Energy Efficiency Optimization with SWIPT in MIMO Broadcast Channels for Internet of Things

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Abstract—Simultaneous wireless information and power transfer (SWIPT) is anticipated to have great applications in fifth-generation (5G) communication systems and the Internet-of-Things (IoT). In this paper, we address the energy efficiency (EE) optimization problem for SWIPT multiple-input multiple-output broadcast channel (MIMO-BC) with time-switching (TS) receiver design. Our aim is to maximize the EE of the system whilst satisfying certain constraints in terms of maximum transmit power and minimum harvested energy per user. The coupling of the optimization variables, namely, transmit covariance matrices and TS ratios, leads to an EE problem which is non-convex, and hence very difficult to solve directly. Hence, we transform the original maximization problem with multiple constraints into a suboptimal min-max problem with a single constraint and multiple auxiliary variables. We propose a dual inner/outer layer resource allocation framework to tackle the problem. For the inner-layer, we invoke an extended SWIPT-based BC-multiple access channel (MAC) duality approach and provide two iterative resource allocation schemes under fixed auxiliary variables for solving the dual MAC problem. A sub-gradient searching scheme is then proposed for the outer-layer in order to obtain the optimal auxiliary variables. Numerical results confirm the effectiveness of the proposed algorithms and illustrate that significant performance gain in terms of EE can be achieved by adopting the proposed extended BC-MAC duality-based algorithm.

Index Terms—Simultaneous wireless information and power transfer (SWIPT), energy efficiency (EE), multiple-input multiple-output (MIMO), Internet-of-Things (IoT).

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I. INTRODUCTION

The pursuit of ever higher data rate in mobile network is justified by the continual increasing demand. Recent studies shown that by 2020, the mobile traffic in mature market is projected to grow from 183 Mb/month/person in 2010 to 16.3 Gb/month/person [1]. Expected to be commercially available in early 2020s, the fifth generation (5G) network will need to deliver ultra high data rates to pave the way for future ultra high rate applications, and the Internet-of-Things (IoT) era. This trend makes spectral efficiency (SE) to be the main performance indicator for the design and optimization of wireless systems, but at the same time constitutes to ever-rising network power consumption which has severe implications in terms of both economic and ecological costs.

Energy harvesting (EH) is considered a prominent solution for prolonging the lifetime of power-constrained wireless devices [2]. In addition, EH enhances sustainability by empowering wireless nodes to collect energy from the surrounding environment. In addition to well-recognized renewable energy sources such as biomass, wind, and solar, wireless power transfer (WPT) has emerged as a new enabler for EH. With WPT, the transmitter can transfer energy to the receivers via ambient radio frequency (RF) electromagnetic waves [3]. The integration of RF-based EH capability in communication systems opens up the possibility for simultaneous wireless information and power transfer (SWIPT). This topic has attracted great attention in both academia and industry recently. The authors in [4] investigated practical beamforming techniques in a multiple-input multiple-output (MIMO) SWIPT system, where two practical receiver approaches, namely time-switching (TS) and power-splitting (PS), were discussed. The work in [5], studied the robust beamforming problem for a multi-antenna SWIPT wireless broadcasting system considering imperfect channel state information (CSI) at the transmitter. In [6], a dynamic switching strategy was proposed for a point-to-point SWIPT link in order to exploit the trade-off between information decoding (ID) and EH based on the TS technique. The authors in [7] evaluated the optimal PS ratio in an amplify-and-forward (AF) wireless cooperative network. The work in [8] investigated the optimal PS strategy which achieves the rate-energy region in a single-input single-output (SISO) SWIPT system. Further, SWIPT was studied in the context of multi-user orthogonal frequency division multiplexing access (OFDMA) in [9], multi-user multiple-input single-output (MISO) in [10], relay systems in [11], ultra dense networks in [12], massive MIMO system in [13] and IoT [14]–[16].

Most research works on SWIPT systems aim to maximize the rate or the harvested energy, or otherwise achieve a certain rate-energy balance. Nevertheless, the stand-alone maximization of the system throughput would inherently constitute to the highest network power consumption. This trend goes against global commitments for tackling the so-called capacity crunch in a sustainable and economically viable manner [17]. On the other hand, the sole goal of maximizing the harvested energy may degrade the delivered information, and in turn quality of service (QoS). An alternative strategy is therefore to consider energy-efficiency (EE), defined as the number of delivered bits per unit energy. Thanks to the rapid resurgence in green radio (GR) research, EE is nowadays considered a fundamental performance metric in the design and deployment of wireless networks [18]. In addition to the many works on the EE optimization problem for conventional communication setups [19], the maximization of EE has been recently considered in the context of SWIPT systems [20]–[24]. The work in [20] provided a resource allocation algorithm for OFDMA-based SWIPT systems considering a PS-based receiver with continuous sets of PS ratios. In [21], the EE optimization problem in the downlink of a multi-user MISO SWIPT cellular setup was studied, where zero-forcing (ZF) beamforming was employed at the base station (BS). In [22], a joint antenna selection and spatial switching (SS) scheme for QoS-constrained EE optimization in a MIMO SWIPT system was provided. The results therein revealed that the SWIPT-based solution is capable of providing additional EE gain compared to conventional systems.

A. Main Contributions

In contrast to previous literature on EE for SWIPT, such as multi-carrier OFDMA systems [20], [25], [26], MISO systems based on a fixed precoder such as ZF [21] or MMSE [27], in this paper, we address the EE optimization problem for SWIPT-based MIMO-broadcast channel (BC) where TS technique is employed at each receiver. Particularly, transmit covariance matrices and TS ratios allocation policies are jointly considered towards optimizing the system EE. In addition, apart from the conventional maximum power condition, per-user minimum harvested energy constraints are explicitly included in the EE maximization problem.

Intuitively, by coupling the optimization variables in terms of transmit covariance matrices and TS ratios, the EE maximization problem under consideration becomes non-convex. Hence, it is very difficult to obtain the system EE solution using direct methods, in particular, considering that the algorithm should be feasible in practice. Hence, to tackle this problem, we transform the original optimization problem with multiple constraints into a min-max problem with a single constraint and multiple auxiliary variables. In order to tackle the transformed problem, we propose a dual inner/outer layer resource allocation framework. By invoking the conventional BC-multiple access channel (MAC) duality principle [28], we formulate a dual SWIPT-based MIMO-MAC EE optimization problem with fixed auxiliary variables and accordingly provide an iterative resource allocation algorithm based on the Dinkelbach method [29] for obtaining the solution. Furthermore,
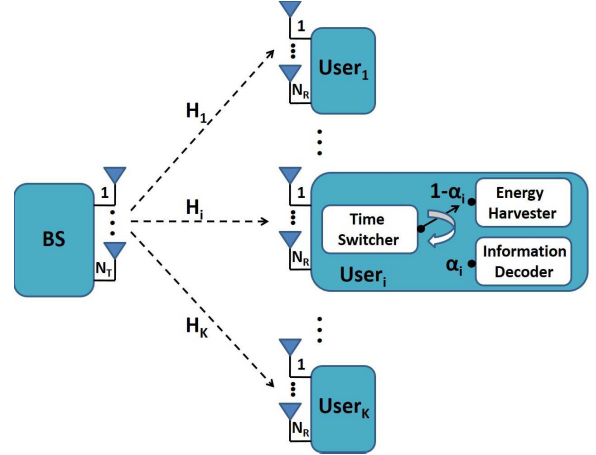

Fig. 1. A downlink SWIPT-based MIMO-BC system with TS-based receivers.

in order to reduce the computational complexity, we prove that there exists a quasi-concave relationship between the EE and the transmit power in the dual MAC EE optimization problem. By exploiting the quasi-concavity of the EE in the transmit power, we develop a low-complexity resource allocation scheme based on the particular EE-power region. A subgradient searching scheme is then proposed in order to reach the optimal auxiliary variables in the outer-layer. Simulation results confirm the validity of the theoretical findings.

B. Paper Organization

The remainder of this paper is organized as follows. The system model and problem formulation are given in Section II. In Section III, the equivalent EE optimization problem based on the extended BC-MAC duality principle is introduced. In Section IV, an iterative resource allocation scheme based on Dinkelbach method is proposed. In Section V, an alternative low-complexity solution based on the quasi-concavity property of EE-power is proposed. A complete solution to the SWIPT-based EE maximization problem is presented in Section VI. Simulation results are provided in Section VII. Finally, conclusions are drawn in Section VIII.

Notation: bold upper and lower case letters are used to denote matrices and vectors; $(\cdot)^{-1}$ stands for the matrix inverse, $(\cdot)^T$ is the matrix transpose; $(\cdot)^H$ corresponds to the matrix conjugate transpose; $\mathbf{I}_{N_t \times N_t}$ is an $N_t \times N_t$ identity matrix; $\mathrm{Tr}(\cdot)$ denotes the trace of a matrix; $[x]^+$ represents $\max(x,0)$; $(\cdot)^b$ and $(\cdot)^m$ correspond to BC and MAC parameters, respectively.

II. PRELIMINARIES

In this section, we introduce the system model of a MIMO-BC with TS-based SWIPT and mathematically formulate the EE optimization problem.

A. System Model

The system consists of a BS with N_t transmit antennas and K users $k \in \{1, 2, \ldots, K\}$ each with N_r receive antennas.

We denote the channel matrix from the BS to the kth user as $\mathbf{H}_k \in C^{N_r \times N_t}$. Channel state information (CSI) is assumed to be perfectly known at the corresponding transmitter and receivers. Note that the CSI at the receivers can be obtained from the channel estimation of the downlink pilots. CSI at the transmitter can be acquired via uplink channel estimation in time division duplex (TDD) systems. We also make a reasonable assumption that the association of users with the BS is fulfilled and fixed during the runtime. Different from the conventional MIMO downlink system, each transmission block in the SWIPT-based MIMO-BC system is divided into two orthogonal time slots, one for ID and the other for EH, per illustrated in Fig. 1. In particular, the TS-based receiver periodically switches its operations between ID and EH. It is assumed that time synchronization has been perfectly built between the transmitter and the receiver. Hence, the received signal from the BS to the kth user before TS can be written as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{x} + \mathbf{n}_k, \tag{1}$$

where $\mathbf{n}_k \in C^{N_r \times 1}$ is the independent zero-mean additive white Gaussian noise (AWGN) with each entry $\mathcal{CN}(0, \sigma^2)$, $\mathbf{x}$ is the transmitted signal on the downlink. In addition, $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2 + \ldots + \mathbf{x}_K$ where $\mathbf{x}_k \in C^{N_t \times 1}$, is the signal transmitted to the kth user.

