Polarization due to rotational distortion in the bright star Regulus

Daniel V. Cotton[1,*], Jeremy Bailey[1], Ian D. Howarth[2], Kimberly Bott[3,4], Lucyna Kedziora-Chudczer[1], P. W. Lucas[5], J. H. Hough[5]

[1]School of Physics, UNSW Sydney, NSW 2052, Australia.

[2]Department of Physics and Astronomy, University College London, Gower Street, London, WC1E 6BT, UK.

[3]Virtual Planetary Laboratory, Seattle, WA 98195, USA.

[4]University of Washington Astronomy Department, Box 351580, UW Seattle, WA 98195, USA.

[5]Centre for Astrophysics Research, School of Physics, Astronomy and Mathematics, University of Hertfordshire, Hatfield AL10 9AB, UK.

*Correspondence to: d.cotton@unsw.edu.au

Polarization in stars was first predicted by Chandrasekhar[1] who calculated a substantial linear polarization at the stellar limb for a pure electron-scattering atmosphere. This polarization will average to zero when integrated over a spherical star but could be detected if the symmetry is broken, for example by the eclipse of a binary companion. Nearly 50 years ago, Harrington and Collins[2] modeled another way of breaking the symmetry and producing net polarization – the distortion of a rapidly rotating hot star. Here we report the first detection of this effect. Observations of the linear polarization of Regulus, with two different high-precision polarimeters, range from +42 parts-per-million (ppm) at a wavelength of 741 nm to −22 ppm at 395 nm. The reversal from red to blue is a distinctive feature of rotation-induced polarization. Using a new set of models for the polarization of rapidly rotating stars we find that Regulus is rotating at $96.5^{+0.6}_{-0.8}$% of its critical angular velocity for breakup, and has an inclination greater than 76.5 degrees. The rotation axis of the star is at a position angle of 79.5±0.7 degrees. The conclusions are independent of, but in good agreement with, the results of previously published interferometric observations of Regulus[3]. The accurate measurement of rotation in early-type stars is important for understanding their stellar environments[4], and course of their evolution[5].

Initial efforts to detect Chandrasekhar's polarization effect proved unsuccessful, leading instead to the discovery of interstellar polarization[6,7]. The predicted polarization in an eclipsing binary has been observed only once, in Algol[8]. The early models of these effects used pure electron scattering atmospheres[1,6]. When more realistic, non-gray stellar-atmosphere models were applied to rotating stars[9,10] the expected polarization at visible wavelengths was found to be much smaller. Indeed, the low predicted polarizations were used to infer that this mechanism could not explain the large polarizations observed[11] in Be stars, which are now attributed to circumstellar decretion disks[12,13].

The development, over the last decade or so, of a new generation of polarimeters[14-16] that can measure stellar polarization at the ppm level brings the small atmospheric polarization effects into the range of observations. Using one of these instruments, PlanetPol[15] on the 4.2-m William Herschel Telescope, Regulus' polarization was measured as 36.7±0.8 ppm[17], which stood out from the much smaller polarizations measured for other stars at similar distance in the same part of the sky. It was concluded that the polarization was likely to be intrinsic. Since Regulus is a rapidly rotating star, polarization due to rotational distortion was considered a possibility. Sonneborn[10] had calculated models for rapidly rotating stars of types B0, B1, B2 and B5 and extrapolation of these to Regulus' B7V spectral type suggested that the polarization was about what was expected for a star rotating at 95% of the critical angular velocity for breakup; very similar to the rotation rate determined from interferometric observations at the time[5]. However, as only one wavelength was observed further observations and modeling were needed to confirm the interpretation.

45 Here we combine the PlanetPol observations with new observations of Regulus obtained with the
46 High Precision Polarimetric Instrument, HIPPI[14] on the 3.9-m Anglo-Australian Telescope at Siding
47 Spring Observatory. These observations were made in four broad wavelength bands with effective
48 wavelengths of 395, 462, 596 and 618 nm. The PlanetPol observations are in a redder band with an
49 effective wavelength of 741 nm. In Figure 1 we show the measured polarizations averaging all
50 observations in each band and correcting for a small amount of interstellar polarization (see
51 Methods). The data are plotted in the form of a QU diagram, where Q/I and U/I are related to the
52 degree of polarization p and position angle θ (measured eastwards from north) through $Q/I = p \cos$
53 2θ and $U/I = p \sin 2\theta$.

54 It can be seen from Figure 1 that the polarization varies strongly with wavelength, and the points lie
55 nearly on a line passing through the diagram's origin. This is exactly what is expected for
56 polarization due to rotational distortion, where gravity darkening toward the equator induces a
57 wavelength dependent asymmetry in the stellar disk. The models of Sonneborn[10] show just this
58 reversal of direction with the polarization being parallel to the star's rotation axis at red
59 wavelengths, and perpendicular to the star's rotation axis in the UV (when the co-ordinate system is
60 rotated to align Stokes Q with the star's rotation axis this corresponds to positive and negative
61 polarization respectively, and we present data by this convention later). In Sonneborn's B5 model
62 the reversal of polarization occurs at ~380 nm. In our Regulus observations it occurs at ~480 nm.
63 However, this is entirely consistent with the trend shown in Sonneborn's models of the reversal
64 shifting to longer wavelengths for later-type stars.

65 Several factors give us confidence in the reliability of the observations. All wavelength bands have
66 been observed multiple times. The repeat observations agree very well even with observations
67 separated by months to years (see Methods). There is no indication of any variability. We can
68 exclude the possibility of Regulus' companion significantly affecting our measurements (see
69 Supplementary Information). The observations obtained with the two different instruments are very
70 consistent, with polarization (after correction for interstellar polarization) in HIPPI's reddest band
71 being $p = 35.4\pm1.6$ ppm at $\theta = 77.1\pm1.3$ degrees, and that measured by PlanetPol being $p =$
72 42.1 ± 1.2 ppm at $\theta = 79.9\pm0.7$ degrees. The increase in p with wavelength is as expected, and the
73 position angles are very similar. These data were obtained 10 years apart – with PlanetPol in 2005-
74 06, and HIPPI in 2016. We have observed several stars at similar distances and in the same region
75 of the sky with both instruments and none show polarizations anywhere near as high as those seen
76 in Regulus. Furthermore when we fit the position angle of the polarization by minimizing U/I (see
77 Methods) it aligns well with the rotational axis of Regulus as determined by interferometry[3] –
78 shown with green dashed lines in Figure 1.

