# A Trivalent Approach to Anaphora and Presupposition*

Daniel Rothschild

University College London
d.rothschild@ucl.ac.uk

## 1 Unified accounts of anaphora and presupposition

The goal of this paper is to present a simple and novel system for capturing core data about anaphora and presupposition projection. With respect to presupposition there is no novelty: I simply use a variant of the strong Kleene trivalent logic to treat presupposition projection.[1] What is new is that I add some apparatus from dynamic semantics to extend the trivalent system to also cover e-type anaphora.

Heim, in her dissertation [1982], gave two treatments of e-type (donkey) anaphora. One (chapter 2) treated anaphora by means of explicit existential quantifiers in a fully static (and very standard) semantic framework, the other (chapter 3) introduced the first compositional dynamic semantics for anaphora. One of Heim's main arguments for adopting the second approach was that her dynamic system provided a unified treatment of anaphora and presupposition, something no other account provided. Heim's account of anaphora and presupposition has been modified and extended by, among others, Beaver [2001] into a unified and powerful system for the treatment of both.[2]

The dynamic treatment of presupposition projection has been criticized by Schlenker [2008, 2009] for its lack of explanatoriness. However, alternative treatments of presupposition projection such as Schlenker's local context approach and the trivalent approach do not obviously integrate well with an account of e-type anaphora.[3] In later work, Heim [1990] suggests integrating a static (presumably trivalent) presuppositional approach to definites with situation semantics and an e-type treatment of pronouns as disguised Fregean descriptions to cover donkey anaphora, a treatment elaborated in Elbourne [2005]. This paper is not the occasion for a full discussion of these semantic theories, but I will pause to note the following:

- Unlike standard dynamic accounts, these proposals have rarely, if ever, been spelled out in large fragments containing sentential connectives and negation.[4]

- The proposals contain complex definitions of quantifiers such as ⌜every⌝ with multiple layers of existential and universal quantification over individuals and situations (a property shared by most dynamic approaches but not by the account I present here).

*I am indebted to Matt Mandelkern for extensive discussion.

[1] It is my view that, when the dust has settles, this remains the simplest viable treatment of presupposition projection on the market. See Peters [1979], Krahmer [1998], George [2007], Fox [2008, 2012] among others.

[2] A different tradition stemming from van der Sandt [1992] uses and Kamp's DRT to unify anaphora and presupposition. Beaver, to my mind, makes convincing arguments against this approach.

[3] I use *e-type anaphora* as a term to describe the general phenomenon in which pronouns are used without c-commanded antecedents, the relation between pronoun and antecedent being inter-sentential, across conditionals, or between the restrictor and matrix of an NP. An *e-type treatment*, by contrast, is a semantic account of such pronouns which treats them as akin to defined descriptions that have Russellian/Fregean semantics [such as, Cooper, 1979, Evans, 1977].

[4] For example the fragment in Elbourne [2005]—often pointed to as one of the most extensive situation-theoretic-cum-descriptive treatments of donkey anaphora—does not contain a semantics for negation, and it is non-trivial to see how one can be added.

- The connections between e-type anaphora and presupposition projection are rarely made explicit in this tradition.

For these and other reasons I do not see the situation-theoretic e-type approach as a particularly promising line for an integrated account of e-type anaphora and presupposition.

As I see the current situation, then, dynamic approaches provide the best unified accounts of presupposition and anaphora. So why should we bother rethinking the framework of dynamic semantics when it is so successful in this respect? Shouldn't we just accept its successes and move on, either just replacing it or expanding it, rather than tweaking? Here I stand with Dekker and Schlenker, in particular, who have suggested that the successes of dynamic semantics may not adequately motivate its foundational ideas.

For instance, a salient feature of standard dynamics semantics is to treat the semantic values of sentences not as truth-conditions but rather as context change potentials (CCPS).[5] In other words, instead of having semantic values be functions from points of a context to truth-values, semantic values are functions from contexts (sets of points) to contexts.[6] There are many obviously inexpressible such functions: for example, we do not have a sentences in any language that expresses the context change that moves any context to one which only accepts the fact that there are pink elephants. There are no knock-down considerations in favor of having lower-type semantic values, but lower types are simpler and, thus, all else equal to be preferred.