It is important to note that it is unnecessary to convert the RF band signal to the baseband for the purpose of collecting the carried energy. However, because of the law of conservation of energy, we could make a reasonable assumption that the total harvested RF-band power (from all receiving antennas) is proportional to that of the received baseband signal. In addition, we note that the structure of an EH-based receiver depends on its specific implementation in practical wireless communication systems. For instance, electromagnetic induction and electromagnetic radiation are capable of transferring wireless energy [30]. On the other hand, the receivers' hardware circuitries as well as the corresponding EH efficiency could be significantly different. Apart from that, due to the practical hardware limitations, the decoding signal cannot be used for collecting energy directly [30]. Consequently, to avoid the impact from the specific hardware implementation details on the resource allocation algorithm design, we do not assume a particular type of EH receiver. In this paper, receivers consisting of a harvesting energy unit and a conventional signal processing unit for concurrent EH and ID is under consideration. Let α_k with $0 \le \alpha_k \le 1$ denote the percentage of transmission time allocated to the ID time slot for user k. Thus, $1 - \alpha_k$ corresponds to the percentage of transmission time allocated to the EH time slot for user k. Hence, the harvested energy at the receiver of user k can be written as

$$E_k = (1 - \alpha_k) \eta_k \text{tr}(\mathbf{G}_k \mathbf{Q}), \tag{2}$$

where η_k is a constant that accounts for the loss in the energy transducer for converting the harvested energy to electrical energy to be stored, $\mathbf{G}_k = \mathbf{H}_k^H \mathbf{H}_k$ is the channel covariance matrix, $\mathbf{Q}$ denotes the total transmit covariance matrix at the BS, $\mathbf{Q} = \sum_{k=1}^K \mathbf{Q}_k^b$, $\mathbf{Q}_k^b = \mathbb{E}(\mathbf{x}_k \mathbf{x}_k^H)$ is the corresponding

transmit covariance matrix, $\mathbf{Q}_k^b \succeq 0$ i.e., $\mathbf{Q}_k^b$ is a positive semidefinite matrix. On the other hand, the total capacity of the MIMO-BC SWIPT system can be expressed as follows

$$C_{BC} = \sum_{k=1}^K \alpha_k R_k^b, \tag{3}$$

where R_k^b denotes the rate achieved by the user k in the downlink. Further, note that dirty paper coding (DPC) is the capacity achieving scheme for Gaussian MIMO-BC [31]. With DPC, the information for different users is encoded in a sequential fashion. It should be noted that the transmit covariance matrices remain the same during each transmission block. This implies that the same transmit covariance matrices are shared in ID and EH models. Without loss of generality, with an encoding order $(1, \cdots, K)$, i.e., the codeword of user 1 is encoded first, the data rate R_k^b for the kth user can be written as [28]

$$R_k^b = W \log \frac{|\mathbf{I}_{N_r \times N_r} + \frac{1}{\sigma^2} \mathbf{H}_k (\sum_{i=k}^K \mathbf{Q}_i^b) \mathbf{H}_k^H|}{|\mathbf{I}_{N_r \times N_r} + \frac{1}{\sigma^2} \mathbf{H}_k (\sum_{i=k+1}^K \mathbf{Q}_i^b) \mathbf{H}_k^H|}. \tag{4}$$

Defining a quantitative power model is very challenging given one needs to consider the particular deployment scenario and components configurations. The following linear power model is widely shown to be a reasonable representative for radio access networks [32]

$$P = \zeta P_T + P_C, \tag{5}$$

where ζ, P_T and P_C are respectively used to denote the reciprocal of the power amplifier drain efficiency, transmission power, and circuit power consumption. However, for the SWIPT-based MIMO-BC system considered in this work, the power consumption model should be extended considering EH devices. In general, small amounts of energy is consumed by the RF EH devices. On the other hand, the system power consumption is intuitively compensated by the harvested energy. It may then be apparent that enabling EH can improve the EE of a wireless communication system. Thus, as in [26], [33], we take the harvested power into consideration in the formulation of the system power consumption model (and hence the EE formulation for the SWIPT-based MIMO-BC system).

Specifically, the total system power consumption is expressed as follows

$$P = \zeta P_T + P_C - \sum_{k=1}^K E_k, \tag{6}$$

where $\sum_{k=1}^K E_k$ represents the harvested power at all the receivers Note that the minus sign denotes that a portion of the power radiated in the RF from the transmitter can be harvested by the K receivers. Recall that P_C is the total circuit power required for supporting reliable communication in our SWIPT-based MIMO-BC

$$P_C = P_{ant}^{BS} N_t + P_{sta} + K P_C^R, \tag{7}$$

where $P_{ant}^{BS} N_t$ denotes the power consumption proportional to the number of transmit antennas, P_{sta} is the constant signal processing circuit power consumption in the transmitters (due

to filters, frequency synthesizer, etc., independent of the power radiated by the transmitter), and P_C^R denotes the total circuit power consumption in each receiver.

It should be noted that the minimum required power to activate the energy harvesting circuit varies depending on the particular technologies and receiver configurations [34]. For example, it has been shown in [35] that the minimum RF input power is 16.7 μW with a novel fully integrated passive transponder integrated circuit. However, in a recent study by Le et al. [36], a RF-DC power conversion system is described which can efficiently convert far-field RF energy to DC voltages for low received power. The presented system can operate at a distance of maximum 44 meters with a received signal power as low as 5.5 μW (-22.6 dBm) from a 4 W Effective Isotropic Radiated Power (EIRP) radiation source. These harvested energy can be used to power a sensor node or to recharge a battery.

B. Problem Formulation

In this work, we employ DPC which achieves the sum rate capacity for SWIPT-based MIMO-BC. It should be noted that due to certain practical constraints, such as pilot overhead, coding and modulation, demodulation and decoding, etc., there exists a performance gap between the channel capacity and the achievable rate. Nevertheless, the sum rate capacity obtained by DPC scheme under perfect CSI represents the information-theoretic upper bound for MIMO-BC, which helps unveil important insights into the problem.

The EE for SWIPT-based MIMO-BC can be defined as the total number of delivered bits per unit energy. The energy consumption includes transmission energy consumption, circuit energy consumption, and harvested energy. Hence, we define EE in a SWIPT-based MIMO-BC as

$$\lambda_{EE} \triangleq \frac{C_{BC}}{P} = \frac{\sum_{k=1}^{K} \alpha_k R_k^b}{\zeta P_T + P_C - \sum_{k=1}^{K} E_k}, \quad (8)$$

where C_{BC} is the total capacity achieved by all users and $P_T = \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^b)$ is the total transmission power.

Given the expression of the system sum rate and power consumption, we can proceed with the problem formulation. The objective of this paper is to maximize the EE in SWIPT-based MIMO-BC whilst meeting two constraints in terms of transmission power and harvested energy. By invoking the linear power model in (6), the optimization problem can be formulated as

$$\max_{\{\mathbf{Q}_k^b, \alpha_k\}_{k \in K}} \frac{\sum_{k=1}^{K} \alpha_k R_k^b}{\zeta P_T + P_C - \sum_{k=1}^{K}(1-\alpha_k)\eta_k \text{tr}(\mathbf{G}_k \mathbf{Q})}, \quad (9)$$

$$\text{s.t.} \quad (1-\alpha_k)\eta_k \text{tr}(\mathbf{G}_k \mathbf{Q}) \geq E_{k,min}, \ \forall \ k \in \mathcal{K}, \quad (10)$$

$$\sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^b) \leq P_{max}, \quad (11)$$

$$\mathbf{Q}_k^b \succeq 0, \ 0 \leq \alpha_k \leq 1, \ \forall \ k \in \mathcal{K}, \quad (12)$$

where P_{max} and $E_{k,min}$ are the maximum total transmit power constraint at the BS and the minimum harvested energy constraint for user $k \in (1, 2, \cdots, K)$, respectively. Note that (12) corresponds to the inherent constraints in terms of TS

ratios. It is easy to see that the coupling of optimization variables leads to the problem (9)-(12) being non-convex and challenging to solve directly. The complexity is considered the main drawback for PHY algorithm design. Therefore, in the following sections, we develop resource allocation schemes for SWIPT-based MIMO-BC to solve the above optimization problem. In particular, to overcome the difficulty, we first transform this multi-constrained EE maximization problem into its equivalent problem that has a single constraint with multiple auxiliary variables. We then, exploit the duality between a SWIPT-based MIMO-BC and a dual MIMO-MAC in the case where the multiple auxiliary variables are fixed. For the dual-MAC problem, we accordingly propose two iterative resource allocation algorithms based on Dinkelbach theory [29] and EE-power quasi-concavity property.

III. EQUIVALENCE AND EXTENDED BC-MAC DUALITY

It is shown in [28] that under a single sum power constraint, the weighted sum rate maximization problem for the MIMO-BC can be transformed to its dual MIMO-MAC problem, which is convex and can be solved in an efficient manner. Furthermore, adding an interference constraint to form a CR MIMO-BC scenario, the authors in [37] proved this BC-MAC duality still holds for a weighted sum rate maximization problem. Nevertheless, in our SWIPT-based MIMO-BC setting, the EE optimization problem in (9)-(12) has not only a sum power constraint but also multiple minimum harvested energy constraints. The imposed multiple constraints further complicates the formulation of an efficient solvable dual problem.

In order to overcome the aforementioned challenges, motivated by the weak duality property obtained from the Lagrange dual problem of non-convex optimization problem, we propose a suboptimal min-max approach where the multi-constrained EE maximization problem in (9)-(12) is transformed into a problem that has a single constraint with multiple auxiliary variables. Based on this, we further develop a duality between a SWIPT-based MIMO-BC and a SWIPT-based dual MIMO-MAC in the case where the multiple auxiliary variables are fixed. In the following, we present an alternative form of the problem (9)-(12) where a suboptimal solution can be obtained.

$$\min_{\chi, \mu_k} \max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} \frac{\sum_{k=1}^{K} \alpha_k R_k^b}{\zeta P_T + P_C - \sum_{k=1}^{K}(1-\alpha_k)\eta_k \text{tr}(\mathbf{G}_k \mathbf{Q})} \quad (13)$$

$$\text{s.t.} \quad \chi(\sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^b) - P_{max})$$

$$+ \sum_{k=1}^{K} \mu_k \left(E_{k,min} - (1-\alpha_k)\eta_k \text{tr}(\mathbf{G}_k \mathbf{Q}) \right) \leq 0, \quad (14)$$

where χ and μ_k are the auxiliary dual variables for the maximum power constraint and the kth minimum harvested energy constraint, respectively. The optimal value of the inner problem (13)-(14), $g(\chi, \boldsymbol{\mu}) = \max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} \lambda_{EE}$, for any given pair of χ and $\boldsymbol{\mu}$, is an upper bound on the optimal value of problem (9)-(12). However, the non-convex property of the original problem (9)-(12) cannot guarantee zero duality gap, and hence problem (13)-(14) achieves a suboptimal solution.

