

**THE CHARACTERIZATION OF
COMMUNAL KNOWLEDGE**

**Case Studies in Knowledge Relevant
to Science and Schooling**

by

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ABSTRACT

This work attempts the task of analysing *communal knowledge* in relation to schooled knowledge. At one level, the thesis concerns a peculiar method of measuring land (*cubação*) used by peasants in Brazil and their understanding of the transformations of *soil*. At another level, it attempts to look outwards all the time to some very general issues so as to discuss questions about the relative valuation of school knowledge and communal knowledge; the distance between educational discourse on the one hand and the teachers and ordinary people's discourse on the other; together with a discussion of knowledge elicitation, representation and acquisition.

The account of the specific communal knowledge described in the thesis is based on a empirical study with adults in a rural community in Brazil and data is *qualitative*. Information is obtained mainly from farm-workers and indigenous primary school teachers. *Teachback*, in the sense proposed by Pask, is the central process around which 'conversations' between participants take place.

Research in Science Education has very largely treated knowledge from an essentially individual point of view. In this thesis, however, *knowledge* is regarded as a social entity realized in individual discursive action. *Knowing* becomes being a participant in a *discourse* and to possess knowledge is turned into to be able to operate a certain kind of discursive process.

The goal of trying to reach *understanding* leads the informants to create new *explanations*, and to think explicitly about the taken-for-granted discourse. This gives the researcher, the possibility of a further level of analysis *about* the discourse (not just of structures *within* the discourse). As an outcome, novel results concerning methods of land measurement serve as an example to place the knowledge of *cubação* in relation to historical knowledge structures and the mechanisms of social transmission and reproduction of knowledge.

To my parents, Elza and Arduino; to my children,
Mônica and Luiz Fernando; and to the memory of
my daughter, Maria Angélica.

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CHAPTER 1

AIMS, PERSPECTIVES AND IMPLICATIONS

1.1 PREAMBLE

This is a thesis about *knowledge* seen from the perspective of a researcher in *science education*. In general terms the thesis takes up the question of trying to characterize and clarify the nature of *communal knowledge* as knowledge in some sense "common" to a community of people and which contrasts with science taught in schools.

The topics I will address are complex and at the same time quite wide ranging in scope. The thesis starts from a concern about schooling, framed in a Brazilian Project's attempt to adapt a science curriculum to the reality of a community of peasants in the North East Region (S. Paulo do Potengi)¹. It concludes by discussing knowledge and schooling in the light of two themes related to *Agriculture* which are analysed throughout the thesis. They are *cubação* (a method of measuring area of tracts of land) and *the cultivation of soil* for planting. It is in relation to *cubação* that most of the analysis about the nature of knowledge itself is carried out.

The background elements, pertinent to the conceptualization of this thesis which arise out of the Project's work are outlined in Appendix 1.A. The importance of studies in *common-sense knowledge* and reasons why a *social* approach would be appropriate in dictating a framework for discussing both knowledge and schooling are there stressed.

The overall aim of the study is to examine *practical-communal* knowledge in relation to the *school-scientific* knowledge; that is, I am concerned with the investigation of possible kinds of relationship it is important to look at between *commonsense* and *science*, so as to discuss questions of the kind tabulated below:

¹ The "Ensino de Ciências a partir de Problemas da Comunidade" (ECPC) Project has been implemented by the Department of Education of the Federal University of Rio Grande do Norte since 1983.

AIMS, PERSPECTIVES AND IMPLICATIONS

Questions about	Examples
(a) the relative valuation of school knowledge and communal knowledge;	"How to affect valuation?" "How to make valuation problematic?"
(b) the distance between educational discourse on the one hand and teachers and ordinary people's discourse on the other;	"What difficulties arise for the planning, choosing the curriculum, communicating?"
(c) the acquiring of knowledge;	"Should one teach practical-communal knowledge?" "In what sense would one do that?" "How would one use it?"
(d) the systematic collection/recording of data.	"How to frame inquiry to . collect the expected . remain open to the unexpected?" "What is data in this case?"

To say that it is my purpose to discuss these questions is not to say that I shall resolve them. The attempt is to give a sense of some of the problematic issues that lie near the forefront of research and the kinds of answer that it may be possible to provide for them, from the perspective of this research. Also, I shall indicate why I think that these questions bear on issues of a more general interest in science education.

The chapter is organized in four sections. First, I indicate the levels at which the argument is developed in the thesis, emphasising the general questions, which, applied to *cubação* and *soil*, will attempt to contribute to the discussion of knowledge and schooling (section 1.2). Secondly, the main distinctive features of the perspective adopted in the investigation of common-sense knowledge are addressed (section 1.3). In section 1.4 the background issues are recapitulated. And finally, in section 1.5, I delineate the theoretical framework.

1.2 THEMES AND LEVELS OF DISCUSSION

It is very important to clarify what are the actual intentions of the work. The thesis is relevant to some very general problems of knowledge but looked at in the context of an extremely *particular object of knowledge*. At one level, this is a

thesis about a peculiar *method of measuring area* used by peasants in Brazil and which is placed and analysed in the context of production in agriculture. People's understanding of the transformation of *soil* for planting constitutes the main link from which specific questions on knowledge and schooling are raised.

But at another level, the thesis attempts to look outwards all the time to some very general questions such as:

- . What is it to speak of knowledge of a community?
- . What is it to attach or deny value to a practice?
- . What are the ways to study it -individual and social?
- . To certain extent we know what knowledge can be in science. Do we even know what are we talking about when we use the word knowledge referring to the communal-practical knowledge?
- . What is it to understand common-sense knowledge relevant to formalized public knowledge, such as science?
- . What is it to consider the relation of *formalized knowledge* -say scientific knowledge in school- to *informal knowledge*?
- . When we talk about communal-practical knowledge, are we studying something which is transparent, there waiting to be described, or is it well hidden and needing to be brought out?
- . What is it to speak of representation of knowledge?
- . Should communal knowledge be conceived of as a *logical* part of people's *tacit* understanding of the world?
- . What difference (difficulties and possibilities) does the existence of a practical discourse make to the problems of establishing a school discourse?
- . When one tries to incorporate elements of people's practical discourse, which areas are selected because they are interpretable, although they are not necessarily relevant?
- . How does one present one discourse to the other and what role does the teacher play in this attempt to move between the two, bringing messages about what happens in the other world?

Thus, the reader will notice that the discussion tries both (a) to set particular and concrete objects/topics/themes in a more general analysis and (b) to refer to and to make use of concrete examples in the discussion of more abstract issues. I have tried to organized the arguments by distinguishing and relating these two levels as clearly as possible.

1.3 ORIENTATIONS AND PERSPECTIVES

In section 1.3.1 I discuss different ways in which the relation of common-sense knowledge to science can be viewed, and in section 1.3.2 I outline the position adopted in this thesis.

1.3.1 Ways of looking at commonsense in science education.

1.3.1.1 *Individually oriented.*

"*Commonsense*" is a broad notion which I am using to indicate a cluster of ideas which in *science education* designate, basically, the ideas prior to teaching that students have, and which in some way contrast with those offered in science lessons. The persistence of these ideas despite instruction and their influence in children's learning of science have made a strong case for the many studies in the field in the last two decades.

It is possible to recognize in some studies, that the term *commonsense* has emerged as a qualification to *ordinary/everyday explanations* which it is possible to describe using the modes of scientific explanations as some sort of control². One way in which researchers have attempted to give their studies greater generality is to focus on *causality*. For example, one finds *causation* being applied as (a) a broad category for designating "a common core for pupils explanations" (Andersson, 1986); (b) an analytical principle (related to the levels of cognitive reasoning of pupils) for submitting curriculum materials to an analysis for the level of demand they make (Shayer, 1970, 1972); and (c) an ontological category for describing theory change in childhood (Carey, 1987).

One theoretical basis for such work has been Piaget's theory of cognitive development, even though what concerns Piaget is not the *actual* individual but the *generic epistemic* individual subject.

Another approach within the individually-oriented point of view has a basis in the work of Kelly (1955). Following Kelly's Personal Construct Theory, it is

² For example, in looking at commonsense ideas about *motion*, researchers would usually take the concept of "*force*" and contrast the Newtonian view with a view which attach to force properties such as "push" and "pull", and which relates to "*objects*" in terms of support, carrying, falling, etc.

assumed that each person constructs for himself or herself representational models of the world which enable him/her to anticipate events, or to plan a course of behaviour. That is, each person develops models of the world which are subject to change over time, since constructions of reality are constantly tested out and modified to allow better predictions in the future (Pope and Keen, 1981). The relevance of encouraging students to reflect upon, and make known their construction of some aspects of reality has been stressed particularly by Pope and Gilbert (1983) who conceive students in terms of Kelly's metaphor of man-the-scientist.

Within the constructivist tradition, Osborne and Wittrock (1983, 1985) propose a generative learning model which emphasises that learning science with understanding is a generative process of constructing meaning from one's memories, knowledge, and experience, and from incoming sensory information. In terms of such a model, teaching involves helping pupils to generate appropriate meanings from incoming information, to link these meanings to other ideas in memory, and to evaluate both newly constructed ideas and the way old ideas are related in memory (Osborne and Wittrock, 1983, p. 505).

1.3.1.2 *Culturally oriented.*

One of the prevailing tendencies in the studies of commonsense in science education holds that any elucidation of students' alternative concepts requires attention to the public conventions and social contexts of their proper use. Researchers have sought to show how the sense of such concepts is connected to the ways in which they are used routinely in communicative situations. The various perspectives within which work is being done -phenomenological (e.g. diSessa, 1987; Arcà, Guidoni, & Mazzoli, 1983, 1984; Hawkins & Pea, 1987; Marton, 1981); linguistic (e.g. Ogborn, 1985); public informative (e.g. Ziman 1980; Lucas, 1983, Ogborn, 1987); ethnographic (e.g. Hewson, 1986; Saxe, 1981); social influential (e.g. Solomon, 1987, 1985)- all attempt to reveal and account for knowledge changes (whether in cognitive or epistemological terms) that are part of ongoing life (in which scientific modes of reasoning constitute concluding goals).

Within this approach, science is contrasted with commonsense, but researchers exhibit a considerable diversity in their concerns about the nature of the relationship. For example, while Solomon regards the social settings as a

meaningful context, Ogborn considers theories of commonsense (understood as a kind of grammar which lies behind the use of commonsense explanations in ordinary discourse), and Lucas focuses on the different sources of scientific literacy (as a complex/set of conditions which would facilitate learning).

While development is usually assumed to occur, the conditions and mechanisms under which changes come about are hardly explained. Expressions such as '*cultural change*', '*cognitive change*', or '*bridging the cultures of everyday and scientific thinking*' are relatively empty. If they have content, it is usually by analogy with Kuhn's idea of scientific revolution; that is, considering the relation between commonsense and science in terms of *theory change*.

A more recent and complementary approach to the 'Kuhnean' view is offered by Harré (1988) who emphasises the *discursive character of explanations*; that is, explanations, whether lay or scientific, are discourses used by human beings to perform communicative acts. The approach brings out the idea of *discursive community* in relation to which the question of "why is this or that discourse explanatory" is discussed by Harré. It is this more *socially* oriented approach which I shall adopt.

1.3.1.3 Oriented to computational models.

Other areas have also informed research involving common-sense explanations. Cognitive Science is a recent one, which combines aspects from psychology and other areas, particularly with Artificial Intelligence (AI)³. Educational implications from research in AI have been widely reported (Self 1985, Ennals 1985, Pask 1976a), and relationships of AI to science education have been proposed (Good 1987, DiSessa 1987, Ogborn 1987).

What is characteristic in studies within this approach is that it is in relation to *formalised common-sense understandings* that explanations are investigated. This is so to the extent to which the approach takes over from AI the traditional problem of trying to clarify the nature of what is meant by *understanding*. That is, "*explaining*", is considered to be an extremely complex phenomenon involving

³ An account of the interdisciplinary nature of Cognitive Science can be found in Stillings, N. *et al.* (1987) *Cognitive Science: An Introduction* Cambridge: The MIT Press.

several facets and features, strategies and conditions for applying knowledge (Schank 1986, Simon 1985). To clarify what is involved in *understanding someone else's thoughts* from a cybernetic approach to cognition, becomes rather important. Attempts can be described, at least, in relation to three different kinds of understanding to which people commonly refer. We can talk about *understanding*: (a) a subject matter (as in Pask 1975a, 1976b); (b) a language (as in Winograd 1972, 1980); or (c) someone else's behaviour (as in Schank 1986). It is possible to see many of the relevant issues in relation to these areas (Figure 1.1).

