MARRIAGE, LABOR SUPPLY, AND HOME PRODUCTION

MARION GOUSSÉ
Department of Economics, Laval University

NICOLAS JACQUEMET
Paris School of Economics, Université Paris 1 Panthéon-Sorbonne, and Centre d'Économie de la Sorbonne

JEAN-MARC ROBIN
Sciences Po, Department of Economics (CNRS UMR8259) and Department of Economics, University College London

We develop a search model of marriage where men and women draw utility from private consumption and leisure, and from a non-market good that is produced in the home using time resources. We condition individual decisions on wages, education, and an index of family attitudes. A match-specific, stochastic bliss shock induces variation in matching given wages, education, and family values, and triggers renegotiation and divorce. Using BHPS (1991–2008) data, we take as given changes in wages, education, and family values by gender, and study their impact on marriage decisions and intra-household resource allocation. The model allows to evaluate how much of the observed gender differences in labor supply results from wages, education, and family attitudes. We find that family attitudes are a strong determinant of comparative advantages in home production of men and women, whereas education complementarities induce assortative mating through preferences.

KEYWORDS: Search-matching, bargaining, assortative mating, collective models, time uses, social norms, gender identity, structural estimation.

1. INTRODUCTION

SINCE WORLD WAR II, the labor force participation of married women has increased dramatically. Many explanations have been proposed: technological change in the household (Greenwood et al. (2016), is a recent contribution), contraception (Goldin and Katz (2002)), changes in wage distributions by gender and experience (e.g., Knowles (2013)), cultural change (Fernández (2013)), structural change in the economy (Galor and Weil (1996)), child care (Attanasio, Low, and Sanchez-Marcos (2008)), divorce laws (Fernández and Wong (2014)). By 1990, the labor supply of married women has reached a plateau, and yet strong differences in time uses persist between men and women, and

between single and married persons. Using British Household Panel Survey (BHPS) data, we thus observe that between 1991 and 2008, married women increase market work and reduce non-market work, but at a very low pace in comparison to the preceding decades (see Section 3 for details). However, during the same period, important changes occur simultaneously in wages: the marriage gap increases for both genders, and more so for men than women; the gender wage gap is slowly closing. Considerable changes in the education of married women are also happening (70% of high school dropouts in 1991, 45% in 2004), and social attitudes are slowly but steadily trending away from the traditional model of the family.

Hence, standard macroeconomic methods focusing on aggregate change are becoming less useful in the post-1990 period. Structural models need to give more room to individual heterogeneity in preferences and home production. In this paper, we shall assume that individuals differ from each other by their wage, their education, and by social attitudes measured by questions like "Do you approve of a married woman earning money in business or industry if she has a husband capable of supporting her?" (Fernández (2013)) or "If a woman earns more money than her husband, it's almost certain to cause problems" (Bertrand, Kamenica, and Pan (2015)).

We build a model of marriage, and time and income sharing where the only marriage externality is a public good that is produced in the household. Home production requires time inputs from both spouses, and depends on spouses' gender, education, and an index of family values, constructed by aggregating a set of questions on social attitudes collected in the BHPS. The home production function exhibits two types of complementarities: first between time uses, second between exogenous individual characteristics (education and family attitudes). The level of home production also depends on a bliss variable that is drawn when a couple meets for the first time, and is infrequently updated after marriage so as to endogenously generate divorce. Men and women share resources by Nash bargaining. Finally, forward-looking singles need to forecast their chances of dating other singles of any type. We thus assume that, in a given year, divorce flows approximately equal marriage flows. This steady-state assumption allows to solve for equilibrium distributions of individual types by gender and marital status. In the empirical application, we further assume that the economy jumps from one 3-year steady state to the next, following exogenous changes in the distributions of wages, education, and family values indices by gender.

As did Bertrand, Kamenica, and Pan (2015), we observe that marriage is much less probable if the woman is paid by the hour more than the man, and when such a marriage occurs, she works fewer hours. However, this pattern is much more pronounced for conservative households than for progressive ones (depending on spouses' family values indices). The distribution of female-to-male earnings ratios is very asymmetric with a mode at 0.1 for conservative couples, whereas it is nearly symmetric for liberal couples with a much higher mode around 0.4.

The model fits these empirical regularities very well and reveals how the comparative advantages of men and women in home production depend on family attitudes. We then ask the following question: what would happen if we were making all individuals culturally liberal? We find that the distribution of labor earnings ratios would become nearly symmetric around a mode that is close to the mode of wage ratios. The labor supply of married women would still be lower than men's by about 10%, a difference that can easily be explained by the gender wage gap (assuming that changing family attitudes would not affect the wage gap).

We also estimate how marriage probabilities depend on spouses' education, family values, and wages. We find strong evidence of education homophily (love of same). However,

for family values, homophily is not uniform: conservative individuals prefer a conservative partner, but progressive individuals are not picky. The model traces the origin of this latter pattern to different comparative advantages in home production of men and women with different family values, whereas for education it is just a matter of tastes. Finally, there is very little homophily based on wages. Women are just more likely to marry richer men. Marriage probabilities barely depend on female wages. Again, this is a direct consequence of the specific structure of the home production function.

We finally quantify the share of income that is privately consumed by male and female spouses (the sharing rule). In the model, the sharing rule is the sum of the bargaining coefficient and a component that reflects the outside options of each spouse. We find that men capture 55% of household total expenditure on private consumption and leisure, which decomposes into 45% for the bargaining coefficient and 10% due to a market advantage of men in the marriage market. The sharing rule is unaffected by family values and exhibits a compensating differential pattern with respect to education, with redistribution operating toward the least educated spouse. On the other hand, we estimate that the richer spouse absorbs a greater share of the rent.

The layout of the paper is as follows. After a brief review of the literature on household behavior, we describe salient features of intrahousehold allocation of time and spouses matching between 1991 and 2008. Then we construct the search-matching-and-bargaining model and we solve the equilibrium for a special specification of preferences. The next section describes the empirical specification, studies identification, and develops the estimation procedure. We then show the results. The Appendix deals with technical details.

2. A BRIEF REVIEW OF THE LITERATURE ON MARRIAGE AND INTRAHOUSEHOLD RESOURCE ALLOCATION

Lundberg and Pollak (1996) ended their insightful survey on bargaining and distribution in marriage by stating that "bargaining models provide an opportunity for integrating the analysis of distribution within marriage with a matching or search model of the marriage market." Since then, the search-matching-and-bargaining framework has been widely used in applied macroeconomics in the perspective of understanding long-term changes such as declining marriage rates or rising female labor supply. Aiyagari, Greenwood, and Guner (2000), Greenwood, Guner, and Knowles (2000), Chiappori (2003), Caucutt, Guner, and Knowles (2002), Gould and Paserman (2003), Fernandez, Guner, and Knowles (2005) are early examples of applications (see the first paragraph for more recent references).

The search-matching-and-bargaining project took time to take off in applied microeconomics because Chiappori (1988, 1992), Apps and Rees (1988) introduced the collective framework in the late 1980s, which has remained dominant for a long time.[1] Their point is that in order to model individual consumption and labor supply in situations where household resources are shared cooperatively, Nash bargaining is an unnecessary restriction. Pareto optimality is sufficient to derive testable restrictions. However, as emphasized by Lundberg and Pollak, policy interventions affecting the distribution of resources within the family can have very different short-run and long-run effects because of marriage market equilibrium feedback. Collective models very successfully describe resource sharing

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[1]Other important developments can be found in Browning et al. (1994), Browning and Chiappori (1998), Chiappori, Fortin, and Lacroix (2002), Blundell et al. (2007). A dynamic version of the collective model was explored by Mazzocco (2004), Chiappori (2007).

within the family given a sharing rule. A matching model—that is, an equilibrium model of the marriage market—is required in order to endogenize the distribution of powers in the family (Chiappori (2012)). For example, if education and wage differentials by gender change over time, this will have an effect on couples' labor supply for a given way of sharing resources, but this may also affect individuals' outside options and the sharing rule itself.

In the standard search model, singles meet mates sequentially; they share resources according to Nash bargaining with singlehood as threat point; marriage is realized if there exists a jointly profitable resource allocation; the distributions of individual types by gender and marital status are calculated at the steady-state equilibrium. Shimer and Smith (2000), Atakan (2006), Lauermann and Nöldeke (2015) and Manea (2017) provided theoretical studies of equilibrium existence.

Search-matching is not the only way of modeling a marriage market. Long after the seminal work of Becker (1973, 1974, 1981), and Grossbard-Shechtman's (1984) introduction of household production, Choo and Siow (2006) revived the perfect-information assignment framework of Shapley and Shubik (1971), and developed an econometric model of matching between heterogeneous men and women that is the stable-matching equivalent of Wong's (2003) empirical application of Shimer and Smith (2000).[2] A series of applications of this framework followed, including Chiappori and Oreffice (2008), Chiappori, Iyigun, and Weiss (2009), Chiappori, Salanié, and Weiss (2016). Choo, Seitz, and Siow (2008) is a first attempt at introducing labor supply in the Choo and Siow model, and Chiappori, Costa Dias, and Meghir (2015) is a big step forward, introducing not only labor supply and endogenous education (like Greenwood et al. (2016)), but also savings and unobserved heterogeneity.

What is the best framework for a marriage market? There is still little literature on this subject. Adachi (2003) showed that, as search costs disappear, the set of equilibrium outcomes of a search-matching model with non-transferable utility converges to the set of stable equilibria of a Gale–Shapley marriage model. Recently, Lauermann and Nöldeke (2015) toned this prediction down by proving that it holds if and only if there is a unique stable matching in the underlying marriage market. With multiple stable matchings, sequences of equilibrium matchings converging to unstable, inefficient matchings can be constructed. We do not know of any result for transferable utility models. Still, the search framework can be thought of as a tâtonnement mechanism, whose equilibrium outcomes are hopefully not too far from the stable matchings of the corresponding frictionless economy. Tâtonnement models are empirically appealing because they are usually easier to simulate, absent of coordination issues and strategic behavior, and they naturally generate the sluggishness that seems to be a general characteristic of many markets.

3. DATA AND FACTS

BHPS Data

We use the original British Household Panel Survey (BHPS) sample of 5,050 British households and 9,092 adults interviewed in the first wave (1991). The panel interviews all adult members of all households comprising either an original sample member or an individual born to an original sample member every year until 2008. It therefore remains

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[2]Useful connections between the Choo–Siow model and optimal transportation theory were made by Galichon and Salanié (2012) and Dupuy and Galichon (2014).

broadly representative of the British population (excluding Northern Ireland and North of the Caledonian Canal) as it changes over time. We only keep individuals who are either single or married to (or cohabiting with) a heterosexual partner, and who are between 22 and 50 years of age at the time of interview. To reduce non response biases, we use the Individual Respondent Weights provided in the survey.

We keep data on usual gross pay per month for the current job, the number of hours normally worked per week (including paid and unpaid overtime hours), and the number of hours spent in a week doing housework (core non-market work excluding child caring and rearing, information not provided by the survey). Hourly wage is the usual gross pay per month divided by the number of hours normally worked per month (without overtime). Wages are deflated by the Consumer Price Index and computed in 2008 pounds.

In order to reduce the number of labor supply corners (zero market hours and missing wages), we replace current observations on wages and market hours by a moving average of past, present, and future observations. Specifically, suppose that we observe wages $w_1, w_2, \ldots$ and hours $h_1, h_2, \ldots$. We replace w_t and h_t by

$$\widehat{w}_t = \frac{\sum\limits_{\tau=-\infty}^{+\infty} w_{t+\tau} \mathbf{1}\{h_{t+\tau} \neq 0\} \phi(\tau/k)}{\sum\limits_{\tau=-\infty}^{+\infty} \mathbf{1}\{h_{t+\tau} \neq 0\} \phi(\tau/k)}, \qquad \widehat{h}_t = \frac{\sum\limits_{\tau=-\infty}^{+\infty} h_{t+\tau} \phi(\tau/k)}{\sum\limits_{\tau=-\infty}^{+\infty} \phi(\tau/k)},$$

where ϕ is the standard normal PDF and k is a smoothing parameter that we arbitrarily choose equal to 2, yielding weights 1, 0.882, 0.607, 0.325, 0.135, 0.044, 0.011 for 0, 1, 2, 3, 4, 5, 6 years apart. Then we trim the 1% top and bottom tails of wage and time use variables. We thus obtain an unbalanced panel of 18 years (1991–2008), whose cross-sectional sizes remain roughly constant over the years with about 1000–1200 couples and 400 singles of each sex, the fraction of married men and women being remarkably stable at around 70%.

Trends

The rather short period of time between 1991 and 2008 has produced some remarkable changes in time uses, wages, and education by gender and marital status.[3] Figure 1 confirms well-known facts about market and non-market work. Men work more paid hours than women, married men work more than single men, and all men, married and single, devote the same amount of time (little) to home production. Married and low-educated women work fewer hours outside the home, and more inside than single and higher-educated women. Education is not a key determinant for men; it is for women. Male hours are remarkably stable over time, while female differences by education and marital status are gradually subsiding.

Interesting composition changes are observable in that period (Figure 2). All groups are converging to the same educational norm. In 1991, about 45% of single men did not have A-levels, while it was 60% for married men and single women, and 70% for married women. By 2004, all groups had reduced the gap to 45%. Wages do not display the same convergence pattern. Married people's wages increase both in absolute terms and with respect to the wages of singles. The gender wage gap is smaller for singles than for married individuals and slowly closes over time for both married and single individuals. Yet, in

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[3]See Ramos (2005) and Gimenez-Nadal and Sevilla (2012) for a more detailed description.

(a) Men, market hours per week

(b) Men, non-market hours/week

(c) Women, market hours

(d) Women, non-market hours/week

Single < HS	Married < HS
Single HS	Married HS
Single > HS	Married >HS
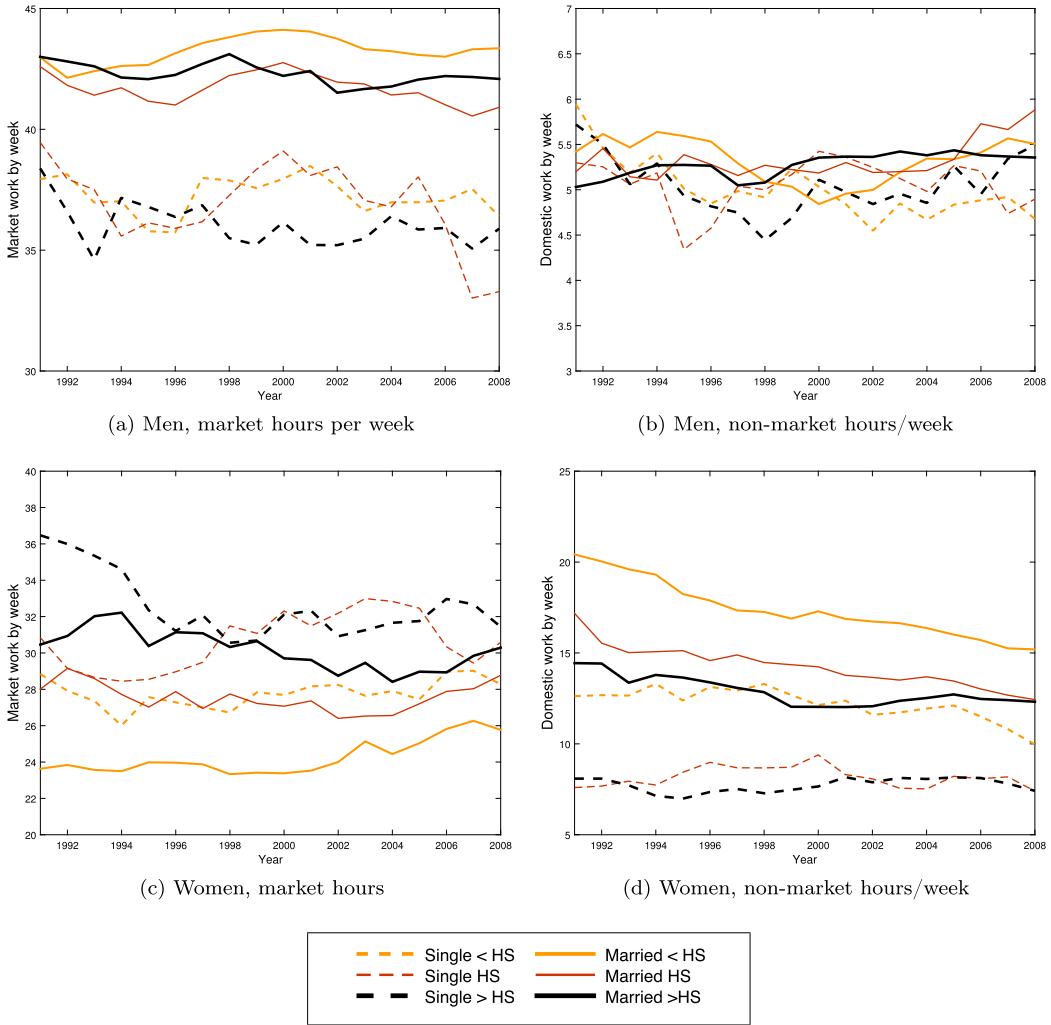
FIGURE 1.—Time use trends.

2008, married men still earn 25% more than married women, and single men about 10% more than single women.

