

# Learning to teach contextualised problem solving in a non-calculus mathematics pathway

Jennie Golding *UCL Institute of Education*, Cathy Smith *Open University* and Mick Blaylock

## Abstract

We report on the professional learning of a small purposive sample of teachers beginning to teach an English post-16 mathematics course centred around contextualised problem solving. We describe their accounts of change, their intentions and justification for those, and how development was supported. The demands of the course require ‘boundary crossing’ between contexts and mathematics that was novel for these teachers and their students, and learning to teach for such approaches is known to be demanding. By drawing on peer support and their own robust subject knowledge, though, these teachers were able to accommodate change to classroom authority and expertise, attention to students’ relationship with mathematics, longer-term planning sequences, and growth in their own management of uncertainty within the classroom.

**Keywords:** Contextualised problem-solving, mathematics in context, teacher beliefs, teacher affect, boundary crossing.

## Introduction

This paper focuses on teacher change and professional learning in an important context that is known to present difficulties. In recent years, England has seen concerted policy moves (e.g. ACME, 2012 a, b) to extend participation in post-16 mathematics beyond the relatively small cohort (Hodgen et al, 2013) taking ‘A-level’ Mathematics, the standard English calculus-rich pre-university course. ‘Core Maths’ was introduced in September 2014. Although there are six versions of the qualification offered, our focus is on features that are shared and potentially new to teachers. All versions are intended to be studied over two years, earning half the credit of A-level Mathematics and accessible to students with a standard level of mathematics qualification (GCSE) at 16. They are designed for enactment around contextualised problem-solving tasks (DfE, 2015), featuring what Ainley, Pratt and Hansen (2006) call *utility* (appreciation of when, how and why mathematics is useful) and *purpose* (meaningful outcomes for the student), and with learning objectives to

- Deepen competence in the selection and use of mathematical methods and techniques.
- Develop confidence in representing and analysing authentic situations mathematically and in applying mathematics to address related questions and issues.
- Build skills in mathematical thinking, reasoning and communication (DfE 2015).

Because Core Maths aims primarily at building up application of mathematical skills rather than enhancing students' repertoire of mathematical facts and procedures, we hypothesised that successful teaching of Core Maths might require teachers to develop new pedagogies and even new mathematical skills. We report a small study which asked, ***'How and why do teachers develop their teaching to accommodate the distinctive demands of Core Maths – and with what support?'*** It contributes to knowledge of the ways that teachers develop their practice to promote mathematical utility and purpose, including its assessment, the support needed, and teachers' capacity for change. Unlike studies of comparable initiatives that report reform-related teacher deficits, we show these teachers were confident to make initial changes to practice within a collaborative support programme that focused on pedagogic resources and only indirectly addressed mathematical knowledge. Moreover, in the absence of formal assessment guidelines or the pressures of high-stakes accountability, teachers developed teaching repertoires that underpinned valued student outcomes and informed their wider practice. And centrally, we show they promoted change to classroom authority and expertise, and paid deep attention to developing students' meaningful relationship with mathematics.

## Locating Core Maths

Globally, the increase of non-traditional mathematics courses (Hodgen et al, 2013) reflects a widely-held aspiration that a greater proportion of the population should be 'mathematically literate.' The interpretation of mathematical literacy is contested, but usually centres on processes such as formulating, employing, and interpreting mathematics in contexts, and reasoning and communicating mathematically to support decisions (OECD, 2013).

In considering the potential demands on teachers, it was instructive to compare Core Maths with other national programmes of study also aimed at mid- to older teenagers, and aspiring to 're-form' school mathematics in ways that promote a content/context duality. This is important because there is substantial evidence (e.g. Greeno et al 1993) that young people find it difficult to transfer knowledge and skills learned in the classroom to other contexts. In this section we review three such initiatives and the research that identifies how teachers have needed to change their practice, and show the connections with definitions of 'readiness to teach Core Maths' in the English context.

There is copious literature focused on RME (*'Realistic Mathematics Education'*) approaches, intensively researched and evaluated in The Netherlands and the USA (e.g. Gravemeijer, 2002; Romberg, 1997). Basic tenets include that students should work with 'experientially-real' (not necessarily genuine) contexts, creating and elaborating symbolic models such as drawings, diagrams and tables, and thereby engaging in 'progressive mathematisation'. Rich, rigorous and challenging peer and teacher-student discussion is needed. Romberg (1997) cites evidence that teaching for an RME approach requires significant changes to traditional instructional practice, authority, and expectations. He describes additional demands on depth of teachers' mathematics knowledge compared with teaching for procedural competence, and points to the need for a coherent vision of mathematics as a discipline (p371).

The challenge of teaching for problem solving more embedded in authentic adult or work situations is well exposed in literature tracking the enactment of South Africa's *'Mathematical Literacy'* (ML) programme from 2006. Here, grade 10-12 students study either Mathematics or ML. However, as in Romberg (1997), shallow teacher knowledge of the related mathematics is quickly exposed (e.g. Bansilal and Mkhwanazi 2014), and teachers may have a variety of beliefs about the relationship between mathematics and context, or find it difficult to put their ML-consistent beliefs into practice (Webb and Webb, 2004). Graven and Venkat (2007) point to a variety of teaching intentions and practices relating to the context/content duality, often shifting over time or with focus and demand of the mathematics. Efforts to transform practice towards ML intentions have been undermined by assessments of limited validity (Venkat et al 2010), despite considerable investment in teacher professional development.

Valid assessment of similar learning intentions also proved a challenge in England, for example in the *'functional skills'* element piloted within the GCSE. Drake et al (2012) show that assessment in this area is often artificial or contrived. However, use of pre-release materials provides an opportunity for teachers to help candidates navigate what might be asked, and what is relevant mathematically and contextually. Such assessment structures have been incorporated into Core Maths – but we know little about the ways in which teachers learn to support students for such practices.

