Reconciling Hayek's and Keynes' Views of Recessions

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Abstract

Recessions often happen after periods of rapid accumulation of houses, consumer durables and business capital. This observation has led some economists, most notably Friedrich Hayek, to conclude that recessions often reflect periods of needed liquidation resulting from past over-investment. According to the main proponents of this view, government spending or any other form of aggregate demand policy should not be used to mitigate such a liquidation process, as doing so would simply result in a needed adjustment being postponed. In contrast, ever since the work of Keynes, many economists have viewed recessions as periods of deficient demand that should be countered by activist fiscal policy. In this paper we reexamine the liquidation perspective of recessions in a setup where prices are flexible but where not all trades are coordinated by centralized markets. The model illustrates why liquidations likely cause recessions characterized by deficient aggregate demand and accordingly suggests that Keynes' and Hayek's views of recessions may be closely linked. In our framework, interventions aimed at stimulating aggregate demand face a trade-off whereby current stimulus postpones the adjustment process and therefore prolongs the recessions, but where some stimulative policies may nevertheless remain desirable.

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1 Introduction

There remains considerable debate regarding the causes and consequences of recessions. Two views that are often presented as opposing, and which created controversy in the recent recession and its aftermath, are those associated with the ideas of Hayek and Keynes.[1] The Hayekian perspective is generally associated with viewing recessions as a necessary evil. According to this view, recessions mainly reflect periods of liquidation resulting from past over-accumulation of capital goods. A situation where the economy needs to liquidate such an excess can quite naturally give rise to a recession, but government spending aimed at stimulating activity, it is argued, is not warranted since it would mainly delay the needed adjustment process and thereby postpone the recovery. In contrast, the Keynesian view suggests that recessions reflect periods of deficient aggregate demand where the economy is not effectively exploiting the gains from trade between individuals. According to this view, policy interventions aimed at increasing investment and consumption are generally desirable, as they favor the resumption of mutually beneficial trade between individuals.[2]

In this paper we reexamine the liquidationist perspective of recessions in an environment with decentralized markets, flexible prices and search frictions. In particular, we examine how the economy adjusts when it inherits from the past an excessive amount of capital goods, which could be in the form of houses, durable goods or productive capital. Our goal is not to offer a novel explanation of why the economy may have over-accumulated in the past,[3] but to ask how it reacts to such an over-accumulation once it is realized. As suggested by Hayek, such a situation can readily lead to a recession as less economic activity is generally warranted when agents want to deplete past over-accumulation. However, because of the endogenous emergence of unemployment risk in our set-up, the size and duration of the recession implied

[1] In response to the large recession in the US and abroad in 2008-2009, a high-profile debate around these two views was organized by Reuters. See `http://www.reuters.com/subjects/keynes-hayek`. See also Wapshott [2012] for a popular account of the Hayek-Keynes controversy.

[2] See Caballero and Hammour [2005] for an alternative view on the inefficiency of liquidations, based on the reduction of cumulative reallocation and inefficient restructuring in recessions.

[3] There are several reason why an economy may over-accumulate capital. For example, agents may have had overly optimistic expectations about future expected economic growth that did not materialize, as in Beaudry and Portier [2004], or it could have been the case that credit supply was unduly subsidized either through explicit policy, as argued in Mian and Sufi [2010] and Mian, Sufi, and Trebbi [2010], or as a by-product of monetary policy, as studied by Bordo and Landon-Lane [2013].

by the need for liquidation is not socially optimal. In effect, the reduced gains from trade between individuals induced by the need for liquidation creates a multiplier process that leads to an excessive reduction in activity. Although prices are free to adjust, the liquidation creates a period of deficient aggregate demand where economic activity is too low because people spend too cautiously due to increased unemployment risk. In this sense, we argue that liquidation and deficient aggregate demand should not be viewed as alternative theories of recessions but instead should be seen as complements, where past over-accumulation may be a key driver of periods of deficient aggregate demand. This perspective also makes salient the trade-offs faced by policy. In particular, a policy-maker in our environment faces an unpleasant trade-off between the prescriptions emphasized by Keynes and Hayek. On the one hand, a policy-maker would want to stimulate economic activity during a liquidation-induced recession because consumers are too cautious. On the other hand, the policy-maker also needs to recognize that intervention will likely postpone recovery, since it slows down the needed depletion of excess capital. The model offers a simple framework where both of these forces are present and can be compared.

One potential criticism of a pure liquidationist view of recessions is that, if markets functioned efficiently, such periods should not be very socially painful. In particular, if economic agents interact in perfect markets and realize they have over-accumulated in the past, this should lead them to enjoy a sort of holiday paid for by their past excessive work. Looking backwards in such a situation, agents may resent the whole episode, but looking forward after a period of over-accumulation, they should nonetheless feel content to enjoy the proceeds of the past excessive work, even if it is associated with a recession. In contrast, in our environment we will show that liquidation periods are generally socially painful because of the multiplier process induced by cautious spending and unemployment risk. In effect, we will show that everyone in our model economy can be worse off when they inherit too many capital goods from the past. This result, whereby abundance creates scarcity, may appear quite counter-intuitive at first pass. To make as clear as possible the mechanism that can cause welfare to be reduced by such abundance, much of our analysis will focus on the case where the inherited capital takes the form of a good that directly contributes to utility, such as houses or durable goods. In this situation we will show why inheriting too many houses or durables can make everyone worse off when decisions are decentralized.

A second potential criticism of a pure liquidationist view of recessions is that it often fails to explain why the economy does not simply reallocate factors to non-durables-producing sectors during the liquidationist period, and thereby maintain high employment. This criticism of the liquidationist view has been made forcefully, among others, by Krugman [1998]. In particular, this line of criticism argues that since recessions are generally characterized by decreased production in almost all sectors, this constitutes clear evidence against the liquidationist view. In this paper we show why the coordination problem that arises initially in the durable goods sector as a result of past over-accumulation can create a contagion effect that causes consumption of both durable *and* non-durable goods to decrease simultaneously. The force that links the markets, and that makes them function as complements instead of substitutes, is precautionary behavior. Once there is less demand in the durable goods sector, agents fearing unemployment reduce demand in both sectors, thereby causing the decreased demand in the durable sector to spillover into non-durables.

The structure of our model builds on the literature related to search models of decentralized trading. In particular, we build on Lucas [1990], Shi [1998], Lagos and Wright [2005] and Rocheteau and Wright [2005] by allowing alternating decentralized and centralized markets to allow for a simple characterization of the equilibrium. However, unlike those papers, we do not have money in our setup. The paper also shares key features with the long tradition of macro models emphasizing strategic complementarities, aggregate demand externalities and multipliers, such as Diamond [1982], Cooper and John [1988] and more recently Angeletos and La'O [2013].

Unemployment risk and its effects on consumption decisions is at the core of our model. The empirical relevance of precautionary saving related to unemployment risk has been documented by many, including Carroll [1992], Carroll and Dunn [1997], Carroll, Sommer, and Slacalek [2012] and Alan, Crossley, and Low [2012]. There are also recent theoretical papers that emphasize how unemployment risk and precautionary savings can amplify shocks and cause business cycle fluctuations. These papers are the closest to our work. For example, Challe and Ragot [2013] have proposed a tractable quantitative model in which uninsurable unemployment risk is the source of wealth heterogeneity. Our model structure is probably most closely related to that presented in Guerrieri and Lorenzoni [2009]. However, their model emphasizes why the economy may exhibit excessive responses to productivity shocks, while our framework offers a mechanism that amplifies demand-type shocks. Our paper also shares many features with Heathcote and Perri [2012], who develop a model in which unemployment risk and wealth impact consumption decisions. They focus on a strong form of demand externality that gives rise to multiple equilibria.[4] Finally, the works by Ravn and Sterk [2012], den Haan, Rendahl, and Riegler [2014] and Challe, Matheron, Ragot, and Rubio-Ramirez [2015] emphasize, as we do, how unemployment risk and precautionary savings can amplify demand shocks, but the mechanisms in these papers differs substantially from ours since they rely on nominal rigidities and constrained monetary policy. While the main mechanism in our model has many precursors in the literature, we believe that our setup illustrates most clearly (*i*) how unemployment risk gives rise to a multiplier process for demand shocks even in the absence of price stickiness or increasing returns, (*ii*) how this multiplier process can be ignited by periods of liquidation, and (*iii*) how policy can and cannot be used to counter the process.

The remaining sections of the paper are structured as follows. In Section 2, we present a few motivating facts regarding the liquidationist view of recessions. In Section 3, we present a static model where agents inherit from the past a given level of capital goods, and we describe how and why high values of inherited capital can lead to deficient demand and poor economic outcomes. We begin the analysis with a model featuring only durable goods. We then extend it to the case of both durable and non-durable goods and show why inheriting many durable goods can also cause a reduction in non-durable purchases even when preferences are separable between the two types of goods. Finally, we extend the static model to the case where the stock of durable goods/houses can be used as collateral when financial markets are imperfect. In Section 4, we extend the model to an infinite-period dynamic setting and emphasize how the economy's behavior is different when it is close to rather than far from the steady state. We then calibrate the model to illustrate how it can provide an explanation of the observations we presented in Section 2. Finally, in Section 5,

[4] The existence of aggregate demand externalities and self-fulfilling expectations is also present in the work of Farmer [2010], Chamley [2014] and Kaplan and Menzio [2013].

we discuss the trade-offs faced by a policy-maker in our dynamic setup. Section 6 concludes.

2 Motivating Observations

The liquidationist view of recessions suggests that the severity and depth of a recession should reflect the extent to which an economy was in a situation of over-accumulated capital prior to the downturn. If the economy had gone through a period of substantial over-accumulation in prior years, then according to the liquidationist view the subsequent recession should be particularly deep and prolonged, as there would be a need to deplete a large amount of capital. In this section, we want to present some suggestive evidence in support of this idea. We first look at a comprehensive measure of capital, "*total capital*", which consists of fixed capital plus durable goods. In the left panel of Figure 1 we present a scatter plot of the relationship between the depth of postwar US recessions[5] and a measure of prior over-investment. To create this measure of capital over-accumulation, we first construct a series of cumulated investment by summing past investments using the perpetual-inventory method over a rolling period of 40 quarters, and then remove a cubic time trend intended to capture the secular changes in growth rates over the period.[6] Our measure of the depth of a recession is the percentage fall in per capita real GDP from the preceding peak to the trough of the recession. Further details about the data and the construction of our measures are presented in Appendix B.

Figure 1

As can be seen in Figure 1, there is a very strong positive correlation between our measure of capital over-accumulation prior to a recession and the subsequent severity of the recession. While this evidence is only suggestive, it does support the notion that severe recessions have generally been preceded by periods of very high investment relative to secular growth. When we correlate our measure of capital accumulation with the length of the recovery – i.e., the

[5] Specifically, our sample runs from 1957Q1 to 2015Q2.

[6] Results are robust to using different low-frequency trend-removal methods, including an HP filter. However, we believe using a low-order polynomial trend to remove low frequency movements in growth rates is more conservative, since the resulting estimated trend it virtually not affected by recessions. In fact, the cubic trend we estimate is very close to linear, with a slight slow down in the 70s and 80s.

time it takes for per capita real GDP to reach its previous peak level – we also find a strong positive and significant relationship, as shown in the right panel of Figure 1. It is worth pointing out that these correlations remain strongly positive and significant even if we exclude the most recent recession.[7]

An interesting aspect of these results is that severe recessions were generally preceded by high accumulation of all three classes of capital that constitute total capital. In Table 1, we report the results of the two following regressions for various measures of capital:

$$x_n = \beta_0 + \beta_1 \widehat{ci}_n + \varepsilon_n, \tag{1}$$

$$x_n = \beta_0 + \beta_1 \widehat{ci}_n + \beta_2 \Delta tfp_n + \varepsilon_n, \tag{2}$$

where n indexes recessions, x is either the depth of the recession or the length of the recovery, $\widehat{ci}$ represents deviations from trend of accumulated investment for either total, durable, non-residential or residential capital and Δtfp_n is the percentage change in TFP from peak to trough.

As shown in Table 1, a positive correlation between our measure of capital-abundance and the subsequent depth and length of recessions holds for each of the four individual types of capital goods. Furthermore, note that this pattern holds even when the change in TFP from peak to trough is included as a control variable (i.e., equation (2)), which implies that the observed pattern is not confounded with effects potentially associated with technological change. In fact, we find that the change in TFP during recessions always enters insignificantly; that is, deeper and longer recessions are not observed to be associated with larger concurrent drops in TFP.[8]

Insert Table 1

Overall, we take this evidence as suggestive of a link between prior accumulation of capital and the severity of a recession, in line with the Hayekian view. It is worth noting that this does not necessarily imply that recessions are efficient adjustments of the economy, as our model will shortly illustrate.

[7] When excluding the most recent recession, the correlations with depth and length of recovery are, respectively, 0.58 and 0.48, with corresponding p-values 0.1 and 0.17.

[8] For our measure of TFP we use Fernald's [2014] series which has been corrected for utilization. Note that TFP growth was positive in all recessions except for the two of the early 80s (see Figure F.1 in Appendix F).

3 Static Model

In this section, we present a stripped-down static model in order to illustrate as simply as possible why an economy may function particularly inefficiently when it inherits a large stock of capital from the past. In particular, we want to make clear why agents in an economy can be worse off when the stock of inherited capital goods is too high. For the mechanism to be as transparent as possible, we begin by making several simplifying assumptions that we eventually relax. For example, in our baseline model, we adopt a random matching setup with a particular matching function and a simple wage-bargaining protocol, while in Appendix D (available online) we extend to more general matching functions and show the robustness of our main results to allowing for different bargaining protocols (including directed search). We also focus on the case where the inherited capital produces services that directly enter agents' utility functions. Accordingly, this type of capital can be viewed as representing houses or other durable consumer goods. In Appendix E (available online) we discuss how the analysis carries over to the case of productive capital.

In our model, trades are decentralized, and there are two imperfections that together cause unemployment risk to be an important factor in household consumption decisions. First, there is a matching friction in the spirit of Diamond-Mortensen-Pissarides, which implies that a household may not be able to find employment. Second, there is limited unemployment insurance, so that households take into account the probability of being unemployed when making consumption decisions. The limited unemployment insurance in our model can be rationalized by invoking a standard adverse selection problem. Since this adverse selection problem can be analyzed separately, in the main body of the text we simply assume that unemployment insurance is not available.[9] The key exogenous variable in the static model will be a stock of consumer durables that households inherit from the past. Our goal is to show why and when high values of this stock can cause the economy to function inefficiently enough to cause a decline in welfare.

The main mechanisms in the model are as follows. Because transactions are not co-

[9] In Appendix D.5 we discuss the adverse selection problem that rationalizes the missing insurance market, and use the concomitant information structure to formulate the social planner's problem and compare it to the decentralized outcome.

ordinated though centralized markets and unemployment is possible, households view the purchase of durable goods as a risky endeavor, since the ease with which one can pay for these goods will depend on whether or not one becomes unemployed. In particular, households in the model purchase goods on credit. If the rate of unemployment is low, agents will be more willing to purchase durable goods since they know that their future labor earnings are likely to be sufficient to cover the cost of these purchases. In contrast, when unemployment is high, agents tend to hold back on making new purchases, since if they become unemployed they will find it difficult (i.e., costly) to service their resulting debts. Unemployment risk therefore causes consumption decisions across households act as "strategic complements":[10] when one household consumes less, this reduces the demand for goods and thereby increases unemployment, which leads other households to also consume less. The result is a potentially inefficiently large reaction to any impetus.

Inheriting from the past a large amount of durable goods is one such impetus. Because of the above mechanism, the economy can respond perversely to inheriting too many durable goods. In particular, if agents inherit a high level of consumer capital (relative to their desired level, as determined for example by the level of technology), then additional consumer capital has a low marginal value to the household; that is, household demand for new durable goods is low. This is the first-round effect. It is important to note that as a result of this first-round effect, households would never be worse off by inheriting more capital. However, because the initial low demand depresses the labor market, households will perceive the expected cost of purchasing new goods as having increased (since they are more likely to end up in the unemployed state where servicing their debt is more costly), and this will lead to a second-round effect whereby agents further reduce their purchases. A negative multiplier process is then set in motion, with the equilibrium outcome being one where agents buy so little, and economic activity is so depressed, that everyone is made worse off. In this case, agents would benefit from a coordinated increased in purchases, which captures the notion that inherited capital causes a situation of deficient demand. As we shall discuss, a social planner would in such a case want to subsidize hiring by firms so as to help restore efficiency and thereby allow the economy to benefit (instead of suffer) from inheriting more capital.

[10] We use here terminology that was popularized in macroeconomic analysis by Cooper and John [1988].

3.1 Setup

Consider an environment populated by a mass L of households indexed by j. In this economy there are two sub-periods, morning and afternoon. In the morning, households may buy a durable good,[11] and try to find employment in the durable-good sector. There will also be a good produced in the afternoon, which we refer to as a service. For now, it is useful to interpret the afternoon period as a reduced-form way of capturing the future. As there is no money in this economy, when the household buys the morning good its bank account is debited, and when (and if) it receives employment income its bank account is credited. Then, in the afternoon, households balance their books by repaying any outstanding debts or receiving a payment for any surplus. These payments are made in terms of the afternoon good (the service), which is also the numéraire in this economy.

Most of the action in the model arises in the morning market. Preferences for the morning are represented by

$$U(c_j) - \nu(\ell_j),$$

where c represents the flow of consumption services derived from the durable good and ℓ is the labor supplied by households in the production of durables. The function $U(\cdot)$ is assumed to be increasing in c, strictly concave and satisfy $U''' > 0$.[12] The dis-utility of work function $\nu(\cdot)$ is assumed to be increasing and convex in ℓ, with $\nu(0) = 0$. Households are initially endowed with X_j units of durables, which they can either consume or trade. We assume symmetric endowments, so that $X_j = X \ \forall j$.[13] In the dynamic version of the model, X will represent the stock of durable goods and will be endogenous.

The key assumption of the model is that trade in the morning is subject to a coordination problem because of frictions in the labor market. In the morning, the household splits up responsibilities between two members. The first member, called the buyer, goes to the durable-good market to make purchases. The second member searches for employment opportunities in the labor market. The market for durables functions in a Walrasian fashion, with both buyers and firms that sell goods taking prices as given. The market for labor in

[11] We refer to this good as "durable" in anticipation of its role in the dynamic version of the model.

[12] We will also assume when needed that the Inada conditions $\lim_{c\to 0} U' = \infty$ and $\lim_{c\to\infty} U' = 0$ hold.

[13] In what follows, we will drop the j index except where doing so may cause confusion.

the morning is subject to a matching friction, with sellers of labor searching for employers and employers searching for labor. The information assumption is that buyers do not know, when choosing their purchases of durables, whether the worker member of the household has yet secured a match. This assumption is an easy modeling device that introduces the possibility of cautionary behavior, as buyers will worry about unemployment risk when making purchases.

There is a large set of potential firms operating in the morning who can decide to search for workers in view of supplying durables to the market. Each firm can hire one worker and has access to a decreasing-returns-to-scale production function $\theta F(\ell)$, where ℓ is the number of hours worked for the firm and $\theta > 0$ is a technology shift factor. Production also requires a fixed cost $\theta\Phi$ in terms of the output good, so that the net production of a firm hiring ℓ hours of labor is $\theta\left[F(\ell) - \Phi\right]$. Φ can be thought of as a vacancy-posting cost, as it is incurred before a search can be conducted. For now, we will normalize θ to one, and will reintroduce θ in its general form when we want to talk about the effects of technological change and balanced growth. We will also assume throughout that $F(0) = 0$ and that $\Omega(\ell) \equiv F'(\ell)\ell$ is strictly increasing in ℓ.[14] Moreover, we will assume that Φ is sufficiently small such that there exists an $\ell > 0$ satisfying $F(\ell) - F'(\ell)\,\ell = \Phi$. These restrictions on the production technology are always satisfied if, for example, $F(\ell) = A\ell^{\alpha}$, with $0 < \alpha < 1$.

We will assume that search is conducted in a random fashion.[15] Given the random search setup, when a firm and a worker match, they need to jointly decide on the number of hours to work and on the wage to be paid. There are many ways the surplus from the match can be divided, as long as it remains within the bargaining set. For the greatest clarity of results, we begin by following Lucas and Prescott [1974] and Guerrieri and Lorenzoni [2009] in assuming that the determination of the wage and of hours worked is done though a competitive pricing process. In effect, one can view a Walrasian auctioneer as calling out a wage w (in units of the afternoon good) that equilibrates the demand for and supply of labor among the two parties in a match. Given that wage, the demand for labor from the firm is therefore given

[14] Because we assume free-entry for morning firms, the quantity $\theta\Omega(\ell)$ will equal net output of durables (after subtracting firms' fixed costs) by a single employed worker.

[15] In Appendix D.4.2 we extend our results to the case of directed search.

by the marginal productivity condition

$$pF'(\ell) = w,$$

where p is the relative price of the morning good in terms of the afternoon one.[16] The supply of labor is chosen optimally by the household in a manner to be derived shortly. This competitive pricing process has the feature of limiting any within-pair distortions that could muddle the understanding of the main mechanisms of the model. In Appendix D.4.1, we show how our results extend to the case where wages and hours worked are instead determined by a Nash bargaining process. As we show, Nash bargaining introduces additional (and somewhat unintuitive) elements into the analysis that are most easily understood after this baseline framework is presented.

Letting N represent the number of firms who decide to search for workers, the number of matches is then given by the constant-returns-to-scale matching function $M(N, L)$, with $M(N, L) \leq \min\{N, L\}$. The equilibrium condition for the durables market is given by

$$L \cdot (c - X) = M(N, L)F(\ell) - N\Phi,$$

where the left-hand side is total purchases of new durables and the right-hand side is the total available supply after subtracting search costs.

Firms will enter the market up to the point where expected profits are zero. The zero-profit condition can be written[17]

$$\frac{M}{N}[pF(\ell) - w\ell] = \frac{M}{N}[pF(\ell) - pF'(\ell)\ell] = p\Phi.$$

At the end of the morning, household j's net financial asset position a_j, expressed in units of the afternoon good, is given by $w\ell_j - p(c_j - X)$, with $\ell_j = 0$ if the household working member has not found a job. We model the afternoon so that it is costly to arrive in that sub-period with debt. For now, we can simply denote the value of entering the afternoon with assets a_j by $V(a_j)$, where we assume that $V(\cdot)$ is increasing and weakly concave, with $V'(a_1) > V'(a_2)$ whenever $a_1 < 0 < a_2$; that is, we assume that the marginal value of a unit

[16] As will become clear, p can be given an interpretation as a gross interest rate between morning and afternoon.

[17] We assume that searching firms pool their ex-post profits and losses so that they make exactly zero profits in equilibrium, regardless of whether they match.

of assets is greater if one is in debt than if one is in a creditor position. As will become clear, this property of V is precisely the one necessary to generate a cautionary response to changes in unemployment risk, which (as noted earlier) is a fundamental part of the main mechanism of the model. In the following sub-section we specify preferences and a market structure for the afternoon that rationalizes a $V(\cdot)$ function of this form.

Taking the function $V(a)$ as given, we can specify the household's morning consumption decision as well as his morning labor-supply decision conditional on a match. The buyer's problem in household j is given by

$$\max_{c_j} \quad U\left(c_j\right) + \mu V\left(w\ell_j - p\left(c_j - X\right)\right) + (1-\mu)V\left(-p\left(c_j - X\right)\right),$$

where μ is the probability that a worker finds a job and is given by $\mu \equiv M(N, L)/L$. From this expression, we can see that the consumption decision is made in the presence of unemployment risk. Note that the equilibrium unemployment rate is given by $1 - \mu$. Conditional on being matched, the worker's problem in household j, taking w as given, can be expressed as choosing a level of hours to supply in the morning so as to solve

$$\max_{\ell_j} \quad -\nu(\ell_j) + V(w\ell_j - p\left(c_j - X\right)).$$

The first-order conditions associated with these problems are given, respectively, by[18]

$$U'\left(c_j\right) = p\left[\mu V'\left(w\ell_j - p\left(c_j - X\right)\right) + (1-\mu)V'\left(-p\left(c_j - X\right)\right)\right],$$

$$\nu'(\ell_j) = wV'(w\ell_j - p\left(c_j - X\right)).$$

From these two equations, we can derive household j's consumption demand as a function of the employment rate μ, the relative prices w and p, and the endowment X. Denoting this function $c^D(\mu, w, p, X)$, it can be verified that $0 \leq \frac{\partial c^D(\mu,w,p,X)}{\partial X} < 1$ and $\frac{\partial c^D(\mu,w,p,X)}{\partial \mu} \geq 0$. In words, (i) j's consumption demand is non-decreasing in X, but its demand for new purchases $(c^D - X)$ is strictly decreasing in X; and (ii) j's consumption demand is increasing in the employment rate μ. From this partial-equilibrium perspective (i.e., if μ ,w, and p were

[18] As can be seen from the worker's first-order condition, the equilibrium wage w will in general be affected by household j's consumption decision c_j. We assume for simplicity that shoppers take the wage as given when making their consumption decision (i.e., they do not try to manipulate the bargaining process through their choice of consumption). Under our later assumptions about V, however, this will not be restrictive.

held constant), an increase in X would lead to fewer new purchases but (weakly) greater consumption. Moreover, it can be easily verified that the partial-equilibrium effect on welfare of inheriting from the past a larger stock of X can only be positive. Thus, for an increase in X to lead to a fall in consumption and a fall in welfare, it will need to come about through general-equilibrium effects. In our setup, the relevant general-equilibrium effect will run though the employment rate μ: the increase in X will lead to fewer new purchases, which in turn will decrease employment and, since $\frac{\partial c^D(\mu,w,p,X)}{\partial \mu} > 0$, this has the potential to reverse the partial-equilibrium effects and lead to a fall in c and a fall in welfare.

3.2 Deriving the Value Function $V(a_j)$

$V(a_j)$ represents the value of entering the afternoon with a net financial asset position a_j. In this subsection, we derive the function V (possessing the key properties assumed in the previous subsection) by specifying primitives in terms of preferences, technology and market organization. We choose to model the afternoon in such a way that if there were no frictions in the morning there would be no trade between agents in the afternoon. Afternoon preferences are given by

$$\widetilde{U}(\widetilde{c}_j) - \widetilde{\nu}(\widetilde{\ell}_j),$$

where $\widetilde{c}_j$ is consumption of afternoon services, $\widetilde{U}(\cdot)$ is increasing and strictly concave in $\widetilde{c}_j$, $\widetilde{\ell}_j$ is the labor used to produce services, and $\widetilde{\nu}(\cdot)$ is increasing and convex in $\widetilde{\ell}_j$.

To ensure that a unit of net assets is more valuable when in debt than when in surplus, let us assume that households in the afternoon can produce services for their own consumption using one unit of labor to produce $\widetilde{\theta}$ units of services. However, if a household in the afternoon has to produce market services – that is, services that can be sold to others in order to satisfy debt – then to produce $\widetilde{\theta}$ units of market services requires them to supply $1 + \tau$ units of labor, $\tau > 0$. To simplify notation, we can set $\widetilde{\theta} = 1$ for now and return to the more general formulation when talking about the effects of technological change. The continuation value function $V(a_j)$ can accordingly be defined as

$$V(a_j) = \max_{\widetilde{c}_j, \widetilde{\ell}_j} \left\{ \widetilde{U}(\widetilde{c}_j) - \widetilde{\nu}(\widetilde{\ell}_j) \right\} \quad \text{subject to}$$

$$\widetilde{c}_j = \begin{cases} \widetilde{\ell}_j + a_j & \text{if } a_j \geq 0, \\ \widetilde{\ell}_j + (1+\tau)a_j & \text{if } a_j < 0. \end{cases}$$

It is easy to verify that $V(a_j)$ is increasing and weakly concave, with a kink at zero. If $\widetilde{\nu}(\widetilde{\ell}_j)$ is strictly convex, then $V(a_j)$ will be strictly concave, with the key property that $V'(a_1) > V'(a_2)$ if $a_1 < 0 < a_2$; that is, the marginal value of an increase in assets is greater if one is in debt than if one is in surplus.[19] Alternatively, if $\widetilde{\nu}(\widetilde{\ell}_j)$ is linear, then $V(a_j)$ will be piecewise linear. Nonetheless, it will maintain the key property that $V'(a_1) > V'(a_2)$ if $a_1 < 0 < a_2$. We will initially work with this latter case, and in particular will assume that $\widetilde{\nu}(\widetilde{\ell}_j) = v\widetilde{\ell}_j$ for a constant v. This formulation will greatly reduce the number of general-equilibrium interactions, allowing us to emphasize the channels we think are most relevant. In Appendix D.2, we discuss how the results are modified when we allow the function $V(\cdot)$ to be a general concave function.

3.3 Equilibrium in the Morning Period

Given the function $V(\cdot)$, a symmetric equilibrium for the morning is represented by five objects: two relative prices (the price of durables p and the wage rate w), two quantities (consumption of durables by each household c and the amount worked in each match ℓ), and a number N of active firms, such that

(i) c solves the buyer's problem taking μ, p, w and ℓ as given;

(ii) the labor supply ℓ solves the worker's problem conditional on a match, taking p, w and c as given;

(iii) the demand for labor ℓ maximizes the firm's profits given a match, taking p and w as given;

(iv) the durable goods market clears;

(v) firms' entry decisions ensure zero profits.

[19] To avoid backward-bending supply curves, we will also assume that $\widetilde{\nu}(\cdot)$ and $\widetilde{U}(\cdot)$ are such that $V'''(a_j) \geq 0$. This assumption is sufficient but not necessary for later results. Note that a sufficient condition for $V'''(a_j) \geq 0$ is that both $\widetilde{U}'''(\cdot) \geq 0$ and $\widetilde{\nu}'''(\cdot) < 0$.