Family Values

In a recent paper, Bertrand, Kamenica, and Pan (2015) observed that "among married couples in the US, the distribution of the share of household income earned by the wife drops sharply at $\frac{1}{2}$." Figure 3(a) shows the distribution of the female wage share ($\frac{w_f}{w_m+w_f}$) in the BHPS data. It is symmetric with a mode between 0.4 and 0.5. The distribution moves a little to the right over time, becoming a bit more equal (i.e., symmetric around 0.5). The distribution of the female share of labor earnings ($\frac{w_f h_f}{w_m h_m+w_f h_f}$) is, however, similar to Bertrand et al.'s U.S. estimate (interestingly, more so in 1991–1993 than later in 2006–2008). Figure 3(b) resembles their Figures I and II insofar as the density puts more mass

(a) Share with O-level or less

(b) Share of higher educated

(c) Mean wages (£)

(d) Female/male wage ratio
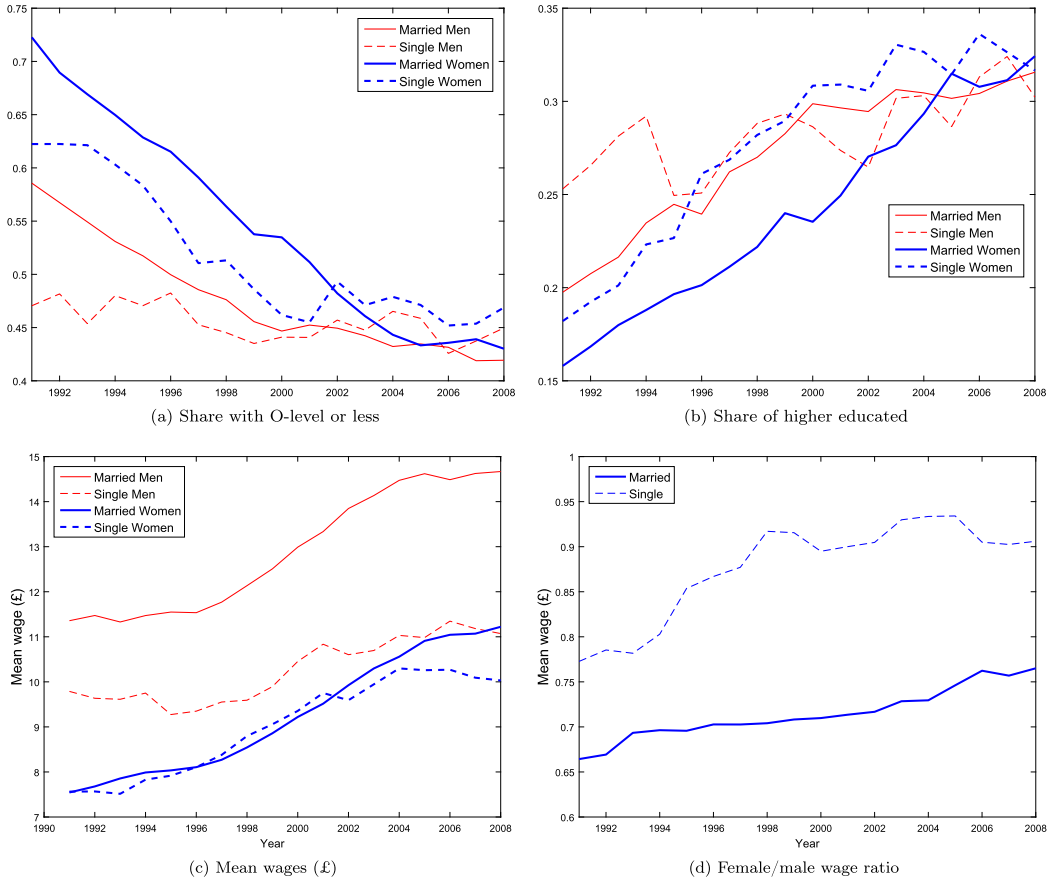
FIGURE 2.—Composition changes in education and wages.

(a) Wage ratio, $\frac{w_f}{w_f + w_m}$

(b) Labor earnings ratio, $\frac{w_f h_f}{w_f h_f + w_m h_m}$

FIGURE 3.—Distribution density of wages and earnings ratios.

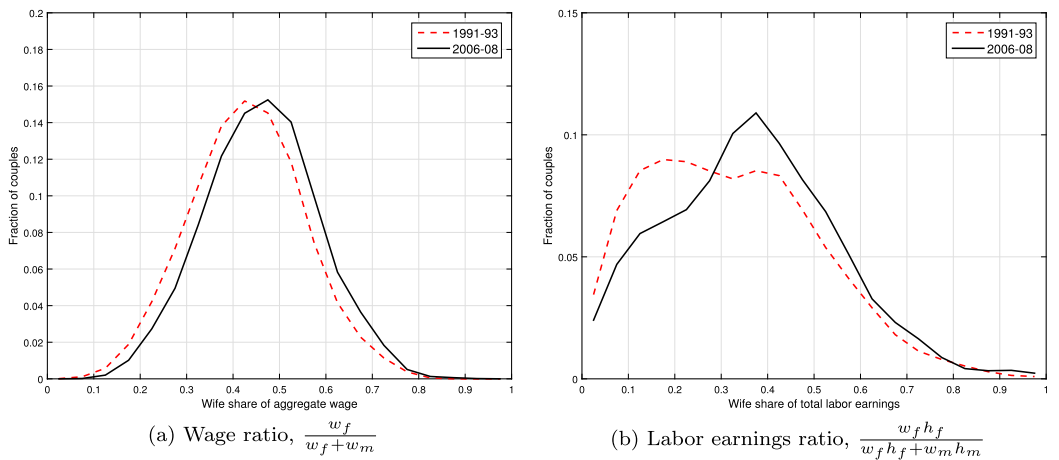to the left of the mode than to its right, and is steeply sloping down beyond 0.4. Females work fewer market hours when they are paid more by the hour than their husband.

So, the gender education gap and, to a lesser extent, the gender wage gap are closing over time, but the division of labor inside the family is adjusting much less rapidly. Bertrand et al. reckoned that this is because "slow-moving identity norms are an important factor that limits further convergence in labor market outcomes." With this interpretation in mind, in addition to wages and education, we looked for independent information on social attitudes influencing time uses. Using the responses to various survey questions about children, marriage, cohabitation, and divorces, we constructed an index of family values by Principal Component Analysis. Table I lists the questions used for this construction and displays the corresponding factor loadings. Given the signs of factor

TABLE I

FAMILY VALUES INDEX

Question	Loading
Pre-school child suffers if mother works	−0.24
Family suffers if mother works full-time	−0.25
Woman and family happier if she works	0.16
Husband and wife should both contribute	0.14
Full-time job makes woman independent	0.12
Husband should earn, wife stay at home	−0.21
Children need father as much as mother	−0.05
Employers should help with childcare	0.12
Single parents are as good as couples	0.17
Adult children should care for parents	−0.07
Divorce better than unhappy marriage	0.12
Attendance at religious services	−0.07
Cohabiting is always wrong	−0.16
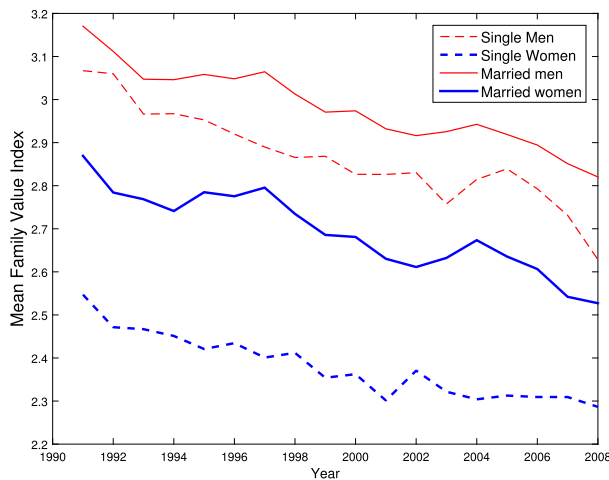
a*Notes*: The Family Values Index is a weighted sum of all responses (1: strongly agree; 2: agree; 3: neither agree nor disagree; 4: disagree; 5: strongly disagree). The variable "Attendance at religious service" is coded: 1: Once a week or +; 2: At least once per month; 3: At least once per year; 4: Practically never; 5: Only weddings, etc. After 1998, the item "Cohabiting is always wrong" became "Cohabiting is alright". So we changed it as "6 − item_answer" to maintain coherency over the years. Weights are estimated by Principal Component Analysis and are displayed in the table. The figure shows the evolution of the mean *FVI* over time by marital status and gender.

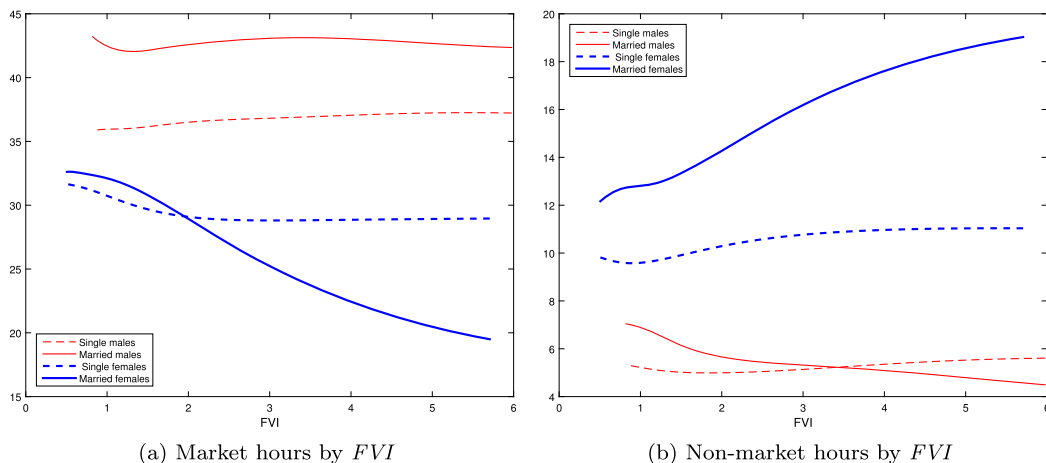(a) Market hours by *FVI*   (b) Non-market hours by *FVI*

FIGURE 4.—Time uses by family values.

loadings, our Family Values Index (*FVI*) is a measure of social conservatism. The Family Values Index is essentially independent of education and wages. More educated and rich individuals are slightly more liberal, but the regression of *FVI* on education and wage by gender has a very low *R*-squared (2.2% and 0.5% for women and men, respectively).

One might worry that the responses to the BHPS questions could be just another way of measuring time uses. However, it is likely that by the time men and women have reached the age of looking for a partner and of choosing an organization of the household, childhood and adolescence have imprinted representations in their minds that these simple survey questions allow us to measure. We want to know how people match on social attitudes, and how different attitudes associate with different time uses.

Men are found to be more conservative than women, and couples are more conservative than singles (see the figure next to Table I).[4] There is a common, steady negative trend, but it is not extremely pronounced (less than half a point in 18 years on a 1-to-5 scale). Figure 4 shows how family values determine the market and non-market hours of married men and women. The effect is stronger for married women's labor supply and for married men and women's work in household. For singles, there is no sizable effect.

The distributions of female wage and earnings ratios vary with family values. In Figure 5, we display the kernel densities of wage and earnings ratios conditional on male and female *FVI*s being above or below their respective medians. All wage ratios are symmetric, but couples with both spouses conservative have a lower and more dispersed wage ratios. Their distribution of earnings ratios is also much more concentrated to the left. At the other extreme, couples with both spouses liberal have a perfectly symmetric distribution of wage ratios, more concentrated around a mode that is closer to 0.5, and their distribution of earnings ratios is the most symmetric of all.

4. MODEL

In this section, we develop a model of marriage and intrahousehold decisions, building on the equilibrium search-bargaining model of Shimer and Smith (2000), which we enrich with labor supply and home production decisions.

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[4]See Fortin (2005) for a study of gender attitudes and labor market outcomes across OECD countries.

(1) Wage ratio, $\frac{w_f}{w_f + w_m}$

(2) Labor earnings ratio, $\frac{w_f h_f}{w_f h_f + w_m h_m}$

(a) Conservative wives and husbands

(b) Liberal wives and husbands
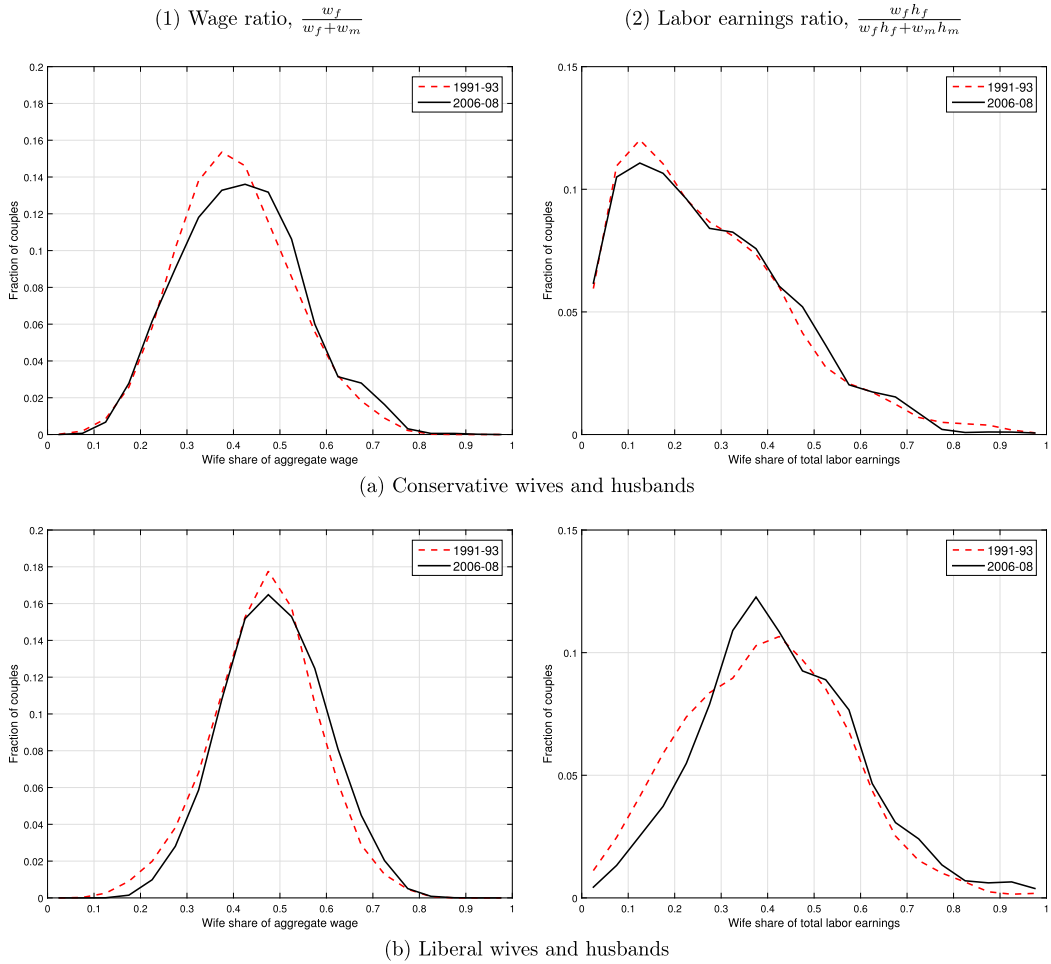
FIGURE 5.—Distribution densities of wage and earnings ratios by family values.

4.1. *The Marriage Market*

Men and women are characterized by their market wage (labor productivity) and other individual characteristics such as education and family values that we call a *type*. Conventionally, we use the label i for male types and j for female types. Let $\ell_m(i)$ and $\ell_f(j)$ denote the density functions of male and female types in the whole population, with $L_m = \int \ell_m(i)\,di$ and $L_f = \int \ell_f(j)\,dj$ denoting the total numbers of men and women in the population. Let $n_m(i), n_f(j), N_m, N_f$ be the corresponding notations for the subpopulations of single men and women. Let $m(i,j)$ and $M = \iint m(i,j)\,di\,dj$ denote the density function of couples' types and the total number of couples. Measures ℓ_m, ℓ_f are exogenous and measures n_m, n_f, m are endogenous.

Assume that only singles search for a partner, ruling out search for an alternative spouse during marriage (voluntary and involuntary). Let λ be the number of meetings between singles per unit of time divided by $N_m N_f$ (the number of potential meetings). The rates at which male and female singles meet a potential partner are equal to λN_f and λN_m. This meeting parameter λ is endogenous and is a function of N_m and N_f. With quadratic

returns, as in Shimer and Smith (2000), λ is constant; but a more reasonable assumption may be constant returns to scale in the meeting function $\lambda N_m N_f$, for example Cobb–Douglas.

When a female single of type j and a male single of type i meet for the first time, a match-specific bliss shock z is drawn from a distribution G. The willingness to marry (say $WtM(i, j, z) = 1$ if yes, $= 0$ if no) depends on all three variables (i, j, z). Denote $\alpha_{ij} = \Pr\{WtM(i, j, z) = 1 | i, j\}$ the probability of marriage given i, j upon meeting.

We also assume that the match-specific bliss shock z is subject to infrequent updates. For simplicity, we assume that in every period, with probability δ, a new value z' is drawn from G independently of the current value z. By changing δ, we can make the stochastic process of shocks (indexed by calendar time) more or less persistent. Divorce occurs if $WtM(i, j, z') = 0$, and $1 - \alpha_{ij}$ is the probability of divorce given a bliss shock occurrence. Matches with a higher probability of marriage thus also have a lower probability of divorce. Conversely, a marriage resulting from an exceptionally large realization of z (love at first sight) should break faster than a marriage based on solid fundamentals.

4.2. *Preferences and Home Production*

The home production of a household public good q is the channel that can make marriage (or cohabitation) preferable to remaining single. Singles have access to a household production technology that requires domestic time as single input: $q = F_i^0(d)$. For married couples, home production depends on the hours spent on domestic chores by both spouses (d_m, d_f), and also on spouses' types (i, j) and a bliss shock $z : q = z F_{ij}^1(d_m, d_f)$.[5] Two types of complementarities will matter: first, in endogenous time inputs d_m, d_f; second, in the particular way exogenous types i, j interact in domestic production. For example, domestic production may be supermodular (or 2-increasing) with respect to spouses' education.

Individuals draw utility from private consumption c (the numeraire), private leisure e, and the public good q. Labor supply is $h = 1 - e - d$, normalizing to 1 the total amount of time available per week to any individual. Let R, R_m, R_f denote the budget expenditures allocated to private consumption and leisure by singles and married males and females. For a single of type i, whose wage is w_i, the budget constraint is

$$c + w_i e = w_i(1 - d) \equiv R. \tag{4.1}$$

For a married couple of male-female type (i, j), we allow for intrahousehold transfers t_m, t_f, that can be positive or negative, such that

$$c_m + w_i e_m = w_i(1 - d_m) - t_m \equiv R_m, \qquad c_f + w_j e_f = w_j(1 - d_f) - t_f \equiv R_f. \tag{4.2}$$

We also assume that households are subject to a living cost (or benefit) C_{ij} that is a function of exogenous characteristics but not wages. The household budget is balanced if $c_m + c_f + C_{ij} = w_i h_m + w_j h_f$, which implies that $t_m + t_f = C_{ij}$. We can think of C_{ij} as a fixed cost for operating the home production technology, or as a proxy for children's consumption.

