

Discriminatively Guided Filtering (DGF) for Hyperspectral Image Classification

Ziyu Wang^{a,b}, Huafeng Hu^b, Lefei Zhang^c, Jing-Hao Xue^{b,*}

^a*Department of Security and Crime Science, University College London, London, UK*

^b*Department of Statistical Science, University College London, London, UK*

^c*Department of Computing, The Hong Kong Polytechnic University, Hong Kong, China*

Abstract

In this paper, we propose a new filtering framework called discriminatively guided image filtering (DGF), for hyperspectral image (HSI) classification. DGF integrates a discriminative classifier and a generative classifier by the guided filtering (GF), considering the complementary strength of these two types of classification paradigms. To demonstrate the effectiveness of the proposed framework, the combination of support vector machine (SVM) and linear discriminative analysis (LDA), which serve as a discriminative classifier and a generative classifier respectively, is investigated in this paper. Specifically, the original HSI is projected into the low-dimensional space induced by LDA to serve as guidance images for filtering the intermediate classification results induced by SVM. Experiment results show the superior performance of the proposed DGF compared with that of the principal component analysis (PCA)-based GF.

Keywords: Hyperspectral image (HSI) classification, guided image filtering (GF), discriminative classifiers, generative classifiers, linear discriminant analysis (LDA), principal component analysis (PCA), support vector machine (SVM).

*Corresponding author.

Email addresses: ziyu.wang.12@ucl.ac.uk (Ziyu Wang), huafeng.hu.15@ucl.ac.uk (Huafeng Hu), cs1fzhang@comp.polyu.edu.hk (Lefei Zhang), jinghao.xue@ucl.ac.uk (Jing-Hao Xue)

1. Introduction

Classification of pixels in hyperspectral images (HSIs) is an important and challenging task in remote sensing [1]. Recently, many classifiers have been developed on the basis of state-of-the-art machine learning techniques such as support vector machine (SVM) [2, 3], manifold and subspace learning [4, 5, 6, 7], sparse representation [8, 9, 10], collaborative representation [11], active learning [12, 13, 14], multitask learning [15], domain adaptation [16], object-based classifiers [17] and deep learning [18].

Because HSIs contain rich information in both spectral and spatial dimensions, the strategies simultaneously exploiting them prevail in the processing and analysis of HSIs [3, 19, 20, 21, 22, 23, 24]. In the pixel-wise classification, it often happens with undesired salt and pepper appearance if the spatial smoothness has not been adequately addressed [25, 26], therefore it is appealing to introduce an image filtering process to relieve this issue. Recently in 2D image processing, a new method called guided filtering (GF) [27] has proved an effective approach to edge-preserving smoothing (now a function *imguidedfilter* in the Image Processing Toolbox of MATLAB, The MathWorks, Inc.). The GF uses the content of a guidance image to guide the smoothing of the input image. It has been pioneered by [28] to facilitate the HSI classification. They first use SVM to classify the HSI resulting in binary classification maps for each class, then adopt GF to filter the classification maps. To build the guidance image, they conduct principal component analysis (PCA) on the HSI, and project the data onto the first principal component to obtain a virtual greyscale guidance image (or onto the first three principal components for a virtual colour guidance image). They show that the resultant PCA-based GF (short as PGF hereafter) can improve the classification performance of SVM on the HSI.

However, we shall show that PGF can be further revised, methodologically and empirically, if we can design a better-founded scheme to produce a superior guidance image for classification. In terms of classification, an ideal guidance image should be as similar as possible to the ground-truth map: it should be

31 able to not only preserve edges, but also provide the between-class discrimina-
32 tive information that is crucial to classification. However, the PCA adopted
33 by PGF is an unsupervised feature-extraction approach which does not con-
34 sider the discriminative information between classes. In contrast, we would
35 consider generative classifiers, which can serve as supervised feature-extraction
36 approaches and have the capability of exploiting the labelling information of the
37 training samples, a capacity that PCA lacks. Moreover, to the discriminative
38 classifiers like the SVM adopted by PGF, the generative classifiers can pro-
39 vide complementary discriminative strength, another capacity that PCA lacks;
40 for the complementarity of these two types of classifiers and the advantages of
41 combining them together, see [29, 30, 31, 32, 33].