Further, existing reports of teachers trying to establish a culture whereby students use mathematics purposefully, either for their own ends or for those communicated by others, suggest they face tensions between mathematics and context, engagement and focus, student and teacher control (Ainley *et al.*, 2006; Wake, 2016). We return to these tensions in discussion.

Taken together, then, the literature suggests that embedding contextualised applications of mathematics can be demanding for both students and their teachers.

From its 2014 introduction until 2017, Core Maths teachers were supported by a government-funded 'Core Maths Support Programme' (CMSP) that identified relevant information and resources, and coordinated a network of local 'Core Maths leads' (CMLs) who facilitated shared practice. CMLs identified initial demands on teachers similar to Romberg's (1997), and by October 2016 the CMSP had formalised what were assumed to be common understandings of 'readiness to teach Core Maths':

'To be confident and competent to teach Core Maths, teachers should:

1. understand, and incorporate into their practice, contextualised mathematical problem-solving approaches;
2. have appropriate subject knowledge for teaching Core Maths and be committed to further developing their subject knowledge;
3. be committed to collaboration both in their classroom practice and for their own professional development.'

(CMSP, 2016).

We designed this exploratory study while enactment was still supported by the CMSP communicating intended curricular aims.

## Theoretical approach

Our overall approach to professional learning draws on Clarke and Hollingsworth's (2002) characterisation of teachers' change environments as multi-domain and non-linear. They highlight the multiplicity of perspectives on teacher change; within these we see the introduction of Core Maths as both an 'adaptation' to the availability of a new course and as 'personal development', aiming for additional skills or strategies (p947). They model teacher growth as having an initial stimulus and then moving amongst different domains: 'the personal domain (teacher knowledge, beliefs and attitudes), the domain of practice (all professional actions, together with the professional context), the domain of consequence (perceived salient outcomes), and the external domain (sources of information, stimulus or support)' (p949). Change within one of these domains can translate to change in another by enaction, and by reflection. The central methodological message we take from this approach is the understanding that teachers learn and change through their professional activity, and within institutional systems. We aim to keep research into teachers' development close to practice by examining their accounts of learning as routes traced within a change environment rather than as a hierarchical progression that culminates in measuring altered teachers, practice or students.

We also note the importance of what Golding (2017) calls teachers' 'capacity for change': a construct that includes the change-related knowledge and skills teachers bring, and also their change-supportive

dispositions and beliefs, their self-efficacy and confidence in relation to the espoused change, and their related resilience and solution focus.

## The Study

We set out to analyse the professional learning of a small purposive and ‘telling’ (Mitchell, 1984) sample of Core Maths teachers (four), identified with the support of the CMSP. In common with the adopted approach to growth, we chose to draw on teacher accounts of their learning in semi-structured hour-long telephone interviews. We acknowledge the threats to validity of such an approach (e.g. Hammersley and Gomm 2008), but addressed those by paying attention to e.g. empathy and rapport, active listening, restatement, and active clarification of interview responses. Nevertheless, it remains the case that we can only access what teachers choose to tell us.

We interviewed one Core Maths teacher and three CMLs from different mathematical and professional backgrounds. Post-16 education in England takes place in schools and post-16 colleges, both represented in the data, which also has regional spread. In three cases the participant was, as is typical, the only Core Maths teacher in their institution for most of their experience. The CML role involved substantial, long-term commitment and also gave an overview of the development of other Core Maths teachers, and we considered this likely to have led to prior reflection on their own, and others’, changing practice, which would enhance the data. However, we felt it was also important to include at least one teacher without that ‘bigger picture’ and motivation:

Table 1: Study participants

Lead (CML) or teacher (CMT)	Context	Taught CM from	Teaching background	Time allocation (hours)	Class size	No of classes taught by Summer 2017
CMT	11-18 school	2015	7 years in 11-18 school, teaching up to A-level	2h per week, for 2 years, now for 1 year	Up to 34	2
CML1	Post-16 college	2014	8 years post-16 teaching, largely up to GCSE	2h per week	Up to 20	3
CML2	Post-16 college	2014	8 years post-16 teaching, up to A-level	2.5h per week then 1h per week over 2 years	12-22	5
CML3	11-18 school	2014	20 years in 11-18 schools, up to A-level	2h per week over 2 years	Up to 20	2 (another teacher has also had 2)

In design, we analysed our research question (*‘How and why do teachers develop their teaching to accommodate the distinctive demands of Core Maths – and with what support?’*) taking into account Clarke

and Hollingsworth's (2002) identification of context, past experience and personal characteristics as germane to teacher growth, and aiming to inquire into change within and across the domains described above.

