

# A Simple Nonlinearity-Tailored Probabilistic Shaping Distribution for Square QAM

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**Abstract:** A new probabilistic shaping distribution that outperforms Maxwell-Boltzmann is studied for the nonlinear fiber channel. Additional gains of 0.1 bit/symbol MI or 0.2 dB SNR for both DP-256QAM and DP-1024QAM are reported after 200 km nonlinear fiber transmission.

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## 1. Introduction

Recently, probabilistic shaping has gained research attention in optical communication systems, in part because it can be used as a rate adaptation mechanism for forward error correction (FEC). However, as well as being flexible in coding rate, probabilistic shaping offers a throughput gain for the additive white Gaussian noise (AWGN) channel. For the AWGN channel, the Maxwell-Boltzmann (MB) distribution is the optimal choice for amplitude ring probabilities [1].

The probability mass function of a MB distribution for a random variable  $X$  is given by

$$\mathbb{P}[X = x_i] = \frac{\exp(-\lambda |x_i|^2)}{\sum_{j=1}^M \exp(-\lambda |x_j|^2)}, \quad (1)$$

where  $\lambda$  is optimized for the channel and  $\{x_1, x_2, \dots, x_M\}$  is the set of constellation symbols. In the field of optical fiber communications, this probabilistically shaped distribution was applied to a 64QAM constellation yielding a reach increase of 40% [2] and to a 1024QAM constellation signal yielding a throughput increase of 13% [3]. Additionally, a successful field trial of probabilistic shaping has recently been conducted [4]. However, this constellation is optimal for only the AWGN channel. For the nonlinear fiber channel, a different distribution can achieve better performance in reach and throughput.

In this work, we include the constellation excess kurtosis induced signal-to-noise ratio (SNR) gain; i.e., that the constellation shape can reduce the nonlinear interference, as analytically predicted by [5–7]. This method yields distributions that introduce less nonlinear interference than a MB distribution and more importantly outperform it in terms of mutual information, as demonstrated in [8]. We initially carry out an optimization of the probability distribution taking into account the SNR dependence on the excess kurtosis. We heuristically derive a general expression without loss of performance. We then present simulation results of a single span 200 km system (following the work of [8]) at  $5 \times 33$  GBd and with the modulation format varying from dual polarization (DP) 64QAM to 1024QAM. The simulation results have shown a mutual information (MI) gain of 0.1 bit/symbol and an additional SNR gain of 0.2 dB.

## 2. Mutual Information Optimization Method

Most analytical models of the nonlinear fiber transmission consider the nonlinear impairments of the channel to behave like AWGN [5–7]. These models have been shown to be accurate in both simulation and experiment [9]. Similar to [10], we assume that the nonlinear interference noise can be written as  $\eta_{\text{tot}} P^3 \approx (\eta_1 + \eta_2 \mathfrak{K}) P^3$  with real numbers  $\eta_1$  and  $\eta_2$ , the launch power  $P$  and the excess kurtosis  $\mathfrak{K} \triangleq \frac{\mathbb{E}[|X|^4]}{\mathbb{E}[|X|^2]^2} - 2$  of the complex constellation. Straightforward calculations yield the following relationship between the SNR at optimum launch power between an input distribution A and an input distribution B:

$$\frac{\text{SNR}_{\text{opt,A}}}{\text{SNR}_{\text{opt,B}}} = \left( \frac{1 + c \mathfrak{K}_B}{1 + c \mathfrak{K}_A} \right)^{\frac{1}{3}}, \quad (2)$$

where  $c \triangleq \frac{\eta_2}{\eta_1}$  is a measure for the relative impact of the modulation format on the nonlinear interference. This work uses a Gaussian constellation as a reference distribution as its excess kurtosis is given by  $\mathfrak{K}_{\text{Gaussian}} = 0$ .