42 Therefore, in this paper we propose a new filtering framework for classifica-
43 tion, which enables the integration of a discriminative classifier, such as SVM,
44 and a generative classifier, which can construct an complementary guidance
45 image for GF to be used in classification. We call this new framework the
46 discriminatively guided filtering (DGF). Specifically, we adopt linear discrimi-
47 native analysis (LDA) as a generative classifier to demonstrate the effectiveness
48 of the proposed DGF. For a C -class HSI classification problem, the LDA-based
49 DGF can be implemented as follows: we first perform multi-class LDA to obtain
50 $C - 1$ directions of projection, then we project the HSI onto the first leading
51 direction to build the virtual greyscale guidance image (or onto the first three
52 leading directions for the virtual colour guidance image). In this way, the ob-
53 tained guidance image preserves the discriminative information among multiple
54 classes. It is also worth noting that the proposed framework of DGF is a prin-
55 cipled framework, which is flexible and can be adapted to the combinations of
56 any discriminative classifiers and generative classifiers that suit the HSI classifi-
57 cation. The combination of SVM and LDA adopted here serves more as a case
58 study to demonstrate the use and effectiveness of DGF. Experimental results
59 show that the proposed DGF outperforms the PCA-based GF of [28], and both
60 of them improve the performance of SVM substantially for HSI classification.

61 2. Guided filtering

62 2.1. Greyscale guided filtering

63 For an input image p (e.g. in our case a classification map resulted from
64 SVM), the guided filtering (GF) [27] is assumed to bridge its filtering output
65 q and the guidance I by using a local linear model. For a greyscale image p ,
66 the output q is assumed to be a linear transformation of I in a local window ω_k
67 centred at the pixel k :

$$q_i = a_k I_i + b_k, \forall i \in \omega_k, \quad (1)$$

68 where i indexes a pixel in ω_k such that q_i and I_i are the (scalar) values of pixel
69 i in q and I , and (a_k, b_k) are coefficients to be estimated for ω_k . This model
70 preserves $\nabla q = a \nabla I$, which ensures that q preserves the edges in I .

71 Although (1) is a simple linear regression model, the coefficients a_k and b_k
72 are solved by a ridge regression model to minimise the following optimisation
73 function:

$$E(a_k, b_k) = \sum_{i \in \omega_k} \{(a_k I_i + b_k - p_i)^2 + \epsilon a_k^2\}, \quad (2)$$

74 where ϵ is the smoothing parameter to penalise a_k , and p_i is the filtering input.

75 The solution of (2) is given by

$$a_k = \frac{\frac{1}{|\omega_k|} \sum_{i \in \omega_k} I_i p_i - \mu_k \bar{p}_k}{\sigma_k^2 + \epsilon}, \quad b_k = \bar{p}_k - a_k \mu_k, \quad (3)$$

76 where μ_k and σ_k^2 are the mean and variance of pixel values in ω_k of the guidance
77 image I , $|\omega_k|$ is the total number of pixels in ω_k , and $\bar{p}_k = \frac{1}{|\omega_k|} \sum p_i$ denotes
78 the mean in ω_k of the input image p . Once a_k and b_k are solved, the filtering
79 output q_i in (1) can be obtained.