Together, these considerations led us to construct questions focused on

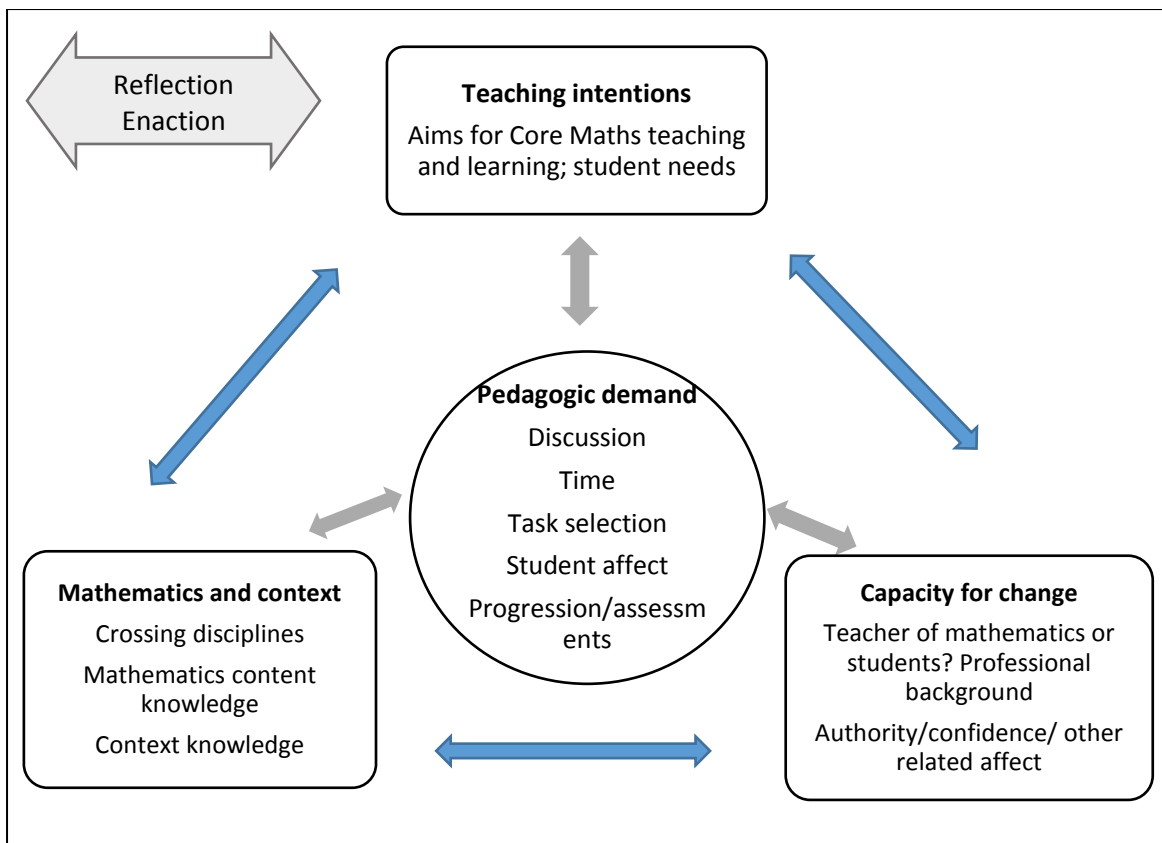
- teaching background, context espoused and mathematics education values
- their own, and the school's, preparation and intentions for teaching Core Maths
- what was the same, and what was different about teaching CM: probing the pedagogical changes pursued, their rationale and outcomes
- the greatest pedagogical challenges encountered, and support for meeting such challenge
- interactions with CMSP and other teachers of Core Maths and any impact of those
- support given to others and advice they'd offer to teachers new to Core Maths (as a proxy for distillation of key professional learning)

Data then consisted of transcriptions of four recorded interviews.

Initial coding of responses was open and descriptive (Miles and Huberman, 1994), based on codes arising from the literature (for example, discussion) and some emergent codes (students' needs, assessment).

Coding was agreed between the first two authors, themselves not Core Maths experts. This analysis was then grouped and interpreted, using four (overlapping) emergent theoretical themes: capacity for change, mathematics/context duality, teaching intentions and teaching demands. Further validity was achieved through interpretative validation by the third author, who had been employed by the CMSP. A recent version of the paper has also been offered to participant teachers for validation. The resultant thematic coding structure appears in Figure 1.

*Figure 1: Analytic themes*



The first theme concerned the **teaching intentions**. Interview data were organised by statements in which teachers expressed their *beliefs* about mathematics or the purpose of the Core Maths or wider mathematics curriculum. We also included here statements in which teachers identified the *needs* of ‘core maths students’: the rationale behind their Core Maths teaching was often framed as meeting those needs. We looked for features within these agendas that are shared by several teachers, and those that have changed over time, or have widened to include other students and courses.

The second element of coding concerns **mathematics and context**: the balance of attention between developing understanding of mathematics and of problem contexts. Clearly, where this relates to the purpose of Core Maths, this element interacts with our coding of teaching intentions. We noted it here particularly in relation to teachers’ statements about their *own* understanding of the range of contexts (perhaps in finance, politics or science), the related mathematical models (Bansilal and Mkhwanazi, 2014), and awareness of their own perspectives to considering what students may know and how they learn in areas, such as economics or science, that would traditionally be part of another teacher’s expertise.

Such considerations give rise to associated **pedagogic demands**. Included here are features of classroom enactment that teachers explained as requiring new emphasis in their Core Maths teaching, such as a change to the profile of *discussion*. That impinges on management of *time* within lessons, critical for teachers when balancing student-led sense-making with teacher-led introduction of new mathematical

ideas. *Task selection* is central to achieving curriculum intentions: teachers need to select tasks both for the scenarios and the mathematics that they want to draw out. For example, when they bring artefacts such as percentage charts into the classroom, they need to be aware of how these will be interpreted. Teachers also, as above, changed the ways they take account of *students' affect*. Lastly when considering pedagogic demands, we coded teachers' talk in relation to progression for *assessment*: an unsurprising emergent sub-theme since Core Maths teachers work with unfamiliar approaches to summative assessment.

Our final theme concerned teachers' **capacity for change**, including their valuing of a shift in emphasis to a mathematics/context duality and the contingent mathematical thinking. Within this aspect we considered evidence of sufficient pre-existing and developing subject and subject pedagogic knowledge (overlapping with our 'mathematics and context' theme), together with statements of how/why they became involved in Core Maths, and any satisfaction or discomfort that resulted from their Core Maths teaching and could be related to their overall professional role, including their confidence related to, and disposition towards, the intentions and demands described above.

## Findings and discussion: how and why do teachers develop professionally?

We wanted to know how, why, and with what support teachers develop professionally to accommodate distinctive demands of Core Maths. We start by considering the 'how' and 'why': what were the espoused changes in practice or personal characteristics that teachers initiated or identified as creating new demands, and why did they make those changes?

