empirical parameters	n_1	0.23	/
empirical parameters	n_1	0.75	/
water density	ρ	970	kg/m ³
thermal conductivity of high temperature side water	λ_h	0.68	W/(m K)
thermal conductivity of low temperature side water	λ_l	0.67	W/(m K)
thermal conductivity of plate	λ_p	15	W/(m · K)
dynamic viscosity of high temperature side water	μ_h	0.00028	Pa · m
dynamic viscosity of low temperature side water	μ_l	0.00041	Pa · m

406 Table 1. Parameters of plate heat exchanger

407 In this section, the operation stability of heating substation was studied. The
408 controller was tuned with the frequency domain approach [12]. In order to illustrate
409 that if the controller is tuned and works well under low primary supply temperature
410 condition, the operation stability may not be ensured at high primary supply
411 temperature, dynamic performances of heating substation under low and high primary
412 supply temperatures with the PI controller tuned at low supply temperature were
413 studied.

414 5.2.1. Dynamic responses in low primary supply temperature

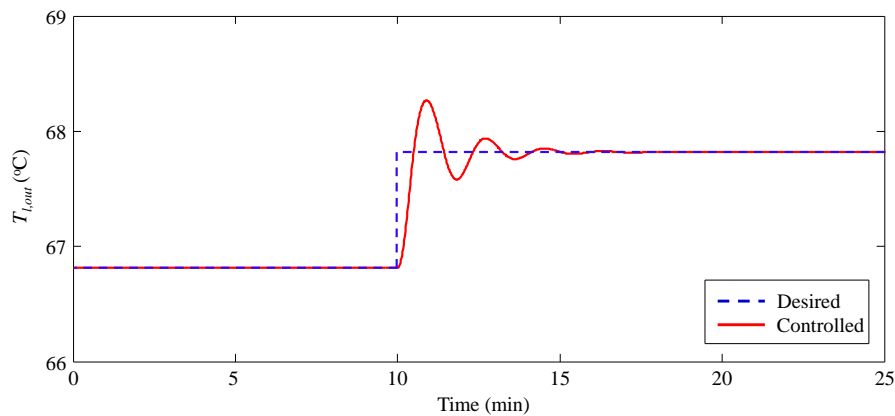
415 The PI controller is usually tuned under a certain operating condition. For the
416 case study in this subsection, the following low primary supply temperature operating
417 condition is chosen for controller tuning: $T_{l,in} = 40$ °C, $T_{h,in} = 70$ °C, $q_l = 0.03$

418 m³/s. The PI controller is tuned by simulating the reference tracking response of
 419 $T_{l,out}$ around the operating condition. The PI controller is tuned as:

$$420 \quad K_1 = 0.004 \frac{s+1}{s} \quad (14)$$

421 The dynamic responses of heating substation control system were calculated with the
 422 nonlinear Simulink model.

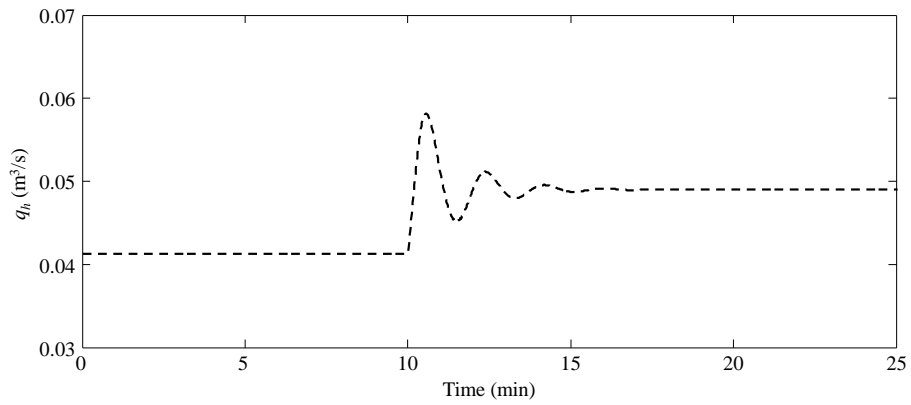
423 Fig. 13 shows the reference tracking responses under the control of K_1 . When
 424 the desired secondary supply temperature $T_{l,out}$ changes from 66.8 °C to 67.8 °C, the
 425 controlled $T_{l,out}$ tracks the new value in about 5 minutes and the overshoot is less
 426 than 0.5 °C. Fig. 14, Fig. 15 and Fig. 16 show the disturbance rejection responses to
 427 step variations of $T_{h,in}$, $T_{l,in}$ and q_l , respectively. Fig. 14 shows that when primary
 428 supply temperature $T_{h,in}$ changes from 70 °C to 71 °C, the deviation of secondary
 429 supply temperature $T_{l,out}$ from the desired value can be controlled within 1 °C. Fig.
 430 15 shows that if secondary return temperature changes from 40 °C to 35 °C, the
 431 deviation of $T_{l,out}$ from desired value can be restricted within 0.5 °C. Fig. 16 shows
 432 that when the secondary flow rate q_l varies from 0.03 m³/s to 0.04 m³/s, the
 433 deviation of $T_{l,out}$ from desired value is within 2 °C.