80 2.2. Multi-band guided filtering

81 The GF has been extended to a colour guidance image (i.e. with three bands)
82 in [27]. Similarly, it has no difficulty to generalise to a multi-band guidance
83 image of d bands by rewriting the local linear model (1) as

$$q_i = \mathbf{a}_k^T \mathbf{I}_i + b_k, \forall i \in \omega_k, \quad (4)$$

84 where \mathbf{I}_i is a d -dimensional vector for pixel i , \mathbf{a}_k is a d -dimensional coefficient
 85 vector, and q_i and b_k are still scalars. Then the GF with a multi-band guidance
 86 image becomes

$$\mathbf{a}_k = (\Sigma_k + \epsilon U)^{-1} \left(\frac{1}{|\omega_k|} \sum_{i \in \omega_k} \mathbf{I}_i p_i - \mu_k \bar{p}_k \right), \quad b_k = \bar{p}_k - \mathbf{a}_k^T \mu_k, \quad (5)$$

87 where μ_k is a $d \times 1$ mean vector, and Σ_k is a $d \times d$ covariance matrix, of the
 88 multi-band guidance image \mathbf{I} in window ω_k , and U is a $d \times d$ identity matrix.

89 3. Discriminatively guided filtering (DGF)

90 As pioneered by [28], the GF not only can be used as an edge-preserving
 91 smoothing operator, but also can help HSI classification. In this direction, we
 92 propose a new filtering framework called discriminatively guided image filtering
 93 (DGF), to combine a discriminative classifier and a generative classifier by the
 94 GF. Specifically, to incorporate the discriminative information from HSI into the
 95 GF, we use LDA as the generative classifier to construct the guidance image.

96 In 2D image processing, the guidance I_i in (1) (or \mathbf{I}_i in (4)) is a greyscale
 97 value (or RGB values) of pixel i in a local window ω_k . However in the PGF [28]
 98 and the proposed DGF, the guidance I_i (or \mathbf{I}_i) is the projection of pixel i in the
 99 lower dimensional space induced by PCA and LDA, respectively.

100 3.1. Methodology of DGF

101 An HSI classification problem usually needs to address multiple classes (i.e. C
 102 classes with $C > 2$). An HSI of N pixels can be denoted by a $B \times N$ matrix
 103 \mathbf{X} with each pixel being of B features, and $N > B$ as usual. For such a
 104 multi-class problem, the multi-class LDA seeks $C - 1$ directions (or say $C - 1$
 105 linear combinations of the B features), the subspace spanned by which can best
 106 separate the classes [34].

107 These directions are the first $C - 1$ leading eigenvectors (corresponding to
 108 the largest eigenvalues) of

$$\mathbf{S}_W^{-1} \mathbf{S}_B, \quad (6)$$

109 where \mathbf{S}_W is the pooled within-class scatter matrix over all C classes, and \mathbf{S}_B
 110 is the between-class scatter matrix. We use \mathbf{W} to denote a $B \times (C - 1)$ matrix
 111 whose columns are these $B \times 1$ eigenvectors. Given \mathbf{W} , each class can have a
 112 unified multi-band guidance image for the GF. To align with the PCA-based
 113 GF [28], we also adopt the two strategies below.

114 *3.1.1. DGF-g: DGF with a greyscale guidance image*

115 We use the projection of the HSI on the first leading eigenvector (\mathbf{w}_g , a $B \times 1$
 116 vector) in \mathbf{W} as the greyscale guidance image:

$$\mathbf{I}_g = \mathbf{w}_g^T \mathbf{X}, \quad (7)$$

117 where \mathbf{I}_g is a $1 \times N$ vector representing the greyscale guidance image of N
 118 pixels. The filtering output for each class is then obtained by the greyscale
 119 guided filtering in (1).

120 *3.1.2. DGF-c: DGF with a colour guidance image*

121 We use the projection of the HSI on the first three leading eigenvectors (\mathbf{W}_c ,
 122 a $B \times 3$ matrix) in \mathbf{W} as the colour guidance image:

$$\mathbf{I}_c = \mathbf{W}_c^T \mathbf{X}, \quad (8)$$

123 where \mathbf{I}_c is a $3 \times N$ matrix representing the colour guidance image. The filtering
 124 output for each class is then obtained through the multi-band guided filtering
 125 in (4).

126 *3.2. Classification algorithm based on DGF*

127 The diagram of DGF for classification of an HSI is illustrated in Fig. 1,
 128 similar to the procedure in [28] but with a different filtering strategy. The
 129 whole HSI is firstly classified by SVM to obtain initial classification results
 130 called classification maps, one for each class, which contains the probability of
 131 the pixels belonging to the class, e.g. \mathcal{C}_j .