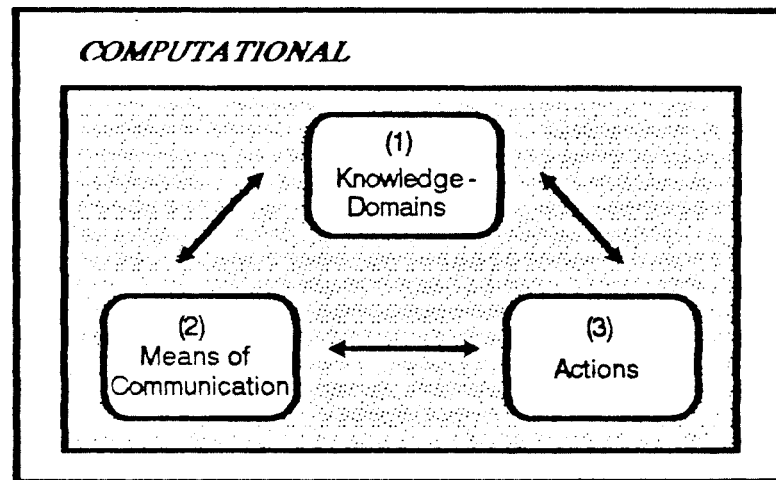


Figure 1.1

For example, in the relation between (1) and (2), the *elicitation*, *representation* and *organization* of knowledge arise as significant issues for investigation. Both *tacit* and *metacognitive* knowledge become a concern in the relation between (2) and (3). The *formalization* of reasoning behind cognitive *performances* would lie in the relation between (1) and (3), while the formalization of knowledge in relation to cognitive *competences* would be better placed between (2) and (3).⁴ In looking at *actions* from a computational perspective one could have questions about *strategic* knowledge. And so on.

So far as learning is concerned, a main focus has been building a *model* of the learner's state of *understanding* (Self, 1985). Thus, attempts have been made to

⁴ The distinction between competence and performance (Chomsky, 1965) is used here in the same perspective as in Bernstein (1970, pp. 160-162) when discussing linguistic codes and speech codes. That is, *knowledge-domains* is taken as a 'contextual system' and *means of communication* as a system of formal properties whose meanings are realised in its use.

diagnose the students' learning difficulties, by developing a picture, or *model*, of obstacles to progress in *understanding* the content in question. The intention is to know both *what* mistakes are being made and *why* (Good 1987). From these perspectives, explanations would be better seen as relevant to the task of *modelling* the learners' level of *understanding*.

One of the most suggestive aspects concerning the teaching and learning of science which squares with the position taken from cognitive science is the developmental *constructive* character of knowledge; that is, it is widely accepted that children are active interpretative learners who bring their prior *understandings* and *frames of interpretations* to making sense of pedagogic presentations of science in schools. *Modelling the students' mental models* and *understanding students' understanding* are characteristic themes with respect to which commonsense is investigated.

1.3.2 Selection of perspectives for this thesis.

1.3.2.1 *Two perspectives of interpretation.*

Taking these approaches together (especially the individual approach), we are led to say that the task of a science teacher is no longer to provide a proper exposition of science, one that lays out a logically organized and complete view of the subject matter and which leaves to the student the sole task of accessibility. Students do not absorb new knowledge in a blank structure. Learning involves making interpretations which one can classify as categories of a natural reasoning system. We can imagine having some attempt to describe these categories and ask: "Is this consonant with what one finds in science?"

For example, we can ask why, in talking about soil and growing plants for example, common-sense reasoning does not use concepts like '*cycle*', '*energy*' or '*transformations*' in the same way as in science? Why do children hold alternative ideas? Why are they so difficult to change? From this point of view it is possible to characterize one approach within which the above question can be framed. Within this approach, the main claim made by researchers could be stated as:

- "(a) We need some theory why common-sense reasoning does not use the 'logic and concepts' of science in the same way as scientific reasoning does.
- (b) Such a theory would seek an account (for example, of a psychological or sociological kind) of how people come to know or to construct their views about the world."

In general, the nature of the required theory is seen as accounting for the discrepancy between scientific and common-sense reasoning. That is, scientific concepts and reasoning are taken as unproblematically given as a reference against which to evaluate commonsense⁵.

To distinguish this point of view from the next, which I now try to characterize, I will call it *perspective 1*, the "*science-referred*" view.

Now, suppose we look at the issue the other way round and say: let us suppose that people have some reasoning resources and that they operate with some categories (which is a perspective whose starting point could be psychology or sociology for example). Suppose also that we know what they are. Then we would ask: "What would this predict or lead us to expect about forms of reasoning?" So, one might say (as a matter of speculation):

"People operate with a notion of cause and a notion of sequence. Cause always presuppose sequence. Effect is always imagined to come after the cause and never before. People also operate with a notion of unconnected coexistence. Things exist simultaneously but have no connection, where connection can be understood in various sorts of ways. And they operate with a notion of conditions. And so on."

Then, suppose one can say that people did think that. Elaborating a little further one would say:

"This makes good sense of the fact that '*energy*' -interpreted as a causal event- includes normally the sequential event. But there is an interpretation of a sequential event which is not intended as a causal event. And in the distinction between those two, it would be interesting to see, for example, if people make some deductions from their mental models of what things are like."

In other words, within this perspective we would be asking to what extent a given reasoning (including scientific reasoning) has exploited the resources available in a common-sense reasoning system. This seems to address a different possibility, one in which the main claim is:

"(a) We need some account of what we can learn from a study of the psychologist's or the sociologist's analysis (in approximately scientific modes) about the nature of common-sense reasoning, (b) so as to take common-sense thinking as something to be understood for itself in the relation to scientific and other reasoning."

⁵ In the worst case, such a view could amount to "physicists doing psychology".

This is then another possibility for us to come to claim that we need to understand how and why people construct their own kinds of reasoning or ideas. To contrast with the former I will call it *perspective 2* - the "*common-sense referred*" view.

1.3.2.2 *The position to be taken in the present work.*

So, given these two perspectives, the general position to be taken in this work squares with the second one. The main reason is that it is within this perspective that the question of what it is to characterize both commonsense and science seems to find a clear role. Also, this seems to be the perspective from which valuation of both communal-knowledge and school-knowledge can be made problematic. I will now discuss some of the implications of adopting such a position.

1.4 BACKGROUND ISSUES

1.4.1 Issues in the relation between structures of knowledge and trying to represent them.

Three general issues in formalizing and representing knowledge emerge for discussion. They arise from a general interest in looking at the relation between structures of knowledge/thought and trying to represent them; and are encapsulated in the following questions:

"What can be the ways to represent communal knowledge?"

"What kind of formal structure can be seen as representing and describing communal knowledge?"

"Can one understand formulations of complex structures of knowledge in terms of explicit rules?"

They are: the 'epistemological', the 'discourse' and the 'tacit' issues.

In the light of the discussion, there are a number of things to say about some of my initial questions (addressed on p. 13). Particularly, I will be looking at five of these questions:

- . What is it to speak of knowledge of the community?
- . When we talk about communal-practical knowledge, are we studying something which is transparent there, waiting to be described, or is it well hidden and needing to be brought out?

- . What is it to consider the relation of 'formalized knowledge' -say scientific knowledge in the school- to 'informal knowledge'?
- . What is it to speak of representation of knowledge?
- . Should communal knowledge be conceived of as a 'logical' part of people's 'tacit' understanding of the world?

1.4.1.1 *The 'epistemological' issue.*

The 'epistemological' issue arises in this research from an interest in trying to clarify how communal knowledge can be conceived as a form of 'representation'. I start with the idea that communal knowledge constitutes what a community of people know; but what exactly does this mean? People make representations, but representations in the sense I intend are public, not private. As Hacking says:

'Representations are external and public, be they the simplest sketch on a wall, or, when I stretch the word 'representation', the most sophisticated theory about electromagnetic, strong, weak or gravitational forces.'
(Hacking, 1983, p. 133)

The position I want to adopt in dealing with communal knowledge in terms of some kind of representation regards knowledge as public and being made by people (accordingly, *reality* is to be seen as an attribute of representation, not as facts). It is knowledge which exists as a discourse, and whose basis is social; that is, of the kind developed in the course of the division of labour and which refers to the particular activities involved.

Communal knowledge refers to a culture and so can be seen as a system of representation with a proper style, which distinguishes it from other kinds of representations like scientific theories or ways of reasoning in the past.

So, considering the questions initially raised, *what would be involved in the representation of communal knowledge given its discursive/cultural character?*

1.4.1.2 *The 'discourse' issue.*

The '*discourse*' issue emerges from general considerations about the nature of a *formal structure* for representing and describing communal knowledge. The question is whether communal knowledge can be seen as a regular system of knowledge whose structure is rule-governed.

One should distinguish two senses of the term '*discourse*'. The first, which is not that intended here, is discourse meaning conversational or other exchanges. The second, which is that intended, is discourse in the structuralist sense, of a system which determines what it is possible to communicate. In the second sense, a discourse is characterised by general rules of formation and of appropriateness.

The structuralist notion of discourse relies heavily on the metaphor of "grammar". As pointed out by Bliss, Monk & Ogborn (1983):

"Probably the force of the metaphor lies in the way actual grammar -the rules of a language- is just what makes meaning possible at all, is the common property of a whole community, is itself an extended and complex structure, and, whilst 'well known' to every native speaker, is in large part sub-conscious."

(Bliss, Monk & Ogborn, 1983, p. 167)

Thus the question which arises is: *"can commonsense be represented as a formal discourse structure?"*

1.4.1.3 The 'tacit' issue.

The 'tacit' issue arises from general considerations about how to understand formulations of complex structures of knowledge in terms of explicit rules. It is widely supposed that communal knowledge must have a large tacit component. That is, if it does have structuring rules, these rules are not consciously available to those who, nevertheless, are regarded as operating within them.

Two questions arise:

- (1) *Can one represent communal knowledge as a structured system of tacit knowledge?*
- (2) *If people's accounts are to be a source of a formalisation of communal knowledge, how can tacit structures be inferred from such data?*

That dealing with such questions is not in principle impossible is shown by the work of, for example, Chomsky (1988) in linguistics. On the other hand, the thinker who has perhaps attended most closely to the nature of tacit knowledge is Polanyi (1958, 1969). For Polanyi, mastering an area of knowledge has a large component of acquisition of skills. These skills, like those of riding a bicycle, are not well captured in any formalisation, in his view.

1.4.2 Issues in the relation between scientific reasoning and common-sense reasoning.

In so far as the school is concerned with a transformation towards forms of scientific reasoning, further questions arise (which have been indicated on p. 13).

- . What difference (difficulties and possibilities) does the existence of a practical discourse make to the problems of establishing a school discourse?
- . When one tries to incorporate elements of people's practical discourse, which areas are taken because interpretable but not necessarily relevant?
- . How does one present one discourse to the other and what role does the teacher play in this attempt of walking and travelling backwards and forwards between the two, bringing messages about what happens in the other world?

It is clear that this last set of questions is not to be answered just in terms of a developmental approach to knowledge. But to take a developmental perspective in which commonsense becomes interesting because viewed as a resource out of which we manufacture 'non-common-sense' knowledge, implies that some questions will turn out to be quite important. For example, what constitutes the *new* aspect which is to be understood as produced by scientific reasoning? What ways of thinking does scientific rationality leave out in constructing scientific forms of reasoning?

1.4.3 Some implications.

In summary, viewed within the second perspective, these arguments suggest:

- (1) That there is the possibility of formalization of communal knowledge.
- (2) That this formalization is not a simple matter of categorizing people's explanations; and so, the research asks what and how it could be.
- (3) That if communal knowledge is to be looked at as being a 'logic' in some sense, this is a 'logic' at the level of text or discourse, and not at the level of assertion.
- (4) That, being described in terms of a formal-knowledge-based kind of representation, such a formalization should account, in some way, for people's explanations. That is, explanations have an important role, but do not constitute the object of formalization of communal knowledge.
- (5) That such a formalization is to be conceived as one stage in a dynamic process of understanding, not as a terminus.

In addition, it is important to try to grasp progression as a *human phenomenon*. What, however, in detail does this second approach look like? Given the questions posed at the start, what kinds of answers can we get from taking this approach? To try to think about these questions through concrete examples turns out to be part of the story to be told in this thesis.