434

435

(a)



436

437

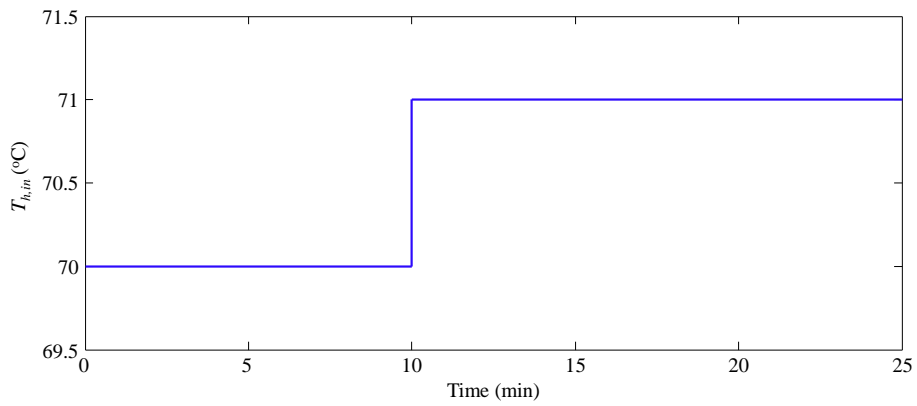
(b)

438 Fig. 13. System responses for tracking desired $T_{l,o}$ under the control of controller K_1 (with

439 $T_{l,in} = 40$ °C, $T_{h,in} = 70$ °C, $q_l = 0.03$ m³/s.). (a) Tracking response of $T_{l,o}$. (b) Tracking

440 response of q_h .

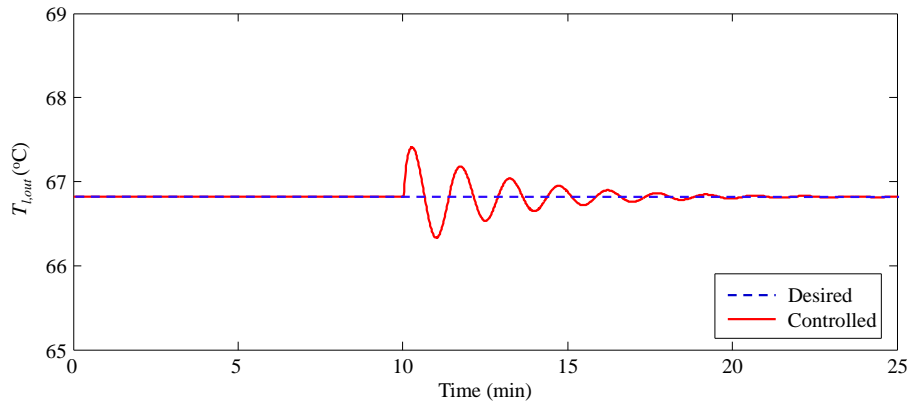
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442

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(a)

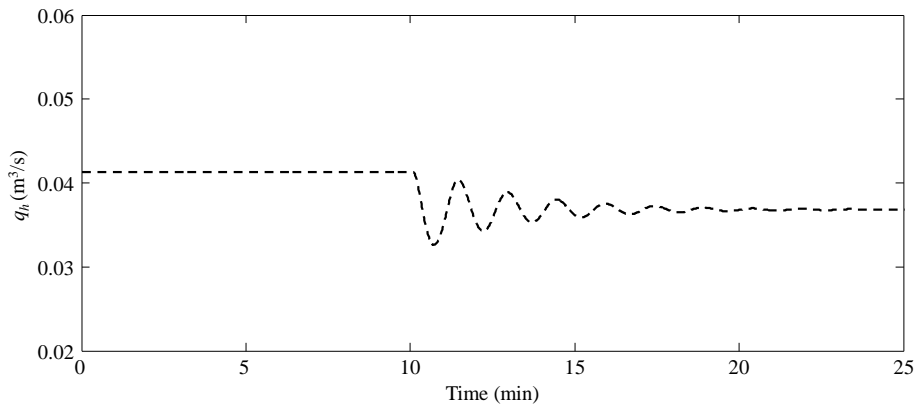


444

445

(b)