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[5]As far as notations are concerned, we index by i and j the variables that are exogenously given, like the wage (w_i is the component of i that is the wage), and we index by m and f the variables of the decision problem.

Let $U_i(c, e, q)$ denote the utility function of an individual with exogenous characteristics i (or j if female). For later use, we also define the conditional indirect utility function

$$\psi_i(R, q, d) = \max_{c,e} U_i(c, e, q) \quad \text{s.t. } c + w_i e \leq R, c \geq 0, 0 \leq e \leq 1 - d. \quad (4.3)$$

4.3. *Marriage Contracts*

We assume that individuals do not make long-term commitments and can walk away from the negotiation at any time. A marriage contract between a male of type i and a female of type j, for a current match-specific shock z, specifies a utility level for both spouses, u_m and u_f, and two promised values $V_m^1(z')$ and $V_f^1(z')$ for any realization z' of the next match-specific shock if the match continues. Let V_i^0 and V_j^0 denote the values of being single.

The present values W_m and W_f of a marriage contract to the male and female spouses for any given choice of (u_m, u_f) follow the Bellman equation,

$$rW_m = u_m + \delta \int \left[\max\{V_i^0, V_m^1(z')\} - W_m \right] dG(z'), \quad (4.4)$$

where r is the time discount rate. The second term of the right-hand side is the option value of divorce after a shock to the match-specific component. If a bliss shock z' accrues, then either the match continuation value $V_m^1(z')$ is greater than the value of singlehood V_i^0 and the match continues, or it is lower and there is a divorce.

Marriage utilities u_m, u_f depend on controls d_m, d_f, t_m, t_f as

$$u_m = \psi_i \left[w_i(1 - d_m) - t_m, q, d_m \right], \qquad u_f = \psi_j \left[w_j(1 - d_f) - t_f, q, d_f \right], \quad (4.5)$$

for $q = zF_{ij}^1(d_m, d_f)$, and these controls are chosen so as to maximize the Nash bargaining criterion

$$\left[W_m - V_i^0 \right]^\beta \left[W_f - V_j^0 \right]^{1-\beta}, \quad (4.6)$$

subject to the feasibility constraint $t_m + t_f = C_{ij}$ and the participation constraint

$$WtM(i, j, z) = 1 \quad \Leftrightarrow \quad V_m^1(z) - V_i^0 \geq 0 \quad \text{and} \quad V_f^1(z) - V_j^0 \geq 0.$$

Individuals draw bargaining power both from their outside options (V_i^0, V_j^0) and from their bargaining coefficients β and $1 - \beta$. A greater β favors the husband in the allocation of resources, for instance because he would be more patient. This is the behavioral source of bargaining power in the household. A greater market value ratio V_i^0/V_j^0 does it also. This is the economic source of that power.

Next, without commitment, the continuation values should align with personal interest. Hence $W_m = V_m^1(z)$ must satisfy the option-value equation,

$$(r + \delta)\left[V_m^1(z) - V_i^0\right] = u_m + \delta \int \left[V_m^1(z') - V_i^0\right]^+ dG(z') - rV_i^0, \quad (4.7)$$

denoting $x^+ \equiv \max\{x, 0\}$, and with a symmetric expression for $V_f^1(z)$.[6] The equilibrium value of a marriage contract is a function of types i, j, and z. We shall use the notation $V_m^1(i, j, z)$, $V_f^1(i, j, z)$ whenever necessary.

Last, the present value of singlehood satisfies the Bellman equation,

$$rV_i^0 = u_i^0 + \lambda \iint \left[V_m^1(i, j, z) - V_i^0 \right] WtM(i, j, z) n_f(j) \, dG(z) \, dj, \qquad (4.8)$$

where $u_i^0 = \max_{d \leq 1} \psi_i[w_i(1 - d), F_i^0(d)]$ and where $n_f(j)$ denotes singles' expectations about type distributions in the future. With anticipated probability $\lambda n_f(j)$, a male single of type i meets a female single of type j. The match-specific bliss shock is z drawn from distribution G. The marriage is consummated if $WtM(i, j, z) = 1$, in which case the male individual i enjoys net continuation value $V_m^1(i, j, z) - V_i^0$ from the next period onward.

4.4. *Steady State*

Calculating the value of being single requires forecasting the chance of meeting a partner of any type in the future. Assuming that the economy is in a steady state easily solves the expectation formation problem. In steady state, flows in and out of the stocks of married couples of each type must exactly balance each other out. This means that, for all (i, j),

$$\delta(1 - \alpha_{ij}) m(i, j) = \lambda n_m(i) n_f(j) \alpha_{ij}. \qquad (4.9)$$

The left-hand side is the flow of divorces. A fraction δ of the $m(i, j)$ married couples draw a new bliss shock. Divorce occurs with probability one minus the marriage probability α_{ij}. The right-hand side is the flow of new (i, j)-marriages. It has two components: a single male of type i, out of the $n_m(i)$ identical ones, meets a single female of type j with probability $\lambda n_f(j)$; the marriage is consummated with probability α_{ij}.

Now, making use of the accounting restrictions,

$$\ell_m(i) = n_m(i) + \int m(i, j) \, dj, \qquad \ell_f(j) = n_f(j) + \int m(i, j) \, di, \qquad (4.10)$$

and replacing $m(i, j)$ by its value from (4.9), i.e. that is, $m(i, j) = \frac{\lambda}{\delta} \frac{\alpha(i, j)}{1 - \alpha(i, j)} n_m(i) n_f(j)$, the equilibrium measures of singles, $n_m(i), n_f(j)$, are solutions to the following fixed-point system:

$$n_m(i) = \frac{\ell_m(i)}{1 + \dfrac{\lambda}{\delta} \displaystyle\int n_f(j) \dfrac{\alpha(i, j)}{1 - \alpha(i, j)} \, dj}, \qquad n_f(j) = \frac{\ell_f(j)}{1 + \dfrac{\lambda}{\delta} \displaystyle\int n_m(i) \dfrac{\alpha(i, j)}{1 - \alpha(i, j)} \, di}. \qquad (4.11)$$

5. EQUILIBRIUM SOLUTION WITH TRANSFERABLE UTILITY

In this section, we solve the equilibrium under a particular specification of preferences that allows to simplify the algebra and avoids complicated numerical solving. We assume

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[6]Notice that the solution to the bargaining problem in c, d_m, d_f, t_m, t_f is invariant to an affine change of indirect utility functions (ψ_i, ψ_j) into $(a_i + b_i \psi_i, a_j + b_j \psi_j)$. Utilities u_m, u_f and flow values $rV_m^1(z), rV_f^1(z)$ are then subject to the same affine transformations.

that the indirect utility is of the form

$$\psi_i(R, q, d) = q \frac{R - A_i}{B_i}, \tag{5.1}$$

where $A_i \equiv A_i(w_i)$ and $B_i \equiv B_i(w_i)$ are individual-specific differentiable, increasing, and concave functions of the wage w_i. We normalize the denominator as $B_i(1) = 1$. The denominator B_i is an individual-specific price index and the numerator $q(R - A_i)$ can therefore be interpreted as a nominal utility level.

Note that the right-hand side of equation (5.1) does not depend on d. This means that we implicitly assume that d is always chosen small enough for the constraint $e \leq 1 - d$ to be never binding. Allowing for corner solutions already complicates the collective model a lot (see Blundell et al. (2007)). Adding marriage formation would complicate things further. We leave this extension for future work. Demands then follow from the indirect utility function by application of Roy's identity:

$$e = -\frac{\partial \psi_i / \partial R}{\partial \psi_i / \partial w_i} = A_i' + \frac{B_i'}{B_i}(R - A_i), \qquad c = R - w_i e. \tag{5.2}$$

Under this assumption, we show that the equilibrium satisfies the following two properties:

1. *Recursivity*. Domestic production inputs are determined first, independently of transfers and values, and transfers and values then follow.

2. *Transferability*. There exists a match surplus that is shared between spouses, and matching requires positive surplus.

5.1. Recursivity

Fix transfers and present values to given levels. For this particular specification of preferences, the solution to the Nash bargaining problem with respect to time inputs d_m, d_f comes out to be independent of these preset transfers and continuation values and can be easily calculated in a first stage.[7]

The first-order conditions of the Nash bargaining problem with respect to domestic production are

$$\frac{1}{w_i} \frac{\partial \ln F_{ij}^1(d_m, d_f)}{\partial d_m} = \frac{1}{w_j} \frac{\partial \ln F_{ij}^1(d_m, d_f)}{\partial d_f} = \frac{1}{R_m - A_i + R_f - A_j},$$

where $R_m - A_i + R_f - A_j$ is *net total private expenditure*, that is, what is left of total family income $w_i + w_j$ to be spent on private consumption and leisure after spending $w_i d_m + w_j d_f + t_m + t_f$ on home production, above and beyond the minimal expenditure $A_i + A_j$.

These conditions deliver two functions $d_m^1(i, j), d_f^1(i, j)$ of observable match characteristics.[8] Let us write X_{ij} and F_{ij}^1 as the equilibrium values of $R_m - A_i + R_f - A_j$ and $F_{ij}^1(d_m^1, d_f^1)$.

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[7]See de Janvry, Fafchamps, and Sadoulet (1991) and Lambert and Magnac (1998) for other examples of recursivity in modeling the decisions of agricultural households.

[8]These demand functions are independent of z because the production function for couples is proportional to z. This is a simplifying condition that is not essential for separability.

5.2. *Transferability*

In Appendix A, we show that the first-order conditions of the Nash bargaining problem with respect to transfers imply the following rent-sharing conditions:

$$B_i\big[V_m^1(z) - V_i^0\big] = \beta S_{ij}(z), \qquad B_j\big[V_f^1(z) - V_j^0\big] = (1 - \beta)S_{ij}(z), \tag{5.3}$$

where the match surplus $S_{ij}(z)$ solves

$$(r + \delta)S_{ij}(z) = zF_{ij}^1 X_{ij} - B_i rV_i^0 - B_j rV_j^0 + \delta \int S_{ij}(z')^+ \, dG(z'). \tag{5.4}$$

The match surplus is equal to the difference of utility flows in marriage and singlehood, $zF_{ij}^1 X_{ij} - B_i rV_i^0 - B_j rV_j^0$, plus a continuation value incorporating the expected effect of a change to the match-specific bliss shock.[9]

Let $\overline{S}_{ij} \equiv \int S_{ij}(z')^+ \, dG(z')$ denote the integrated surplus. Let $\mathcal{G}(s) \equiv \int (z - s)^+ \, dG(z)$. The function $\mathcal{G}$ is decreasing and invertible on the support of G, with $\mathcal{G}' = -(1 - G)$. By integrating equation (5.4), we obtain that $\overline{S}_{ij}$ solves

$$(r + \delta)\overline{S}_{ij} = F_{ij}^1 X_{ij} \mathcal{G}\left(\frac{B_i rV_i^0 + B_j rV_j^0 - \delta \overline{S}_{ij}}{F_{ij}^1 X_{ij}}\right). \tag{5.5}$$

The matching probability becomes

$$\alpha_{ij} \equiv \Pr\{S_{ij}(z) > 0\} = 1 - G\left(\frac{B_i rV_i^0 + B_j rV_j^0 - \delta \overline{S}_{ij}}{F_{ij}^1 X_{ij}}\right) \tag{5.6}$$

$$= 1 - G\left[\mathcal{G}^{-1}\left(\frac{(r + \delta)\overline{S}_{ij}}{F_{ij}^1 X_{ij}}\right)\right]. \tag{5.7}$$

The first equality results from equation (5.4) and the second one uses equation (5.5). Moreover, equation (4.8) for the value of singlehood becomes

$$B_i rV_i^0 = B_i u_i^0 + \lambda \beta \int \overline{S}_{ij} n_f(j) \, dj, \qquad B_j rV_j^0 = B_j u_j^0 + \lambda(1 - \beta) \int \overline{S}_{ij} n_m(i) \, di, \tag{5.8}$$

respectively for single men and women.

5.3. *Transfers*

Knowing present values, we can finally determine equilibrium transfers. We also show in Appendix A that, because of the lack of commitment, equilibrium transfers $t_m(i, j, z)$ and $t_f(i, j, z)$ can be obtained as the solution to the static Nash bargaining problem

$$\max_{t_m, t_f}\big[u_m - rV_i^0\big]^\beta \big[u_f - rV_j^0\big]^{1-\beta}.$$

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[9]Note that the function $(i, j, z) \mapsto qX_{ij} = zF_{ij}^1 X_{ij}$ is indeed the equilibrium value of aggregate nominal utility $B_i u_m + B_j u_f = q[w_i(1 - d_m) - t_m + w_j(1 - d_f) - t_f] = qX_{ij}$.

That is, since $B_i u_m + B_j u_f = q X_{ij} = z F^1_{ij} X_{ij}$,

$$B_i u_m = B_i r V^0_i + \beta \big[z F^1_{ij} X_{ij} - B_i r V^0_i + B_j r V^0_j \big]. \tag{5.9}$$

Alternatively, dividing by $q = z F^1_{ij}$, we obtain net private incomes as a share of net total private expenditure X_{ij}:

$$R_m - A_i = w_i \big(1 - d^1_m \big) - t_m - A_i = \beta_{ij}(z) X_{ij}, \tag{5.10}$$

$$R_f - A_j = w_j \big(1 - d^1_f \big) - t_f - A_j = \big[1 - \beta_{ij}(z) \big] X_{ij}, \tag{5.11}$$

where the *collective sharing rule* $\beta_{ij}(z)$ can be derived from equation (5.9) as

$$\beta_{ij}(z) = \beta + \frac{(1 - \beta) B_i r V^0_i - \beta B_j r V^0_j}{z F^1_{ij} X_{ij}}. \tag{5.12}$$

In this model, the share of the household private consumption budget that is allocated to the male spouse is the sum of the bargaining coefficient β and the market advantage in the marriage market that results from the relative values of singlehood, $B_i r V^0_i$, $B_j r V^0_j$.

5.4. *Equilibrium*

The equilibrium is fully characterized by the following functions of individual types: $\overline{S}_{ij}$, $B_i r V^0_i$, $B_j r V^0_j$, $n_m(i)$, $n_f(j)$, α_{ij}. They are obtained as a fixed point of the system of equations (4.11), (5.5), (5.6), and (5.8) for $\lambda \equiv \lambda(N_m, N_f)$ (to be specified) with $N_m = \int n_m(i)\, di$ and $N_f = \int n_f(j)\, dj$. Proving equilibrium existence is difficult (see Shimer and Smith (2000), Atakan (2006), Lauermann and Nöldeke (2015), Manea (2017)) and the equilibrium is likely not unique.

To calculate the equilibrium numerically, we discretize the value and density functions on a Chebyshev grid and we use Clenshaw–Curtis quadrature to approximate the integrals. Then, the equilibrium is obtained by iterating the fixed-point operator (see Appendix D for details). We tried different choices of starting values and the algorithm always converged to the same limit.

6. SPECIFICATION, IDENTIFICATION, AND ESTIMATION

In this section, we describe the parametric specification of the model used in estimation and how the parameters depend on individual types (wage, gender, and other exogenous characteristics). Then, we study identification and explain the estimation procedure.

6.1. *Parametric Specification*

The way parameters depend on exogenous variables (gender g_i, education Ed_i, wage w_i, and family values index FVI_i) is specified as follows. Let $x_i = (Ed_i, FVI_i)$ denote the vector of exogenous individual characteristics in addition to wage and gender.

Meeting Rates

The meeting function is Cobb–Douglas: $\lambda(N_m, N_f) = \xi (N_m N_f)^{-1/2}$.

Preferences

Males' indirect utility for consumption and leisure is such that

$$A_i = a_{0i} + a_{1i}w_i + \frac{1}{2}a_{2i}w_i^2, \qquad \ln B_i = b_i \ln w_i. \tag{6.1}$$

All parameters may depend on x_i.[10] However, in the estimation, we restrict parameters a_{0i}, a_{1i}, and b_i to be linear functions of characteristics x_i, and we set a_{2m} constant given gender.

Leisure expenditure follows from equation (5.2) as

$$w_i e_m = a_{1i}w_i + a_{2m}w_i^2 + b_i(R_m - A_i), \qquad R_m = w_i(1 - d_m) - t_m, \tag{6.2}$$

given domestic time d_m and transfer t_m ($t_m = 0$ for singles). Consumption is then

$$c_m = R_m - w_i e_m = a_{0i} - \frac{1}{2}a_{2m}w_i^2 + (1 - b_i)(R_m - A_i). \tag{6.3}$$

For women, we obtain similar expressions with j and f in place of i and m.

Domestic Production

The domestic production functions are Stone–Geary:

$$[couples] \qquad F_{ij}^1(d_m, d_f) = Z_{ij}\big(d_m - D_i^1\big)^{K_m^1}\big(d_f - D_j^1\big)^{K_f^1}, \tag{6.4}$$

$$[singles] \qquad F_i^0(d_m) = \big(d_m - D_i^0\big)^{K_m^0}, \qquad F_j^0(d_f) = \big(d_f - D_j^0\big)^{K_f^0}. \tag{6.5}$$

Note the absence of TFP parameter in front of the home production for singles. This is a normalization that is rendered necessary by the ordinal nature of preferences (see footnote 6). The TFP parameter Z_{ij} measures the quality of the public good that is produced in the household and subsumes all complementarities with respect to exogenous male and female types in domestic production. The Stone–Geary specification forces complementarity between time inputs and $K_m^1 = K_f^1 \to 0$ corresponds to a Leontieff technology with $d_m = D_i^0, d_f = D_j^0$. Given the limited variability of non-market time uses across calendar time and types, we thought that a small departure from Leontieff would not be a bad assumption. But more work is needed to test alternative specifications (such as CES).