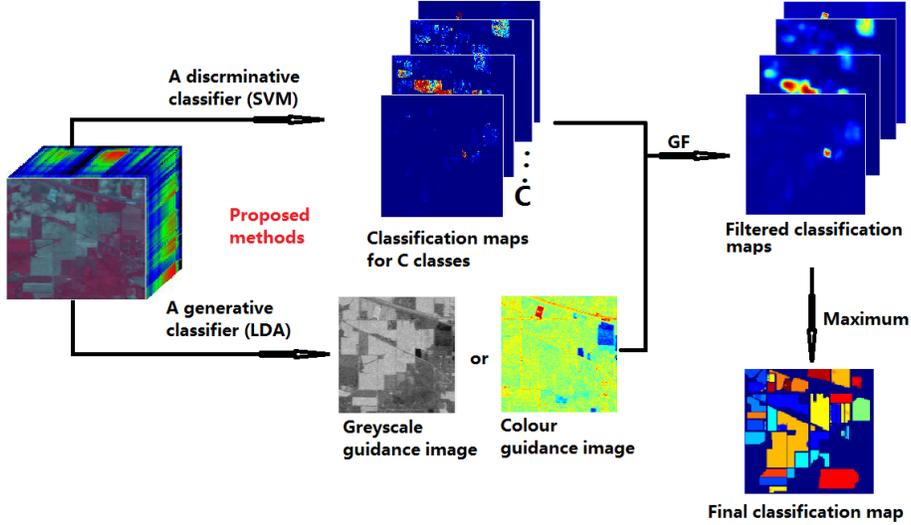


Figure 1: The proposed DGF methods for HSI classification.

132 Then to improve these initial spectrum-based classification results, the pro-
 133 posed DGF aims to incorporate spatial structure information, by using edge-
 134 preserving GF, and discriminative information, by using LDA to generate the
 135 guidance image.

136 Finally the C filtered classification maps are merged into a final classification
 137 map: The label of a test pixel \mathbf{x}_{test} is

$$l(\mathbf{x}_{test}) = \underset{j}{\operatorname{argmax}} f_j(\mathbf{x}_{test}), \text{ for } j = 1, \dots, C, \quad (9)$$

138 where $f_j(\mathbf{x}_{test})$ is the filtered classification results of \mathbf{x}_{test} for class \mathcal{C}_j , meaning
 139 that \mathbf{x}_{test} is classified into class \mathcal{C}_j if $f_j(\mathbf{x}_{test})$ has the highest value among all
 140 the C classes.

141 The DGF-based classification algorithm is in Algorithm 1.

Algorithm 1 Classification of an HSI based on **DGF**

Input:

A vectorised HSI $\mathbf{X} \in \mathbb{R}^{B \times N}$; training HSI pixels $\mathbf{X}^{train} \in \mathbb{R}^{B \times N_{train}}$ and their labels $\mathbf{y}^{train} \in \mathbb{R}^{N_{train} \times 1}$.

The radius of the local window size r and the smoothing parameter ϵ , for GF.

Output: A classification map $L(\mathbf{X})$, a $N \times 1$ vector.

Training phase:

- Train a classifier Φ (e.g. SVM) using $\{\mathbf{X}^{train}, \mathbf{y}^{train}\}$.
- Train a multi-class LDA model by (6).