All else is never equal, though, and type-theoretical considerations are not my only ones. Another way in which my semantics is simpler is that the definitions of the quantifiers and connectives I use are essentially their classical definitions: the dynamic effects of these really do follow from their classical definitions (and the strong Kleene logic). Thus, I share the motivations for Schlenker's non-dynamic account of presupposition projection which relies on a classical understanding of connectives and quantifiers. This, again, adds to the simplicity of the semantic system and relatedly its learnability. More significant, perhaps, are empirical advantages: I handle the behavior of anaphora under double negation and through disjunctions in a straightforward way, something dynamic accounts tend to struggle with.[7]

My account is in the spirit of the constructive criticisms of dynamic semantics put forward by Dekker [1994, 2012] and Schlenker [2008, 2009]. The account is similar to Dekker's Predicate Logic with Anaphora (and is directly inspired by it), in that it also uses many of the conceptual innovations of dynamic semantics without resorting to a context change potential-based semantics. On the other hand, the account is parallel to Schlenker's static accounts of presupposition (transparency theory and his local context theory) in that it uses more standard, non-stipulative definitions of all quantifiers and connectives, including conjunction. From a broader perspective, while Dekker treats e-type anaphora but not presuppositon, and Schlenker treats presupposition but not e-type anaphora, I try to treat both.

[5] Or in extensional fragments such as Groenendijk and Stokhof [1991] as functions from assignment functions to assignment functions rather than assignment functions to truth values.

[6] I am assuming here that contexts are sets whose elements I call points, these points can be worlds as in Stalnaker's framework or world-assignment function pairs as in many dynamic accounts.

[7] In some aspects of this, I follow Krahmer [1998], except that Krahmer combines DRT with trivalence, rather than simply having a trivalent system and he does not cover all the aspects of disjunction that I do.

## 2 Some rules of the game

We will adopt a Heimian notion of context according to which a context is a set of pairs of assignment functions and worlds.[8] This conception, of course, does not commit us to higher-type semantic values, just the (fairly) uncontroversial idea that speaker and hearers keep track of possible discourse references for certain 'variables' introduced in discourse.[9]

Truth conditions—certain forms of irrelevant context-sensitivity aside—are simply functions from elements of such contexts (pairs of assignment functions and worlds) to truth values. Our semantics will be static or truth-conditional in that the semantic value of sentences will be such truth-conditions.[10]

The update rule associated with a sentence $\phi$ will be Stalnakerian [Stalnaker, 1970, Rothschild and Yalcin, 2015]. When a sentence $\phi$ is asserted in a context $c$ we remove from $c$ every element on which $\phi$ is not true.[11]

## 3 E-type pronouns and presuppositions

The following examples illustrate a small part of the connection between presupposition projection and e-type anaphoric relations.

(1) a. John used to smoke and he hasn't stopped smoking.
b. ?John hasn't stopped smoking and he used to smoke.
c. ?John didn't used to smoke and he hasn't stopped smoking.

(2) a. A man walked in and he wasn't wearing a hat.
b. ?He wasn't wearing a hat and a man walked in.
c. ?A man didn't walk in and he was wearing a hat.

It is worth sketching an aspect of the empirical connection between presupposition and anaphora. Consider a case where we have a complex sentence $s$ with constituents $\phi$ and $\psi$ such that $\phi$ presupposes $X$ and $\psi$ classically entails $X$. For example in (1-a), $\phi = \ulcorner$he hasn't stopped smoking$\urcorner$, $\psi = \ulcorner$he used to smoke$\urcorner$ and $X = \ulcorner$he used to smoke.$\urcorner$ If $s$ does not itself presuppose $X$, and this is because of $\psi$, then we'll say that $\psi$ allows for local satisfaction of the presupposition of $\phi$ (it filters it out). This is the case in (1-a). Very roughly speaking, the same configurations that allow for local satisfaction of presuppositions also allow for e-type anaphora. This is what is illustrated by the examples above.

[8] This is also the approach explored *inter alia* in Groenendijk et al. [1996]: context includes information about variable assignments, not just worlds

[9] This is in the spirit of Lewis [1983] and even seems to be countenanced in an unsystematic pragmatic way by Stalnaker [1998]. Note that this notion can lead to a flavor of discourse dynamism within a completely static semantic system as discussed by Rothschild and Yalcin [2016, §5].

[10] It is important to distinguish this kind of static compositional semantics from what Rothschild and Yalcin [2015] call dynamics at the *conversational level*. That is, the CCPs associated with assertion in our semantic/pragmatic whole may be dynamic in various senses, but the sentences themselves are static in that they are not functions from contexts to contexts but rather simply truth assignments for individual points.

[11] Of course, the effect of this update rule will not always be what Stalnaker had in mind in the classic papers where he suggests these updates since the context includes not just worlds but also assignment functions. I share with the dynamic tradition a lack of interest in the question of whether sentences express propositions. Since contexts are not just sets of worlds we cannot simply identify the semantic value of a sentence with the set of worlds it is true in. But for reasons addressed already in Lewis [1980] that is not generally viable in semantic theorizing. Certainly we can define various different notions of content in contexts, but we need not take a stand on these.

Given this connection between local satisfaction of presupposition and e-type anaphora we might expect a theoretical connection. Our account, like the dynamic account stemming from Heim [1982], tries to make good on that expectation.