1.5 THE THEORETICAL FRAMEWORK

Research in Science Education has very largely treated knowledge from an essentially individual, that is psychological point of view, as a possession of or a property of people taken one at a time -whether in development or learning, whether with respect to content ('concepts') or skills. The social dimension has been taken as context; as influential but as a set of factors to be held constant rather than to be studied in its own right.

There is a different perspective, taken in this thesis, which regards the *social* as fundamental. Knowledge can be understood and defined at the social level, when questions about it concern its means of social construction, reproduction and sustenance. Individual variation is now seen as context. In this sense '*knowledge*' is not regarded as possessed by individuals, but rather as a social entity realised in individual discursive action. '*Knowing*' becomes being a participant in a *discourse* and to possess knowledge is turned into being able to operate a certain kind of discursive process. Taking this view, the possessing of knowledge (*connaissance*) is not a fundamental characteristic of some individual. What the individual has is some set of *competences* in relation to that knowledge which are essentially *discursive* competences -the ability to join and to participate in a discourse. Thus, the criteria for somebody to be knowledgeable in something is the extent to which he is a functioning member of a discursive community in relation to which a field of knowledge (*savoir*) can be characterized. And this can have degrees. One can be a beginner, another can be more involved, another can be an expert. As soon as we accept this stance we are led to ask questions about roles that people play in such a community.

There are two broad 'forms of knowledge' which have particular interest for this research. One is *science*. The other is supposed to exist as knowledge supporting most human regularities in thought, feelings and behaviour, and which I am calling *commonsense*. While the former exists and is formalized to be transmitted as an abstract result of human inquiry through history, the latter is supposed to be

found locally in human *praxis*. This does not mean that science exists detached from human praxis. Actually, what makes commonsense similar to science is the social perspective in which a kind of *discursive practice* can be seen in operation. Also, there is the possibility of describing them in relation to a discursive community.

In this work, the relevant community is a community of Brazilian peasants, in which case commonsense is best seen in relation to *communal* knowledge.

From this perspective, '*knowledge*' is not pure, detached from other considerations such as power, social relations and social-historical change. These become part of its meaning and definition. There is not, for example, knowledge on the one hand and power on the other, linked by 'accident', but, as Foucault would say, a single entity '*knowledge-power*'.

To view knowledge in this way is to open it to politicisation. One cannot avoid some more or less explicit political stance which arises in the relation between knowledge and power, such as "*in what ways knowledge is used to differentiate people*." Particularly, if one adopts a Freirean position, *knowledge* is to be conceived of as making problematic aspects of people's living, and *teaching* is to be understood as both *supportive* of the existing community and *subversive* of aspects of social structure. If schooling tries to take account of the communal practical knowledge of people, then it will inevitably find itself confronting knowledge conceived of as a social entity and not knowledge conceived of as something which a given individual does (or does not) possess. In addition, the question arises of how science looks problematic as a body of knowledge which contains in itself *problems to be searched* and not bits of information to be transmitted; and so, it becomes imperative -for the purpose of application to schooling- to bring knowledge into discussion, in relation to power.

As far as the cases concerning knowledge (*cubação* and *soil*) are concerned in this thesis, communal knowledge will be looked at as an entity:

- . of a large historical scale,
- . on a small social scale,
- . in relation to practical activities,
- . in relation to relations of power and ownership,

and seeing it as implicated in the whole fabric of *living, being* and *knowing*.

AIMS, PERSPECTIVES AND IMPLICATIONS

The attempt is to try to understand knowledge in relation to schooling not as an entity looked just from one perspective, but as one seen from a variety of kinds of perspectives and which are brought together. That is, as a 'fragment of knowledge', but from a multivariate point of view.

CHAPTER 2

THEME: AGRICULTURE

2.1. INTRODUCTION

As stated previously, this is a thesis about *knowledge*. It starts from a concern about schooling, framed in a Science Project in Brazil trying to adapt a science curriculum to the reality of a community of peasants in the North East Region; and concludes by discussing knowledge and schooling in the light of themes related to *Agriculture*. The reader will discover that out of the many different things which could be treated in relation to such a theme, one has been selected for intensive study: *cubação*. In this sense, the main empirical material for the thesis turns out to be common knowledge about measuring land.

The reasons for choosing this particular topic are many. An important one relates to the way in which *Agriculture* itself is treated in this research. Following the thematic approach adopted by the ECPC-Project (see Appendix 1.A), *land* (through an analysis of the cultivation of *soil* for growing crops) is taken as the starting point for investigating knowledge related to production and, as the study proceeds, *the knowledge of cubação* is shown to be relevant knowledge for description as *communal knowledge*. The community which constitutes the empirical scenario of the narrative about *Agriculture* is São Paulo do Potengi (SPP), a small rural district of approximately 14000 inhabitants in the State of Rio Grande do Norte.

There are three kinds of sources supporting the present account.¹ *First*, there are reports from the Project which provide the main issues to be addressed. The *second* kind of source informing the present description refers to some economic and sociological studies in Agriculture in Brazil. From these studies a more secure theoretical background is sought which helps to give shape to the formal account of issues suggested by the Project, in the way I see them.

In addition I make use of a *third* kind of source which can be regarded as "information from farm-workers" and which comes from my own empirical study

¹ A list of references is given in Appendix 2.A.

with adults in the community. In this sense, part of what is described in this chapter represents one first layer of *results* from this study. The detailed description of how this information was elicited from people and its relation to the main body of empirical data is presented in chapters 3 and 4.

The chapter is in four parts. The first part tells how *production* is organized in S. Paulo do Potengi (SPP). In it, following some geographical background information, I describe very briefly the nature of production, the productive cycles of the main crops, and the ways in which both production and work are organized. The second part discusses the *appropriation of the land*, in which historical peculiarities of the occupation of the territory, of the structure of land holding and of forms of land tenure in the Region, are addressed. The third part discusses the relation between technology and work, in which the value that land acquires in the course of its appropriation is emphasised, and some implications for discussing science from the community point of view are raised. Finally, I summarize the main points addressed in the chapter and indicate their relevance to the characterization of communal knowledge as developed in the thesis.

2.2 THE ORGANIZATION OF PRODUCTION IN S. PAULO DO POTENGI

2.2.1 Geographical background.

S. Paulo do Potengi is a '*município*' of about 420 km², located in the geographical Region of the NE known as '*Agreste Potiguar*'² (Fig. 2.1). *Agreste* is a region of transition between two others: the '*Zona da Mata*' (with a warm climate and two well defined seasons, one wet and other dry) and the '*Sertão*' (which is also warm, but dry, and is exposed to severe periodic droughts). It is characteristic of the *Agreste* to present a diversity of landscapes in a very short distance and to function very like a complex mixture of the other two regions, with the alternation of wet ('*brejos*') and dry ('*caatinga*') areas.

A visitor approaching the Region will notice that there are several hills interposed with level land. The landscape is arid and flat (mainly in the hot season), but isolated characteristic trees (such as '*algarobeiras*' and '*juazeiros*') in the fields break the view. The ground is uneven with narrow and medium layers of soil, whose

² '*Potiguar*' is an adjective which qualifies entities "from the State of Rio Grande do Norte".

composition provides farmers with four basic kinds of substratum for planting: one extremely thick, with bad drainage, saturated with sodium, easily affected by erosion, soaking-wet during the winter and drying up during the summer; a second, mineral based, presenting natural fertilization and good drainage; a third, relatively thick, with minor problems of drainage, susceptible to erosion, and not too fertile; and a porous, well drained layer, erosion resistant, but with a low degree of natural fertility.

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Figure 2.1

SPP is a land of 'submerged rivers' whose water is extremely 'salted' (due to a characteristic chemical composition of the soil and subsoil), and so unsuitable for the basic needs of living organisms (men, animals and plants). Life in the community depends strongly on the rain which falls from February to July³. During this period, all the possible ways of using and storing water *at the superficial*

³ The climate in SPP is usually characterized by two distinct 'seasons': one wet (the winter) and another dry (the summer). The annual average rainfall is about 530 mm (21 in.) and most of the rain (~ 90%) falls during the winter. The period from October to January is comparatively dry and castigating. Temperatures stay usually around 26 °C, raising up to 35 °C during hot summers and falling down to 18 °C during the winter.

level must be made effective. These include, for example, capturing water from the roofs of houses and large buildings; storing it in impermeable receptacles (home vessels, purpose-built reservoirs or natural stone slots); and organizing seasonally the tasks of cultivating crops. Deep ground water is used only in extreme cases such as periods of intense drought, and potable water needs frequently to be transported from other regions of the State to the town (which is done by water-trucks). Geological studies developed in the region by SUDENE⁴ indicated that hard water is an irreversible reality for SPP, and this fact has become part of the discourse of people in the community (it is frequently and routinely referred to).

'Rivers' (in the ordinary sense that the word is used in SPP) are classified as permanent or temporary depending on how deep, from the surface, water can be found during the summer season (permanent rivers preserve a cursory stream near to the surface, some of them staying apparent only in parts; temporary rivers staying down in the subsoil). Usually, the banks of rivers (alluvial streams) and lakes are transformed into transitory planting fields (the roots' access to water and the accumulation of sediment laid down by the waters during the winter provide good conditions for growing crops).

It is the Potengi river which gives the most characteristic note to the landscape. It is considered to be the main perennial river crossing the *município*, and in 1985 a dam was constructed as part of a governmental plan for developing agriculture in the Region. The dam stands near to the main town, and is intended to benefit, mainly, the population living along the river sides. In addition to the Potengi, there is the perennial Camaragibe river and other sporadic rivers, some of them of great importance such as the 'Riacho Salgado' and the 'Riacho Pedra Branca'.

In 1983, 35% of the population of SPP was estimated to live in the main town and the rest in small villages, usually located in farms or small farms (Figure 2.2). The town itself is the collecting centre of the basic produce from the villages, and is connected to Natal (the capital of RN, 80 km away) and to other cities, from where manufactured goods are brought⁵.

⁴ SUDENE: *Superintendência de Desenvolvimento do Nordeste*.

⁵ The exchange of products for subsistence and goods takes place in a Sunday open market in the proper town. Most cotton is bought while in the farms by intermediary traders who sell it for treatment in distant localities.

There is no railway system in the town (some neighbouring *municípios* have), but good motor-roads exist for intra-regional communication. Contact between villages and the town are not always satisfactory, and during the rainy periods it can become difficult to get around some distant places by car.

2.2.2 Agricultural production.

Production in the area is basically agricultural. More than two thirds of the households are engaged in cultivating crops and nearly ten months are intensively spent in work on the land. Cattle are raised on big farms, by landowners who usually do not live in the 'município'.

Agriculture is the unique activity of small land holdings⁶. Cotton has been the main crop selected for cultivation for two reasons: its economic relevance and its suitability to the climate and environmental conditions of the Region. It is also largely practised in big farms by peasants with no land, who establish agreements of land tenure (the actual forms that land tenure can take are described in section 2.3.3).

In addition to cotton, the majority of small producers grow crops for subsistence: manioc (a root from which flour is produced), beans, corn, broad beans and sweet potatoes. They call them 'lavouras' (particularly in the case of manioc) or 'roças'. Some features of these crops are displayed in Table 2.1.

Crop	Main Destiny	Cycle	Harvest
cotton	market	annual	one
corn (sweet)	subsistence	annual	one
corn (dried)	market	annual	one
beans	market/subsistence	annual	one
manioc	market/subsistence	>biannual	>two

Table 2.1

⁶ Their properties represent 80.1 percent of the total of properties in SPP (taken those with 1 to 20 hectares in Table 2.3, page 39).

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The productive cycle for annual crops begins in December/January (Figure 2.3), when the ground starts to be prepared for the seeds, or '*manivas*' (small pieces of the manloc's stem).

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Figure 2.3

Preparation consists of breaking, refining, marking the planting places on the ground, and in rare cases manuring or levelling the soil; and then of 'waiting for the first rain' (the substitute for irrigation). All the work is done by human labour with the help of oxen, tractors being used mainly only by big farmers. Seeds are sown directly in pits dug in the field immediately after the rain (there is not a nursery/transplantation system for young shoots), and in a number greater than the actual number of plants which will stay in the pits.⁷ The extra stalks are picked as soon as the bunches become 10-20 cm high, and this initiates a period of clearing the ground of weeds (which is a task to be maintained nearly until the harvesting time, and for which children are brought in to help). Harvesting starts in May/June with sweet corn (and some green beans) and finishes in November/December with cotton (an activity in which children and women participate).