The morning equilibrium can therefore be represented by the following system of five equations:

$$M(N,L)F(\ell) = L(c - X) + N\Phi, \tag{3}$$

$$M(N,L)[pF(\ell) - w\ell] = Np\Phi, \tag{4}$$

$$\begin{aligned} U'(c) = p\Bigg\{ &\frac{M(N,L)}{L} V'\left(w\ell - p\left(c - X\right)\right) \\ &+ \left[1 - \frac{M(N,L)}{L}\right] V'\left(-p\left(c - X\right)\right) \Bigg\}, \end{aligned} \tag{5}$$

$$\nu'(\ell) = V'\left(w\ell - p\left(c - X\right)\right) w, \tag{6}$$

$$pF'(\ell) = w. \tag{7}$$

In the above system,[20] equations (5) and (6) represent the first-order conditions for the household's choice of consumption and supply of labor. Equations (4) and (7) represent a firm's entry decision and its labor demand condition. Finally, (3) is the goods market clearing condition.

At this level of generality it is difficult to derive many results. Nonetheless, we can combine (5), (6) and (7) to obtain the following expression regarding one characteristic of the equilibrium,

$$\frac{\nu'(\ell)}{U'(c)} \left\{ 1 + (1 - \mu) \left[\frac{V'\left(-p\left(c - X\right)\right)}{V'\left(w\ell - p\left(c - X\right)\right)} - 1 \right] \right\} = F'(\ell). \tag{8}$$

From equation (8), we see that as long as $\mu < 1$, the marginal rate of substitution between leisure and consumption will not be equal to the marginal productivity of work; that is, the economy will exhibit a labor market wedge given by

$$(1 - \mu) \left[\frac{V'\left(-p\left(c - X\right)\right)}{V'\left(w\ell - p\left(c - X\right)\right)} - 1 \right].$$

Note that both unemployment risk ($\mu < 1$) and imperfect insurance ($V'\left(-p\left(c - X\right)\right) \neq V'\left(w\ell - p\left(c - X\right)\right)$) are needed for the wedge to be non-zero. In this environment, contemplating the possibility of being unemployed agents hold back on purchases, which in turn causes the marginal rate of substitution between leisure and consumption to be low relative

[20] To ensure that an employed worker's optimal choice of labor is strictly positive, we assume that $\lim_{c \to 0} U'(c) > \lim_{\ell \to 0} \frac{\nu'(\ell)}{F'(\ell)}$.

to the marginal productivity of labor. As we will see, changes in X will cause this wedge to vary, which will cause a feedback effect on economic activity.

Our main goal now is to explore the effects of changes in X on equilibrium outcomes. In particular, we are interested in clarifying why and when an increase in X can actually lead to a reduction in consumption and/or welfare. The reason we are interested in this comparative static is that we are interested in knowing why periods of liquidation – that is, periods where agents have inherited high levels of durable goods from the past – may be socially painful.

To clarify the analysis, we make two simplifying assumptions. First, we assume that the matching function takes the form $M(N, L) = \Lambda \min\{N, L\}$, with $0 < \Lambda \leq 1$. The attractive feature of this matching function is that it has two regimes: one where congestion externalities are concentrated on workers, and one where the externalities are concentrated on firms. In the case where $N > L$, which we refer to as a tight labor market, workers are the more scarce factor and only the firm matching rate is affected by changes in the ratio N/L. In the case where $L > N$, which we refer to as a slack labor market, jobs are scarce and only the worker matching rate is affected by changes in the ratio N/L. While this particular matching function is not necessary for our main results, it allows us to cleanly compare the case where congestion externalities are stronger for firms to the case where they are stronger for workers. We will also assume that $V(a)$ is piece-wise linear, with $V(a) = va$ if $a \geq 0$ and $V(a) = (1 + \tau)va$ if $a < 0$, where $\tau > 0$ and $v > 0$. This form of the $V(\cdot)$ function corresponds to the case discussed in Section 3.2 where the dis-utility of work in the afternoon is linear. The important element here is τ. In effect, $1 + \tau$ represents the ratio of the marginal value of an extra unit of assets when one is in debt relative to its value when one is in surplus. A value of $\tau > 0$ can be justified in many ways, one of which is presented in Section 3.2. Alternatively, $\tau > 0$ could reflect a financial friction that produces a wedge between borrowing and saving rates. In what follows, we refer to τ as the "marginal cost of debt".

Under these two functional-form assumptions, the equilibrium conditions can be reduced to the following:

$$\frac{\Lambda \min\{N, L\}}{L} = \frac{c - X}{F'(\ell)\ell}, \tag{9}$$

$$\Phi = \frac{\Lambda \min\{N, L\}}{N}[F(\ell) - F'(\ell)\ell], \tag{10}$$

$$U'(c) = \frac{\nu'(\ell)}{F'(\ell)}\left(1 + \tau - \frac{\Lambda \min\{N, L\}}{L}\tau\right), \tag{11}$$

$$w = \frac{\nu'(\ell)}{v}, \tag{12}$$

$$p = \frac{\nu'(\ell)}{vF'(\ell)}. \tag{13}$$

This system of equations now has the feature of being block-recursive, in that equations (9), (10) and (11) can be solved for c, ℓ and N, with equations (12) and (13) then providing the wage and the price. From equations (9) and (11), one may clearly see the complementarity that can arise between consumption and employment in the case where $N < L$ (i.e., where the labor market is slack). From (11) we see that, if $N < L$, agents will tend to increase their consumption if they believe there are many firms looking for workers (N expected to be large). Then from equation (9) we see that, to meet anticipated demand, more firms will need to enter if they believe that consumption will be high, which results in more hiring. Thus, greater consumption favors greater employment, which in turn reinforces consumption. This feedback effect arises as the result of consumption and employment playing the role of strategic complements. Workers demand higher consumption when they believe that many firms are searching to hire, as they view a high N as reducing their probability of entering the afternoon in debt. It is important to note that this complementarity argument is implicitly taking ℓ, the number of hours worked by agents, as fixed. But, in the case where the economy is characterized by a slack labor market, this is precisely the right equilibrium conjecture. From (10) we can see that if the labor market is slack, then ℓ is simply given by $\ell^\star$, the solution to the equation $\Lambda[F(\ell^\star) - F'(\ell^\star)\ell^\star] = \Phi$, and is therefore locally independent of X or c. Hence, in the slack labor market regime, consumption and firm hiring will act as strategic complements. As is common in the case of strategic complements, multiple equilibria can arise. This possibility is stated in Proposition 1.[21]

[21] The proofs of all propositions are presented in Appendix C.

Proposition 1. *There exists a marginal cost of debt $\bar{\tau} > 0$*[22] *such that (i) if $\tau < \bar{\tau}$, then there exists a unique equilibrium for any value of the durables stock X; and (ii) if $\tau > \bar{\tau}$, then there exists a range of X for which there are multiple equilibria.*

While situations with multiple equilibria may be interesting, in this paper we will focus on cases where the equilibrium is unique. Accordingly, Proposition 1 tells us that our setup will have a unique equilibrium if the marginal cost of debt is not too large. For the remainder of this section, we will assume that $\tau < \bar{\tau}$. Proposition 2 focuses on this case and provides a first step in the characterization of the equilibrium.

Proposition 2. *When $\tau < \bar{\tau}$, there exists an $X^\star$ such that if $X \leq X^\star$ then the equilibrium is characterized by a tight labor market ($N \geq L$), while if $X > X^\star$ it is characterized by a slack labor market ($N < L$). Furthermore, there exists an $X^{\star\star} > X^\star$ such that if $X > X^{\star\star}$, then employment is zero and agents simply consume their endowment (i.e., $c = X$).*[23]

The content of Proposition 2 is very intuitive as it simply reflects that when households have a low endowment of the durable good, they have a high marginal utility. This high marginal utility leads them to purchase many new goods. The high demand is met by new firms entering the market, which leads to a tight labor market. In contrast, if the endowment is high, marginal utility will be low which will reduce the demand for new goods, thereby creating a slack labor market. In this case, variation in demand induces variation in entry decisions, and therefore unemployment, because the free-entry condition acts to create a supply curve for goods that is perfectly elastic. Finally, if X is extremely high, all trade among agents will stop, as people are content to simply consume their endowment. Proposition 3 complements Proposition 2 by indicating how consumption is determined in each regime.

Proposition 3. *When the labor market is slack ($X^{\star\star} > X > X^\star$), the level of consumption is given as the unique solution to*

$$c = U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\left[1 + \tau - \frac{c - X}{F'(\ell^\star)\,\ell^\star}\tau\right]\right).$$

[22] $\bar{\tau} = -U''\left(U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right)\right)\frac{F'(\ell^\star)[F(\ell^\star)-\Phi]}{\nu'(\ell^\star)}$.

[23] $X^\star = U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right) - F'(\ell^\star)\,\ell^\star$ and $X^{\star\star} = U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau)\right)$.

When the labor market is tight ($X \leq X^{\star}$), consumption is the unique solution to

$$c = U'^{-1}\left(\frac{\nu'(\Omega^{-1}(c-X))}{F'(\Omega^{-1}(c-X))}\left[1+\tau(1-\Lambda)\right]\right).$$

Finally, when $X \geq X^{\star\star}$, consumption is given by $c = X$.

Given the above propositions, we are now in a position to examine an issue of main interest, which is how an increase in X affects consumption. In particular, we want to ask whether an increase in X, which acts as an increase in the available supply of goods, can lead to a decrease in the actual consumption of goods. Proposition 4 addresses this issue.

Proposition 4. *If $X^{\star\star} > X > X^{\star}$, then consumption c is decreasing in the durables stock X. If $X \leq X^{\star}$ or $X > X^{\star\star}$, then c is increasing in X.*

The content of Proposition 4 is illustrated in Figure 2. Proposition 4 indicates that, starting at $X = 0$, consumption will continuously increase in X as long as X is compatible with a tight labor market. Then, when X is greater than $X^{\star}$, the economy enters the slack labor market regime and consumption starts to decrease as X is increased. Finally, beyond $X^{\star\star}$ trade collapses and consumption becomes equal to X and hence it increases with X.

The key result is the existence of a region where total consumption decreases as X increases. The reason why consumption decreases with a higher supply of X in the slack region is because consumption decisions across households play the role of strategic complements in this region. Note that, under our current functional form assumptions, an increase in X leads in partial equilibrium to a one-for-one fall in expenditures, where we define expenditures as $e = c - X$. If there were no further general-equilibrium effects, the net effect on consumption would be zero. However, there *are* general-equilibrium effects, and these change depending on whether the labor market is tight or slack. When the labor market is slack, the partial-equilibrium decrease in expenditures reduces the demand for goods as perceived by firms. Less firms then search for workers, which increases the risk of unemployment. The increase in unemployment risk leads households to cut their expenditures further, which further amplifies the initial effect of an increase in X on expenditures. It is because of this multiplier process that an increase in the supply of the good leads to decrease in its total consumption ($X + e$). Note that such a negative effect does not happen when the labor market is tight, as a decrease in e does not cause an increase in incentives to save, which is the key mechanism at play causing consumption to fall. It is relevant to note that when the labor market is slack, both w and p are invariant to changes in X, and hence there are no general equilibrium effects working though prices in this regime. In contrast, when the labor market is tight, an increase in X leads to a fall in prices which tends to favor an increase in consumption. In this sense, the consumption decisions of households play the role of strategic substitutes when the labor market is tight, so that increased consumption by one households tends to increase prices which in turn causes other households to *decrease* their consumption.

Insert Figure 2

The link noted above between household j's expenditure, which we can denote by $e_j = c_j - X_j$, and expenditures by other agents in the economy, which we denote by e, can be captured by rewriting the relations determining e_j implied by the elements of Proposition 3 as

$$e_j = Z(e) - X, \tag{14}$$

with

$$Z(e) \equiv U'^{-1}\left(Q(e)\right) \tag{15}$$

and

$$Q\left(e\right) \equiv \begin{cases} \frac{\nu'(\ell^\star)}{F'(\ell^\star)}\left(1+\tau-\tau\frac{e}{e^\star}\right) & \text{if } 0 < e < \Lambda e^\star, \\ \frac{\nu'\left(\Omega^{-1}(e)\right)}{F'(\Omega^{-1}(e))}[1+\tau(1-\Lambda)] & \text{if } e \geq \Lambda e^\star. \end{cases} \tag{16}$$

Here, $e^\star \equiv \Omega(\ell^\star)$ is the level of output (net of firms' search costs) that would be produced if all workers were employed, with hours per employed worker equal to $\ell^\star$. In equilibrium we have the additional requirement that $e_j = e$ for all j.

The equilibrium determination of e is illustrated in Figure 3. In the Figure, we plot the function $e_j = Z(e) - X$ for two values of X: a first value of X that places the equilibrium in the slack labor market regime, and a second value of X that places it in a tight labor market regime. An equilibrium in this figure corresponds to the point where the function $e_j = Z(e) - X$ crosses the 45° line. Note that changes in X simply move the $e_j = Z(e) - X$ curve vertically.

Insert Figure 3

There are several features to note about Figure 3. First, in the case where $X \in (X^\star, X^{\star\star})$, so that the equilibrium of the economy corresponds to a slack labor market with positive trade (i.e., $0 < e < \Lambda e^\star$), the diagram is similar to a Keynesian cross. We can see graphically how an increase in X by one unit shifts down the $Z(e) - X$ curve and, since the slope of $Z(e) - X$ is positive and less than one, a multiplier process kicks in that causes e to fall by more than one. Because of this multiplier process, total consumption of durables, which is equal to $e + X$, decreases, which is the essence of the first part of Proposition 4. Second, when $X < X^\star$, so that the labor market is tight (i.e., the equilibrium is such that $e > \Lambda e^\star$), the diagram is different from the Keynesian cross. The most notable difference is the negative slope of the function $Z(e) - X$ for values of $e > e^\star$. This reflects the fact that unemployment risk is not increasing in this regime. In fact, when X is sufficiently small so that the labor market is tight, an increase in X by one unit leads to a decrease in e that is less than one, compared to a decrease of greater than one as exhibited in the slack regime. Here, expenditure by others plays the role of a strategic substitute with one's own expenditure – as opposed to playing the role of a strategic complement as is the case in the unemployment regime – through its effects on real wages and prices. Accordingly, in this region, an increase in X leads to an increase in total consumption of durables. Another more subtle difference with the Keynesian cross is in how the intercept of $Z(e) - X$ is determined. The intercept is given by $U'^{-1}(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau)) - X$. The X term in the intercept can be interpreted as capturing a pure aggregate-demand effect, whereby higher values of X reduce aggregate demand. However, the remaining term, $U'^{-1}(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau))$, reflects technology and preferences. In particular, we can generalize this term by re-introducing the technology parameter θ, in which case the intercept becomes $U'^{-1}(\frac{\nu'(\ell^\star)}{\theta F'(\ell^\star)}(1+\tau))$.[24] In this case, we see that an improvement in technology shifts up the intercept, and will lead to an increase in expenditures. This feature of the $Z(e) - X$ curve illustrates its equilibrium nature, which incorporates both demand and supply effects, as opposed to a Keynesian cross that only reflects demand effects.

[24] Recall that an increase in θ is associated with a proportional change in the search cost, so that $\ell^\star$ remains unchanged.

3.4 Is There Deficient Demand When the Labor Market is Slack?

In the case where X is large enough for the economy to be in the slack labor market regime ($X^\star < X < X^{\star\star}$), we would like to establish whether this regime should be characterized as suffering from deficient aggregate demand. For this, we need to first define the concept of deficient demand. In our definition we want to focus on a situation where economic activity is inefficiently low and where that low level of activity can be traced back to a lack of demand by others. In particular, we want our definition to exclude a situation where economic activity is inefficiently low simply because of price distortions that are unrelated to a lack of demand by others. For this reason, we define deficient demand as follows.

Definition. Deficient demand *is a situation where increased demand by one agent would favor increased demand by other agents, and where a coordinated increased in demand by all agents would leave everyone better off.*

Using this definition, Proposition 5 indicates that the slack labor market regime is in fact characterized by deficient demand, while the full employment regime is not.

Proposition 5. *When the labor market is slack ($X^\star < X < X^{\star\star}$) it exhibits deficient demand for all $\tau > 0$, while if the labor market is tight ($X < X^\star$) the economy does not exhibit deficient demand.*

3.5 Effects of Changes in X on Welfare

We have shown that when X is high enough, then the labor market will be slack and a local increase in X will cause consumption to fall. We now want to ask how expected welfare is affected in this case, where expected welfare is defined as

$$U(c) + \mu\left[-\nu(\ell) + V(w\ell - p(c - X))\right] + (1 - \mu)V(-p(c - X)).$$

In particular, we want to ask whether welfare can decrease when the economy is endowed with more goods. Proposition 6 answers this question in the affirmative. Proposition 6 actually goes a step further and indicates two sufficient conditions for there to exist a range of X in the slack labor market regime where an increase in X leads to a fall in welfare.

Proposition 6. *An increase in X can lead to a fall in expected welfare. In particular, if either (i) τ is close enough to $\bar{\tau}$ or (ii) the average cost of work $\frac{\nu(\ell^\star)}{\ell^\star}$ is low enough relative to the marginal cost of work $\nu'(\ell^\star)$, then there is always a range of $X \in [X^\star, X^{\star\star}]$ such that an increase in X leads to a decrease in expected welfare.*

Proposition 6 provides an answer to whether more goods can make everyone worse off. In effect, the proposition indicates that the economy can function in a very perverse fashion when households have inherited many goods. We saw from Proposition 4 that an increase in X always leads to a decrease in consumption when the economy is in the slack regime. In comparison, Proposition 6 is weaker as it only indicates the possibility of a fall in welfare in the slack region when X rises. In response to a rise in X in the slack regime, there are three distinct channels through which expected welfare is affected. First, as discussed above, consumption falls, which tends to directly decrease welfare. Second, this fall in consumption is associated with a fall in the probability of being employed. It can be verified that the net benefit of being employed is strictly positive, so that this second effect also tends to decrease welfare. Finally, a rise in X means that a given quantity of consumption can be obtained with a lower level of expenditure, which increases assets for the employed and decreases debt for the unemployed, and therefore tends to increase welfare. Whether this final effect is outweighed by the first two depends on the factors discussed in Proposition 6.

3.6 Adding a Non-Durable Goods Sector in the Morning Period

In the previous section we showed that when trade in goods and labor is not simultaneous and there is risk of unemployment, the economy can function in a rather perverse fashion. In particular, we showed that inheriting a large amount of goods can result in deficient demand, with increases in inherited goods reducing both consumption and welfare. In this section we want to briefly explore the robustness of these results to allowing for a second sector in the morning that can potentially expand when the market for durables contracts. This exploration is especially relevant given a common criticism of the liquidation view of recessions, as expressed for example by Krugman [1998], that suggests that liquidationist models cannot explain why we see falls in the consumption of both durable and non-durable goods during most recessions. In order to explore this issue, let us extend the previous setup by making a few small changes. First, let us allow morning utility to take the form $U^d(c^d)+U^n(c^n)-\nu(\ell)$, where c^d is durable consumption and is equal as before to $X+e$, c^n is the added morning non-durable consumption good, and ℓ is hours worked. We will assume that there is only one labor market, so that workers can switch frictionlessly across sectors of production. Both durable goods producers and non-durable goods producers will search for workers in this labor market. The markets for durable goods and non-durable goods are assumed to be distinct, with each functioning in a Walrasian fashion. We treat producers of the different goods symmetrically, with production functions in the respective sectors denoted by $F^d(\ell)$ and $F^n(\ell)$ (maintaining the assumptions that $F^{s\prime}(\ell)\ell$ is increasing in ℓ in both sectors $s \in \{d, n\}$), and assuming that both types of firms face the fixed cost of entering the market given by Φ. Otherwise, we maintain the same structure as before, including the functional-form assumptions for $M(N, L)$ and $V(\cdot)$. The issue we want to examine is the relationship between X and equilibrium outcomes for this economy. Proposition 7 establishes this.

Proposition 7. *In the economy with both a durable good and a non-durable good in the morning, if τ is not too large, then for any value of $X > 0$ there exists a unique equilibrium. Moreover, if Φ is not too small, there exists an $X^\star$ and an $X^{\star\star} > X^\star$, such that:*

(i) For $X < X^\star$, the labor market is tight and the consumption of both durables ($c^d = X+e$) and non-durables (c^n) increases with X.

(ii) For $X \in [X^\star, X^{\star\star}]$, the labor market is slack with both c^d and c^n decreasing with X.

(iii) For $X > X^{\star\star}$, $c^d = X$ and c^n is invariant to X.

The most interesting aspect of this proposition from our point of view is the existence of a slack labor market regime (when $X \in [X^\star, X^{\star\star}]$), where the consumption of durables, purchases of new durables, and purchases of non-durables all decrease in response to an increase in X. Although workers can be hired by non-durable goods firms in response to an increase in X, the proposition tells us that this substitution does not happen when the labor market is slack. To understand why this does not arise, it is helpful to examine the equilibrium conditions in the case where the labor market is slack, which can be combined to obtain

$$U^{d\prime}\left(c^{d}\right)=\frac{\nu^{\prime}\left(\ell^{d\star}\right)}{F^{d\prime}\left(\ell^{d\star}\right)}\left[1+\tau-\left(\frac{c^{d}-X}{F^{d\prime}\left(\ell^{d\star}\right)\ell^{d\star}}+\frac{c^{n}}{F^{d\prime}\left(\ell^{n\star}\right)\ell^{n\star}}\right)\tau\right], \tag{17}$$

$$U^{d\prime}\left(c^{n}\right)=\frac{\nu^{\prime}\left(\ell^{n\star}\right)}{F^{n\prime}\left(\ell^{n\star}\right)}\left[1+\tau-\left(\frac{c^{d}-X}{F^{d\prime}\left(\ell^{d\star}\right)\ell^{d\star}}+\frac{c^{n}}{F^{d\prime}\left(\ell^{n\star}\right)\ell^{n\star}}\right)\tau\right], \tag{18}$$

where $\ell^{d\star}$and $\ell^{n\star}$ (hours worked in the unemployment regime) are defined implicitly by the conditions $\Lambda[F^{n}(\ell^{d\star})-F^{n\prime}(\ell^{d\star})\ell^{d\star}]=\Phi$ and $\Lambda[F^{n}(\ell^{n\star})-F^{n\prime}(\ell^{n\star})\ell^{n\star}]=\Phi$.

From equations (17) and (18), we see that when the labor market is slack consumption in the two sectors act as strategic complements, and therefore they move in the same direction in response to a change in X. As long as X is high enough to push the labor market into slack, any further increase in X decreases employment in the durable goods sector, which increases overall unemployment risk. Since this initial increase in unemployment risk causes households to hold back on all of their purchases, demand for both durable and non-durable goods falls, further increasing unemployment risk, and so on. Hence, this version of the model offers an explanation for why inheriting a high level of durables can lead to low activity in both the durable and non-durable goods sectors.[25]

3.7 Adding a Collateral Constraint

In the previous sections we have illustrated how high levels of inherited durable goods can depress the economy and create a situation of deficient demand. The effects emphasized in the model run in the opposite direction of those emphasized in much of the literature on collateral constraints, wherein higher levels of capital may relax such constraints and favor expenditure by households. In this section we briefly explore how adding a collateral constraint to our model changes the characterization of equilibrium outcomes.

Recall that households in our model were allowed to debit their bank account in order to buy goods in the morning market without any limitations. Now suppose we return to

[25] A key condition for the unemployment regime to emerge with high X is that Φ not be too small. Specifically, we require that the value of c^n that solves

$$U^{d\prime}(c^{n})=\frac{\nu^{\prime}(\ell^{n\star})}{F^{n\prime}(\ell^{n\star})}\left[1+\tau-\frac{c^{n}}{F^{d\prime}(\ell^{n\star})\ell^{n\star}}\tau\right]$$

be such that $\frac{c^{n}}{F^{d\prime}(\ell^{n\star})\ell^{n\star}}<\Lambda$. This property is guaranteed if Φ is large enough.

our baseline model (without non-durable goods in the morning) and change it along the several dimensions. First, let us assume that inherited capital goods X also give utility in the afternoon period.[26] Second, we allow creditors to seize capital if debt is not repaid in the afternoon. Third, we assume that a fraction $\tau^\star > \tau$ of capital is lost when transferring it to a creditor in payment of debt.[27] In this case, a collateral constraint of the form $(1+\tau)pe \leq X$ will guarantee that households honor their obligations. How does the presence of such a constraint affect how a change in X impacts economic activity? Proposition 8 provides an answer to this question.

Proposition 8. *In the presence of a collateral constraint of the form $(1+\tau)pe \leq X$, there exist X^+ and X^{++} satisfying $0 < X^+ \leq X^{++} < X^{\star\star}$ such that equilibrium outcomes are characterized by*

(i) If $X^{++} < X < X^{\star\star}$, then the labor market is slack, there is deficient demand and $\frac{\partial c}{\partial X} < 0$.

(ii) If $0 < X < X^+$, then the labor market is slack and $\frac{\partial c}{\partial X} > 1$.

Proposition 8 indicates that the presence of a collateral constraint causes the effect of X on c to become richer when compared to the baseline model. In particular, if X is not too large ($0 < X < X^+$), then, despite high desired consumption, the economy will now exhibit unemployment because the binding collateral constraint limits the amount that households can actually purchase. In this situation, consumption is highly positively responsive to an increase in X, since this relaxes the credit constraint. There is not a coordination problem in this regime, since an increase in the demand of others has no effect on a household's collateral constraint. This regime did not arise in our baseline model. In contrast, when X is high enough ($X^{++} < X < X^{\star\star}$), we once again find that more X leads to less consumption because of the multiplier process discussed previously. In this region, even though agents have enough collateral to spend more, their purchases are still inefficiently low since they do not internalize the effect that their purchasing decisions have on others. It is interesting to

[26] Utility in the afternoon is now given by $\widetilde{U}\left(X+\widetilde{c}\right) - v\widetilde{\ell}$, where $\widetilde{c}$ is the amount of the afternoon good that is consumed by the household and $\widetilde{\ell}$ is the labor used to produce services.

[27] The assumption that $\tau^\star > \tau$ guarantees that agents will pay back their debts by working more instead of defaulting.

note that we could have $X^+ = X^{++}$, in which case the labor market would always be slack, but the reasons for high unemployment would be very different depending on whether X were high or low. If it were low, unemployment would be high because agents would be limited in how much they can borrow in order to purchase goods that would create employment. In contrast, if X were high, unemployment would be high not because of limited access to credit, but because of the low initial impetus to go out and buy new goods.

4 Dynamics

In this section we explore a dynamic extension of our static model in which morning consumption contributes to the accumulation of X. In particular, we want to consider the case where the evolution of X obeys the accumulation equation

$$X_{t+1} = (1-\delta)X_t + \gamma e_t \qquad 0 < \delta \leq 1 \quad , \quad 0 < \gamma \leq 1-\delta, \tag{19}$$

where the parameter γ represents the fraction of morning consumption expenditures, $e_t = c_t - X_t$, that enters the stock of durable goods. Since we do not want to allow heterogeneity between individuals to expand over time, we allow individuals to borrow and lend only within a period but not across periods; in other words, households are allowed to spend more than their income in the morning, but must repay any resulting debt in the afternoon of the same period. The problem facing a household in the morning of a period is therefore to choose how many durables to buy and, conditional on a match, how much labor to supply. We model the afternoon of a period as in Section 3.2, where households use labor to produce services either for their own consumption or, at a level of productivity that is lower by a factor $1+\tau$, for the consumption of others. In each afternoon, then, the household chooses how many services to consume and to produce to both satisfy its needs and to pay back any accumulated debt. In order to keep the model very tractable, we will continue to assume that dis-utility of work in the afternoon of a period is linear (i.e., equal to $v\widetilde{\ell}$). Under this assumption, all households will choose the same level of consumption of services in each afternoon, while the production of services will vary across households depending on whether they enter the afternoon in debt or in surplus. Since there are no interesting equilibrium interactions in afternoons, we can maintain most of our focus on equilibrium outcomes in the sequence of morning periods.

Relative to the static case, the only difference in equilibrium relationships (aside from the addition of the accumulation equation (19)) is that the first-order condition associated with the household's choice of consumption is now given by the Euler equation

$$U'\left(X_t + e_t\right) - Q\left(e_t\right) = \beta\left[(1-\delta-\gamma)U'\left(X_{t+1} + e_{t+1}\right) - (1-\delta)Q\left(e_{t+1}\right)\right], \tag{20}$$

where Q is as defined in equation (16). In this dynamic setting, an equilibrium will be represented as a sequence of the previous equilibrium conditions (9), (10), (12) and (13), plus the accumulation equation (19) and the Euler equation (20).

There are many complications that arise in the dynamic version of this model, which makes characterizing equilibrium behavior more difficult. In particular, there can be multiple equilibrium paths and multiple steady-state solutions. Luckily, the problem can be simplified if we focus on cases where δ is small; that is, on cases where the durability of goods is high. In addition to simplifying the analysis, focusing on the low-δ case appears reasonable to us, as many consumer durables are long-lived, especially if we include housing in that category. In the case where δ is sufficiently small, as stated in Proposition 9, the economy will have only one steady state and that steady state will have the property that the labor market will be slack.

Proposition 9. *If δ is sufficiently small, then the model has a unique steady state and this steady state is characterized by a slack labor market.*

Proposition 9 is very useful, as it will allow us to analyze the equilibrium behavior around the unique steady state. Accordingly, for the remainder of this section, we will assume that δ is sufficiently small so that Proposition 9 applies.

4.1 Local Dynamics

In this subsection, we explore the local dynamics assuming that δ is small enough that the steady state is unique and in the slack labor market regime. The first question we address is whether equilibrium dynamics can exhibit local indeterminacy. In other words, can forward-looking behavior give rise to an additional potential local source of multiple equilibria in our setup? Proposition 10 indicates that this is not possible; that is, the roots of the system around the unique steady state can not both be smaller than one.[28]

Proposition 10. *The local dynamics around the steady state can either exhibit monotonic convergence in c and X, convergence with oscillations, or divergence. Locally indeterminacy is not possible.*

Proposition 10 is useful as it tells us that the decision rule for consumption around the steady state is a function.[29] Accordingly, we can now examine the sign of the derivative of this function; that is, whether the decision rule for consumption around the steady state has the property that a larger X leads to a lower level of consumption, as was the case in our static model when in the slack labor market regime. Proposition 11 indicates that if τ is not too large, then local dynamics will exhibit this property. Note that the condition on τ is a sufficient condition only.