However, it is still very difficult to directly find an efficiently

solvable dual problem for (13)-(14). Extending the decomposition approach proposed in [37], we develop a two-layer approach to solve problem (13)-(14). In particular, the inner-layer is used to solve problem (13)-(14) with fixed χ and μ_k whilst the outer-layer is to update χ and μ_k through a sub-gradient approach. Thus, in the following, we first investigate the EE maximization problem considering fixed auxiliary dual variables χ and μ_k. The problem in (13)-(14) can hence be reduced to

$$\max_{\mathbf{Q}_k^m \succeq 0, 0 \le \alpha_k \le 1} \frac{\sum_{k=1}^K \alpha_k R_k^b}{\zeta P_T + P_C - \sum_{k=1}^K (1-\alpha_k)\eta_k \text{tr}(\mathbf{G}_k\mathbf{Q})} \quad (15)$$

$$\text{s.t.} \quad \chi\left(\sum_{k=1}^K \text{tr}(\mathbf{Q}_k^b) - P_{\max}\right)$$

$$+ \sum_{k=1}^K \mu_k(E_{k,min} - (1-\alpha_k)\eta_k\text{tr}(\mathbf{G}_k\mathbf{Q})) \le 0. \quad (16)$$

The solution of the above problem is unfortunately non-trivial given the objective function is non-concave even under fixed auxiliary dual variables χ and μ_k. Thus, we exploit an extended SWIPT-based BC-MAC duality principle based on results from [28] and [37]. Consequently, the weighted sum rate maximization problem in SWIPT-based MIMO-BC under constraints in (14) can be formulated as

$$\max_{\mathbf{Q}_k^b \succeq 0, 0 \le \alpha_k \le 1} \sum_{k=1}^K \alpha_k R_k^b \quad (17)$$

$$\text{s.t.} \quad \chi \sum_{k=1}^K \text{tr}(\mathbf{Q}_k^b) - \sum_{k=1}^K \mu_k(1-\alpha_k)\eta_k\text{tr}(\mathbf{G}_k\mathbf{Q}) \le P_{all}, \quad (18)$$

where $P_{all} := \chi P_{\max} - \sum_{k=1}^K \mu_k E_{k,min}$. Since χ and μ_k are fixed, P_{all} is a constant in (17)-(18). Hence, by extending the general BC-MAC duality principle from [28] to our SWIPT-based MIMO-BC scenario, we have the following SWIPT-based dual MAC problem corresponding to the original SWIPT-based BC problem in (17)-(18).

Proposition 1: The SWIPT-based dual MAC problem of (17)-(18) is given by

$$\max_{\boldsymbol{Q}_k^m \succeq 0, 0 \le \alpha_k \le 1} \sum_{k=1}^K \alpha_k R_k^m \quad (19)$$

$$\text{s.t.} \quad \sum_{k=1}^K tr(\boldsymbol{Q}_k^m) \le P_{all}, \quad (20)$$

where $\boldsymbol{Q}_k^m$ is the transmit signal covariance matrix of the kth user, and R_k^m is the rate achieved by the kth user of the dual MAC defined as

$$R_k^m = W \log \frac{|\boldsymbol{N} + \sum_{k=1}^i \boldsymbol{H}_k^H \boldsymbol{Q}_k^m \boldsymbol{H}_k|}{|\boldsymbol{N} + \sum_{k=1}^{i-1} \boldsymbol{H}_k^H \boldsymbol{Q}_k^m \boldsymbol{H}_k|}, \quad (21)$$

with the noise covariance at the BS denoted with $\mathbf{N} = \chi\boldsymbol{I} - \sum_{k=1}^K \mu_k(1-\alpha_k)\eta_k\boldsymbol{G}_k$.
Proof: See Appendix A.

Proposition 1 indicates that the capacity region of a SWIPT-based MIMO-BC with power constraint P_T is equal to the union of capacity regions of the SWIPT-based dual-MAC with power constraints such that $\sum_{k=1}^K \text{tr}(\mathbf{Q}_k^m) = P_T$. However, *Proposition 1* describes the rate region for SWIPT-based MIMO-BC system and its duality relationship with SWIPT-based MIMO-MAC, meaning the EE aspect is still an open question. Hence, in order to tackle the problem in (15)-(16), we develop the following proposition (EE aspect) based on the results in *Proposition 2*.

Proposition 2: The solution of the SWIPT-based dual MAC EE maximization problem, namely,

$$\max_{\boldsymbol{Q}_k^m \succeq 0, 0 \le \alpha_k \le 1} \frac{\sum_{k=1}^K \alpha_k R_k^m}{\zeta \sum_{k=1}^K tr(\boldsymbol{Q}_k^m) + P_C} \quad (22)$$

$$\text{s.t.} \quad \sum_{k=1}^K tr(\boldsymbol{Q}_k^m) \le P_{all}, \quad (23)$$

is an upper-bound of the solution to the problem in (15)-(16).
Proof: See Appendix B.

Consequently, instead of directly tackling the EE maximization problem in (15)-(16), in this work, we have provided a dual-MAC upper bound solution of (22)-(23). Therefore, the original problem (9)-(12) can be solved through an efficient iterative method using the following processes:

(i) Inner-loop: Finds the solution of the SWIPT-based dual MAC EE maximization problem (22)-(23), under fixed auxiliary dual variables χ and μ_k.

(ii) Outer-loop: Obtains the optimal dual variables χ and μ_k.
It should be noted that due to the additional minimum energy harvesting constraints and the TS ratio variables α_k, the optimization problem is a non-linear fractional programming and hence the problem in (22)-(23) cannot be solved via our previous duality result provided in [19], in which only a single sum power constraint was considered. Besides, the TS ratio α_k in the SWIPT system plays a very important role in our optimization problem, hence further increases the complexity for obtaining a solution. For the optimization problems of such nature, it is generally helpful to relate it to a concave program by separating numerator and denominator with the help of parameter β, this is what is known as the Dinkelbach method [29]. In the following sections, we propose an iterative resource allocation algorithm based on the Dinkelbach method to obtain the upper-bound solution to the problem in (15)-(16).

IV. ITERATIVE RESOURCE ALLOCATION SCHEME BASED ON DINKELBACH METHOD

Recall that the optimization problem in (22)-(23) belongs to a family of non-linear fractional programming problems which are non-convex and difficult to solve directly. Nevertheless, by invoking the theory of non-linear fractional programming in [29], we can use the Dinkelbach method to solve this non-convex non-linear fractional programming problem. Specifically, we transform the fractional-form objective function into a numerator-denominator subtractive form as discussed in the following proposition.

Proposition 3: The maximum EE β^* can be achieved if and only if

$$\max_{\mathbf{Q},\boldsymbol{\alpha}} U_R(\mathbf{Q}, \boldsymbol{\alpha}) - \beta^* U_T(\mathbf{Q}, \boldsymbol{\alpha})$$
$$= U_R(\mathbf{Q}^*, \boldsymbol{\alpha}^*) - \beta^* U_T(\mathbf{Q}^*, \boldsymbol{\alpha}^*) = 0$$

1) Initialize $\beta = 0$, and δ as the maximum tolerance;
2) **REPEAT**
3)   For a given β, obtain an intermediate resource allocation policy $\{\mathbf{Q}, \boldsymbol{\alpha}\}$ by solving the problem in (27)-(28);
4)   **IF** $U_R(\mathbf{Q}, \boldsymbol{\alpha}) - \beta U_T(\mathbf{Q}, \boldsymbol{\alpha}) \leq \delta$
5)   Convergence = **TRUE**;
6)   **RETURN** $\{\mathbf{Q}^*, \boldsymbol{\alpha}^*\} = \{\mathbf{Q}, \boldsymbol{\alpha}\}$ and $\beta^* = \frac{U_R(\mathbf{Q}, \boldsymbol{\alpha})}{U_T(\mathbf{Q}, \boldsymbol{\alpha})}$;
7)   **ELSE**
8)   Set $\beta = \frac{U_R(\mathbf{Q}, \boldsymbol{\alpha})}{U_T(\mathbf{Q}, \boldsymbol{\alpha})}$ and $n = n + 1$, Convergence = **FALSE**;
9)   **END IF**
10) **UNTIL** Convergence = **TRUE**.

TABLE I
PROPOSED ITERATIVE RESOURCE ALLOCATION ALGORITHM BASED ON DINKELBACH METHOD

for $U_R(\mathbf{Q}, \boldsymbol{\alpha}) \geq 0$ and $U_T(\mathbf{Q}, \boldsymbol{\alpha}) \geq 0$, where

$$U_R(\mathbf{Q}, \boldsymbol{\alpha}) = \sum_{k=1}^{K} \alpha_k R_k^m, \qquad (24)$$

$$U_T(\mathbf{Q}, \boldsymbol{\alpha}) = \zeta \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) + P_C, \qquad (25)$$

$$\text{and} \quad \beta^* = \frac{U_R(\mathbf{Q}^*, \boldsymbol{\alpha}^*)}{U_T(\mathbf{Q}^*, \boldsymbol{\alpha}^*)}. \qquad (26)$$

Proof: Please refer to [29] for a proof of *Proposition 3*.

Proposition 3 offers a sufficient and necessary condition for designing the optimal resource allocation strategy. Particularly, we can find an equivalent optimization problem with an objective function in subtractive form, e.g. $U_R(\mathbf{Q}, \boldsymbol{\alpha}) - \beta^* U_T(\mathbf{Q}, \boldsymbol{\alpha})$ in the considered case, corresponding to the original optimization problem with an objective function in fractional form, such that both problems share the same optimal solution. In addition, the optimality guarantees the equality in (24), and thus we could apply this equality condition to verify the optimality of the solution. Therefore, we develop a resource allocation strategy for the equivalent objective function (subtractive form) whilst satisfying the condition stated in *Proposition 3*.