Three aspects of the organization of the productive work in agriculture deserve attention.

(1) The first concerns the kinds of *crop-system* around which the productive work is organized: either a farmer plants *cotton*, corn and beans; or he plants *manioc*, corn and beans (other crops are complementary and used mainly for subsistence). Contrasting with cotton, corn and beans, manioc is not an annual crop, some varieties having a life cycle of four or more years⁸. Also, manioc can be harvested on more than one occasion in a year. There are appropriate times for garnering good roots for more substantial flour production (usually 2 to 4 in a year), but to pick up sporadic roots is harmless. In this sense, manioc is considered to be a kind of 'cash crop' as compared to the others.

Thus, to plant cotton or manioc regulates for farm-workers the actual limits in which both their time and harvest can be more 'freely' used, and in this sense they represent two distinct alternatives within the system of production. Cotton is economically more valuable, but given the present stage of agricultural development in the Region (still highly influenced by natural events), manioc appears as a supporting alternative. Table 2.2 shows the productivity of the main crops from 1970 until 1984, in which the effect of the 1983 drought is evident.

⁷ When seeds are planted and do not germinate due to insufficient rain, a second or third attempt is made.

⁸ A variety of cotton with similar characteristic is cultivated in some other Regions of the North East, but its production in SPP is insignificant.

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(2) The second point concerns the way in which *tracts* of land are organized for cultivation within the farm. Basically, there are two conditions to consider. One is the nature of the available *soil*, in relation to which certain possibilities of cultivation are defined, such as those expressed in the following statements currently made by farm-workers.

- (a) Manioc is *only* planted in '*arisco*' (a sandy soil).
- (b) Sweet potatoes are *always* planted in '*arisco*', but *can* be planted in the *bank of rivers*.
- (c) Cotton is *always* planted in '*barro*' (a clay soil) or '*várzea*' (a rich clay soil), but *can* be planted in other kinds of soil.
- (d) Beans are *always* planted in '*barro*' or '*arisco*', but *can* be planted in the bank of rivers.
- (e) Corn *can* be planted in any kind of soil.

Year	Cotton	Corn	Beans	Manioc	Sweet Potatoes	Broad Beans
1970	210	142	172	3761	4000	---
1971	150	900	720	6000	----	---
1972	150	660	600	8000	6000	---
1973	375	480	498	8000	----	---
1974	270	479	349	6000	5000	481
1975	310	262	225	5806	----	---
1976	269	480	459	6000	5000	300
1977	270	480	408	6000	5000	300
1978	270	360	299	6000	5000	300
1979	189	239	215	6000	5000	301
1980	330	388	234	7110	----	---
1981	212	080	148	10212	----	---
1982	192	139	240	10000	5000	---
1983	100	180	120	8500	----	---
1984	400	400	600	8000	6000	600

Table 2.2: PRODUCTIVITY OF THE MAIN CROPS IN SPP (kg/ha)
1970-1984 (IBGE)

In other words, as far as decisions about how to apportion tracts for planting need to be made, the kind of soil available constitutes one relevant factor for setting profitable possibilities.

Another relevant factor is the state of the tract defined in terms of its use in the *configuration of the farm*. It is a practice, particularly in big farms, to apportion tracts and to designate them with different functions due to their characteristic conditions. There are four kinds of tract: the 'virgin tract', the 'field', the 'new tract' ('terreno novo') and the 'resting tract'.

A '*virgin tract*' is a piece of land covered with vigorous natural vegetation, usually trees and shrubs. If it was used for cultivation in the past, this trace is not clearly identified. A special designation exists for a tract covered with small shrubs ('*caatinga*') and this is said to have been already cultivated, given its undeveloped kind of flora.

A '*field*' is a delimited piece of land covered by crops in a square-grid system. In it, a certain crop (or a combination of crops) is grown for four or five years and then replaced by a different one (a practice traditionally called 'rotation of crops'). The 'field' is the productive/profitable tract.

The transformation of a virgin tract into a 'field' designates the condition of a '*new tract*'. For the case of temporary crops, this transformation embraces two consecutive phases⁹, each taking two years to be completed: the clearing of the superficial vegetation, and the preparation of the ground. In the first, trees are cut and shrubs hoed. Corn and beans are planted in the open gaps, on a irregular spacing-basis (it is not possible to manage with the oxen-pulled '*capinadeira*' in such a tract). In the second phase, the remaining trunks and roots are dug out; and then burned. The ground is then ploughed, raked and pricked out. It is possible for the farm-worker (particularly in the fourth year) to plant the tract on the basis of a 'field', but this will not yet receive the main profitable crop.

A '*resting tract*' is an old 'field' whose productivity has diminished in consequence of a production which does not restore its initial fertility. It is left uncultivated for some years, when the cattle is allowed to graze in the tract. It can develop towards a '*caatinga*'.

Thus, there is variety in the way tracts are used as exploitable devices in the functioning of the farm. As such, they resemble components of the farm's formation and sustenance. Virgin tracts are transformed into 'fields'; 'fields' rotate crops; crops reduce fertility; fertility needs to be restored. For the farmer, the outcome is the profitable production of crops to go to market. For the farm-worker who establishes a contract of land tenure for cultivating a 'field' or a 'new tract', the outcome is the subsistence of his family and an eternal and constant 'rotation': from a 'new tract' to 'another', or from one 'field' to 'another'. Because farm-

⁹ In the case of more permanent crops the formation of a field would include the formation of bunches, up to their productive stage.

workers tend to oppose this system of rotation, they are usually judged as naïvely misunderstanding the scientific relevance of crop rotation.

(3) Finally, the third aspect refers to the ways in which the lay-out of the '*field*' is settled. Seeds of a given crop are sown at the crossing points of two perpendicular sets of parallel lines (1 '*braça*' = 2.2 m apart) traced in the ground by an ox-pulling '*capinadeira*'; and internal pits are pricked out between crossing points (Figure 2.4). The same crop or different ones can be sown in internal pits, but certain combinations are preferred by farmers and farm-workers (beans and corn are one of the most frequent associations). A third set of pits can be inserted between internal pits, to receive one of the already chosen crops (usually, a third crop is not planted in the same tract¹⁰), such as the one shown in Figure 2.5.

When two crops are planted in the same '*field*' (understood as a tract with a square-grid), the same tract of land is referred to in terms of two fields, which are then qualified in terms of their crops. Figures 2.5 and 2.6 show a tract with a *field of corn* and a *field of beans*. When bunches of one crop alternate with those of a different crop in parallel lines, the tract has two fields arranged in *rows* (Figure 2.6). Equal tracts in area can happen to embrace fields of different '*areas*'. Figure 2.7 shows a tract equal in area to the tracts of Figures 2.5 and 2.6, but whose field of beans is bigger.¹¹

Single fields with pits a half '*braça*' apart are recognized to be more profitable, but only cotton and manioc have such a privilege (eventually beans). Corn is usually interposed with the previous ones, and beans are never planted with cotton or manioc. It is a common practice for a farm-worker to have two different tracts (one for the main crop for the market, and other for subsistence), but he would say that this increases the amount of work required in all the phases of cultivation.¹² Because farm-workers plant crops for subsistence in association with the main crop for the market, it is said that their agriculture is based on a *variety of crops*.

¹⁰ Except the broad beans which are always planted later in the same hollows as corn, whose stem (when '*dead*') serves to support the broad beans' plant.

¹¹ Thus, as far as the measurement of *areas* in SPP is concerned, there is a distinction between *tracts* and *fields* that must be considered.

¹² Amount of work is usually indicated in terms of the size of a '*field*' that a man can cultivate on his own. In SPP, farm-workers estimate that a man works a '*field*' of 6 '*mil covas*' (~ 3 hectares) if with a '*capinadeira*' or 25 '*mil covas*' if with a tractor. An old man would work a '*new tract*' of 3 '*mil covas*'.

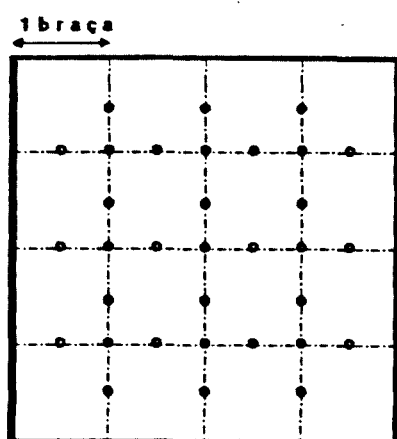


Figure 2.4

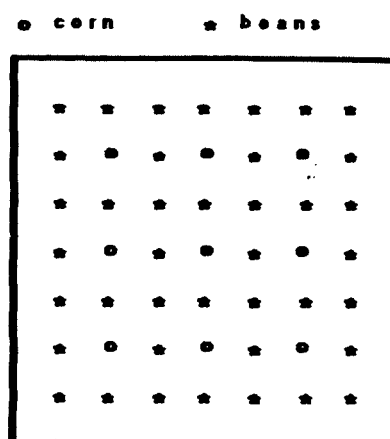


Figure 2.5

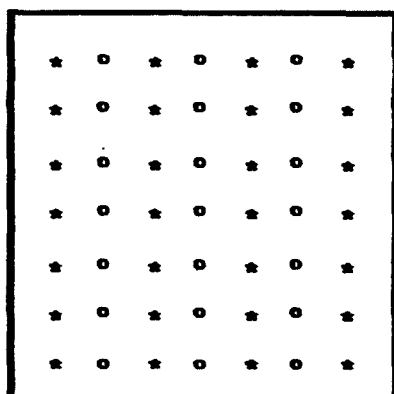


Figure 2.6

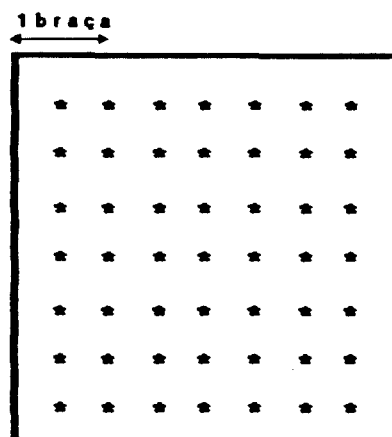


Figure 2.7

2.3 THE APPROPRIATION OF THE LAND

If we look at the distribution of landed property in S. Paulo do Potengi today, we will see, at one extreme, a small number of large land owners; and at the other, we will find a large number of small land holdings. From Table 2.3 it is possible to see that 20808 ha (57.4% of the total area) is appropriated by 36 (3.0%) owners while 5157 ha (13.8%) is shared by 928 (81.1%) owners.

A sociologist or an agrarian economist would easily, and not surprisingly, recognize this high concentration of the land as characteristic of two aspects of the history of the area: its colonial economic origin and the development in Brazil of a capitalist mode of production.

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Groups of areas (ha)	Number of properties	% of the total prop.	Area (ha)	% of total area
<1 to <5	538	46.5	1241	3.4
5 to <10	238	20.5	1725	4.7
10 to <20	152	14.1	2091	5.7
20 to <50	122	10.5	3757	10.3
50 to <100	45	3.8	3048	8.4
100 to <200	25	2.1	3471	9.6
200 to <500	21	1.8	6592	18.2
500 to <1000	10	0.8	6814	18.8
>1000	5	0.4	7402	20.4
Total	1156	100.0	36145	100.0

Table 2.3: LAND DISTRIBUTION: S. Paulo do Potengi, 1980 (IBGE)

In so far as a colonial origin is concerned, it is common to regard the economy as a result of the articulation of three sectors: the external market, the internal market, and the sector of subsistence. The external market, because most of what is produced in the colonial areas is sent overseas for the central economy; the internal market, as the life of some segments of the colonial society depends on the commercial transactions of importation and exportation; and the sector of subsistence, as it is necessary to produce food and goods to keep alive the population which is responsible for the production of the basic products to be sent to the international market.

It is characteristic of the colonial economy that, being a dependent-economy, its possibility of expansion is regulated by the development and growth of industries established in those countries which produce goods from raw material. In this sense, it is possible to identify in the Brazilian economy, certain kinds of product which have, in a given period, experienced growth, strengthening and falling; and which have constituted, in that period, the main ordering factor for structuring production towards both market and subsistence.