### Teaching intentions

All four of our participants expressed beliefs that were aligned with the DfE (2015) stated aims, enacted in social constructivist ways. They believed that 'all students deserve to be able to study some maths' [CML2], that the idea of Core Maths is to develop students who are 'users of mathematics rather than just being doers of mathematics' [CML3] and who 'understand the maths in their daily life' [CML1]. Teaching Core Maths had reinforced these beliefs, in one case making them 'see maths teaching differently' [CML1], and for the others facilitating a shift to teaching 'more aligned to my core values than any other course I teach' [CML3]. Our first analysis showed that all teachers claimed enhanced knowledge of the students in the Core Maths class, an outcome which they valued because it enabled them to identify students' personal and mathematical needs, and plan how to meet them. To this end, they reported considerable effort invested in balancing students' engagement with a focus that supported meaningful use of appropriate mathematics-in-context. Participants talked of two main groups of students whose needs were met by Core Maths: three teachers described students who were motivated to study mathematics, perhaps in order to support their



other A-levels, but who found Mathematics A-Level too abstract or too difficult. For these students, Core Maths functioned as a 'back-up' [CML2], but, teachers suggested, was a course which served students' long-term goals by providing a qualification and/or a new orientation to using mathematics that supported wider or future study. Another group, also identified by three teachers, consisted of many students who had 'been taught to jump through hoops to get enough marks' at GCSE. Core Maths enabled teachers to provide the 'useful maths that we should have been teaching [them] all along' [CML3] and the students were described as, for the first time, learning to function mathematically in effective ways. Similar messages had been promoted by CMSP as 'for study, work and life'. Interestingly, while these teachers mentioned difficulties of timetabling to include both groups, only one teacher described differentiation as a challenge in planning lessons, suggesting that Core Maths can indeed suit both audiences.

In articulating their rationales for Core Maths, all interviewees emphasised the meaningful learning of mathematics. They argued predominantly for a 'broader understanding of maths' [CML3] that was achieved by using target content in a variety of contexts. Three teachers also explicitly expressed a goal that students would function better in their adult life because they were prepared to take a mathematical view of 'real world' issues such as money and information. This goal extended beyond acquiring context-specific skills. CML1 argued this most strongly: 'I want them to have learnt scepticism.' We identified, however, a third element in their articulation of goals, which prioritised neither a better understanding of mathematics, nor an understanding of real world contexts, but instead students' confidence and sense of agency over mathematics, including an emergent belief that mathematics should, and can, make sense. CML2 was typical in approving when colleagues commented about his students: 'how confident they are to work mathematically, to function mathematically, even if it's not quite explicitly what they've been working on in my lessons.' This attention to mathematical self-efficacy is linked in part to teachers' reflections on perceived weaknesses in previous examination-dominated courses.

The espoused teaching intentions were therefore consistent with teachers' pre-existing professional beliefs, but successful enactment of the course deepened and refined those. Comparing these teachers' accounts with Clarke and Hollingsworth's (2002) framework, intentions appeared to emanate from teachers' beliefs about the aims of Core Maths or what is important more widely, but then to develop almost continuously through cycles of enaction, reflection and developing outcomes of salience.

### Mathematics and Context

Consistent with the approaches articulated in Wake (2016) and with Ainley, Pratt and Hansen (2006)'s 'purpose and utility', teachers emphasised that Core Maths attempts to take authentic contexts, often as experienced in the workplace or in adult life, and transpose them into the classroom, where they become a source of authentic enquiry through which meaningful mathematical ideas can be developed. Models and

representations of the context then become 'boundary objects' between the context and mathematics. Teachers described students' sense-making as entailing both students and teacher in articulating emergent meaning and actively listening to others' articulation. Teachers needed to be aware of multiple perspectives on the mathematical objects in a context – and ready to draw out clarification of assumptions, definitions and communications: 'There are a lot of other things that you needed to discuss, to prompt, to talk about, to ask questions about. It's not simply 'here are five questions on confidence intervals; do them' ' [CML1]. But alongside that, Wake (2016) also points to students' reframing of themselves in relation to mathematics, and that was reflected by these teachers: 'They become users of mathematics rather than just being doers of mathematics' [CML3].

For our teachers, there was little reported difficulty with the mathematics content – and neither did they report tensions between teaching for content and teaching for context, though that involved new challenges in their planning. As discussed below, they perceived learning about problems in context as the initial motivator for students – but part of their job was to plan also for sufficient focus on both context and mathematics. They did not feel they lacked specialised knowledge – the mathematical models required were within their general knowledge, or what they could read up on. Instead, they reported drawing on both students and colleagues in order to understand new contexts, then harnessing their content knowledge in deep and flexible ways in order to optimise the opportunities for students to encounter the mathematical potential of a context. They pointed also to the challenge this would pose to teachers with less deep subject-knowledge and confidence. One exception to that was in financial mathematics, where two teachers had felt challenged by APRs and similar – and sought relevant advice from external experts, only to find they could not explain them either.

Teachers reflected that students were now the experts in some contexts, particularly those closely related to their other areas of study, with teachers supplying the nuances of the related areas of mathematics. They did not see this as a tension, but welcomed the shift in aspects of classroom authority as evidence of students' growing ownership of the course. One fruitful approach to the widening of remit was to liaise with teachers for students' other subjects, identify where maths is needed and devise 'boundary tasks' that cross the curriculum.

All four teachers expressed a personal interest in the 'bigger picture' of what mathematics is used for, and were re-inspired to give it more time in this and also in their other teaching. CML3 reflected that although teachers of other subjects would already have some contextual knowledge, they might find it difficult to respond as flexibly or deeply to students' mathematical questions as a mathematics expert. In this respect, it should be remembered that the sample teachers, and those they worked with in CMSP-supported meetings, were self-selecting early adopters, with most of their students opting in to the course, whereas the

aspiration (DfE, 2015) is for the vast majority of eligible students to be participating, necessarily drawing on a wide range of teacher backgrounds, experiences – and probably beliefs.