Proposition 11. *If τ is sufficiently small, then in a neighborhood of the unique steady state, consumption is decreasing in X, with the dynamics for X converging monotonically to the steady state.*

From Proposition 11 we now know that, as long as τ is not too big, our model has the property that when the economy has over-accumulated relative to the steady state (i.e., if X slightly exceeds its steady-state value), then consumption will be lower than in the steady state throughout the transition period, which we may refer to as a period of liquidation. In this sense, the economy is overreacting to its inherited excess of capital goods during this liquidation period, since it is reducing its expenditures to such an extent that people are consuming less even though there are more goods available to them in the economy. While such a response is not socially optimal, it remains unclear whether it is so excessive as to make people worse off in comparison to the steady state, since they are also working less during the liquidation phase. It turns out that, as in the static case, the welfare effect of such a liquidation period depends, among other things, on whether the average dis-utility

[28] In this section we only consider local dynamics around a unique unemployment-regime steady state. Nonetheless, it is straightforward to show that if the unique steady state is in the full-employment regime, then the local dynamics necessarily exhibit monotonic convergence.

[29] This is a slight abuse of language since Proposition 10 does not rule out the existence of other equilibrium paths away from the steady state.

of work is small enough relative to the marginal dis-utility. For example, if the average dis-utility of work is sufficiently low relative to its marginal value, then it can be verified that a liquidation period induced by inheriting an excess of X will make average utility in all periods of the transition lower than the steady-state level. This result depends in addition on the unemployment rate not being too large in the steady state.

While we do not have a simple characterization of the global dynamics of the model, Propositions 10 and 11 suggest that, starting from $X = 0$, the economy will likely go though a phase with a tight labor market, with both X and c increasing over time. The economy then enters into a range with a slack labor market once X is large enough. Then, as long as τ is not too great, X will continue to monotonically increase, converging toward its steady state. In contrast to X, upon entering the slack regime, consumption starts to decrease as unemployment risk increases and causes household's to hold back on purchases, which further depresses activity. Eventually, the economy will reach a steady state where consumption, employment, and possibly period welfare are below the peak levels reached during the transition.

4.2 A Quantitative Model of Booms and Busts with Irrational Exuberance

In this section, we illustrate the possibility of generating boom-bust cycles with the type of mechanism that we have put forward. In particular, we evaluate the model's ability to match the suggestive evidence presented in Section 2. As mentioned earlier, our aim is not to propose a novel explanation as to why the economy may embark on periods of rapid accumulation that turn out to be *ex post* excessive. Possible explanations could be noisy news, financial shocks (which could be modeled as changes in the marginal cost of debt, τ, in our model), uncertainty, waves of optimism, etc. For the purposes of illustration, in this section we suppose that the economy randomly undergoes episodes of excessive optimism about the growth of the economy, which we call irrational exuberance.

To derive some quantitative implications, we make several functional-form assumptions and then calibrate the model to reproduce salient features of the U.S. postwar business cycle. The model is written in per capita terms, and the length of a period is taken to be a quarter. We assume that the period utility and production functions are given, respectively, by $\log(c) - \nu\frac{\ell^{1+\omega}}{1+\omega}$ and $\theta_t A\ell^{\alpha}$, where total factor productivity θ_t follows a deterministic trend with growth rate γ_θ. On top of the parameters ν, ω, A, γ_θ and α, we need to calibrate the discount factor β, the depreciation rate δ, the share of durable goods in total expenditure γ, the vacancy-posting cost Φ, the marginal cost of debt τ, and the number of participating workers L.

We set $\gamma_\theta = 1.0048$ to match average per capita GDP growth and normalize θ_0 to 1. We set the discount factor to the commonly chosen level $\beta = 0.99$. We interpret X as the stock of durable goods. Expenditures on durable goods are on average over the postwar period 8.5% of total expenditures, so that we set $\gamma = 0.085$.[30] Similarly, the depreciation rate is calibrated to the the average ratio of "Current-Cost Depreciation of Consumer Durable Goods" over the "Current-Cost Net Stock of Consumer Durable Goods", so that $\delta = 0.045$. We set $\alpha = 2/3$ to obtain a labor income share of 2/3. The Frisch elasticity of labor supply in the intensive margin ω is set to 0.5 following Chetty, Guren, Manoli, and Weber [2011]. A and L are normalized to one. ν and Φ are then set to target the postwar average unemployment rate of 6% and to yield the normalized level of hours per capita $\ell = 1$.

Episodes of irrational exuberance are modeled in the following way. The economy can be either in normal times or in an episode of exuberance, but actual TFP always grows at factor γ_θ. In normal times, actual growth is correctly anticipated by economic agents when making spending decisions. With probability $1 - p_1$, the economy remains in the normal regime in the subsequent period, while with probability p_1 it enters into an episode of irrational exuberance. When the economy is exuberant, agents initially optimistically believe that TFP grew at a higher-than-usual factor $\kappa\gamma_\theta$ from last period, where $\kappa > 1$ measures the degree of optimism. Shoppers make their purchase decisions according to this optimistic belief. On the other hand, upon entering the labor market workers learn the true level of TFP and receive a wage commensurate with it. Accordingly, households "overspend" and "overborrow" in the morning, thereby "overaccumulating" the durable good X. When in an

[30] We interpret consumption in the model as reflecting both the services from existing durable goods, plus the felicity from current expenditures, where current expenditures can be on both durable and non-durable goods. For this reason, we set $\gamma < (1 - \delta)$, which implies that agents know that only a fraction γ of these expenditures will become durable goods.

exuberant period, the economy remains exuberant into the next period with probability p_2, and returns to normal with probability $1 - p_2$. Given the mechanisms of our model, when the economy returns to normal times it does so with a high stock of durable goods X, which is the main force that triggers a recession.[31] The simulated model will thus feature booms and busts, even though actual TFP never deviates from trend.

There are four parameters that are specific to our model and shock structure that cannot be taken from the literature: the marginal cost of debt parameter τ, the level of optimism in irrational exuberance episodes κ, and the transition probabilities p_1 and p_2. We calibrate these parameters to match four moments of the U.S. business cycle:[32] the standard deviation of output growth (0.96%), the average depth of a recession (-2.9%), the average length of a recovery (6 quarters) and the number of recession episodes per 100 quarters (3.3 episodes) (see Table 2).[33] Using this procedure, the calibrated values of the parameters are as follows. The degree of optimism is $\kappa = 1.22$, meaning that in an optimistic quarter, agents believe that the growth rate of TFP is 0.585% instead of the average rate of 0.48%. The marginal cost of debt is given by $\tau = 0.065$. The probability of entering into an optimistic episode is $p_1 = 0.3$, and the probability of remaining in an exuberant episode is $p_2 = 0.48$. With this calibration, exuberant episodes last for an average of two quarters, and it takes at least three quarters of exuberance to produce a recession.[34]

Insert Table 2

Next, using data simulated from our calibrated model, we perform the same empirical analysis conducted on actual data in Section 2: we compute cumulated investment, and

[31] An alternative way to model over-optimistic beliefs would be to assume rational (Bayesian) learning by economic agents in an environment with permanent and transitory shocks to TFP and/or noisy signals, as in, for example Beaudry and Portier [2004], Jaimovich and Rebelo [2009] or Blanchard, L'Huillier, and Lorenzoni [2013] (see Beaudry and Portier [2014] for a survey of this literature). Because the reaction of the economy to excessive accumulation in our model is quite independent of the precise mechanism that generates optimistic episodes, we adopt here the analytically simpler approach of irrational exuberance.

[32] To obtain moments for the model we average the relevant statistics from 10,000 simulations of the model of 270 quarters each in length.

[33] To detect peaks and troughs, we use Harding and Pagan's [2002] version of Bry and Boschan's [1971] algorithm, as provided by James Engel. Episodes of decreasing output are classified as recessions if total output growth is smaller than -2% in total.

[34] Recall that a recession is an episode in which output per capita decreases. Given the secular trend in TFP, under our calibration short periods of exuberance generate negative deviations from trend output per capita, but not a negative growth rate.

obtain the correlations between it and both the depth of recessions and the length of recoveries. Note that neither of these correlations were targeted in the calibration exercise. Nonetheless, as shown in Table 2, the model does a very good job at matching both of these moments.[35] As an illustration, Figure 4 shows scatter plots of depth and length against cumulated investment for a typical simulation of our model.

Insert Figure 4

To illustrate the mechanism at play in a boom-bust cycle driven by irrational exuberance, Figure 5 displays the path of an economy that is on a balanced growth path until period 4, and then, beginning in period 5, enters into an 8-quarter episode of exuberance. As shown in the upper panel of the figure, GDP, as measured by total expenditure, increases faster and above trend during the exuberance episode, which translates into a corresponding over-accumulation of durable goods (lower panel). When agents realize their error in period 12, they cut back on their expenditure to start a liquidation process.[36]

Insert Figure 5

5 Dynamic Policy Trade-Offs

In this final section, we turn to one of our motivating questions and ask whether or not stimulative policies should be used when an economy is going through a liquidation phase characterized by high unemployment. In particular, we consider the case where the economy has inherited from the past a level of X above its steady-state value and, in the absence of intervention, would experience a period of liquidation, with consumption below its steady-state level throughout the transition. Obviously, the first-best policy in this environment would be to remove the sources of frictions or to perfectly insure agents against unemployment risk. However, for reasons such as adverse selection or moral hazard, such first-best policies

[35] Note that, since TFP in the model does not fluctuate around its trend, controlling for TFP growth will have no impact, just as we found in the data (Table 1 in Section 2).

[36] In Appendix G, we study the pattern of fluctuations generated by a simple RBC model subject to irrational exuberance shocks. Periods of optimism are associated with high levels of activity but low investment, so that cumulated investment is below trend at the onset of a recession, and its correlation with depth of recessions and length of recoveries is negative. We also model the case of TFP shocks, and show that those correlations are small and insignificant.

may not be possible. We therefore consider the value of a more limited type of policy: one that seeks only to temporarily boost expenditures. In particular, we are interested in asking whether welfare would be increased by stimulating expenditures for one period, knowing that this would imply a higher X (and therefore lower consumption) in all subsequent periods, thereby prolonging the recession. This question is aimed at capturing the tension between the Keynesian and Hayekian policy prescriptions discussed earlier.[37]

In order to understand the policy we are considering, recall that the law of motion for X is given by

$$X_{t+1} = (1-\delta)X_t + \gamma e(X_t), \qquad 0 < \gamma < 1-\delta,$$

where the function $e(X_t)$ is the equilibrium policy function for e_t. Now, beginning from steady state, suppose at $t = 0$ we stimulate expenditures by ϵ for one period such that the stock at $t = 1$ is now given by

$$\widetilde{X}_1 = (1-\delta)X_0 + \gamma\left[e(X_0) + \epsilon\right].$$

As as result of this one-period stimulus, the path of expenditures for all subsequent periods will be changed even if there is no further policy intervention. The new sequence for X, which we denote $\widetilde{X}_t$, will be given by $\widetilde{X}_{t+1} = (1-\delta)\widetilde{X}_t + \gamma e(\widetilde{X}_t)$ for all $t \geq 1$. We now examine the effect on welfare of such a policy. To do so, we focus on the case where the steady state of the system is stable and δ is sufficiently small that the labor market is slack at the steady state. In this case, we have the following result.

Proposition 12. *Suppose the economy is undergoing a liquidation (i.e., X, and therefore the unemployment rate, are above the steady state). Then a small temporary stimulus will increase welfare even though it delays the return of unemployment to its steady state level.*

Proposition 12 indicates that when the economy is experiencing a liquidation, a one-period policy of stimulating household expenditures can increase welfare even though it postpones the recovery. This arises because the gain in utility coming from the initial

[37] In answering this question, we will be examining the effects of such a policy without needing to be very explicit about the precise policy tools used to engineer the stimulus, which could come from a number of different sources. For example, the stimulus we consider could be engineered by a one-period subsidy to consumption financed by a tax on the employed.

stimulus is greater than the loss in utility associated with a more prolonged recession. With respect to the policy debate between the followers of Hayek and Keynes, we take the results from Proposition 12 as indicating that Hayek may have been right to emphasize the cost of stimulus policies in terms of prolonging a recession, but it may nevertheless still be desirable to stimulate demand in such a case – at least by a small amount– since in the absence of intervention the market outcome will tend to generate a recession that is excessively deep.

6 Conclusion

There are three elements that motivated us to write this paper. First, there is the observation that many recessions arise after periods of fast accumulation of capital goods, either in the form of houses, consumer durables, or productive capital. This, in our view, gives plausibility to the hypothesis that recessions may often reflect periods of liquidation where the economy is trying to deplete excesses from past over-accumulation.[38,39] Second, during these apparent liquidation-driven recessions, the process of adjustment seems to be socially painful and excessive, in the sense that the level of unemployment does not seem to be consistent with the idea that the economy is simply "taking a vacation" after excessive past work. Instead, the economy seems to be exhibiting some coordination failure that makes the exploitation of gains from trade between individuals more difficult than in normal times. These two observations capture the tension we believe is often associated with the Hayekian and Keynesian views of recessions. Finally, even when monetary authorities try to counter such recessions by easing policy, this does not seem to be sufficent to eliminate the problem. This leads us to believe that there are likely mechanisms at play beyond those related to nominal rigidities. Hence, our objective in writing this paper was to offer a framework that is consistent with these three observations, and accordingly to provide an environment where the policy trade-offs inherent to the Hayekian and Keynesian views could be discussed.

A central contribution of the paper is to provide a simple macro model that explains

[38] Note that this is a fundamentalist view of recessions, in that the main cause of a recession is viewed as an objective fundamental (in this case, the level of capital relative to technology) rather than a sunspot-driven change in beliefs.

[39] An alternative interpretation of this observation is that financial imbalances associated with the increase in capital goods are the main source of the subsequent recessions.

why an economy may become particularly inefficient when it inherits an excessive amount of capital goods from the past. The narrative behind the mechanism is quite straightforward. When the economy inherits a high level of capital, this decreases the desire for trade between agents in the economy, leading to less demand. When there are fixed costs associated with employment, this will generally lead to an increase in unemployment. If the risk of unemployment cannot be entirely insured away, households will react to the increased unemployment by limiting purchases and thereby further depressing demand. This multiplier process will cause an excess reaction to the inherited goods and can be large enough to make society worse off even if – in a sense – it is richer since it has inherited a large stock of goods. Within this framework, we have shown that policies aimed at stimulating activity will face an unpleasant trade-off, as the main effect of stimulus will simply be to postpone the adjustment process. Nonetheless, we find that such stimulative policies may remain desirable even if they postpone recovery.

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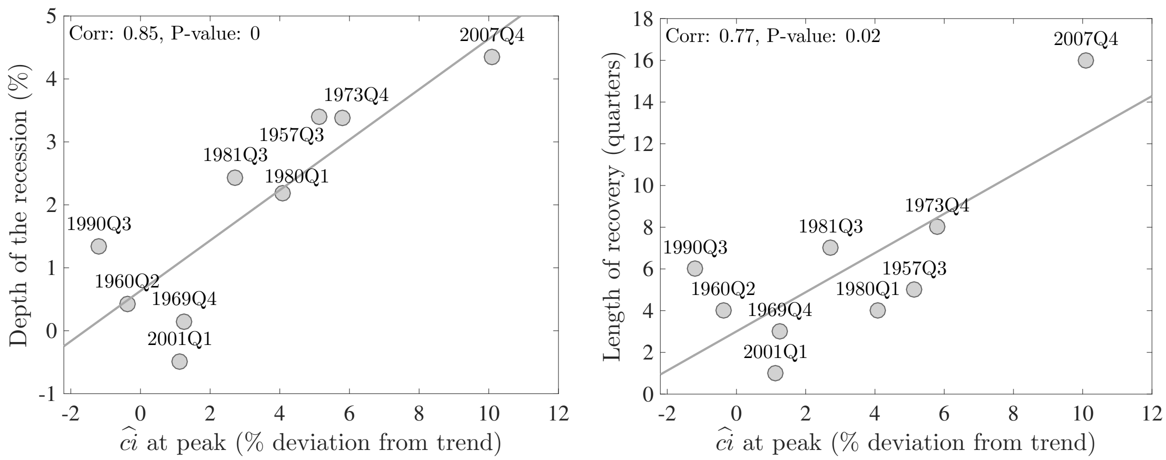
Appendix

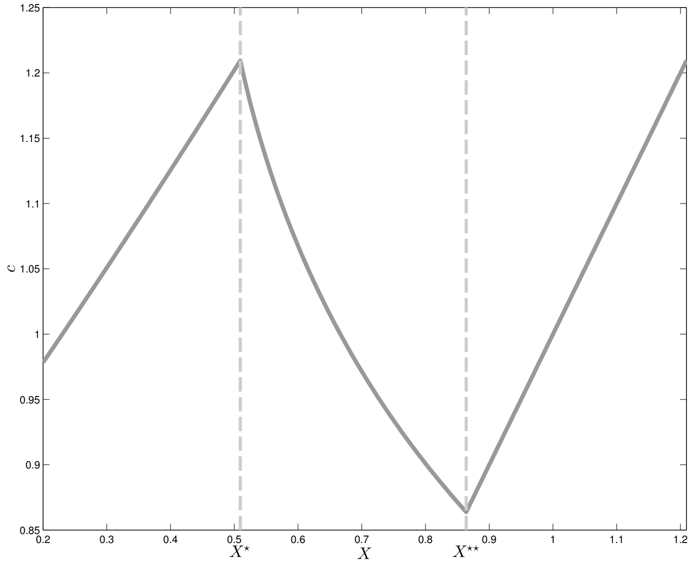
A Figures and Tables

Figure 1: Depth of Recession and Length of Recovery *vs.* Cumulated Total Investment



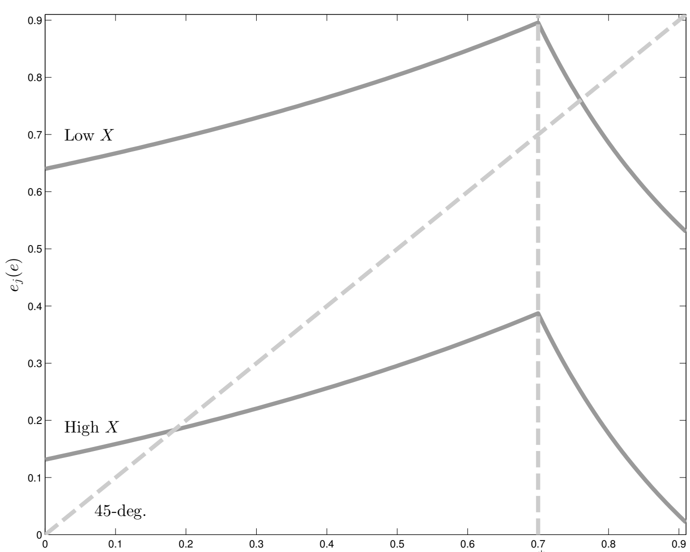
Note: Horizontal axis is capital over-accumulation, measured as cumulated per capita investment over previous 10 years, detrended using a cubic trend. Vertical axis is either depth of recession, measured as percentage fall in real per capita GDP from peak to subsequent trough, or length of recovery, measured as the number of quarters it takes for real per capita GDP to reach its previous peak. Dates correspond to peak quarters.

Figure 2: Consumption as Function of X



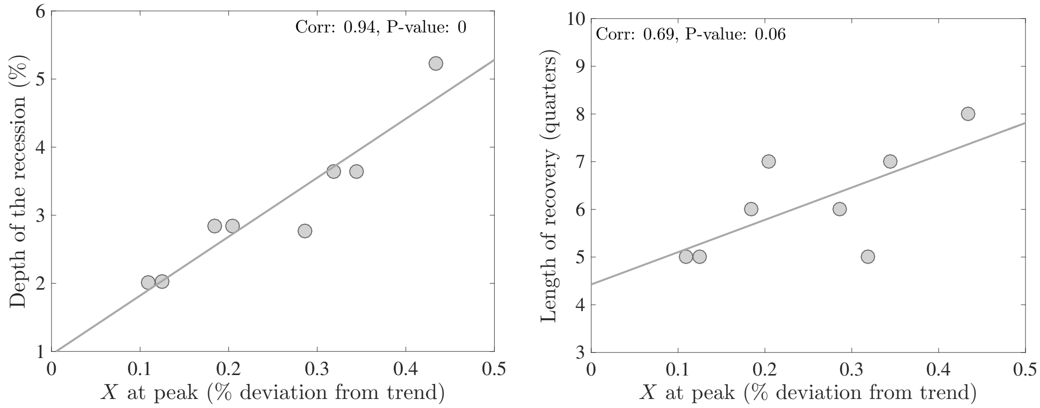
Note: Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \frac{\nu\ell^{1+\omega}}{1+\omega}$ *and* $F(\ell) = A\ell^{\alpha}$, *with parameters* $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$, $\Lambda = 1$ *and* $\tau = 0.4$.

Figure 3: Equilibrium Determination



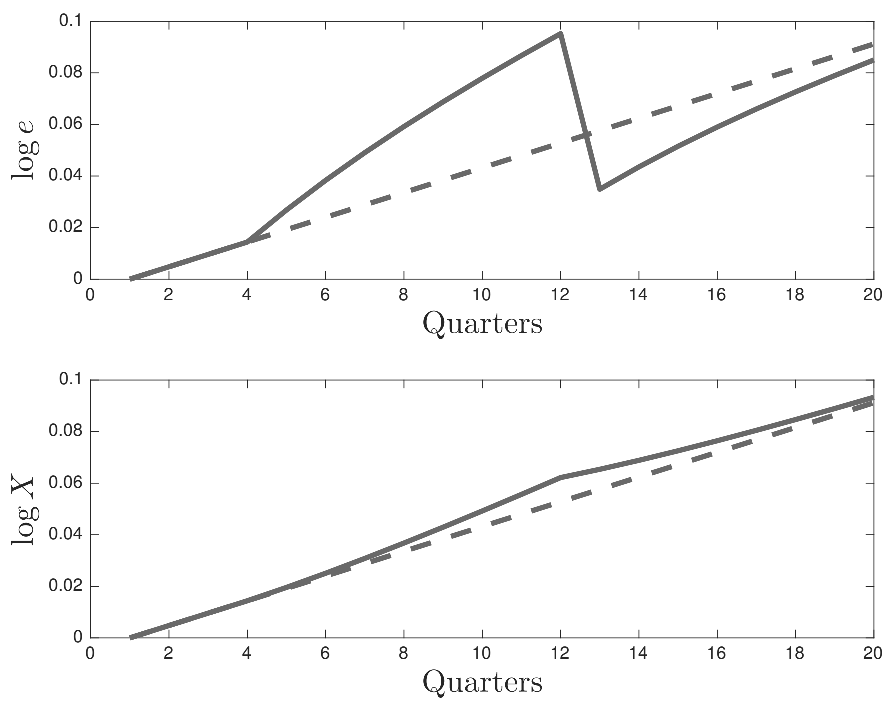
Note: Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \frac{\nu\ell^{1+\omega}}{1+\omega}$ *and* $F(\ell) = A\ell^{\alpha}$, *with parameters* $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$, $\Lambda = 1$ *and* $\tau = 0.4$. *Values of* X *used were* $X = 0$ *for the full-employment equilibrium and* $X = 0.69$ *for the unemployment equilibrium.*

Figure 4: Depth of Recession and Length of Recovery *vs.* Cumulated Total Investment in One Simulation of The Irrational Exuberance Model



Note: Horizontal axis is capital accumulation, measured as cumulated per capita investment over previous 10 years, detrended using a cubic trend. Vertical axis is either depth of recession, measured as percentage difference in real per capita GDP from peak to subsequent trough, or length of recovery, measured as the number of quarters it takes for real per capita GDP to reach its previous peak.

Figure 5: An Episode of Irrational Exhuberance



Note: Dashed line represents the path of the variable along the balanced growth path. Solid line is associated with an 8-quarter episode of irrational exuberance starting in period 5. During this episode, agents believe that the TFP growth rate is 0.585% per quarter instead of 0.48%. e represents total expenditures on goods, and X is the stock of durables. See the text for functional forms and parameter calibration.

Table 1: The Relation Between Depth of Recession or Length of Recovery and Cumulated Investment for US Postwar Recessions

Dep. Variable	:	**Depth of recession**							
Measure of capital	:	Tot. Capital		Resid. Capital		N-Res. Capital		Dur. Goods	
Equation	:	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
constant		3.01	3.22	4.41	3.94	3.50	3.71	3.08	3.19
		(0.06)	(0.09)	(0.01)	(0.04)	(0.13)	(0.14)	(0.09)	(0.13)
$\widehat{ci}_t$		0.94	0.97	0.48	0.48	0.82	0.96	0.74	0.75
		(0.02)	(0.03)	(0.02)	(0.03)	(0.15)	(0.17)	(0.03)	(0.05)
Δtfp		-	-0.19	-	0.30	-	-0.42	-	-0.10
		-	(0.76)	-	(0.63)	-	(0.65)	-	(0.89)
Dep. Variable	:	**Length of Recovery**							
Measure of capital	:	Tot. Capital		Resid. Capital		N-Res. Capital		Dur. Goods	
Equation	:	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
constant		0.63	0.88	1.33	1.34	0.63	0.84	0.47	0.70
		(0.19)	(0.10)	(0.02)	(0.08)	(0.38)	(0.21)	(0.25)	(0.11)
$\widehat{ci}_t$		0.40	0.43	0.17	0.17	0.42	0.56	0.36	0.38
		(0.00)	(0.00)	(0.03)	(0.05)	(0.04)	(0.02)	(0.00)	(0.00)
Δtfp		-	-0.22	-	-0.01	-	-0.42	-	-0.20
		-	(0.25)	-	(0.98)	-	(0.14)	-	(0.19)

Notes: Capital is measured as cumulated per capita investment over previous 10 years, detrended using a cubic trend. Dependent variable is either depth of recession, measured as percentage difference in real per capita GDP from peak to subsequent trough, or length of recovery, measured as the number of quarters it takes for real per capita GDP to reach its previous peak. TFP is Fernald's [2014] series which has been corrected for utilization. Numbers in parentheses are p-values.

Table 2: Targeted and Non-Targeted Moments

Moment	Data	Model
Targeted Moments		
s.d of output growth	0.96%	0.95%
Average depth of a recession	-2.9%	-3.0%
Average length of a recovery (quarters)	6	5.7
Number of recession episodes (per 100 quarters)	3.3	3.3
Non-Targeted Moments		
corr(ci,length)	0.77	0.78
corr(ci,depth)	0.85	0.87

Note: "Data" refers to postwar US data, and "Model" to 10,000 simulations of our model of 270 quarters each. ci stands for capital accumulation, measured as cumulated per capita investment over previous 10 years, detrended using a cubic trend. "Depth" represents the depth of recession, measured as percentage difference in real per capita GDP from peak to subsequent trough, and "length" is the length of recovery, measured as the number of quarters it takes for real per capita GDP to reach its previous peak. In the model simulations, peaks and troughs are detected using Harding and Pagan's [2002] version of Bry and Boschan's [1971] algorithm.

B Data

In sections 2 and 4.2, we use a sample that runs from 1947Q1 to 2015Q2. Series definitions are:

- Peaks and troughs dates are taken from the NBER website.
- Population : FRED, Total Population: All Ages including Armed Forces Overseas (POP), Thousands, Quarterly, Not Seasonally Adjusted, starts in 1952. For 1948 to 1952, we use log linearly interpolated annual data from the Current Population Reports, Series P-25, downloaded: 01/2016.
- Output: BEA, Table 1.1.4. for nominal level and 1.1.5. for price index, Gross Domestic Product, real level constructed as nominal level divided by price index, 1947Q1-2015Q2, seasonally adjusted, downloaded: 01/2016.
- Durable goods: BEA, Table 1.1.4. for nominal level and 1.1.5. for price index, Gross Domestic Product, real level constructed as nominal level divided by price index, 1947Q1-2015Q2, seasonally adjusted, downloaded: 01/2016.
- Non Residential Investment: BEA, Table 1.1.4. for nominal level and 1.1.5. for price index, Gross Domestic Product, real level constructed as nominal level divided by price index, 1947Q1-2015Q2, seasonally adjusted, downloaded: 01/2016.
- Residential Investment: BEA, Table 1.1.4. for nominal level and 1.1.5. for price index, Gross Domestic Product, real level constructed as nominal level divided by price index, 1947Q1-2015Q2, seasonally adjusted, downloaded: 01/2016.
- Fixed Investment: BEA, Table 1.1.4. for nominal level and 1.1.5. for price index, Gross Domestic Product, real level constructed as nominal level divided by price index, 1947Q1-2015Q2, seasonally adjusted, downloaded: 01/2016.
- Total Investment: BEA, Table 1.1.4. for nominal level and 1.1.5. for price index, computed as the sum of real fixed investment and real consumption in durable goods, 1947Q1-2015Q2, seasonally adjusted, downloaded: 01/2016.
- TFP: Utilization-adjusted quarterly-TFP series for the U.S. Business Sector, produced by John Fernald, series ID: dtfp_util, 1947Q1-2015Q2, downloaded: 01/2016.
- Depreciation rates for consumer durables, fixed, residential and non residential investment: computed as the average of the ratio Current-Cost Depreciation of Fixed Assets and Consumer Durable Goods / Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods, over 1948-2014 using annual data, BEA, Tables 1.1. and 1.3, downloaded: 01/2016.
- Unemployment : Civilian Unemployment Rate, Percent, Annual, Seasonally Adjusted (`UNRATE`), FRED database, 1948Q1-2015Q1, downloaded: 01/2016.