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447

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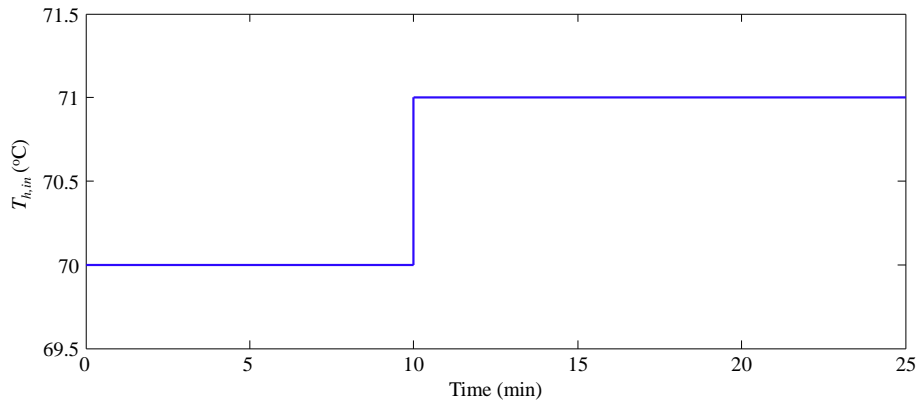
(c)

449 Fig. 14. System responses for rejecting the variation of $T_{h,in}$ under the control of K_1 (with

450 $T_{l,in} = 40$ °C, $q_l = 0.03$ m³/s and desired $T_{l,out} = 66.8$ °C). (a) Variation of $T_{h,in}$. (b)

451 Response of controlled $T_{l,out}$. (c) Response of q_h .

452

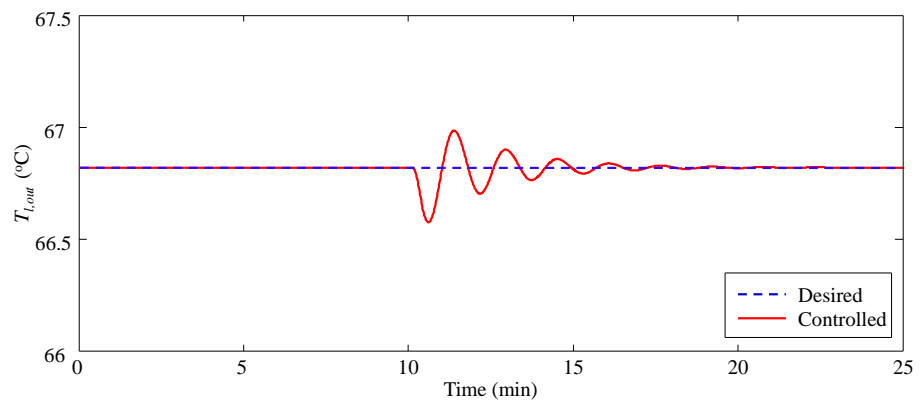


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(a)

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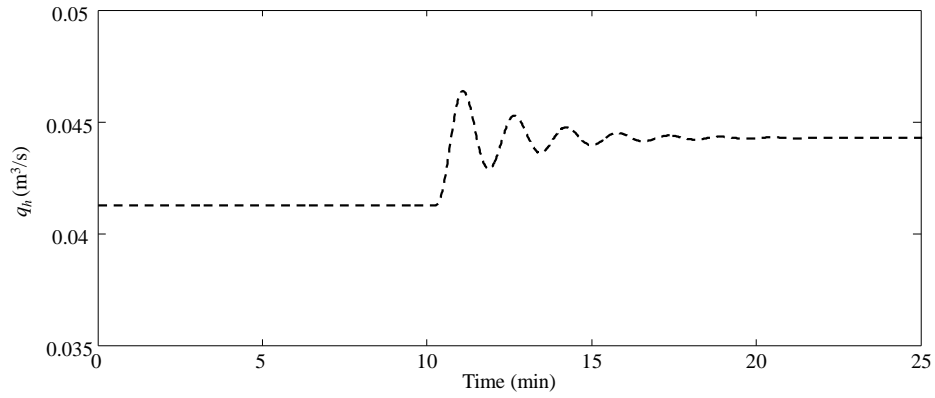


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(b)

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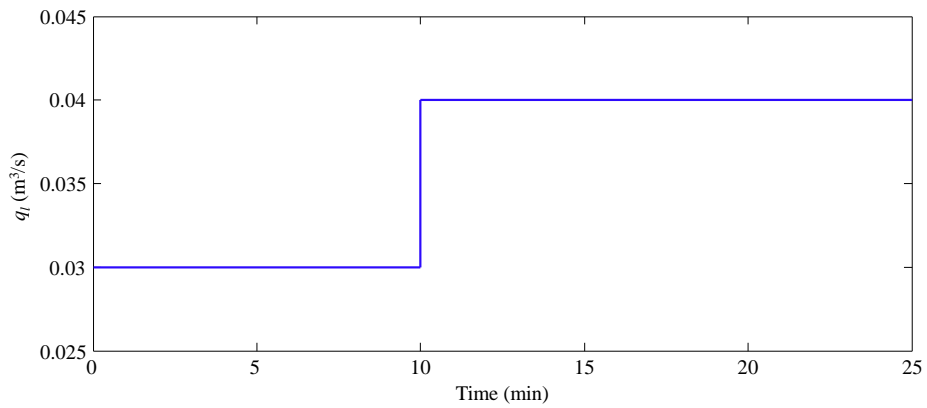
(c)