### Pedagogic demands

In their narratives of the demands of teaching Core Maths, a first focus for interviewees was **task selection**. This had a high profile for curricular reasons: teachers needed to select the specification to follow, any options within it (for example a focus on finance or modelling), and then the contexts, artefacts and questions that would be used, strategically selecting those so that over time, the (relatively small quantity of) mandated mathematical content had been drawn out. It also had a high profile for affective reasons. Teachers were concerned – particularly in the early weeks - to choose tasks that would motivate students by signalling a change: ‘something new that they haven’t done before so they’re not thinking it’s just GCSE again’ [CMT]. All the teachers commented on the need to build students’ initial commitment to Core Maths, because it lacked status as a new qualification, outside typical university entrance requirements. Some teachers chose contexts deemed of general interest, such as climate change, while others paid particular attention to matching contexts to students’ interests: ‘there’s a big investment in planning, in thinking round what will work for your group of students, and you try to hit everyone’s interests over not too long a period.’ [CML2].

Teachers often cited the CMSP task bank as invaluable in providing tasks that they could trust to be curriculum-appropriate. Tasks were also sourced from colleagues teaching students’ other subjects. Ainley, Pratt and Hansen (2006) show such task *development* for younger children is highly demanding, and although these late teenagers bring a substantial fund of both mathematics and contextual experience or second-hand knowledge, teachers reported they still had to devote considerable time and expertise to task selection, essentially to meet both *purpose* and *utility* requirements for the range of their students. Textbooks, though valued, were used largely to support or enrich teachers’ knowledge rather than provide initial tasks.

Changes in aspects of **classroom authority** and expertise were noted as a demand for new Core Maths teachers who need to be comfortable that they are ‘not always, or even often, the expert in the room, just the ...the coordinator I suppose, of the learning’ [CML2] – and that they are more than teachers of mathematics: ‘(You’re)...not only teaching the mathematics – students also need to understand the situation that is being explored. They are often curious and ask about what interests them in the contexts’ [CML3]. This was not usually, however, framed as a tension, but rather as a constructive and purposeful change associated with attending to the students’ needs and relationship with mathematics. The exception to this was CML1’s early experiences of resentment among students, apparently influenced by mandatory

participation in her college. This, and her awareness of coming new to contextualised problem solving, meant that she initially felt her classroom authority, and confidence, were undermined.

All the interviewees described Core Maths as distinctive in that an initial task merited development into an **extended learning sequence** involving a range of mathematics and/or contexts. Thus, one variation in their use of time was in the granularity of their planning. CML3 described rethinking his planning of content coverage: 'it's almost like a spider's web that comes out from the task. And you might tick off different aspects...' Teachers needed flexibility, since students could ask unexpected questions of the context or related mathematics. Teachers described adapting to a more fluid pace as a welcome challenge, which they recognised as furthering a necessary and valuable outcome for students: 'their mentality about numbers and stats around them, how they change... It does take a while' [CML1]. Wake (2016) points to the extended time needed for students to become familiar with artefacts that can appear routine for those already embedded in the context. Here, teachers considered the Core Maths curriculum to be well designed, with enough learning hours (if enacted as intended) and sufficiently few new mathematics topics to allow 'genuine learning' [CML2].

**Discussion** was highlighted by three teachers as central to contextualised problem-solving, although only one explicitly claimed to have developed associated skills: 'you have to learn how to do it, and how to use all their ideas and respond to whatever you get in a constructive way' [CML2]. Others noticed it as a source of initial discomfort, but implicitly an area they'd developed: 'One of the things is having the confidence that when the worms are coming out of the can, what's going to happen, and not getting too stressed over it. [...] It's something that I feel I can still develop in terms of how to ...put a ring round it a bit better and how to get people communicating better mathematically' [CML1]. It appears that what concerns these teachers at this stage is relinquishing control, rather than primarily use of time or, as suggested in Wake (2016), the need to develop students' skills of listening and collaborative sense-making. It may be that, as mathematics teachers, they have not previously been required to attend to dialogic teaching, for example in helping students to respond to each other in situations where there are many reasonable answers.

Interviewees identified the unusual position that Core Maths initially placed them in: they were teaching a new course without detailed knowledge of the **assessment** strategy that would determine success for them, the school and the students. At the interview teachers could evaluate outcomes from at most two cohorts, sometimes from a small number of students of variable commitment, and they made no claims for developing reliable schema of progression or summative assessment in relation to knowledge/demands of mathematics or context: 'I predicted all Bs and Cs and Ds and then on down. I think there were [...] three As, two Bs, six Cs...' [CML3]. Instead, we identified an informal criterion for success (and hence implicitly for progression, as we note below) as being used across teachers' accounts: students' readiness to interrogate a

situation mathematically. Although one, very experienced, teacher felt secure that this approach would benefit students' grades, others felt under-equipped and responded either with experimentation, 'tweaking' tasks from the CMSP network into internal assessment instruments, or frustration: 'I didn't feel that I had the support to turn that - following the Core Maths approach - into preparing them for the exam.' [CML2]. Incorporating pre-release context-rich examinable material into class tasks was mentioned as a source of particular anxiety, because of its novelty for teachers.

Finally in this section we return to teachers' attention to the ***affective reactions*** of their students, specifically motivation and confidence, which influenced teachers' decisions about tasks, discussion, time and assessment. The attention to affect was framed as initially required in order to 'build commitment' [CML2] to a new course, and to promise more 'positive experiences' than at GCSE. However, it was also noted as a *result* of experiencing teaching the Core Maths curriculum. By listening to students, and giving time to their questions and approaches, teachers found they developed strong relationships that could be used to support learning. Two teachers noticed the comparison with getting to know their students in other classes: 'that doesn't have to be different from A-level, in fact I now do that at A-level too, but you can't do without it at Core Maths.' [CML2]. They reflected that the pressure on time and fuller content at A-level made it more difficult to adopt the dialogic approach. Thus, the structure and content/context duality of Core Maths may allow teachers to enact their core beliefs about the importance of both respecting and harnessing student affect. Our teachers mirror experiences reported in Graven and Venkat's (2007) study - that use of authentic contexts provides both motivation and opportunity for learners' increased participation and confidence. Wake (2016) warns that the change in emphasis to mathematics in context requires students to reconceptualise themselves as *users of* mathematics, and that this emotional work needs recognition and support by teachers: an aspiration embraced by our teachers.