Denoting i_{Xt} real investment of quarter t, where X is either consumer durables, fixed, non residential and residential and δ_X the depreciation rate, cumulated investment ci_{Xt} between t and $t-T+1$ is computed as $ci_{Xt} = \sum_{j=0}^{T-1} \delta_X^j i_{Xt-j}$.

C Proofs of Propositions

For simplicity, when the matching function is of the form $M(N, L) = \Lambda \min\{N, L\}$ we prove results only for the case where $\Lambda = 1$. It is straightforward to extend all proofs to the more general case.

Proof of Proposition 1

We first establish that there always exists an equilibrium of this model. Substituting equation (9) into equation (11) and letting $e \equiv c - X$ yields

$$U'(X+e) = \frac{\nu'(\ell)}{F'(\ell)}\left(1+\tau-\tau\frac{e}{\Omega(\ell)}\right), \tag{C.1}$$

where $\Omega(\ell) \equiv F'(\ell)\ell$, which is assumed to be strictly increasing. When $N < L$, equation (10) implies that $\ell = \ell^\star$, and equation (9) implies that $e < e^\star$, where $e^\star \equiv \Omega(\ell^\star)$. On the other hand, when $N > L$, equation (9) implies that $\ell = \Omega^{-1}(e)$. Further, since $\min\{N, L\} < N$ and $F(\ell) - F'(\ell)\ell$ is strictly increasing in ℓ, equation (10) implies that $\ell > \ell^\star$, and thus, by strict increasingness of Ω, we also have $e > e^\star$. Substituting these results into equation (C.1) yields that $e > 0$ is an equilibrium of this model if it satisfies

$$U'(X+e) = Q(e), \tag{C.2}$$

where the function $Q(e)$ is defined in equation (16). Note that Q is continuous, strictly decreasing on $[0, e^\star]$, and strictly increasing on $[e^\star, \infty)$.

Lemma C.1. *If $U'(X) \leq Q(0)$, then there is an equilibrium with $e = 0$. If $U'(X) > Q(0)$, then there is an equilibrium with $e > 0$.*

Proof. For the first part, suppose aggregate conditions are that $e = 0$. Then the household's marginal utility of consumption from simply consuming its endowment is no greater than its expected marginal cost. Households thus respond to aggregate conditions by making no purchases, which in turn validates $e = 0$. For the second part, we have that $\min Q(e) = \nu'(\ell^\star)/F'(\ell^\star) > 0$. Since $\lim_{c\to\infty} U'(c) \leq 0$ by assumption, it follows that, for any X, there exists an e sufficiently large that $U'(X+e) < \min Q(e)$, and therefore there must exist a solution $e > 0$ to equation (C.2). $\square$

Lemma C.1 implies that an equilibrium necessarily exists. We now to showing under what conditions this equilibrium is unique for all values of X. As in equation (14), let $e_j(e) = U'^{-1}(Q(e)) - X$ denote j's optimal expenditure when aggregate expenditure is e, so that equilibrium is a point $e_j(e) = e$. $e_j(e)$ is continuous everywhere, and differentiable everywhere except at $e = e^\star$, with

$$e_j'(e) = \frac{Q'(e)}{U''\left(U'^{-1}(Q(e))\right)}.$$

Note that $e_j'(e)$ is independent of X, strictly increasing on $[0, e^\star]$ and strictly decreasing on $[e^\star, \infty)$.

Lemma C.2. *If*

$$\lim_{e\uparrow e^\star} e_j'(e) < 1, \tag{C.3}$$

then $e_j'(e) < 1$ for all e.

Proof. For $e > e^\star$, $e_j'(e) < 0$, so that $e_j'(e) < 1$ is satisfied. For $e < e^\star$, note that

$$e_j''(e) = \frac{Q''(e) - U'''\left(X + e_j(e)\right)\left[e_j'(e)\right]^2}{U''\left(X + e_j(e)\right)}.$$

Since $Q''(e) = 0$ on this range and $U''' > 0$, we have $e_j''(e) > 0$, and thus $e_j'(e) < \lim_{e \uparrow e^\star} e_j'(e)$, which completes the proof. □

Lemma C.3. *Inequality* (C.3) *holds if and only if*

$$\tau < \bar{\tau} \equiv -U''\left(U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right)\right)\frac{F'(\ell^\star)\left[f(\ell^\star) - \Phi\right]}{\nu'(\ell^\star)}.$$

Proof. We have that

$$\lim_{e \uparrow e^\star} e_j'(e) = \frac{\nu'(\ell^\star)\tau}{-U''\left(U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right)\right)F'(\ell^\star)\left[f(\ell^\star) - \Phi\right]},$$

which is clearly less than one if and only if $\tau < \bar{\tau}$. □

Lemma C.4. *If $\tau < \bar{\tau}$, then there always exists a unique equilibrium regardless of the value of X. If $\tau > \bar{\tau}$, then there exists values of $X \in \mathbb{R}$ such that there are multiple equilibria.*

Proof. Note that an equilibrium in (e, e_j)-space is a point where the $e_j = e_j(e)$ locus intersects the $e_j = e$ locus. To see the first part of the lemma, suppose $\tau < \bar{\tau}$ so that inequality (C.3) holds. Then since the slope of the $e_j = e$ locus is one, and the slope of the $e_j = e_j(e)$ locus is strictly less than one by Lemma C.2, there can be only one intersection, and therefore the equilibrium is unique.

To see the second part of the lemma, suppose that $\tau > \bar{\tau}$ and thus (C.3) does not hold. Then by strict convexity of $e_j(e)$ on $(0, e^\star)$, there exists a value $\underline{e} < e^\star$ such that $e_j'(e) > 1$ on $(\underline{e}, e^\star)$. Define $\widetilde{X}(e) \equiv U'^{-1}(Q(e)) - e$, and note that e is an equilibrium when $X = \widetilde{X}(e)$. We show that there are at least two equilibria when $X = \widetilde{X}(e)$ with $e \in (\underline{e}, e^\star)$. To see this, choose $e_0 \in (\underline{e}, e^\star)$, and note that, for $X = \widetilde{X}(e_0)$, $e_j(e_0) = e_0$ and $e_j'(e) > 1$ on $(e_0, e^\star)$. Thus, it must also be the case that $e_j(e^\star) > e^\star$. But since $e_j(e)$ is continuous everywhere and strictly decreasing on $e > e^\star$, this implies that there exists some value $e > e^\star$ such that $e_j(e) = e$, which would represent an equilibrium. Since $e_0 < e^\star$ is also an equilibrium, there are at least two equilibria. □

This completes the proof of Proposition 1.

Proof of Proposition 2

Lemma C.5. *If $\tau < \bar{\tau}$ and X is such that $e > 0$, then $de/dX < 0$.*

Proof. Differentiating equilibrium condition (C.2) with respect to X yields

$$\frac{de}{dX} = \frac{U''(X + e)}{Q'(e) - U''(X + e)}. \tag{C.4}$$

From Lemma C.3, $Q'(e) > U''\left(U'^{-1}(Q(e))\right)$. In equilibrium, $U'^{-1}(Q(e)) = X + e$, so that this inequality becomes $Q'(e) > U''(X + e)$, and thus the desired conclusion follows by inspection. □

Given Lemma C.5 and the fact that the economy exhibits unemployment when $e < e^\star$ and full employment when $e \geq e^\star$, the economy will exhibit unemployment if and only if $X \leq X^\star$, where

$$X^\star \equiv U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right) - F'(\ell^\star)\,\ell^\star$$

is the level of X such that $e = e^\star$ is the equilibrium. This establishes the first part of the proposition.

Next, from Lemma C.1, we see that there is a zero-employment equilibrium if and only if $U'(X) \leq \frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau)$, which holds when $X \geq X^{\star\star}$, where

$$X^{\star\star} \equiv U'^{-1}\left(\frac{\nu'(\ell^\star)}{F'(\ell^\star)}(1+\tau)\right).$$

This completes the proof of Proposition 2.

Proof of Proposition 3

If $X < X^{\star\star}$, we know from Proposition 2 that $e > 0$, and therefore e solves equation (C.2). Substituting $e = c - X$ for e yields the desired result in this case. From Proposition 2, we also know that if $X \geq X^{\star\star}$ then $e = 0$, in which case $c = X$, which completes the proof.

Proof of Proposition 4

If $X > X^{\star\star}$ then $e = 0$ and $c = X$, so that c is increasing in X. Suppose instead $X < X^{\star\star}$, so that $e > 0$. Differentiating $c = X + e$ with respect to X and using equation (C.4), we obtain

$$\frac{dc}{dX} = \frac{Q'(e)}{Q'(e) - U''(X+e)}. \tag{C.5}$$

Since the denominator of this expression is positive (see proof of Lemma C.5), the sign of dc/dX is given by the sign of $Q'(e)$, which is negative if $e < e^\star$ (i.e., if $X^\star < X < X^{\star\star}$) and positive if $e > e^\star$ (i.e., if $X < X^\star$). This completes the proof.

Proof of Proposition 5

Letting $\mathcal{U}(e)$ denote welfare conditional on the coordinated level of e, we may obtain that

$$\mathcal{U}(e) = U(X+e) + \mu(e)\left[\mathcal{L}^\star - \frac{\nu'(\ell^\star)}{F'(\ell^\star)}e\right] - [1-\mu(e)](1+\tau)\frac{\nu'(\ell^\star)}{F'(\ell^\star)}e,$$

where $\mu(e) = e/[F'(\ell^\star)\,\ell^\star]$ denotes employment conditional on e and $\mathcal{L}^\star \equiv \nu'(\ell^\star)\ell^\star - \nu(\ell^\star) \geq 0$. Using the envelope theorem, the only welfare effects of a marginal change in e from its decentralized equilibrium value are those that occur through the resulting change in employment. Thus,

$$\mathcal{U}'(e) = \left[\mathcal{L}^\star + \tau\frac{\nu'(\ell^\star)}{F'(\ell^\star)}e\right]\mu'(e) > 0,$$

and therefore a coordinated rise in e would increase expected utility of all households.

Proof of Proposition 6

Denote welfare as a function of X by

$$\mathcal{U}(X) \equiv U(X+e) + \mu\left[-\nu(\ell) + V(w\ell - pe)\right] + (1-\mu)V(-pe).$$

If $X < X^{\star}$ (full-employment regime) or $X > X^{\star\star}$ (zero-employment regime), $\mathcal{U}'(X) > 0$ clearly always holds. Thus, we focus on the case where $X \in (X^{\star}, X^{\star\star})$, in which case some algebra yields

$$\mathcal{U}(X) = U(X+e) + \left\{\ell^{\star}\left[\nu'(\ell^{\star}) - \frac{\nu(\ell^{\star})}{\ell^{\star}}\right] + \frac{\nu'(\ell^{\star})}{F'(\ell^{\star})}\tau e\right\}\mu - \frac{\nu'(\ell^{\star})}{F'(\ell^{\star})}(1+\tau)e.$$

Using the envelope theorem, we may differentiate this expression with respect to X to obtain

$$\mathcal{U}'(X) = U'(X+e) + \left[\mathcal{L}^{\star} + \frac{\nu'(\ell^{\star})}{F'(\ell^{\star})}\tau e\right]\frac{d\mu}{dX}, \tag{C.6}$$

where $\mathcal{L}^{\star} \equiv \nu'(\ell^{\star})\ell^{\star} - \nu(\ell^{\star}) \geq 0$.

Lemma C.6. *$\mathcal{U}''(X) > 0$ on $(X^{\star}, X^{\star\star})$.*

Proof. Substituting the equilibrium condition (C.2) into (C.6) and using the fact that $d\mu/dX = \left[F'(\ell^{\star})\,\ell^{\star}\right]^{-1} de/dX$, after some algebra we obtain

$$\mathcal{U}'(X) = \frac{\nu'(\ell^{\star})}{F'(\ell^{\star})}\left[1 + \tau + \tau\mu\left(\frac{de}{dX} - 1\right)\right] + \frac{\mathcal{L}^{\star}}{F'(\ell^{\star})\,\ell^{\star}}\frac{de}{dX}. \tag{C.7}$$

From (C.4), we may also obtain that

$$\frac{de}{dX} = \left(\frac{\nu'(\ell^{\star})\tau}{-U''(X+e)\left[F'(\ell^{\star})\right]^2\ell^{\star}} - 1\right)^{-1},$$

$$\frac{d^2e}{dX^2} = \frac{U'''(X+e)}{U''(X+e)}\frac{de}{dX}\left[\frac{dc}{dX}\right]^2 > 0,$$

and therefore

$$\mathcal{U}''(X) = \frac{\nu'(\ell^{\star})}{F'(\ell^{\star})}\tau\frac{d\mu}{dX}\left(\frac{de}{dX} - 1\right) + \left[\frac{\nu'(\ell^{\star})}{F'(\ell^{\star})}\tau\mu + \frac{\mathcal{L}^{\star}}{F'(\ell^{\star})\,l^{\star}}\right]\frac{d^2e}{dX^2}.$$

Since $de/dX < 0$, $d\mu/dx < 0$ and thus the first term is positive, as is the second term, and the proof is complete. ☐

Lemma C.7. *If $\tau > \underline{\tau} \equiv \frac{\nu(\ell^{\star})}{\nu'(\ell^{\star})\ell^{\star}}\frac{\bar{\tau}}{1+\bar{\tau}}$ then there exists a range of X such that $\mathcal{U}'(X) < 0$.*

Proof. Since $\mathcal{U}$ is convex by Lemma C.6, $\mathcal{U}'(X) < 0$ for some X if and only if $\lim_{X\downarrow X^{\star}}\mathcal{U}'(X) < 0$. Taking limits of equation (C.7), and using the facts that $\lim_{X\downarrow X^{\star}}\frac{de}{dX} = -\bar{\tau}/(\bar{\tau} - \tau)$ and $\lim_{X\downarrow X^{\star}}\mu = 1$, we obtain that

$$\lim_{X\downarrow X^{\star}}\mathcal{U}'(X) = \frac{\nu'(\ell^{\star})}{F'(\ell^{\star})}\left(1 - \frac{\tau\bar{\tau}}{\bar{\tau} - \tau}\right) - \frac{\mathcal{L}^{\star}}{F'(\ell^{\star})\,\ell^{\star}}\left(\frac{\bar{\tau}}{\bar{\tau} - \tau}\right).$$

Substituting in from the definition of $\mathcal{L}^{\star}$, straightforward algebra yields that this expression is less than one if and only if $\tau > \underline{\tau}$. ☐

Note that, by convexity of $\nu(\ell)$ and the fact that $\nu(0) = 0$, we have $\nu(\ell^{\star}) \leq \nu'(\ell^{\star})\ell^{\star}$, and thus $\underline{\tau} < \bar{\tau}$, so that there always exists values of τ such that $\underline{\tau} < \tau < \bar{\tau}$. From the definition of $\underline{\tau}$, we also see that, holding τ and $\nu'(\ell^{\star})$ constant, if $\nu(\ell^{\star})/\ell^{\star}$ is small, this inequality is more likely to be satisfied.

Proof of Proposition 7

In the extended model with non-durables, the equilibrium conditions for quantities can be written[40]

$$U^{d\prime}\left(c^{d}\right)=\frac{\nu'\left(\ell^{d}\right)}{F^{d\prime}\left(\ell^{d}\right)}\left(1+\tau-\frac{\Lambda\min\{N^{d}+N^{n},L\}}{L}\tau\right), \tag{C.8}$$

$$\frac{\Lambda\min\{N^{d}+N^{n},L\}}{L}=\left[\frac{c^{d}-X}{F^{d\prime}\left(\ell^{d}\right)\ell^{d}}\right]\left[\frac{N^{d}+N^{n}}{N^{d}}\right], \tag{C.9}$$

$$\Phi=\frac{\Lambda\min\{N^{d}+N^{n},L\}}{N^{d}+N^{n}}\left[F^{d}\left(\ell^{d}\right)-F^{d\prime}\left(\ell^{d}\right)\ell^{d}\right], \tag{C.10}$$

$$U^{n\prime}(c^{n})=\frac{\nu'\left(\ell^{n}\right)}{F^{n\prime}\left(\ell^{n}\right)}\left(1+\tau-\frac{\Lambda\min\{N^{d}+N^{n},L\}}{L}\tau\right), \tag{C.11}$$

$$\frac{\Lambda\min\{N^{d}+N^{n},L\}}{L}=\left[\frac{c^{n}}{F^{n\prime}\left(\ell^{n}\right)\ell^{n}}\right]\left[\frac{N^{d}+N^{n}}{N^{n}}\right], \tag{C.12}$$

$$\Phi=\frac{\Lambda\min\{N^{d}+N^{n},L\}}{N^{d}+N^{n}}\left[F^{n}\left(\ell^{n}\right)-F^{n\prime}\left(\ell^{n}\right)\ell^{n}\right]. \tag{C.13}$$

We first consider equilibria within three regimes separately, then establish the relationship between these regimes and different ranges of X. We assume that $\lim_{c^{n}\to 0}U^{n\prime}\left(c^{n}\right)=\infty$, so that we will always have $N^{n}>0$. In our notation, the superscript s will index the sector, with $s\in\{d,n\}$.

Equilibrium Regime 1: $N\equiv N^{d}+N^{n}\geq L$

Suppose $N\equiv N^{d}+N^{n}\geq L$, so that $\mu=1$. Assuming Φ is small enough to ensure the existence of a regime with $\mu<1$ (see footnote 25), we must have $N^{d}>0$ when $\mu=1$. Letting $\psi\equiv N^{d}/N$, and letting $p^{s}\left(\ell\right)\equiv\nu'\left(\ell\right)/F^{s\prime}\left(\ell\right)$, the equilibrium conditions can be written

$$U^{d\prime}\left(X+e\right)=p^{d}\left(\ell^{d}\right), \tag{C.14}$$

$$U^{n\prime}\left(c^{n}\right)=p^{n}\left(\ell^{n}\right), \tag{C.15}$$

$$F^{d}\left(\ell^{d}\right)-F^{d\prime}\left(\ell^{d}\right)\ell^{d}=\frac{N}{L}\Phi, \tag{C.16}$$

$$F^{n}\left(\ell^{n}\right)-F^{n\prime}\left(\ell^{n}\right)\ell^{n}=\frac{N}{L}\Phi, \tag{C.17}$$

$$e=\psi\Omega^{d}\left(\ell^{d}\right), \tag{C.18}$$

$$c^{n}=\left(1-\psi\right)\Omega^{n}\left(\ell^{n}\right). \tag{C.19}$$

Lemma C.8. *There exists at most one equilibrium in Regime 1.*

Proof. The zero-profit conditions (C.16) and (C.17) can be solved to obtain $\ell^{s}=\ell^{s}\left(N\right)$, with $\ell^{s\prime}\left(N\right)>0$. Substituting the resource constraint equations (C.18) and (C.19) into the demand equations (C.14) and (C.15) for e and c^{n}, we may then reduce the system to two equations,

$$U^{d\prime}\left(X+\psi\Omega^{d}\left(\ell^{d}\left(N\right)\right)\right)=p^{d}\left(\ell^{d}\left(N\right)\right), \tag{C.20}$$

$$U^{n\prime}\left(\left(1-\psi\right)\Omega^{n}\left(\ell^{n}\left(N\right)\right)\right)=p^{n}\left(\ell^{n}\left(N\right)\right), \tag{C.21}$$

[40] The prices and wages are given by $w^{d}=\frac{\nu'(\ell^{d})}{v}$, $p^{d}=\frac{\nu'(\ell^{d})}{vF^{d\prime}(\ell^{d})}$, $w^{n}=\frac{\nu'(\ell^{n})}{v}$ and $p^{n}=\frac{\nu'(\ell^{n})}{vF^{n\prime}(\ell^{n})}$.

in two unknowns, N and ψ. Each of these equations gives a locus of points in (N, ψ)-space, with intersections representing equilibria. Totally differentiating with respect to N yields, respectively,

$$\begin{aligned}
\frac{d\psi}{dN} &= \frac{\ell^{d\prime}(N)}{\Omega^d(\ell^d(N))}\left[-\frac{p^{d\prime}\left(\ell^d(N)\right)}{-U^{d\prime\prime}\left(X+\psi\Omega^d\left(\ell^d(N)\right)\right)}-\psi\Omega^{d\prime}\left(\ell^d(N)\right)\right]<0,\\
\frac{d\psi}{dN} &= \frac{\ell^{n\prime}(N)}{\Omega^n(\ell^n(N))}\left[\frac{p^{n\prime}\left(\ell^n(N)\right)}{-U^{n\prime\prime}\left((1-\psi)\Omega^n\left(\ell^n(N)\right)\right)}+(1-\psi)\Omega^{n\prime}\left(\ell^n(N)\right)\right]>0,
\end{aligned}$$

so that the locus associated with (C.20) is downward-sloping, and the locus associated with (C.21) is upward-sloping. Thus, if an equilibrium exists in this region, it is clearly unique.

□

Lemma C.9. *In an equilibrium in Regime 1, we have $de/dX < 0$, $dc^d/dX > 0$ and $dc^n/dX > 0$.*

Proof. A rise in X results in a shift down of the locus associated with equation (C.20), so that $d\psi/dX < 0$ and $dN/dX < 0$. Since a fall in N results in a fall in ℓ^s and thus also p^s, it follows from equations (C.14) and (C.15) that $dc^d/dX > 0$ and $dc^n/dX > 0$. Further, since ψ and ℓ^d both fall, it follows from equation (C.18) that $de/dX < 0$. □

Equilibrium Regime 2: $N < L$ and $N^d > 0$

Suppose now that $N \equiv N^d + N^n < L$ but with a positive output of durables ($N^d > 0$).

Lemma C.10. *For τ sufficiently small, there exists at most one equilibrium in Regime 2.*

Proof. Let

$$\begin{aligned}
Z^n(e, c^n) &\equiv U^{n\prime-1}\left(\frac{\nu'(\ell^{n\star})}{F^{n\prime}(\ell^{n\star})}\left[1+\tau-\tau\left(\frac{e}{\Omega^d(\ell^{d\star})}+\frac{c^n}{\Omega^n(\ell^{n\star})}\right)\right]\right)\\
&= U^{n\prime-1}\left(\frac{\nu'(\ell^{n\star})}{F^{n\prime}(\ell^{n\star})}(1+\tau-\tau\mu)\right),
\end{aligned}$$

where, as before, $\Omega^s(\ell) \equiv F^{s\prime}(\ell)\,\ell$. For a given e, c^n solves $c^n = Z^n(e, c^n)$. This solution will always be unique if $Z_2^n(e, c^n) < 1$ for all combinations of e and c^n such that $N \leq L$. Since

$$Z_2^n(e, c^n) = \frac{1}{-U''(Z^n(e, c^n))}\frac{\nu'(\ell^{n\star})}{F^{n\prime}(\ell^{n\star})}\frac{\tau}{\Omega^n(\ell^{n\star})},$$

$Z_2^n(e, c^n)$ is maximized when $Z^n(e, c^n)$ is maximized, which in turn occurs when $\mu = 1$. Thus, a sufficient condition to ensure that c^n is always uniquely determined given e is that

$$\tau < \bar{\tau}^n \equiv -U''\left(U^{n\prime-1}\left(\frac{\nu'(\ell^{n\star})}{F^{n\prime}(\ell^{n\star})}\right)\right)\Omega^n(\ell^{n\star})\frac{F^{n\prime}(\ell^{n\star})}{\nu'(\ell^{n\star})}.$$

Assume henceforth that this is true, and let $c^n(e)$ denote the unique value of c^n that solves $c^n = Z^n(e, c^n)$. Note that

$$c^{n\prime}(e) = \frac{\frac{\nu'(\ell^{n\star})}{F^{n\prime}(\ell^{n\star})}\frac{\tau}{\Omega^d(\ell^{d\star})}}{-U^{n\prime\prime}(c^n(e)) - \frac{\nu'(\ell^{n\star})}{F^{n\prime}(\ell^{n\star})}\frac{\tau}{\Omega^n(\ell^{n\star})}}.$$

Since $\tau < \bar{\tau}^n$, it may be verified that the denominator of this expression is strictly positive, so that $c^{n\prime} > 0$. Further,

$$c^{n\prime\prime}(e) = \frac{\frac{\nu'(\ell^{n\star})}{F^{n\prime}(\ell^{n\star})}\frac{\tau}{\Omega^d(\ell^{d\star})}U^{n\prime\prime\prime}(c^n(e))\,c^{n\prime}(e)}{\left[-U^{n\prime\prime}(c^n(e)) - \frac{\nu'(\ell^{n\star})}{F^{n\prime}(\ell^{n\star})}\frac{\tau}{\Omega^n(\ell^{n\star})}\right]^2} > 0.$$

Next, let

$$\begin{aligned} Z^d(e) &\equiv U^{d\prime-1}\left(\frac{\nu'\left(\ell^{d\star}\right)}{F^{d\prime}\left(\ell^{d\star}\right)}\left[1+\tau-\tau\left(\frac{e}{\Omega^d\left(\ell^{d\star}\right)}+\frac{c^n(e)}{\Omega^n\left(\ell^{n\star}\right)}\right)\right]\right) \\ &= U^{d\prime-1}\left(\frac{\nu'\left(\ell^{d\star}\right)}{F^{d\prime}\left(\ell^{d\star}\right)}(1+\tau-\tau\mu)\right), \end{aligned}$$

so that the equilibrium solves $e = Z^d(e) - X$. This equilibrium is unique for all X if $Z^{d\prime}(e) < 1$ for all e. We have

$$Z^{d\prime}(e) = \frac{1}{-U^{d\prime\prime}(Z^d(e))}\frac{\nu'\left(\ell^{d\star}\right)}{F^{d\prime}\left(\ell^{d\star}\right)}\tau\left[\frac{1}{\Omega^d\left(\ell^{d\star}\right)}+\frac{c^{n\prime}(e)}{\Omega^n\left(\ell^{n\star}\right)}\right] > 0.$$

Since

$$Z^{d\prime\prime}(e) = \frac{U^{d\prime\prime\prime}\left(Z^d(e)\right)\left[Z^{d\prime}(e)\right]^2}{-U^{d\prime\prime}(Z^d(e))} + \frac{1}{-U^{d\prime\prime}(Z^d(e))}\frac{\nu'\left(\ell^{d\star}\right)}{F^{d\prime}\left(\ell^{d\star}\right)}\tau\frac{c^{n\prime\prime}(e)}{\Omega^n\left(\ell^{n\star}\right)} > 0,$$

it follows that $Z^{d\prime}(e)$ is maximized at the maximum value of e such that the economy is still in the unemployment regime, i.e., at the value of e such that

$$\frac{e}{\Omega^d\left(\ell^{d\star}\right)}+\frac{c^n(e)}{\Omega^n\left(\ell^{n\star}\right)} = 1.$$

Let $e^\star$ denote the value of e for which this is true, and note that

$$Z^d(e^\star) \equiv U^{d\prime-1}\left(\frac{\nu'\left(\ell^{d\star}\right)}{F^{d\prime}\left(\ell^{d\star}\right)}\right).$$

Then a sufficient condition for there to always exist a unique equilibrium is that

$$\tau < \bar{\tau}^d \equiv \min\left\{\bar{\tau}^n, -U^{d\prime\prime}\left(Z^d(e^\star)\right)\left[\frac{1}{\Omega^d\left(\ell^{d\star}\right)}+\frac{c^{n\prime}(e^\star)}{\Omega^n\left(\ell^{n\star}\right)}\right]^{-1}\frac{F^{d\prime}\left(\ell^{d\star}\right)}{\nu'\left(\ell^{d\star}\right)}\right\}.$$

□

Lemma C.11. *If $\tau < \bar{\tau}^d$, then in an equilibrium in Regime 2, we have $de/dX < 0$, $dc^d/dX < 0$ and $dc^n/dX < 0$.*

Proof. Since $c^{n\prime}(e) > 0$, it follows that $dc^n/dX < 0$ as long as $dc^d/dX < 0$. Differentiating the equilibrium condition $e = Z^d(e) - X$ with respect to X and solving yields

$$\frac{de}{dX} = -\frac{1}{1-Z^{d\prime}(e)}.$$

Since $\tau < \bar{\tau}^d$, we have $0 < Z^{d\prime}(e) < 1$, so that $de/dX < -1$ and thus $dc^d/dX < 0$.

□

Equilibrium Regime 3: $N \equiv N^d + N^n < L$ and $N^d = 0$

Suppose finally that there is unemployment ($\mu < 1$), but that there is no output of durables ($N^d = 0$). Clearly then, $e = 0$, and thus $c^d = X$.

Lemma C.12. *If $\tau < \bar{\tau}$, then there exists at most one equilibrium in Regime 3, and this equilibrium is independent of X.*

Proof. An equilibrium in this case is given by a solution to $c^n = Z^n(0, c^n)$. As argued in the proof of Lemma C.10, this solution is unique if $\tau < \bar{\tau}$ and given in that case by $c^n(0)$. Further, since $Z^n(0, c^n)$ does not depend on X in any way, this equilibrium is independent of X.

□

Equilibrium as a function of X

Define $X^{\star\star} \equiv Z^d(0)$ and $X^\star \equiv Z^d(e^\star) - e^\star$. It is straightforward to verify that if $X < X^\star$ then the equilibrium is in Regime 1, if $X^\star \leq X < X^{\star\star}$ then the equilibrium is in Regime 2, and if $X \geq X^{\star\star}$ then the equilibrium is in Regime 3. The properties of Proposition 7 then follow immediately.

Proof of Proposition 8

Household j's demand for new goods is given by

$$e_j = \min\left\{U'^{-1}(Q(e)) - X, \frac{X}{(1+\tau)\,p(e)}\right\}$$

where

$$p(e) = \begin{cases} \frac{\nu'(\ell^\star)}{vF'(\ell^\star)} & \text{if } e \leq e^\star \\ \frac{\nu'(\Omega^{-1}(e))}{vF'(\Omega^{-1}(e))} & \text{if } e > e^\star \end{cases}$$

Note that $p(e)$ is constant (and equal to $p^\star$) for $e < e^\star$, while $p'(e) > 0$ for $e > e^\star$. Thus, $X/[(1+\tau)\,p(e)]$ is constant on $e < e^\star$ and decreasing on $e > e^\star$.