461 Fig. 15. System responses for rejecting the variation of $T_{l,in}$ under the control of K_1 (with

462 $T_{h,in} = 70$ °C, $q_l = 0.03$ m³/s and the desired $T_{l,out} = 66.8$ °C). (a) Variation of $T_{l,in}$. (b)

463 Response of controlled $T_{l,out}$. (c) Response of q_l .

464

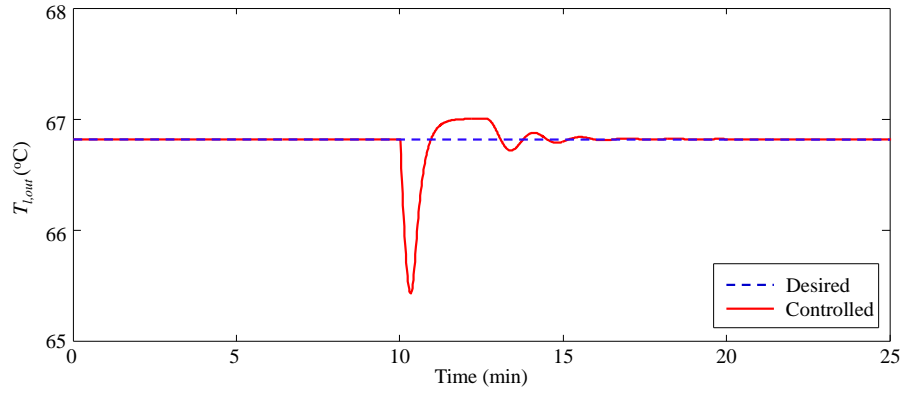


465

466

(a)

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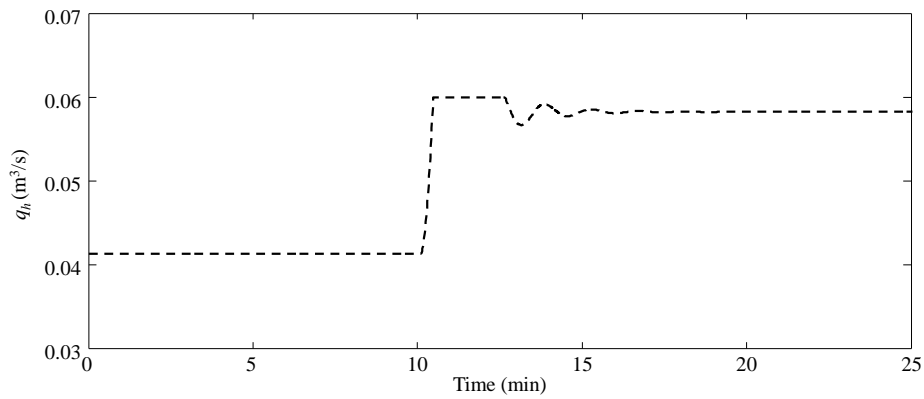


468

469

(b)

470



471

472

(c)

473 Fig. 16. System responses for rejecting the variation of q_l under the control of K_1 (with $T_{h,in} =$

474 $70\text{ }^\circ\text{C}$, $T_{l,in} = 40\text{ }^\circ\text{C}$ and desired $T_{l,out} = 66.8\text{ }^\circ\text{C}$). (a) Variation of q_l . (b) Response of

475 controlled $T_{l,out}$. (c) Response of q_h .

476 These reference tracking and disturbance rejection responses indicate that under

477 the control of K_1 , the heating substation system is stable around the operating

478 condition of $T_{h,in} = 70\text{ }^\circ\text{C}$, $T_{l,in} = 40\text{ }^\circ\text{C}$, $q_l = 0.03\text{ m}^3/\text{s}$. Hence, the tuned PI

479 controller K_1 seems to be suitable for the control of the heating substation with the

480 proposed parameters. However, the simulation test of responses is only conducted
481 around the operating condition of $T_{h,in} = 70$ °C, $T_{l,in} = 40$ °C, $q_l = 0.03$ m³/s. When
482 the operating condition changes largely, instability may occur as the measured data
483 shown in Fig. 3.

484

485 5.2.2. Robust stability test and retuning of the controller

486 In order to investigate the stability of the heating substation under the control of
487 controller K_1 , the stability criterion proposed in subsection 4.3 was adopted. Fig.
488 17-(a) shows the Nyquist curves of the heating substation under the control of K_1 at
489 all possible operating conditions. The possible operating conditions were defined as
490 conditions that satisfy: 65 °C $\leq T_{h,in} \leq 95$ °C, 30 °C $\leq T_{l,in} \leq 50$ °C, 0.01 m³/s \leq
491 $q_h \leq 0.06$ m³/s and 0.03 m³/s $\leq q_h \leq 0.04$ m³/s. This range can cover most of the
492 operation conditions of the heating substation proposed in this paper. The proposed
493 method in section 4.3 can be used to study the operation stability at these possible
494 operating condition.