Test phase:

- Classify \mathbf{X} using the trained classifier Φ and obtain the initial classification maps M_1, \dots, M_C .
 - Obtain the DGF guidance image:
 - either \mathbf{I}_g of \mathbf{X} using DGF-g (7), or
 - \mathbf{I}_c of \mathbf{X} using DGF-c (8).
 - Filter M_1, \dots, M_C by \mathbf{I}_g as the greyscale guided filtering (1), or by \mathbf{I}_c as the colour guided filtering (4).
 - Classify each test pixel in \mathbf{X} by (9) and obtain $L(\mathbf{X})$.
-

142 4. Experiments

143 4.1. Data and compared methods

144 The experiments are carried out on a real hyperspectral dataset: the AVIRIS
145 Indian Pines dataset, which is publicly available [35] and has been widely used as
146 a benchmark dataset for HSI classification including those involving GF [28, 36].
147 The dataset (Fig. 4(a)) consists of 145×145 pixels with 200 spectral bands after
148 removing the water absorption bands. To make fair comparison with [28], for
149 each of the 16 ground-truth classes (Fig. 4(b)), we randomly select the same
150 number of labelled pixels as training samples and the rest as test samples which
151 is utilised in [28], as listed in Table 1.

152 In our experiments, five methods are compared, including SVM, PGF-g [28],
153 PGF-c [28], DGF-g and DGF-c. Among these methods, PGF-g represents the
154 method using SVM as a spectral classifier and adopting the first principal com-
155 ponent of the HSI as a virtual greyscale guidance image for GF (referred to as
156 EPF-G-g in [28]). Similarly, PGF-c represents the method using the first three
157 principal components as a virtual colour guidance image (EPF-G-c in [28]).
158 Correspondingly, DGF-g and DGF-c represent our proposed methods in (7)
159 and (8).

160 As with [28], the LIBSVM toolbox [37] is used to execute SVM. A dimen-
161 sion reduction toolbox (<http://lvdmaaten.github.io/drtoolbox>) is adopted to
162 perform PCA and LDA. The Image Processing Toolbox of MATLAB is used to
163 run GF. We also employ three standard performance measures: the overall ac-
164 curacy (OA), the average accuracy (AA) and the κ coefficient [38] for evaluating
165 the classification performances of the compared methods.

166 LDA has a limitation with the potential singularity of the pooled within-
167 class scatter matrix \mathbf{S}_W [39]. The singularity occurs when the data dimension
168 B is larger than the number of training samples N_{train} . In our case, B is
169 much smaller than N_{train} , and the singularity problem does not happen in our
170 experiments. Nevertheless, we note that our approach is applicable when there
171 are sufficient training samples. In the case of $B > N_{train}$, we can perform a

172 preliminary dimension reduction before applying LDA to avoid the singularity
 173 issue.