Let $\tilde{X}_1 \equiv (1+\tau)\,p^\star e^\star$ and let $\tilde{X}_0$ be the unique solution satisfying $\tilde{X} \geq \tilde{X}_1$ to

$$U'^{-1}\left(Q\left(\frac{\tilde{X}}{(1+\tau)\,p^*}\right)\right) - \tilde{X} = \frac{\tilde{X}}{(1+\tau)\,p^\star}$$

provided such a solution exists. $\tilde{X}_0$ is the value of X (if it exists) that places the equilibrium in the slack region with a collateral constraint that just binds. It can be verified that the solution to $e = U'^{-1}(Q(e)) - X$ satisfies $e < \min\{e^\star, X/[(1+\tau)\,p^\star]\}$ for $\tilde{X}_0 < X < X^{\star\star}$[41] when $\tilde{X}_0$ exists, and for $X^\star < X < X^{\star\star}$ when $\tilde{X}_0$ does *not* exist. Thus, letting $X^{++} = \tilde{X}_0$ if $\tilde{X}_0$ exists, and $X^{++} = X^\star$ otherwise, we have that, for $X^{++} < X < X^{\star\star}$, the equilibrium is in the slack region and features a non-binding collateral constraint. Part (i) of the Proposition then follows immediately.

Next, note that the equilibrium is in the slack region with a binding collateral constraint for $X < \tilde{X}_0$ if $\tilde{X}_0$ exists, and for $X < \tilde{X}_1$ if $\tilde{X}_0$ does not exist. Thus, letting $X^+ = \tilde{X}_0$ if $\tilde{X}_0$ exists, and $X^+ = \tilde{X}_1$ otherwise, we have that, for $X < X^+$, the equilibrium is in the slack region and features a binding collateral constraint. Further, since $e = X/[(1+\tau)\,p^\star]$ in this case, we have $\partial e/\partial X > 0$, and thus $\partial c/\partial X > 1$, which completes the proof of part (ii) of the Proposition.

[41] Note that we must have $\tilde{X}_0 < X^{\star\star}$. Suppose not, i.e., suppose $\tilde{X}_0 \geq X^{\star\star}$. Then if $X = \tilde{X}_0$ agents would not want to make any purchases in equilibrium ($e = 0$), in which case the collateral constraint $e \leq X/[(1+\tau)\,p^*]$ would not bind, contradicting the definition of $\tilde{X}_0$.

Proof of Proposition 9

It can be verified that the steady-state level of purchases e solves

$$U'\left(\frac{\delta+\gamma}{\delta}e\right) = \zeta Q(e), \tag{C.22}$$

where $\zeta \equiv \frac{1-\beta(1-\delta)}{1-\beta(1-\delta)+\beta\gamma} \in (0,1)$.

Lemma C.13. *For δ sufficiently small, a steady state exists and is unique.*

Proof. Similar to in the static case, we may express individual j's optimal choice of steady-state expenditure e_j given aggregate steady-state expenditure e as $e_j(e) = \frac{\delta}{\delta+\gamma}U'^{-1}(\zeta Q(e))$. As before, we can verify that $e_j'(e) < 0$ for $e > e^\star$, while $e_j'(e) > 0$ and $e_j''(e) > 0$ for $e < e^\star$. Thus, an equilibrium necessarily exists and is unique if $e_j'(e) < 1$ for $e < e^\star$, which is equivalent to the condition that $\lim_{e\uparrow e^\star} e_j'(e) < 1$. This is in turn equivalent to the condition $\tau < \widetilde{\tau}$, where

$$\widetilde{\tau} \equiv -\frac{\delta+\gamma}{\delta\zeta}U''\left(U'^{-1}\left(\zeta\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\right)\right)\frac{F'(\ell^\star)\left[F(\ell^\star)-\Phi\right]}{\nu'(\ell^\star)}. \tag{C.23}$$

As $\delta \to 0$, $\widetilde{\tau}$ approaches infinity, and thus it will hold for any τ, which completes the proof. □

Note for future reference that if $e_j'(e) < 1$ then

$$(\delta+\gamma)U''(X+e) < \delta\zeta Q'(e). \tag{C.24}$$

Lemma C.14. *For δ sufficiently small, there exists a steady state in the unemployment regime.*

Proof. Since $U'(0) > Q(0)$ by assumption, we also have $U'(0) > \zeta Q(0)$. Thus, if $U'\left(\frac{\delta+\gamma}{\delta}e^\star\right) < \zeta Q(e^\star)$ then equation (C.22) holds for at least one value of $e < e^\star$. Note that $\lim_{\delta\to 0}\frac{\delta+\gamma}{\delta}e^\star = \infty$ and $\lim_{\delta\to 0}\zeta t = (1-\beta)/(1-\beta+\beta\gamma) > 0$. Thus, since $\lim_{c\to\infty}U'(c) \leq 0$ by assumption, it follows that $\lim_{\delta\to 0}U'\left(\frac{\delta+\gamma}{\delta}e^\star\right) \leq 0 < \lim_{\delta\to 0}\zeta Z(e^\star)$, and thus the desired property holds for δ close enough to zero. □

Lemmas C.13 and C.14 together prove the proposition.

Proof of Proposition 10

Linearizing the system in e_t and X_t around the steady state, letting variables with hats denote deviations from steady state and variables without subscripts denote steady-state values, we have

$$\widehat{X}_{t+1} = (1-\delta)\widehat{X}_t + \gamma\widehat{e}_t,$$

$$\widehat{e}_{t+1} = -\frac{[1-\beta(1-\delta)(1-\delta-\gamma)]U''X+e)}{\beta\left[(1-\delta)Q'(e)-(1-\delta-\gamma)U''(X+e)\right]}\widehat{X}_t + \frac{Q'(e)-\left[1-\beta\gamma(1-\delta-\gamma)\right]U''(X+e)}{\beta\left[(1-\delta)Q'(e)-(1-\delta-\gamma)U''(X+e)\right]}\widehat{e}_t,$$

or

$$\widehat{x}_{t+1} \equiv \begin{pmatrix}\widehat{X}_{t+1}\\ \widehat{e}_{t+1}\end{pmatrix} = \begin{pmatrix}1-\delta & \gamma\\ a_{eX} & a_{ee}\end{pmatrix}\begin{pmatrix}\widehat{X}_t\\ \widehat{e}_t\end{pmatrix} \equiv A\widehat{x}_t,$$

where a_{eX} and a_{ee} are the coefficients on $\widehat{X}_t$ and $\widehat{e}_t$ in the expression for $\widehat{e}_{t+1}$. The eigenvalues of A are then given by

$$\lambda_1 \equiv \frac{1-\delta+a_{ee}-\sqrt{(1-\delta+a_{ee})^2-4\beta^{-1}}}{2},$$

$$\lambda_2 \equiv \frac{1-\delta+a_{ee}+\sqrt{(1-\delta+a_{ee})^2-4\beta^{-1}}}{2}.$$

We may obtain that

$$\lambda_1\lambda_2 = \beta^{-1} > 1, \tag{C.25}$$

so that $|\lambda_i| > 1$ for at least one $i \in \{1,2\}$. Thus, this system cannot exhibit local indeterminacy (see, e.g., Blanchard and Kahn [1980]), which completes the proof.

Proof of Proposition 11

Note that (C.25) implies that if the eigenvalues are real then they are of the same sign, with $\lambda_2 > \lambda_1$.

Lemma C.15. *The system is saddle-path stable if and only if*

$$|1-\delta+a_{ee}| > \frac{1+\beta}{\beta}, \tag{C.26}$$

in which case the eigenvalues are real and of the same sign as $1-\delta+a_{ee}$.

Proof. To see the "if" part, suppose (C.26) holds, and note that this implies $(1-\delta+a_{ee})^2 > [(1+\beta)/\beta]^2 > 4\beta^{-1}$, and therefore the eigenvalues are real. If $1-\delta+a_{ee} > (1+\beta)/\beta$, then this implies that $\lambda_2 > \lambda_1 > 0$, and therefore the system is stable as long as $\lambda_1 < 1$, which is equivalent to the condition

$$(1-\delta+a_{ee}) - 2 < \sqrt{(1-\delta+a_{ee})^2-4\beta^{-1}}. \tag{C.27}$$

Since $1-\delta+a_{ee} > (1+\beta)/\beta > 2$, both sides of this inequality are positive, and therefore, squaring both sides and rearranging, it is equivalent to

$$1-\delta+a_{ee} > \frac{1+\beta}{\beta}, \tag{C.28}$$

which holds by hypothesis. A similar argument can be used to establish the claim for the case that $-(1-\delta+a_{ee}) > (1+\beta)/\beta$.

To see the "only if" part, suppose the system is stable. If the eigenvalues had non-zero complex part, then $|\lambda_1| = |\lambda_2| > 1$, in which case the system would be unstable. Thus, the eigenvalues must be real, i.e., $(1-\delta+a_{ee})^2 > 4\beta^{-1}$, which in turn implies that $|1-\delta+a_{ee}| > 2\sqrt{\beta^{-1}}$. If $1-\delta+a_{ee} > 2\sqrt{\beta^{-1}}$, then, reasoning as before, $\lambda_2 > \lambda_1 > 0$, and therefore if the system is stable then (C.27) must hold. Since $(1-\delta+a_{ee}) > 2\sqrt{\beta^{-1}} > 2$, then again both sides of (C.27) are positive, and thus that inequality is equivalent to (C.28), which in turn implies (C.26). Similar arguments establish (C.26) for the case where $-(1-\delta+a_{ee}) > 2\sqrt{\beta^{-1}}$. □

Lemma C.16. *The system is saddle-path stable with positive eigenvalues if and only if*

$$(1-\delta-\gamma)U''(X+e) < (1-\delta)Q'(e). \tag{C.29}$$

Proof. Note that the system is stable with positive eigenvalues if and only if (C.28) holds. We have that

$$1-\delta+a_{ee}-\frac{1+\beta}{\beta}=\frac{[1-\beta(1-\delta-\gamma)][\delta\zeta Q'(e)-(\delta+\gamma)U''(X+e)]}{\beta[(1-\delta)Q'(e)-(1-\delta-\gamma)U''(X+e)]}.$$

Since the numerator is positive by (C.24), inequality (C.28) holds if and only if (C.29) holds. $\square$

Lemma C.17. *If*

$$\tau<\widetilde{\tau}^{\star}\equiv-\frac{1-\delta-\gamma}{1-\delta}U''\left(U'^{-1}\left(\zeta\frac{\nu'(\ell^{\star})}{F'(\ell^{\star})}\right)\right)\frac{F'(\ell^{\star})\left[F(\ell^{\star})-\Phi\right]}{\nu'(\ell^{\star})},$$

then the system is saddle-path stable with positive eigenvalues.

Proof. Note that condition (C.29) always holds around a full-employment steady state. If the steady state is in the unemployment regime, then condition (C.29) holds if and only if $e_j'(e)<\frac{\delta}{\delta+\gamma}\zeta\frac{1-\delta-\gamma}{1-\delta}\in(0,1)$, where $e_j(e)$ is as defined in Lemma C.13. As before, this condition holds for all e if it holds for $\lim_{e\uparrow e^{\star}}e_j'(e)$, which it can be verified is equivalent to the condition $\tau<\widetilde{\tau}^{\star}$. Note also that $\widetilde{\tau}^{\star}<\widetilde{\tau}$, where $\widetilde{\tau}$ was defined in equation (C.23), so that this condition is strictly stronger than the one required to ensure the existence of a unique steady state. $\square$

Lemmas C.16 and C.17 together establish that, for τ sufficiently small (e.g., $\tau<\widetilde{\tau}^{\star}$), the system converges monotonicaly to the steady state. It remains to show that consumption is decreasing in the stock of durables. Assuming τ is sufficiently small so that the system is saddle-path stable with positive eigenvalues, it is straightforward to obtain the solution $\widehat{X}_t=\lambda_1^t\widehat{X}_0$, $\widehat{e}_t=\psi\widehat{X}_t$, $\widehat{c}_t=(1+\psi)\widehat{X}_t$, where $\psi\equiv-(1-\delta-\lambda_1)/\gamma$. Thus, consumption is decreasing in the stock of durables if and only if $\psi<-1$.

Lemma C.18. *If* (C.29) *holds and the steady state is in the unemployment regime, then* $\psi<-1$.

Proof. We may write

$$\begin{aligned}&1-\delta-\gamma-\lambda_1\\&\qquad=\frac{\sqrt{[a_{ee}+2\gamma-(1-\delta)]^2+4\beta^{-1}[\beta(1-\delta-\gamma)(a_{ee}+\gamma)-1]}-[a_{ee}+2\gamma-(1-\delta)]}{2}.\end{aligned}$$

Now, $a_{ee}+2\gamma-(1-\delta)>a_{ee}-(1-\delta)>0$, so that $1-\delta-\gamma-\lambda_1$ is positive if and only if $\beta(1-\delta-\gamma)(a_{ee}+\gamma)>1$. We have

$$\beta(1-\delta-\gamma)(a_{ee}+\gamma)=\frac{[1+\beta\gamma(1-\delta)]Q'(e)-U''(X+e)}{\left(\frac{1-\delta}{1-\delta-\gamma}\right)Q'(e)-U''(X+e)}.$$

Note by earlier assumptions that this expression is strictly positive, and that

$$\frac{1-\delta}{1-\delta-\gamma}-[1+\beta\gamma(1-\delta)]=\gamma\frac{1-\beta(1-\delta)(1-\delta-\gamma)}{1-\delta-\gamma}>0$$

Thus, if $Q'(e)<0$ (i.e., the steady state is in the unemployment regime) then $\beta(1-\delta-\gamma)(a_{ee}+\gamma)>1$, in which case $1-\delta-\gamma-\lambda_1>0$ and therefore $\psi<-1$. $\square$

Proof of Proposition 12

Let $\widetilde{e}_t(X_0,\epsilon)$ and $\widetilde{X}_t(X_0,\epsilon)$ denote the paths for expenditure and the stock of durables, respectively, when the initial stock of durables is X_0 and there is stimulus of ϵ in period 0. Letting $e(X_t)$ denote the (de-centralied) equilibrium policy function for e, we thus have $\widetilde{X}_0(X_0,\epsilon) = X_0$, $\widetilde{e}_0(X_0,\epsilon) = e(X_0) + \epsilon$, and, for $t \geq 1$, $\widetilde{X}_t(X_0,\epsilon) = (1-\delta)\widetilde{X}_{t-1}(X_0,\epsilon) + \gamma\widetilde{e}_{t-1}(X_0,\epsilon)$ and $\widetilde{e}_t(X_0,\epsilon) = e\left(\widetilde{X}_t(X_0,\epsilon)\right)$. Let $\mathcal{U}(X_0,\epsilon)$ denote the corresponding welfare as a function of X_0 and ϵ. If $\mathcal{U}_\epsilon(X,0) > 0$ (where X is the steady state stock of durables), then it follows that, for X_0 greater than but sufficiently close to X (so that the economy is undergoing a liquidation), we will have $\mathcal{U}_\epsilon(X_0,0) > 0$, i.e., a small one-period stimulus will enhance welfare. We thus turn now to establishing that $\mathcal{U}_\epsilon(X,0) > 0$.

Using the envelope condition, it is straightforward to obtain that

$$\mathcal{U}_\epsilon(X,0) = \frac{\mathcal{L}^\star + \tau\frac{\nu'(\ell^\star)}{F'(\ell^\star)}e}{F'(\ell^\star)\,\ell^\star}\sum_{t=0}^{\infty}\beta^t\frac{\partial\widetilde{e}_t(X,0)}{\partial\epsilon}$$

where e is the steady state level of purchases. Thus, $\mathcal{U}_\epsilon(X,0)$ is positive if the summation in this expression is positive. One may obtain that

$$\frac{\partial\widetilde{e}_t(X,0)}{\partial\epsilon} = \begin{cases} 1 & \text{if } t = 0, \\ \gamma e'(X)\left\{\prod_{i=1}^{t-1}\left[1-\delta+\gamma e'(X)\right]\right\} & \text{if } t \geq 1, \end{cases}$$

Since $e'(X) = -(1-\delta-\lambda_1)/\gamma$, where λ_1 is the smallest eigenvalue of the dynamic system (in modulus), we have that $\partial\widetilde{e}_t(X,0)/\partial\epsilon = -(1-\delta-\lambda_1)\lambda_1^{t-1}$ for $t \geq 1$. Thus,

$$\sum_{t=0}^{\infty}\beta^t\frac{\partial\widetilde{e}_t(X,0)}{\partial\epsilon} = \frac{1-\beta(1-\delta)}{1-\beta\lambda_1}$$

If the system is saddle-path stable, then $|\lambda_1| < 1$, and thus this expression is strictly positive, which completes the proof.

For online appendix

D Robustness of Results to Alternative Assumptions

In our baseline model we have made several restrictive assumptions. For example, we worked with a matching function of the "min" form and we adopted a very particular process for the determination of wages and hours worked. These choices have allowed us to present the main mechanisms of interest in their simplest form. In this section we aim to highlight how our results carry over to more general frameworks. We start by discussing how these results can be extended to more general matching functions, then consider how the analysis changes as we adopt alternative processes for the determination of wages and hours worked. We complete this section by making explicit the informational constraint that limits unemployment insurance in our model. This will allow us to use the explicit information constraint to formulate the social planner's problem and compare it to the market solution.[42]

D.1 Allowing for a More General Matching Technology

One of the important simplifying assumptions of our model is the use of a matching function of the "min" form. This specification has the nice feature of creating two distinct regimes: one where congestion externalities are on the worker's side and one where they are on the firm's side. However, this stark dichotomy, while useful, is not central to the main results of the model. In fact, as we now discuss, the important feature for our purposes is that there be one regime in which expenditures by individual agents play the role of strategic substitutes and another in which they play the role of strategic complements. To see this, it is helpful to re-examine the equilibrium condition for the determination of expenditure for a general matching function. This is given by

$$U'(X+e_j) = vp(e)\left[1+\tau-\frac{M(N(e),L)}{L}\tau\right] \equiv r(e), \tag{D.1}$$

where $M(N,L)$ is a CRS matching function satisfying $M(N,L) \le \min\{N,L\}$. Equation (D.1) equalizes the marginal utility of morning consumption to its cost in terms of afternoon goods. We will refer to $r(e)$ as the cost of funds that the households faces in the morning when taking consumption decisions. In (D.1) we have made explicit the dependence of r on e, where this dependence comes from viewing the other equilibrium conditions as determining N, p w and ℓ as functions of e.[43] Note that the other equilibrium conditions imply that $p(e)$ and $N(e)$ are always weakly increasing in e. Higher morning demand e (weakly) increases the price of morning goods and therefore the cost of funds, while also (weakly) increasing firm entry which *decreases* the cost of funds, since the probability of being unemployed in the morning – and therefore of being in

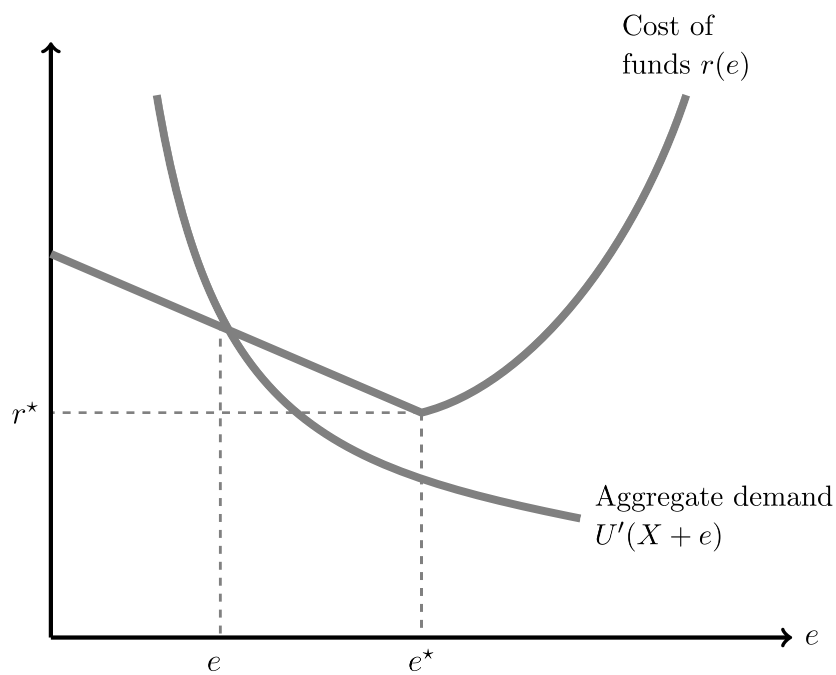
[42] Throughout this section we will be assuming that we are in a region of the parameter space that guarantees uniqueness of equilibrium.

[43] These remaining four equilibrium conditions can be written

$$\begin{aligned}
\nu'(\ell) &= vw, \\
pF'(\ell) &= w, \\
M(N,L)F(\ell) &= L(c-X)+N\Phi, \\
M(N,L)[pF(\ell)-w\ell] &= Np\Phi.
\end{aligned}$$

debt in the afternoon with a premium τ to pay – is lower. From equation (D.1), which relates the determination of expenditure for agent j, e_j, to the average expenditure of all agents, e, we see that average expenditure can play the role of either strategic substitute or strategic complement to the expenditure decision of agent j. In particular, through its effect on the price p, e can play the role of a strategic substitute (if $p'(e) > 0$), while through its effect on firm entry and, in turn, unemployment, it can play the role of strategic complement (if $N'(e) > 0$). The sign of the net effect of e on e_j therefore depends on whether the price effect or the unemployment effect dominates. In the case where $M(N, L) = \Lambda \min\{N, L\}$, the equilibrium features the stark dichotomy whereby $p'(e) = 0$ and $\partial M(N(e), L)/\partial e > 0$ for $e < \Lambda e^{\star}$, while $p'(e) > 0$ and $\partial M(N(e), L)/\partial e = 0$ for $e > \Lambda e^{\star}$. In other words, for low values of e the expenditure of others plays the role of strategic complement to j's decision (since the price effect is not operative), while for high values of e it plays the role of strategic substitute (since the increased-risk-of-unemployment channel is not operative). This reversal in the role of e from acting as a complement to acting as a substitute can be seen in Figure 3 as a change in the curve from being upward- to downward-sloping. These properties can

Figure D.1: Cost of Funds and Aggregate Demand



also be clearly seen in the alternative representation of the equilibrium shown in Figure D.1, where we have plotted the cost-of-funds schedule $r(e)$ and the marginal utility schedule $U'(X + e)$.[44] The important element to note in this figure is that the cost-of-funds schedule $r(e)$ is first decreasing and then increasing in e. Over the range $e < e^{\star}$, the cost of funds to an agent is declining in aggregate e, since N is increasing while p is staying constant. Therefore, in the range $e < e^{\star}$, a rise in e reduces unemployment risk and makes borrowing less costly to agents. This is the complementarity zone. In contrast, over the range $e \geq e^{\star}$, the effect of e on the cost of funds is

[44] Note that the latter is always downward-sloping since U is concave.

positive since the unemployment risk channel is no longer operative, while the price channel is. This is the strategic substitute zone. In the figure, a change in X moves the implicit aggregate demand curve $U'(X+e)$ horizontally, without affecting the cost-of-funds curve. A change in X therefore has the equilibrium property $\partial e/\partial X < -1$ when $e < e^\star$ because the cost-of-funds curve is downward-sloping in this region, while $\partial e/\partial X > -1$ in the region $e > e^\star$ because the cost-of-funds curve is upward-sloping.

From the above discussion it should be clear that our main results depend on the existence of two regions: one (associated with low levels of e) where the cost of funds is decreasing because the negative unemployment-risk channel dominates any potentially positive price channel, and a second (associated with high levels of e) where the price channel dominates the unemployment-risk channel. As we now show, this characterization of the economy holds for a larger class of matching functions, with one caveat: there may also exist an intermediate range of e with mixed properties. The class of matching functions we consider are those that satisfy the following assumption.

Assumption D.1. The matching function $M(N,L) \le \min\{N,L\}$ is continuous, is weakly increasing and concave in both arguments, exhibits constant returns to scale, and satisfies

$$\lim_{N\to 0} \frac{\partial M\left(1, \frac{L}{N}\right)}{\partial N} = 0 \quad \text{and} \quad \lim_{N\to\infty} \frac{\partial M\left(\frac{N}{L}, 1\right)}{\partial L} = 0. \tag{D.2}$$

While many of the properties of Assumption D.1 are fairly standard, there are two worth emphasizing. First, a matching function clearly needs to satisfy $M(N,L) \le \min\{N,L\}$ in order to be admissible. Note that this rules out, for example, matching functions of the Cobb-Douglas type.[45] Second, since $M(1,L/N) = M(N,L)/N$ is the firm matching rate, $\partial M(1,L/N)/\partial N \le 0$ captures the congestion effect that additional firm entry has on that matching rate. The first part of equation (D.2) requires that this firm congestion effect disappears as N becomes small. Similarly, $\partial M(N/L,1)/\partial L \le 0$ denotes the *worker* congestion effect, so that the second part of equation (D.2) requires that worker congestion disappears as N becomes large. Simple examples of matching functions that satisfy Assumption D.1 include the "min" function used above and the ball-urn matching function given by $M(N,L) = N(1-\exp\{-L/N\})$.[46]

In order to characterize equilibrium outcomes for the class of matching functions satisfying Assumption D.1, it is useful to first define the cut-off level of X, denoted $X^{\max}$, that would just cause trade in the economy to fall zero. This value is defined by $X^{\max} = U'^{-1}(\frac{\nu'(\ell^\star)}{F'(\ell^\star)})(1+\tau)$, where $\ell^\star$ is defined implicitly by $[F(\ell^\star) - F'(\ell^\star)\ell^\star]M_1(0,L) = \Phi$. We can now examine how the economy behaves with low versus high values of X for a more general specification of the matching function. This is given by Proposition D.1.

Proposition D.1. *For any matching function satisfying Assumption D.1,*[47] *if $F(\ell) = A\ell^\alpha$ then there exist X^+ and X^{++} satisfying $X^+ \le X^{++} < X^{\max}$, such that equilibrium outcomes are characterized by*

(i) If $X < X^+$, then there is not deficient demand and $\frac{\partial c}{\partial X} \ge 0$.

(ii) If $X \in (X^{++}, X^{\max})$, then there is deficient demand and $\frac{\partial c}{\partial X} < 0$.

[45] More generally, this rules out any matching function of the CES form with elasticity of substitution greater than or equal to one.

[46] See, for example, Hall [1979] for the use of the ball-urn matching function, and Pissarides and Petrongolo [2001] for a more general discussion of matching functions.

[47] We are again assuming that τ is suffciently small to guarantee a unique equilibrium

See Section D.6 for proofs of all Appendix D propositions.

Proposition D.1 generalizes results of Section 3 by indicating that, for our class of matching functions, the behavior of our model economy will again differ depending on whether the economy inherits a small or a large amount of goods from the past.[48] In particular, the proposition states that, for large values of X, the economy will again exhibit deficient demand – in the sense that agents' purchases are strategic complements, and a coordinated increase in c would increase welfare – and that in such a region the economy acts rather perversely with $\frac{\partial c}{\partial X} < 0$. In contrast, the economy would not exhibit deficient demand or act perversely if X were small.

The main difference between Proposition D.1 and the results of Section 3 is that, with the "min" matching function, two regions spanned all possible values of $X < X^{\max}$. However, this is not the case with Proposition D.1. Implicit in Proposition D.1 is the possible existence of a third region between X^{+} and X^{++} where properties may be mixed. We have not been able to exclude the possibility of such a third region for this general class of matching functions. However, for most parametric examples, we have been able to find simple sufficient conditions that guarantee the simple dichotomy, so that this third potential region is in fact empty (or, in other words, that $X^{+} = X^{++}$). For example, if we assume that disutility of labor ν is of the constant-elasticity form $\nu(\ell) = \ell^{1+\omega}$ and that the matching function is of the ball-urn type, then the third region is empty. A second interesting example is the case where the matching function is CES with elasticity of substitution strictly less than one, i.e., where $M(N,L) = (N^{-\gamma} + L^{-\gamma})^{-\frac{1}{\gamma}}$ with $\gamma > 0$.[49] If $\gamma > 1$, it can be verified that this matching function satisfies Assumption D.1, and further that our simple dichotomy (where $X^{+} = X^{++}$) also holds.[50] Although the ball-urn and CES matching functions are special parametric cases, these examples nicely illustrate that many of the results obtained using the simpler "min" matching function are not knife-edge, as they carry over to these alternate cases.[51]

D.2 Generalizing the $V(\cdot)$ Function

In deriving our previous results we assumed that the function $V(a)$, which represents the continuation value of entering the afternoon market with assets a, took the form of a piecewise-linear function with a kink at zero. This allowed $V(a)$ to possess the property that $V'(a_1) > V'(a_2)$ for $a_1 < 0 < a_2$ – which is of fundamental importance in generating the precautionary behavior that is central to the main mechanism – while maintaining analytical tractability. In this section we discuss how our result regarding the effect of X on c generalizes when we assume simply that $V(a)$ is increasing and concave in a, without restricting it to be piecewise linear. It can be again shown in this case that there exist a $X^{\star}$ and and $X^{\star\star}$ such that if $X^{\star} < X < X^{\star\star}$ then the labor market

[48] The restriction in Proposition D.1 that $F(\ell) = A\ell^{\alpha}$ is not necessary for the result, but it greatly simplifies presentation.

[49] This matching function was used in den Haan, Ramey, and Watson [2000].

[50] This result is based on maintaining the functional form assumption $\nu(\ell) = \ell^{1+\omega}$.