We let the D_i and D_j parameters be general functions of characteristics x_i, x_j. Moreover, C_{ij} and Z_{ij} are functions of both spouses' characteristics x_i, x_j. In the estimation, to make type complementarities easier to interpret, we will let C_{ij} be linear in x_i and x_j, with no interaction terms between x_i and x_j.

The times used in home production are therefore, for singles,

$$d_m^0 = D_i^0 + \frac{K_m^0}{1 + K_m^0}\big(1 - D_i^0 - A_i/w_i\big), \qquad d_f^0 = D_j^0 + \frac{K_f^0}{1 + K_f^0}\big(1 - D_j^0 - A_j/w_j\big), \tag{6.6}$$

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[10]The consumption good is the numeraire. With a non-unitary price p for c, A and B become $A = a_0 p + a_1 w + \frac{1}{2}a_2 w^2/p$ and $\ln B = b \ln w + (1 - b) \ln p$. Concavity of the cost function implies $a_2 < 0$.

and for couples,

$$d_m^1 = D_i^1 + K_m^1 X_{ij}/w_i, \qquad d_f^1 = D_j^1 + K_f^1 X_{ij}/w_j, \qquad (6.7)$$

with net total private expenditure being

$$X_{ij} = \frac{w_i(1 - D_i^1) + w_j(1 - D_j^1) - C_{ij} - A_i - A_j}{1 + K_i^1 + K_j^1}. \qquad (6.8)$$

The equilibrium domestic productions are, for singles,

$$F_i^0 = \left[\frac{K_m^0}{1 + K_m^0}(1 - D_i^0 - A_i/w_i)\right]^{K_m^0}, \qquad F_j^0 = \left[\frac{K_f^0}{1 + K_f^0}(1 - D_j^0 - A_j/w_j)\right]^{K_f^0}, \qquad (6.9)$$

and for couples,

$$F_{ij}^1 = Z_{ij}\left(\frac{K_m^1}{w_i}\right)^{K_m^1}\left(\frac{K_f^1}{w_j}\right)^{K_f^1} X_{ij}^{K_m^1 + K_f^1}. \qquad (6.10)$$

This specification is quite flexible. Suppose that some exogenous factor increases D_j^1 (say family values). Then the wife will increase her non-market hours and the husband will reduce his own input to home production (because of the induced decrease of X_{ij}). Minimal inputs D_i^1, D_j^1 thus govern home production specialization. At the same time, domestic output F_{ij}^1 should fall following the decrease of X_{ij}. It is therefore important to allow the factors determining minimal inputs to determine public good quality Z_{ij} or living cost C_{ij} at the same time in order to decouple the level of home production from the division of labor in home production.

Last, the distribution of match-specific shocks z is assumed to be log-normal: $G(z) = \Phi(\ln z/\sigma)$, where Φ is the standard normal CDF and σ is the standard deviation of z. We assume a zero mean as any nonzero would be absorbed into Z_{ij}. We then have

$$\mathcal{G}(s) = \int (z - s)^+ \, dG(z) = -s\Phi\left(-\frac{\ln s}{\sigma}\right) + e^{\frac{\sigma^2}{2}}\Phi\left(-\frac{\ln s}{\sigma} + \sigma\right).$$

6.2. *Identification*

The details of the identification proof are relegated to Appendix B. We assume that we observe the time uses, marital status, and characteristics of the whole population of men and women over a fixed period of time in which the economy is in a steady state (i.e., the distributions remain fixed over time because divorces offset new marriages). Although identification may hold under far less restrictive assumptions, we only discuss identification under the preceding parametric restrictions.

We first show that rates λ and δ determining dating and divorce risk, and the conditional matching probability α_{ij} are identified from marriage and divorce flows given couples' types. The main source of identification here is that a common factor, the bliss shock z, is determining both marriage and divorce: $\lambda\alpha_{ij}$ is the marriage rate and $\delta(1 - \alpha_{ij})$ is the divorce rate.

Then, we show that the parameters of preferences and home production are identified in much the same way as in standard labor supply models, the other spouse's earnings

acting as a source of non-earned income allowing to identify income effects. For this to work, we need the sharing rule to exhibit residual variation conditional on spouses' socio-demographic characteristics and wages. This additional source of variation is the match-specific bliss variable z unobserved to the econometrician. One can then identify b_m/b_f by regressing $w_i e_m^1$ on $w_j e_f^1$, keeping wages and socio-demographics constant.

This is yet not enough to guarantee the identification of preference and home production parameters. Time uses must exhibit sufficient nonlinear variation in wages, in particular to separately identify b_m from b_f. It is thus required that the demand system be not linear in wages ($a_{2m} \neq 0$ and $a_{2f} \neq 0$) and that home production time uses not remain constant as wages vary ($K_m^1 + K_f^1 \neq 0$).

Third, the bargaining parameter β and the variance of match-specific shocks are identified from the first- and second-order moments of married couples' leisure demands. Intuitively, β sets the level of the sharing rule $\beta_{ij}(z)$ and z its dispersion given observed match types.

Last, the public good quality parameter Z_{ij} is identified from the structural link between public good quality and matching probability.

6.3. Estimation Strategy

We use household data on time uses, gender, wages, family values, and education covering the period 1991–2008. We drop all individual observations corresponding to young individuals aged less than 22 and older individuals aged more than 50.[11] We split the whole sample into six 3-year periods: 91–93, 94–96, 97–99, 00–02, 03–05, 06–08. We assume that each sub-sample is a draw from a steady-state economy characterized by different distributions of male and female types. By contrast, structural parameters are assumed to remain the same throughout the entire observation period. That is, we expect the model to fit the data both in cross-sections and across time.

The index i now refers to an observation unit of the sample of male singles, j refers to female singles, and (i, j) refers to couples. For singles, we observe domestic time use d_i^0, labor supply h_i^0, and education $Ed_i \in \{L, M, H\}$ (O-level, A-level, and higher education), wages w_i, and family values indices FVI_i. For couples, the corresponding time use observations are d_{mij}^1, d_{fij}^1, h_{mij}^1, and h_{fij}^1. Leisure is deduced as $e = 1 - d - h$. The estimation procedure is iterative and goes through the following steps.

Step 1: Reduced-Form Estimation of Meetings Rates and Matching Probabilities

For each sub-period, pooling the three cross-sections, we compute kernel density estimates of $n_m(i)$, $n_f(j)$, and $m(i, j)$ from the stocks of male singles, female singles, and married couples on a grid of values for i and j comprising three education categories, 16 Chebyshev nodes for wages, and 8 Chebyshev nodes for the family values index (see Appendix D).[12] We then calculate marriage and divorce flows $MF(i, j)$, $DF(i, j)$ by education only because of the small sample sizes of flows. For example, we calculate the number of new marriages for 1994 as the number of singles in 1994 who are married in 1995, and the flow of divorces for 1994 as the number of singles in 1994 who were married

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[11]We experimented with narrower age ranges without changing the results substantially.

[12]The fact that the same individuals may be present in different periods does not matter under the assumption that the distribution of the first year data is the same as the one in the second year, etc. Kernel density estimators as well as least squares only require the application of the law of large numbers for consistency. Correlations between observations matter yet for asymptotic standard errors.

in 1993. Then we add the numbers for 1994, 1995, and 1996 to get the aggregate flows for the period 1994–1996.

The number of new marriages (or cohabitations) of type (i, j) per unit of time is linked to λ and α_{ij} by the relation

$$MF(i, j) = \lambda n_m(i) n_f(j) \alpha_{ij}. \tag{6.11}$$

Divorce occurs when the last draw of z (occurring at rate δ) ceases to satisfy the matching rule (with probability $1 - \alpha_{ij}$). It follows that the flow of divorces per unit of time is

$$DF(i, j) = m(i, j) \delta (1 - \alpha_{ij}). \tag{6.12}$$

Define $MR(i, j) \equiv \frac{MF(i,j)}{n_m(i) n_f(j)}$, the marriage rate (by potential match type), and $DR(i, j) \equiv \frac{DF(i,j)}{m(i,j)}$, the divorce rate. Eliminating α_{ij} from equations (6.11) and (6.12) yields

$$\frac{MR(i, j)}{\lambda} + \frac{DR(i, j)}{\delta} = 1. \tag{6.13}$$

Using equation (6.13) and the Cobb–Douglas specification of the meeting rate $\lambda = \xi (N_m N_f)^{-1/2}$, we estimate $1/\xi$ and $1/\delta$ by OLS, pooling all four intermediate sub-periods 94–96, 97–99, 00–02, 03–05 together and eliminating the two extreme periods (91–93 and 06–08) to avoid boundary problems (no divorce flow can be calculated in 1991 and no marriage flow in 2008).[13] We use this approach to estimating rates instead of a duration model because of censoring (right-censoring for marriage and left- and right-censoring for singlehood).

Then we could estimate a different matching probability α_{ij} for each sub-period using flows. However, the model assumes that households anticipate steady-state equilibrium distributions. If we impose equal inflows and outflows, $MF(i, j) = DF(i, j)$, matching probabilities follow from equation (4.9) as

$$\alpha_{ij} = \frac{\delta m(i, j)}{\delta m(i, j) + \lambda n_m(i) n_f(j)}. \tag{6.14}$$

Equation (6.14) is not only consistent with the way households are supposed to form expectations, it also makes the estimation of matching probabilities much more precise as stock samples are much bigger than flow samples.[14] Figure 6 displays the empirical link between marriage and divorce flows $MF(i, j)$ and $DF(i, j)$. There is clearly a lot of noise and the regression line is a bit off the 45 degree line. But there is no clear structure that would indicate that equation (6.14) is not a good approximation of the true marriage probability.

Step 2: Preferences, Home Production, Bargaining Power, and Bliss Shock Distribution

We then estimate the parameters of preferences and domestic productions, as well as the Nash bargaining parameter β and match dispersion σ, by nonlinear least squares, with

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[13]Note also that running the regression with or without weighting observations (i, j) by their sample size $\ell_m(i) \ell_f(j)$ gives close estimates.

[14]Estimates of $1/\lambda$ and $1/\delta$ rely on flows; there is no other way. But they are more precisely estimated than matching probabilities because the regression procedure averages over types (i, j), which smoothes out small sample errors.
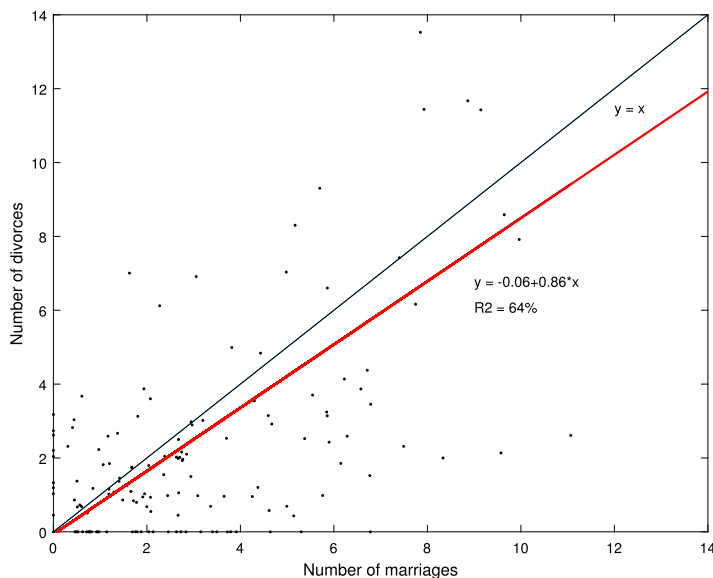
FIGURE 6.—Link between new marriages and divorces by periods and match types. *Notes*: (1) To make this graph, we calculated marriage and divorce flows by education (3 categories) and *FVI* (2 categories: above or below the median *FVI*), and we considered 4-year subperiods (92–95, 96–99, 00–03, 04–07). We obtain $3 \times 3 \times 2 \times 2 \times 4 = 144$ points. (2) The regression line is obtained by regressing $DF(i, j)$ on $MF(i, j)$, weighting each point (i, j) by their sample representativeness, measured as $\ell_m(i)\ell_f(j)$.

time uses as dependent variables and wages, education, and *FVI*s as observed regressors. The model allows to calculate all the relevant conditional expectations, unfortunately not in closed form. The bliss shock must be integrated out, and the sharing rule, the matching probabilities, and the distributions of individual types by gender and marital status must be recalculated at each new parameter iteration. The formulas for the residuals are provided in Appendix C, and the numerical techniques used to approximate them are exposited in Appendix D.

Note that we could plug the pre-estimated distributions of types and the matching probabilities in the residuals instead of using the model to calculate them. However, when we proceed to the counterfactual analyses, we have to predict the distributions and marriage probabilities. The benchmark economy is the one that is predicted using the estimated parameters. It thus seems preferable that we try to estimate the parameters by fitting the benchmark economy to the data.

Step 3: Public Good Quality

Public good quality Z_{ij} is estimated as a flexible high-order polynomial in x_i, x_j, allowing for all interactions. That is, we invert the theoretical link between Z_{ij} and matching probabilities α_{ij}. This is a standard minimum distance procedure based on consistent estimates of matching probabilities. See Appendix C for details.

7. EMPIRICAL RESULTS

In this section, we first describe parameter estimates and the fit. Then we discuss the implied sharing rule. We end the section by a counterfactual analysis.

7.1. *Parameter Estimates and fit*

Meeting Rates

We estimate $\xi = 0.151$ and nearly identical meeting rates for singles ($\lambda N_m = \xi(N_m/N_f)^{1/2}$ is close to $\lambda N_f = \xi(N_f/N_m)^{1/2}$ as $N_m \simeq N_f$). This implies a median duration between two consecutive datings of about 4.5 years and a first quartile of 10 to 11 months. We estimate the yearly probability of a bliss shock to be $\delta = 0.0378$, or a median duration between two consecutive shocks of 18 years (first quartile: 7 to 8 years). This may seem a long time between datings and bliss shocks, but remember that we select only labor-active individuals.[15]

Unconstrained Marriage Probabilities

We start with the marriage probabilities estimated using equation (6.14). These estimates only rely on flows and stocks, and not on the economic mechanism determining values. Table II displays mean matching probabilities by education, *FVI*, and wages quartiles for three periods. We shall refer to these estimates of matching probabilities as the unconstrained, or actual, ones. The first observation is that the sorting patterns are very stable. Second, the sorting patterns are quite different by education, family values, or wages. Yet, all three patterns express different forms of positive assortative mating.

The matching probability matrix for education is symmetric with a strong main diagonal. This is the usual form of homophily. The matching probability matrix for *FVI* is also symmetric, but with diagonal elements that are not all dominant. The Q1 row and the Q1 columns are identical, with nearly identical entries. This means that liberal individuals are rather indifferent to the family values of their partner. On the other end, the Q4 row and the Q4 column are also similar, but with decreasing entries. Conservative individuals value conservative partners.

For wages, it is yet again different. There is no symmetry. The female wage does not seem to matter much for matching (though richer women marry richer men more often). The male wage comparatively matters a lot; males with a high wage (in the Q4 interval) marry twice more frequently than males with a low wage (Q1).

This is interesting in relation to the social norm discussed by Bertrand, Kamenica, and Pan (2015). The solid line in Figure 7 plots the estimated matching probability α_{ij} as a function of the female wage ratio ($\frac{w_f}{w_m+w_f}$). The matching probability is decreasing with the female wage ratio for values of this ratio above 0.5, whereas it is more or less constant for values less than 0.5. However, matching probability estimates indicate that this is likely because low-wage males are just not attractive in general, and not because of a social norm preventing marriages with the husband earning less than the wife.

The strong, symmetric homophily with respect to education explains why married women's educational attainment has so much increased over the last 25 years and why the gender-and-marriage gap in education has disappeared. The particular form of matching with respect to wages also explains why the marriage gap is more pronounced for men than for women.

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[15]There is a bit of attrition across time in the panel. If we keep in the stocks only singles who are still in the panel the following year, and couples who were already there the year before, we estimate $\xi = 0.163$ and $\delta = 0.0391$. The results are not substantially affected. We also investigated a more general specification with a different distribution G_0 for the first bliss shock and another distribution G_1 for the shocks during marriage. The results were not fundamentally altered.

TABLE II

SORTING BY EDUCATION AND WAGES

		Female									
		1991–1993			2000–2002			2006–2008			
		<HS	HS	>HS	<HS	HS	>HS	<HS	HS	>HS
Male	<HS	0.49	0.31	0.21	0.49	0.42	0.23	0.47	0.46	0.26
	HS	0.34	0.35	0.23	0.41	0.42	0.26	0.44	0.47	0.32
	>HS	0.21	0.17	0.39	0.24	0.32	0.42	0.18	0.33	0.43

(a) By education

		Female											
		1991–1993				2000–2002				2006–2008			
		Q4	Q3	Q2	Q1	Q4	Q3	Q2	Q1	Q4	Q3	Q2	Q1
Male	Q4	0.51	0.49	0.41	0.30	0.59	0.47	0.42	0.31	0.58	0.49	0.46	0.36
	Q3	0.50	0.48	0.42	0.32	0.49	0.44	0.41	0.34	0.49	0.46	0.44	0.36
	Q2	0.45	0.44	0.41	0.35	0.45	0.42	0.40	0.35	0.45	0.44	0.43	0.36
	Q1	0.37	0.37	0.38	0.37	0.38	0.38	0.38	0.36	0.39	0.39	0.39	0.35

(b) By *FVI*

		Female											
		1991–1993				2000–2002				2006–2008			
		Q4	Q3	Q2	Q1	Q4	Q3	Q2	Q1	Q4	Q3	Q2	Q1
Male	Q4	0.50	0.49	0.48	0.42	0.52	0.50	0.48	0.44	0.59	0.55	0.52	0.44
	Q3	0.43	0.44	0.43	0.39	0.50	0.49	0.48	0.45	0.51	0.49	0.47	0.40
	Q2	0.41	0.42	0.41	0.37	0.40	0.42	0.42	0.41	0.41	0.42	0.41	0.35
	Q1	0.29	0.31	0.31	0.30	0.25	0.28	0.30	0.31	0.27	0.30	0.30	0.27

(c) By wage quartile

Notes: This table displays the estimated unconstrained marriage probabilities α_{ij} separately projected on male-female educations, *FVI*s, and wages. Q1 denotes the interval below the first quartile, Q2 between first and second quartile, etc.