79 To provide further confirmation of the mechanism of polarization and to allow constraints on the
80 properties of the star, we have developed a new set of models for the polarization of rotating stars,
81 verifying the code by comparison with the model results of Harrington[18] and Sonneborn[10] (see
82 Methods).

83 Each model uses as a starting point a value for the angular velocity expressed as ω/ω_c where ω_c is
84 the critical angular velocity for breakup of the star. Given this and a chosen value for the effective
85 temperature (T_p) and gravity (g_p) at the pole we can calculate the distribution of local effective
86 temperature, T_{eff} and g over the star. The effective gravity g is a function of the colatitude Θ and
87 can be determined by methods described in the literature[2,9]. The variation of T^I_{eff} with colatitude
88 depends on adopting a gravity-darkening law. For our Regulus models we use the gravity-darkening
89 law of Espinosa Lara and Rieutord[19]. This law has been developed specifically for rapidly rotating
90 stars and provides results that agree well with two-dimensional models of rotating stars as well as
91 with interferometric observations, which show generally less variation of T^I_{eff} over the stellar
92 surface than predicted by the classic von Zeipel law[20].

93 We then create a set of 46 ATLAS9[21] stellar-atmosphere models each calculated for the pair of T^I_{eff}
94 and log g values that apply at colatitudes from 0 to 90 degrees with 2 degree intervals. We calculate

95   the emergent specific intensity (radiance) and polarization as a function of μ (cosine of the viewing
96   zenith angle) and wavelength for each of these models using a version of the SYNSPEC spectral-
97   synthesis code[22] that we have modified to do radiative transfer with full polarization using the
98   Vector Linearized Discrete Ordinate (VLIDORT) code from RT Solutions[23].

99   We then select an inclination at which to view the star, and set up a grid of "pixels" covering the
100 observed view, spaced at intervals of 0.01 of the polar radius. For each pixel that overlaps the star
101 we calculate the colatitude Θ and viewing angle μ. We can then interpolate in our grid of 46 models
102 and 21 μ values to obtain the specific intensity and polarization for that pixel. These can then be
103 plotted to create images such as those in Figure 2 for a specific wavelength range, or integrated
104 across the star to provide the flux and polarization spectrum at high resolution.

105 The polarization wavelength dependence is a complex combination of effects arising from the
106 change in polarization as a function of μ (e.g. Supplementary Figure 4) and the change in relative
107 contributions of equatorial and polar regions at different wavelengths (e.g. Figure 2) and
108 temperatures according primarily to the Planck function[9]. Figure 3 shows how the modeled
109 parameters affect the polarization wavelength dependence. All four main parameters are significant.
110 Increasing the polar temperature (T_p) shifts the whole curve upwards, and therefore shifts the
111 crossover from negative to positive polarization to shorter wavelengths. Decreasing the inclination
112 reduces the positive polarization at red wavelengths while having little effect on the negative
113 polarization in the blue. Increasing ω/ω_c or decreasing log g_p increases all polarization, making the
114 red polarization more positive, and the blue more negative. As a result, the effects of ω/ω_c and
115 gravity cannot be readily distinguished.

116 It is not, therefore, possible to uniquely determine all four parameters from polarization data alone.
117 To progress further we need additional constraints. Using an inversion of the method for finding
118 distance described by Howarth and Smith[24], we can use the known distance and hence absolute
119 magnitude ($M_V = -0.58$) and spectroscopically determined projected equatorial rotational velocity, v
120 sin i, for which we use 318±8 km/s (see Methods), to derive the global effective temperature, T_{eff}
121 (and hence T_p) and log g_p given values of ω/ω_c and i. This provides a two-dimensional parameter
122 space which we have explored by calculating a grid of models, in the range ω/ω_c = 0.9 to 0.99 and i
123 = 65 to 90 degrees; Figure 4 shows the results of fitting these models to our data.

124 From Figure 4, and after additionally taking account of errors in v sin i and F_{UV} (see Methods), we
125 obtain $\omega/\omega_c = 0.965^{+0.006}_{-0.008}$ and $i > 76.5$ degrees with a best-fit value of 80.0 degrees, which
126 corresponds to $T_p = 14375^{+215}_{-181}$, and log $g_p = 3.992^{+0.022}_{-0.026}$. The results are in excellent agreement with
127 interferometry[3] where $\omega/\omega_c = 0.962^{+0.013}_{-0.028}$ and $i = 86.3^{+1.7}_{-2.3}$ degrees was obtained. The bottom left
128 panel of Figure 3 shows that polarization is particularly sensitive to ω/ω_c near critical velocity,
129 consequently we are able to constrain ω/ω_c more tightly than interferometry. Our determination of
130 inclination is not as precise as that from interferometry. It can be seen from Figure 3 that the models
131 are not very sensitive to inclination near 90 degrees. This difficulty would not be encountered if the
132 star were less inclined; in the present instance it might be overcome by high-precision
133 spectropolarimetry. Whilst U/I is negligible across the broad band, our models show significant
134 polarizations in spectral lines due to the Öhman Effect[25] (see Supplementary Information). This
135 polarization is zero at 90 degrees and increases rapidly with decreasing inclination.

136 Particularly in extreme stars such fundamental parameters as we've determined here are of principal
137 importance. They are needed to, for instance, accurately track the star's evolutionary path along the
138 H-R diagram[5] and therefore its end state (e.g. supernova; compact remnant), or to determine the
139 limits of the Be phenomenon. Rotation also impacts how much mass is lost back into the ISM and
140 the local space weather in the form of stellar winds[4]. Yet, to date only half a dozen rapidly rotating
141 stars have been resolved with interferometry, whereas our initial estimates indicate we may be able
142 to determine parameters for thrice that number with our present instrumentation and facilities. This
143 result thus represents a watershed moment for stellar linear polarimetry. Previously the field has

144 been largely restricted to studying material external to stars or those with extreme magnetic fields[12].
145 Now we are able to probe fundamental parameters of the stellar atmosphere itself.