495 As is shown, the black curve, which denotes the Nyquist curve of the $T_{h,in} = 70$
496 °C, $T_{l,in} = 40$ °C, $q_l = 0.03$ m³/s condition, does not encircle or cross the point (-1,
497 0). This is the reason that the heating substation operates stably under the control of
498 K_1 around the condition of $T_{h,in} = 70$ °C, $T_{l,in} = 40$ °C, $q_l = 0.03$ m³/s. However,
499 there are still many cases don't satisfy this criterion. This means that the heating
500 substation will be unstable under the control of K_1 in some operating conditions. Fig.
501 18-(a) shows the variation of $T_{h,in}$ and desired $T_{l,out}$, which will lead to instability

502 of the heating substation (with $T_{l,in} = 40$ °C, $q_l = 0.03$ m³/s). As is shown in Fig.
503 18-(b) and 18-(c), the dynamic responses of $T_{l,out}$ and q_h under the control of K_1
504 (drawn in dark blue) become oscillatory when the primary supply temperature $T_{h,in}$
505 increases from 70 °C to 85 °C. The oscillation form of the primary flow rate q_h ,
506 shown in Fig. 18-(b), is very similar to the measured primary flow rate data shown in
507 Fig. 3-(a). The heating substation becomes unstable because the increase of primary
508 supply temperature $T_{h,in}$ makes the loop gain $|L(i\omega)|$ larger and causes the curve of
509 $L(i\omega)$ to encircle point (-1, 0). This phenomenon also indicates that the high primary
510 supply temperature conditions are worse than low supply temperature conditions. This
511 also demonstrates that controllers tuned at a certain operating condition cannot ensure
512 stability for all operating conditions.

513 In order to stabilize the heating substation, controller K_1 should be tuned again
514 by considering all the possible operating conditions. The red Nyquist curve in Fig.
515 17-(a) denotes the worst operation condition. If the worst condition Nyquist curve
516 doesn't encircle or cross the point (-1, 0), the heating substation will be stable at all
517 operating condition. Therefore, according to Fig. 17-(a), to make the red curve do not
518 encircle or cross the point (-1, 0), the controller gain k_c should be smaller. Fig. 17-(b)
519 shows the Nyquist curves of the heating substation under the control of K_2 at all
520 possible operating conditions, where

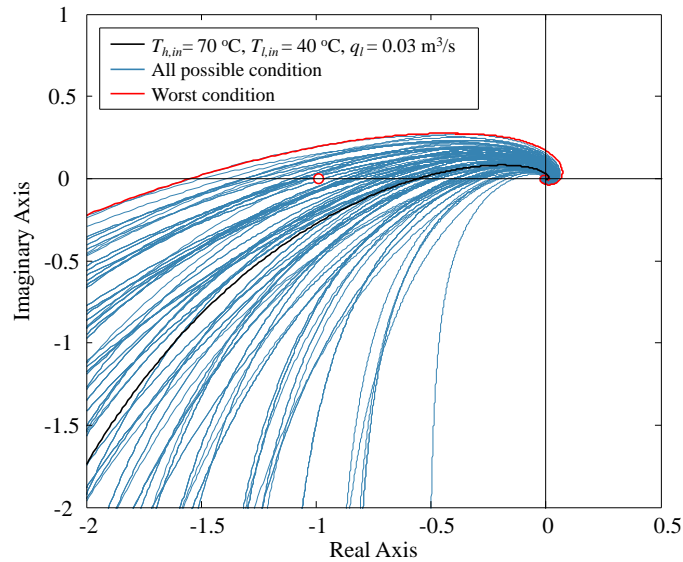
$$521 \quad K_2 = 0.002 \frac{s+1}{s}$$

522 (18)

523 As is shown in Fig. 17-(b), the Nyquist curves of all the possible conditions do not

524 encircle or cross the point (-1, 0). Therefore the heating substation under the control
 525 of controller K_2 will be stable even in worse condition. In Fig. 18-(b) and Fig. 18-(c),
 526 the responses drawn in red are under the control of K_2 . As is shown, the operation of
 527 heating substation remains stable even when primary supply temperature increases to
 528 very high.