Marriage probabilities explain sorting, which explains selection by gender, type, and marital status. We now have to understand how matching probabilities relate to preferences and home production.

Preferences and Home Production Estimates

Preference and home production parameter estimates are displayed in Table III. The impact of education on preferences and home production (a, b, D) is similar for men and women. More educated individuals value consumption more than leisure (parameter a_0). Education also increases the income effect for leisure (parameter b). There is little effect of education on home production specialization (parameters D_{1m}, D_{1f}). Education increases the living cost C: more educated individuals spend more on the public good.

Family values, on the contrary, have little influence on preferences. The only significant parameter is $a_{1f}[FVI]$: conservative females supply less market work; but even for
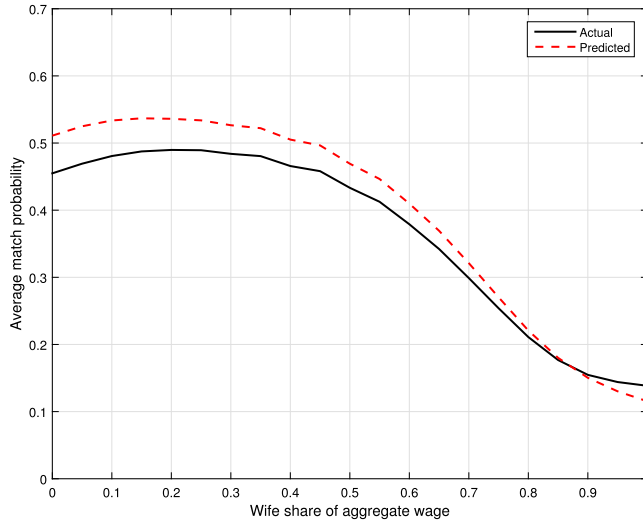
FIGURE 7.—Matching probability by female wage ratio (1991–2008 mean).

TABLE III

ESTIMATED PREFERENCE AND HOME PRODUCTION PARAMETERS

Preferences			Home production, singles		
$a_{0f}[Ed=L]$	−21.46	(7.80)	$D_f^0[Ed=L]$	0.0524	(0.0177)
$a_{0m}[Ed=L]$	−26.63	(8.09)	$D_m^0[Ed=L]$	0.0467	(0.0188)
$a_{0f}[Ed=H]$	−12.74	(5.01)	$D_m^0[Ed=H]$	0.0481	(0.0142)
$a_{0m}[Ed=H]$	−13.58	(5.50)	$D_f^0[Ed=H]$	0.0363	(0.0142)
$a_{0f}[FVI]$	0.7455	(0.7355)	$D_f^0[FVI]$	0.0096	(0.0041)
$a_{0m}[FVI]$	0.6168	(0.6101)	$D_m^0[FVI]$	0.0031	(0.0039)
$a_{1f}[Ed=L]$	0.4403	(0.0175)	K_f^0	0.0177	(0.0070)
$a_{1m}[Ed=L]$	0.3939	(0.0197)	K_m^0	0.0002	(0.0034)
$a_{1f}[Ed=H]$	0.4350	(0.0215)			
$a_{1m}[Ed=H]$	0.3812	(0.0258)	Home production, couples		
$a_{1f}[FVI]$	0.0184	(0.0046)	$D_f^1[Ed=L]$	0.0701	(0.0105)
$a_{1m}[FVI]$	−0.0001	(0.0050)	$D_m^1[Ed=L]$	0.0660	(0.0095)
			$D_f^1[Ed=H]$	0.0564	(0.0094)
a_{2f}	−0.0031	(0.0007)	$D_m^1[Ed=H]$	0.0708	(0.0085)
a_{2m}	−0.0008	(0.0005)	$D_f^1[FVI]$	0.0159	(0.0029)
			$D_m^1[FVI]$	−0.0073	(0.0025)
$b_f[Ed=L]$	0.0303	(0.0119)			
$b_m[Ed=L]$	0.0345	(0.0122)	$C[constant]$	37.14	(10.05)
$b_f[Ed=H]$	0.0721	(0.0248)	$C[FVI_f]$	−1.251	(0.787)
$b_m[Ed=H]$	0.0940	(0.0340)	$C[FVI_m]$	−3.896	(1.180)
$b_f[FVI]$	−0.0023	(0.0020)	$C[Ed_f=L]$	−4.023	(2.328)
$b_m[FVI]$	−0.0000	(0.0021)	$C[Ed_m=L]$	−9.105	(3.742)
			K_f^1	0.0183	(0.0038)
			K_m^1	0.0056	(0.0026)

Notes: Standard errors in parentheses. $Ed=H$ indicates "high-school graduate" and $Ed=L$ indicates an education level that is less than A-level.

$FVI = 5$, the effect is lower than 0.1 hours. However, family values determine home production quite differently for men and women. Conservative women demand more leisure (parameter a_{1f}) and home production time ($D_f^{0,1}$), while conservative men, when married, want to spend less time in home production (D_m^1). The living cost C is also higher for liberal couples, which could mean that liberal couples prefer to buy home services instead of doing the work themselves.

There is thus a correlation between family attitudes, as they can be measured by the questionnaire, and task specialization, some individuals being more prone than others to adopt the traditional family model, where the wife specializes in home production and the husband in market work.

Public Good Quality, Sorting, and Complementarities

The standard deviation of the logged bliss shock is estimated at 0.268 (with a standard error of 0.111), which is far from negligible. It moves home production in a $[-41\%, +69\%]$ interval around its equilibrium value F_{ij}^1 with 95% probability.

Next, we look at how the model fits marriage probabilities. Given parameters, we predict marriage probabilities as indicated in Section 5 and compare them with the unconstrained matching probabilities. In Table IV, we calculate the measure of fit: $R^2 = 1 - \text{Var}(\alpha_{ij} - \widehat{\alpha}_{ij})/\text{Var}(\alpha_{ij})$, where α_{ij} and $\widehat{\alpha}_{ij}$ denote unconstrained and predicted matching probabilities. We obtain an R^2 close to 80% in all years, meaning that a fixed set of structural parameters for the whole period costs 20% in prediction error. Figure 7 (dotted line) shows that the model fits matching probabilities by female wage ratio well.

Table IV also shows the fit obtained with the quadratic projection of the nonparametric estimates of $\ln Z_{ij}$. By using the quadratic approximation, the fit is reduced by about 7 percentage points. This shows that high-order polynomial terms could be neglected. Then we predict marriage probabilities with a quadratic projection of Z_{ij} without male-female interaction terms. This reduces the fit by 33%. So gender complementarities matter. Interestingly, only one complementarity really does matter: education homophily. Moreover,

TABLE IV

FIT OF MATCHING PROBABILITIES (R^2 OF REGRESSION OF UNCONSTRAINED ON PREDICTED α_{ij})

	1991	1994	1997	2000	2003	2006	Mean
Nonparametric estimation of Z_{ij}	0.72	0.83	0.86	0.87	0.82	0.78	0.81
Quadratic projection of $\ln Z_{ij}$	0.68	0.77	0.80	0.80	0.75	0.72	0.75
Quadratic projection of $\ln Z_{ij}$							
without interactions	0.38	0.51	0.60	0.62	0.57	0.53	0.54
Quadratic projection of $\ln Z_{ij}$ with no interactions but…							
$w_m * w_f$	0.43	0.54	0.63	0.65	0.59	0.56	0.57
$FVI_m * FVI_f$	0.37	0.51	0.61	0.63	0.57	0.54	0.54
$Ed_m * Ed_f$	0.66	0.72	0.76	0.75	0.69	0.67	0.71
$w_m * Ed_f$	0.43	0.55	0.64	0.66	0.61	0.57	0.58
$w_f * Ed_m$	0.53	0.61	0.68	0.67	0.63	0.58	0.62
$w_m * FVI_f$	0.38	0.51	0.60	0.62	0.57	0.54	0.54
$w_f * FVI_m$	0.38	0.51	0.60	0.62	0.57	0.54	0.54
$FVI_f * Ed_m$	0.38	0.51	0.60	0.62	0.57	0.54	0.54
$FVI_m * Ed_f$	0.37	0.52	0.60	0.63	0.59	0.55	0.54

despite important changes in women's educational attainment since the early 1990s, the strength of education homophily does not seem to vary over time in any specific way.[16]

Fit of Selection and Time Uses

We next show how our model fits selection, that is, the dynamics of wages and individual types by marital status. We take the changes in the distributions of wages, family values, and education by gender as given and we simulate different steady-state equilibria for the six time periods, keeping the model parameters fixed.

We first note that the difference between the actual number of married couples and its prediction is never greater than 4%, and the errors on the number of singles are correspondingly small. Then the first two rows of Figure 8 show how the model fits selection. The wages of married men are slightly overestimated and the wages of single men are slightly underestimated. Yet the different marriage wage gaps (married versus single) of men and women are well replicated by the model. The fit of family values indices and education by marital status over time is very good. The last row of Figure 8 shows the fit of time use trends. The trends in the market hours of married and single women are slightly overestimated, the model predicting a bit more change for women than is actually observed. The marriage differentials of market and non-market hours are very well fitted.

Last, Figure 9 shows how the model fits the distributions of wage and earnings ratios (within couples) for the middle period 2000–2002, first for all couples, then for conservative couples (both spouses' *FVI*s below their medians), and finally for liberal couples (*FVI*s above medians). The fit is a bit worse for wages than for labor earnings, but the model reproduces the observed patterns well, both overall and by family values.

7.2. *The Sharing Rule*

We estimate the bargaining power parameter β to be 0.45 with a standard deviation of 0.10. However, this does not imply that men get less than half of the surplus. The share of the surplus that each spouse reclaims also depends on his/her outside option. The sharing rule $\beta_{ij}(z)$, that is, the share of the net total private expenditure X_{ij} that is appropriated by the husband, is a better indicator of market power.

Its estimated mean value across all couples' types is remarkably stable—between 0.55 and 0.56 throughout the whole period. Hence, better outside options compensate for a bargaining coefficient that is estimated slightly less than 0.5 and give the husband a share of the rent slightly bigger than 0.5. The division of labor in the family is gender-specific; nevertheless, the total income that is left for private consumption and leisure is split between spouses in a way that is quite egalitarian.

In Figure 10, we show the evolution of the sharing rule over the period, separately for various household types. There is evidence of compensating differentials in education. The husband gets a bigger share of the rent if he is less educated and if the wife is more educated. Note, however, that educated females always get less than the fair share. The only case where the wife gets more than the husband is for uneducated females married to educated males. Family values have little effect on income sharing. They affect work in household but not private income sharing. There are no compensating differentials in wages. Figure 10(d) summarizes the effect of within-household wage inequality on income

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[16]We also find that Z_{ij} depends very little on wages in general, as we would expect. However, Z_{ij} does depend on family values, although not much on interacted family values.

(a) Mean wage

(b) Mean family values index

(c) Share of high school dropouts

(d) Share of college educated

(e) Mean market hours

(f) Mean non-market hours

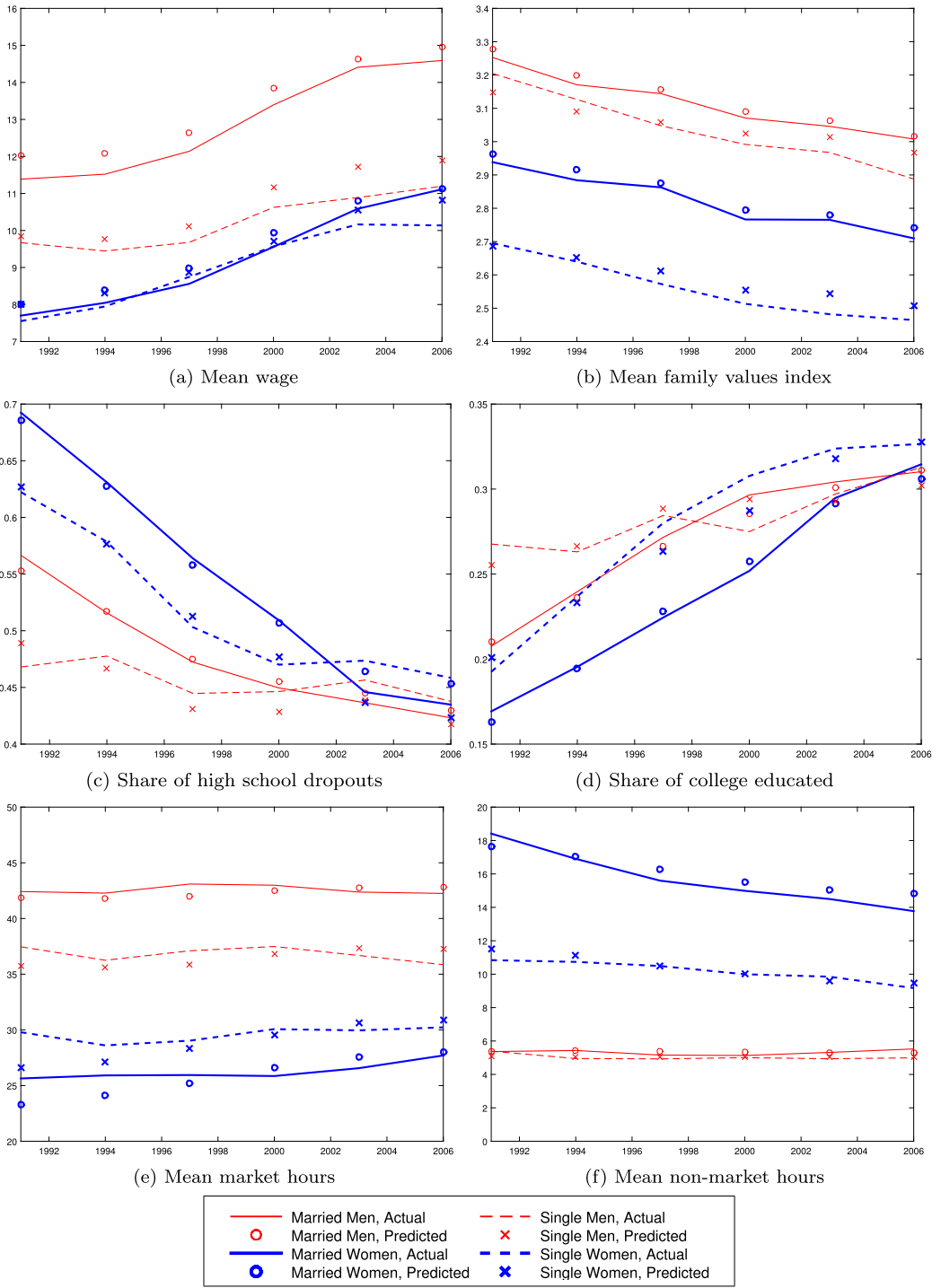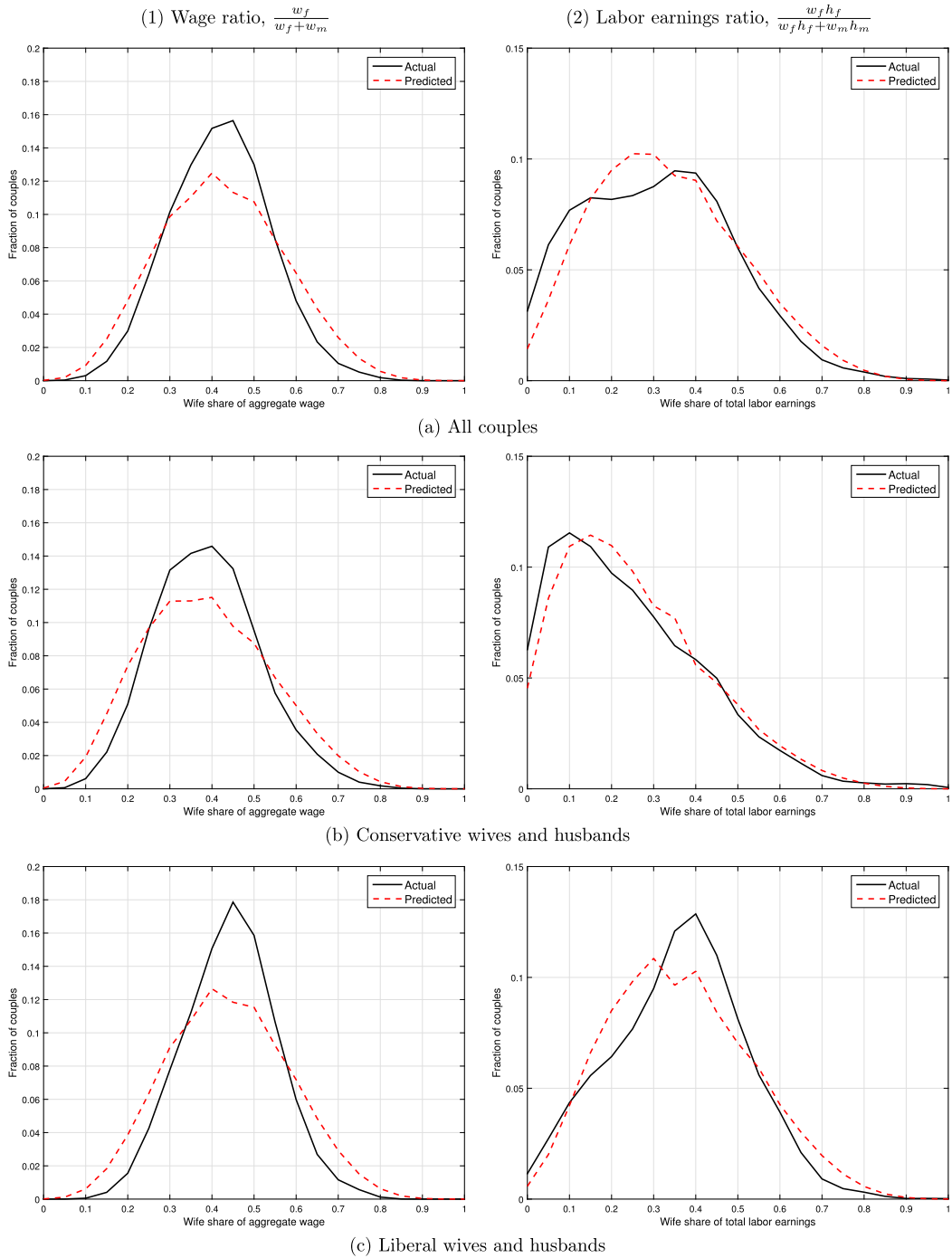| Married Men, Actual | Single Men, Actual |
Married Men, Predicted	Single Men, Predicted
Married Women, Actual	Single Women, Actual
Married Women, Predicted	Single Women, Predicted

FIGURE 8.—Fit of hours and selection.