151 **Author contributions:** J.B., D.V.C., L.K-C., K.B., P.W.L. and J.H.H. drafted the initial proposals
152 to observe Regulus with HIPPI, this followed an extended discussion of the PlanetPol results dating
153 back to 2010 that included I.D.H., J.H.H., J.B. and P.W.L. All authors contributed to the discussion
154 and drafting of the final manuscript. The HIPPI observations were carried out and directed by J.B.,
155 D.V.C., L.K-C. and K.B. In addition the following authors made specific contributions to the work:
156 D.V.C. contributed the initial data analysis, telescope polarization subtraction, the stellar
157 atmosphere modeling, the interstellar subtraction, model comparison to data and other calculations
158 including the position angle calculation and the calculations related to Regulus' companions. J.B.
159 contributed the polarized radiative transfer modeling and verification, gravity darkening
160 calculations and PlanetPol bandpass model calculations and code. I.D.H. contributed rotational
161 velocity calculations, the knowledge and calculations needed to constrain parameter space, and
162 other miscellaneous calculations. K.B. contributed HIPPI bandpass model calculations and code,
163 and research on Regulus' companions. P.W.L. contributed details of the PlanetPol observations not
164 otherwise available.

165 **Competing interests:** The authors declare no competing financial interests.

References:

167 1   Chandrasekhar, S. On the radiative equilibrium of a stellar atmosphere. X. *Astrophys. J.* **103**, 351-
168   370 (1946).
169 2   Harrington, J. P. & Collins, G. W. Intrinsic polarization of rapidly rotating early-type stars.
170   *Astrophys. J.* **151**, 1051-1056 (1968).
171 3   Che, X. *et al.* Colder and hotter: Inteferometric imaging of β Cassiopeiae and α Leonis. *Astrophys. J.*
172   **732**, 68 (2011).
173 4   Cranmer, S. R. & Owocki, S. P. The effect of oblateness and gravity darkening on the radiation
174   driving winds from rapidly rotating B stars. *Astrophys. J.* **440**, 308-321 (1995).
175 5   McAlister, H. A. *et al.* First results from the CHARA array. I. An interferometric and spectroscopic
176   study of the fast rotator α Leonis (Regulus). *Astrophys. J.* **628**, 439-452 (2005).
177 6   Hiltner, W. A. Polarization of radiation from distant stars by the interstellar medium. *Nature* **163**,
178   283 (1949).
179 7   Hall, J. S. Observations of the polarized light from stars. *Science* **109**, 166-168 (1949).
180 8   Kemp, J. C., Henson, G. D., Barbour, M. S., Kraus, D. J. & Collins, G. W. Discovery of eclipse
181   polarization in Algol. *Astrophys. J. Lett.* **273**, L85-L88 (1983).
182 9   Collins, G. W., II. Intrinsic polarization in nongray atmospheres. *Astrophys. J.* **159**, 583-591 (1970).
183 10   Sonneborn, G. Theoretical surface brightness and continious polarization of rapidly rotating B stars.
184   in *Be stars.* (eds M. Jaschek & H.-G. Groth) 493-496 (D. Reidel Publishing Co., 1982).
185 11   Coyne, G. V. & Kruszewski, A. Wavelength dependence of polarization. XVII. Be-Type stars.
186   *Astron. J.* **74**, 528-532 (1969).
187 12   Clarke, D. *Stellar Polarimetry*. (WILEY-VCH Verlag GmbH & Co., 2010).
188 13   Porter, J. M. & Rivinius, T. Classical Be Stars. *Publ. Astron. Soc. Pacific* **115**, 1153-1170 (2003).
189 14   Bailey, J. *et al.* A high-sensitivity polarimeter using a ferro-electric liquid crystal modulator. *Mon.*
190   *Not. R. Astron. Soc.* **449**, 3064-3073 (2015).
191 15   Hough, J. H. *et al.* PlanetPol: A very high sensitivity polarimeter. *Publ. Astron. Soc. Pacific* **120**,
192   1282-1297 (2006).
193 16   Wiktorowicz, S. J. & Matthews, K. A high-precision optical polarimeter to measure inclinations of
194   high-mass X-Ray binaries. *Publ. Astron. Soc. Pacific* **118**, 1302-1318 (2008).