## 4 A trivalent account of presupposition

Let me give a brief outline of how the facts about presupposition projection can be accounted for on a trivalent framework. On a trivalent semantics, sentences can be either true, false, or undefined (1,0, or #). The connectives handle undefinedness according to the strong Kleene truth tables, the guiding principle of which is to give truth values when those are determined by what is defined. We can also add order effects, more closely matching standard theories of presupposition projection by using Peter's truth tables [Peters, 1979, Krahmer, 1998, George, 2007].

The relationship between the trivalent truth tables and context is important here. A sentence $\phi$ is acceptable in a context $c$ iff $\phi$ is true or false at every element of $c$. This, sometimes called Stalnaker's principle, was proposed by him as an intuitive principle, but has since been recognized to be rather a kind of stipulation [for discussion, see Soames, 1989, Rothschild, 2008b, Fox, 2012].

To give an example, let us treat ⌜John stopped smoking⌝ as undefined if John didn't use to smoke and true or false otherwise. Then, given the strong Kleene understanding of conjunction, (1-a) will be defined in any context. This is because when the presupposition of the second conjunct is not satisfied the first conjunct is false and so the entire sentence is undefined.

## 5 E-type Anaphora and Content

What has been called the problem of the formal link [Heim, 1990] puts a particularly sharp constraint on how we treat anaphoric connections. Here are types of examples due to Partee and Heim respectively:

(3) a. ? Nine of the ten marbles are on the floor. . . . ? It's on the couch.
b. One of the ten marbles is not on the table. . . . It's on the couch.

(4) a. ? Every married man loves her.
b. Every man with a wife loves her.

The lesson here is that pronouns without appropriately marked NP antecedents are difficult, and often result in infelicity. This suggests that if we are to use a presuppositional approach we cannot rely on simply presuppositions about the state of the world. Rather we also need presuppositions that somehow involve variables.[12]

[12]The modern e-type treatment of anaphora rather attempts to cover such facts by positing syntactic conditions on the licensing of covert descriptions, such as Elbourne's [2005] NP-deletion. My view is that such conditions face serious challenges as they separate the syntactic licensing conditions from the semantics of the pronouns and will inevitable make bad predictions. Here is one example:

(i) ?Everyone who doesn't have a home but knows a home-owner, stays in it.

(ii) Every who isn't a home-owner but knows someone who has a home, stays in it.

The problem is that it would seem that ⌜a home⌝ in (i) should license the description ⌜the home⌝ whose presupposition is then satisfied by restrictor state.

## 6 Anaphoric presuppositions: intersentential case

Let's focus for a moment on the *inter*-sentential case, such as (3-a) and (3-b). The question is how the context differs between these two examples to make the assertions different: that is, what effect does the first sentence have on the context to make the pronoun in the second sentence felicitous. Let us assume, as in the Heim's approach, that pronouns are simply variables. What we need to explain the contrasts in the section above is to posit special constraints on the use of variables. For example, Heim puts a *familiarity presupposition* on the use of definite variables—a special structural condition on the local context of a pronoun. In this section I outline a related approach that fits more naturally with a standard trivalent treatment of presuppositions.

I will assume something like the partiality of assignment functions, though in a slightly non-standard way. What I will assume is that assignment functions are functions from variables onto the usual domain $D$ as well as an absurd object $\perp$. Empty contexts include all assignments.[13]

The presupposition of the use of a definite variable $x$ is simply that $x$ does not refer to $\perp$. To get this to work we need the harmless assumption that the extension of every predicate is undefined when applied to $\perp$.

What about our treatment of indefinites? How do they ensure that intersentential anaphora works? Nothing special is needed, except assuming, as Heim does, that indefinites put constraints on variables rather than having existential force in the usual sense. ⌜One marble$_x$ is not on the table⌝ is true at $\langle f, w\rangle$ iff $f(x)$ is a man who walked in at $w$. Any context on which it is true at every point, thus, will make ⌜it$_x$'s on the couch⌝ defined.

Let's go through a simple example with truth conditions spelled out:

$$[\![\text{A man}_x \text{ walked in.}]\!]^{f,w} = \begin{cases} 1 \text{ iff } f(x) \text{ is a man who walked in in } w \\ 0 \text{ otherwise} \end{cases}$$

$$[\![\text{He}_x \text{ had a drink}]\!]^{f,w} = \begin{cases} \# \text{ if } f(x) = \perp \\ 1 \text{ if } f(x) \text{ had a drink in } w \\ 0 \text{ otherwise} \end{cases}$$

The point is that once the context absorbs the first sentence, then the second sentence is guaranteed to be acceptable since all the points at which $f(x) = \perp$ have been eliminated. In terms of truth-conditions, this theory so far matches Heim: indefinites have existential force given that the empty assignment allow all possible assignments.[14]

## 7 Conjunction

The story for inter-sentential anaphora extends to a treatment of e-type anaphora across a conjunction:

(5) A man$_x$ walked in and he$_x$ ordered a drink.