174 *4.2. Parameter settings*

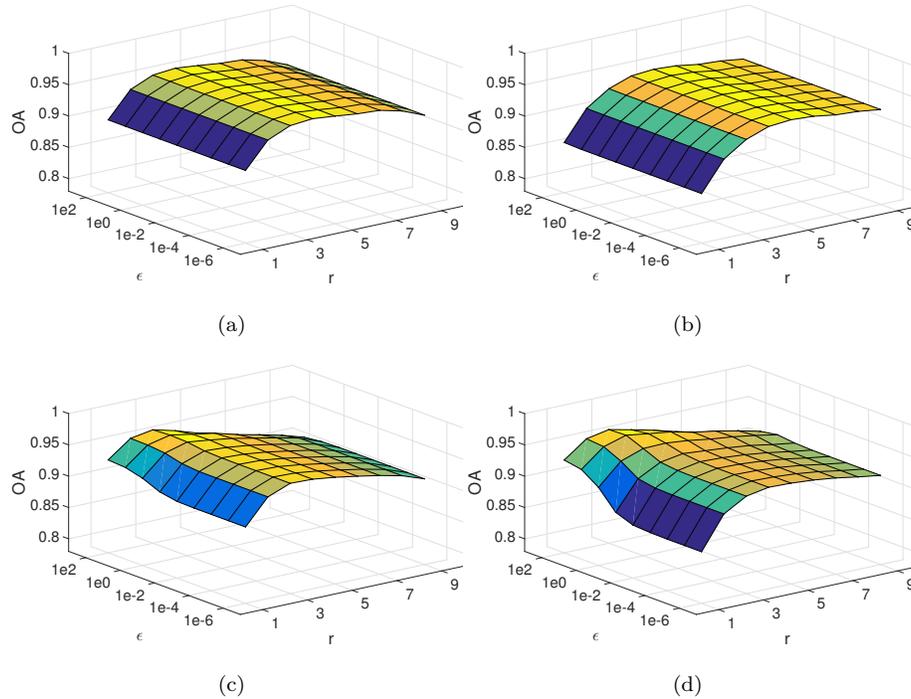


Figure 2: Overall classification accuracies over window radius r and smoothing parameter ϵ for (a) PGF-g [28], (b) PGF-c [28], (c) DGF-g, and (d) DGF-c.

175 There are parameters of SVM and GF. For SVM, we use the polynomial
 176 kernels with 20-fold cross validation to tune the parameters. The optimal values
 177 of parameters C and γ of the kernel function are tuned to be 5.66 and 0.16,
 178 respectively.

179 For GF, two parameters should be tuned: the radius r of the local window ω
 180 and the smoothing parameter ϵ . The influence of these two parameters on the
 181 overall classification performances of the compared methods are demonstrated
 182 in Fig. 2. The range of r is from 1 to 9 and the range of ϵ covers 10^{-6} to 10^2 . We

183 can observe that the optimal performances of PGF-g and PGF-c occur at about
 184 $r = 4$ (shown in Fig. 2(a)-2(b)). Hence we set $r = 4$ for PGF-g and PGF-c, and
 185 as with [28] we set $\epsilon = 0.01$ for them. For the proposed DGF-g and DGF-c, we
 186 set $r = 3$ and $\epsilon = 10$ as the optimal performances roughly occur there (shown
 187 in Fig.2(c)-2(d)).

188 *4.3. Classification results*

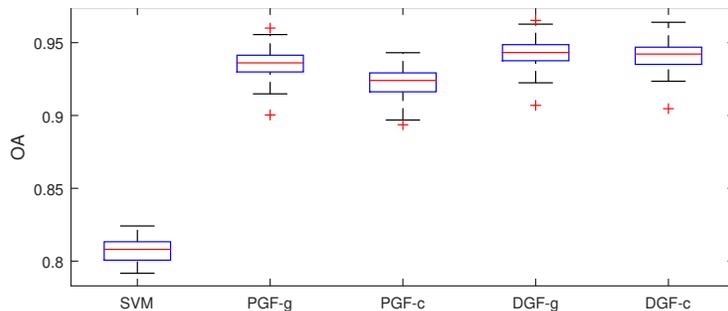


Figure 3: Boxplots of the overall classification accuracies on Indian Pines.

189 To have a reliable evaluation and fair comparison, we repeat the experi-
 190 ments under these parameter settings for 50 times through performing 50 ran-
 191 dom training-test splits while keeping the same numbers of samples for training
 192 and testing. For illustrative purposes, the classification results for one of the
 193 50 experiments are given in Table 1 and depicted in Figs. 4(c)-4(g), respec-
 194 tively. Moreover, all of methods' overall classification accuracies are recorded
 195 and boxplotted in Fig. 3.

196 From these results, we can observe at least two clear patterns. Firstly, all
 197 PGF-g, PGF-c, DGF-g and DGF-c improve the performance of SVM substan-
 198 tially, which confirms that incorporating the guided filtering process can help
 199 the spectral-based classifier. Secondly, the proposed DGF-g and DGF-c out-
 200 perform PGF-g and PGF-c in OA and κ . It indicates that the discriminative
 201 information provided by LDA (but unable by PCA) to GF can further improve
 202 classification performance.

Table 1: Indian Pines: Ground-truth label, training set, test set, and the classification accuracies (%) obtained by SVM, PGF-g [28], PGF-c [28], DGF-g and DGF-c. The best performance is in bold.