195 17 Bailey, J., Lucas, P. W. & Hough, J. H. The linear polarization of nearby bright stars measured at the
196   parts per million level. *Mon. Not. R. Astron. Soc.* **405**, 2570-2578 (2010).
197 18 Harrington, J. P. Polarized continuum radiation from stellar atmospheres. in *Polarimetry: From the*
198   *Sun to stars and stellar envionments.* 395-400 (Cambridge University Press, 2015).
199 19 Espinosa Lara, F. & Rieutord, M. Gravity darkening in rotating stars. *Astron. Astrophys.* **533**, A43
200   (2011).
201 20 von Zeipel, H. The radiative equilibrium of a rotating system of gaseous masses. *Mon. Not. R.*
202   *Astron. Soc.* **84**, 665-683 (1924).
203 21 Kurucz, R. ATLAS9 Stellar Atmosphere Programs and 2 km/s grid. *Kurucz CD-ROM No. 13* **13**
204   (1993).
205 22 Hubney, I., Stefl, S. & Harmanec, P. How strong is the evidence of superionization and large mass
206   outflows in B/Be stars. *Bull. Astron. Inst. Czech.* **36**, 214-230 (1985).
207 23 Spurr, R. J. D. VLIDORT: A linearized pseudo-spherical vector discrete ordinate radiative transfer
208   code for forward model and retrieval studies in multilayer scattering media. *J. Quant. Spec. Rad.*
209   *Trans.* **102**, 316-342 (2006).
210 24 Howarth, I. D. & Smith, K. C. Rotational mixing in early-type main-sequence stars. *Mon. Not. R.*
211   *Astron. Soc.* **327**, 353-368 (2001).
212 25 Öhman, Y. On the Possibility of Tracing Polarization Effects in the Rotational Profiles of Early-
213   Type Stars. *Astrophys. J.* **104**, 460-462 (1946).
214 26 Cotton, D. V. *et al.* The linear polarization of Southern bright stars measured at the parts-per-million
215   level. *Mon. Not. R. Astron. Soc.* **455**, 1607-1628 (2016).
216 27 Marshall, J. P. *et al.* Polarization Measurements of Hot Dust Stars and the Local Interstellar
217   Medium. *Astrophys. J.* **825**, 124 (2016).
218 28 Cotton, D. V. *et al.* The intrinsic and interstellar broad-band linear polarization of nearby FGK
219   dwarfs. *Mon. Not. R. Astron. Soc.* **467**, 873-897 (2017).
220 29 Thureau, N. D. *et al.* An unbiased study of debris discs around A-type stars with Herschel. *Mon. Not.*
221   *R. Astron. Soc.* **445**, 2558-2573 (2014).
222 30 Frisch, P. C. *et al.* Following the interstellar magnetic field from the heliosphere into space with
223   polarized starlight. *J. Phys. Conf. Ser.* **767** (2016).
224 31 van Leeuwen, F. Validation of the new Hipparcos reduction. *Astron. Astrophys.* **474**, 653-664
225   (2007).
226 32 Serkowski, K., Mathewson, D. S. & Ford, V. L. Wavelength dependence of interstellar polarization
227   and ratio of total to selective extinction. *Astrophys. J.* **196**, 261-290 (1975).
228 33 Wilking, B. A., Lebofsky, M. J. & Rieke, G. H. The wavelength dependence of interstellar linear
229   polarization - Stars with extreme values of λ_{max}. *Astron. J.* **87**, 695-697 (1982).
230 34 Hiltner, W. A. Interstellar Polarization. *Vistas Astron.* **2**, 1080-1091 (1956).
231 35 Slettebak, A., Collins, G. W., II, Parkinson, T. D., Boyce, P. B. & White, N. M. A system of
232   standard stars for rotational velocity determinations. *Astrophys. J. Supp. Ser.* **29**, 137-159 (1975).
233 36 Zorec, J. & Royer, F. Rotational velocities of A-type stars. IV. Evolution of rotational velocities.
234   *Astron. Astrophys.* **537** (2012).
235 37 Dekker, H., D'Odorico, S., Kaufer, A., Delabre, B. & Kotzlowski, H. Design, construction, and
236   performance of UVES, the echelle spectrograph for the UT2 Kueyen Telescope at the ESO Paranal
237   Observatory. in *SPIE Proceedings: Optical and IR Telescope Instrumentation and Detectors.* 534-
238   545 (2000).
239 38 Pepe, F. *et al.* HARPS: a new high-resolution spectrograph for the search of extrasolar planets. in
240   *SPIE Proceedings: Optical and IR Telescope Instrumentation and Detectors.* 582-592 (2000).
241 39 Kaufer, A. & Pasquini, L. FEROS: the new fiber-linked echelle spectrograph for the ESO 1.52-m
242   telescope. in *SPIE Proceedings: Optical Astronomical Instrumentation.* 844-854 (1998).
243 40 Baranne, A. *et al.* ELODIE: A spectrograph for accurate radial velocity measurements. *Astron.*
244   *Astrophys. Supp. Ser.* **119**, 373-390 (1996).
245 41 Gavrilović, N. & Jankov, S. Rotation of Achernar and Regulus through Fourier Transform Analysis
246   of High Resolution Spectra. in *Active OB-Stars: Laboratories for Stellar and Circumstellar Physics,*
247   *ASP Conference Series.* 425-427 (Astronomical Society of the Pacific, 2007).
248 42 Townsend, R. H. D., Owocki, S. P. & Howarth, I. D. Be-star rotation: how close to critical? *Mon.*
249   *Not. R. Astron. Soc.* **350**, 189-195 (2004).
250 43 Gray, D. F. *The Observation and Analysis of Stellar Photospheres*. 3rd edn, (Cambridge University
251   Press, 2005).

252   44   Howarth, I. D. & Morello, G. Rapid rotators revisited: absolute dimensions of KOI-13. *Mon. Not. R.*
253   *Astron. Soc.* **470**, 932-939 (2017).
254   45   Howarth, I. D. New limb-darkening coefficients and synthetic photometry for model-atmosphere
255   grids at Galactic, LMC and SMC abundances. *Mon. Not. R. Astron. Soc.* **413**, 1515-1523 (2011).
256   46   Castelli, F. & Kurucz, R. New Grids of ATLAS9 Model Atmospheres. *arXiv Astrophysics e-prints*
257   (2004).
258   47   Grevesse, N. & Sauval, A. J. Standard Solar Composition. *Space Sci. Rev.* **85**, 161-174 (1998).
259   48   Hubeny, I. & Lanz, T. Non-LTE line-blanketed model atmospheres of hot stars. 1: Hybrid complete
260   linearization/accelerated lambda iteration method. *Astrophys. J.* **439**, 875-904 (1995).
261   49   Gies, D. R. *et al.* A Spectroscopic Orbit for Regulus. *Astrophys. J. Lett.* **682**, L117-L120 (2008).
262

263

264

265

Figure 1 | Polarization observations of Regulus on a Q-U diagram. Polarization observations of Regulus averaged for each band (grey points) and the same observations corrected for interstellar polarization (red points) – 1-sigma error bars for the observations are derived from the internal measurement statistics and those of the telescope polarization; the interstellar corrected points also include a contribution derived from the scatter in interstellar polarization measured in nearby stars. Labels show wavelength in nm. The blue line is the fitted orientation for the rotation axis of the star (see Methods). The green dashed lines show the bounds on the rotation axis of the star as determined by interferometry[3].