At any point in the context in which $x$ is assigned $\perp$ the sentence is false on the strong Kleene understanding of conjunction since the first conjunct is false.[15] If we wish to explain why the

[13] On the standard use of partial assignment functions in dynamic semantics the empty assignment is that in which no variable has a defined assignment, not one in which every possible assignment (including empty/absurd ones) is in the context.

[14] While we have not yet put any condition on the use of indefinites, we might assume that indefinite should not be reused because of some variation of a maximize presupposition rule [Heim, 1991].

[15] See the appendix for details including the strong Kleene conjunction.

reverse order, as in (6), is infelicitous will need to use the Peters [1979] version of the connectives or apply an order constraint.

(6) ?He$_x$ ordered a drink and a man$_x$ walked in.

# 8 Taking stock

Let us take stock of where we are so far. Here are the salient features of the semantics and pragmatic that have posited so far: a) Heim's basic understanding of context as sets of pairs of worlds and assignment functions, b) a particular type of assignment functions that includes absurd objects, c) a variable-based semantics for both indefinites and pronouns, d) a trivalent logic and strong Kleene connectives, e) Stalnaker's updates rule (i.e. pointwise) and principle for presupposition felicity (i.e. a sentence is felicitous if it is defined at every point in the context). With these resources we give a reasonable treatment of presupposition, intersentential anaphora as well as anaphora across conjunctions. These are the easy cases, however: the challenge will be to give adequate treatments of negation, disjunctions, quantifiers, and adverbs of quantification.

(One thing to note is that this semantics does not behave well when the same index is reused by a new quantifier. Different dynamic systems have treated reused indices in different ways. My attitude is that as there is no empirical evidence that indices are ever reused in natural language not choose systems on the basis of how they respond to reused indices.)[16]

# 9 Negation?

Let us start with negation. The most obvious problem is with the negation of indefinites. Recall that we wanted ⌜A man$_x$ walks in⌝ to be true at a point iff $x$ is a man who walks in at that point. This is necessary in order that the sentence does the job of satisfying later presuppositions of variables: the context, once the sentence has been asserted, needs to be one that includes the fact that $x$ picks out a man who walked in. What about the (wide-scope) negation of ⌜A man$_x$ walks in⌝? In an empty context this eliminates any world in which any man walks in.

If we're going to treat truth-value gaps as presupposition-invoking we also need ⌜A man$_x$ walks in⌝ to be true or false everywhere. So, if we stick with a trivalent logic with a strong Kleene negation we are in a bind, since our truth-conditions have in fact forced our hands with our falsity-conditions, and these are not what we want. It is exactly these kinds of considerations which led Heim in her static fragment of chapter 2 to propose an existential closure operation under negation.

We cannot, of course, simply existentially close all variables, since pronouns do not undergo existential closure under negation. One option, which we will take here, is to simply existentially close those variables that are not at risk of causing presupposition failures. There are some subtleties here, but we will use the following definitions which, with some relatively harmless auxiliary assumptions, should prove adequate. We will say a sentence $\phi$ is *definedness-sensitive* to a variable $x$ iff there exists a world $w$ and an assignment function $f$ s.t.$[\![\phi]\!]^{f_{x\to\perp},w} = \#$ and for all $o$, $[\![\phi]\!]^{f_{x\to o},w} \neq \#$.[17] We say that $f'[\phi]f$ iff $f'$ agrees with $f$ on all definedness-sensitive variables.

[16] Here I'm in agreement with an unpublished paper by Charlow [2016]. Note also that we can probably explain why do not generally coindex two indefinite quantifiers by means of a maximize presupposition rule.

[17] A problem is this: ⌜$\exists x(x = x \vee$ John knows 1+1 = 2)⌝.

We can now define our dynamic existential closure operator, † as follows:

$$[\![\dagger\phi]\!]^{f,w} = \begin{cases} 1 \text{ if there is an } f'[\phi]f \text{ s.t. } [\![\phi]\!]^{f',w} = 1 \\ 0 \text{ if for all } f'[\phi]f,\ [\![\phi]\!]^{f',w} = 0 \\ \# \text{ otherwise} \end{cases}$$

Now, if we have a strong Kleene negation we can treat the negated sentences as follows:

$$[\![\neg \dagger \text{ A man}_x \text{ walked in}]\!]^{f,w} = \begin{cases} 1 \text{ iff no man in } w \text{ walked in} \\ 0 \text{ otherwise} \end{cases}$$

The closure or assertion operator † will prove very useful in many places (including with disjunctions and under quantifiers), so we need not think of it as merely required for negation. For now, we will assume it can be freely placed under other operators (at the root level it only eliminates anaphoric potentials so is not useful).