Class	Train	Test	SVM	PGF-g	PGF-c	DGF-g	DGF-c
1	25	21	100.00	100.00	100.00	100.00	100.00
2	83	1345	77.70	93.68	92.49	95.46	95.17
3	78	752	73.40	93.88	91.76	94.02	94.68
4	68	169	87.57	100.00	98.82	100.00	100.00
5	79	404	93.81	98.27	96.29	97.03	96.53
6	78	652	95.86	99.85	99.69	100.00	100.00
7	4	24	4.17	0.00	0.00	0.00	0.00
8	66	412	99.51	100.00	100.00	100.00	100.00
9	2	18	0.00	0.00	0.00	0.00	0.00
10	81	891	74.86	97.87	97.64	98.77	98.77
11	99	2356	66.47	92.19	90.32	94.27	94.10
12	73	520	90.00	100.00	99.23	100.00	99.81
13	70	135	99.26	99.26	100.00	100.00	99.26
14	90	1175	90.21	98.13	97.62	97.45	97.62
15	65	321	79.44	99.07	94.39	99.07	99.07
16	46	47	95.74	97.87	93.62	87.23	93.62
Total	1007	9242					
OA			79.81	95.55	94.31	96.27	96.25
AA			76.75	85.63	84.49	85.21	85.54
κ			0.770	0.949	0.945	0.957	0.957

Table 2: Overall classification accuracies (%). Methods with * indicates that their OAs are obtained from [28] under their optimal parameters settings via 5-fold cross-validation.

WLS*	NC*	JBF-g*	PGF-g	DGF-g	JBF-c*	PGF-c	DGF-c
94.93	95.20	95.42	95.55	96.27	95.41	94.31	96.25

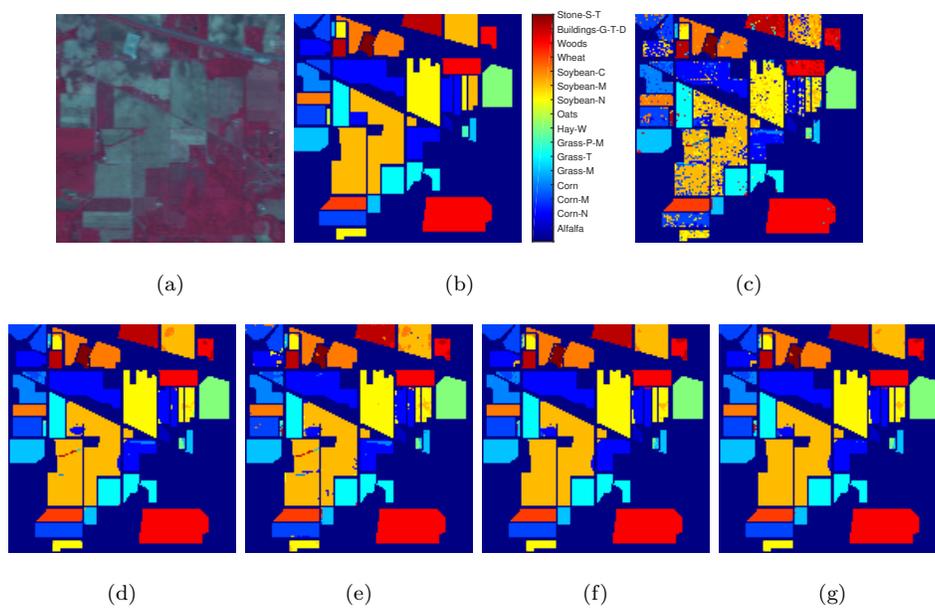


Figure 4: Indian Pines: (a) mean image shown in the false colour; (b) ground-truth labels. Classification maps (and OA) of (c) SVM (79.81%); (d) PGF-g (95.55%); (e) PGF-c (94.31%); (f) DGF-g (96.27%); (g) DGF-c (96.25%).