Based on Dinkelbach method [29], here, we propose an iterative algorithm for solving (22)-(23) with an equivalent objective function such that the obtained solution satisfies the conditions stated in *Proposition 3*. The proposed algorithm is summarized in Table I.

It can be observed from Table I that the key step for the proposed iterative algorithm concerns the solution to the following optimization problem for a given parameter β in each iteration (i.e., step 3),

$$\max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} \sum_{k=1}^{K} \alpha_k R_k^m - \beta(\zeta \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) + P_C) \quad (27)$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) \leq P_{all}. \qquad (28)$$

We define $f(\mathbf{Q}_1^m, \cdots, \mathbf{Q}_K^m, \alpha_1, \cdots, \alpha_K) =$

$\sum_{i=1}^{K} \Delta_k \log |\mathbf{N} + \sum_{k=1}^{i} \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k|$, where $\Delta_k = \alpha_k - \alpha_{k+1}$, and thus the optimization problem in (27)-(28) can be reformulated as

$$\max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} f(\mathbf{Q}_1^m, \cdots, \mathbf{Q}_K^m, \alpha_1, \cdots, \alpha_K)$$

$$-\beta(\zeta \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) + P_C) \quad s.t. \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) \leq P_{all}. \quad (29)$$

To solve the above problem, Lagrangian dual method can be employed. However, it should be noted that globally solving the auxiliary problem in each Dinkelbachs iteration (step 3) is a critical requirement to be able to claim the optimality. Therefore, to prove the Lagrangian dual problem has zero duality gap, we obtain the following *Lemma*.

Lemma 1. The objective function of problem (29) is jointly concave in $\mathbf{Q}$ and $\boldsymbol{\alpha}$.
Proof: See Appendix C.

Thus, the problem (29) is a concave maximization problem with a convex constraint, and zero duality gap is guaranteed. The corresponding Lagrangian function can be expressed as

$$L(\mathbf{Q}_1^m, \cdots, \mathbf{Q}_K^m, \alpha_1, \cdots, \alpha_K, \tau) := f(\mathbf{Q}_k^m, \alpha_k)$$

$$-\beta(\zeta \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) + P_C) - \tau[\sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) - P_{all}], \quad (30)$$

where $\tau \geq 0$ is the Lagrangian multipliers associated with the maximum power constraint. The dual objective function of (27) is written as

$$g(\tau) = \max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} L(\mathbf{Q}_1^m, \cdots \mathbf{Q}_K^m, \alpha_1, \cdots, \alpha_K, \tau), \quad (31)$$

and the dual problem is given by

$$\min_{\tau} g(\tau) \quad s.t. \quad \tau \geq 0. \qquad (32)$$

In this work, an iterative approach is used here in order to achieve the optimum $\mathbf{Q}_k^m$ and α_k for the dual MIMO-MAC problem. In particular, we update $\mathbf{Q}_k^m$ through the gradient of the Lagrangian function (30) with respect to $\mathbf{Q}_k^m$ and α_k as follows [37]

$$\nabla_{\mathbf{Q}_k^m} L :=$$

$$\frac{\partial f[\mathbf{Q}_1^m(n), \cdots, \mathbf{Q}_{k-1}^m(n), \mathbf{Q}_k^m(n-1), \cdots, \mathbf{Q}_K^m(n-1)]}{\partial \mathbf{Q}_k^m(n-1)}$$

$$-\beta \zeta \tau \mathbf{I}_{N_r \times N_r}, \quad (33)$$

$$\nabla_{\alpha_k} L :=$$

$$\frac{\partial f[\alpha_1(n), \cdots, \alpha_{k-1}(n), \alpha_k(n-1), \cdots, \alpha_K(n-1)]}{\partial \alpha_k(n-1)}, \quad (34)$$

$$\mathbf{Q}_k^m(n) = [\mathbf{Q}_k^m(n-1) + t \nabla_{\mathbf{Q}_k^m} L]^+, \quad (35)$$

$$\alpha_k(n) = \alpha_k(n-1) + t \nabla_{\alpha_k} L, \quad (36)$$

where t represents the step size, and the notation $[\mathbf{A}]^+$ is defined as $[\mathbf{A}]^+ := \sum_i [q_i]^+ \mathbf{v}_i \mathbf{v}_i^H$, with q_i and $\mathbf{v}_i$ denote the ith eigenvalue and the corresponding eigenvector of $\mathbf{A}$ respectively. It should be noted that the existence of the gradient of the Lagrangian function (30) with respect to $\mathbf{Q}_k^m$ depends on two parts: the existence of $\frac{\mathrm{d} \log |\mathbf{N} + \sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k|}{\mathrm{d} \mathbf{Q}_k^m}$ and $\frac{\mathrm{d} \, \text{tr}(\mathbf{Q}_k^m)}{\mathrm{d} \mathbf{Q}_k^m}$.

It has been shown in [38] (Proposition 3.4) that $d\,\mathrm{tr}(\mathbf{Q}_k^m)$ exists and that $d\,\mathrm{tr}(\mathbf{Q}_k^m) = \mathrm{tr}(d\mathbf{Q}_k^m)$. Furthermore, it has also been shown in [38] (Proposition 3.14) that $d\ln(\det)(\mathbf{Z})$ exists if $\mathbf{Z} \in \mathcal{C}^{N \times N}$ is invertible. Since every positive definite matrix is invertible and its inverse is also positive definite [39], $\frac{d\log |\mathbf{N}+\sum_{k=1}^{K} \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k|}{d\mathbf{Q}_k^m}$ exists. Therefore, the gradient of the Lagrangian function (30) with respect to $\mathbf{Q}_k^m$ exists, and we can compute the gradient in (33) and (34) as follows

$$\frac{\partial f(\mathbf{Q}_1^m, \cdots, \mathbf{Q}_K^m)}{\partial \mathbf{Q}_k^m}$$

$$= \sum_{j=k}^{K} \Delta_j \mathbf{H}_k (\mathbf{N} + \sum_{i=1}^{j} \mathbf{H}_i^H \mathbf{Q}_i^m \mathbf{H}_i)^{-1} \mathbf{H}_k^H, \qquad (37)$$

$$\frac{\partial f(\alpha_1, \cdots, \alpha_K)}{\partial \alpha_k}$$

$$= \sum_{i=1}^{K} \Delta_i \mathrm{tr}[\mu_i \eta_i \mathbf{G}_i (\mathbf{N} + \sum_{k=1}^{i} \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k)^{-1})] + R_k^m. \quad (38)$$

After we obtain the optimum $\mathbf{Q}_k^m$ and α_k, our next task is to find out the optimal τ. Given that the Lagrangian function $g(\tau)$ is a convex function with respect to τ, we can achieve the optimal τ through a one-dimensional searching approach. Nevertheless, it is not guaranteed that $g(\tau)$ is differentiable, and thus the gradient approach may not available in this case. On the other hand, the well-known sub-gradient approach can be applied here to search the optimal solution where τ is updated in accordance with the sub-gradient direction as the following Lemma.

Lemma 2. $P_{all} - \sum_{k=1}^{K} tr(\mathbf{Q}_k^m)$ *is the sub-gradient of the dual objective function* $g(\tau)$*, where* $\mathbf{Q}_k^m, k = 1, 2, \cdots, K$ *are the corresponding optimal covariance matrices under fixed* τ*.*
Proof: The proof of *Lemma 2* is similar to that in [37], and thus is omitted for brevity.

Upon convergence of the transmit covariance matrices $\mathbf{Q}_k^m, k = 1, 2, \cdots, K$ and the TS ratios $\alpha_k, k = 1, 2, \cdots, K$, the current consumption power is saved in order to compare with P_{all}. In particular, as stated in *Lemma 1*, the value of τ should be increased if $\sum_{k=1}^{K} \mathrm{tr}(\mathbf{Q}_k^m) \geq P_{all}$, and decrease otherwise. This procedure is continued until convergence, i.e., $|\tau_{\min} - \tau_{\max}| \leq \varepsilon$.

We can now present the algorithm to solve the optimization problem for a given parameter β in each iteration, namely the bisection-based resource allocation algorithm, as in Table II.

A. Special Case with Equal TS Ratios

In the previous section, we investigated the case in which different SWIPT-equipments (receivers) are employed by the users, and hence the optimal TS ratio factors for users are assumed to vary (depending on their channel gain and the minimum EH constraints). However, in a practical communications system, it is very likely that all served users are equipped with the same SWIPT-equipments, resulting in $\alpha = \alpha_1 = \alpha_2 \cdots = \alpha_K$. As a result, here, we develop an efficient solution for solving the equal TS ratio case.

Motivated by the iterative approach proposed in [40], we here separate the process of determining the TS ratio α and

1) Initialize $\tau_{\min}$ and $\tau_{\max}$;
2) **REPEAT**
3) $\quad\tau = (\tau_{\min} + \tau_{\max})/2$;
4) $\quad$**REPEAT** Initialize $\mathbf{Q}_1^m(0), \cdots, \mathbf{Q}_K^m(0)$,
$\quad\quad\alpha_1(0), \cdots, \alpha_K(0)$, $n = 1$;
5) $\quad\quad$**FOR** $k = 1, \cdots, K$
6) $\quad\quad\quad\mathbf{Q}_k^m(n) = [\mathbf{Q}_k^m(n-1) + t\nabla_{\mathbf{Q}_k^m} L]^+$;
7) $\quad\quad\quad\alpha_k(n) = \alpha_k(n-1) + t\nabla_{\alpha_k} L$;
8) $\quad\quad$**END FOR**
9) $\quad\quad n = n + 1$;
10) $\quad$**UNTIL** $\mathbf{Q}_k^m$ and α_K for $k = 1, \cdots, K$ converge, i.e., $||\nabla_{\mathbf{Q}_k^m} L||^2 \leq \epsilon$, $||\nabla_{\alpha_k} L||^2 \leq \epsilon$ for a small ϵ;
11) $\quad$if $\sum_{k=1}^{K} \mathrm{tr}(\mathbf{Q}_k^m) \geq P_{all}$, $\tau_{\min} = \tau$,
$\quad\quad$elseif $\sum_{k=1}^{K} \mathrm{tr}(\mathbf{Q}_k^m) \leq P_{all}$, $\tau_{\max} = \tau$;
12) **UNTIL** $|\tau_{\min} - \tau_{\max}| \leq \varepsilon$.