The idea of an 'economic cycle' is an analytic tool which is especially relevant to this colonial approach to the economy, and is generally used by historians to describe those aspects of economic development in which links exist between the production and commercialization of a given product, and the international market. From this perspective, the succession wood/sugar/ gold/coffee is said to express

the main cycles of the Brazilian economy up to the first half of the XX Century. In the context of its formation, the NE region has been usually a peripheral component, but which participates in the whole economy, in a specific series of cycles. In the establishment of possibilities of actual economic events, geographical and physical conditions have played a fundamental role. In the NE region, droughts are the best known events of this kind. As far as the appropriation of land is concerned, economic cycles are a good tool to describe the settlement of a given region, not only in the sense of a geographical location of inhabitants in a given place and time, but as a historical process which relates to external events, and particularly to the market.

At this point, it is necessary to recall that this historical process has taken place within the context of a developing capitalist mode of production. Historically related to international capitalism, the dependent/peripheral Brazilian development has also been characterized by the creation of non-capitalist forms of relations of production. Namely, in the case of agriculture, there are cases of 'backwardness' (in terms of both techniques and of 'archaic' relations of work); or cases in which work is not directly subordinated to capital (expressed by the incomplete spread of wage forms of payment in the rural areas: payment in harvest, goods or living place is still a custom).

As a result of the implementation of new social relations and technologies, which arise as imposed by the accelerated ingress of capitalism into Brazilian agriculture, substantial changes towards modernization have taken place in the rural world in the last five decades. This process, which has been complex and non-linear, has introduced a multiplicity of relations of work, as well as new forms of land tenure. In it, the reproduction of the old forms of social relations are still allowed, but now under the influence of the more general 'logic' of capital.

One possibility for interpreting this diversity, is to conceive of the Brazilian 'campesinato' (peasant mode of production) as constituted by 'pequenos produtores' (small producers), defined as such not only by the juridical expression of land tenure, but also by social/cultural practices and by representations which are proper to this social group.

Corresponding to this diversity, a complex net of forms of land tenure can be described, which do not necessarily refer to cases of legal ownership. For the majority of categories of 'pequenos produtores', family-based production is still

the common mode of organization. In this sense, it is possible to think of these people as peasants working for their families' subsistence. But this description would be incomplete. A distinction between the commercial entrepreneur and the non entrepreneur should also be made; in which the role of the latter in the production of food, should be addressed.

Emerging as a 're-creation' of non-capitalist forms of production, the presence of these categories of producers generates contradictions and not solutions to the problems of capitalism, and so should not be interpreted as merely functional to commercial and/or industrial capital. In the case of SPP, conflicts have their basis in the antagonism between the patterns of reproduction of the lives of the small producers, and the operation of the market economy. It is in this context of contention, that the forms of appropriation of the land should be understood.

Therefore, despite having strayed too far beyond my immediate aims in discussing such complex issues, they are worth mentioning, because they indicate relevant limits and boundaries for any attempt to describe how ownership has been established in SPP, in relation to the context of the Brazilian capitalist mode of production.

Summarizing, in the analysis of the appropriation of land in SPP, there are two important aspects which deserve attention: one refers to the occupation of its territory; and the other to the structure of land holding. The above perspective suggests that an appropriate analytical posture would require one not to look at the formation in Table 2.3 as given, but as it has emerged within a concurrent process of development in which broad events (external to the State), allied to specific events and to particular physical and geographical conditions, have promoted its configuration. It also suggests that one should look at the 'pequenos produtores', who may be defined as those owning tract(s) of less than 10 hectares (first two entities in Table 2.3). In which case, the concepts of subordination and specificity are basic to any study of this social class and of its position in the general society in which it finds itself.

The rest of section 3.3 is concerned with the second of the aspects mentioned above, for which results of the empirical study were available (namely, the structure of land holding, in the context of which forms of land tenure in SPP can be defined). An account of the occupation of the territory is, however, given in Appendix 2.B.

2.3.1 The structure of land holding.

Land, in the initial stages of the occupation of the territory, was divided into great tracts, which were concentrated in the hands of few large families. Small tracts were held by 'vaqueiros', 'moradores', and by small families (see Appendix 2.B for the origin these terms). This concentration of ownership in a few hands became a source of social power, constituting a basis for what is generally known in Brazil as a 'rural oligarchy'.

The control that big farmers have exercised over peasants in relation to the act of voting in public elections, over the definition of places and conditions for living and over the possibilities for getting jobs, relates to the social formation of these 'oligarchies'. Also this concentration has played an economic function as the owner could make use of the land to raise money: by making land productive or not.

As the capitalist mode of production became established in rural areas, two correlated processes have taken place, which deserve our attention. The first concerns how land has been partitioned in SPP. As a consequence of inheritance¹³ and by reasons of a characteristic division of the tracts, the fragmentation of properties has recently reached a point which renders impossible any profitable exploitation of the soil: either because the area of the parcels is too small, or because the shape of the parcels makes them unsuitable for agricultural purposes. Nowadays, the majority of the small parcels are too long and very narrow, composing a configuration of numerous parallel strips; so narrow that a car would not be able to run within one single tract. Sometimes one can find strips of ground, the length of which extends to a kilometre or more, while their width does not exceed a few metres. Take as an example the fields along the Potengi River. The map of this region (Figure 2.8) resembles a linear mosaic.

Arguments can be raised about the reasons for dividing the tracts in such a characteristic way. One possibility can be addressed in terms of a general preference among inheritors for a tract facing a river or a road; which is an argument that applies well when this is the case, but which does not explain the same practice being used -as it is- far from rivers and roads.

¹³ In Brazil, by force of law, all children share equivalent parts of the inheritance.

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PHOTO REDACTED DUE TO THIRD PARTY RIGHTS OR OTHER LEGAL ISSUES



Another possibility -which does not exclude the previous one- is to think of it as a practice from ancient times in which farmers were accustomed to measuring two sides of the tract with a rope or a 'braça', and afterwards to divide the tract into 'mil covas' or 'braças', corresponding to the shares of the owners.

In other words, to divide the tract by this method would correspond to the most simple way of calculating the area which should be apportioned to each owner, accordingly to their rights in the inheritance. In the cadastre-register of SPP it is still possible to find descriptions of tracts made in terms of a 'front-number' of 'braças' (frontage), whose depth is given in terms of integer numbers (usually one) or half parts of a 'légua' (in English, 'league', which is an old measure of road-distance, usually about 3 miles); when, by the rules of a surveying-description, at least, all the boundaries of the tract should be described; their location identified; and the area of the tract indicated. Whatever the reason, the consequence is that small farmers have emigrated to the town itself, constituting today a large group of wage earners who have little or no land for their own use. Without land, they are obliged to offer their labour-force as the only good they own and whose negotiation is an essential condition for living.

The second process concerns the way in which land has been used. Some of the original big landowners still manage their 'fazendas' in the old 'oligarchic-style' and much of their land is left uncultivated. The land can be used as collateral to raise money, so that the owners of land can afford to devote only a small proportion of it to production. On these 'fazendas' it is still possible to see work patterns reminiscent of slavery and of a plantation ('latifúndio') agriculture. For substantial periods of time, peasants work on the landowner's main crop. In return, they can receive some wages; but they can also be allocated small tracts of land which they can cultivate for home consumption, for sale and/or for sharing with the owner. This has given people the opportunity to change towards or to create other forms of land tenure. Thought of as totally undeveloped, this proportion of the land has constituted actually, not a great under-used space, but a real means of subsistence of a large group of peasants.

Thus, interposed within the big 'fazendas' one will find 'arrendamentos' (renting tracts), 'moradias' (living-in places) and 'posses'. This last form -the 'posse'- is a claim on the ownership of land by right of having occupied and used it for a given period of time and now is established in law.

But not all 'fazendas' have remained traditional. Many have moved (or are in a process of moving) over to a commercial system of agriculture. Again, the land itself may be used as collateral to raise money. But in these commercial 'fazendas', that money is used for mechanization and for development. The main importance of this land is that it can be used to make a profit. Characteristic of this type of 'fazenda' is the fact that a high proportion of the land is worked, and crops tend to become specialized monocultures. In S. Paulo do Potengi, cotton has been -potentially- the standard monoculture; while in some neighbouring 'municípios', sugar cane extends for mile after mile. Also, cattle are raised which require basically the production of large areas of grass. So, these modern 'fazendas' are a great contrast to the traditional ones. They convert the land into a kind of factory-farming, where monoculture replaces crop variety, and farm-workers are factory-workers.

It is exactly in this movement from the traditional 'fazendas' to the commercial ones that the different forms of land use come into competition, and conflicts can result. They pose questions about whose land it is, how people can claim ownership and how their land should be used. These questions have been central to rural development in Brazil, particularly when different perspectives of relating to the land come in direct conflict at the personal level.

2.3.2 Forms of land tenure in SPP.

In such a complex context, land ownership can take a diversity of forms. Basically, there are four elements that distinguish them: (a) the mode of payment, which is characterized by the kind and amount of payment; (b) the period of holding, which can be limited and pre-established (defined); or limited but variable; or definitive; or temporary; (c) the extension of the tract, which are distinguished as 'minifúndio' (small extensions) or 'latifúndio' (large extensions); and (d) the origin of the ownership, in which case inheritance, commercial transactions, 'posse', rent, or personal agreements constitute the possibilities.

Among them, it is possible to recognize three types of 'owners': the 'legal owner', one who has rights over the tract by force of law; the 'owner by using', who has no legal rights but who acquires ownership by reasons of making the tract productive; and the 'not owner', who has no rights at all, despite spending labour-force in the production.

The network in Figure 2.9 summarizes these elements and types of ownership, some of them being divided into more delicate instances (see Bliss, Monk & Ogborn, 1983, for the network notations, terminology and concepts).