### Teacher capacity for change

Here, we consider teachers' knowledge, skills and affect for supporting the espoused change. We noted above that our teachers were both experienced and well-placed mathematically. They described teaching Core Maths as an opportunity for professional development, with two CMLs and the CMT framing it as a site for them to consolidate or explore an existing belief about teaching. Their narratives confirm the professional satisfaction they felt in designing and supporting the course. CML1 viewed Core Maths as a more mixed experience: her account includes the experienced threats of a 'horrible first cohort'- 'crashing and burning' - and 'daunting' changes, although the report is of eventual enjoyment and growth. Two participants (CMT and CML3) strongly identified themselves as 'teachers of mathematics', but, unlike Graven and Venkat's (2007) teachers, none showed evidence of being particularly challenged by foregrounding context rather than mathematics: all participants said that although an academic mathematics route has suited them, it was not for everyone – and two participants argued that A-level students should also

experience mathematics in authentic contexts. Three identified an empowering impact on students of doing so: 'Core Maths is centred around applying mathematics, taking information and understanding how that informs you of the world around you' [CML3].

More long-lasting threats to teacher confidence were, as in other related initiatives, associated with their loss of expertise in assessment: course consisting entirely of contextualised problem solving, no matter how highly valued, came without knowledge of markers of student progression that could be used formatively. Further, CML2 and CML3 started teaching Core Maths before its summative assessment structures had been finalised. They reported finding lack of available sample assessments simultaneously liberating ('a rather luxurious first couple of months' [CML2]) and daunting (CML2:) '(using) pre-release material ...(is) a very particular skill, and we banged our heads against a brick wall, it felt, for ages, before we came up with some reasonable ideas for making that work' [CML2]. Although all the teachers commented that Core Maths was widely perceived as less high stakes than A-levels or GCSEs, their professional role required monitoring students so as to predict examination performance.

Three teachers mentioned the isolation often experienced by teachers of Core Maths, who at present are often the only such teacher in their institution - yet by 2017 all had moved to a position where they talked positively about how they had developed as a teacher. All had tried to adopt aspects of their Core Maths approach to their other teaching, but commented that they were limited by both more high stakes assessment and greater density of content in other courses.

Finally in this section, all perceive themselves to have been *successful* in their enactment of Core Maths, instantiating that with examples of how their students have slowly learned powerful ways of interacting with mathematical aspects of their everyday lives. This has reinforced their belief in the value of the approaches adopted to such an extent that, so far as they believe possible, they have incorporated those into their other teaching. Our sample teachers came to Core Maths with well-developed capacity for this change, and indeed their accounts of early adoptions by other teachers do not offer anything very different. However, it is interesting to note that in adapting to accommodate a content/context duality, teachers developed competence in coming to understand a variety of novel contexts and analyse those for their mathematical potential, often harnessing local peer or student expertise. They thus increased their knowledge base and adapted their practice in significant ways, but clearly had *sufficient* initial expertise, with supportive affect, to achieve that. It is likely to prove more challenging to scale up successful enactment to the degree envisaged nationally, because of the limitations on teacher capacity within the system (Smith, 2017).

## How has development been supported?

Participants pointed to the breadth and depth of the demands on teachers: 'I think they have to know that it's going to be the most intensive preparation-wise of any course they're going to teach' [CMT]; 'lots of teaching about effective communication.... It needs really big changes in your practice.... you've got to have that peer support' [CML2]; 'You have to have people you can hold on to at the beginning....and to be as creative as you can' [CML1]. Our findings thus agree with Hough (2015, p3) who signalled 'how much support teachers will need in changing their practices to accommodate the demands of teaching Core Maths. This requires training and reflection opportunities beyond the initial CPD phases'. However, in contrast to Graven and Venkat's (2007) teachers, who received extended and intense development for their task, the initial CPD of these teachers was very limited: 'Preparation?? There wasn't any. Because no-one had done it before, and when we started most of the specifications hadn't even been accredited' [CML2].

Apart from a small number of CMSP-provided tasks, teachers drew on the expertise of students and of colleagues in other disciplines, as well as a variety of resources identified individually. The onus of identifying the mathematical potential of those, and of developing that in appropriate and context-authentic ways, lay with individual teachers. Textbooks were perceived to function in new ways, used, if at all, only at the end of extended learning sequences: 'The textbook is fantastic as well....I use it... as a reinforcing tool once my own contextualised teaching has been done and we've had that period of time where they've been just playing around with the maths that I've been pointing them in the direction of' [CML3].

The most telling support for all our teachers, though, seemed to be the termly regional Core Maths workshops facilitated by the CMLs, who were paid to invest time, preparation and follow-up of these. Here teachers pooled individual experiences and collaboratively developed approaches and resources. Such approaches were seen to support an autonomy and ownership of enactment, but to be critical in building up and retaining confidence when attempting significant change: 'There were regular physical meetings and they were incredibly useful.... just being in a room with like-minded people and discussing approaches. And it's about honing your approach and seeing different perspectives and people's experience. And that was invaluable' [CMT]; 'The networks... are crucial, and they've been great' [CML2]; 'I think...most of my best ideas have been other people's ideas' [CML3].