[51] The ball-urn function and CES function with $\gamma > 1$ belong to a more general class of matching functions for which we may obtain a simple sufficient condition to ensure the simple dichotomy when $\nu(\ell) = \ell^{1+\omega}$. In particular, let $\eta(N,L) \equiv N/M(N,L)$ be the inverse of the firm matching rate, and suppose η is convex in N (as is the case for the above functions). Then it may be verified that a sufficient (but not necessary) condition for Proposition D.1 to hold with $X^{+} = X^{++}$ is that $\alpha \geq (1+\omega)/2$.

is slack. The equilibrium condition determining $e = c - X$ in this range can be written as

$$U'\left(X+e\right) = \frac{\nu'(\ell^\star)}{F'\left(\ell^\star\right)}\left\{\mu(e) + \left[1-\mu(e)\right]\frac{V'\left(-\frac{w(e)}{F'(\ell^\star)}e\right)}{V'\left(w(e)\ell^\star - \frac{w(e)}{F'(\ell^\star)}e\right)}\right\} \tag{D.3}$$

where $\ell^\star$ continues to be defined by the condition $F(\ell^\star) - F'\left(\ell^\star\right)\ell^\star = \Phi$, $\mu(e) = e/\left[F'\left(\ell^\star\right)\ell^\star\right]$ is the employment rate, and the function $w(e)$ is implicitly defined by the condition

$$\nu'(\ell^\star) = w(e)V'\left(w(e)\ell^\star - \frac{w(e)}{F'\left(\ell^\star\right)}e\right). \tag{D.4}$$

Our previous formulation had a very similar form to equation (D.3) except that the term

$$\frac{V'\left(-\frac{w(e)}{F'(\ell^\star)}e\right)}{V'\left(w(e)\ell^\star - \frac{w(e)}{F'(\ell^\star)}e\right)} \equiv 1+\tau(e)$$

was constant and equal to $1+\tau$. Hence, the implication of adopting a piecewise-linear function was to allow us to treat $\tau(e)$ as a constant. In the more general case, we need to take into account how $\tau(e)$ changes with e.

As we noted in the previous section, the right-hand side of equation (D.3) can be interpreted as the marginal cost of funds to the household and can again be represented by a function $r(e)$. The necessary condition for an increase in X to lead to a fall in c is that $r'(e) < 0$. We can see that e will now affect $r(e)$ through two channels. As in the piecewise-linear case, there is the effect operating through $\mu(e)$, which is always negative. In addition, there is the effect operating through the function $\tau(e)$. The combined effect will be negative as long as $\tau'(e)$ is not too positive. It is important to recognize that the effect of e on $\tau(e)$ is quite involved, since when $V(a)$ is not piecewise linear, changes in e will affect wages, which in turn affect $\tau(e)$. Sufficient conditions for $\tau'(e)$ to be negative are given in the following proposition.

Proposition D.2. *The function* $\tau(e) = \frac{V'\left(-\frac{w(e)}{F'(\ell^\star)}e\right)}{V'\left(w(e)\ell_j - \frac{w(e)}{F'(\ell^\star)}e\right)} - 1$ *will be decreasing in* e *if (1) the elasticity of* $V'(a)$ *is greater than -1 when* $a > 0$*, and (2)* $V'''(a)$ *is not too positive.*[52]

Proposition D.2 implies that our result regarding the negative effect of X on c is robust to allowing for a more general $V(\cdot)$ function. While the result does not generalize to any function $V(a)$ that is increasing and concave in a, Proposition D.2 indicates that the curvature of $V(a)$ must change sufficiently with changes in a for the result to be potentially reversed. Let us also emphasize that Proposition D.2 provides sufficient conditions for a change in e to decrease the marginal cost of fund $(r(e))$; these conditions are not in general necessary.[53]

[52] These conditions can be translated into conditions on the functions $\tilde{U}$ and $\tilde{\nu}$. For example, the required properties on $V(a)$ will be met if the elasticity of $\tilde{\nu}$ is smaller than one and if neither $\tilde{\nu}'''$ nor $\tilde{U}'''$ are too positive.

[53] If the function $V(a)$ satisfy the conditions of Proposition D.2, it implies that $\partial c/\partial X$ cannot be between 0 and 1. However, it does not rule out the possibility of the perverse case where $\partial c/\partial X$ is greater than 1. Since the case where $\partial c/\partial X$ generally implies multiple equilibria, and we are not interested in cases with multiple equilibria, we do not explore this possibility further here.

D.3 Allowing X to Enter $V(\cdot)$

Up to now, we have assumed that a household's stock of durables X cannot be used to help repay one's debt in the afternoon period. In this section, we briefly discuss how allowing X to enter $V()$ could change the results. For example, suppose the continuation value of entering the afternoon period with assets a is given by a function of the form $V(a, X)$ with $V_a > 0$, $V_{aa} < 0$, $V_X > 0$ and, most importantly, $V_{aX} < 0$; that is, having more X would reduce the costs associated with taking on more debt. There are two cases to consider here depending on the partial-equilibrium effect of X on demand for new purchases. First, consider the case where V_{aX} is such that the partial-equilibrium response of a household (holding μ, p, and w fixed) to an increase in X continues to be a decrease in purchases. In this case, our analysis is not much changed, since the first-round effect is a decrease in e, and then as long as the conditions expressed just above for the more general $V(\cdot)$ function hold, then the multiplier process with take effect and the net general-equilibrium effect can still lead to a decrease in consumption. However, if V_{aX} is sufficiently negative for the partial equilibrium effect of X on e to be positive,[54] then the multiplier effect will be set in motion, but in this case the net general-equilibrium effect will be an increase in c. Hence, if V_{aX} is allowed to be sufficiently negative, the main results can be reversed.

D.4 Changing the Search and Bargaining Protocol

In this section, we return to the "min" specification of the matching function and examine how results are modified when we maintain random matching but change the bargaining protocol to Nash bargaining. We then discuss the implications of adopting directed search. In the remaining sections of the paper, we will simplify notation by setting $\Lambda = 1$ and thereby have $M(N, L) = \min\{N, L\}$. This implies that a tight labor market will be characterized by full employment and a slack labor market with be characterized by unemployment. As should be clear from our previous analysis, the important distinction between the two regimes is not that the presence or absence of unemployment, but is instead the fact that in a slack labor market ($N < L$) workers experience cogestion effects while in a tight labor market ($N > L$) it is the firms that experience congestion effects.

D.4.1 Nash Bargaining

In our baseline model we assumed that, upon a successful search, wages and employment were determined by a Walrasian protocol in the spirit of that used by Lucas and Prescott [1974]. This protocol gave rise to two results. First, it implied that hours worked satisfies a pair-wise efficient condition given by

$$pF'(\ell) = \frac{\nu'(\ell)}{v},$$

and second it implied that the wage is equal to the marginal dis-utility of work;[55] that is,

$$w = \frac{\nu'(\ell)}{v}.$$

Suppose we replace this "competitive" determination of w and ℓ (within a match) by Nash bargaining.

[54] This arises when the effect of the decreased marginal cost of debt from having more X dominates its effect on the marginal utility, U'.

[55] We could alternatively say here that wages are given by the marginal product condition $w = pF'(\ell)$.

The gain from a match for a firm is $pF(\ell) - w\ell$ while outside option is zero. The gain for the household is $-\nu(\ell)+V(w\ell-p(c-X)$ while the outside option is $V(-p(c-X))$. Using the piecewise linear specification for V, the Nash-Bargaining criterion $\mathcal{W}$ is:

$$\mathcal{W} = \Big(pF(\ell) - w\ell\Big)^{1-s}\Big(- \nu(\ell) + vw\ell + v\tau p(c-X)\Big)^{s}.$$

Maximizing $\mathcal{W}$ w.r.t. ℓ and w gives the following F.O.C.:

$$\begin{aligned}
\frac{(1-s)\mathcal{W}}{pF(\ell)-w\ell}\Big(pF'(\ell)-w\Big) &= \frac{s\mathcal{W}}{-\nu(\ell)+vw\ell+v\tau p(c-X)}\Big(vw-\nu'(\ell)\Big),\\
\frac{(1-s)\mathcal{W}}{pF(\ell)-w\ell} &= \frac{s\mathcal{W}}{-\nu(\ell)+vw\ell+v\tau p(c-X)}v.
\end{aligned}$$

Rearranging gives the two equations

$$\begin{aligned}
vpF'(\ell) &= \nu'(\ell),\\
vw\ell &= svpF(\ell)+(1-s)\nu(\ell)-(1-s)v\tau p(c-X).
\end{aligned}$$

Thus, an equilibrium is given by a solution to the five equations:

$$\begin{aligned}
u'(c) &= \frac{\nu'(\ell)}{F'(\ell)}\left(1+\tau-\frac{M(N,L)}{L}\tau\right), && \text{(D.5)}\\
w\ell &= spF(\ell)+\frac{1-s}{v}\nu(\ell)-(1-s)\tau p(c-X), && \text{(D.6)}\\
vpF'(\ell) &= \nu'(\ell), && \text{(D.7)}\\
M(N,L)F(\ell) &= L(c-X)+N\Phi, && \text{(D.8)}\\
M(N,L)\big(pF(\ell)-w\ell\big) &= pN\Phi. && \text{(D.9)}
\end{aligned}$$

As shown above, under Nash bargaining the within-pair efficiency condition $pF'(\ell) = \nu'(\ell)/v$ remains. However, the determination of wages changes. In particular, under Nash bargaining the wage is given by (D.6), that we can rewrite as

$$w = \frac{\nu(\ell)-\tau p(c-X)+s\,[pF(\ell)-\nu(\ell)+\tau p(c-X)]}{l}, \qquad 0\leq s<0, \tag{D.10}$$

where s reflects the share of the match surplus that is attributed to the worker (an additional parameter). The total wage payment now reflects the reservation utility of the worker, which is given by $\nu(\ell)-\tau p(c-X)$, plus a share of the match surplus, which is given by $pF(\ell)-\nu(\ell)+\tau p(c-X)$. An important implication of this is that the wage is now decreasing in c. If a worker enters a match with a greater c, he is in a less desirable bargaining position since, if negotiations were to break down, the worker would be left with costly debt. This causes the worker to settle for a lower wage when he has committed to a high level of consumption. Our baseline formulation ruled out this mechanism in the determination of wages. As we shall now show, this mechanism will tend to amplify a number of our of previous results, since it will imply that p will be a decreasing function of c, which will in turn cause the cost-of-funds schedule (plotted in Figure D.1) to have an even more negative slope in the unemployment regime.

With Nash bargaining, the equilibrium determination of c, N and ℓ reduces to the solution to equations (11), (9), and the zero-profit condition given (D.9) that we can rewrite

$$(1-s)\frac{\min\{N,L\}}{N}\left[F(\ell)-\frac{\nu(\ell)F'(\ell)}{\nu'(\ell)}+\tau(c-X)\right]=\Phi, \tag{D.11}$$

where we have used the new wage equation (D.10).[56] Proposition D.3 states that under Nash Bargaining we get a similar characterization of equilibrium outcomes as that obtained in our baseline model.[57]

Proposition D.3. *When wages and hours worked are determined by Nash bargaining, there again exists an $X^\star$ and an $X^{\star\star} > X^\star$ such that*

(i) If $X < X^\star$, then the labor market will be tight ($N > L$), there is not deficient demand, and $\frac{\partial c}{\partial X} \geq 0$.

(ii) If $X \in (X^\star, X^{\star\star})$, then the labor market will be slack ($N \leq L$), there is deficient demand, and $\frac{\partial c}{\partial X} < 0$.

(iii) If $X \geq X^{\star\star}$, then $c = X$.

Although Proposition D.3 indicates that many equilibrium properties remain unchanged as we switch from our baseline bargaining protocol to Nash bargaining, this does not imply that the equilibrium values of c and ℓ do not change. For example, in the slack regime of our baseline model, the equilibrium had a recursive structure. The zero-profit condition determined ℓ, and hence the price p, and then the optimal consumption decision was determined by the condition $U'(c) = \frac{\nu'(\ell)}{F'(\ell)}[1 + \tau - \tau \frac{c-X}{F(\ell)-\Phi}]$. Accordingly, in our baseline setup, the price of goods and hours worked did not vary as we changed X when the labor market was slack. In the case of Nash bargaining this recursive property is lost. The optimal consumption decision and the zero-profit condition must be solved jointly for ℓ and c, which in turn implies that the price also changes as X changes. These effects are characterized in Proposition D.4.

Proposition D.4. *If wages and hours worked are determined by Nash bargaining, then*

(i) When the labor market is tight (i.e., for $X < X^\star$), $\frac{\partial p}{\partial X} < 0$ and $\frac{\partial \ell}{\partial X} < 0$.

(ii) When the labor market is slack (i.e., for $X \in (X^\star, X^{\star\star})$), $\frac{\partial p}{\partial X} > 0$ and $\frac{\partial \ell}{\partial X} > 0$.

Propositions D.3 and D.4 together indicate that, when the labor market is slack, an increase in X will lead to lower consumption and higher prices. The reason for this is that, as one cuts back on consumption due to higher X, the worker's bargaining position improves, which puts downward pressure on firm profits. In order to maintain zero expected profits, matched firms must then hire more hours of labor, which in turn results in higher prices. This contrasts with our baseline model where an increase in X in a slack market led to lower consumption at an unchanged price. The extra mechanism induced by Nash bargaining can therefore be seen as increasing the strength of the complementarity between the consumption decisions of the households in comparison to our baseline setup. Note that, in the slack regime, the consumption decision for household j satisfies the relationship $U'(c_j) = p[1 + \tau - \tau \frac{c-X}{F(\ell)-\Phi}]$, where c is the average consumption level of other agents. Recall that this condition holds regardless of whether we have Nash bargaining or the Walrasian protocol of our baseline model. In the slack regime of our baseline model, an exogenous

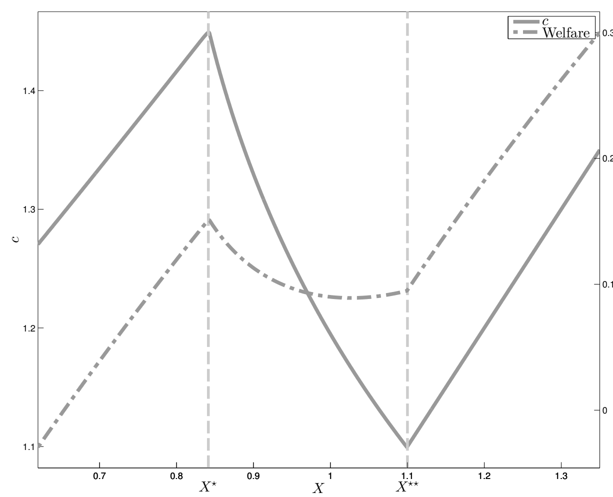
[56] We will assume that the function $sF(\ell) + (1-s)\frac{\nu(\ell)F'(\ell)}{\nu'(\ell)}$ is always increasing in ℓ, as can be easily verified to be the case under standard functional forms.

[57] We have also derived sufficient conditions for an increase in X to lead to a decrease in welfare in the presence of Nash Bargaining. However, the expressions are rather complicated and not very informative, so we have omitted them here. Using numerical simulations, we have found it rather easy to find regions in the unemployment regime where an increase in X leads to a decrease in welfare.

increase in c would not change p or ℓ, so from the household's optimal consumption decision we can see easily that the consumption of other agents acts as a complement to one's own consumption. In the case of Nash bargaining, this complementarity becomes even stronger, as in addition to the direct effects of others' consumption on the probability of employment, when the consumption of other agents increases that tends to decrease the price and lower hours worked, and hence further increases one's desire to consume. Since this additional mechanism is rather subtle, we opted to focus on the simpler and more direct mechanism in the baseline model, leaving us to clarify this additional channel here.

To conclude this section, we present in Figure D.2 the behavior of consumption and welfare as a function of X for an example with Nash bargaining. The parameters used in this example are similar to those used in in Figures 2 and **??** for our baseline model. As can be seen, consumption and welfare are both increasing until X reaches $X^\star$, after which they begin to decline as the economy enters a region where the labor market is slack.

Figure D.2: Consumption and Welfare as Functions of X (Nash Bargaining)



Note: Solid line is consumption (left axis). Dash-dot line is welfare (right axis). Example is constructed assuming the functional forms $U(c) = \log(c)$, $\nu(\ell) = \frac{\nu \ell^{1+\omega}}{1+\omega}$ *and* $F(\ell) = A\ell^\alpha$, *with parameters* $\omega = 1$, $\nu = 0.5$, $\alpha = 0.67$, $A = 1$, $\Phi = 0.35$, $\Lambda = 1$, $\tau = 0.1$ *and* $s = 0.5$.

D.4.2 Directed Search

Up to now, we have focused on environments where search is done in a random fashion. In this section, we explore how our results would change if we allowed for directed search.[58] In particular, we examine whether the emergence of deficient demand when X is high and the property that $\frac{\partial c}{\partial X} < 0$ in such cases are driven by the assumption of random search, or whether they are robust to allowing for directed search.

In the case of directed search, we can view the household's problem as simultaneously choosing both a consumption level and a particular labor market in which to search for a job. There is a

[58] Directed search is also known as competitive search; see Moen [1997].

potential continuum of job markets, each specified as a triple composed of a wage, a number of hours worked, and a tightness level, where tightness level $\theta \equiv N/L$ translates into a job-finding rate for workers of $M(\theta, 1)$. The potential job markets available to households in equilibrium are all of the triples of characteristics w, ℓ and θ that leave firms with zero profits. The equilibrium outcome for this economy therefore maximizes household utility subject to the firm's zero-profit condition (taking the price p as given); that is, it solves

$$\max_{c,w,\ell,\theta} U(c) + M(\theta,1)\left[-\nu(\ell) + V(w\ell - p(c-X))\right] + \left[1 - M(\theta,1)\right] V(-p(c-X))$$

subject to

$$M\left(1, \theta^{-1}\right)\left[F(\ell) - \frac{w}{p}\ell\right] = \Phi.$$

Maintaining the usual assumption on the form of $V(\cdot)$, solving this maximization problem yields the now-familiar conditions $U'(c) = pv(1+\tau-\frac{M(N,L)}{L}\tau)$, $pF'(\ell) = \frac{\nu'(\ell)}{v}$, and $\frac{M(N,L)}{N}[F(\ell)-\frac{w}{p}\ell] = \Phi$. The only new condition is the wage-determination equation, which is now given by

$$w = \frac{\nu(\ell) - \tau p(c-X) + \xi\left[pF(\ell) - \nu(\ell) + \tau p(c-X)\right]}{\ell}, \tag{D.12}$$

where ξ is the elasticity of the matching function with respect to L, as given by $\xi = \frac{M_2(N,L)L}{M(N,L)}$. The set of equilibrium conditions is then completed with the usual market-clearing condition for the goods market, $M(N,L)F(\ell) = L(c-X) + N\Phi$.

If we combine the wage determination equation (D.12) with the zero-profit condition, we get

$$(1-\xi)\frac{M(N,L)}{N}\left[F(\ell) - \frac{\nu(\ell)F'(\ell)}{\nu'(\ell)} + \tau(c-X)\right] = \Phi. \tag{D.13}$$

This zero-profit condition is identical to the one obtained under Nash bargaining (equation (D.11)), except the fixed bargaining power s from the Nash bargaining setup has been replaced by the (endogenous) elasticity of the matching function ξ, which is a standard result in the case of directed search. If we assume that the matching function is once again of the "min" form, we can derive similar results to those obtained for the random-search Nash bargaining case, as stated in Proposition D.5.

Proposition D.5. *The properties stated in Proposition D.3 also hold under directed search.*

Proposition D.5 indicates that, under directed search, the economy will again have a tendency to exhibit deficient demand and behave perversely when X is high, while this will not be the case if X is low.

D.5 Justifying the Absence of Unemployment Insurance and Formulating the Social Planner's Problem

In our analysis thus far, we have assumed that agents do not have access to unemployment insurance. It may be thought that allowing for the private provision of unemployment insurance would necessarily eliminate the mechanisms we have highlighted. For this reason, in this subsection we want to indicate how our analysis can be extended to allow for the private provision of unemployment insurance, but where the provision of this insurance will be constrained by an adverse selection problem. Once we have presented this adverse selection problem, we can then examine the more interesting question of how a social planner would allocate resources in an economy subject to two frictions: a search friction that creates unemployment, and an information friction that limits unemployment insurance. The solution to this social planner's problem will then be used to clarify the fundamental nature of the inefficiency that arises in the de-centralized case.

D.5.1 Adverse Selection as a Constraint on Unemployment Insurance

To explore the role of adverse selection, we return to our baseline model of Section 3, but now suppose that only a fraction ρ of households behave as the households we modeled in that Section. We will refer to these households as participant households. Suppose the remaining $(1-\rho)$ fraction of households, which we call the non-participant households, do not value consumption in the morning and are unwilling to work at the market wage, but value consumption in the afternoon in exactly the same way as the participant households. Now suppose that some private insurer wanted to offer unemployment insurance before the matching process, but could not differentiate between the two types of households. In this case, the insurer will not be able to offer insurance contracts that are only attractive to participant households, because any contract with a positive net payment to unemployed individuals will be desirable to non-participants. Therefore, as indicated in Proposition D.6, as long as $1-\rho$ is sufficiently high this type of adverse selection problem implies that the only equilibrium outcome is one where no insurance is offered. Accordingly, in this setup, the mechanisms we have emphasized regarding the economy's behavior when unemployment insurance is assumed not to exist will apply equally in an environment where the private provision of unemployment insurance is allowed but is constrained by an adverse selection problem.

Proposition D.6. *In the presence of both participant households and non-participant households, if $1-\rho \geq \frac{1}{1+\tau}$, i.e., if the fraction of non-participant households is sufficiently high, then no unemployment-insurance contracts are traded in equilibrium.*

D.5.2 The Constrained Social Planner's Problem

In this section we want to show how a social planner would allocate resources in our environment if it simultaneously faces the search friction and the adverse selection problem presented previously. We will take the goal of the social planner to be the maximization of utility of participant households subject to the constraint that it offers contracts to participant households that are not attractive to non-participant households. We implicitly assume here that the social planner cannot distinguish between the two types of agents, and that non-participant households are sufficiently important in number that it is not socially optimal to include them in any transfer scheme where they would be pure beneficiaries. We view the planner as offering a contract specifying four elements: an amount of the morning good e that is not contingent on morning labor market status; a number of hours worked conditional on finding employment, ℓ; an amount of the afternoon good to receive if one is employed in the morning, d^e; and an amount of the afternoon good that must be produced if one is unemployed in the morning, d^u. In the absence of the adverse selection problem, the social planner problem would choose these elements, plus the number of firms to enter the market, by solving

$$\max_{c,l,N,d^e,d^u} U(X+e) + \frac{M(N,L)}{L}\left[vd^e - \nu(\ell)\right] - \left[1 - \frac{M(N,L)}{L}\right] v(1+\tau)d^u,$$

subject to the resource constraint in the morning $M(N,L)F(\ell) = Le + NK$ and the resource constraint in the afternoon $\frac{M(N,L)}{L}d^e = [1 - \frac{M(N,L)}{L}]d^u$. It is easy to verify that the solution to this problem has $d^e = d^u$; that is, household would not bear any risk associated with unemployment.

The problem with this solution is that a non-participant household would in general want to participate in this scheme by accepting the offered morning goods, and then trading these goods to participant households in return for promises of afternoon goods. In fact, a non-participant households will want to take part in any scheme offered by the planner if it can manage to end up with a positive amount of afternoon goods (after deducting d^u, the number of goods transferred to the planner) by making such trades. Recognizing this, the social planner will be constrained to choose values of e and d^u that will be unattractive to non-participant households.[59] For this to be the case, it must be that a non-participant household cannot, by trading away all e of their morning good, obtain enough afternoon goods so that they are left with a positive amount after paying the required d^u to the planner. It can be verified that the relevant incentive compatibility constraint is given by[60]

$$\frac{U'(x+e)}{1+\tau-\frac{M(N,L)}{L}\tau}e \leq d^u. \tag{D.14}$$

On the left-hand side of this constraint is the number of afternoon goods that a non-participant could obtain by selling all of their morning goods. Here, $\frac{U'(x+e)}{1+\tau-\frac{M(N,L)}{L}\tau}$ is the expected marginal value to participant households of morning goods, expressed in terms of afternoon goods. Accordingly, a non-participant household would accept any contract offered by the planner that satisfies $d^u < \frac{eU'(x+e)}{1+\tau-\frac{M(N,L)}{L}\tau}$, since that household could guarantee itself a positive amount of afternoon goods after repaying the amount d^u agreed upon in the contract.

Once the constraint (D.14) is taken into account, the solution to the social planner's problem is given by the two resource constraints, the incentive compatibility condition at equality (since it always binds), plus the two new conditions

$$U'(X+e) = \frac{\nu'(\ell)}{F'(\ell)} \cdot \frac{1+\tau-\frac{M(N,L)}{L}\tau}{1+\left[1-\frac{M(N,L)}{L}\right]\frac{-U''(X+e)}{U'(X+e)}e\tau}, \tag{D.15}$$

$$\begin{aligned}(1-\xi)\frac{M(N,L)}{N}\left\{F(\ell)-\frac{\nu(\ell)F'(\ell)}{\nu'(\ell)}+\frac{\tau e}{\left\{1+\left[1-\frac{M(N,L)}{L}\right]\frac{-U''(X+e)}{U'(X+e)}e\tau\right\}}\right\} = \\ \Phi+(1-\xi)\frac{M(N,L)}{N}\frac{\tau^2 e\left[1-\frac{M(N,L)}{L}\right]}{1+\tau\left[1-\frac{M(N,L)}{L}\right]}\cdot\frac{1}{1+\left[1-\frac{M(N,L)}{L}\right]\frac{-U''(X+e)}{U'(X+e)}e\tau},\end{aligned} \tag{D.16}$$

where, as before, ξ is the elasticity of the matching function with respect to L. Equation (D.15) can be interpreted as the socially optimal condition for the determination of expenditures, while equation (D.16) can be interpreted as the socially optimal zero-profit condition. It is interesting to compare these two conditions to those we derived for a decentralized economy. One can see that, for all cases we considered, the decentralized conditions determining expenditures and entry decisions differ from the solution to the planner's problem as long as $\tau > 0$. The environment that most

[59] Since they never search for jobs, non-participant households do not care about the levels of ℓ or d^e specified in the contract.

[60] We assume that any trade between households occurs before the resolution of unemployment uncertainty in the morning. Further, we assume that the realized employment state of one household cannot be verified by another, so that any promise of afternoon goods made by a household in the morning cannot be contingent on the realized state of their employment.

closely resembles the social planner's solution is the decentralized economy with directed search. Accordingly, we will focus our comparison here between directed search and the social optimum, noting that much of what we say also extends to our baseline analysis and to the case with Nash bargaining.

In comparison to the solution under directed search, the social planner would want to encourage more expenditure by households in the morning while simultaneously limiting firm entry. This can be seen by the fact that the term in the denominator on the right-hand side of equation (D.15) is greater that one, and by the fact that the last term in equation (D.16) is positive. The social planner could implement his preferred outcome in a directed-search environment by the use of a subsidy on the purchase of morning goods,[61] and by a tax on entry.[62] If we go a step further and focus on the case where the matching function is of the "min" form, then we can see that the social planner's solution and the decentralized solution with directed search actually become identical in the case of a tight labor market.[63] However, they continue to differ when the labor market is slack, which will happen for high values of X. This comparison highlights that economic activity will be inefficiently low in the directed search environment when X is high, but not when it is low. In other words, if the value of trading in the market is high, as is the case when X is low, the decentralized economy can overcome the frictions and trade at the socially efficient level. However, when the gains from trade between individuals are rather low, as is the case when X is high, the decentralized outcome under directed search will be inefficient relative to the constrained social optimum.

The reason that the social planner's problem and the directed-search equilibrium do not coincide is due to a pecuniary externality. When agents are deciding how much to consume in the decentralized environment, they do not take into account the effect of their consumption on employment and prices. They do not recognize that, by consuming more, they would reduce wage demands, thereby favoring lower prices and more production. Since the extra production is socially desirable, as the economy tends to be in a situation of deficient demand with high X, it would be in the interest of agents to coordinate action by consuming more and favoring more entry.

D.6 Proofs of Appendix D Propositions

Proof of Proposition D.1

Let $\theta \equiv N/L$ be labor-market tightness and $\mu(\theta) \equiv M(\theta, 1)$ be the resulting employment rate. Then we can obtain

$$\ell = \ell(\theta) \equiv \left[\frac{\Phi\theta}{(1-\alpha) A\mu(\theta)}\right]^{\frac{1}{\alpha}},$$

$$e = \frac{\alpha\Phi}{(1-\alpha)}\theta,$$

$$p = p(\theta) \equiv \frac{\nu'(\ell(\theta))}{\alpha A [\ell(\theta)]^{\alpha-1}},$$

$$w = w(\theta) \equiv \nu'(\ell(\theta)).$$

[61] In order to implement the social optimum, the subsidy would need to depend on the value of X.

[62] The budget can be balanced by imposing a lump-sum tax on the employed as needed.

[63] With the matching function of the "min" form, the free entry condition implies that $N = L$.

Equilibrium is then given by a solution to the equation

$$U'\left(X+\frac{\alpha\Phi}{(1-\alpha)}\theta\right)=Q(\theta)\equiv p(\theta)\left[1+\tau-\tau\mu(\theta)\right]$$

for θ.

Lemma D.1. *For X sufficiently small, $dc/dX \geq 0$. For $X < X^{\max}$ sufficiently large, $dc/dX < 0$.*

Proof. Assuming τ is small enough so that the equilibrium is unique, it can be easily verified that $dc/dX < 0$ if $Q'(\theta) < 0$ and $dc/dX > 0$ if $Q'(\theta) > 0$. Further, since $d\theta/dX < 0$, showing that Lemma D.1 holds is equivalent to showing that $Q'(\theta) < 0$ for θ sufficiently small (i.e., X sufficiently large) and $Q'(\theta) > 0$ for θ sufficiently large (i.e., X sufficiently small).