274
275

276   **Figure 2 | Example of polarization modeling of Regulus.** The upper panel shows the distribution
277   of effective gravity over the surface of the star, in this case for ω/ω_c = 0.962 and inclination *i* =
278   86.3 degrees (the interferometry parameters determined by Che et al.[3]). The lower panels show
279   the derived intensity distribution over-plotted with polarization vectors at wavelengths of 400 nm
280   and 700 nm. The intensity scale is specific intensity (or radiance) I_ν in units of erg.cm^{-2}.s^{-1}.Hz^{-1}.sr^{-1}.
281

282

Figure 3 | Observations of Regulus vs. model results. Observations of Regulus (black points with error bars) compared with smoothed model results showing the effect of varying different parameters (polar gravity, polar temperature, rotation and inclination). The observations have been rotated into the plane of Regulus' determined rotational axis (i.e. by 79.5 degrees), and the error bars show both the formal error in Q/I (as per Figure 1) and the full-width-half-maximum of the spectral bands. Solid colored circles show the best-fit model averaged over the observed bandpass. The upper two model sets use the interferometrically determined parameters ω/ω_c = 0.962 and inclination i = 86.3. The lower two models use parameters for a fixed v sin i of 318 km s^{-1} as explained in the text.

292
293

294 **Figure 4 | Results of model fitting.** Results of a grid of models calculated for $v \sin i$ = 318 km s^{-1}
295 fitted to the polarization observations as a function of wavelength. The resulting χ^2 is plotted as a
296 function of rotation rate and inclination. Models near ω/ω_c = 0.96 at 90 degrees, 0.965 at 80
297 degrees, or slightly higher values for lower inclinations are the best fits. The contour lines mark
298 the formal 1-sigma and 2-sigma uncertainties using a reduced χ^2 statistic. The point with error bars
299 shows the parameters from interferometry and the associated 1-sigma uncertainty[3].
300

Methods

Multi-band high-precision polarimetry.

We made observations with two high-precision polarimeters – PlanetPol[15] and HIPPI[14]. High-precision polarimetry requires the use of high-frequency modulation to eliminate the effects of astronomical seeing. PlanetPol uses Photo-Elastic Modulators (PEMs) oscillating at tens of thousands of Hertz. HIPPI uses Ferro-electric Liquid Crystal (FLC) technology to provide modulation at 500 Hz.

The PlanetPol (PP) observations were made in a single broad red band that runs from 575 to 1025 nm. With HIPPI we utilised three different filters, those being a 425 nm short-pass filter (425SP), and SDSS g' and r' filters. All three filters were used in combination with the blue-sensitive Hamamatsu H10720-210 ultra bialkali photocathode PMT (b). We also made observations with the r' filter paired with the red-sensitive Hamamatsu H10720-20 infrared-extended multialkali photocathode PMT (r), resulting in 5 bands in total.

Because of the filter and PMT profiles, and the modulator performance, the effective wavelength and polarization efficiency varies with the spectral type of the target. We have calculated these for each band using the bandpass model described by Bailey et al.[14]. Some, but not all, of these have been reported previously[14,26,27], so for completeness we list the bandpass calculations for each filter/PMT combination used here in Supplementary Table 1, along with the corresponding PlanetPol bandpass calculations given by Hough et al.[15]. The stars we observed for this work – Regulus and various calibration standards – are within 60 pc, and so no interstellar reddening has been applied in the bandpass model. We have also made bandpass calculations specifically for Regulus. The stellar model used for this is amongst the suite of models described in the Polarization Radiative Transfer section; specifically it is the one with $\omega/\omega_c = 0.95$ and $i = 90$ deg. The various band sensitivities and effective wavelengths as applied to the Regulus stellar model are shown in Supplementary Figure 1.

The r' (b) observations in May 2014 used a different modulator to all the other HIPPI observations. This gave a 1.5% higher efficiency than the data in Supplementary Table 1 (see notes in the table).

Observations and calibration.

The PlanetPol observations, taken in April 2005 and February 2006 at the 4.2 m William Herschel Telescope (WHT) in La Palma, have been detailed previously[17]. There the standard A0 efficiency correction from Supplementary Table 1 was applied[17]. Here we have instead applied a bandpass model specific to Regulus, and error weighted the two observations; the details are otherwise unchanged.

The HIPPI observations were acquired in four observing runs over three years, on the Anglo-Australian Telescope (AAT) at Siding Spring in Australia and are reported here for the first time. For each observation we took measurements at four different position angles of the AAT's Cassegrain rotator (0, 45, 90 and 135 degrees). The effects of the background sky are typically removed through the subtraction of a sky measurement, obtained at a separation of 2 arcmin for each PA setting and for each observation. The duration of the sky measurements was 3 minutes per Stokes parameter. The observing, calibration and data-reduction methods are described in full detail by Bailey et al.[14].

Observations of high-polarization standards ($p\sim$1-5%) were used to calibrate position angle. A list of the high-polarization standards observed (in the g' band but occasionally checked against similar measurements in other bands) is given in Supplementary Table 2. The precision of each determination is taken to be ±0.5 degrees, based on the mean consistency of the calibration provided by the different reference stars, which themselves have uncertainties of this order.

350   We use observations of stars previously measured with negligible polarizations to determine
351 the zero-point or telescope polarization (TP). Deviations from zero polarization in the standards
352 chosen can therefore be a source of error. To minimize this complication we keep a very short list
353 of low-polarization standards, about which we can be most certain. Even so, variance in the low-
354 polarization standards could produce differences up to a few ppm in the assumed telescope zero
355 point between runs. To combat this problem here – where exquisite precision is required – we have
356 assigned each low-polarization standard a non-zero polarization based on the best available data.
357   The assigned polarization values for each standard star are given in Supplementary Table 3, for
358 each band in which they were observed. The assigned values and associated errors for α Ser, β Leo,
359 and β Vir come from PlanetPol observations[17], and are adjusted for other bands based on the
360 polarimetric color of the ISM (explained in more detail in the next section). By assuming only
361 interstellar polarization we can employ the method outlined in Cotton et al.[28] to determine values
362 for Sirius and β Hyi; this method is described in full in its application to the interstellar subtraction
363 for Regulus in the next section. QU plots made to inter-compare our low-polarization measurements
364 are sparsely populated but seem to corroborate the assignments.
365   The assigned values are subtracted from each standard measurement, and their uncertainties
366 propagated. The error-weighted mean of the standard measurements is then used determine the TP.
367 For the error in TP we use the greater of either the root-mean-square propagated errors, or the error-
368 weighted mean of the scatter (standard deviation) in the individual measurements, whichever is
369 greater. This methodology has its greatest consequence for the May 2014 observations, as there is a
370 significant difference between measurements of α Ser and β Leo. β Leo hosts a debris disk[29], and
371 while the polarization determined with PlanetPol is suitably small, there may be an intrinsic
372 component resulting in anomalous deviation at shorter wavelengths. Subsequent observations of α
373 Ser suggest very-low-level variability may be present; our data are as yet insufficient to test our
374 suspicions. The May 2014 run is the only one where α Ser and β Leo were used as standards in this
375 work. In other subsequent work we have tried to avoid using either of these stars as standards.
376 Given these circumstances, the larger error assigned according to the scatter is appropriate.
377   The observations of the low-polarization standards, rotated into the QU-frame and corrected
378 according to their assigned values (from Supplementary Table 3) and the resulting TP are given in
379 Supplementary Table 4.
380   Observations of Regulus were made during three different observing runs. The exposure times
381 varied according to target availability, weather and other considerations. The observations lasted
382 approximately half an hour to an hour, with second-stage chopping employed to swap the sign of
383 the modulation every 40 to 80 seconds. The observations are detailed in Supplementary Table 5.
384 We also observed HD 99028 as an interstellar control; the details of this along with the PlanetPol
385 observations are also given in Supplementary Table 5.
386
387 **Interstellar polarization subtraction.**