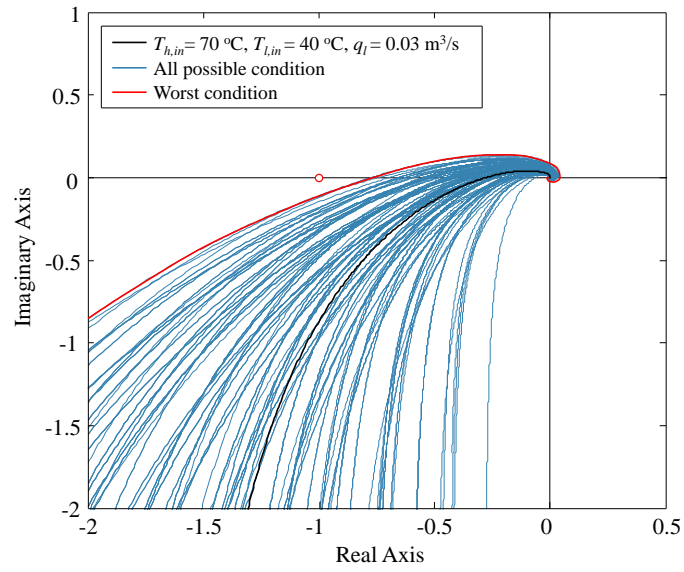
529 Hence, the heating substation controller tuned at certain operating conditions
 530 may be unstable when operating condition changes in large range. To ensure operation
 531 stability of heating substation at all conditions, the operation stability should be tested
 532 when operating condition changes, and the proposed method can be used as a tool for
 533 analyzing the operation stability of heating substation at all possible operating
 534 conditions.



535

536

(a)



537

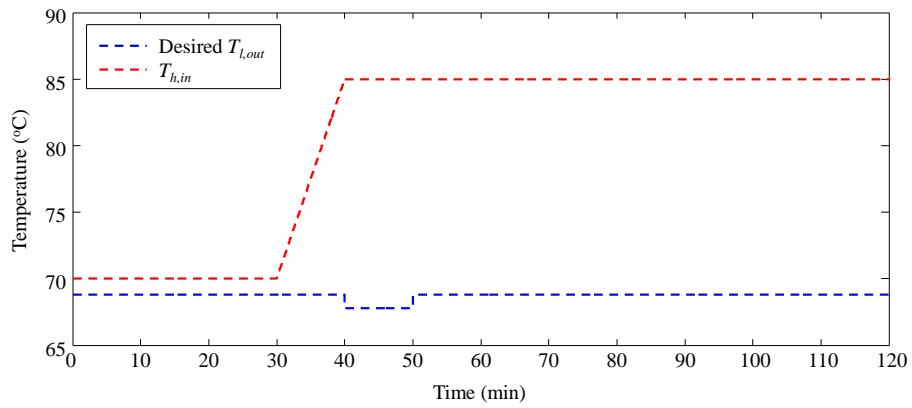
538

(b)

539 Fig. 17. Nyquist curves of heating substation system at all possible operating conditions. (a) Under

540 the control of K_1 . (b) Under the control of K_2 .

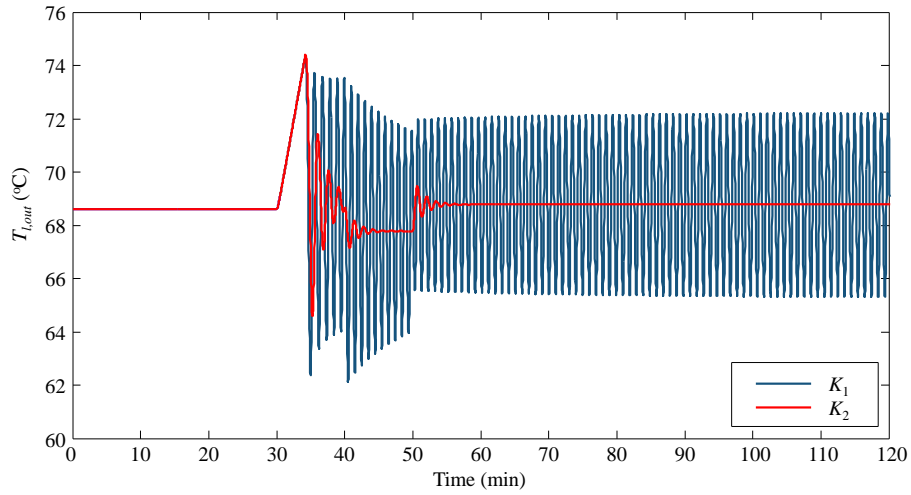
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542

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(a)

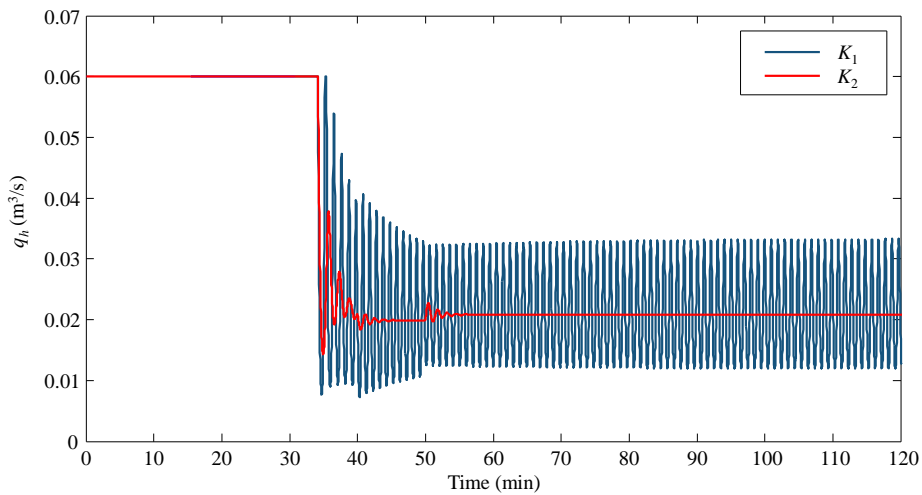


544

545

(b)