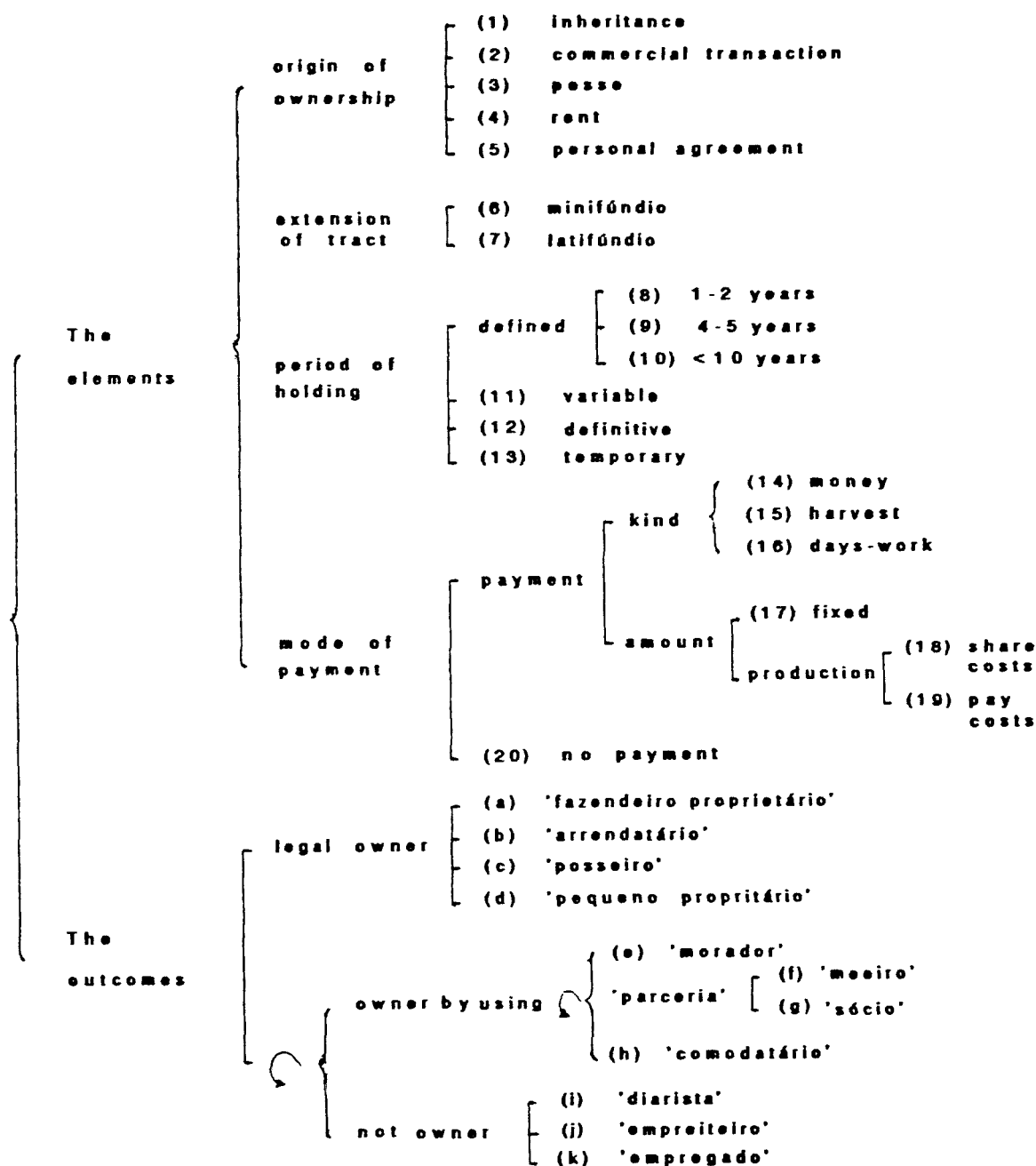


Figure 2.9: TERMS FOR DESCRIBING POSSIBILITIES OF FORMS OF OWNERSHIP

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Outcome	Origin	Extension	Period	Mode
(a)	(1)	(7)	(12)	(20)
(a)	(2)	(7)	(12)	(14)+(17)
(b)	(4)	(6)	(9)	(14)+(17)
(b)	(4)	(6)	(9)	(15)+(17)
(c)	(3)	(6)	(10)	(20)
(d)	(1)	(6)	(12)	(20)
(d)	(2)	(6)	(12)	(14)+(17)
(d)	(3)	(6)	(12)	(20)
(e)	(5)	(6)	(11)	(20)
(e)	(5)	(6)	(11)	(15)+(19)
(e)	(5)	(6)	(11)	(17)+(19)
(f)	(5)	(6)	(11)	(15)+(19)
(g)	(5)	(6)	(11)	(15)+(18)
(h)	(5)	(6)	(8)	(20)
(i)	(5)	(6)	(13)	(16)+(17)
(j)	(5)	(6)	(13)	(20)
(k)	(5)	(6)	(11)	(16)+(17)

Table 2.4: ACTUAL FORMS OF LAND TENURE

By taking paths in this network, forms of land tenure can be described, the most common in SPP being those represented in Table 2.4. As some variations do happen in practice, different paths for the same 'outcome' are represented. For example, taking the first two forms in the Table, we have that 'fazendeiro' possesses the tract by force of the law. He can become owner via inheritance, when no money is required (first path); or he can pay a fixed amount of money for it in a commercial transaction (second path). His tract is of a large extension and he keeps it -in principle- for the rest of his life. In the same way, all the other forms can be read.

In the way they are described, these forms of ownership only represent single "self-consistent" possibilities, recognized as categories of tenure. What this network -and consequently this Table- does not do is to represent the social contradictions which actually arise within the system which uses these categories. Namely, cases in which a given person can be regarded either, (a) as not strictly well-defined according to the characteristics of a category; or (b) at the same time, as two things which give him a conflicting position or state. An example of the former situation can be easily recognized in relation to the 'comodatário' who appears as a 'free' producer, but who actually constitutes an extremely exploited category in regard to the consumption of labour-force: the hardship involved in preparing a 'terreno novo' ('new tract', completely virgin) is too costly, probably much more than in cases which require explicit payments for the land (like 'arrendatário' or 'meeiro'). One farm-worker explained:

"There are some owners who give native land to the farm-worker, for 2 or 3 years. So, this is the peasant who works others' land, without payment. But this free-payment is a favour that nobody should desire. Because it is necessary to put down the native wood... [...] Hard wood. And this is very expensive and the owner will receive the tract back... clear. [...] Without wood ('destocado'). They (the owners) always draw up the transaction: - In the end, in the last year, you give me the tract back, 'destocado'."

Another example can be referred in relation to the 'arrendatário'. As he has a legal agreement for holding a tract for four or five years, he seems to have the right of keeping it for use for his own purposes. However, it is a recognized procedure -at the end of any annual production- for the original and 'true' owner of the land to destroy the fence which delimits and protects the 'arrendatário's' crops, to let his cattle live upon the scraps of the crops.

In SPP, this usually happens between December and February, when the next productive cycle starts. Also, to plant immediately after the first rain-period (February/March) is a crucial condition for obtaining a good harvest. Problems

arise for the 'arrendatário' when both: (a) he has not yet completed the harvest because of late production, in December; or (b) the rainy period happens to start before he gets his tract back, in February. In this way, the farm-worker feels that he has the right, but it is taken away. As expressed by a farm-worker:

"The most important things one farm-worker should know about his work are how to do the work, when to do it, and when it is necessary to do it. Lets say: a person has a large tract but, when it rains... what he must do at first is: if it rains in the afternoon, he should plant in the next morning. The rain comes in the afternoon, he plants in the morning: this is something which is well done. What he should not do is to wait for 3, 4 or 5 days... Once... Sometimes people wait for more than one month... There was a time when I happened to work in Mr. X's farm... That was one year plenty of cotton and we worked until late. And so the cattle stayed in the field and we were there, waiting. One month passed... It is only when the cattle is grazed that they will drive it away. This is wrong. Because the farm-worker rented the tract and so he has the right. Well, but things happen as I am telling you, indeed. In my view, it should be like that: it rains today, I will plant tomorrow; and everything is all right. Now, we should not wait all this time to be able to plant. Because what happens is that the winter is sure to come and if it rains... In some situations, when the person plants, the crops will not grow any more, you see? But if he plants as soon as the rain... Here, if we have two months of winter... even if the land is weak, having two months of rain we will harvest, do you know? I have never seen one year... only in 83 I saw nothing; but in all the others..."

Another right that the 'arrendatário' sees as suppressed refers to the acquisition -from the owner- of a letter of permission which allows him to get agricultural mortgages from the bank. One farm-worker said:

"[...] Now, there are some people -avaricious people- who do not want to give...they do not provide a letter of permission... But they always do it, you see? After all there are many sharpsters. And so it is the individual who is...he does not do his best to please the owner. Easily they give the letter. And then, people can work well, can't they?"

Examples of the latter situation -namely, when a person would be seen in a conflicting position- can also be pointed out: there is the case of a farm-worker who can in fact be both a 'morador' and a 'meeiro'. He is a 'morador' by reasons of holding a tract within the 'fazendeiro's' farm; a fact that necessarily implies some kind of subordination to the 'fazendeiro', since the farm-worker will receive a house and a tract for his subsistence (for example, he will have to give days-of-work to the owner, to buy goods in his shop and to sell him his production). A possible form of subordination is exactly one which characterizes the 'meeiro'. 'Meeiro' is a kind of 'parceria' which acquires this particular denotation 'mela' (half), in order to connote the equal parts in which the harvest must be divided with the owner, as a condition for holding the tract. In this form,

costs of planting, growing and cropping are not shared, the onus for these expenses remaining on the farm-worker. As such, 'meeiro' is understood -and must be treated- as independent of 'morador'. This 'overlap' happens because for the farm-worker to be a 'morador' implies both that he is tied to the owner and that he has a tract of his own and works for himself. As expressed by a farm-worker:

"To work as a 'morador' is to work in a 'fazenda'; to work for the farm-worker himself and to work for the 'fazendeiro'."

The same happens in relation to 'empregado' and 'morador'. In the transition towards factory-farming, 'morador' should give place to 'empregado', who, in the new system, is expected to become a factory-worker; and so without any possibility of holding a tract of his own. What is actually happening is that the legal formalities concerning the characterization of 'empregado' (such as contractual agreements and fixed salary) have been introduced, but still admitting the existence -in relation to the same person- of the old forms of 'morador'; that is, some 'concessions' (from both sides) which are prohibited in law, still exist.

In summary, the best that the network in Figure 2.9 can do is (a) to set out the formal distinctions one can establish between categories; and (b) to allow to a person to be seen as belonging to more than one category.

A different network would be needed which sets out to express contradictions regarding land tenure. Certainly, this network would not address categories of tenure but categories of contradictions which are to be understood as social facts. But to try to frame in a network the intricate picture of social relations that arises in the Brazilian rural world is, at least, a hard task. Passing by this task there is a complex and controversial question concerning the interpretation of the diversity of ways in which the agricultural production is organized within the country and made specific in each particular region. On the other hand, the variety of situations in which actual categories of relationship or of contradiction can be seen as undone, reverted, replaced or exchanged, create a texture of such an interconnected set of relationships -antagonistic at the level of analysis but mutually reinforcing as experienced- that any attempt to describe them at a superficial level, in terms of experience, fails. Elements of contradictions become disguised as natural features which make up the conditions of work. They can be recognized by individuals as wrong, but they do not raise problems which challenge new postures. The farm-workers' accounts introduced above give an indication of this. In this sense, 'reality' presents itself as obvious and unproblematic, and so,

difficult to be grasped in its contradictions. Also, there are political questions related to the way in which the Agrarian Reform has been proposed and implemented in recent times.

To tackle these questions is beyond the scope of the present analysis. What seems important here, is to mark out the social connotation that land acquires in the process of its appropriation, since it is in this way that different perspectives of cultivating the ground and practising *cubação* can be distinguished and discussed later in this thesis.

2.4 TECHNOLOGY AND WORK

2.4.1 The value of land.

It is currently known that the capitalist development of agriculture presents particularities in relation to the development of industry. One example is that in principle, the fundamental means of production of agriculture -namely, the land- is not susceptible to be multiplied of man's own free will, as happens to be the case for machines and tools of work. It is exactly because land constitutes a means of production relatively non-reproducible, that the historical appropriation of the land acquires especial significance. The agrarian structure becomes in this way, the background against which the productive process develops in the rural world.

In Brazil, what is peculiar about this process, is the fact that the capitalist development started at a time by which not all the land had been appropriated. There was a permanent frontier of movement, with free land and no owner. It is possible to say that since the early times of colonization until the middle of the XIX Century, land was of relatively free-access, for it existed in profusion as a natural resource. However, if we go back to the colonial times, this "relatively free-access" needs qualification: because of the characteristic development of productive labour, the ownership of slave labour and tools of work constituted a necessary condition for holding land. Thus, the large and wealthy blocks of appropriated land became the property of the 'grandes senhores' (big owners) and not of the few 'homens livres' (free citizens); namely, there was a direct relation between the number of slaves and the area of land occupied by each owner¹⁴.

¹⁴ This relation can be seen, also, transposed to the level of production for subsistence. During the Dutch invasion, for example, the owners were obliged to plant 1 'mil cova' (~0.3 ha) of manioc for each slave of his property not directly

But in the presence of free-land, there was always the possibility for citizens to set up upon their own; and this constituted one of the major problems faced by the 'grandes senhores' during the period preceding the slave liberation (which happened in 1888). By transforming themselves into owners, citizens would have their subsistence guaranteed, and high salaries would have to be paid by the 'grandes senhores' to compensate, in the citizens' eyes, this alternative of self-subsistence. It is in this sense that free-land constituted a threat to the existence of a cheap labour force, and so, its monopoly became imperative. Free land could exist while slaves constituted the labour force; but as soon as labour became formally free, land needed to become formally captive.

Mechanisms for making tenure difficult for citizens were required and, anticipating slave liberation, the 'Lei das terras' (Law of the land) was promulgated in 1850. This law enshrined the notion of private property as needed by capitalism, since it only allowed the tenure of land through mechanisms of commercial transactions; and created a necessary juridical system to compel free labour to sell labour force. But the 'Lei das terras' did not eliminate either the existence of free-land or the moving frontiers; it only regulated the way in which free-land became incorporated in the productive system.

As is known, within capitalism, the productivity of investment demands an intensification of production. If land is available, the incorporation of new land is one natural tendency and an extensive kind of development usually follows. But if land is not available, any possible expansion is conceived of through an industrialization of agriculture, in which case the limitations imposed by nature have reduced importance as barriers to production: men need be able to generate the necessary land, and they do so by using developed techniques (irrigation, machines, fertilizers, etc.) and by performing certain relations which are established between different agents of production. *Technology* and *work* can then be understood as mechanisms for creating 'new land' where it does not exist; and land can become a 'reproducible' means of production.

involved with planting, or 500 'covas' for each of those involved with the cultivation of profitable crops. 1 mil cova is a unit of area considered to be -in colonial times- the amount of sugar cane transformed into sugar in a mill, in 1 day.

An analysis of how this process took place (particularly from the end of the XIX Century until now), and in which land could reveal its real value, would be of great importance; but is beyond the purpose of this study. It is worth mentioning because there is one aspect in the relation between technology and work which can be of some help for this thesis. It refers to the *rationalization of work* which arises out of the actual forms in which the intensification of production takes place.