FIGURE 9.—Fit of the distributions of wages and earnings ratios, 2000–2002.
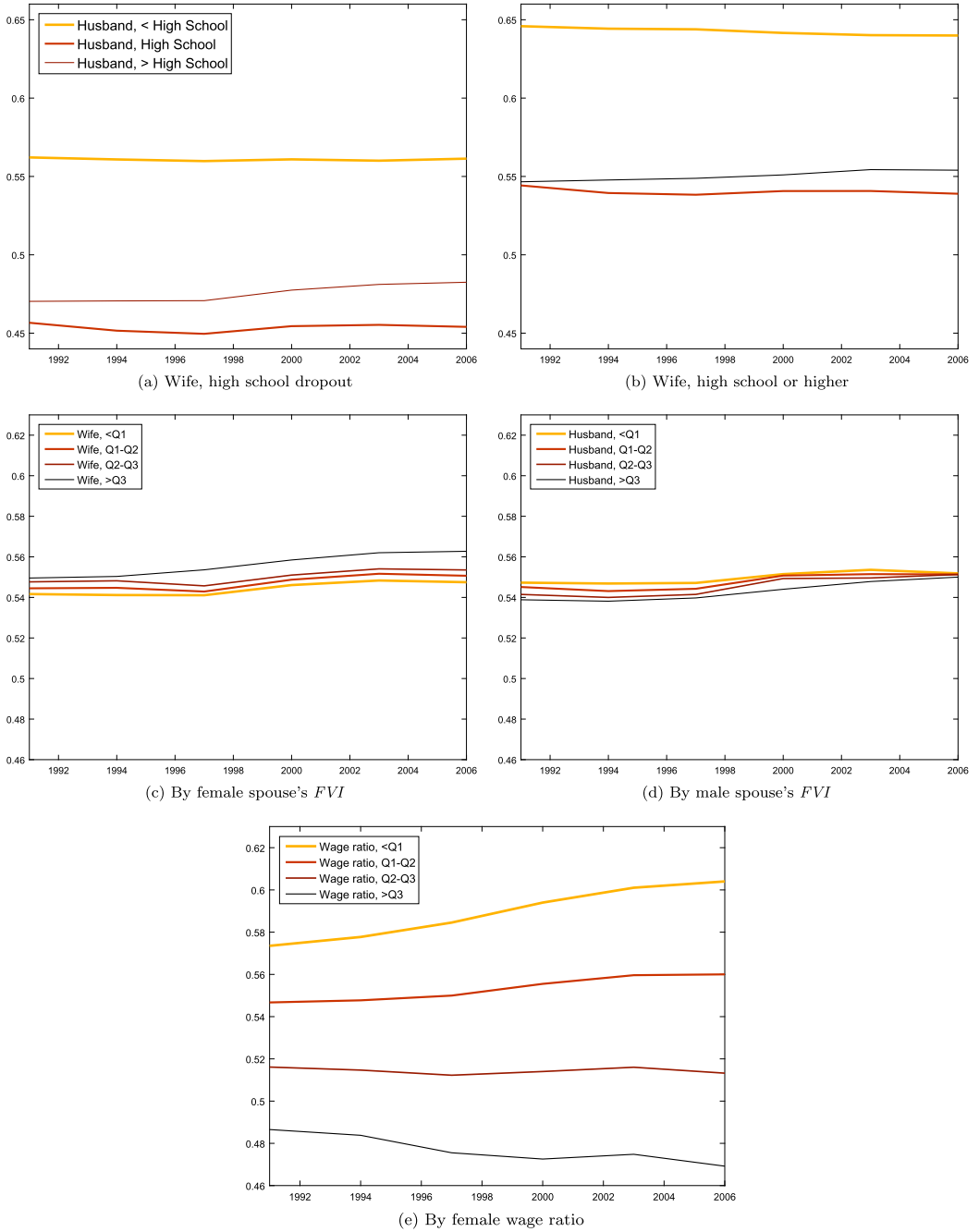
FIGURE 10.—Mean sharing rule by type.

sharing. The share of income that is appropriated by one spouse is higher if his/her wage ratio is higher. Moreover, this link is getting stronger over time. Hence, a consequence of the (slow) cultural change away from the traditional model of the family may be that there is progressively less redistribution between spouses.

7.3. *Counterfactual Changes*

Wage Elasticities

In the first counterfactual analysis, we increase all wages in the 2000–2002 sample by 10%, separately for men and women, and simulate changes in market and non-market hours.[17] We first run the simulations with unchanged distributions of types by gender and marital status. Then, we simulate the new equilibrium including distributional changes. The results on hours are reported in Table V. The first column displays the actual numbers. The second column displays the baseline simulation with the estimated parameters.

Estimates of uncompensated or Marshallian elasticities vary a lot across publications (see Blundell and Macurdy (1999), Meghir and Phillips (2010)). Our estimates of female own-wage labor supply elasticities (0.38 for married and single women) are similar to the ones of Blundell et al. (2016). They estimated an elasticity of participation rates (extensive margin) equal to 0.47 and an elasticity of hours worked (intensive margin) equal to 0.22.

Female own-wage elasticities are more than twice as large as male own-wage elasticities. Cross-wage elasticities are moderate, although women respond to a rise in husband wage by reducing their market hours and by increasing their non-market hours. The elasticities of home production hours are similar in absolute value to labor supply elasticities.

Finally, the general equilibrium effects of a change in female wage on hours are small because female wage is not a strong determinant of matching. A wage rise, however, renders males more attractive in the marriage market; there are more marriages and increased specialization in paid work for men and housework for women.

TABLE V

WAGE ELASTICITIES

	Actual 2000-02	Baseline sim.	Equilibrium distributions				Fixed distributions			
			$\frac{\Delta w_m}{w_m} = 10\%$		$\frac{\Delta w_f}{w_f} = 10\%$		$\frac{\Delta w_m}{w_m} = 10\%$		$\frac{\Delta w_f}{w_f} = 10\%$	
Labor supply										
Married Men	42.99	42.49	43.02	1.3%	42.44	−0.1%	43.03	1.3%	42.39	−0.2%
Single Men	37.48	36.83	37.78	2.6%	36.78	−0.1%	37.70	2.4%	36.78	−0.1%
Married Women	25.86	26.63	26.46	−0.7%	27.64	3.8%	26.32	−1.2%	27.58	3.5%
Single Women	30.07	29.56	29.48	−0.3%	30.68	3.8%	29.89	1.1%	30.96	4.7%
Home production time										
Married Men	5.13	5.33	5.25	−1.5%	5.34	0.3%	5.24	−1.6%	5.35	0.4%
Single Men	5.00	5.04	5.04	−0.1%	5.04	0.0%	5.03	−0.2%	5.03	−0.1%
Married Women	14.99	15.52	15.64	0.8%	15.08	−2.9%	15.72	1.3%	15.11	−2.7%
Single Women	10.00	10.01	10.04	0.3%	9.73	−2.8%	9.88	−1.3%	9.61	−4.0%
Population										
# single men	1416	1468	1384	−5.7%	1466	−0.1%				
# single women	1452	1509	1458	−3.4%	1495	−0.9%				
# couples	3802	3745	3814	1.8%	3753	0.2%				

[17]We use this sample because it is in the middle of the period. It is also one of the biggest sub-samples, and by being away from the borders, the moving averages of wages and hours are more accurate.

The Determinants of the Gender Labor Supply gap

We then proceed to the simulation of structural changes to the economy so as to better understand the forces driving the gender differences in labor supply. First, we remove the gender wage gap conditional on education and family values. Specifically, we multiply each female wage w_j by $\exp x_j(\gamma_m - \gamma_f)$, where γ_m, γ_f denote the regression coefficients of the regressions of log wages on education and *FVI* separately for men and women. Second, we keep the distribution of characteristics fixed and give all females in the sample the preferences of males with the same characteristics. Third, we do the same for home production (same home production functions for male and female individuals; same coefficients for men and women in the home production for couples). Last, we consider an economy with only liberal individuals ($FVI = 1$).

Table VI displays the results. First, wage equalization has very little demographic effects. This is because matching is essentially independent of female wages. Female market hours increase by about 2h40 (10%) and non-market hours are reduced by 1h10 (7.5%) irrespective of marital status. The effect on male hours is negligible. Second, preference equalization has moderate effects on time uses. Women increase labor supply and work in home by about 7% (+2 h and +1 h). The number of singles falls by about 14%, which is the biggest counterfactual demographic effect. Third, removing female comparative advantage in home production goes a long way toward removing all differences in time uses between men and women (+9 h market and non-market). The number of singles is also reduced but much less than in the preceding case.

Fourth and finally, changing attitudes also has dramatic effects. The number of singles increases, and married women massively substitute paid work to house work (+8 h and −4 h). Married men increase the number of hours devoted to home production by about one hour per week. Figures 11 and 12 show the effect of the last two counterfactuals

TABLE VI

COUNTERFACTUAL SIMULATIONS

	Actual 2000-02	Baseline sim.	(1) No wage gap		(2) Same preferences		(3) Same domestic production		(4) *FVI* = 1 for all	
Labor supply										
Married Men	42.99	42.49	42.37	−0.3%	41.77	−1.7%	42.55	0.2%	43.24	1.8%
Single Men	37.48	36.83	36.71	−0.3%	36.87	0.1%	36.78	−0.1%	36.49	−0.9%
Married Women	25.86	26.63	29.27	9.9%	28.66	7.6%	35.77	34.3%	34.53	29.6%
Single Women	30.07	29.56	32.29	9.3%	30.00	1.5%	34.46	16.6%	30.64	3.7%
Home production time										
Married Men	5.13	5.33	5.37	0.8%	5.56	4.3%	5.35	0.5%	6.26	17.5%
Single Men	5.00	5.04	5.04	−0.1%	5.06	0.3%	5.04	0.1%	4.54	−9.8%
Married Women	14.99	15.52	14.35	−7.5%	16.45	6.0%	6.10	−60.7%	11.45	−26.2%
Single Women	10.00	10.01	9.30	−7.1%	10.63	6.2%	4.90	−51.0%	9.23	−7.8%
Population										
# single men	1416	1468	1475	0.5%	1258	−14.3%	1418	−3.4%	1586	8.0%
# single women	1452	1509	1498	−0.7%	1305	−13.5%	1457	−3.4%	1633	8.2%
# couples	3802	3745	3747	0.1%	3943	5.3%	3786	1.1%	3627	−3.2%

Notes: In simulation (1) (no gender wage gap), we replace each female wage w_j by $w_j \times e^{x_j(\gamma_m - \gamma_f)}$. In simulation (2) (same preferences for men and women), we give females the preferences of males. In simulation (3) (same domestic production for men and women), we give females the domestic production parameters of males. In simulation (4), we make every man and woman in the sample liberal ($FVI = 1$). All percentage changes are with respect to baseline simulation.
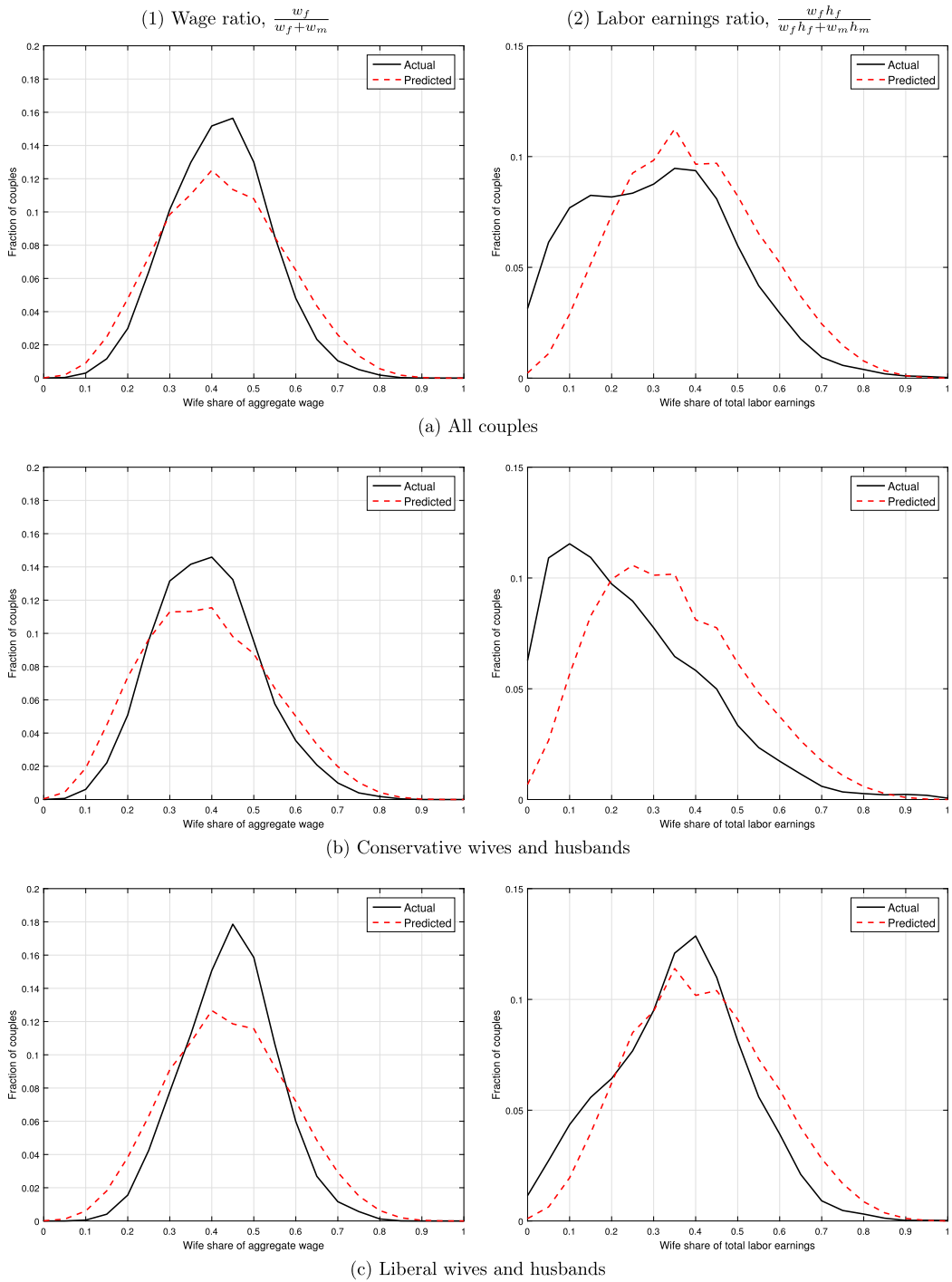
FIGURE 11.—Counterfactual distributions of wages and earnings ratios, 2000–2002—No female home production advantage.

(1) Wage ratio, $\frac{w_f}{w_f + w_m}$ (2) Labor earnings ratio, $\frac{w_f h_f}{w_f h_f + w_m h_m}$
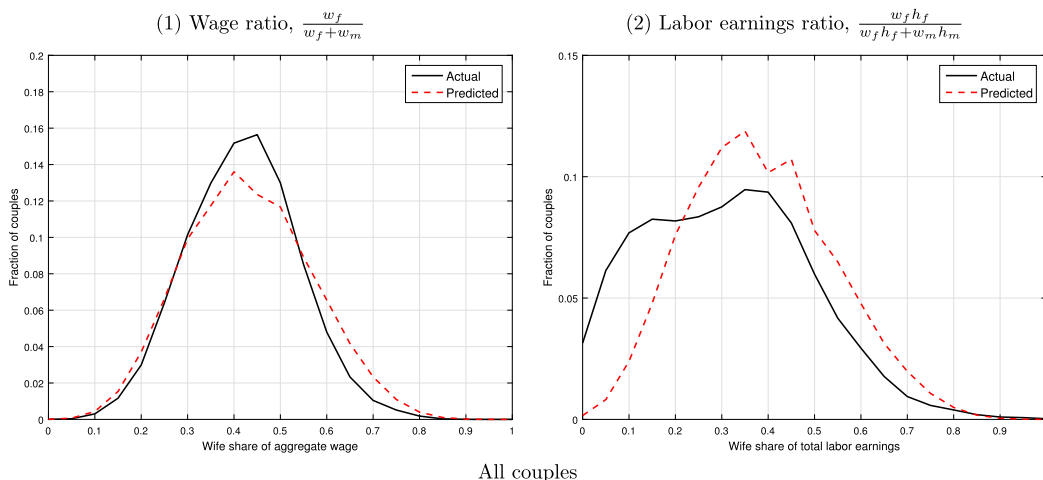

All couples

FIGURE 12.—Counterfactual distributions of wages and earnings ratios, 2000–2002—All liberal.

on wage and earnings ratios. Making everybody liberal comes out to be quantitatively equivalent to removing females' comparative advantage in home production.

8. CONCLUSION

In this paper, we build a model of marriage formation and within-household allocation of resources. By endogenizing marriage formation, we endogenize the sharing rule and its link to distribution factors. The model allows to explain how exogenous changes to wages, education, and family attitudes are transmitted to labor supply and home production time inputs. An important innovation of our work is that, in addition to standard wage and education characteristics, we condition behavior by an index of family values.