546



547

548

(c)

549 Fig. 18. Dynamic responses in worse operating condition under the control of K_1 and K_2 (with

550 $T_{l,in} = 40$ °C, $q_l = 0.03$ m³/s). (a) Variation of desired $T_{l,out}$ and $T_{h,in}$. (b) Responses of q_h . (c)

551 Responses of controlled $T_{l,out}$.

552

553 **6. Conclusions**

554 In this paper, the nonlinear ODE model of plate heat exchanger was developed.
555 Based on the nonlinear ODE model, the linearized model of plate heat exchanger for
556 controller design and stability analysis was derived. The nonlinear ODE model of
557 plate heat exchanger was solved with Simulink. In order to validate the nonlinear
558 plate heat exchanger model, the parameters and measured data provided in Ref. [16]
559 were adopted in simulation. The simulated results were compared with the measured
560 data provided in the literature. Associated with the equal percentage valve model and
561 controller model, the Nyquist stability criterion was proposed for analyzing the
562 operation stability criterion of district heating substation at all operating conditions.
563 The dynamic responses of heating substation under the control of a PI controller tuned
564 at a certain operating condition were analyzed. And the operation stability of heating
565 substation was also studied. And the following conclusions have been drawn:

566 (1) Comparison of the measured data and simulated results of the plate heat
567 exchanger shows that the proposed nonlinear ODE model has satisfactory
568 accuracy in describing the thermal dynamics of plate heat exchanger. Relative
569 errors of the two outlet temperatures: $T_{h,out}$ and $T_{l,out}$ are both varying within
570 $\pm 8\%$.

571 (2) Simulation results of heating substation control system indicate that the controller
572 tuned at a certain operating condition may be unstable, when operating condition
573 changes in large range. For example, the operation instability of district heating
574 substation may occur at the high primary supply temperature, if the controller is
575 tuned at low primary supply temperature.

576 (3) With the proposed stability criterion for heating substation operation, the
 577 controller can be retuned to be stable at all operating conditions.

578 Since operation stability of heating substation is the basic requirement of the
 579 operation and is of great importance to energy conservation of pumping system,
 580 reducing the failure rate of control valve and improving the heating quality of the
 581 secondary system, the proposed method will be very helpful and applicable to heating
 582 substation controller tuning and operation stability analysis for stable operation.

583

584 **Appendix A. Calculation of heat transfer coefficient k**

585 The overall heat transfer coefficient of plate heat exchanger can be determined
 586 by the following formula:

$$587 \quad k = \left(\frac{1}{k_h} + \frac{1}{k_p} + \frac{1}{k_l} \right)^{-1} \quad (\text{A-1})$$

588 where $k_p = \frac{\lambda_p}{d_p}$ is the heat transfer coefficient of the plate. k_h and k_l are
 589 determined by the following formulas [16]:

$$590 \quad k_h = \frac{Nu_h \lambda_h}{D}, \quad Nu_h = C_{Nu} \cdot Re_h^{n_1} \cdot Pr_h^{n_2}, \quad Re_h = \frac{\rho D q_h}{\mu_h M b d}, \quad Pr_h = \frac{\mu_h c_p}{\lambda_h} \quad (\text{A-2})$$

$$591 \quad k_l = \frac{Nu_l \lambda_l}{D}, \quad Nu_l = C_{Nu} \cdot Re_l^{n_1} \cdot Pr_l^{n_2}, \quad Re_l = \frac{\rho D q_l}{\mu_l M b d}, \quad Pr_l = \frac{\mu_l c_p}{\lambda_l} \quad (\text{A-3})$$

592 where $D = 2d$ is the hydraulic diameter; λ_h and λ_l are the heat conductivities of
 593 the high temperature side water and the low temperature side water, respectively;
 594 Nu_h and Nu_l are the Nusselt numbers of the two sides; C_{Nu} , n_1 and n_2 are
 595 empirical parameters provided by the manufacturer; Re_h and Re_l are the Reynolds
 596 numbers of the two sides; Pr_h and Pr_l are the Prandtl numbers of the two sides; μ_h
 597 and μ_l are the dynamic viscosities of the two sides.