TABLE II
BISECTION BASED RESOURCE ALLOCATION ALGORITHM

the covariance matrix $\mathbf{Q}$ as follows

$$\underbrace{\boldsymbol{\alpha}[0] \rightarrow \mathbf{Q}[0]}_{\text{Initialization}} \rightarrow \cdots \underbrace{\boldsymbol{\alpha}[t] \rightarrow \mathbf{Q}[t]}_{\text{Iteration t}} \rightarrow \underbrace{\boldsymbol{\alpha}^{opt} \rightarrow \mathbf{Q}^{opt}}_{\text{Optimal Solution}}. \quad (39)$$

In particular, a bi-section approach is proposed on the basis of the following proposition.

Proposition 4: *For any given transmit covariance matrices* $\boldsymbol{Q}_k^m, k = 1, 2, \cdots, K$*, that satisfies the constraints in (28), the following optimization problem,*

$$\max_{\alpha} \ \vartheta(\alpha) = \max_{\alpha} \ \sum_{k=1}^{K} \alpha R_k$$

$$-\beta(\zeta \sum_{k=1}^{K} tr(\boldsymbol{Q}_k^m) + P_C) \qquad (40)$$

$$s.t. \ 0 \leq \alpha \leq 1, \qquad (41)$$

is concave in α*.*
Proof: Proposition 4 follows immediately from *Lemma 1*; thus a similar proof to that in Appendix C can be applied here.

Consequently, *Proposition 4* guarantees the existence and uniqueness of the global maximum solution. Furthermore, $\vartheta(\alpha)$ either strictly decreases or first increases and then strictly decreases with α. Therefore, problem (27)-(28) can be decomposed into two layers and solved iteratively through the bi-section processes, per detailed in Table III.

V. ALTERNATIVE SOLUTION BASED ON QUASI-CONCAVITY PROPERTY

The proposed iterative resource allocation scheme for the problem in (22)-(23) is based on the Dinkelbach method, and hence the convergence speed for β may be slow for some special cases, i.e., when large number of users exists in the network. To facilitate practical implementation of the optimal resource efficient design, we will exploit and prove the quasi-concave relation between the maximum EE $\lambda_{EE}^*(P_T^m)$ and transmit power $P_T^m = \sum_{k=1}^{K} \mathrm{tr}(\mathbf{Q}_k^m)$. In particular, we first

1) Initialize $\alpha_{\min} = 0$ and $\alpha_{\max} = 1$;
2) **REPEAT**
3) $\alpha = (\alpha_{\min} + \alpha_{\max})/2$;
4)   **REPEAT**, Initialize $\mathbf{Q}_1(0), \cdots, \mathbf{Q}_K(0)$, $n = 1$;
5)   **FOR** $k = 1, \cdots, K$
6) $\mathbf{Q}_k(n) = [\mathbf{Q}_k(n-1) + t\nabla_{\mathbf{Q}_k}L]^+$;
7)   **END FOR**
8) $n = n + 1$;
9)   **UNTIL** $\mathbf{Q}_k$ for $k = 1, \cdots, K$ converge;
10)   if $\nabla_\alpha \vartheta \geq 0$, $\alpha_{\min} = \alpha$, elseif $\nabla_\alpha \vartheta \leq 0$, $\alpha_{\max} = \alpha$;
11) **UNTIL** $|\alpha_{\min} - \alpha_{\max}| \leq \varepsilon$.

TABLE III
BISECTION BASED RESOURCE ALLOCATION ALGORITHM FOR THE EQUAL
TS RATIO CASE.

demonstrate the quasi-concavity of EE in transmit power, and then develop a dual-layer resource allocation scheme based on the EE-power relationship.

A. Fundamentals of EE-Power Relationship

We proceed by providing a fundamental study of the EE-power relationship.

Proposition 5: *With transmit covariance matrices $\mathbf{Q}_k^m$, $k = 1, 2, \cdots, K$ and TS ratios α_k, $k = 1, 2, \cdots, K$, that satisfies the constraints in (23), i.e., $P_T^m \leq P_{max}$, the maximum EE, $\lambda_{EE}^* = \max\limits_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} \lambda_{EE}(P_T^m)$, is strictly quasi-concave in P_T^m.*
Proof: See Appendix D.

Proposition 6: *For any given transmission power in the region $[P_{min}, P_{max}]$, the maximum EE, $\lambda_{EE}^*(P_T^m)$, is*
(i) strictly decreasing with P_T^m and is maximized at $P_T^m = P_{min}$ if

$$\left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m = P_{min}} \leq 0,$$

(ii) strictly increasing with P_T^m and is maximized at $P_T^m = P_{max}$ if

$$\left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m = P_{min}} > 0$$

$$and \quad \left.\frac{d\lambda_{EE}^*(P)}{dP_T^m}\right|_{P_T^m = P_{max}} \geq 0,$$

(iii) first strictly increasing and then strictly decreasing with P_T^m and is maximized at $P_T^m = \frac{\bar{\lambda}_{EE}}{C_{BC}(\bar{\lambda}_{EE})} - P_C$ if

$$\left.\frac{d\lambda_{EE}^*(P_T^m)}{dP}\right|_{P_T^m = P_{min}} > 0$$

$$and \quad \left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m = P_{max}} \leq 0,$$

(iv) infeasible if

$$P_{min} > P_{max},$$

where $C_{BC}(\bar{\lambda}_{EE})$ represents the capacity under maximum

achievable EE.
Proof: See Appendix E.

Note that a quasi-concavity property guarantees the existence of a unique maximum solution, and thus *Proposition 5* ensures the existence and uniqueness of the maximum solution. Moreover, the quasi-concavity further indicates that $\lambda_{EE}(P_T^m)$ either strictly decreases or first increases and then strictly decreases with P_T^m. *Proposition 6* further reveals that there exists a maximum point at a finite power region. Thus, the optimization problem in (22)-(23) can be solved through a dual-layer decomposition method using the following processes,
(i) inner-layer: For a fixed transmit power, P_T^m, determines the maximum EE $\lambda_{EE}^*(P_T^m)$;
(ii) outer-layer: obtains the optimal EE, $\bar{\lambda}_{EE}$, through a gradient-based approach.
Note that the key challenge of the proposed dual-layer decomposition method lies in the inner-layer, where $\lambda_{EE}^*(P_T^m)$ is to be obtained. This is discussed in detail as analysis proceeds.

B. Resource Allocation for the Dual MAC Problem

Recall that the inner-layer is concerned with finding the maximum EE, $\lambda_{EE}^*(P_T^m)$, based on a given transmission power, i.e., any P_T^m in the power region $[P_{min}, P_{max}]$. Hence, the optimization problem with a given transmission power P_T^m can be expressed as

$$\max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1} \quad \sum_{k=1}^{K} \alpha_k R_k^m \tag{42}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) \leq P_T^m. \tag{43}$$

Hence, the inner-layer of the proposed algorithm has been transformed to solve the optimization problem in (42)-(43) based on a given transmission power P_T^m. Apparently, the optimization problem in (42)-(43) is similar to the problem in (29), and hence they share the same solution structure. In other words, the proposed bisection-based resource allocation algorithm in Section IV can be applied here to solve the sum rate maximization problem in (42)-(43), and hence the detailed methodology is omitted here.

We next design a searching algorithm in order to solve the outer-layer of the EE optimization problem in (22)-(23). We first initialize the transmit power as $P_T^m(0)$. Based on the fixed transmit power, we determine the maximum EE $\lambda_{EE}^*(P_T^m)$ using the proposed bisection-based resource allocation algorithm in Section IV. We then develop a searching scheme based on *Proposition 6* to update the transmit power P_T^m as follows

$$P_T^m(n) = \begin{cases} \frac{P_T^m(n-1)}{\varsigma} & \left.\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\right|_{P_T^m(n-1)} < 0 \\ \varsigma P_T^m(n-1) & \text{otherwise} \end{cases}, \tag{44}$$

where $\varsigma > 1$ denotes the searching step. In addition, the value of ς should be reduced if the sign of gradient $\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}$ is changed as in

$$\varsigma(n) = \frac{\varsigma(n-1)}{2} \tag{45}$$

and (44) is repeated until convergence, i.e., $|P_T^m(n+1) - P_T^m(n)| \leq \epsilon$ or either P_{max} or P_{min} is achieved. In other words, the proposed resource allocation algorithm for the dual MAC problem will converge to the optimal point or the boundary point. It should be noted that the computational complexity of the outer-layer algorithm depends on the number of iterations and is linear with $\frac{1}{\varsigma^2}$ [41]. Therefore, choosing an appropriate ς to balance the convergence speed and complexity is very important.

VI. SOLUTION TO THE SWIPT-BASED EE MAXIMIZATION PROBLEM

Here, we provide a complete solution to the EE optimization problem in (13)-(14), namely an extended BC-MAC duality-based EE maximization algorithm. Under fixed χ and $\boldsymbol{\mu}$, the problem can be reformulated as follows

$$x(\chi, \boldsymbol{\mu}) =$$
$$\max_{\mathbf{Q}_k^b, \alpha_k} \frac{\sum_{k=1}^{K} \alpha_k R_k^b}{\zeta P_T^m + P_C - \sum_{k=1}^{K} \sum_{k=1}^{K} (1-\alpha_k)\eta_k \mathbb{E}[\mathrm{tr}(\mathbf{G}_k \mathbf{Q})]} \quad (46)$$

$$\text{s.t. } \chi(\sum_{k=1}^{K} \mathrm{tr}(\mathbf{Q}_k^b) - P_{\max})$$

$$+ \sum_{k=1}^{K} \mu_k(E_{k,min} - (1-\alpha_k)\eta_k \mathbb{E}[\mathrm{tr}(\mathbf{G}_k\mathbf{Q})]) \leq 0, \quad (47)$$

$$\mathbf{Q}_k^b \succeq 0, \ 0 \leq \alpha_k \leq 1, \ \forall \ k \in \mathcal{K}. \quad (48)$$

In addition, the problem (13)-(14) is equivalent to the following

$$\min_{\chi, \boldsymbol{\mu}} \ x(\chi, \boldsymbol{\mu}) \quad (49)$$
$$\text{s.t.} \quad \chi \geq 0 \text{ and } \mu_k \geq 0, \ \forall k \in \mathcal{K}. \quad (50)$$

By applying the BC-MAC duality in Section III together with the proposed Dinkelbach method-based iterative resource allocation scheme in Section IV, or the alternative solution based on quasi-concavity property in Section V, one can achieve $x(\chi, \boldsymbol{\eta})$. We then apply the BC-MAC covariance mapping approach from [37] to obtain the corresponding BC transmit covariance matrices $\mathbf{Q}_k^b, k = 1, \cdots, K$. Once we have obtained the solution for a given χ and $\boldsymbol{\mu}$, we can use the following lemma to update χ and $\boldsymbol{\mu}$ through a sub-gradient approach.