Family values are an important factor of gender identity (Akerlof and Kranton (2000)). They definitely shape the way labor supply or work in the household vary by gender. Whereas Bertrand, Kamenica, and Pan (2015) showed that gender differences in labor supply or marriage duration cannot be explained without advocating some strong and slow-changing gender identity norm, we demonstrate that this norm is not uniformly shared by all individuals and households. The most liberal households behave very differently from the most conservative ones. Our structural model allows us to simulate a counterfactual economy where everybody would be liberal. The marriage rate would decline and married women would increase labor market participation very substantially. A two-hour difference remains that can be explained by the gender wage gap.

In future work, it will be interesting to explore further classic topics in family economics, such as fertility and child rearing, divorce laws, the evaluation of family tax credits, etc. In addition, our description of matching can and should be improved by introducing aging and other aspects of the life cycle in the analysis. Allowing for other types of shocks and uncertainty than the bliss shock, such as wage and unemployment shocks, is also important. More generally, our framework should be extended to better understand the link between marriage and labor markets. For example, women who specialize in home production may be losing human capital.

APPENDIX A: EQUILIBRIUM SOLUTION WITH TRANSFERABLE UTILITY

Match Surplus

Spouses solve the Nash bargaining problem

$$\max_{c,d_m,d_f,t_m,t_f} \left[W_m - V_i^0\right]^\beta \left[W_f - V_j^0\right]^{1-\beta}$$

subject to $t_m + t_f = C_{ij}$, where

$$(r+\delta)\left[W_m - V_i^0\right] = u_m + \delta \int \left(V_m^1(z') - V_i^0\right)^+ dG(z') - rV_i^0 \equiv u_m + v_m \quad \text{(say),}$$

$$(r+\delta)\left[W_f - V_j^0\right] = u_f + \delta \int \left(V_f^1(z') - V_j^0\right)^+ dG(z') - rV_j^0 \equiv u_f + v_f,$$

with

$$u_m = q\frac{w_i(1-d_m) - t_m - A_i}{B_i}, \qquad u_f = q\frac{w_j(1-d_f) - t_f - A_j}{B_j}, \qquad q = zF_{ij}^1(d_m, d_f).$$

The first-order conditions for transfers t_m, t_f,

$$\frac{\beta}{(r+\delta)\left[W_m - V_i^0\right]} \frac{\partial u_m}{\partial R} = \frac{1-\beta}{(r+\delta)\left[W_f - V_j^0\right]} \frac{\partial u_f}{\partial R},$$

yield the rent-sharing conditions

$$\frac{\beta}{u_m + v_m}\frac{q}{B_i} = \frac{1-\beta}{u_f + v_f}\frac{q}{B_j} \equiv \frac{q}{(r+\delta)S_{ij}(z)} \quad \text{(say).}$$

It follows that

$$(r+\delta)S_{ij}(z) = B_i(u_m + v_m) + B_j(u_f + v_f) = B_i(\underline{u}_m + v_m) + B_j(\underline{u}_f + v_f) - qC_{ij},$$

using $t_m + t_f = C_{ij}$ and with

$$\underline{u}_m = q\frac{w_i(1-d_m) - A_i}{B_i}, \qquad \underline{u}_f = q\frac{w_j(1-d_f) - A_j}{B_j}.$$

Domestic Production Inputs

The first-order condition for d_m, d_f is

$$\frac{w_i d_m}{\epsilon_m} = \frac{w_j d_f}{\epsilon_f} = q\left[\frac{\partial u_m/\partial q}{\partial u_m/\partial R} + \frac{\partial u_f/\partial q}{\partial u_f/\partial R}\right]$$

$$= w_i(1-d_m) - A_i + w_j(1-d_f) - A_j - C_{ij},$$

for elasticities $\epsilon_m = \frac{\partial \ln F_{ij}^1(d_m,d_f)}{\partial \ln d_m}, \epsilon_f$. The optimal home production inputs d_m, d_f and home production output q are thus simple functions of match characteristics. Let X_{ij} be the equilibrium value of $w_i(1-d_m) - A_i + w_j(1-d_f) - A_j - C_{ij}$ obtained from the above first-order conditions. Because home production depends on z multiplicatively, $q = zF_{ij}^1(d_m, d_f)$, X_{ij} only depends on i, j but not on z.

Continuation Values

Making use of the promise keeping constraints, $W_m = V_m^1(z)$, we have

$$V_m^1(z) - V_i^0 = u_m + v_m = \beta \frac{S_{ij}(z)}{B_i}, \tag{A.1}$$

$$V_f^1(z) - V_j^0 = u_f + v_f = (1 - \beta) \frac{S_{ij}(z)}{B_j}. \tag{A.2}$$

Hence

$$v_m \equiv \delta \int \left(V_m^1(z') - V_i^0 \right)^+ \mathrm{d}G(z') - rV_i^0 = \delta \beta \frac{\overline{S}_{ij}}{B_i} - rV_i^0,$$

$$v_f \equiv \delta \int \left(V_f^1(z') - V_j^0 \right)^+ \mathrm{d}G(z') - rV_j^0 = \delta(1 - \beta) \frac{\overline{S}_{ij}}{B_j} - rV_j^0,$$

for $\overline{S}_{ij} \equiv \int S_{ij}(z')^+ \, \mathrm{d}G(z')$, and

$$(r + \delta)S_{ij}(z) = B_i \left(\underline{u}_m - rV_i^0 \right) + B_j \left(\underline{u}_f - rV_j^0 \right) - qC_{ij} + \delta \overline{S}_{ij}.$$

Thus $S_{ij}(z)$ solves the integral equation

$$(r + \delta)S_{ij}(z) = qX_{ij} - B_i rV_i^0 - B_j rV_j^0 + \delta \overline{S}_{ij}, \tag{A.3}$$

as

$$B_i u_m + B_j u_f = B_i \underline{u}_m + B_j \underline{u}_f - qC_{ij} = qX_{ij} = zF_{ij}^1 X_{ij}.$$

Transfers

Transfers follow from the above rent-sharing equations (equations (A.1) and (A.2)). Indeed, using equation (A.3),

$$B_i u_m = \beta(r + \delta)S_{ij}(z) - B_i v_m = \beta \left[qX_{ij} - B_i rV_i^0 - B_j rV_j^0 \right] + B_i rV_i^0.$$

Hence,

$$q \left[w_i(1 - d_m) - t_m - A_i \right] = B_i rV_i^0 + \beta \left[qX_{ij} - B_i rV_i^0 - B_j rV_j^0 \right], \tag{A.4}$$

with a similar expression for t_f. Note that we can then write

$$w_i(1 - d_m) - t_m - A_i = \beta_{ij}(z)X_{ij}, \qquad w_j(1 - d_f) - t_f - A_j = \left[1 - \beta_{ij}(z) \right]X_{ij},$$

for $\beta_{ij}(z) = \beta + \frac{B_i rV_i^0 - \beta[B_i rV_i^0 + B_j rV_j^0]}{zF_{ij}^1 X_{ij}}$.

Value of Singlehood

It remains to work out the value for singles, that is,

$$rV_i^0 = \max_d \psi_i \left[w_i(1 - d), F_i^0(d) \right] + \beta \lambda \frac{1}{B_i} \iint S_{ij}(z)^+ \, \mathrm{d}G(z)n_f(j) \, \mathrm{d}j, \tag{A.5}$$

with a similar expression for females.

Solving for Values

Let $\overline{S}_{ij} \equiv \int S_{ij}(z)^+ \, dG(z)$ denote the integrated surplus. The following fixed-point equation thus defines $\overline{S}_{ij}$:

$$(r + \delta)\overline{S}_{ij} = F_{ij}^1 X_{ij} \mathcal{G}\left(\frac{B_i r V_i^0 + B_j r V_j^0 - \delta \overline{S}_{ij}}{F_{ij}^1 X_{ij}} \right), \tag{A.6}$$

with $\mathcal{G}(s) \equiv \int (z - s)^+ \, dG(z) = \int_s^{+\infty} z \, dG(z) - s[1 - G(s)]$. Note that $\mathcal{G}'(s) = -[1 - G(s)] \in (-1, 0)$ for all interior point s.[18] Moreover,

$$B_i r V_i^0 = B_i u_i^0 + \lambda \beta \int \overline{S}_{ij} n_f(j) \, dj,$$

and

$$B_j r V_j^0 = B_j u_j^0 + \lambda(1 - \beta) \int \overline{S}_{ij} n_m(i) \, di.$$

Then $S_{ij}(z)$ follows from solving equation (A.6) after substitution of the values for singles.

APPENDIX B: IDENTIFICATION

In this appendix, we develop the identification arguments in the text.

Step 1: Meetings and Matching Probability

The number of new marriages (or cohabitations) of type (i, j) per unit of time is

$$MF(i, j) = \lambda n_m(i) n_f(j) \alpha_{ij}. \tag{B.1}$$

We observe the flows $MF(i, j)$ and the stocks $n_m(i), n_f(j)$. We are thus faced with the usual inference problem of disentangling λ from α_{ij}, the meeting rate from the matching probability.

The solution lies in the common mechanism linking marriage and divorce flows. Divorce occurs when the last draw of z (occurring at rate δ) ceases to satisfy the matching rule (with probability $1 - \alpha_{ij}$). It follows that the flow of divorces per unit of time is

$$DF(i, j) = m(i, j)\delta(1 - \alpha_{ij}). \tag{B.2}$$

Define $MR(i, j) \equiv \frac{MF(i,j)}{n_m(i) n_f(j)}$, the marriage rate (by potential match type), and $DR(i, j) \equiv \frac{DF(i,j)}{m(i,j)}$, the divorce rate. Eliminating α_{ij} from equations (B.1) and (B.2) yields

$$\frac{MR(i, j)}{\lambda} + \frac{DR(i, j)}{\delta} = 1. \tag{B.3}$$

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[18]In the particular log-normal case of $G(s) = \Phi(\ln s / \sigma)$ and $G^{-1}(x) = e^{\sigma \Phi^{-1}(x)}$, then

$$\mathcal{G}(s) = -s\Phi\left(-\frac{\ln s}{\sigma} \right) + e^{\frac{\sigma^2}{2}} \Phi\left(-\frac{\ln s}{\sigma} + \sigma \right).$$

Under the assumption that α_{ij} is not a constant independent of couple characteristics, then $MR(i, j)$ and $DR(i, j)$ are not collinear and this equation identifies $1/\lambda$ and $1/\delta$. Then, α_{ij} follows as

$$\alpha_{ij} = \frac{\delta MR(i, j)}{\delta MR(i, j) + \lambda DR(i, j)}. \tag{B.4}$$

Step 2: Preferences and Home Production

Fix socio-demographics (x_i, x_j). Given x_i, x_j, all parameters are thus only gender-specific or constant. All remaining variation is due to wages (w_i, w_j).

Claim 1. The match-specific source of variation in the sharing rule $\beta_{ij}(z)$, namely z, allows to identify b_m/b_f by regressing $w_i e_m^1$ on $w_j e_f^1$, keeping wages constant.

PROOF: Consider married couples. The leisure expenditures of husband and wife are

$$w_i e_m^1 = a_{1m} w_i + a_{2m} w_i^2 + b_m \beta_{ij}(z) X_{ij}, \tag{B.5}$$

$$w_j e_f^1 = a_{1f} w_j + a_{2f} w_j^2 + b_f [1 - \beta_{ij}(z)] X_{ij}, \tag{B.6}$$

with

$$X_{ij} = \frac{w_i(1 - D_m^1) + w_j(1 - D_f^1) - C - A_i - A_j}{1 + K_m^1 + K_f^1}. \tag{B.7}$$

The sharing rule $\beta_{ij}(z)$ is a function of wages w_i, w_j and z given observed spouse types x_i and x_j. Eliminating $\beta_{ij}(z)$ out of equations (B.5) and (B.6), we obtain the following structural link between the spouses' leisure supplies:

$$w_i e_m^1 + \frac{b_m}{b_f} w_j e_f^1 = a_{1m} w_i + a_{2m} w_i^2 + \frac{b_m}{b_f}(a_{1f} w_j + a_{2f} w_j^2) + b_m X_{ij}. \tag{B.8}$$

This equation implies that conditional on all observable characteristics and wages, the right-hand side of this equation is constant and residual variation with respect to z in leisure expenditures identifies b_m/b_f.   *Q.E.D.*

Claim 2. Parameters $a_{2m}, a_{2f}, b_m, b_f, K_m^1, K_f^1, a_{1m}, a_{1f}, D_m^1, D_f^1$ are identified from the nonlinear link between expenditures $w_i d_m^1, w_j d_f^1, w_i e_m^1 + \frac{b_m}{b_f} w_j e_f^1$ and wages w_i, w_j if $K_m^1 + K_f^1, a_{2m}$, and $a_{2f} \neq 0$.

PROOF: Consider domestic time expenditures:

$$w_i d_m^1 = w_i D_m^1 + K_m^1 X_{ij}, \qquad w_j d_f^1 = w_j D_f^1 + K_f^1 X_{ij}. \tag{B.9}$$

As D_i^1, D_j^1, and C_{ij} do not depend on wages, denoting $k_m^1 = \frac{K_m^1}{1 + K_m^1 + K_f^1}$,

$$\frac{\partial w_i d_m^1}{\partial w_i} = D_m^1 + K_m^1 \frac{\partial X_{ij}}{\partial w_i} = D_m^1 + k_m^1(1 - D_m^1 - a_{1m} - a_{2m} w_i), \tag{B.10}$$

$$\frac{\partial w_i d_m^1}{\partial w_j} = K_m^1 \frac{\partial X_{ij}}{\partial w_j} = k_m^1(1 - D_f^1 - a_{1f} - a_{2f} w_j). \tag{B.11}$$

The variation of these derivatives with respect to wages identifies $k_m^1 a_{2m}$ and $k_m^1 a_{2f}$. By symmetry, $k_f^1 a_{2m}$ and $k_f^1 a_{2f}$ are also identified (with $k_f^1 = \frac{K_f^1}{1+K_m^1+K_f^1}$).

Moreover, differentiating equation (B.8),

$$\frac{\partial w_i e_m^1}{\partial w_i} + \frac{b_m}{b_f} \frac{\partial w_j e_f^1}{\partial w_i} = a_{1m} + 2a_{2m}w_i + b_m \frac{\partial X_{ij}}{\partial w_i}$$

$$= a_{1m} + 2a_{2m}w_i + b_m \frac{1 - D_m^1 - a_{1m} - a_{2m}w_i}{1 + K_m^1 + K_f^1},$$

which identifies $[2 - b_m(1 - k^1)]a_{2m}$, denoting $k^1 = k_m^1 + k_f^1$. By symmetry, that is, differentiating with respect to the other wage, $[2 - b_f(1 - k^1)]a_{2f}$ is also identified.

Let $x_m = [2 - b_m(1 - k^1)]a_{2m}$ and $y_m = k^1 a_{2m}$, with symmetric notations for x_f, y_f. Let $t = b_m/b_f$. Knowing b_m/b_f, $k_m^1 a_{2m}$, $k_m^1 a_{2f}$ and $k_f^1 a_{2m}$, $k_f^1 a_{2f}$, if y_m and $y_f \neq 0$, we have $k^1 = \frac{2(1-t)}{x_m/y_m - 2x_f/y_f}$. Then a_{2m} and b_m follow. For y_m and y_f to be different from 0, it is required that $a_{2m}, a_{2f}, k^1 \neq 0$. The demand system must not be linear and home production time uses must respond to wages. Under this condition, $k_m^1, k_f^1, a_{2m}, a_{2f}, b_m, b_f$ are identified.

Next, the intercept of $\partial w_i d_j^1 / \partial w_i$ identifies $1 - D_m^1 - a_{1m}$ (equation (B.11)); $\partial w_i d_m^1 / \partial w_i$ identifies D_m^1 (equation (B.10)); $\partial(w_i e_m^1 + \frac{b_m}{b_f} w_j e_f^1)/\partial w_i$ identifies a_{1m}. A symmetric argument applies for a_{1f} and D_f^1.

Finally, parameter X_{ij} is identified from equation (B.8), and parameter C will follow from equation (B.7) if we can identify a_{0m} and a_{0f}. We do that by proving the next claim. *Q.E.D.*

Claim 3. Parameters K_m^0, K_f^0 and a_{0m}, a_{0f} are identified from the time uses (leisure and home production) of singles.

PROOF: The leisure equation for singles,

$$w_j e_f^0 = a_{1f} w_j + a_{2f} w_j^2 + b_f \big[w_j (1 - d_j^0) - A_j \big],$$

$$A_j = a_{0f} + a_{1f} w_j + \frac{1}{2} a_{2f} w_j^2, \tag{B.12}$$

identifies $b_f a_{0f}$, hence a_{0f}, all other parameters being known at this stage. The RHS of home production time,

$$w_j d_f^0 = w_j D_f^0 + \frac{K_f^0}{1 + K_f^0} \big[w_j (1 - D_f^0) - A_j \big], \tag{B.13}$$

is a quadratic function of w_j, which identifies K_f^0 and D_f^0. *Q.E.D.*

Step 3: Sharing Rule and the Distribution of Match-Specific Shocks

At this stage, the sharing rule $\beta_{ij}(z)$ is identified from leisure expenditures using equations (B.5) or (B.6). Identification in step 2 was based on the co-variations of expenditures $w_i e_m^1$ and $w_j e_f^1$ across different match types (i, j). The exact levels are determined by the sharing rule $\beta_{ij}(z)$.