598

599 **Appendix B. Linearized plate heat exchanger model**

600 Do the following parameter replacement:

$$601 \quad a_1 = -\left(\frac{\bar{k}}{\rho c_p d} + \frac{N\bar{q}_h}{MbdL}\right), \quad a_2 = \frac{N\bar{q}_h}{MbdL}, \quad a_3 = \frac{\bar{k}}{\rho c_p d}, \quad a_4 = \frac{\bar{k}}{\rho c_p d}, \quad a_5 = \frac{Nq_l}{MbdL},$$

$$602 \quad a_6 = -\left(\frac{N\bar{q}_l}{MbdL} + \frac{\bar{k}}{\rho c_p d}\right), \quad b_{11,j} = \frac{N(\bar{T}_{h,j+1} - \bar{T}_{h,j})}{MbdL} + \frac{(\bar{T}_{l,j} - \bar{T}_{h,j})}{\rho c_p d} \cdot \left(\frac{\partial k}{\partial q_h}\right)_{\bar{q}_h},$$

$$603 \quad b_{12,j} = \frac{\bar{T}_{l,j} - \bar{T}_{h,j}}{\rho c_p d} \cdot \left(\frac{\partial k}{\partial q_l}\right)_{\bar{q}_l}, \quad b_{21,j} = \frac{\bar{T}_{h,j} - \bar{T}_{l,j}}{\rho c_p d} \cdot \left(\frac{\partial k}{\partial q_h}\right)_{\bar{q}_h},$$

$$604 \quad b_{22,j} = \frac{N(\bar{T}_{l,j-1} - \bar{T}_{l,j})}{MbdL} + \frac{(\bar{T}_{h,j} - \bar{T}_{l,j})}{\rho c_p d} \cdot \left(\frac{\partial k}{\partial q_l}\right)_{\bar{q}_l}$$

605 where \bar{k} , \bar{q}_h , \bar{q}_l , $\bar{T}_{l,j}$ and $\bar{T}_{h,j}$ are the equilibrium point values of the nonlinear

606 ODE model of plate heat exchanger Eq. (8) and (9). Then define

$$607 \quad A_{11} = \begin{pmatrix} a_1 & a_2 & & & & & \\ & a_1 & a_2 & & & & \\ & & a_1 & a_2 & & & \\ & & & \ddots & \ddots & & \\ & & & & a_1 & a_2 & \\ & & & & & & a_1 \\ & & & & & & a_1 \end{pmatrix}, \quad A_{12} = \begin{pmatrix} a_3 & & & & & & \\ & a_3 & & & & & \\ & & a_3 & & & & \\ & & & \ddots & & & \\ & & & & & & a_3 \\ & & & & & & & a_3 \end{pmatrix}$$

$$608 \quad A_{21} = \begin{pmatrix} a_4 & & & & & & \\ & a_4 & & & & & \\ & & a_4 & & & & \\ & & & \ddots & & & \\ & & & & & & a_4 \\ & & & & & & & a_4 \end{pmatrix}, \quad A_{22} = \begin{pmatrix} a_6 & & & & & & \\ a_5 & a_4 & & & & & \\ & a_5 & a_4 & & & & \\ & & a_5 & a_4 & & & \\ & & & a_5 & \ddots & & \\ & & & & \ddots & a_4 & \\ & & & & & \ddots & a_4 & a_5 & a_4 \end{pmatrix}$$

609 and

$$610 \quad A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$

$$611 \quad T = (T_{h,1} \quad T_{h,2} \quad \dots \quad T_{h,N} \quad T_{l,1} \quad T_{l,2} \quad \dots \quad T_{l,N})^T$$

$$612 \quad B_1 = (b_{11,1}, b_{11,2}, \dots, b_{11,j}, \dots, b_{11,N}, b_{21,1}, b_{21,2}, \dots, b_{21,j}, \dots, b_{21,N})^T$$

$$613 \quad B_2 = (b_{12,1}, b_{12,2}, \dots, b_{12,j}, \dots, b_{12,N}, b_{22,1}, b_{22,2}, \dots, b_{22,j}, \dots, b_{22,N})^T$$

$$614 \quad B_3 = (O_{1 \times (N-1)}, a_2, O_{1 \times N})^T$$

$$615 \quad B_4 = (O_{1 \times N}, a_5, O_{1 \times (N-1)})^T$$

616 $C = (O_{1 \times N}, O_{1 \times (N-1)}, 1)$

617 where A_{11} , A_{12} , A_{21} and A_{22} are $N \times N$ matrices. T , B_1 , B_2 , B_3 and B_4 are
618 $2N$ -dimensional vectors.

619 The transfer function form of the linearized plate heat exchanger model is:

620 $T_{l,out} = G(s)q_h + G_{d,1}(s)q_l + G_{d,2}(s)T_{h,in} + G_{d,3}(s)T_{l,in}$

621 where

622 $G(s) = C(sI - A)^{-1}B_1$

623 $G_{d,1}(s) = C(sI - A)^{-1}B_2$

624 $G_{d,2}(s) = C(sI - A)^{-1}B_3$

625 $G_{d,3}(s) = C(sI - A)^{-1}B_4$

626

627

628 **Acknowledgement**

629 This work was supported by the State Oceanic Administration of China (Grant
630 No. cxsf-43); and the National Natural Science Foundation of China (No. 51106110).

631

632 **Reference**

633

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