Based on equation (2), the SNR gain as a function of the input distribution can be included in an AWGN channel model. The mutual information is then calculated using Gauss-Hermite quadrature, as shown in [11, Section III]. A numerical optimization method has been implemented that maximizes the MI for a fixed input distribution and its respective SNR by changing the ring probabilities. The trade off between the SNR gain from excess kurtosis and the MI loss from suboptimal shaping for AWGN will yield a net MI gain. Heuristically, we found that the independent ring optimizations did not find any improvements over

$$\mathbb{P}[X = x_i] = \frac{\exp(-v_1|x_i|^2 - v_2|x_i|^4)}{\sum_{j=1}^M \exp(-v_1|x_j|^2 - v_2|x_j|^4)}, \quad (3)$$

where the optimization parameters  $v_1$  and  $v_2$  significantly reduce the computational complexity compared to the unconstrained problem.

We first evaluated the MI for different square quadrature amplitude modulation (QAM) modulation formats and optimized the shaped formats for  $c = 0.69$ , a value obtained using equation (2) from simulation (see Section 3). We assume an AWGN channel with the noise variance corrected by the constellation format excess kurtosis. The results of the optimization process are plotted as MI versus the optimum SNR for a Gaussian-modulated signal as shown in Fig 1. The plotted MI values are calculated for an AWGN with the SNR deviation according to equation (2). The results for DP 64QAM, 256QAM and 1024QAM are shown for uniform distribution, the MB distribution and the proposed optimized distribution. For both the MB and the proposed optimized distributions, the ring powers are numerically optimized for each value of Gaussian-modulated SNR.

For all modulation formats, the proposed optimized distribution outperforms the MB and the uniform distributions for all SNR values. For an optimal Gaussian-modulated SNR of 18 dB, the 64QAM results are already converged. On the other hand, with both DP 256QAM and the 1024QAM, the optimized distribution outperforms the MB distribution by approximately 0.1 bit/symbol and 0.2 dB SNR.

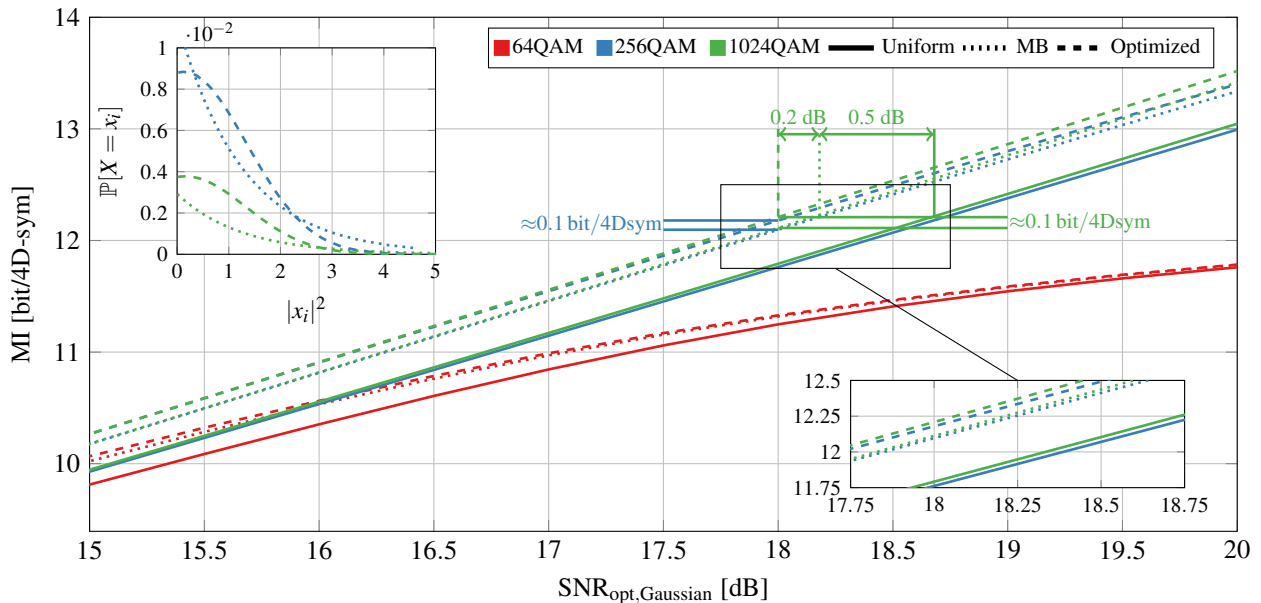


Fig. 1. MI as a function of SNR at optimal launch power a for Gaussian-modulated signal. The optimized distribution provides an additional 0.2 dB SNR gain on top of 0.5 dB SNR gain achieved by the MB distribution. Inset: The probability mass function of the optimized and MB distribution tailored to  $\text{SNR}_{\text{opt,Gaussian}}=18$  dB.