Moreover, we also now can get anaphora across double negations by not using † at all, e.g. $[\![\neg\neg\text{A man}_x \text{ walked in}]\!] = [\![\text{A man}_x \text{ walked in}]\!] \neq [\![\neg\neg \dagger \text{ A man}_x \text{ walked in}]\!]$

What about $[\![\neg \dagger \neg\text{A man}_x \text{ walked in}]\!]$? This has the following truth-conditions:

$$= \begin{cases} 1 \text{ if some man in } w \text{ walked in} \\ 0 \text{ otherwise} \end{cases}$$

It simply lacks anaphoric potential.

What about negation without the †-operator: $\ulcorner\neg$A man$_x$ walked in$\urcorner$? While this does successfully put conditions on $x$, it does not ensure that $x \neq \perp$. In addition, when asserted in a context without any variable information it does not put any worldly conditions on the context. We might hope to eliminate such parses on pragmatic grounds, or postulate syntactic constraints to remove them.

# 10 Disjunction

## 10.1 Partee disjunction

We have already seen that our theory accounts naturally for anaphoric connections across double negations. What about the related question of how the theory accounts for this kind of disjunction example, due to Partee:

(7) There isn't a bathroom here, or it's under the stairs.

We don't naturally get a coherent reading. For on the parse in (8) the entire sentence will presuppose that $x$ is assigned.

(8) ($\neg$ † there is a bathroom$_x$ ) $\vee$ (it$_x$'s under the stairs)

It is easy to check that there is no way to place the † operator to yield the desired reading.

However, if we are allowed to insert logically redundant material (in a classical sense), then we *can* get the desired reading. Note that from the perspective of propositional logic $\ulcorner\phi \vee \psi\urcorner$ is equivalent to $\ulcorner\phi \vee (\neg\phi \wedge \psi)\urcorner$. So from a classical perspective $\ulcorner$($\neg$there is a bathroom$_x$) $\vee$ (it$_x$'s under the stairs)$\urcorner$ is equivalent to $\ulcorner$($\neg$there is a bathroom$_x$ ) $\vee$ (there is a bathroom$_x$ $\wedge$ it$_x$'s under the stairs)$\urcorner$. Now if we just add a † operator we get the correct reading: $\ulcorner$($\neg$ † there is a bathroom$_x$ ) $\vee$ (there is a bathroom$_x$ $\wedge$ it$_x$'s under the stairs)$\urcorner$.

What I am proposing is that we can tweak logical forms not only by adding the †-operator, but also by adding (classically) logically redundant conjunctions. The combination of these two free operations will then give us the desired readings under disjunctions. Of course, such free operations provide a significant divergence between overt syntactic form and that which finally makes up the meaning, but the proposed operations are sufficiently constrained, I believe, to be plausible.[18]

### 10.2 Stone disjunction

Another aspect of the dynamics of disjunction can be handled in my system without modification. Consider disjunctions that can serve as anaphoric antecedents to donkey anaphora:

(9) Either a man will bring a comb or a woman will bring a brush. In either case, ask them to leave it for me.

One natural suggestion is that the pronouns are linked to both antecedents as follows:[19]

(10) Either a man$_x$ will bring a comb$_y$ or a woman$_x$ will bring a brush$_y$. In either case, ask them$_x$ to leave it$_y$ for me.

Stone [1992] posed examples of this general form as a particular problem for dynamic semantics. What we see, though, is that in our semantics with a simple classical semantics for disjunction we have no problem with these examples. For any context in which the first sentence is accepted both $x$ and $y$ will not refer to $\perp$ but rather to either a man or woman or a comb or brush, respectively. Thus, the presuppositions of the pronouns in the second sentence will be satisfied, yielding the desired interpretation. Again, we see that by using a trivalent semantics without stipulative accessibility rules we eliminate problems that plague traditional dynamics semantics.

### 10.3 From disjunction to conditional

Consider this kind of anaphoric connection:

(11) Either it's a holiday or a customer$_x$ will come in. And if it's not a holiday, he$_x$'ll want to be served.

To my knowledge, cases such as (11) have not been discussed in the literature, though they resemble, in some respects, cases of modal subordination. Standard dynamic accounts have no natural resources to account for them, while e-type approaches can easily treat them since the presuppositions of a definite description such as ⌜the customer⌝ is satisfied in the local context of the consequent of the conditional. Likewise the account I am advocating here naturally captures such examples since the special presupposition of the pronoun (that $x$ does not pick out $\perp$) is conditionally satisfied once the context is updated with the first disjunct.[20]

[18] Kamp and Reyle [1993] also consider adding extra material to the second disjunct to get the desired reading, but they do not give an explicit system. I am recycling the basic idea from [Rothschild, 2008a] of facilitating dynamic effects by allowing reconstructions of logical form according to classical equivalence. While I describe such operations as free here, I believe we will need some constraints on them in order not to generate unattested readings. The viability of this proposal will ultimately depend on the nature of these constraints.