203 For further assess whether the difference of the performances of DGF and
204 PGF is statistical significant, we also perform the Wilcoxon signed-rank test,
205 a widely-used non-parametric statistical hypothesis test for paired samples. In
206 this paper, the test is designed for testing whether the paired classification
207 performances, i.e. 50 OAs (or 50 AAs) of PGF and that of DGF, differs at the
208 1% significance level. Specifically, for OAs (or AAs), we conduct two tests: one
209 for PGF-g versus DGF-g and the other for PGF-c versus DGF-c. The obtained
210 p -values of the two tests of OAs are $8.0e-10$ and $7.5e-10$; this indicates strong
211 evidence that DGF is significantly better than PGF in terms of OA, confirming
212 the observation from Fig. 3 and Table 1. The corresponding p -values for AAs
213 are 0.011 and $7.6e-10$; this provides strong evidence that DGF-c is significantly
214 better than PGF-c in terms of AA, while no strong evidence that DGF-g is
215 significantly worse than PGF-g, also in line with what is revealed by Table 1.

216 From Table 1, we also note that all GF-based methods (and SVM itself)
217 fail for classes 7 and 9. This is due to the lack of training samples for these
218 two class, which are only 4 for class 7 and 2 for class 9. Also, classes 7 and
219 9 cover a narrow region in the dataset, and the filtering of these two classes
220 can be dominated by other classes adjacent and thus misclassified. The bad
221 performance of the methods in identifying classes 7 and 9 make a big influence
222 on AA but little influence on OA, because the number of test pixels of these
223 two class are also small. This explains why OA are higher than AA for all the
224 compared methods listed in Table 1.

225 For further evaluation, we also compare DGF with some other modern
226 edge-preserving smoothing methods, such as the weighted least squares (WLS)
227 method, the normalised convolution (NC) filter and the joint bilateral filter
228 (JBF). These methods are also compared in [28]. Since our experiment settings
229 are exactly the same as [28], we compare the performance of our proposed DGF
230 directly with those reported in [28], as listed in Table 2. We can observe that
231 our proposed method DGF-g and DGF-c still outperform the other compared
232 edge-preserving methods, which shows the superiority of the proposed idea to
233 combine the discriminative classifier and the generative classifier by GF for the

234 HSI classification.

235 **5. Summary and future work**

236 In this paper, a new filtering framework called discriminatively guided filter-
237 ing (DGF) has been proposed, which integrates a discriminative classifier and a
238 generative classifier by the guided filtering for hyperspectral image classification.
239 The combination of SVM and LDA has been adopted illustrating the effective-
240 ness of DGF, which also inspires us to investigate the performance of other
241 generative-discriminative combinations as a direction of our future research.

242 Furthermore, there are other reports using the Indian Pines dataset for eval-
243 uation of classification performance, for example, our recent work called JSM-
244 DKSVD which focuses on dictionary learning [10]. We shall note that it may
245 not be fair to directly compare the proposed DGF to such a dictionary learning-
246 based classification method, since the latter has an extra learning process to
247 learn the dictionary. Nevertheless, we believe that the proposed GDF frame-
248 work can be improved by incorporating such a learning process, which leads to
249 another direction meriting our future research.

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Ziyu Wang received the M.Sc. degree in statistics in 2012, the M.Res. degree in security science in 2013 and the Ph.D. degree in security science and statistical science in 2017, all from University College London. Her research interests include hyperspectral image analysis, sparse representation and statistical classification.



Huafeng Hu received the B.S. degree in applied mathematics in 2015 from Xi'an Jiaotong-Liverpool University and the M.Sc. degree in statistics in 2016 from University College London. Her research interests include hyperspectral image classification and convolutional neural network for image classification.



Lefei Zhang received the B.S. and Ph.D. degrees from Wuhan University in 2008 and 2013, respectively. He is an Associate Professor with the School of Computer, Wuhan University, and also a Hong Kong Scholar with the Department of Computing, The Hong Kong Polytechnic University. His research interests include pattern recognition, image processing and remote sensing.



Jing-Hao Xue received the Dr.Eng. degree in signal and information processing from Tsinghua University in 1998 and the Ph.D. degree in statistics from the University of Glasgow in 2008. He is a Senior Lecturer in the Department of Statistical Science, University College London. His research interests include statistical classification, high-dimensional data analysis, pattern recognition and image analysis.