### 3. Numerical Simulation Demonstration

We simulated a single span transmission link based on 200 km ultra-low-loss single-mode fiber with an attenuation of 0.165 dB/km, a dispersion coefficient of 16.3 ps/nm/km and a nonlinear parameter of 1.2 /W/km, followed by

an erbium doped fiber amplifier with a noise figure of 5 dB. The transmitter generates a dual-polarization 33 GHz-spaced  $5 \times 33$  Gbd wavelength division multiplexing (WDM) signal, yielding  $5 \times 400$  Gbit/s. The optimum transmission performance is achieved by sweeping the optical launch power per channel. For Gaussian modulation the simulated transmission system achieves an SNR of 18 dB at optimal launch power.

The simulation results are shown in Fig. 2. The predicted gain from Fig. 1 matches well with the results for both 256QAM and 1024QAM. In Fig. 3 it can be observed that the MI gains based on relation (2) are similar to the results obtained by numerical simulation. This gain is expected to hold for a larger range in SNR.

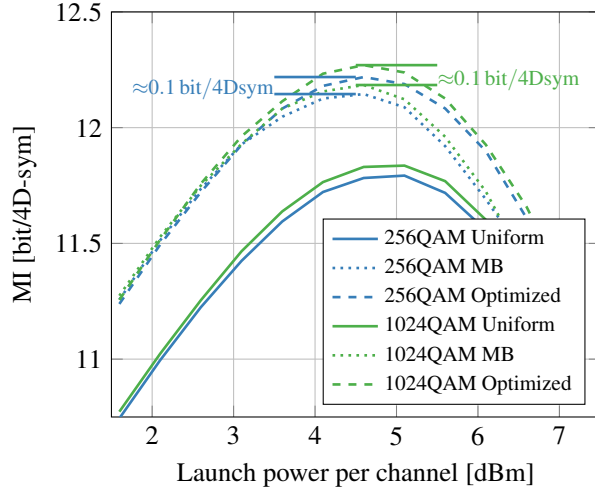


Fig. 2. Simulation results based 200 km single span transmission over ultra low loss fiber.

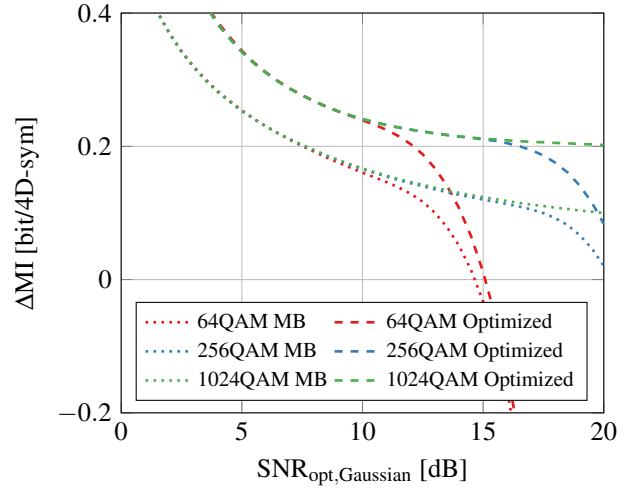


Fig. 3. MI offset from Gaussian-modulated signals for both MB and optimized distributions.

#### 4. Conclusion

We proposed an optimized shaping distribution which provides improved performance over the Maxwell-Boltzmann distribution for the nonlinear channel. We found a simple expression giving an optimized distribution for the nonlinear channel, enabling the optimization of higher order QAM constellations. For both DP-256QAM and DP-1024QAM, the proposed optimized distribution outperforms the Maxwell-Boltzmann distribution by 0.1 bit/symbol and provides an additional 0.2 dB SNR. This gain is in addition to the 0.5 dB SNR gain achieved by shaping using the Maxwell-Boltzmann distribution.

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