Lemma 3. The sub-gradient of $x(\chi, \boldsymbol{\mu})$ is $[P_{max} - \sum_{k=1}^{K} tr(\boldsymbol{Q}_k^b), (1 - \alpha_k)\eta_k[tr(\boldsymbol{G}_k\boldsymbol{Q})] - E_{k,min}]$, $k = 1, 2, \cdots, K$, where $\chi \geq 0$, $\boldsymbol{\mu} \geq \boldsymbol{0}$ and $\boldsymbol{Q}_k, \alpha_k, k = 1, 2, \cdots, K$, respectively denote the corresponding optimal transmit covariance matrices and TS ratios for a fixed χ and $\boldsymbol{\mu}$ in (49).
Proof: The proof of *Lemma 3* is similar to that in [37], and thus is omitted for brevity.

It should be noted that with a constant step size, the sub-gradient approach will converge to a point that is very close to the optimal value [42], i.e.,

$$\lim_{n \to \infty} |\chi^n - \chi^*| < \epsilon, \text{ and } \lim_{n \to \infty} |\mu_k^n - \mu_k^*| < \epsilon, k = 1, \cdots, K, \quad (51)$$
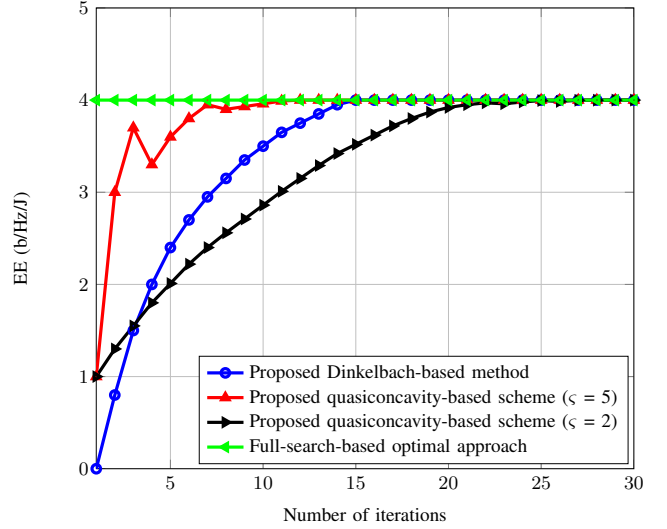

Fig. 2. Convergence behavior of the proposed Dinkelbach method-based scheme and quasi-concavity-based approach.

where χ^* and μ_k^* are the optimal values, and χ^n and μ_k^n are the values of χ and μ_k at the nth iteration of the sub-gradient approach, respectively. This result indicates that the sub-gradient approach determines an ϵ-suboptimal point in a finite number of iterations.

In Table IV, the computational complexities of the aforementioned Dinkelbach method-based solution and quasiconcavity-based solution based on the number of floating points [43] are listed for comparison. It should be noted that the computational complexity of DPC depends on QR decomposition, which is approximately of order $\mathcal{O}(KN_t^2 N_r)$ [44]. Furthermore, the computational complexity of the Dinkelbach method with stopping criteria ε is $\mathcal{O}(\frac{1}{\varepsilon^2}\log(K))$ [29], whereas the computational complexity of the quasiconcavity-based solution is linear with $\frac{1}{\varsigma^2}$, where ς is the searching step [41]. Therefore, as it can be seen from Table IV, the proposed quasiconcavity-based solution has a lower computational complexity compared to the proposed Dinkelbach method-based solution.

Algorithm	Complexity
Dinkelbach-Method	$\mathcal{O}(\frac{1}{\epsilon^2}\frac{1}{\varepsilon^2}\log(K)KN_t^2 N_r)$
Quasiconcavity-Solution	$\mathcal{O}(\frac{1}{\epsilon^2}\frac{1}{\varsigma^2}KN_t^2 N_r)$

TABLE IV
COMPLEXITY COMPARISON FOR THE PROPOSED ALGORITHMS

VII. SIMULATION RESULTS

In this section, we present simulation results to verify the theoretical findings and analyze the effectiveness of the proposed approaches. In our simulation, the BS employs $N_t = 4$ transmit antennas and is surrounded by uniformly-distributed users. Each user is equipped with $N_r = 2$ receive antennas, and the total number of users is set to $K = 4$. The dynamic power consumption proportional to the number of antennas P_{ant}^{BS} is set to be 1 W [45]. The path-loss is calculated using
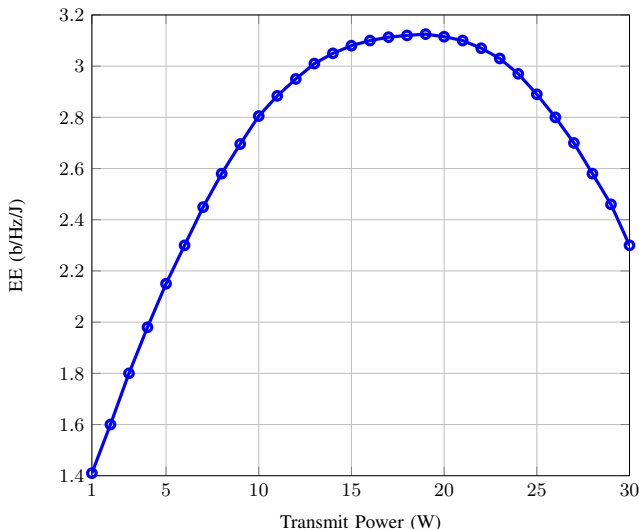
Fig. 3. λ_{EE}^*-versus-P_T^m curve using the proposed quasi-concavity-based resource allocation approach.

Fig. 4. Convergence behavior of the proposed extended BC-MAC duality-based EE maximization algorithm in terms of EE.

$128.1 + 37.6 \log_{10} d$ with distance d (in Kilometers) [46], and the radius of the cell is set to 500 m. The drain efficiency of the power amplifier ζ is set to 38% in our simulation whilst the energy harvesting efficiency is set to $\eta = 50\%$. The power budget for each BS is considered to be 46 dBm. The minimum harvesting energy E_k is set to 10% of the maximum transmit power. In addition, a larger stopping criteria ε and ς will increase the convergence speed of the proposed algorithms, but this comes at the cost of reduced accuracy in the achievable EE. Hence, to guarantee a precise convergence with all these factors, ε and ς are set to $1e^{-3}$ for our simulations. It is noted that these system parameters are merely chosen to demonstrate the EE optimization in an example and can easily be modified to any other values to address different scenarios.

In the first simulation, the performance of the proposed algorithms for the SWIPT-based dual MAC EE maximization problem is studied. The convergence behavior of the proposed Dinkelbach method-based scheme and quasi-concavity-based approach are first evaluated by illustrating how the EE performance behaves with the number of iterations. As shown in Fig. 2, both the proposed Dinkelbach method-based iterative resource allocation scheme and the proposed quasi-concavity-based scheme converge to the optimal value. In particular, the EE converges after approximately 30 iterations when $\varsigma = 2$, but is reduced to 12 iterations when a larger step size is chosen, e.g., $\varsigma = 5$. This result coincides with our theoretical findings where the computational complexity is inversely proportional to the square of the step size ς^2. Hence, compared to the proposed Dinkelbach method-based scheme, the proposed quasi-concavity-based scheme is more computationally efficient when an appropriate ς is selected. Similar results can be observed in a MIMO-BC scenario [41], where ς is set to 3 for improving the convergence speed. Next, we study the relationship between the achievable EE and transmission power. It can be seen from Fig. 3 that the EE-transmission power relationship has a bell shape curve. This quasi-concavity is the foundation of the proposed methodology and infers
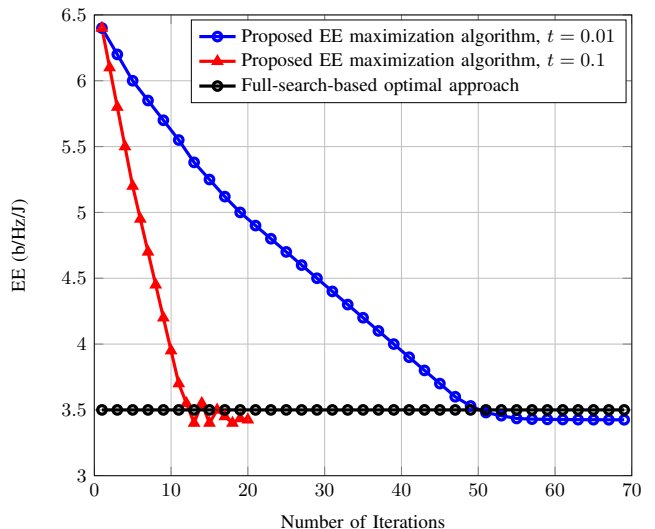
that the proposed quasi-concavity-based resource allocation algorithm always leads to the maximum EE performance. In addition, the convergence behavior of the proposed extended BC-MAC duality-based resource allocation algorithm is also studied. Fig. 4 depicts the EE versus the number of iterations for step sizes 0.1 and 0.01. As it can be seen from the figure, the proposed extended BC-MAC duality-based EE maximization algorithm converges to a stable value, and the step size affects the accuracy and convergence speed of the algorithm. Moreover, the EE achieved by the proposed solution is very close to the optimal EE with only a 3% loss. It should be noted that due to the min-max nature of the problem in (13)-(14), the initial iterations can have a higher EE than the optimal one. However in those iterations, the outer minimization is not yet completed and hence although the numerical EE value is higher, those solution is not valid. It is only until the outer minimization converges. A similar trend is also reported in [37] where the sum rate maximization problem is solved using a min-max approach. This is the reason for the EE in the initial iterations to be above the global optimum.