388   Within the Local Hot Bubble (LHB) interstellar polarization is small (roughly 0.2 – 2 ppm/pc).
389 Yet even at this level accurate subtraction is crucial to measuring intrinsic polarization induced by
390 rapid rotation. Only recently has sufficient precision been obtained in measurements of stars within
391 the LHB[17,26,28,30] to enable accurate interstellar subtraction in this region.
392   To determine the contribution to our measurements from interstellar polarization we have, in
393 the first instance, used the same method as in past work[28]. Regulus is north of Galactic latitude b =
394 +30. From measurements of around 30 stars, the relationship between the magnitude of polarization
395 and distance for this part of the ISM, in the g' band, has been determined, on average, to be[28] p_i =
396 $(0.261\pm0.017)d$, where p_i is in ppm and d is in pc. Regulus lies at a distance of 24.3 pc[31], which
397 gives p_i = 6.3±0.4 ppm. Because the interstellar medium can be clumpy we have adopted a more
398 conservative estimate of the error in p_i, which we describe below.
399   Interstellar polarization varies as a function of wavelength. The best current estimate of
400 interstellar polarization color within 100 pc is described by Marshall et al.[27] as likely peaking at 470

401 nm, having been calculated using the empirically-determined Serkowski Law[32] as modified by
402 Wilking[33]; we have used the relation given there to adjust the magnitude of interstellar polarization
403 in the other bands using their effective wavelengths. The most extreme difference is between the g'
404 and PlanetPol bands and is ~1 ppm.
405   In nearby space it has been shown[28] that the position angles of pairs of stars polarized by the
406 interstellar medium are correlated at angular separations up to 35 degrees; with the correlation
407 being greater at closer separations. In that work an angular separation weighted average of
408 intrinsically unpolarized A-K stars within 50 pc of the Sun was used to determine interstellar
409 polarizations for nearby FGK dwarfs. Here we use the same approach and list of stars (with the
410 addition of HD 99028) but extend the distance limit to 70 pc from the Sun, since Regulus lies at a
411 greater distance than most of the targets in the earlier study. The control stars from the list with
412 angular separation to Regulus less than 35 degrees are listed in Supplementary Table 6. For each of
413 these stars we have determined a debiased polarization with distance (where the debiasing is a
414 standard way to account for the effects of measurement error on the positive definite quantity p),
415 and calculated the equivalent polarization at Regulus' distance. The error-weighted scatter in these
416 measurements from the determined interstellar polarization (i.e. 6.3 ppm) has been taken as the
417 error in our interstellar determination for Regulus; the adopted value thus being 6.3±1.4 ppm.
418   The position angles of all the assumed intrinsically unpolarized stars near Regulus are shown
419 in Supplementary Figure 2. Within 35 degrees three stars meeting the required criteria were
420 observed with PlanetPol[17], another with HIPPI in the g' band during a bright-star survey[26] and we
421 have observed one further star with HIPPI in the g' band; these are listed in Supplementary Table 6.
422 Taking the separation-weighted average of the position angles of these stars gives the interstellar
423 position angle at Regulus as 175±12 degrees, where the weight was calculated as 1-(s/35), where s
424 is the angular separation to Regulus in degrees. We assume no angular dispersion with wavelength
425 and subtract the interstellar polarization at the same angle in every band. Our Regulus observations
426 before and after interstellar subtraction are shown in Supplementary Table 7.

Interstellar reddening.

429   In our bandpass model and in the stellar modeling that follows we adopt $E_{(B-V)} = 0$. This
430 assumption is justified on the basis of the very low level of interstellar polarization. Within the
431 LHB, photometry is not sensitive enough to accurately measure interstellar reddening[17]. However,
432 past studies have looked at the correlation between $E_{(B-V)}$ and p_i beyond the LHB, and we can use
433 these to infer a likely range for $E_{(B-V)}$ here. A summary of these studies given by Clarke[12], who
434 mentions extremes of $p_i/E_{(B-V)} \leq 9.0$[32] and 0.18[34]. Although we do not know what the specific
435 relationship is for nearby space, it is reasonable to conclude that $E_{(B-V)}$ is probably much less than
436 0.005.

Determination of Regulus' rotation-axis position angle.

439   A determination of Regulus' rotation-axis position angle, θ , can be made after subtraction of
440 the interstellar polarization from the raw measurements. This is accomplished by assuming any
441 intrinsic polarization is due to rotational oblateness, and rotating Q/I and U/I into a new reference
442 frame (Q_r/I, U_r/I) that minimizes U_r/I. In practice we use Python's 'scipy.optimize.curve_fit' routine
443 with its 'trf method' and error weight each of the bands. This gives θ as 79.5±0.7 degrees in good
444 agreement with the interferometric determination of 77 to 80 degrees[3]. The polarization in each
445 band after rotation by this angle is given in Supplementary Table 7.