Over time, these meetings spawned online networks using social media and Dropbox to share ideas and resources; additionally, CMSP has validated and standardised presentation of teachers' adopted resources at a national level. Teachers noted these remain accessible after the closure of CMSP, implicitly communicating key intentions of the course. However, all our sample teachers talked about a need to continue developing their practice for new groups of students with different needs and emphasised that the face to face nature of network meetings was distinctively valuable. One CML expressed a concern that without key messages

provided centrally, there was a danger that future adopters would start – and end - their preparation with a textbook and lose the approach of designing extended learning sequences based on students' affective and mathematical responses. However good the textbook may be, he believed that this approach would lose much of the flexibility and enterprise of early adopters, and was key to developing confidence in contextualised problem solving.

Teacher development to date has therefore been a product of a low-stakes assessment environment and committed teachers, coupled with local collaboration and funding to support that. There is, though, concern among these teachers. Future provision of teacher support should continue to be sufficient in both quality and quantity to communicate a deep understanding of the aspirations of the course, if those are to be met.

## Conclusion

Core Maths was developed as an attempt to improve young people's contextual mathematical functioning in robust ways, and eventually, at scale. These early indications of teachers' belief in its initial success suggest that working with such a content/context duality can be fruitful in moving young people towards effective mathematical functioning. However, these well-placed teachers also point to considerable challenges inherent in developing teaching that supports their perceived course intentions. The sample teachers claim to have needed to change their classroom practice in a number of ways, most notably in the location of authority and expertise, the granularity of their planning and flexibility in enactment, and their attention to students' relationship with mathematics. They have had to learn to teach with pre-release assessments, while also appreciating the enhanced validity offered over traditional examinations. However, in contrast to some of the difficulties and issues reported in similar situations elsewhere, these teachers' accounts of professional development suggest a cautiously optimistic assessment of the potential for wider valid enactment of Core Maths.

We have shown above that, contrary to the expectations of some other initiatives to promote contextual mathematics (e.g. Bansilal and Mkhwanazi, 2014; Romberg, 1997; Wake, 2016), the teachers in this sample did not report concerns either about developing their own knowledge of context or mathematics-in-context, nor about balancing students' attention – for themselves or for other early adopters. There was less discomfort than we expected with changes that concerned teachers' authority, and a close alignment between teachers and the CMSP around the intention that students' learning about contexts is as important as their learning about mathematics. We conclude that institutional and cultural settings are highly significant in considering teachers' needs in preparation for change. The support provided by the CMSP was largely through example tasks, planning guidance and peer support, and did not directly address mathematical subject knowledge. For this sample, this resource was sufficient to allow teachers themselves to work on applying their own relevant mathematical knowledge – and particularly their knowledge of



mathematics-in-context, as well as of those contexts. Moreover, they used the knowledge of students and teachers of other subjects to conceptualise Core Maths as functional mathematics underpinning much of the 16-18 school curriculum. We suggest that future support programmes should not *assume* that significant mathematical subject knowledge development is an initial priority.

A second way in which our study challenges existing thinking is by demonstrating that, even in the English culture of performativity, teachers are able to conceptualise learning outcomes and indicators of progression that do not depend on an externally-provided grading system. These teachers have developed new salient outcomes – new ways of formatively evaluating their teaching and students’ progress towards mathematical literacy - by paying attention to student’s participation, their willingness to apply mathematics and their self-efficacy as users of mathematics. This is in contrast to usual practice in England (Ofsted 2012) and their own concerns about teaching an examination course with no exemplar assessment material. We must emphasise how strongly teachers reported a need for support, such as given by the CMSP, in providing information and collaboration around these new forms of formative and summative assessment. There are also caveats around this finding. These teachers were early adopters of Core Maths, opting into it and with beliefs well aligned with its intentions, as well as benefiting from central investment in the CMSP. They were thus unusually well placed in terms of capacity (Golding, 2017) to make such changes.

For our sample teachers this has been so successful that they have all incorporated related changes into their other teaching in ways they had previously not thought possible, apparently supported by the low-stakes and low-density nature of Core Maths, and by the CMSP-initiated support, both of which allowed experimentation with assessment and evaluation practices. We know that collaborative peer networks, particularly those involving external expertise, can be very productive for teacher development (e.g. Joubert and Sutherland 2009). We suggest a need for further research on how mathematics teachers’ development might be hampered if they teach only high-stakes, high-density courses.

Comparing our findings with the CMSP’s articulation of ‘teacher readiness to teach Core Maths’, (CMSP, 2016) the characteristics identified were certainly evident. However, teachers claimed much more was needed. For example, they talked not just about collaboration but about professional conversations that helped them understand changes in authority and power in the classroom, leading to shared teacher/student responsibility for defining what is made available to learn and shared expertise. They drew on not just deep and flexible subject knowledge but growing understanding of contexts. ‘Contextualised mathematical problem solving approaches’ were described not just as pulling the mathematics from contexts and using it to answer related questions, but as deeply responsive modes of facilitating students to become effective users of mathematics. They became both attuned and confident to persist in pursuing the potential of mathematics to support human curiosity and activity in a variety of authentic contexts. Teachers

claimed that such approaches are not only desirable, but necessary, for Core Maths. Expansion and embedding of contextualised problem solving depends on similar approaches being developed by teachers who might come with less experience, less deep and flexible disciplinary knowledge, or initial beliefs less aligned with the intentions of Core Maths. National aspirations (Smith, 2017) would also entail the enrolment of many more students not at present opting for the course, and perhaps currently indifferent to its potential benefits. If this apparently initially successful approach is to thrive, central and vibrant communication of the emerging benefits therefore needs to be addressed, and sustained collaborative support maintained.

In conclusion, then, for these teachers, Core Maths has developed their teaching by fostering consideration of how to shift power and agency in the classroom between teacher and student with a consequential transition in how the teacher conceptualises the mathematics and what mathematical student outcomes are valued. Teachers described professional growth in relation to this dispersion of expert authority when balancing content/context, they developed attention to students as users of mathematics, granularity and flexibility of planning, and management of classroom unpredictability. Their teaching intentions combined clear values, strategies for enacting these and identification of outcomes for evaluation. They ascribed this to a combination of a low stakes Core Maths policy context, CMSP support, and a low-density curriculum.