In the next simulation, the proposed extended BC-MAC duality-based EE maximization algorithm under different maximum transmit power allowance is evaluated and presented in Fig. 5. To show the EE gain achieved by TS-based SWIPT system, we compare our proposed scheme with the scheme that maximize the EE without EH [19] and the scheme that aims for maximizing the system sum rate [47]. It is observed that the EE achieved by our proposed extended BC-MAC duality-based EE maximization algorithm is monotonically non-decreasing with respect to the maximum transmit power allowance P_{max}, and tends to be saturated in the higher transmit power constraint region, i.e., $P_{max} > 25$ dBm. This is because in the higher transmit power constraint region, a balance between the system EE and the total power consumption can be achieved. On the other hand, all the algorithms achieve similar performance in terms of the system EE criterion in the lower transmit power constraint region, i.e., $5 < P_{max} < 15$
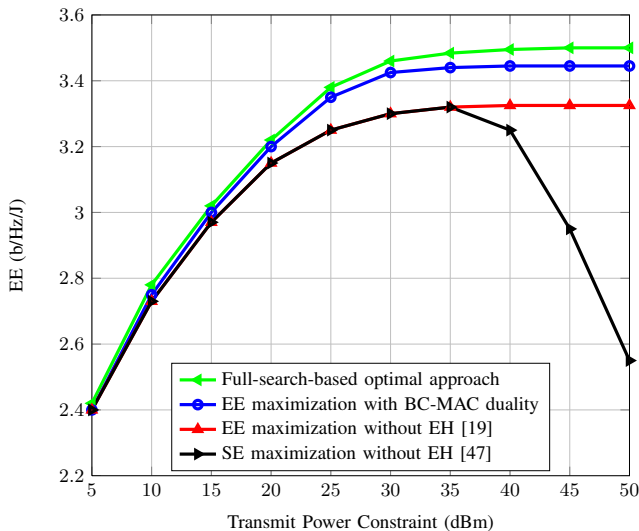
Fig. 5. The performance of the proposed extended BC-MAC duality-based EE maximization algorithm.
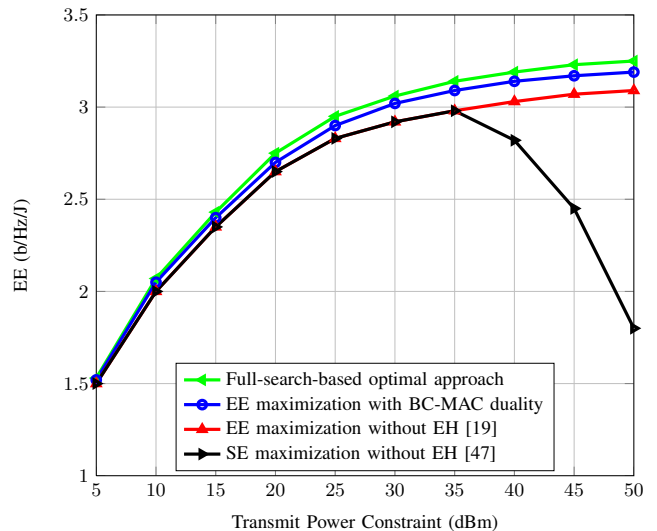

Fig. 6. The performance of the proposed extended BC-MAC duality-based EE maximization algorithm under Rayleigh fading and Log-Normal shadowing with standard deviation of 8 dB.

dBm. Besides, due to the fact that the received power of the desired signal may not be sufficiently large for delivering information and energy harvesting at the same time, the system with TS-based energy harvesting receivers achieves a small performance gain compared to the system without energy harvesting receivers. Nevertheless, in the region of higher transmit power, the proposed extended BC-MAC duality-based EE maximization algorithm outperforms the other two schemes substantially. In particular, there is about a 5% gain in terms of EE can be achieved by our proposed extended BC-MAC duality-based EE maximization algorithm compared to the scheme that without energy harvesting receivers [19]. Furthermore, due to the fact that the increasing sum rate of the system cannot offset the consumption of the transmit power, the sum rate maximization scheme without energy harvesting [47] achieves a very low EE.

Furthermore, the proposed extended BC-MAC duality-based EE maximization algorithm under different channel model is also studied. To show the effects of wireless fading channels, we consider Rayleigh fading and Log-Normal shadowing with standard deviation of 8 dB. As it can be seen from Fig. 6, the EE gain achieved by the proposed extended BC-MAC duality-based EE maximization algorithm over the scheme that maximizes EE without EH [19] and the one that maximizes SE without EH [47] is reduced compared to that over non-fading channels[1] in Fig. 5. This is because under Rayleigh fading and Log-Normal shadowing, additional power is required to satisfy the minimum harvested energy constraint for each user, resulting in the EE loss in a SWIPT system. Hence, the range of profitable operation for the TS-based SWIPT system is at moderate to high transmit power constraint region if Rayleigh fading and Log-Normal shadowing is considered.

We then investigate the average system EE for the proposed extended BC-MAC duality-based EE maximization algorithm

[1]No-fading channels refers to those that suffers only path loss degradation but without shadowing and multipath fading
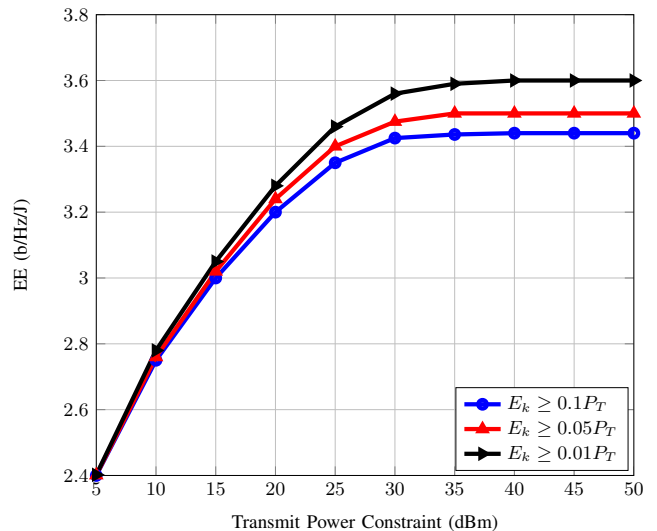

Fig. 7. Energy efficiency versus the maximum transmit power allowance for the proposed extended BC-MAC duality-based EE maximization algorithm with different minimum required power transfer.

with different level of minimum required harvested energy. As shown in Fig. 7, the increasing level of minimum required harvested energy will not always lead to an increasing system EE. Furthermore, jointly considering the results in Fig. 7 and Fig. 5, we can conclude that there exists an optimal minimum required harvested energy value for the EE optimization problem. As a result, the performance of system EE can be further improved if the TS ratios and the minimum required harvested energy are jointly considered, and that would be investigated in our future works.

Finally, we evaluate the achievable EE under different number of users in the SWIPT-based MIMO-BC. To show the impact of number of users, we fix the number of antennas at the BS and each user to $N_t = 8$ and $N_r = 2$, respectively. As shown in Fig. 8, the EE first increases then decreases with increasing number of users for the proposed extended BC-
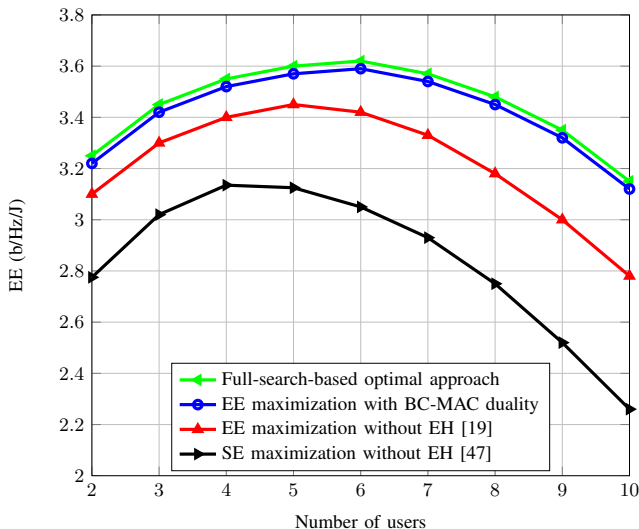
Fig. 8. Comparison of the achievable EE for different number of users in the network, where $N_t = 8$ and $N_r = 2$.

MAC duality-based EE maximization algorithm. The best EE performance in this simulation scenario is achieved when there exists $K = 6$ users. Since the spatial dimensions for DPC is $\min(N_r \times K, N_t)$, the maximum multiplexing gain under these simulation parameters is eight, which also indicates the maximum number of users achieving the maximum spatial dimensions is $K = 4$. Nonetheless, there exists multiuser diversity in BC scenario; in other words, the sum rate capacity will still increase marginally if the number of admitted users is increased beyond 4. On the other hand, since each user should achieve a minimum harvested energy, the total power consumption increases with K. Hence, when the number of users is further increased, the increase in power consumption will outgrow the gain in sum-rate from multiuser diversity. This explains why the optimal number of user is 6 in this scenario. Furthermore, the advantage of having SWIPT in a MIMO-BC scenario is clearly demonstrated, where the EE gain is further increased with increasing number of users in the network.

VIII. Conclusions

In this paper, we addressed the EE optimization problem for SWIPT-based MIMO-BC with TS receiver. Our aim was to maximize the EE of the system whilst satisfying constraints in terms of maximum power and minimum harvested energy for each user. The corresponding EE maximization problem from the coupling of the optimization variables, namely the transmit covariance matrices and TS ratios, is non-convex. Hence, to tackle the problem, we transform the original maximization problem with multiple constraints into a suboptimal min-max problem with single constraint and multiple auxiliary variables. For the min-max problem with single constraint, a dual-layer resource allocation strategy is proposed. We incorporate an extended SWIPT-based BC-MAC duality principle in order to simplify the inner-layer problem, and accordingly provide two different iterative resource allocation algorithms for solving

the dual MAC problem with fixed auxiliary variables. A sub-gradient-based searching scheme is then proposed to obtain the optimal auxiliary variables in the outer-layer. Numerical results validate the effectiveness of the proposed algorithms and show that significant performance gain in terms of EE can be achieved by our proposed extended BC-MAC duality-based EE maximization algorithm.