We have

$$Q'(\theta)=p(\theta)\left\{\frac{p'(\theta)}{p(\theta)}\left[1+\tau-\tau\mu(\theta)\right]-\tau\mu'(\theta)\right\}.$$

We may obtain that

$$\frac{p'(\theta)}{p(\theta)}=\left[\omega(\theta)+1-\alpha\right]\frac{\ell'(\theta)}{\ell(\theta)},$$

where

$$\omega(\theta)\equiv\frac{\nu''(\ell(\theta))\,\ell(\theta)}{\nu'(\ell(\theta))}\geq 0,$$

and

$$\frac{\ell'(\theta)}{\ell(\theta)}=\frac{1-\mathcal{E}_{\mu\theta}(\theta)}{\alpha\theta},$$

where the notation $\mathcal{E}_{fx}(x)$ denotes the elasticity $f'(x)\,x/f(x)$. Thus

$$Q'(\theta)=\frac{p(\theta)}{\theta}\Gamma(\theta), \tag{D.17}$$

where

$$\Gamma(\theta)\equiv\frac{\omega(\theta)+1-\alpha}{\alpha}\left[1-\mathcal{E}_{\mu\theta}(\theta)\right]\left[1+\tau-\tau\mu(\theta)\right]-\tau\mu'(\theta)\,\theta.$$

Consider first the case where $\theta\to\infty$. From (D.17), we see that the sign of $Q'(\theta)$ is equal to the sign of $\Gamma(\theta)$. Since, by Assumption D.1,

$$\lim_{N\to\infty}\frac{\partial M\left(\frac{N}{L},1\right)}{\partial L}=0,$$

we may obtain that

$$\lim_{\theta\to\infty}\mu'(\theta)\,\theta=0,$$

which in turn implies that $\lim_{\theta\to\infty}\mathcal{E}_{\mu\theta}(\theta)=0$. Thus, letting $\bar{\mu}\equiv\lim_{\theta\to\infty}\mu(\theta)$, we have

$$\lim_{\theta\to\infty}\Gamma(\theta)=(1+\tau-\tau\bar{\mu})\lim_{\theta\to\infty}\frac{\omega(\theta)+1-\alpha}{\alpha}>0,$$

so that for θ sufficiently large (X sufficiently small) we have $Q'(\theta)>0$, and thus $dc/dX>0$.

Next, consider the case where $\theta\to 0$. From (D.17), we see that the sign of $Q'(\theta)$ is equal to the sign of $\Gamma(\theta)/\theta$, so that $\mathrm{sgn}(Q'(0))=\mathrm{sgn}(\lim_{\theta\to 0}\Gamma(\theta)/\theta)$. We have

$$\lim_{\theta\to 0}\frac{\Gamma(\theta)}{\theta}=\frac{\omega(0)+1-\alpha}{\alpha}(1+\tau)\lim_{\theta\to 0}\left[\frac{1-\mathcal{E}_{\mu\theta}(\theta)}{\theta}\right]-\tau\mu'(0).$$

Note that the basic restrictions on the matching function imply that $\mu(0) = 0$ and $0 < \mu'(0) \leq 1$.[64] Thus,

$$\lim_{\theta\to 0} \mathcal{E}_{\mu\theta}(\theta) = \lim_{\theta\to 0} \frac{\mu'(\theta)}{\mu(\theta)/\theta}.$$

Since the limit of the numerator is non-zero and bounded, and the limit of the denominator is equal to $\mu'(0)$ by definition (since $\mu(0) = 0$), which is also non-zero and bounded, we have $\lim_{\theta\to 0} \mathcal{E}_{\mu\theta}(\theta) = 1$. By Assumption D.1,

$$\lim_{N\to 0} \frac{\partial M\left(1, \frac{L}{N}\right)}{\partial N} = 0,$$

from which we may obtain

$$0 = -\frac{1}{L} \lim_{\theta\to 0} \frac{\mu(\theta)}{\theta} \frac{1 - \mathcal{E}_{\mu\theta}(\theta)}{\theta} = -\frac{\mu'(0)}{L} \lim_{\theta\to 0} \frac{1 - \mathcal{E}_{\mu\theta}(\theta)}{\theta}.$$

Since $\mu'(0) > 0$, this can only be true if $\lim_{\theta\to 0} [1 - \mathcal{E}_{\mu\theta}(\theta)]/\theta = 0$. Thus,

$$\lim_{\theta\to 0} \frac{\Gamma(\theta)}{\theta} = -\tau\mu'(0) < 0,$$

so that for θ sufficiently small (X sufficiently large) $Q'(\theta) < 0$, and thus $dc/dX < 0$, which completes the proof.

□

Lemma D.2. *For X sufficiently small, there is not deficient demand. For $X < X^{\max}$ sufficiently large, there is deficient demand.*

Proof. Conditional on θ, equilibrium welfare is given by

$$\mathcal{U}(X, \theta) \equiv U\left(X + \frac{\alpha\Phi}{(1-\alpha)}\theta\right) + \mu(\theta)[w(\theta)\ell(\theta) - \nu(\ell(\theta))] - Q(\theta)\frac{\alpha\Phi}{(1-\alpha)}\theta.$$

Using the envelope theorem and other results from above, at the equilibrium level of θ we may obtain that

$$\mathcal{U}_2(X, \theta) = \mu'(\theta)\left[\nu'(\ell)\ell - \nu(\ell)\right] + \mu(\theta)\nu''(\ell)\frac{\ell^2}{\alpha\theta}[1 - \mathcal{E}_{\mu\theta}(\theta)] - Q'(\theta)\frac{\alpha\Phi}{(1-\alpha)}\theta.$$

If $Q'(\theta) < 0$, which occurs when X is sufficiently large (see Lemma D.1), then clearly $\mathcal{U}_2(X, \theta) > 0$, so that we have deficient demand. Suppose instead that $Q'(\theta) > 0$, which occurs when X is sufficiently small. Substituting in for $Q'(\theta)$ we may obtain

$$\begin{aligned}\mathcal{U}_2(X, \theta) = \frac{\mu(\theta)}{\theta}\nu'(\ell)\ell \Bigg\{ &\mathcal{E}_{\mu\theta}(\theta)\left[1 + \tau\mu(\theta) - \frac{\nu(\ell)}{\nu'(\ell)\ell}\right] \\ &- [1 - \mathcal{E}_{\mu\theta}(\theta)]\left[\frac{\omega(\theta) + 1 - \alpha}{\alpha}\tau(1 - \mu(\theta)) + \frac{1-\alpha}{\alpha}\right]\Bigg\}.\end{aligned}$$

[64] Technically, Asssumption D.1 does not rule out the possibility that $\mu'(0) = 0$. However, since μ is concave and non-decreasing, if $\mu'(0) = 0$ this would imply that $\mu(\theta) = 0$ for all θ, i.e., employment is always zero. We ignore this uninteresting case.

Since $\nu(\ell)/\nu'(\ell)\ell \leq 1$ and $\mathcal{E}_{\mu\theta}(\theta) \to 0$ as $\theta \to \infty$, the first term in braces approaches zero as $\theta \to \infty$, while the second term approaches

$$-\frac{1}{\alpha}\left\{\tau \lim_{\theta\to\infty} \omega(\theta)[1-\mu(\theta)] + 1 - \alpha\right\} < 0,$$

so that for θ sufficiently large (X sufficiently small) $\mathcal{U}_2(X,\theta) \leq 0$, so that there is not deficient demand.

□

Proof of Proposition D.2

We may write the right-hand side of (D.3) as

$$r(e,w,\mu) = \frac{\nu'(\ell^\star)}{F'(\ell^\star)}\left[\mu + (1-\mu)\frac{V'\left(-\frac{w}{F'(\ell^\star)}e\right)}{V'\left(w\ell^\star - \frac{w}{F'(\ell^\star)}e\right)}\right].$$

There are thus three effects of e on r: the direct effect, the effect through w, and the effect through μ. As noted in the text, the effect through μ is always negative. Next, we have that

$$r_w = -\frac{\nu'(\ell^\star)}{F'(\ell^\star)}\ell^\star \frac{\mu V''\left(-\frac{w}{F'(\ell^\star)}e\right) + (1-\mu)\frac{V'\left(-\frac{w}{F'(\ell^\star)}e\right)}{V'\left(w\ell^\star - \frac{w}{F'(\ell^\star)}e\right)}V''\left(w\ell^\star - \frac{w}{F'(\ell^\star)}e\right)}{V'\left(w\ell^\star - \frac{w}{F'(\ell^\star)}e\right)} > 0,$$

by concavity of V, so that the effect of e on r through w is of the same sign as $w'(e)$. Implicitly differentiating (D.4) with respect to e, we may obtain

$$w'(e) = \frac{\frac{w^2}{F'(\ell^\star)}V''(w\ell^\star(1-\mu))}{V'(w\ell^\star(1-\mu)) + w\ell^\star(1-\mu)V''(w\ell^\star(1-\mu))}.$$

Since $w\ell^\star(1-\mu) > 0$, $w'(e)$ will be negative if condition (1) from the Proposition holds, in which case the effect of e on r through w will be negative.

Next, we may obtain the direct effect of e as

$$r_e = \frac{\nu'(\ell^\star)(1-\mu)w}{[F'(\ell^\star)]^2[V'(w\ell^\star(1-\mu))]^2}\left\{V'(-w\ell^\star\mu)V''(w\ell^\star(1-\mu)) - V'(w\ell^\star(1-\mu))V''(-w\ell^\star\mu)\right\}$$

Thus, r_e is negative only if the term in braces is negative. This in turn is the case if

$$\frac{\partial[V''(a)/V'(a)]}{\partial a} < 0,$$

which is true when $V''' \cdot V' - (V'')^2 < 0$. This holds when condition (2) from the Proposition holds, which completes the proof.

Proof of Proposition D.3

The arguments establishing that $de/dX \leq 0$ (with strict equality as long as $N > 0$) are nearly identical to in the Walrasian bargaining case, and are therefore omitted. Thus, there exist $X^\star$ and $X^{\star\star}$ such that for $X < X^\star$ the equilibrium satisfies $N \geq L$ (labor market is tight), for $X \in (X^\star, X^{\star\star})$ the equilibrium satisfies $0 < N < L$ (labor market is slack), and for $X \geq X^{\star\star}$ we have $N = 0$.

Lemma D.3. *Suppose $X \in (X^\star, X^{\star\star})$, so that the labor market is slack but there is strictly positive employment, i.e., $0 < N < L$. Then $dc/dX < 0$ and there is deficient demand.*

Proof. As before, let $\theta \equiv N/L$ be labor-market tightness and $\mu(\theta) \equiv \min\{\theta, 1\}$ be the resulting employment rate, and note that since $N < L$ we have $\mu(\theta) = \theta$. Conditional on e, we may obtain from equation (D.11) that

$$\ell = \ell(e) \equiv \Omega^{-1}\left(\frac{\Phi}{1-s} - \tau e\right),$$

where

$$\Omega(\ell) \equiv F(\ell) - \frac{\nu(\ell) F'(\ell)}{\nu'(\ell)}.$$

It may be easily verified that $\Omega'(\ell) > 0$, so that Ω^{-1} is well-defined.

Letting $\mathcal{W} \equiv w\ell$ be the total wage bill, given e we may obtain

$$p = p(e) \equiv \frac{\nu'(\ell(e))}{F'(\ell(e))},$$
$$\theta = \theta(e) \equiv \frac{e}{F(\ell(e)) - \Phi},$$
$$\mathcal{W} = \mathcal{W}(e) \equiv \nu(\ell) + p\frac{s\Phi}{1-s} - \tau p e.$$

Equilibrium is then a solution to

$$U'(X+e) = Q(e) \equiv p(e)\left[1 + \tau - \tau\theta(e)\right],$$

for e (provided this solution satisfies $\theta(e) \leq 1$). As usual, we will have $dc/dX < 0$ if and only if $Q'(e) < 0$.[65] Since $\ell'(e) < 0$, it is easily verified that $p'(e) < 0$ and $\theta'(e) > 0$, so that $Q'(e) < 0$ necessarily holds.

Next, welfare conditional on e is given by

$$\mathcal{U}(X, e) = U(X+e) + p(e)\left[\frac{s\Phi}{1-s}\theta(e) - (1+\tau)e\right].$$

Taking the derivative with respect to e and evaluating at the equilibrium, we may obtain

$$\mathcal{U}_2(X, e) = Q(e) + p'(e)\left[\frac{s\Phi}{1-s}\theta - (1+\tau)e\right] + p(e)\left[\frac{s\Phi}{1-s}\theta'(e) - (1+\tau)\right].$$

It may be verified that

$$\ell'(e) = -\frac{\tau p \ell}{\nu(\ell)\mathcal{E}_{pl}(\ell)},$$
$$\theta'(e) = \frac{p}{\mathcal{W}}\left[1 + \tau\theta\frac{\mathcal{E}_{\nu l}(\ell)}{\mathcal{E}_{pl}(\ell)}\right],$$
$$p'(e) = -\frac{\tau p^2}{\nu(\ell)},$$

[65] Note that this necessarily follows only under the maintained assumption that τ is sufficiently small such that a unique equilibrium exists.

where $\mathcal{E}_{pl}(\ell) \equiv \frac{\nu''(\ell)\ell}{\nu'(\ell)} - \frac{F''(\ell)\ell}{F'(\ell)} > 0$ and $\mathcal{E}_{\nu l}(\ell) = \frac{\nu'(\ell)\ell}{\nu(\ell)} > 0$. Using these relationships, plus the fact that $\theta = pe/\mathcal{W}$, yields (after some algebra)

$$\mathcal{U}_2(X,e) = \frac{\tau^2 p^2 e}{\nu(\ell)}(1-\theta) + \frac{s\Phi}{1-s}\frac{p^2}{\mathcal{W}}\left[1 + \tau\theta\frac{\mathcal{E}_{\nu l}(\ell)}{\mathcal{E}_{pl}(\ell)}\right] > 0,$$

so that there is deficient demand. $\square$

Lemma D.4. *Suppose $X < X^\star$, so that the labor market is tight, i.e., $N > L$. Then $dc/dX > 0$ and there is not deficient demand.*

Proof. Using the notation defined in the proof of Lemma D.3, we may obtain

$$\ell = \ell(e) \equiv \bar{\Omega}^{-1}([1+\tau(1-s)]e),$$
$$p(e) \equiv \frac{\nu'(\ell(e))}{F'(\ell(e))},$$
$$\theta = \theta(e) \equiv \frac{F(\ell(e)) - e}{\Phi},$$
$$\mathcal{W} = \mathcal{W}(e) \equiv p(e)e,$$

where

$$\bar{\Omega}(\ell) \equiv sF(\ell) + (1-s)\frac{\nu(\ell)F'(\ell)}{\nu'(\ell)},$$

and, by assumption, $\bar{\Omega}'(\ell) > 0$ (see footnote 56) so that $\bar{\Omega}^{-1}$ is well-defined. Equilibrium is then given by a solution to

$$U'(X+e) = p(e),$$

for e (provided this solution satisfies $\theta(e) > 1$). We will clearly have $dc/dX > 0$ if and only if $p'(e) > 0$. Since $\bar{\Omega}'(\ell) > 0$, it is easily verified that $p'(e) > 0$, so that indeed $dc/dX > 0$.

Next, equilibrium welfare conditional on e is given by

$$\mathcal{U}(X,e) = U(X+e) - \nu(\ell(e)).$$

Taking derivatives with respect to e and evaluating at the equilibrium, we may obtain

$$\mathcal{U}_2(X,e) = -\frac{(1-s)p\left[\tau + \frac{\mathcal{E}_{pl}(\ell)}{\mathcal{E}_{\nu l}(\ell)}\right]}{1-(1-s)\frac{\mathcal{E}_{pl}(\ell)}{\mathcal{E}_{\nu l}(\ell)}}.$$

It may be verified that the assumption $\bar{\Omega}'(\ell) > 0$ implies that the denominator of this expression is strictly positive, and thus $\mathcal{U}_2(X,e) < 0$ so that there is not deficient demand.

$\square$

Proof of Proposition D.4

The proofs of Lemmas D.3 and D.4 establish that when $X \in (X^\star, X^{\star\star})$ we have $dp/de < 0$ and $d\ell/de < 0$, while when $X < X^\star$ we have $dp/de > 0$ and $d\ell/de > 0$. Proposition D.4 then follows immediately.

Proof of Proposition D.5

The arguments establishing that $de/dX \leq 0$ (with strict equality as long as $N > 0$) are nearly identical to in the Walrasian bargaining case, and are therefore omitted. Thus, there exist $X^\star$ and $X^{\star\star}$ such that for $X < X^\star$ the equilibrium satisfies $N \geq L$ (labor market is tight), for $X \in (X^\star, X^{\star\star})$ the equilibrium satisfies $0 < N < L$ (labor market is slack), and for $X \geq X^{\star\star}$ we have $N = 0$.

It is easily verified that, when $X > X^\star$, $\xi = 0$ and thus the system is identical to the Nash bargaining case with $s = 0$, and thus the desired properties in this case follow directly from Proposition D.4. We thus focus only on the case where $X < X^\star$.

Lemma D.5. *There does not exist an equilibrium with $N > L$.*

Proof. When $N > L$, we have $\xi = 1$. Since $\Phi > 0$, from the zero-profit condition (D.13) we see that this value of ξ cannot be consistent with an equilibrium.

□

Thus, for $X < X^\star$ we must have $N = L$. This implies

$$\ell = \ell(e) \equiv F^{-1}(e + \Phi), \tag{D.18}$$

$$p(e) = \frac{\nu'(\ell(e))}{F'(\ell(e))}. \tag{D.19}$$

Since the matching function is not differentiable at the point $N = L$, any value $\xi < 1$ of the worker's share of the match surplus can be consistent with zero firm profits as long as e is appropriately chosen. Equivalently, given e, we may obtain a worker's share consistent with zero profit via

$$\xi = \xi(e) \equiv \frac{(1+\tau)e - \frac{\nu(\ell(e))}{p(e)}}{(1+\tau)e - \frac{\nu(\ell(e))}{p(e)} + \Phi} \in [0, 1). \tag{D.20}$$

Thus, an equilibrium is a solution to

$$U'(X + e) = p(e),$$

for e (provided this solution satisfies $e \geq e^\star$, where $e^\star$ is the maximum value of e below which the equilibrium features $N < L$[66]), in which case w is given by (D.12) with $\xi = \xi(e)$, i.e., $w = pe/\ell$. Since $\ell'(e) > 0$, we have $p'(e) > 0$, and thus $dc/dX > 0$.

Next, equilibrium welfare conditional on e is given by

$$\mathcal{U}(X, e) = U(X + e) - \nu(\ell(e)).$$

Taking derivatives with respect to e and evaluating at the equilibrium, it is straightforward to show that $\mathcal{U}_2(X, e) = 0$, so that there is not deficient demand, which completes the proof.

Proof of Proposition D.6

We suppose there is a competitive insurance industry offering a menu of unemployment insurance contracts. A typical contract is denoted (h, q), where h is the premium, paid in all states, and q is the coverage, which the purchaser of the contract receives if and only if he is unemployed. Both h and q are expressed in units of good 1. Since insurance is only potentially useful when

[66] This value is given implicitly by $\Omega\left(F^{-1}(e^\star + \Phi)\right) + \tau e^\star = \Phi$, where $\Omega(\ell) \equiv F(\ell) - \frac{\nu(\ell)F'(\ell)}{\nu'(\ell)}$.

$0 < \mu < 1$, we henceforth assume that this is true. Note also that zero profit of insurers requires that $h = (1 - \mu\widehat{\rho})q$, where $\widehat{\rho}$ is the fraction of purchasers of the contract that are participant households. This implies that non-participant households will not purchase any such zero-profit contract featuring $q < 0$.

Lemma D.6. *In any separating equilibrium, no contracts are purchased by participant households.*[67]

Proof. Suppose there is a separating equilibrium, and let (h_p, q_p) denote the contract purchased by participant households, and (h_n, q_n) that purchased by non-participant households. From the insurer's zero-profit condition, we must have $h_p = (1 - \mu)q_p$ and $h_n = q_n$. Since non-participant households will always deviate to any contract with $h_p < q_p$, this implies that we must have $q_p < 0$ in such an equilibrium.

Next, for any zero-profit separating contract, the assets of employed participant households are given by $A_e = w\ell - p[(1 - \mu)q_p + e]$ and of unemployed participant households by $A_u = p(\mu q_p - e)$. Note that, since $q_p < 0$ and from the resource constraint $wl > pc$, we must have $A_u < 0 < A_e$. Also, the derivative of the household's objective function with respect to q_p along the locus of zero-profit contracts is given by

$$\frac{\partial \mathcal{U}}{\partial q_p} = p\mu(1 - \mu)\left[V'(A_u) - V'(A_e)\right] > 0,$$

wherever such a derivative exists. Since $A_u < 0 < A_e$, this derivative must exist at the candidate equilibrium, and therefore in a neighborhood of that equilibrium the objective function is strictly increasing on $q_p < 0$. Thus, given any candidate zero-profit equilibrium contract with $q_p < 0$, there exists an alternative contract (h'_p, q'_p) with $q'_p > q_p$ which satisfies that $h'_p - (1 - \mu)q'_p$ is strictly greater than but sufficiently close to zero so that participant households would choose it over (h_p, q_p), while non-participant households would not choose it, and therefore insurers could make a positive profit selling it. Thus, (h_p, q_p) cannot be an equilibrium contract. Since this holds for all $q_p < 0$, it follows that no separating equilibrium exists in which contracts are purchased by participant households. □

Next, consider a pooling equilibrium, so that $\widehat{\rho} = \rho$. As argued above, we must have $q \geq 0$ in any such equilibrium. Assets of an employed worker when choosing a zero-profit pooling contract $(h, q) = ((1 - \mu\rho)q, q)$ are given by $A_e = w\ell - p[(1 - \mu\rho)q + e]$, while $A_u = p(\mu\rho q - e)$ are those of an unemployed worker. Let $\mathcal{U}(q)$ denote the value of the household's objective function when choosing such a zero-profit pooling contract.

Lemma D.7. *If $\mathcal{U}(q)$ is strictly decreasing in q whenever $A_e > A_u$, then a pooling equilibrium does not exist.*

Proof. Note first that if $A_e \leq A_u$, then being unemployed is always strictly preferred to being employed by participant households, so that this cannot represent an equilibrium. Furthermore, as argued above, we must have $q \geq 0$ in any pooling equilibrium. Thus, suppose $A_e > A_u$ and $q > 0$. We show that such a q cannot represent an equilibrium. To see this, let (h', q') denote an alternative contract with $0 < q' < q$ and $h' = (1 - \mu\rho)q'$. Since $\mathcal{U}$ is strictly decreasing in q, this contract is strictly preferred by participant households. Furthermore, since non-participant households would get net payment $\mu\rho(q' - q) < 0$ from deviating to this new contract, only participant households would deviate to it, and therefore the expected profit to an insurer offering it would be $(1-\rho)\mu q' > 0$.

[67] Technically, agents are always indifferent between not purchasing a contract and purchasing the trivial contract $(0, 0)$. For ease of terminology, we will assume that this trivial contract does not exist.

Thus, this deviation is mutually beneficial for participants and insurers, and so q cannot be an equilibrium. ☐

Lemma D.8. *If $\rho < 1/(1+\tau)$, then there is no equilibrium in which an insurance contract is purchased by participant households.*

Proof. Note that $\mathcal{U}(q)$ is continuous, with

$$\mathcal{U}'(q) = p\mu\left[(1-\mu)\rho V'(A_u) - (1-\mu\rho)V'(A_e)\right],$$

wherever this derivative exists (i.e., whenever $A_e A_u \neq 0$). If $A_e A_u > 0$, then $V'(A_e) = V'(A_u)$, and therefore $\mathcal{U}'(q) = -p\mu(1-\rho)V'(A_e) < 0$. Suppose on the other hand that $A_e A_u < 0$. If in addition $A_e > A_u$, we must have $A_u < 0 < A_e$, and therefore $\mathcal{U}'(q) = -pv\mu\{1-\rho[1+\tau(1-\mu)]\}$. Since $\rho < 1/(1+\tau)$, it follows that $\mathcal{U}'(q) < 0$. Thus, $\mathcal{U}(q)$ is strictly decreasing whenever $A_e > A_u$, and therefore by Lemma D.7, no pooling equilibrium exists. Since, by Lemma D.6, there does not exist a separating equilibrium either, no equilibrium exists. ☐

E A Version with Productive Capital

E.1 Setup and Main Results

We have shown how a rise in the supply of the capital good X, by decreasing demand for employment and causing households to increase precautionary savings, can perversely lead to a decrease in consumption. While thus far we have considered the case where X enters directly into the utility function, in this section we show that Proposition 4 can be extended to the case where X is introduced as a productive capital good. To explore this in the simplest possible setting, suppose there are now two types of firms and that the capital stock X no longer enters directly into the agents' utility function. The first type of firm remains identical to those in the first version of the model, except that instead of producing a consumption good they produce an intermediate good, the amount of which is given by $\mathcal{M}$. There is also now a continuum of competitive firms who rent the productive capital good X from the households and combine it with goods purchased from the intermediate goods firms in order to produce the consumption good according to the production function $g(X, \mathcal{M})$. We assume that g is strictly increasing in both arguments and concave, and exhibits constant returns to scale. Given X, it can be verified that the equilibrium determination of $\mathcal{M}$ will then be given as the solution to

$$g_{\mathcal{M}}(X, \mathcal{M})U'(g(X, \mathcal{M})) = Q(\mathcal{M}), \tag{E.1}$$

where $Q(\cdot)$ is defined in equation (16).

Note the similarity between condition (E.1) and the corresponding equilibrium condition for the durable-goods version of the model, which can be written $U'(X+e) = Q(e)$. In fact, if $g(X, \mathcal{M}) = X + \mathcal{M}$, so that the elasticity of substitution between capital and the intermediate good $\mathcal{M}$ is infinite, then the two conditions become identical, and therefore X affects economic activity in the productive-capital version of the model in exactly the same way as it does in the durable-goods model. Thus, a rise in X leads to a fall in consumption when the economy is in the unemployment regime. In fact, as stated in Proposition E.1, this latter result will hold for a more general g as long as g does not feature too little substitutability between X and $\mathcal{M}$.[68]

[68] We assume throughout this section that an equilibrium exists and is unique. Conditions under which this is true are similar to the ones obtained for the durable-goods model, though the presence of non-linearities in g makes explicitly characterizing them less straightforward in this case.

Proposition E.1. *If the equilibrium is in the full-employment regime, then an increase in productive capital leads to an increase in consumption. If the equilibrium is in the unemployment regime, then an increase in productive capital leads to a decrease in consumption if and only if the elasticity of substitution between X and $\mathcal{M}$ is not too small.*

The reason for the requirement in Proposition E.1 that the elasticity of substitution be sufficiently large relates to the degree to which an increase in X causes an initial impetus that favors less employment. If the substitutability between X and $\mathcal{M}$ is small, so that complementarity is large, then even though the same level of consumption could be achieved at a lower level of employment, a social planner would nonetheless want to increase employment. Since the multiplier process in our model simply amplifies – and can never reverse – this initial impetus, strong complementarity would lead to a rise in employment and therefore a rise in consumption, rather than a fall. In contrast, if this complementarity is not too large, then an increase in X generates an initial impetus that favors less employment, which is in turn amplified by the multiplier process, so that a decrease in consumption becomes more likely.[69]

Let us emphasize that the manner in which we have just introduced productive capital into our setup is incomplete – and possibly unsatisfying – since we are maintaining a static environment with no investment decision. In particular, it is reasonable to think that the more interesting aspect of introducing productive capital into our setup would be its effect on investment demand. To this end, we now consider extending the model to a simple two-period version that features investment. The main result from this endeavor is to emphasize that the conditions under which a rise in X leads to a fall in consumption are weaker than those required for the same result in the absence of investment. In other words, our results from the previous section extend more easily to a situation where X is interpreted as physical capital if we simultaneously introduce an investment decision. The reason for this is that, in the presence of an investment decision, a rise in X is more likely to cause an initial impetus in favor of less activity.

To keep this extension as simple as possible, let us consider a two-period version of our model with productive capital (where there remains a morning and an afternoon in each period). In this case, it can be verified that the continuation value for household j for the second period is of the form $R(X_2) \cdot X_{2,j}$, where $X_{2,j}$ is capital brought by household j into the second period and X_2 is capital brought into that period by all other households. In order to rule out the possibility of multiple equilibria that could arise in the presence of strategic complementarity in investment, we assume we are in the case where $R'(X_2) < 0$. The description of the model is then completed by specifying the capital accumulation equation,

$$X_2 = (1 - \delta)X_1 + i, \tag{E.2}$$

where i denotes investment in the first period and X_1 is the initial capital stock, as well as the new first-period resource constraint,

$$c + i = g(X_1, \mathcal{M}). \tag{E.3}$$

Given this setup, we need to replace the equilibrium condition from the static model (equation (E.1)) with the constraints (E.2) and (E.3) plus the following two first-order conditions,

$$g_{\mathcal{M}}(X_1, \mathcal{M})U'(c) = Q(\mathcal{M}), \tag{E.4}$$

$$U'(c) = R(X_2). \tag{E.5}$$

[69] Note that a rise in X also increases output for any given level of employment. To ensure that consumption falls in equilibrium, we require that the substitutability between X and $\mathcal{M}$ be large enough so that the drop in employment more than offsets this effect.

Equation (E.4) is the household's optimality condition for its choice of consumption, and is similar to its static counterpart (E.1), while equation (E.5) is the intertemporal optimality condition equating the marginal value of consumption with the marginal value of investment.