[19] Schlenker [2011] gives evidence from sign language that this is the logical form of anaphora with disjunctive antecedents.

[20] I make the simplifying assumption here that the conditional in the second sentence is just a material conditional with strong Kleene semantics.

## 11 Quantifiers

With respect to generalized quantifiers such as ⌜every⌝ we will assume a classical behavior. The syntactic/semantic assumptions of our quantifiers are relatively simple: a quantifier $Q_x$ takes two arguments of sentential type, which are assumed to contain the free variable $x$. $Q_x$ then expresses a conservative relationship between the objects satisfying the two arguments (i.e. the objects that when the assignment function assigns $x$ to those objects makes the arguments true), as in standard generalized quantifier theory [Barwise and Cooper, 1981].

The critical problem we face is how to understand anaphoric relationships between the restrictor and matrix of quantifiers. Consider donkey sentences:

(12) Every (man who owns a donkey, beats it)

Given our reformation rules across logical equivalence, these sentences present no problem. For conservativity ensures that conjoining the restrictor to the matrix in the standard fragment makes no difference to truth conditions. So we switch the logical form of (12) to (13).

(13) Every († man who owns a donkey, † (man who owns a donkey and beats it))

This gets us what is called the 'weak' reading of donkey anaphora, namely that every man who owns a donkey beats at least one donkey he owns. We will need to avail ourselves of one of the many strategies in the literature for also obtaining the other reading, but I leave that for another occasion.[21]

It is notable that the same technique, using classically equivalent logical forms to capture anaphoric relations, works for both Partee-disjunction and classic donkey anaphora under quantifiers.

## 12 Dynamic adverbs of quantification

As it happens I think the correct treatment of adverbs of quantification requires situational quantifiers, for roughly the reasons discussed in von Fintel [1994]. However, if we want to define a Lewisian adverb of quantifier that behaves appropriate for examples like these there is no technical obstacle. I give a definition in the appendix which follows the usual dynamic definitions of adverbs of quantifiers as Lewisian [1975] unselective quantifiers.

## 13 Summary and comparative remarks

Our proposed semantics took a number of important ideas from the literature on dynamic semantics, particularly Heim's dissertation.

- Contexts have a Heimian file structure: they are sets of assignment function world/pairs.
- Pronouns and indefinites put conditions on variables.
- There is kind of default existential quantification at the sentence level and under operators such as negation.

[21] In my view the weak reading is the right one to get as it is always attested whereas some sentences with donkey anaphora have no ambiguity:

(i) No man who owns a donkey beats it.

It is worth comparing this to a dynamic semantics that also covers anaphora and presupposition. As I noted earlier Beaver's [2001] ABLE is the obvious comparison point as it covers presupposition and anaphora (as well as epistemic modals) and builds on much other work in dynamic semantics from Heim [1982], Kamp [1981] onwards. There are a number of significant differences between my system and Beaver's (and other dynamic systems):

My system has semantic values that are functions from assignment world pairs to truth values. Beaver's are functions from sets of assignment worlds pairs to sets of assignment worlds pairs (or relations in type, but he only uses functional relations in his fragment). Beaver's definitions of connectives and quantifiers all make reference explicitly to order effects of accessibility relations. My quantifiers and connectives are simply those from a strong Kleene logic.

On the other hand, my system allows free insertion of †-operators, giving existential closures as well restricted additions of conjunctives where they do not (classically) affect the truth-conditions.

My system, in terms of type, is close to that of Dekker [1994, 2012] who also assigns truth-conditions as semantic values rather than CCPs. However, unlike Dekker, I provide a treatment of presupposition and have a more classical treatment of quantification and connectives. In my rigid use of standard (trivalent) quantifier definitions this proposal is in the spirit of Schlenker's work on presupposition.

# A Syntax

The sets $V$ of variables: $x, y, z \dots$
Relational predicates, $P, R, Q \dots$
Where $P$ is a relational predicate and $\gamma_1 \dots \gamma_n$ are variables, $\phi$ and $\psi$ are arbitrary wff, we form wff as follows:
$P(\gamma_1 \dots \gamma_n) | \text{some }_{\gamma_1}(\phi, \psi) | \text{every}_{\gamma_1}(\phi, \psi) | \phi \wedge \psi | \phi \vee \psi | \neg\phi | \text{always}(\phi, \phi) | \dagger \phi$

# B Semantics

Let $D$ be the domain of objects and $W$ be a set of worlds. Let an assignment function be a function from $V$ to $D \cup \perp$, where $\perp$ is a special object not in the domain. An interpretation $I$ is a mapping from relational predicates and worlds to $n$-tuples of $D$. The denotation function $[\![\cdot]\!]$ is a function from a wff, an interpretation, an assignment function and world to the set $\{0, 1, \#\}$. (We generally do not refer to the interpretation function, but just refer to predicates holding in worlds as usual.)