Now, remember that

$$\beta_{ij}(z) = \beta + \frac{1}{z}\frac{(1-\beta)B_i rV_i^0 - \beta B_j rV_j^0}{F_{ij}^1 X_{ij}},$$

where

$$B_i rV_i^0 = B_i u_i^0 + \lambda\beta\int \overline{S}_{ij} n_f(j)\,\mathrm{d}j, \qquad B_j rV_j^0 = B_j u_j^0 + \lambda(1-\beta)\int \overline{S}_{ij} n_m(i)\,\mathrm{d}i.$$

In the text, we derived the following formulas for $\overline{S}_{ij}$ and α_{ij}:

$$(r+\delta)\overline{S}_{ij} = F_{ij}^1 X_{ij}\mathcal{G}\left(\frac{B_i rV_i^0 + B_j rV_j^0 - \delta\overline{S}_{ij}}{F_{ij}^1 X_{ij}}\right),$$

which gives $\frac{B_i rV_i^0 + B_j rV_j^0}{F_{ij}^1 X_{ij}}$ given $\frac{\overline{S}_{ij}}{F_{ij}^1 X_{ij}}$,

$$\alpha_{ij} = 1 - G\left[\mathcal{G}^{-1}\left(\frac{(r+\delta)\overline{S}_{ij}}{F_{ij}^1 X_{ij}}\right)\right], \tag{B.14}$$

which gives $\frac{\overline{S}_{ij}}{F_{ij}^1 X_{ij}}$ given α_{ij}. These two equations yield the following link between $\overline{S}_{ij}$ and $B_i rV_i^0 + B_j rV_j^0$:

$$\delta\overline{S}_{ij} = \left(B_i rV_i^0 + B_j rV_j^0\right)\theta_{ij}, \tag{B.15}$$

with

$$\theta_{ij} \equiv \frac{\mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right]}{\frac{r+\delta}{\delta}G^{-1}(1-\alpha_{ij}) + \mathcal{G}\left[G^{-1}(1-\alpha_{ij})\right]}.$$

Parameter θ_{ij} is identified up to G or σ, as the matching probability α_{ij} is identified in step 1.

This in turn implies that $(B_i rV_i^0, B_j rV_j^0)$ is the solution to the Fredholm equation of the second kind:

$$B_i rV_i^0 = B_i u_i^0 + \frac{\lambda}{\delta}\beta\int \left(B_i rV_i^0 + B_j rV_j^0\right)\theta_{ij} n_f(j)\,\mathrm{d}j, \tag{B.16}$$

$$B_j rV_j^0 = B_j u_j^0 + \frac{\lambda}{\delta}(1-\beta)\int \left(B_i rV_i^0 + B_j rV_j^0\right)\theta_{ij} n_m(i)\,\mathrm{d}i. \tag{B.17}$$

There will be a unique solution for $(B_i rV_i^0, B_j rV_j^0)$ to system (B.16)–(B.17) if the right-hand side's linear integral operator is contracting. A sufficient condition for that is $\lambda\beta\int \theta_{ij} n_f(j)\,\mathrm{d}j < \delta$ and $\lambda(1-\beta)\int \theta_{ij} n_m(i)\,\mathrm{d}i < \delta$. We have $0 \le \theta_{ij} \le 1$. We will estimate meeting rates $\lambda N_m \simeq \lambda N_f \simeq 0.15$, $\delta = 0.038$, and $\beta \simeq 0.45$. So $\lambda N_m \beta/\delta \simeq 2$. We will estimate the mean of θ_{ij}, averaged across i and j among singles, below $1/2$. This is sufficient for identification.

Hence values $B_i rV_i^0, B_j rV_j^0$ can be identified up to β and G and given the distributions of types amongst singles and the matching probabilities, which have already been identified in step 1. Then we get $\overline{S}_{ij}$ from equation (B.15) and $F_{ij}^1 X_{ij}$ from equation (B.14).

Although we cannot formally prove identification given the amount of nonlinearities, we can yet safely base the identification of β and σ (the variance of z) on first- and second-order moments of the distribution of the sharing rule.

Step 4: Public Good Quality

The last parameter to identify is public good quality Z_{ij}. For this, we can use equation (B.15) to deduce $\overline{S}_{ij}$ from $B_i r V_i^0 + B_j r V_j^0$. Then equation (B.14) allows to identify $F_{ij}^1 X_{ij}$. The identification of Z_{ij} finally results from equilibrium home production F_{ij}^1:

$$F_{ij}^1 = Z_{ij} \left(\frac{K_m^1}{w_i} \right)^{K_m^1} \left(\frac{K_f^1}{w_j} \right)^{K_f^1} X_{ij}^{K_m^1 + K_f^1}. \tag{B.18}$$

APPENDIX C: ESTIMATION DETAILS

We first define the following residuals.

1. For single men, the residuals are

$$\varepsilon_i^0 = d_i^0 - D_i^0 - K_m^0 X_i^0 / w_i,$$

$$\eta_i^0 = e_i^0 - a_{1i} - a_{2m} w_i - b_i X_i^0 / w_i,$$

with $X_i^0 = \frac{w_i[1 - D_i^0] - A_i}{1 + K_m^0}$. Similar expressions can be obtained for single women.

2. For couples, the residuals for conditional means are

$$\varepsilon_{mij}^1 = d_{mij}^1 - D_i^1 - K_m^1 X_{ij} / w_i,$$

$$\varepsilon_{fij}^1 = d_{fij}^1 - D_j^1 - K_f^1 X_{ij} / w_j,$$

$$\eta_{mij}^1 = e_{mij}^1 - a_{1i} - a_{2m} w_i - b_i \overline{\beta}_{ij} X_{ij} / w_i,$$

$$\eta_{fij}^1 = e_{fij}^1 - a_{1j} - a_{2f} w_j - b_j (1 - \overline{\beta}_{ij}) X_{ij} / w_j,$$

where

$$\overline{\beta}_{ij} = \mathbb{E}\big[\beta_{ij}(z) | z \geq G^{-1}(1 - \alpha_{ij}) \big]$$

$$= \beta + \mathbb{E}\left[\frac{1}{z} \Big| z \geq G^{-1}(1 - \alpha_{ij}) \right] \frac{(1 - \beta) B_i r V_i^0 - \beta B_j r V_j^0}{F_{ij}^1 X_{ij}}. \tag{C.1}$$

We solve the Fredholm equation for $B_i r V_i^0, B_j r V_j^0$ on the Chebyshev grids mentioned earlier. This allows to approximate the integrals by finite sums using Clenshaw–Curtis quadrature. The Fredholm equations

$$B_i r V_i^0 = B_i u_i^0 + \frac{\lambda}{\delta} \beta \int \big(B_i r V_i^0 + B_j r V_j^0 \big) \theta_{ij} n_f(j) \, dj, \tag{C.2}$$

$$B_j r V_j^0 = B_j u_j^0 + \frac{\lambda}{\delta} (1 - \beta) \int \big(B_i r V_i^0 + B_j r V_j^0 \big) \theta_{ij} n_m(i) \, di \tag{C.3}$$

thus become a standard linear system that can be solved by matrix inversion. See Appendix D for details.

3. To identify σ, we use second-order moments for leisure. Specifically,

$$\nu^1_{mmij} = \left(\eta^1_{mij}\right)^2 - b_i^2 \sigma_{ij}^2 X_{ij}^2 / w_i^2,$$

$$\nu^1_{ffij} = \left(\eta^1_{fij}\right)^2 - b_j^2 \sigma_{ij}^2 X_{ij}^2 / w_j^2,$$

$$\nu^1_{mfij} = \eta^1_{mij} \eta^1_{fij} + b_i b_j \sigma_{ij}^2 X_{ij}^2 / w_i / w_j,$$

where

$$\sigma_{ij}^2 = \mathbb{V}\left[\beta_{ij}(z) | z \geq G^{-1}(1 - \alpha_{ij})\right]$$

$$= \mathbb{V}\left[\frac{1}{z} \bigg| z \geq G^{-1}(1 - \alpha_{ij})\right]\left(\frac{(1-\beta)B_i r V_i^0 - \beta B_j r V_j^0}{F^1_{ij} X_{ij}}\right)^2.$$

(C.4)

We minimize the sum of squared residuals ε, η, and ν with respect to the parameters of preferences, home production, β, and σ. Once all these parameters have been estimated, we estimate public good quality Z_{ij} from equilibrium home production F^1_{ij}:

$$Z_{ij} = F^1_{ij} X_{ij}\left[\left(\frac{K_m^1}{w_i}\right)^{K_m^1}\left(\frac{K_f^1}{w_j}\right)^{K_f^1} X_{ij}^{1 + K_m^1 + K_f^1}\right]^{-1}.$$

APPENDIX D: COMPUTATIONAL DETAILS

This appendix shortly describes the numerical tools used in estimation. The best reference here is Trefethen (2013).

D.1. *Chebyshev Nodes*

We discretize continuous functions on a compact domain using Chebyshev grids. For example, let $[\underline{x}, \overline{x}]$ denote the support of male wages; we construct a grid of $n + 1$ points as

$$x_k = \frac{\overline{x} + \underline{x}}{2} + \frac{\overline{x} - \underline{x}}{2}\cos\frac{k\pi}{n}, \quad k = 0, \ldots, n.$$

D.2. *Clenshaw–Curtis Quadrature*

Many equations involve integrals. Given Chebyshev grids, it is natural to use Clenshaw–Curtis quadrature to approximate these integrals:

$$\int_{\underline{x}}^{\overline{x}} f(x)\,\mathrm{d}x \simeq \frac{\overline{x} - \underline{x}}{2}\sum_{k=0}^{n}\omega_k f(x_j),$$

where the weights ω_j can be easily computed using Fast Fourier Transform (FFT). The following MATLAB code can be used to implement CC quadrature (Waldvogel (2006)):

```
function [nodes,wcc] = cc(n)
nodes = cos(pi*(0:n)/n);
N=[1:2:n-1]'; l=length(N); m=n-1;
```

```
v0=[2./N./(N-2); 1/N(end); zeros(m,1)];
v2=-v0(1:end-1)-v0(end:-1:2);
g0=-ones(n,1); g0(1+l)=g0(1+l)+n; g0(1+m)=g0(1+m)+n;
g=g0/(n^2-1+mod(n,2)); wcc=real(ifft(v2+g));
wcc=[wcc;wcc(1)];
```

Note that, although Gaussian quadrature provides exact evaluations of integrals for higher-order polynomials than CC, in practice CC works as well as Gaussian. On the other hand, quadrature weights are much more difficult to calculate for Gaussian quadrature. See Trefethen (2008).

D.3. *Integral Equations*

We need to solve functional fixed-point equations. The standard algorithm to calculate the fixed point $u(x) = T[u](x)$ is to iterate $u_{p+1}(x) = Tu_p(x)$ on a grid. If the fixed-point operator T involves integrals, we simply iterate the finite-dimensional operator $\widehat{T}$ obtained by replacing the integrals by their approximations at grid points.

Example 1: Equilibrium

Using the previous approximations, an equation like

$$u(x) = T[u](x) = \frac{\ell(x)}{1 + \rho \displaystyle\int_{\underline{x}}^{\overline{x}} u(y)\alpha(x, y)\, \mathrm{d}y}$$

becomes

$$\boldsymbol{u} = \big[u(x_k)\big]_{k=0,\dots,n} = \widehat{T}(\mathbf{u}) = \left[\frac{\ell(x_k)}{1 + \rho \displaystyle\sum_{\ell=0}^{n} \omega_\ell u(x_\ell)\alpha(x_k, x_\ell)}\right]_{k=0,\dots,n}.$$

It was sometimes necessary to "shrink" steps by using iterations of the form $u_{p+1} = u_p + \theta(Tu_p - u_p)$ with $\theta \in (0, 1]$. A step size $\theta < 1$ may help if T is not everywhere strictly contracting.

Example 2: Singles' Values

$(B_i r V_i^0, B_j r V_j^0)$ solve the inhomogeneous Fredholm system

$$B_i r V_i^0 = B_i u_i^0 + \frac{\lambda}{\delta}\beta \int \big(B_i r V_i^0 + B_j r V_j^0\big)\theta_{ij} n_f(j)\, \mathrm{d}j,$$

$$B_j r V_j^0 = B_j u_j^0 + \frac{\lambda}{\delta}(1 - \beta) \int \big(B_i r V_i^0 + B_j r V_j^0\big)\theta_{ij} n_m(i)\, \mathrm{d}i.$$

This is a linear system that can be solved for after discretizing the state space, or value function iteration. Suppose that i and j are discrete variables with weights ω_i, ω_j. Define the matrices

$$\Theta_m = -\frac{\lambda\beta}{\delta}\big[\theta_{ij} n_f(j)\omega_j\big]_{i \times j}, \qquad \Theta_f = -\frac{\lambda(1 - \beta)}{\delta}\big[\theta_{ij} n_m(i)\omega_i\big]_{i \times j}^{\top},$$

and

$$\Delta_m = 1 - \frac{\lambda\beta}{\delta} \operatorname{diag}\left(\sum_j \theta_{ij} n_f(j)\omega_j\right), \qquad \Delta_f = 1 - \frac{\lambda(1-\beta)}{\delta} \operatorname{diag}\left(\sum_i \theta_{ij} n_m(i)\omega_i\right).$$

Then

$$\begin{bmatrix} B_i r V_i^0 \\ B_j r V_j^0 \end{bmatrix} = \begin{pmatrix} \Delta_m & \Theta_m \\ \Theta_f & \Delta_f \end{pmatrix}^{-1} \begin{bmatrix} B_i u_i^0 \\ B_j u_j^0 \end{bmatrix}.$$

D.4. *Interpolation*

We can interpolate functions very easily between points $y_0 = f(x_0), \ldots, y_n = f(x_n)$ using Discrete Cosine Transform (DCT):

$$f(x) = \sum_{k=0}^n Y_k \cdot T_k(x), \tag{D.1}$$

where Y_k are the OLS estimates of the regression of $y = (y_0, \ldots, y_n)$ on Chebyshev polynomials

$$T_k(x) = \cos\left(k \arccos\left(\frac{x - \dfrac{\overline{x} + \underline{x}}{2}}{\dfrac{\overline{x} - \underline{x}}{2}}\right)\right),$$

but are more effectively calculated using FFT. A MATLAB code for DCT is, with $y = (y_0, \ldots, y_n)$:

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
f = @(x) cos(acos((2*x-(xmin+xmax))/(xmax-xmin))
*(0:n))*Y(1:n+1);
```

A bidimensional version is

```
Y = y([1:n+1 n:-1:2],:);
Y = real(fft(Y/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)];
Y = Y(:,[1:n+1 n:-1:2]);
Y = real(fft(Y'/2/n));
Y = [Y(1,:); Y(2:n,:)+Y(2*n:-1:n+2,:); Y(n+1,:)]';
f=@(x,y)cos(acos((2*x-(xmin+xmax))/(xmax-xmin))*(0:n))...
*Y(1:n+1,1:n+1)...
*cos((0:n)'*acos((2*y'-(ymin+ymax))/(ymax-ymin)));
```

The fact that the grid $(x_0, \ldots, x_n)$ is not uniform and is denser towards the edges of the support interval allows to minimize the interpolation error and thus avoids the standard problem of strong oscillations at the edges of the interpolation interval (Runge's phenomenon).

Another advantage of DCT is that, having calculated $Y_0, \ldots, Y_n$, then polynomial projections of $y = (y_0, \ldots, y_n)$ of any order $p \le n$ are obtained by stopping the summation in equation (D.1) at $k = p$. Finally, it is easy to approximate the derivative f' or the primitive

$\int f$ simply by differentiating or integrating Chebyshev polynomials using

$$\cos(k \arccos x)' = \frac{k \sin(k \arccos x)}{\sin(\arccos x)},$$

and

$$\int \cos(k \arccos x)\, \mathrm{d}x = \begin{cases} x & \text{if } k = 0, \\ \dfrac{x^2}{2} & \text{if } k = 1, \\ \dfrac{\cos(k+1)x}{2(k+1)} - \dfrac{\cos(k-1)x}{2(k-1)} & \text{if } k \geq 2. \end{cases}$$

In calculating an approximation of the derivative, it is useful to smoothen the function by summing over only a few polynomials. Derivatives are otherwise badly calculated near the boundary. Moreover, our experience is that the approximation

$$\int_{\underline{x}}^{x} \mathbf{1}\{t \leq x\} f(x)\, \mathrm{d}x \simeq \sum_{k=0}^{n} w_k \mathbf{1}\{t \leq x_k\} f(x_k)$$

gave similar results as integrating the interpolated function.

D.5. *Kernels*

In the application, we had to calculate the density of variables such as the wage ratio $y_{ij} = \frac{w_j}{w_i + w_j}$ across couples (i, j). A kernel density estimator of the distribution of M observations y_{ij} is

$$\widehat{f}(y) = \frac{1}{M} \sum_{(i,j)} K_h(y - y_{ij}),$$

for a kernel K_h with width h. When we use the model to predict this density, we use our estimate of the match distribution $m(w_m, w_f)$ (omitting socio-demographic characteristics for simplicity, which we average over), which we have tabulated on the Chebyshev grid (using the same notation for the data and the grid points):

$$w_i = \frac{\overline{w}_m + \underline{w}_m}{2} + \frac{\overline{w}_m - \underline{w}_m}{2} \cos \frac{i\pi}{n}, \quad i = 0, \dots, n,$$

$$w_j = \frac{\overline{w}_f + \underline{w}_f}{2} + \frac{\overline{w}_f - \underline{w}_f}{2} \cos \frac{j\pi}{n}, \quad j = 0, \dots, n.$$

Then we calculate

$$\widehat{\widehat{f}}(y) = \frac{\overline{w}_m - \underline{w}_m}{2} \frac{\overline{w}_f - \underline{w}_f}{2} \sum_{i,j=0}^{n} K_h\left(y - \frac{w_j}{w_i + w_j}\right) m(w_i, w_j).$$

We do the same for earnings ratios.

To calculate the conditional mean matching probability given wage ratio, we use a Nadaraya–Watson estimator on data:

$$\widehat{\mathbb{E}}\left[\alpha_{ij}\,\bigg|\,\frac{w_j}{w_i + w_j} = y\right] = \frac{\sum\limits_{(i,j)} \alpha_{ij} K_h\left(y - \dfrac{w_j}{w_i + w_j}\right)}{\sum\limits_{(i,j)} K_h\left(y - \dfrac{w_j}{w_i + w_j}\right)}.$$

The prediction is

$$\widehat{\widehat{\mathbb{E}}}\left[\alpha(w_i, w_j)\,\bigg|\,\frac{w_j}{w_i + w_j} = y\right] = \frac{\sum\limits_{(i,j)} \alpha(w_i, w_j) K_h\left(y - \dfrac{w_j}{w_i + w_j}\right) m(w_i, w_j)}{\sum\limits_{(i,j)} K_h\left(y - \dfrac{w_j}{w_i + w_j}\right) m(w_i, w_j)},$$

where $\alpha(w_i, w_j)$ is the predicted matching probability evaluated on the Chebyshev grid.

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