Determination of projected equatorial rotation velocity.

448   As discussed below, Regulus' equatorial rotation velocity, $v \sin i$, provides a useful constraint
449 on system parameters. The literature reports a rather wide range of values for the former parameter
450 (~260 – 350 km/s; e.g.,[35,36]), so to make a new determination we de-archived high-resolution
451 spectra obtained with several echelle spectrographs (UVES[37], HARPS[38], FEROS[39], and Elodie[40];

452 resolving powers $\sim 5 - 10 \times 10^4$, signal:noise ratios ~ 500 for each instrument in merged datasets). The
453 spectrum of Regulus is dominated by strong hydrogen-Balmer lines (consistent with its spectral
454 type), with the star's rapid rotation leading to blending of other characteristic features (specifically,
455 the HeI/MgII 447.1/448.1 nm classification lines). After examining the spectrum, we chose the
456 isolated, unsaturated Si II 634.7 nm line for analysis (as did Gavrilović & Jankov[41]). This is a 'cold'
457 line (its equivalent width increases monotonically going from equator to pole in our models), and
458 continuum gravity darkening is not severe at this wavelength (half the observed 650 nm continuum
459 flux originates from colatitudes $\gtrsim 70$ degrees); consequently Si II $\lambda 634.7$ does not suffer strongly
460 from the degeneracy between line width and equatorial rotation velocity discussed by Townsend,
461 Owocki & Howarth[42].

462   While direct modeling of the spectral-line profile can yield estimates of $v \sin i$, its Fourier
463 transform encodes this information in a more straightforwardly accessible manner, with the first
464 minimum in the FT scaling linearly with $v \sin i$ for a spherical star and wavelength-independent
465 limb darkening (e.g.[43]). However, rotational distortion, gravity darkening, and wavelength-
466 dependent limb darkening all modify the scaling factor, and noise can influence the precise location
467 of the minimum. Synthetic spectra were therefore generated for full rotating-star models, using
468 Roche geometry and ATLAS9 model atmospheres as described below, for an appropriate range of v
469 $\sin i$ values. (The results are insensitive to the precise values adopted for other stellar parameters, as
470 we confirmed through sensitivity tests.) The model spectra were convolved with approximate
471 (Gaussian) instrumental profiles; and noise, determined directly from the observations, added. The
472 Fourier transforms of the observed and model spectra were then compared in order to determine v
473 $\sin i$. The modeling fully accommodates any small residual nonlinearity between line width and
474 rotation velocity, and we verified, through sensitivity tests, that the results are insensitive to the
475 precise values adopted for other parameters (inclination, effective temperature, polar gravity, and
476 ω/ω_c).

477   The Fourier transforms of the observed and model spectra were then compared in order to
478 determine $v \sin i$. We estimate $v \sin i \approx 336, 318, 310, 318$ km/s from the Elodie, FEROS, HARPS,
479 and UVES spectra, respectively, and adopt 318±8 km/s (where the estimated error is dominated by
480 the inter-instrument differences, and accommodates both the small individual uncertainties and the
481 weak sensitivity to other parameters). Comparison of observed and model spectra confirms that this
482 value leads to a satisfactory match, and our result is consistent with the other direct measurements
483 published this century of 300±20 km/s[41], and 317±3 km/s[5].

484

485 **Constraining stellar parameter space.**

486   Any modeling of rotating stars (including our polarization calculations) requires a number of
487 stellar parameters as inputs: three to characterize the basic structure (e.g, effective temperature,
488 mass, and polar radius, or surrogates) and two that describe the rotation (e.g., $v \sin i$ and inclination,
489 i, or surrogates).

490   The effective temperature of Regulus is such that the bulk of its radiant-energy output is
491 directly observable, and hence the spectral-energy distribution gives a strong constraint on the
492 global effective temperature T_{eff}. Furthermore, the distance to Regulus is precisely determined from
493 *Hipparcos* astrometry[31]: $d = 24.3 \pm 0.2$ pc. Given T_{eff}, the apparent brightness gives the angular size
494 of the star as seen from Earth, whence d yields the stellar radius (e.g., polar or equatorial radii r_p,
495 r_{eq}), for any given values of ω/ω_c and i.

496   Given ω/ω_c and i, the observed $v \sin i$ establishes a value for ω_c, which in turn yields the stellar
497 mass (within the framework of the Roche model; see, e.g., Howarth & Morello[44]) for the
498 corresponding r_{eq}. Although the line profiles and flux distribution (especially the size of the Balmer
499 jump) potentially offer further leverage on parameters such as polar gravity, g_p, in practice
500 observational uncertainties limit the additional utility of those diagnostics.

501   These principles can therefore be used to determine the effective temperature and polar
502 gravity (along with the stellar mass and radius) from the known distance plus the observed flux

503 distribution and projected equatorial rotation velocity, given values for ω/ω_c and i (which are
504 constrained by the polarization modeling described below).
505   In practice, we determined temperatures and radii, for specified values of $v \sin i$, ω/ω_c, and i,
506 by requiring rotating-star model fluxes to reproduce simultaneously the observed magnitude in the
507 Johnson V band (effective wavelength ~550 nm) and the UV flux integrated over the range 132.5 –
508 194.5 nm. The UV flux was measured from the archival low-dispersion spectrum SWP 33624 (the
509 same spectrum used by McAlister et al.[5]), obtained with the *International Ultraviolet Explorer*
510 satellite and downloaded from the IUE Final Archive at the Mikulski Archive for Space Telescopes.
511 The required model-atmosphere fluxes and magnitude normalization were taken from Howarth[45]
512 and are consistent with those used in the polarization calculations.
513   The projected equatorial rotation velocity is quite well determined, leaving ω/ω_c and i as the
514 principal 'free' parameters. Supplementary Figure 3 shows results of an initial grid, sampled at 0.01
515 in ω/ω_c and every 5 degrees in i. We calculated additional points on a finer grid (0.005 in ω/ω_c and
516 2.5 degrees in i) around the region of the best-fitting solution from the polarization analysis. Final
517 numerical results of our analysis of the basic stellar parameters are incorporated into Supplementary
518 Table 8 (where the uncertainties are generated from Monte-Carlo simulations, sampling the
519 estimated errors in the input parameters).
520
521 **Modeling.**