We let $f[\phi]f'$ iff $f$ agrees with $f'$ on all variables $x$ such that there exists a world $w$ and an assignment function $g$ s.t. $[\![\phi]\!]^{g_{x\to\perp}, w} = \#$ and for all $o$, $[\![\phi]\!]^{g_{x\to o}, w} \neq \#$

$$[\![P(\gamma_1, \dots \gamma_n)]\!]^{f,w} = \begin{cases} \# \text{ if any of } f(\gamma_1) \dots f(\gamma_n) = \perp \\ 1 \text{ iff } \langle f(\gamma_1), \dots f(\gamma_n)\rangle \text{ is in the extension of } \phi \text{ at } w \\ 0 \text{ otherwise} \end{cases}$$

$$[\![\phi \wedge \psi]\!]^{f,w} = \begin{cases} 1 \text{ if } [\![\phi]\!]^{f,w} = 1 \text{ and } [\![\psi]\!]^{f,w} = 1 \\ 0 \text{ if } [\![\phi]\!]^{f,w} = 0 \text{ or } [\![\psi]\!]^{f,w} = 0 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\phi \vee \psi]\!]^{f,w} = \begin{cases} 1 \text{ if } [\![\phi]\!]^{f,w} = 1 \text{ or } [\![\psi]\!]^{f,w} = 1 \\ 0 \text{ if } [\![\phi]\!]^{f,w} = 0 \text{ and } [\![\psi]\!]^{f,w} = 0 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\neg\phi]\!]^{f,w} = \begin{cases} 1 \text{ if } [\![\phi]\!]^{f,w} = 0 \\ 0 \text{ if } [\![\phi]\!]^{f,w} = 1 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\text{some}_x(\phi, \psi)]\!]^{f,w} = \begin{cases} 1 \text{ if } [\![\phi]\!]^{f,w} = 1 \text{ and } [\![\psi]\!]^{f,w} = 1 \\ 0 \text{ if } f(x) = \perp \text{ or } [\![\phi]\!]^{f,w} = 0 \text{ or } [\![\psi]\!]^{f,w} = 0 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\text{every}_x(\phi, \psi)]\!]^{f,w} = \begin{cases} 1 \text{ if } \forall o \in D : [\![\phi]\!]^{f_{x\to o},w} = 1 \text{ and } [\![\psi]\!]^{f_{x\to o},w} = 1 \\ 0 \text{ if } \exists o \in D : [\![\phi]\!]^{f_{x\to o},w} = 1 \text{ and } [\![\psi]\!]^{f_{x\to o},w} = 0 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\text{always}(\phi, \psi)]\!]^{f,w} = \begin{cases} 1 \text{ if } \forall f'[\phi]f \text{ such that } [\![\phi]\!]^{f',w} = 1, \exists f''[\psi]f' \text{ such that } [\![\psi]\!]^{f'',w} = 1 \\ 0 \text{ if } \exists f'[\phi]f \ [\![\phi]\!]^{f',w} = 1 \text{ and } \forall f''[\psi]f' \ [\![\psi]\!]^{f'',w} = 0 \\ \# \text{ otherwise} \end{cases}$$

$$[\![\dagger\phi]\!]^{f,w} = \begin{cases} 1 \text{ if } \exists f'[\phi]f \text{ such that } [\![\phi]\!]^{f',w} = 1 \\ 0 \text{ if } \forall f'[\phi]f \ [\![\phi]\!]^{f',w} = 0 \\ \# \text{ otherwise} \end{cases}$$

# C Transformation mechanisms

In moving from expressed logical forms to the form interpreted we allow the following two alterations:

**†-insertion:** Replace any instance of a wff $\phi$ inside a wff with $\dagger\phi$

**Adding redundant conjunctions:** if a wff contains the wffs $\phi$ and $\psi$ replace any instance of $\psi$ with $\phi \wedge \psi$ if the replacement is a classically equivalent.( Definition of classical equivalence: formulas $\alpha$ and $\beta$ are classically equivalent if when all † operators are removed for every interpretation $I$ and assignment function $f$ $[\![\phi]\!]^{f,w} = 1$ iff $[\![\psi]\!]^{f,w} = 1$.)

# References

John Barwise and Robin Cooper. Generalized quantifiers and natural language. *Linguistics and Philosophy*, pages 159–219, 1981.