APPENDIX A
PROOF OF PROPOSITION 1

We can rewrite the constraint (16) (left hand side) as the following linear formulation

$$
\chi \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^b) - \sum_{k=1}^{K} \mu_k (1 - \alpha_k) \eta_k [\text{tr}(\mathbf{G}_k \mathbf{Q})]
$$
$$
= \chi \text{tr}(\sum_{k=1}^{K} \mathbf{Q}_k^b) - \text{tr}(\sum_{k=1}^{K} \mu_k (1 - \alpha_k) \eta_k \mathbf{G}_k \mathbf{Q}))
$$
$$
= \chi \text{tr}(\mathbf{Q}) - \text{tr}(\mathbf{G}\mathbf{Q})), \tag{52}
$$

where $\mathbf{G} = \sum_{k=1}^{K} \mu_k (1 - \alpha_k) \eta_k \mathbf{G}_k$. It is clear that constraint (16) satisfies the general linear power constraint $\text{tr}(\mathbf{A}\mathbf{Q}) \leq P$. Furthermore, it has been shown in [37] that the weighted factors are shared between the MAC and the BC, thus the TS ratios $\boldsymbol{\alpha}$ are sharing by the MAC and BC. On the other hand, if there exists another solution set $\check{\boldsymbol{\alpha}}$ that maximizes the MAC problem but different from the optimal TS ratio $\boldsymbol{\alpha}$ in the BC, we can also obtain another optimal covariance matrices set in dual MAC. This is because the noise covariance at the BS in the dual MAC is written as $\boldsymbol{N} = \chi \mathbf{I} - \sum_{k=1}^{K} \mu_k (1 - \alpha_k) \eta_k \mathbf{G}_k$. Apparently changing the TS factors in the dual MAC problem has impact on determining the optimal transmit covariance matrices. Therefore, this contradicts the uniqueness of the covariance mapping from the dual MAC to BC. Hence, we conclude that the duality relationship between SWIPT-based MIMO-BC system and its dual SWIPT-based MIMO-MAC stills holds. ∎

APPENDIX B
PROOF OF PROPOSITION 2

According to *Proposition 1*, the capacity region of a SWIPT-based MIMO-BC with power constraint P_T is equal to the union of capacity regions of the SWIPT-based dual-MAC with power constraints such that $\sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^m) = P_T$. Substitute the duality result to problem (22)-(23), the objective function can be reformulated as

$$
\max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1}
$$
$$
\frac{\sum_{k=1}^{K} \alpha_k R_k^b}{\zeta \chi \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^b) - \zeta \sum_{k=1}^{K} \mu_k (1 - \alpha_k) \eta_k \text{tr}(\mathbf{G}_k \mathbf{Q}) + P_C}.
$$
$$
\geq \max_{\mathbf{Q}_k^m \succeq 0, 0 \leq \alpha_k \leq 1}
$$
$$
\frac{\sum_{k=1}^{K} \alpha_k R_k^b}{\zeta \chi \sum_{k=1}^{K} \text{tr}(\mathbf{Q}_k^b) - \sum_{k=1}^{K} \mu_k (1 - \alpha_k) \eta_k \text{tr}(\mathbf{G}_k \mathbf{Q}) + P_C}.
$$

$$= g(\chi, \boldsymbol{\mu}). \tag{53}$$

Therefore, the solution of the SWIPT-based dual MAC EE maximization problem (22)-(23) is an upper-bound of the solution to the problem in (15)-(16). ∎

APPENDIX C
PROOF OF LEMMA 1

In order to prove the concavity, we define $l(\mathbf{Q}, \boldsymbol{\alpha}) = \sum_{k=1}^{K} \alpha_k R_k^m - \beta(\zeta \sum_{k=1}^{K} \operatorname{tr}(\mathbf{Q}_k^m) + P_C)$. We first investigate the relationship between $\boldsymbol{\alpha}$ and the objective function l. Thus, we obtain the following

$$\nabla^2 l(\alpha_k) = 2\frac{dR_k^m}{d\alpha_k} + \alpha_k \frac{d^2 R_k^m}{d\alpha_k^2}. \tag{54}$$

Substitute (21) into (54), we obtain

$$\frac{dR_k^m}{d\alpha_k} = \mu_k \eta_k \mathbf{G}_k tr[(\mathbf{N} + \sum_{k=1}^{i} \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k)^{-1}$$

$$-(\mathbf{N} + \sum_{k=1}^{i-1} \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k)^{-1}], \tag{55}$$

$$\frac{d^2 R_k^m}{d\alpha_k^2} = \mu_k^2 \eta_k^2 \mathbf{G}_k^2 tr[(\mathbf{N} + \sum_{k=1}^{i-1} \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k)^{-2}$$

$$-(\mathbf{N} + \sum_{k=1}^{i} \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k)^{-2}]. \tag{56}$$

Thus, defining $\mathbf{Z}_i = \sum_{k=1}^{i} \mathbf{H}_k^H \mathbf{Q}_k^m \mathbf{H}_k$, we can rewrite (54) as

$$\nabla^2 l(\alpha_k) = tr[2\mu_k \eta_k \mathbf{G}_k((\mathbf{N} + \mathbf{Z}_i)^{-1} - (\mathbf{N} + \mathbf{Z}_{i-1})^{-1})$$

$$+\mu_k^2 \eta_k^2 \mathbf{G}_k^2((\mathbf{N} + \mathbf{Z}_{i-1})^{-2} - (\mathbf{N} + \mathbf{Z}_i)^{-2})]$$

$$= tr[\mu_k \eta_k \mathbf{G}_k((\mathbf{N} + \mathbf{Z}_i)^{-1} - (\mathbf{N} + \mathbf{Z}_{i-1})^{-1})$$

$$(2\mathbf{I} - \mu_k \eta_k \mathbf{G}_k((\mathbf{N} + \mathbf{Z}_{i-1})^{-1} + (\mathbf{N} + \mathbf{Z}_i)^{-1}))]$$

$$\leq tr[2\mu_k \eta_k \mathbf{G}_k((\mathbf{N} + \mathbf{Z}_i)^{-1} - (\mathbf{N} + \mathbf{Z}_{i-1})^{-1})] \tag{57}$$

Given that $(\mathbf{Z}_i - \mathbf{Z}_{i-1}) = \mathbf{H}_i^H \mathbf{Q}_i^m \mathbf{H}_i$ is a positive semi-definite matrix, $((\mathbf{N} + \mathbf{Z}_i)^{-1} - (\mathbf{N} + \mathbf{Z}_{i-1})^{-1})$ is a negative semi-definite matrix. Therefore, applying the property of trace operator, i.e., $tr(\mathbf{AB}) = tr(\mathbf{BA})$, and replacing the channel covariance matrix $\mathbf{G}_k = \mathbf{H}_k^H \mathbf{H}_k$, we obtain

$$tr[2\mu_k \eta_k \mathbf{G}_k((\mathbf{N} + \mathbf{Z}_i)^{-1} - (\mathbf{N} + \mathbf{Z}_{i-1})^{-1})]$$

$$= tr[2\mu_k \eta_k \mathbf{H}_k((\mathbf{N} + \mathbf{Z}_i)^{-1} - (\mathbf{N} + \mathbf{Z}_{i-1})^{-1})\mathbf{H}_k^H] \leq 0 \tag{58}$$

Hence, the objective function of problem (29) is concave with respect to $\boldsymbol{\alpha}$. Furthermore, it has been shown in [37] that a sum rate maximization problem with users covariance matrices as the variables is concave. In this case, the role of the splitting factors $\boldsymbol{\alpha}$ is formally played by the weighted parameters, and these are not variables in the dual problem ($\boldsymbol{\alpha}$ are fixed in this case). Thus, the objective function of problem (29) is concave with respect to $\mathbf{Q}$. As a result, we can conclude that the objective function of problem (29) is jointly concave in $\mathbf{Q}$ and $\boldsymbol{\alpha}$. ∎

APPENDIX D
PROOF OF PROPOSITION 5

To prove $\lambda_{EE}^*(P_T^m)$ is a quasi-concave function, we denote the superlevel sets of $\lambda_{EE}^*(P_T^m)$ as

$$\mathcal{S}_\kappa = \{P_T^m \leq P_{all} | \lambda_{EE}^*(P_T^m) \geq \kappa\}. \tag{59}$$

In accordance with [48], for any real number κ, if the convexity for $\mathcal{S}_\kappa$ holds, $\lambda_{EE}^*(P_T^m)$ is strictly quasi-concave in P_T^m. Therefore, we here divide the proof into two cases. For the case of $\kappa < 0$, since EE is always positive and hence there are no points on the counter, $\lambda_{EE}^*(P_T^m) = \kappa$. For the case of $\kappa \geq 0$, λ_{EE} can be rewritten as

$$\lambda_{EE} = \frac{C_{MAC}(P_T^m)}{\zeta P_T^m + P_C}, \tag{60}$$

and hence $\mathcal{S}_\kappa$ is equivalent to $\kappa\zeta P_T^m + \kappa P_C - C_{MAC}(P_T^m) \leq 0$. Since it has been proven that $C_{MAC}(P_T^m)$ is convex in P_T^m [28], therefore the convexity of $\mathcal{S}_\kappa$ holds and $\lambda_{EE}^*(P_T^m)$ is strictly quasi-concave in P_T^m. ∎

APPENDIX E
PROOF OF PROPOSITION 6

To prove the statement in *Proposition 6*, the limit of $\lambda_{EE}^*(P_T^m)$ is analyzed as follows

$$\lim_{P_T^m \to 0} \lambda_{EE}^*(P_T^m) = \lim_{P_T^m \to 0} \max_{\mathbf{Q}_k^m \succeq 0, \ 0 \leq \alpha_k \leq 1} \frac{C_{MAC}(P_T^m)}{\zeta P_T^m + P_C}.$$

$$= \lim_{P_T^m \to 0} \frac{o(P_T^m)}{\zeta P_T^m + P_C} = 0. \tag{61}$$

Thus, given that $\lambda_{EE}^*(P_T^m)$ is strict concave (Appendix F), beginning with $P_T^m = P_{min}$, $\lambda_{EE}^*(P_T^m)$ either strictly decreases with P_T^m while $\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\Big|_{P_T^m = P_{min}} \leq 0$, or first strictly increases and then strictly decreases with P_T^m while $\frac{d\lambda_{EE}^*(P_T^m)}{dP_T^m}\Big|_{P_T^m = P_{min}} > 0$. In addition, we can conclude that the maximum EE achieved in the power region $[P_{min}, P_{max}]$ is straightforward as indicated in *Proposition 6*. ∎

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