2.4.2 Implications for discussing science from the community point of view.

As was said earlier, the direction of agricultural changes points towards the implementation of a kind of factory-farming whose characteristics can be abstractly seen as typically capitalist, but whose development should be understood within the limits represented by the social formations which give it its shape. By analogy with contradictions which arise as social facts, *practices* such as the high degree of land partition; the use of a variety of crops; the avoidance of crop rotation; and the absence of systematic fertilization; are consequences of such a development which stand as contradictions from a scientific/technological perspective. In this respect, there are some points to consider.

(1) As far as small properties are concerned, particularly those along the Potengi river, plans of land re-allocation between families have been proposed by the government, in an attempt to transform the valley into a profitable one. Studies suggested that a typical-tract of 11.8 hectares (2.2 ha of irrigated land: alluvial; and 9.6 ha of non-irrigated: 'tabuleiro') would be adequate for a family with five members. In the alluvial part, the cultivation of cotton, beans, grass, rice, garlic and bananas would be given incentives; while in the 'tabuleiro' the recommendation would be for the combination corn/beans, 'algaroba' (a resistant tree which produces forage for cattle) and natural grass.

One aspect pointed out by the government report as striking, refers to the difficulties faced by researchers in establishing the actual area of the properties as they are at present. The area declared by the owners very rarely matched the estimated area of specialists who used, in their calculations, both an aerophotographic method and information from the Register Office. In some cases the difference is notable; sometimes the declared area exceed the estimated one but in others the opposite occurs. A list of examples is given in Appendix 2.C.

(2) As far as the modernization of agriculture is concerned, programmes of integrated development have been proposed by the central government for rural areas in which financial institutions (such as Banks) and institutions for communicating new technologies have taken an important function. In SPP, the Brazilian Bank and the EMATER¹⁵ have fulfilled an important role. Despite being distinct in administrative terms, in the community these institutions articulate efforts towards the development of production. To raise the productivity of investment is the common goal: while the Brazilian Bank manages supervisory credit, the EMATER advises farmers and farm-workers through educational programmes and activities of extension. The transmission and implementation of technology constitute the basis of their programmes: financial support is oriented towards specific systems of production and techniques; technical advice is given for developing and testing 'technological packages'.

(3) As far as the intensification of cotton production is concerned, there is an external/natural factor to consider. The occurrence of a pest known as "bicudo" (*Anthonomus grandis Boheman*) has reduced the cultivation of cotton since 1985, but its economic importance has not yet been completely dismissed. As is known, cotton farming in the presence of "bicudo" requires a large scale production and the use of advanced technologies, and can be extremely productive and economically worthwhile for big farmers. Fields of cotton are supposed to be laid out within an 'anticipating-cotton-belt' which functions as a kind of alarm against the "bicudo", and fields have to be cultivated successively in a slightly dislocated sequence of cycles. This certainly requires a less weather-dependent kind of production. It also requires the transformation of a large number of 'virgin tracts' into 'fields', since an intensification of investment demands all possible forms of increasing production.

Thus, it is possible to say that cotton production becomes evidently dependent on the ways that the productive system as a whole develops towards a 'modern' kind of agriculture. In 1986, it was a prevailing statement among farm-workers that the number of 'new tracts' being offered by the farmers for cultivation had increased. At the same time, farm-workers were advised by the EMATER's technicians to suspend cotton production for three years; and the Brazilian Bank was particularly

¹⁵ EMATER: "*Empresa de Assistência Técnica e Extensão Rural do Rio Grande do Norte*".

interested in giving credit for infrastructure improvement of farms. On the other hand, a significant contingent of farm-workers have moved to a parallel kind of activity which is already established in neighbouring 'municipios', that is, of mining.

It is necessary to make very clear, once more, that my intention in stressing the above points is not to raise a discussion about the perspectives of development for the Region. The point I am trying to make relates to the implications which follow from recognizing that underlying the kind of development proposed for the Region there is embedded a notion of *rationalization* which, from the point of view of the *practical world*, brings together science, social sciences and technology as co-referents. From this perspective, the discussion of *science* or *social sciences* is, essentially, a discussion through a *perception of technology*; which may be very simple technology. What is relevant in the argument is that it is a discussion through a problematization of artefacts, of effects. It is an awareness of consequences rather than an induction into a discourse. Questions such as those posed by the ECPC-Project are a good example:

- . "What does it mean to regard agriculture as the main economic occupation of people in the community; and to regard cotton as the basic economic product?"
- . "What kinds of change do the actual practices of transforming the soil imply? What are the consequences?"
- . "Why certain kinds of crops are planted in particular types of soil?"
- . "Why do people say that it is necessary to give up production of cotton for three years in order to exterminate the "bicudo"?"
- . "How can we know how much profit a small owner will lose in a year of drought?"
- . "Why should monoculture replace crop varieties?"

The Project tries both to distinguish and integrate science and social sciences, but the discussion is still bounded by technological issues. Which is right. Looked at in terms of primary schooling, the teachers come from the practical world and so do the pupils. "Relevance" is the main criterion from which problems and practices of the community are brought to schooling; and not "being a participant in a discourse", in the sense one would understand the joining of scientists in a discursive scientific community¹⁶.

¹⁶ I am not here referring to pedagogic discourse, which is something to be analyzed in terms of the educational system.

2.5 CONCLUSION

The description presented in this chapter has tried to set out an *understanding* of the theme *Agriculture* which is relevant in making it a *general case in knowledge*. Land was discussed in terms of the main distinctions which arise from a social/historical analysis of the Region/community which was chosen as scenario of the empirical study of the relation of communal knowledge to schooling (the empirical study is described in chapters 3, 4 and 7). Among the variety of forms that land can take, special attention was given to entities such as *farm*, *tract*, *'field'*, *'new tract'*, *territory*, *'município'*, *'posse'*, *'moradia'*, *'arrendamento'*, *ground*, *soil*.

Three of the problems which arise in characterizing the relations between commonsense knowledge and science are those of differentiation, development and contextuality. The use of the analysis offered in this chapter is intended to help with these problems, by providing an intelligible and reasonably well founded structure against which to judge the participants' accounts about similar issues (that is, farm-workers' and teachers' accounts); and to initiate a discussion of the specific knowledge of *'cubação'*, in relation to some large, difficult, but important questions concerning the social/historical relations of knowledge (chapters 5-8).

Such problems are particularly acute in the present case, in which a characterization of farm-workers' and teachers' understandings is intended to be made in relation to formalized/structured bodies of knowledge. It cannot be presupposed that farm-workers and primary teachers have very clearly articulated views about how and why things happen in agriculture, at the same time in which it should be wrong to assumed a-priori that they do not have any. As argued in chapter 1, the position I am taking says that commonsense explanations are not readily encapsulated in short statements to which one gives assent or refutation; rather, that they generally hang together in a structure of arguments and presuppositions.

Suppose, by analogy, that someone wishes to study the technological knowledge of factory-workers in a developed country *for the purpose of deriving/analysing problems of application to schooling*. What my argument stresses is that would be a mistake just to define some technological knowledge and set a questionnaire, particularly if the questions are strictly and directly set by reference to the logic of the scientific content. One should first of all try to discover the actual mode of

activity of working in the factory; that is, to look at what actually happens; at what people have to do and say while they work; at what people understand about what they are doing. Also, one should be looking at the systems of transmission and construction of knowledge before one focuses on something to investigate. Certainly, one could not eschew an analysis of industrial development at a more theoretical level (whether in a social, economic, or scientific sense); but following the "common-sense referred" alternative already mentioned in chapter 1, more abstract categories and concepts would have to be taken as suggesting hypothesis about reasoning structures, and not as a reference against which to evaluate commonsense.

Whether in relation to agriculture or industry, science, discussed from the point of view of everyday practices, is fundamentally a discussion through an understanding of technology, in which case no assumption is made about ordinary subjects being inducted into a scientific discourse.

But as the empirical investigation develops, a distinctive knowledge (cubação) concerning methods of measuring land in the Region of study arises, in relation to which the necessity of making assumptions about the role of farm-workers in a discursive community becomes imperative. Cubação is then taken as a discursive practice, and a parallel is established between the functioning community of expert farm-workers and the discursive scientific community. Communal knowledge turns out to be the main object of investigation. In this attempt, ordinary individuals are seen as social individuals and the "forms of ownership" described by the network in Figure 2.9 in this chapter, are then treated in terms of "social forms of relating to the land".

In this sense, 'cubação' and soil are brought together as case studies in knowledge relevant to science and schooling.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

Like most research in the social sciences, the work described in this thesis can be regarded as a process of systematic inquiry that leads to knowledge stated in propositions, and involves, particularly, an element of interaction with persons in order to offer some kind of empirical evidence for the research conclusions.

One aspect of interacting with persons is the role they are permitted to play. At one extreme we have researches where subjects are taken to make no direct contributions; and so the inquiry is all on the side of the researcher, and the action being inquired into is all on the side of the subject. At the other extreme we have more cooperative inquiries, in which case it is for the researcher to interact with subjects so that they contribute more directly in all stages of knowledge production¹. As stated by Heron:

"This contribution may be strong, in the sense that the subject is co-researcher and contributes to creative thinking at all stages. Or it may be weak, in the sense that the subject is thoroughly informed of the research propositions at all stages and is invited to assent or dissent, and if there is dissent, then the researcher and subject negotiate until agreement is reached. In the complete form of this approach, not only will the subject be fully fledged co-researcher, but the researcher will also be co-subject, participating fully in action and experience to be researched."

(Heron, 1981, p. 19-20)

In so far as this research has been strongly motivated by Freire's ideas (Freire, 1972), I would say that it can be located at the *cooperative* extreme, and so, the intention is to do research *with* people and not *on* people.

A second aspect is that of *reliability*, which is serious when data derives from such human interaction. Obviously, the same protocol can be analysed in a large number of ways, depending on the investigator's interests. In so far as each method, each

¹ A discussion of these positions which puts forward the relevance of the latter approach can be found in Reason, P. & Rowan, J. (1981).

way of knowing, gives us a kind of knowledge, it is feasible to presume that findings are very *specific* to the method or methods used. In a very pragmatic sense, it is possible to say that knowledge produced in this way is fragmented and composed of multiple discrete packages, characterizing what Fiske (1986) calls "method specificity". Problems arise for research when knowledge in one package cannot be firmly related to that in other packages. Fiske suggests three reasons for this:

"First, the data and the findings obtained by one measuring procedure typically fails to be duplicated by those from another procedure, even when applied to the same protocol. Second, a single kind of protocol commonly yields data and findings that cannot be coordinated with those from another kind of protocol. Third, the conditions under which the protocol is obtained ordinarily affect the data and the findings."

(Fiske, 1986, p. 62)

One of the main factors associated with problems of specificity, is the absence of testable *theories* that can encompass a group of stable bodies of knowledge. An essential role of theory is in bridging protocols to descriptions of results (through an analysis of protocol which "leads to" *data*²). Questions are inevitably raised about how much we ought to believe of what knowledge from research can tell us about human behaviour, such as: "Can we have data on which we can rely? Can we generalize from it? When can we do this?"

There are two kinds of methodological error that I will try to avoid: one says that if I have the right methodology and lots of data, there must be some way to get the answers; the other says that if I have a well formulated theory, then there must surely be some correct method of checking it.

It is also worth repeating that this research focuses on communal knowledge for the purpose of discussing questions about knowledge *and* schooling. As such, it must be regarded as seeking an *application* to pedagogy and curriculum design; application which requires the researcher to communicate to an audience (science educators and teachers), her findings about other's thinking.

² Which means, finding some method of analysing protocols so that the output of the analysis can be regarded as data for a given purpose. That is, I am claiming (a) that protocols are not yet data (they do not purport to describe anything, they simply exist); and (b) that data is a construction out of protocols, by analysis. In describing results we manipulate that which was constructed as data.

This means that it will not be enough simply to describe common-sense explanations of particular events, but it will be necessary to explicate them. Thus, there is a level at which I shall not be able to claim that subjects would be prepared to validate my account of their explanations/reasons. In other words, there is an intention to "study" something. Using Latour's words "we go from place A, where we find "colleagues", to a place B they do not know very well, where we find "informants"; then after a long process of 'becoming native', we go back to B with a bilingual glossary of some sort that translate the native concepts into a vocabulary understandable in A; finally, it is with our colleagues left in A that we argue about meanings, translation, rendering, faithfulness, and so on." (Latour, 1986, p. 544).