David Beaver. *Presupposition and Assertion in Dynamic Semantics*. CSLI, 2001.

Simon Charlow. Where is the destructive update problem? unpublished manuscript, Rutgers, 2016.

Robin Cooper. The interpretation of pronouns. In Frank Heny and Helmut S. Schnelle, editors, *Syntax and Semantics*, volume 10, pages 61–92. Academic Press, 1979.

Paul Dekker. Predicate logic with anaphora (seven inch version). In Lynn Santelmann and Mandy Harvey, editors, *Proceedings of SALT IV*, pages 79–95. Ohio State University, 1994.

Paul Dekker. *Dynamic Semantics*. Springer, 2012.

Paul Elbourne. *Situations and Individuals*. MIT Press, 2005.

Gareth Evans. Pronouns quantifiers and relative clauses (i). *Canadian Journal Of Philosophy*, 7(3):467–536, 1977.

Danny Fox. Two short notes on Schlenker's theory of presupposition projection. *Theoretical Linguistics*, 34:237–252, 2008.

Danny Fox. Presupposition projection from quantificational sentences: trivalence, local accommodation, and presupposition strengthening. manuscript, HUJI and MIT, 2012.

Benjamin George. Predicting presupposition projection: some alternatives in the strong Kleene tradition. manuscript, UCLA, 2007.

J. Groenendijk and M. Stokhof. Dynamic predicate logic. *Linguistics and Philosophy*, 14: 39–100, 1991.

Jeroen Groenendijk, Martin Stokhof, and Frank Veltman. Corefrence and modality. In Shalom Lappin, editor, *Handbook of Contemporary Semantic Theory*. Blackwell, 1996.

Irene Heim. *The Semantics of Definite and Indefinite Noun Phrases*. PhD thesis, University of Massachusetts, Amherst, 1982.

Irene Heim. E-type pronouns and donkey anaphora. *Linguistics and Philosophy*, 13:137–177, 1990.

Irene Heim. Artikel und definitheit. In A v. Stechow and D. Wunderlich, editors, *Semantics: An International Handbook of Contemporary Research*. de Gruyter, 1991.

Hans Kamp. A theory of truth and semantic representation. In J. Groenendijk, T.M.V. Janssen, and M.B.J. Stokhof, editors, *Formal Methods in the Study of Language*, pages 277–322. Mathematish Centrum, 1981.

Hans Kamp and Uwe Reyle. *From Discourse to Logic*. Kluwer, 1993.

Emiel Krahmer. *Presupposition and Anaphora*. CSLI, 1998.

David Lewis. Adverbs of quantification. In Edward L. Keenan, editor, *Formal Semantics of Natural Language*. Cambridge University Press, 1975.

David Lewis. Index, context, and content. In S. Kranger and S. Ohman, editors, *Philosophy and Grammar*, pages 79–100. Reidel, 1980.

David Lewis. Scorekeeping in a language game. In *Philosophical Papers, Vol.I.* Oxford University Press, 1983.

Stanley Peters. A truth-conditional formulation of Karttunen's account of presupposition. *Synthese*, 40:301–316, 1979.

Daniel Rothschild. Making dynamics semantics explanatory. manuscript, Columbia University, 2008a.

Daniel Rothschild. Presupposition projection and logical equivalence. *Philosophical Perspectives*, 22(1):473–497, 2008b.

Daniel Rothschild and Seth Yalcin. On the dynamics of conversation. *Noûs*, 2015.

Daniel Rothschild and Seth Yalcin. Three notions of dynamicness in language. *Linguistics and Philosophy*, 39(4):333–355, 2016.

Philippe Schlenker. Be articulate: A pragmatic theory of presupposition projection. *Theoretical Linguistics*, 34(3):157–212, 2008.

Philippe Schlenker. Local contexts. *Semantics and Pragmatics*, 2(3):1–78, 2009.

Philippe Schlenker. Donkey anaphora: the view from sign language (ASL and LSF). *Linguistics and Philosophy*, 34(4):341–395, 2011.

Scott Soames. Presuppositions. In D. Gabbay and F. Guenther, editors, *Handbook of Philosophical Logic*, volume IV, pages 553–616. Dordrecht, 1989.

Robert Stalnaker. Pragmatics. *Synthese*, 22:272–289, 1970.

Robert Stalnaker. On the representation of context. *Journal of Logic, Language and Information*, 7(1):3–19, 1998.

Matthew Stone. *Or* and anaphora. In Chris Barker and David Dowty, editors, *SALT 2*, page ß, 1992.

Rob van der Sandt. Presuppostion projection as anaphora resolution. *Journal of Semantics*, 9 (4):333–377, 1992.

Kai von Fintel. *Restrictions on Quantifier Domains*. PhD thesis, University of Massachusetts, Amherst, 1994.