Of immediate interest is whether, in an unemployment-regime equilibrium, a rise in X_1 will produce an equilibrium fall in consumption and/or employment in the first period. As Proposition E.2 indicates, the conditions under which our previous results extend are weaker than those required in Proposition E.1 for the static case, in the sense that lower substitution between X and $\mathcal{M}$ is possible.

Proposition E.2. *In the two-period model with productive capital,*[70] *an increase in capital leads to a decrease in both consumption and investment if and only if the elasticity of substitution between X and $\mathcal{M}$ is not too small. Furthermore, for a given level of equilibrium employment, this minimum elasticity of substitution is lower than that required in Proposition E.1 in the absence of investment decisions.*

The intuition for why consumption and investment fall when the elasticity of substitution is high is similar to in the static case. The addition of the investment decision has the effect of making it more likely that an increase in X leads to a fall in consumption because the increase in X decreases investment demand, which in turn increases unemployment and precautionary savings.

E.2 Proofs of Appendix E Propositions

Proof of Proposition E.1

The following result will be useful.

Lemma E.1. *Let $\mathcal{E}^g_{X\mathcal{M}}$ denote the elasticity of substitution between X and $\mathcal{M}$ embodied in g. Then*

$$\mathcal{E}^g_{X\mathcal{M}} = \frac{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})}{g_{X\mathcal{M}}(X,\mathcal{M})g(X,\mathcal{M})}. \tag{E.6}$$

Proof. Letting H^k denote homogeneity of degree k, note first that, since g is H^1, for $a, b \in \{X, \mathcal{M}\}$, g_a is H^0 and g_{ab} is H^{-1}.

Next, by definition, we have

$$\mathcal{E}^g_{X\mathcal{M}} \equiv \left[\frac{d\log\left(g_X(X,\mathcal{M})/g_{\mathcal{M}}(X,\mathcal{M})\right)}{d\log\left(\mathcal{M}/X\right)}\right]^{-1}.$$

Letting $\widetilde{\mathcal{M}} \equiv \mathcal{M}/X$ and using H^0 of g_X and $g_{\mathcal{M}}$, we may obtain

$$\begin{aligned}
\mathcal{E}^g_{X\mathcal{M}} &= \frac{g_X(1,\widetilde{\mathcal{M}})}{g_I(1,\widetilde{\mathcal{M}})\widetilde{\mathcal{M}}}\left[\frac{d}{d\widetilde{\mathcal{M}}}\left(\frac{g_X(1,\widetilde{\mathcal{M}})}{g_{\mathcal{M}}(1,\widetilde{\mathcal{M}})}\right)\right]^{-1} \\
&= \frac{g_X(1,\widetilde{\mathcal{M}})g_{\mathcal{M}}(1,\widetilde{\mathcal{M}})}{\widetilde{\mathcal{M}}\left[g_{X\mathcal{M}}(1,\widetilde{\mathcal{M}})g_I(1,\widetilde{\mathcal{M}}) - g_X(1,\widetilde{\mathcal{M}})g_{\mathcal{M}\mathcal{M}}(1,\widetilde{\mathcal{M}})\right]} \\
&= \frac{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})}{\mathcal{M}\left[g_{X\mathcal{M}}(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M}) - g_X(X,\mathcal{M})g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})\right]},
\end{aligned}$$

[70] We are again assuming that the equilibrium exists, is unique, and is in the unemployment regime.

where the last line follows from H^0 of g_a and H^{-1} of g_{ab}. Adding and subtracting $g_{X\mathcal{M}}(X,\mathcal{M})g_X(X,\mathcal{M})X$ in the denominator and grouping terms yields

$$\mathcal{E}^g_{X\mathcal{M}} = \frac{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})}{g_{X\mathcal{M}}(X,\mathcal{M})\left[g_X(X,\mathcal{M})X + g_{\mathcal{M}}(X,\mathcal{M})\mathcal{M}\right] - g_X(X,\mathcal{M})\left[g_{X\mathcal{M}}(X,\mathcal{M})X + g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})\mathcal{M}\right]}.$$

The first bracketed term in the denominator equals $g(X,\mathcal{M})$ by H^1 of g, while the second bracketed term equals 0 by H^0 of $g_{\mathcal{M}}$, and thus equation (E.6) follows. □

Next, let $W(X,\mathcal{M}) \equiv U(g(X,\mathcal{M}))$. Then the equilibrium condition (E.1) can be written

$$W_{\mathcal{M}}(X,\mathcal{M}) = Q(\mathcal{M}). \tag{E.7}$$

Note that

$$W_{\mathcal{M}\mathcal{M}}(X,\mathcal{M}) = \left[g_{\mathcal{M}}(X,\mathcal{M})\right]^2 U''(g(X,\mathcal{M})) + g_{\mathcal{M}\mathcal{M}}U'(g(X,\mathcal{M})) < 0,$$

so that the left-hand side of equation (E.7) is strictly decreasing in $\mathcal{M}$. To ensure the existence of an equilibrium with $\mathcal{M} > 0$, we assume that $W_{\mathcal{M}}(X,0) > Q(0)$. We further assume that $g_{\mathcal{M}\mathcal{M}\mathcal{M}}(X,\mathcal{M}) \geq 0$, which ensures that $Q_{\mathcal{M}\mathcal{M}\mathcal{M}} > 0$, and therefore, similar to in the durable-goods model, there are at most three equilibria: at most two in the unemployment regime, and at most one in the full-employment regime. Additional conditions under which we can ensure that there exists a unique equilibrium are similar in flavor to in the durable-goods case, though less easily characterized explicitly. We henceforth simply assume conditions are such that the equilibrium is unique, and note that this implies that

$$W_{\mathcal{M}\mathcal{M}}(X,\mathcal{M}) < Q'(\mathcal{M}) \tag{E.8}$$

at the equilibrium value of $\mathcal{M}$. Define also

$$\mathcal{E}^Q_{\mathcal{M}} \equiv \frac{Q'(\mathcal{M})\mathcal{M}}{Q(\mathcal{M})}$$

as the elasticity of Q with respect to $\mathcal{M}$.

Lemma E.2. *$dc/dX < 0$ if and only if*

$$-\mathcal{E}^Q_{\mathcal{M}}\mathcal{E}^g_{X\mathcal{M}} > 1. \tag{E.9}$$

Proof. Differentiating the equilibrium condition (E.7) with respect to X yields that

$$\frac{d\mathcal{M}}{dX} = \frac{W_{X\mathcal{M}}(X,\mathcal{M})}{Q'(\mathcal{M}) - W_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})}. \tag{E.10}$$

Doing the same with the equilibrium condition $c = g(X,\mathcal{M})$ yields

$$\begin{aligned}\frac{dc}{dX} &= g_X(X,\mathcal{M}) + g_{\mathcal{M}}(X,\mathcal{M})\frac{d\mathcal{M}}{dX}\\ &= \frac{g_X(X,\mathcal{M})\left[Q'(\mathcal{M}) - W_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})\right] + g_{\mathcal{M}}(X,\mathcal{M})W_{X\mathcal{M}}(X,\mathcal{M})}{Q'(\mathcal{M}) - W_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})},\end{aligned}$$

where the second line has used (E.10). By (E.8), the denominator is positive, so that this expression is of the same sign as the numerator. Substituting in for $W_{\mathcal{M}\mathcal{M}}$ and $W_{X\mathcal{M}}$ and using the equilibrium condition (E.7), we may obtain that $dc/dX < 0$ if and only if

$$\left[\frac{g_{X\mathcal{M}}(X,\mathcal{M})}{g_X(X,\mathcal{M})} - \frac{g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})}{g_{\mathcal{M}}(X,\mathcal{M})}\right]\mathcal{M} < -\mathcal{E}^Q_{\mathcal{M}}. \tag{E.11}$$

The term in square brackets, meanwhile, can be written as

$$\frac{g_{X\mathcal{M}}(X,\mathcal{M})\left[g_X(X,\mathcal{M})X + g_{\mathcal{M}}(X,\mathcal{M})\mathcal{M}\right] - g_X(X,\mathcal{M})\left[g_{X\mathcal{M}}(X,\mathcal{M})X + g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})\mathcal{M}\right]}{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})\mathcal{M}}.$$

By H^0 of $g_{\mathcal{M}}$, the second term in the numerator equals zero, and thus by H^1 of g, we have that

$$\frac{g_{X\mathcal{M}}(X,\mathcal{M})}{g_X(X,\mathcal{M})} - \frac{g_{\mathcal{M}\mathcal{M}}(X,\mathcal{M})}{g_{\mathcal{M}}(X,\mathcal{M})} = \frac{g_{X\mathcal{M}}(X,\mathcal{M})g(X,\mathcal{M})}{g_X(X,\mathcal{M})g_{\mathcal{M}}(X,\mathcal{M})\mathcal{M}}.$$

Substituting this into (E.11) and using (E.6) yields (E.9). □

If the economy is in the full-employment regime, $\mathcal{E}^Q_{\mathcal{M}} > 0$ and therefore, since $\mathcal{E}^g_{X\mathcal{M}} > 0$, condition (E.9) cannot hold. Thus, from Lemma E.2, if the economy is in the full-employment regime, $dc/dX > 0$. If instead the economy is in the unemployment regime, then $\mathcal{E}^Q_{\mathcal{M}} < 0$, and therefore condition (E.9) can hold as long as $\mathcal{E}^g_{X\mathcal{M}}$ is sufficiently large, which completes the proof of the proposition.

Proof of Proposition E.2

Let $y = g(X_1, \mathcal{M})$ denote output of the final good in the first period. Furthermore, let $B(X_2) \equiv U'^{-1}(R(X_2)) + X_2$ denote the total resources (output plus undepreciated first-period capital) that would be required for the choice X_2 to satisfy the constraints (E.2) and (E.3) as well as the intertemporal optimality condition (E.5), and note that

$$B'(X_2) = \frac{R'(X_2)}{U''(c)} + 1 > 1, \tag{E.12}$$

where the inequality follows from the assumption made that $R'(X_2) < 0$. Since total resources actually available are $(1-\delta)X_1 + g(X_1,\mathcal{M})$, we have $X_2 = B^{-1}((1-\delta)X_1 + g(X_1,\mathcal{M}))$, and therefore from condition (E.4) equilibrium can be characterized by a solution to

$$G(X_1, \mathcal{M}) = Q(\mathcal{M}), \tag{E.13}$$

for $\mathcal{M}$, where $G(X,\mathcal{M}) \equiv g_{\mathcal{M}}(X,\mathcal{M})R(B^{-1}((1-\delta)X + g(X,\mathcal{M})))$. Note that

$$G_{\mathcal{M}}(X_1,\mathcal{M}) = g_{\mathcal{M}\mathcal{M}}(X_1,\mathcal{M})R(X_2) + \frac{R'(X_2)\left[g_{\mathcal{M}}(X_1,\mathcal{M})\right]^2}{B'(X_2)} < 0.$$

Similar to in the static case, we assume that $G(X, 0) > Q(0)$ so that there is an equilibrium with $\mathcal{M} > 0$, and further, conditions are such that this equilibrium is unique, which implies that

$$G_{\mathcal{M}}(X_1,\mathcal{M}) < Q'(\mathcal{M}) \tag{E.14}$$

at the equilibrium value of $\mathcal{M}$.

Lemma E.3. *If $dX_2/dX_1 < 0$ then $dc/dX_1 < 0$ and $di/dX_1 < 0$.*

Proof. Since in equilibrium $c + X_2 = B(X_2)$, we have that

$$\frac{dc}{dX_1} = \left[B'(X_2) - 1\right] \frac{dX_2}{dX_1}.$$

Since $B'(X_2) > 1$, if $dX_2/dX_1 < 0$ then $dc/dX_1 < 0$. Further, if X_2 falls when X_1 rises, from the capital accumulation equation (E.2) we see that i must also fall. □

Lemma E.4. *$dX_2/dX_1 < 0$ if and only if*

$$\left\{-\mathcal{E}^Q_{\mathcal{M}} + \frac{(1-\delta)g_{X\mathcal{M}}(X,\mathcal{M})}{g_X(X,\mathcal{M})\left[g_X(X,\mathcal{M}) + 1 - \delta\right]}\right\} \mathcal{E}^g_{X\mathcal{M}} > 1. \tag{E.15}$$

Proof. Differentiating the equilibrium condition (E.13) with respect to X_1 yields that

$$\frac{d\mathcal{M}}{dX_1} = \frac{G_X(X_1,\mathcal{M})}{Q'(\mathcal{M}) - G_{\mathcal{M}}(X_1,\mathcal{M})}. \tag{E.16}$$

Doing the same with $y = g(X,\mathcal{M})$ yields

$$\frac{dy}{dX_1} = g_X(X_1,\mathcal{M}) + g_{\mathcal{M}}(X_1,\mathcal{M})\frac{d\mathcal{M}}{dX_1}, \tag{E.17}$$

while differentiating $X_2 = B^{-1}((1-\delta)X_1 + g(X_1,\mathcal{M}))$ yields

$$\begin{aligned}\frac{dX_2}{dX_1} &= \frac{1}{B'(X_2)}\left(1 - \delta + \frac{dy}{dX_1}\right)\\ &= \frac{\left[1 - \delta + g_X(X_1,\mathcal{M})\right]\left[Q'(\mathcal{M}) - G_{\mathcal{M}}(X_1,\mathcal{M})\right] + g_{\mathcal{M}}(X_1,\mathcal{M})G_X(X_1,\mathcal{M})}{B'(X_2)\left[Q'(\mathcal{M}) - G_{\mathcal{M}}(X_1,\mathcal{M})\right]},\end{aligned}$$

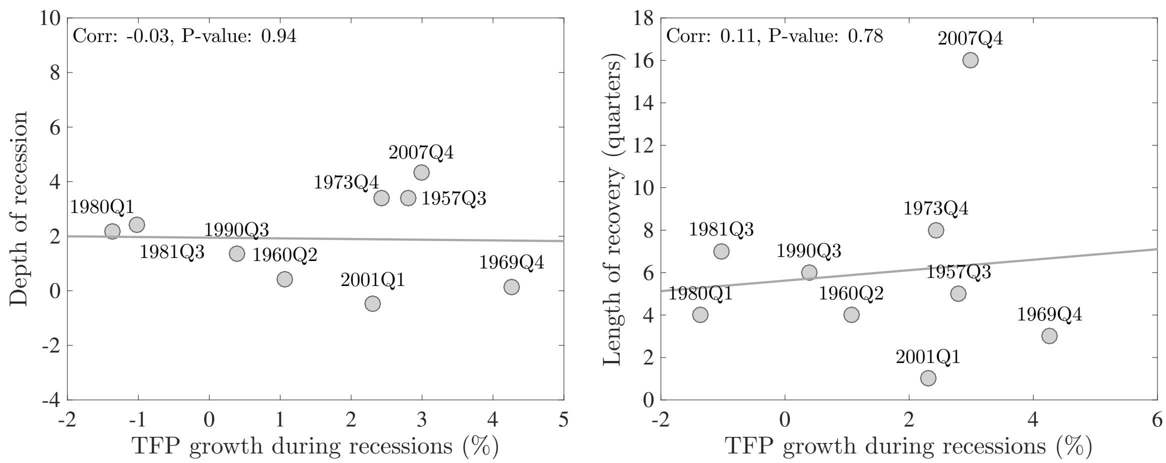
where the second line has used equations (E.16) and (E.17). Since the denominator of this expression is positive by (E.12) and (E.14), the sign of dX_2/dX_1 is given by the sign of the numerator. Substituting in for $G_{\mathcal{M}}$ and G_X and using (E.13), some algebra yields that this expression is negative if and only if condition (E.15) holds. □

Lemmas E.3 and E.4 together indicate that $dc/dX_1 < 0$ and $di/dX_1 < 0$ both hold if and only if condition (E.15) holds. Further, for a given equilibrium level of $\mathcal{M}$, it is clear that the minimum level of $\mathcal{E}^g_{X\mathcal{M}}$ needed to satisfy (E.15) is (weakly) greater than that needed to satisfy (E.9) in the static case.

F TFP Growth During Recessions

Figure F.1 plots for US postwar period the depth of recessions and length of Recoveries against TFP growth during the Recession. We find no significant relationship. Furthermore, TFP growth is indeed positive in 7 out of the 9 recessions we have observed since 1958.

Figure F.1: Depth of Recession and Length of Recovery *vs.* TFP Growth During the Recession



Note: Horizontal axis is TFP growth from peak to trough. Vertical axis is either depth of recession, measured as percentage difference in real per capita GDP from peak to subsequent trough, or length of recovery, measured as the number of quarters it takes for real per capita GDP to reach again the peak level.

G A simple RBC Model with Irrational Exuberance or TFP shocks

Here we contrast our mechanism with the one of a simple RBC model.

G.1 Model

Preferences are given by
$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t + \nu \frac{\ell_t^{1+\omega}}{1+\omega} \right).$$
Technology is given by
$$Y_t = AK_{t-1}^{\alpha} \left(\theta_t \ell_t\right)^{1-\alpha},$$
and the law of motion of capital is
$$K_{t+1} = (1-\delta)K_t + I_t,$$
with the resource constraint
$$I_t = Y_t - C_t.$$
We assume that θ is a stochastic technological drift that follows the process
$$\theta_t = \gamma_\theta^t \widehat{\theta}_t.$$
When we simulate the model with irrational exuberance, $\widehat{\theta}_t$ is a constant. In the case of TFP shocks, $\widehat{\theta}$ is a random walk:
$$\widehat{\theta}_t = \widehat{\theta}_{t-1} e^{\varepsilon_t},$$
where ε is normally *i.i.d.* with mean zero and variant σ^2.

The model calibration follows the same logic than in the main text, but K stands here for total capital. We set $\gamma_\theta = 1.0048$ to match average per capita GDP growth. We set discount factor and depreciation rate to the commonly chosen levels $\beta = .99$ and $\delta = .025$. We set $\alpha = 2/3$ to obtain a labor income share of 2/3. The Frish elasticity of labor supply in the intensive margin ω is set to .5 following Chetty, Guren, Manoli, and Weber [2011]. A is normalized to 1 and ν is set to reach the normalized level of hours per capita $\ell = 1$.

G.2 Irrational Exuberance

Episodes of irrational exuberance are modeled as in the main text. The economy can be either in normal times or in an episode of exuberance, but actual TFP always grows at factor γ_θ. In normal times, actual growth is anticipated by economic agents when making spending decisions. With probability $1 - p_1$, the economy stays in the normal time regime the next period, whereas with probability p_1 it enters in an episode of irrational exuberance. When the economy is exuberant, agents optimistically believe that TFP is for this period growing at a higher factor $\kappa\gamma_\theta$, where $\kappa > 1$ measure the degree of optimism. In contrast with our model, there is here a representative household, but we assume as in our model that it splits between a shopper and a worker that do not communicate, so that consumptions decisions are taken according to expected TFP, whereas labor supply decision is taken according to observed TFP. Investment is then residually pinned down by the household budget constraint.

We then need to set the level of optimism in irrational exuberance κ and the transition probabilities p_1 and p_2. We calibrate these parameters to minimize the distance between model and data for three of the four moments of the U.S. business cycle that we have used for our model: the standard deviation of output growth (0.96%), the average length of a recovery (6 quarters) and the number of recessions episodes over an history of 270 quarters (9 episodes). [71] The resulting parameters are presented in Table G.1. The parameters values we obtain are much less appealing than in our model, as it takes a lot of irrational exuberance to match the data. The degree of optimism takes the value $\kappa = 3.78$, meaning that in an optimistic quarter, agents believe that the growth rate of TFP is 1.33% instead of the average rate of .48%. The probability of entering in an optimistic episode is $p_1 = .89$, and the probability of staying one more period is a exuberant episode is $p_2 = .89$. With this calibration, an exuberant episode last for an average of ten quarters. As shown in Table G.1, the correlation of cumulated investment and depth of recessions or length

Table G.1: Targeted and Non Targeted Moments, RBC Model with Exuberance Shocks

Moment	Data	Model
Targeted Moments		
s.d of output growth	.96%	.94%
Average length of a recovery	6	7.6
Number of recession episodes	3.3	3.2
Non Targeted Moments		
Average depth of a recession	-2.9%	-3.8%
cor(ci,length)	.77	-.88
cor(ci,depth)	.85	-.91

Note: "Data" refers to postwar US data, and "Model" to 10,000 simulations of 270 quarters of the simple RBC model with irrational exuberance. ci stands for capital accumulation, measured as cumulated per capita investment over past 10 years and detrended using a cubic trend. "Depth" represents the depth of recession, measured as percentage difference in real per capita GDP from peak to subsequent trough, and "length" is the length of recovery, measured as the number of quarters it takes for real per capita GDP to reach again the peak level. The number of recession episodes is per 100 quarters. In the model simulations, peaks and troughs are detected using Harding and Pagan's [2002] version of Bry and Boschan's [1971] algorithm.

of recoveries is close to -1, as oppose to close to 1 in the data. This is illustrated on Figure G.1, that shows the scatter plot of depth and length against cumulated investment for one simulation of the RBC model.

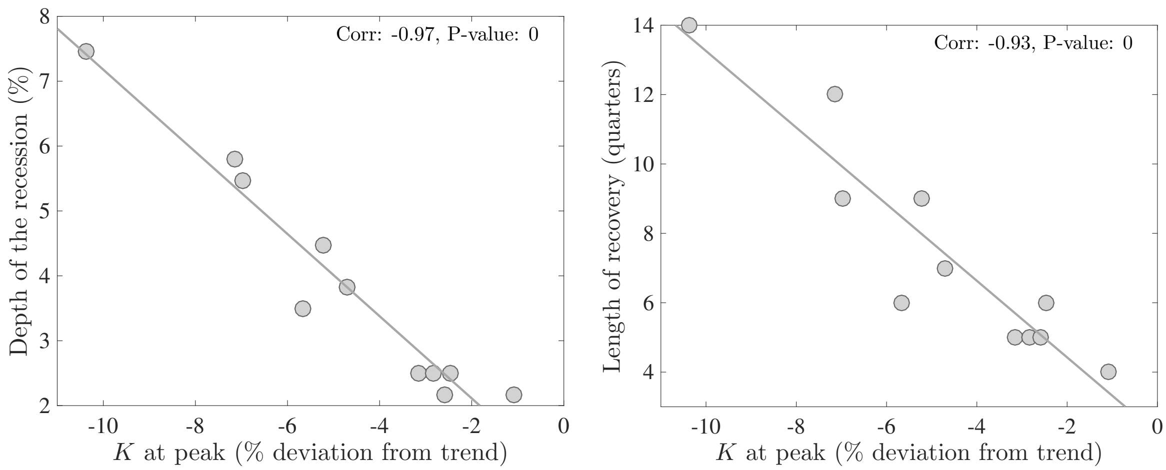
Finally, G.2 displays the path of an economy that is on a balanced growth path until period 4, and that enters in period 5 into a 8-quarters episode of exuberance. In contrast with our model, output jumps back on the balanced growth path (and not below) at the end of the boom episode, and capital is below trend during the boom.

G.3 TFP Shocks

Finally, we consider the simple RBC model with TFP shocks only. The variance of shocks innovation is calibrated to match the standard deviation of output growth, which is obtained for $\sigma = .774\%$.

[71] As we have only three parameters, we do not target the average depth of a recession (-2.9%)

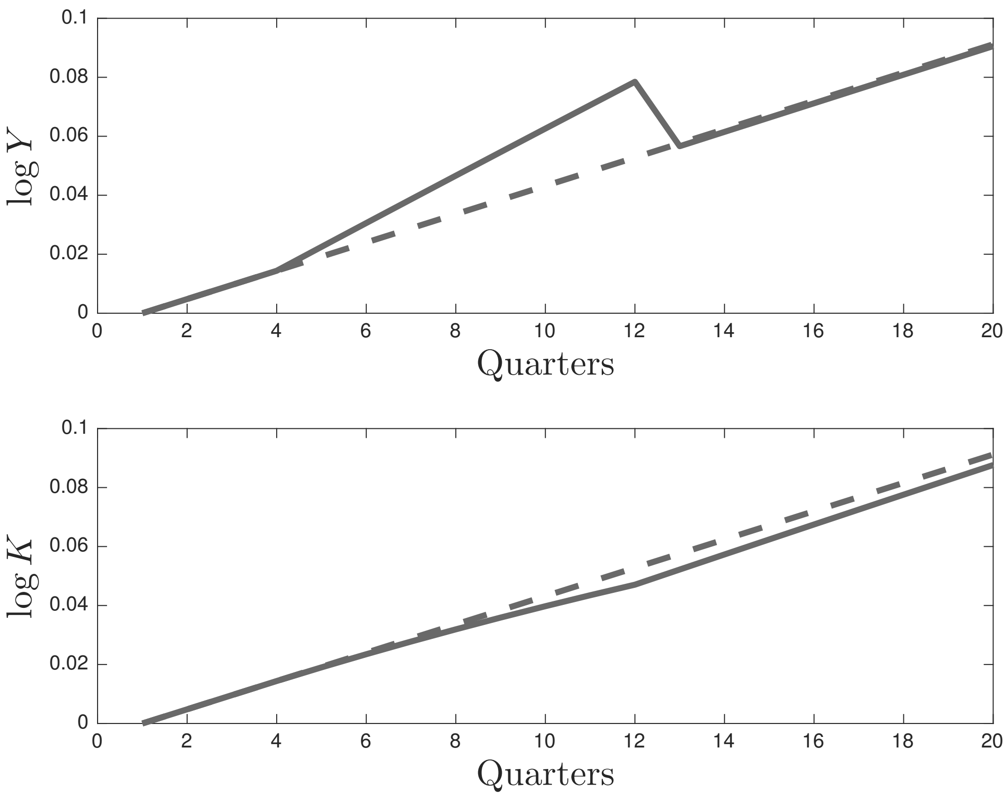
Figure G.1: Depth of Recession and Length of Recovery *vs.* Cumulated Total Investment in One Simulation of a simple RBC Model with Irrational Exuberance



Note: Horizontal axis is capital accumulation, measured as cumulated per capita investment over past 10 years and detrended using a cubic trend. Vertical axis is either depth of recession, measured as percentage difference in real per capita GDP from peak to subsequent trough, or length of recovery, measured as the number of quarters it takes for real per capita GDP to reach again the peak level.

The other moments, which are not calibrated, are displayed in Table G.2. The correlation of cumulated investment and depth of recessions or length of recoveries is small and positive, but never significant. This is illustrated for one simulation in Figure G.3.

Figure G.2: An Episode of Irrational Exuberance in a Simple RBC Model



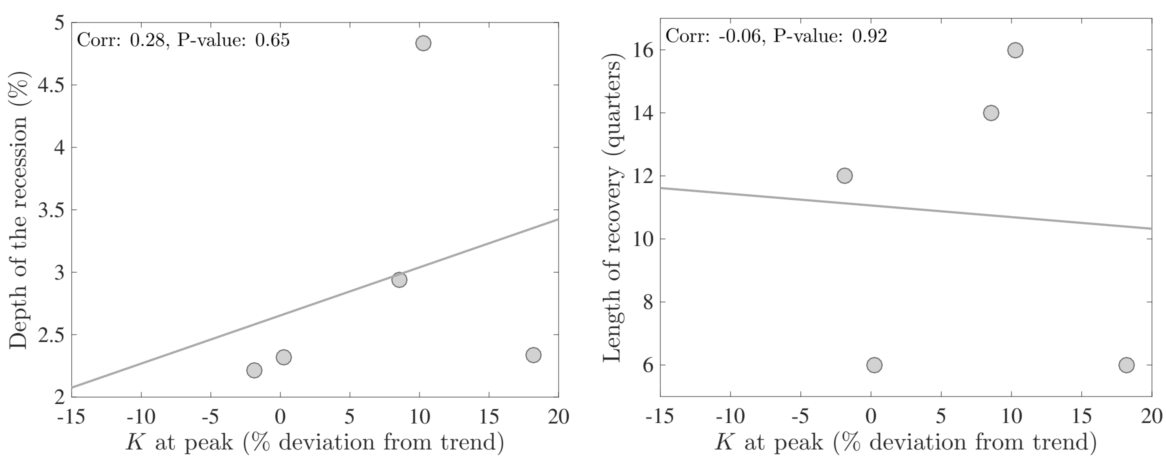
Note: Note: The dashed line represents the path of the variable along the balanced growth path. The plain line is associated with 8-quarters episode of irrational exuberance that starts in period 5. During this episode, agents believe that TFP growth rate is 1.33% per quarter instead of .48%. e represents total expenditures on goods and $\widehat{X}_t$ *is the stock of durables. See the text for functional forms and parameters calibration.*

Table G.2: Targeted and Non Targeted Moments, RBC Model with TFP Shocks

Moment	Data	Model
Targeted Moments		
s.d of output growth	.96%	.96%
Non Targeted Moments		
Average length of a recovery	6	9.3
Number of recession episodes	3.3	2.5
Average depth of a recession	-2.9%	-5.1%
cor(ci,length)	.77	.3
cor(ci,depth)	.85	.24

Note: "Data" refers to postwar US data, and "Model" to 10,000 simulations of 270 quarters of the plain RBC model with TFP shocks. ci stands for capital accumulation, measured as cumulated per capita investment over past 10 years and detrended using a cubic trend. "Depth" represents the depth of recession, measured as percentage difference in real per capita GDP from peak to subsequent trough, and "length" is the length of recovery, measured as the number of quarters it takes for real per capita GDP to reach again the peak level. The number of recession episodes is per 100 quarters. In the model simulations, peaks and troughs are detected using Harding and Pagan's [2002] version of Bry and Boschan's [1971] algorithm.

Figure G.3: Depth of Recession and Length of Recovery *vs.* Cumulated Total Investment in One Simulation of a simple RBC Model with TFP Shocks



Note: Horizontal axis is capital accumulation, measured as cumulated per capita investment over past 10 years and detrended using a cubic trend. Vertical axis is either depth of recession, measured as percentage difference in real per capita GDP from peak to subsequent trough, or length of recovery, measured as the number of quarters it takes for real per capita GDP to reach again the peak level.