Finally, the theoretical approaches adopted suggest questions that we argue should be asked about more traditional mathematics courses: in particular, what is their purpose and utility to students? The relative attention paid to mathematics and to context, and to learning to cross those boundaries, are central to becoming a user of mathematics and important wherever the intentions of a course that includes applications.

## References

- Advisory Committee for Mathematics Education (2012a) *Post-16 mathematics: A strategy for improving provision and participation* [http://www.acme-uk.org/media/10520/20121217acme\\_post\\_16\\_](http://www.acme-uk.org/media/10520/20121217acme_post_16_)
- Advisory Committee for Mathematics Education (2012b) *Post-16: Planning for success*, at [http://www.acme-uk.org/media/10523/20121217acme\\_post\\_16\\_planning.pdf](http://www.acme-uk.org/media/10523/20121217acme_post_16_planning.pdf)
- Ainley, J., Pratt, D. & A. Hansen (2006) Connecting engagement and focus in pedagogic task design *British Educational Research Journal* 32/1, 23-3.
- Bansilal, S., & T.W. Mkhwanazi (2014). Mathematical Literacy teachers' engagement with contextualised income tax calculations. *Pythagoras*, 35/2, Art. #246
- Clarke, D., & H. Hollingsworth. 2002. Elaborating a model of teacher professional growth. *Teaching and Teacher Education* 18, 947-967.
- CMSP (2016). *Core Maths Support 2016-17 briefing paper* (unpublished).
- Department for Education (2015) *Core Maths Technical Guidance* at [https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/450294/Core-Maths-technical-guidance.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/450294/Core-Maths-technical-guidance.pdf), accessed 12 October 2017

- Drake, P., Wake, G. & A. Noyes (2012). Assessing 'functionality' in school mathematics examinations: what does being human have to do with it? *Research in Mathematics Education* 14/3, 237-252
- Golding, J. (2017). Mathematics teacher capacity for change *Oxford Review of Education* 43/4, 502-517
- Gravemeijer, K. (2002) Preamble: from models to modelling in K. Gravemeijer et al (eds) *Symbolising, modelling and tool use in mathematics education* 7-22. Springer, Dordrecht
- Graven, M. & H. Venkat (2007) Emerging pedagogic agendas in the teaching of Mathematical Literacy *African Journal of Research in Mathematics, Science and Technology Education*, 11/2, 67-84
- Greeno, J. G., Moore, J. L., & Smith, D. R. (1993). Transfer of situated learning. In D. K. Detterman & R. J. Sternberg (Eds.), *Transfer on trial: Intelligence, cognition and instruction* 99-167. Norwood, NJ: Ablex.
- Hammersley, M. & Gomm, R. (2008). Assessing the radical critiques of interviews. In: M. Hammersley, (Ed.), *Questioning Qualitative Inquiry: Critical Essays* (pp. 89-100) London: Sage.
- Hodgen, J., Marks, R. and D. Pepper (2013). *Towards universal participation in post-16 mathematics* Nuffield Foundation, London.
- Hough, S. (2015) *Changes in teacher identity in the teaching of Core Maths* unpublished.
- Joubert, M. & Sutherland, R. (2009) *A perspective on the literature: CPD for teachers of mathematics*. Sheffield: NCETM
- Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis* (2nd ed.). London: Sage Ltd.
- Mitchell, C. J. (1984). Typicality and the Case Study. In *Ethnographic Research: A Guide to General Conduct*, edited by R.F. Ellen, 238-241. New York: Academic Press.
- OECD (2013): *OECD Skills Outlook 2013: First Results from the Survey of Adult Skills* available at <http://skills.oecd.org/skillsoutlook.html>
- OFSTED (2012) *Mathematics made to measure* HMSO, London.
- Romberg, T. A. (1997). Mathematics in context: Impact on teachers. In E. Fennema & B. S. Nelson (eds.), *Mathematics teachers in transition*. Mahwah, NJ: Erlbaum.
- Smith, A. (2017) *Review of post-16 mathematics in England* DfE, London.
- Venkat, H., Graven, M., Lampen, E., and P. Nalube (2010). Critiquing the Mathematical Literacy assessment taxonomy: Where is the reasoning and the problem solving? *Pythagoras* 70/43-56
- Webb, L. AND Webb, P. (2004). Eastern Cape teachers' beliefs of the nature of mathematics: implications for the introduction of in-service mathematical literacy programmes for teachers *Pythagoras* 60/13-19
- Wake, G. (2016). Mathematics, modelling and students in transition *Teaching Mathematics and its applications* 35, 172-186

**Jennie Golding** is a lecturer in Mathematics Education at University College London Institute of Education. Her background is in the teaching of mathematics from pre-school to university entrance level and in teacher initial and continuing education. Her research foci are mathematics education policy enactment and the development of teacher capacity. [J.golding@ucl.ac.uk](mailto:J.golding@ucl.ac.uk)

**Cathy Smith** is the Head of Mathematics Education and a Senior Lecturer at The Open University, UK. Her current work involves mathematics teacher professional development, supervision of research students, and re-research. She has a longstanding research interest in pedagogy of advanced mathematics education and in studying discourses of participation in mathematics. [cathy.smith@open.ac.uk](mailto:cathy.smith@open.ac.uk)

**Mick Blaylock** is a part-time tutor at MMU as well as a freelance mathematics education researcher, consultant, presenter and CPD provider. He led the DfE funded Core Maths Support Programme to introduce the new Core Maths qualifications. [M.Blaylock@mmu.ac.uk](mailto:M.Blaylock@mmu.ac.uk) [mjblaylock@aol.com](mailto:mjblaylock@aol.com)