522   All our modeling is based on the assumption of a Roche model and uniform rotation. On this
523 basis the geometry of a rapidly-rotating star follows equations given by Harrington & Collins[2] that
524 specify how the radius and other properties vary as a function of colatitude (Θ) and rotational
525 velocity expressed as ω/ω_c. Each colatitude has a different value of gravity (g) and local effective
526 temperature (T'_{eff}). To represent this variation we create for each model (where a model is defined
527 by its polar temperature T_p and gravity, its ω/ω_c and its gravity darkening law) a set of 46 ATLAS9
528 model atmospheres representing the T'_{eff} and log g of the star at 2 degree intervals of colatitude from
529 0 to 90 degrees. The ATLAS9 models use as a basis the Castelli & Kurucz model grid[46] for solar
530 abundances[47].
531
532 **Polarized radiative transfer.**

533   To calculate the polarization of the emission from these models we use a modified version of
534 the SYNSPEC spectral synthesis code of Hubeny – briefly described by Hubeny, Stefl and
535 Harmanec[22]. We have modified the code to do polarized radiative transfer using the Vector
536 Linearized Discrete Ordinate (VLIDORT) code[23] which is a comprehensive implementation of the
537 discrete-ordinate method of radiative transfer.
538   From the emission, absorption and scattering properties of each layer of the atmosphere
539 available in SYNSPEC it is straightforward to provide the inputs needed by VLIDORT for each
540 atmospheric layer. We assume that all scattering is described by a Rayleigh scattering matrix, which
541 is appropriate for Thompson scattering from electrons. Our modified version of SYNSPEC outputs
542 the intensity and polarization as a function of viewing angle (μ = cosine of the local zenith angle)
543 and wavelength. SYNSPEC uses a non-uniform wavelength scale in order to fully sample line
544 structure, which we re-bin to a uniform 0.01 nm spacing for subsequent processing.
545   To apply the model to a rotating star we use the geometric model to determine the colatitude Θ
546 and μ as a function of coordinates x, y in the observed image plane for a given inclination i. We
547 divide the observed image into a large number of "pixels" using a spacing of 0.01 in units of the
548 polar radius of the star. We then interpolate in the grid of models to obtain intensity and
549 polarization for each pixel. We use linear interpolation in Θ and cubic spline interpolation in μ. The
550 polarizations require rotation through an angle ξ as defined by Harrington and Collins[2] to rotate into
551 a coordinate system defined by the rotation axis of the star.
552   The results of this process are intensity and polarization values for each pixel, at the full
553 spectral resolution of the model, that can be plotted as an image and vector map as in Figure 2, or

554 can be integrated over the star to give the total polarization. Because we work in the observed
555 image plane and use a large number of pixels (typically about 40,000) the integration is a simple
556 sum over pixels for each Stokes parameter. To affect a comparison to observation, as in Figure 4,
557 the integrated intensity and polarization is fed into the bandpass model at the full spectral resolution
558 of the model.
559
560 **Tests of the modeling.**

561 We have tested our polarization version of SYNSPEC by calculating polarization for a number
562 of stellar-atmosphere models that are included in the results of Harrington[18]. Supplementary Figure
563 4 shows an example for TLUSTY[48] models for $T_{eff} = 15000$ K and log $g = 2.5$ and 3.5 at a
564 wavelength of 400 nm. The polarizations from the two models agree very well. There is a small
565 difference in specific intensity between Harrington's results and those from SYNSPEC/VLIDORT
566 but our results agree very well with intensity from SYNSPEC's standard radiative transfer. This is
567 probably due to the fact that Harrington's calculations are for continuum only, whereas our models
568 include spectral lines. Similar agreement has been found for other wavelengths and different
569 temperature models.
570 To test our rotating-star models we have compared with results reported by Sonneborn[10].
571 Supplementary Figure 5 shows a comparison of our modeling with Sonneborn's results for B1, B2
572 and B5 stars with $\omega/\omega_c = 0.95$ and $i = 90$ degrees. Sonneborn does not specify the temperature and
573 gravity values used for these models. However, we found that we could match his results closely
574 with $T_p = 28000$ K (B1), 23000K (B2) and 18000 K (B5) with a log g_p of 4.07 in all cases. For these
575 models we used a von Zeipel gravity darkening law[20] consistent with Sonneborn's original work. In
576 a future work we plan to examine stars with different spectral types in more detail.
577
578 **Regulus final parameters.**

579 The result of fitting our model grid to the data is shown in Figure 4. From this we obtain a best
580 fit in χ^2 and the 1-sigma confidence region in reduced χ^2 that constitutes the model error. To obtain
581 the final errors we must also consider the errors in T_{eff} and log g_p due to the uncertainty in the
582 parameters obtained from the literature, significantly, the UV flux, F_{UV}, (which we take as 5%) and
583 $v \sin i$. These are shown diagrammatically in the bottom left corner of Supplementary Figure 3 and
584 are simply added as root-mean-squares to the model errors to obtain the final uncertainties in T_{eff}
585 and log g_p. To calculate the errors in ω/ω_c and i the observational uncertainties in T_{eff} and log g_p
586 must first be transformed into a corresponding change in polarization via an interpolation of the
587 upper two panels of Figure 4; the change in polarization can then be equated to errors in ω/ω_c and i
588 through the lower two panels of the same figure, and finally added as root-mean squares to the
589 model errors. Supplementary Table 8 lists all the parameters adopted and calculated in modeling
590 Regulus, and the final values determined. For other derived physical parameters, given at the
591 bottom of Supplementary Table 8, the errors are propagated from the uncertainties on the model-
592 determined parameters using Monte-Carlo methods.
593
594 **Data availability:** All processed data generated during this study is included in this published
595 article (and its supplementary information files), the raw data files are available upon reasonable
596 request. All other data analysed in this work comes from public repositories, where this is the case
597 the origin of the data is indicated in the text.

598

599

600

$\omega/\omega_c = 0.962$ inc = 86.3 deg

relative gravity

400 nm

1000 ppm

700 nm

1000 ppm