It also means that the research must necessarily involve ordinary language (Harré and Secord, 1972). This is so, first because it is through ordinary language that commonsense is built. But it is also so, because of the intention to communicate results, as something *applicable*.

On the other hand, the organization of common-sense knowledge presumes both indexicality and intersubjectivity of aspects of a cognitive schema. Indexicality refers to the location of utterances in a context of time, space and ultimately, of tacit rules. Meanings are held to be situationally determined, dependent upon the concrete context in which they appear in the sense that they are construed, not given, from arbitrary signs. They constitute something imposed in the world -it is arbitrary, not natural. Intersubjectivity refers to a state of affairs in which two or more people understand that they are experiencing events the same way (D'Andrade, 1986, p. 31). It is a necessary assumption on the part of the members of a group or culture, and, in terms of knowledge, it refers to that which needs not to be referenced explicitly, knowledge that is assumed to be shared by participants of a particular setting (Cicourel, 1986, p. 262). In summary, as far as communicative acts are concerned, things have their conventional meanings and are known tacitly. And this holds for both expert cultures (such as science), and everyday communicative practices as well.

For these reasons it seems more sensible to conceive methodological inquiry as a process of confronting problems and information. Both are a function of the other. So, if I am posing a question, I am also required to give a reason for posing that question. I will try to answer the question with some data, and try to expose the prior assumptions that the question itself makes.

3.2 DELIMITATION OF THE EMPIRICAL STUDY

The empirical study on which this work is based arose from an interest in the practices and knowledge of agriculture in community relevant to science and schooling. In practical terms this meant that I had already some questions related to the application of the ECPC-Project's programme of *Agriculture* which could be clarified by investigating both communal knowledge and the peasants' understandings related to such a knowledge. Particularly, these questions characterized a problem of *representation* located at the level of implementation of the science programme with pupils (the problem and its justification are treated in chapter 4).

Methodologically, information would be *elicited* from people within a perspective in which the researcher did not know what exactly what she wanted to find out. A goal of trying to reach *understanding* was set; and *teachback*, in the sense proposed by Pask (1975a, 1975b, 1976a, 1976b), was used as a process for reaching understanding.

3.2.1 Understanding the unknown.

One should distinguish two senses of the expression "*the researcher did not know exactly what she wanted to find out*". The first expresses the intention of the researcher -in trying to focus on the construction of *explanations*- to consider the possibility of inferring tacit structures from data. As suggested in chapter 1, p. 22, communal knowledge is supposed to have a large tacit component represented in terms of structuring rules (which are not consciously available to those who are regarded as operating within them). This sense of the "*unknown*" pervaded the whole empirical study and set the original motivation for seeking information from people in terms of an attempt to reach *understanding* (not only to look at the content of people's accounts). As stated, the empirical study started focusing on how farm-people understand the transformations of soil for cultivating crops and related issues.

The second sense, which arose out of the attempt mentioned above, expressed the idea that, having explored some issues on soil, I came to know that "*I did not understand well enough*" how farm-people thought about the cultivation of the soil. Worse, I had "*discovered*", in cubação, a topic about they knew everything and I understood nothing; and which seemed to be relevant to a clarification of the

main contingencies in terms of which farm-people think about the cultivation of the soil (the reasons I had to think so are introduced in chapter 4). Thus, the central question for me became:

"By what means can a researcher be assured that she understands an expertise which belongs to others?"

My position had to be one which considered farm-workers to be *expert-informants* from whom knowledge has to be *elicited* for further representation. In such an attempt, the researcher has to *learn* with farm-workers and teachers, and so becomes faced with the sense of incomprehensibility between systems of knowledge or models of reasoning.

In dealing with these questions, I took a basis in an adaptation of some ideas of Pask about the nature of *understanding* and *explanation*, as his theory proposes criteria for someone to say that one had understood an ordinary expert. In other words, Conversation Theory provides, also, a foundation for eliciting knowledge from an ordinary person (even though this person is an illiterate farm-worker or a naïve teacher), as if he/she were playing the role of an expert. In this discussion, emphasis is placed on '*teachback*' as a heuristic device which links elicitation to the requirements of representing knowledge.

In summary, Conversation Theory can be seen as providing a general and appropriate methodological framework for both:

- (a) eliciting knowledge proper to a group/community of people; and
- (b) describing it for the purpose of communicating an understanding which can be contrasted with science.

3.2.2 Adapting Pask to problems of elicitation and representation.

One essential virtue of Pask's position, for understanding, is that the traditional position that knowledge is only a property of one individual at a time, is modified. For Pask, understanding is not necessarily a property of a person, but of some structure (*conversation*) which may (or may not) be attached to more than one person.

Conversation is a formal structure within which there is dialog but which should not be identified with dialog in its common/ordinary sense. It always takes place

between 'participants' who are individuated differently from the distinction between ordinary individuals, in such a way that an individual of interest need not be one person but can be two or more. As Ogborn & Johnson say:

"Pask distinguishes M-individuals, as individuals looked at as objects (whether people with skin boundaries or machines with metal cases), from P-individuals, as stable, self-reproducing conversations. About P-individuals one can meaningfully say that they know and understand; a P-individual may or may not inhabit one M-individual; an M-individual may or may not have the capacity to become a P-individual. On this view, intelligence is not a property of a person qua M-individual, but of a conversation (so that a person may have several intelligences, and several people together may have an intelligence); it makes sense to say that (for example) physics is known and made by the group "physicists" acting together, besides being known (often differently) by people individually who are physicists."

(Ogborn & Johnson, 1984, p. 16)

In a Conversation, *understanding* depends on the ability to *reconstruct concepts* on the basis of *explanations*. The demonstration of understanding is required to be carried out in a particular way, in two stages:

Level 0: which places emphasis on processes and the knowledge embedded within these processes (what, how, when, to do things); and so provides a stable but adaptative framework within which to understand a problem or absorb new knowledge. In this sense, questions asked at this level provide conditions for procedural knowledge to be discussed, as they bring about the settings in which explanations should be placed; and

Level 1: which requires the demonstration of the learner's ability to make explicit the internal structure of the way knowledge is represented. This is particularly important when the learner must apply new knowledge to domains that require him to go beyond the way knowledge was originally intended to be used (as, for example, in analogical or metacognitive thinking). In this case, by being able to make explicit rules of inference and to compare knowledge, the learner can be seen as approaching knowledge declaratively.

There are two main pre-requisites imposed by Pask's theory for an understanding to take place within a conversation, which are:

- (a) that the thing understood becomes a *shared* notion, a *public* entity;
and
- (b) that a *common agreement* about what count as concepts and explanations is achieved.

For Pask, agreements can be made in natural language, but the demonstration of understanding presupposes that a *conversational language* should be used. In this way, a third constraint is added:

(c) that participants should *agree* to obey the *rules* of a conversational language *L* which is stratified into two levels $L = L^1, L^0$ (which bear a relation to the levels 0 and 1 already mentioned above).

While L^0 reveals the *rules* for arriving at a stabilised understanding (for reconstructing levels 0 and 1), L^1 *explanations* account for the *reasons for the rules*. *Understanding* is reconstructive³.

The subtle assumption underlying Pask's position, and which makes it possible for us to see the participants in a conversation, as gradually achieving understanding, is, I believe, the idea that structural properties of an entity (in this case, a conversation), along with certain conditions, endow it with distinctive capacities⁴ to perform or behave in the observed fashion (in this case, accumulating knowledge with understanding). Thus it is that, as indicated by Johnson (1983), and Ogborn & Johnson (1984), Pask concentrates more on specifying the necessary mould of an individual capable of learning and less on giving a description of the steps involved in learning.

However, it must be understood that Pask starts from a perspective of a machine implementation of understanding⁵. For this reason, to suggest the relevance of his theory to a problem such as: "What is it to elicit knowledge which may be tacit from ordinary people seen as experts?", necessarily implies the need to *adapt* it considerably.

³ This is fundamental in Conversation Theory. Particularly, it has implications for how memory and learning are conceived. For Pask, there is no static memory; all memory is dynamic, always building itself, all the time. As he says: "... a memory is a procedure that reconstructs or reproduces a concept. We contend that stable concepts, for all practical purposes the concepts existing in a mental repertoire, are those which can be reconstructed or reproduced by at least one (usually many) memory-procedures in the same repertoire. It follows that learning is an evolutionary type of process in which concepts and memories are constructed, *ab initio*, and an understanding signifies the generation and existence of a stable concept, i.e., a concept associated with a memory which either exists or is created in the process." (Pask, 1976.b, p. 5)

⁴ The key implication for teaching is that capacities are not necessarily activated in the open world and thus need to be facilitated.

⁵ Probably for this reason he is led to greater explicitness and clarity; which is certainly valuable for the present argument, even though the 'mechanization' of the process of understanding in no way constitutes my concern.

As stated earlier, the problems raised by adapting these ideas for the present research are, firstly, what it means to say that the expert's knowledge can be replicated by the elicitor, and secondly, what are the requirements for an ordinary person to become engaged in a conversation in which he/she plays the role of an expert. In other words, the former concerns when/how we can say that we have understood *any* expert *at all*; and the latter asks in which sense is it possible for a *naïve/novice* person to be seen *as* an expert. "*Understanding*" turns out to be the key concept for discussing both subproblems.

Pask's answer to the first subproblem contains, basically, two components. As Bliss & Ogborn (1987) explain:

"The first essential is to externalise knowledge at both levels in the form of descriptions of that knowledge. The descriptions are not the knowledge itself, but the means for the person who is trying to understand to build a parallel (not identical) knowledge system. The second essential is that the knowledge elicitor explains back to the expert, as if the elicitor were now an expert, the knowledge that has been acquired. We can say that the knowledge elicitor has understood the expert if the expert agrees with the fed-back knowledge descriptions."

(Ogborn & Bliss, 1987, p.44)

Together, these two components constitute what Pask calls *teachback*, and function as a *heuristic* device for eliciting and representing understandings. Teachback is a heuristic, in that the procedure makes no guarantee that it will produce a solution or a correct solution to the problem of achieving understanding (by comparison with truth-preserving algorithms or with other procedures for which they might be substituted), but instead gives us some advice about how to solve the problem. In this sense, *teachback* characterizes a procedure for stabilizing knowledge in a conversational language $L = L^0, L^1$, and so becomes an appropriate device for providing descriptions of knowledge to be represented as knowables (conversational domains).

Pask's position about the second problem presumes that both expert or naïve reasoning are subjected to formal reasoning. Thus, expertise does not mean simply to be knowledgeable in some subject area (in which case expertise is a-priori defined) but to be able to generate and process explanations within the constraints imposed by Conversation Theory (which requires that the expert is able to perform; but some crucial conditions should somehow be guaranteed). The outcome of a conversation, as an achievement, can be seen -or not- as a description of a given subject area where understanding took place.

3.2.2.4 Conclusion.

As I have explained, the problem of "eliciting knowledge" was compounded by the fact of trying to "understand the unknown". There was, for example, the question of the discovery of areas of knowledge which were not predicted. In particular, I started to discover something about a topic which I did not even know existed.

Contrasting with more traditional researches where the disciplined way one needs to go about eliciting is clear, in research such as this it is not trivial to know what it is to be disciplined or systematic. The difficulty is to know whether the researcher has found out as much as she could, that what she has found is 'really there'; and so on. And the researcher has, then, to use a methodological framework within which she can be what it is traditionally meant to be organized, and yet be open to what the informants have to say.

It was in relation to this second problem that Pask was particularly useful, in that he offered a framework within which I could be what it means to be disciplined and yet to be open to the unexpected. In particular, the distinction he draws between levels L^0 and L^1 in the demonstration of understanding fitted neatly the procedure of elicitation demanded by the kind of problems I had (see chapter 4). In addition, by establishing kinds of questions to ask when understanding is involved (how and why an event takes place, or what would happen if some feature of the situation changed), Conversation Theory provided a practical way of conducting the Conversation in such a way as to promote both descriptions and inferences also during the interviews themselves. Thus, the task of making inferences about the practice would not rest solely on the researcher's hands. But would be shared with the informants in the sense that they (the informants) would be invited to set explanations (and conditions) as part of their description.

Thus, from the point of view of the researcher, inferences would not refer only to the content of what the informants say about their practice, but about the explanation of what they say (explanations would constitute a 'second-order-content'). It was at this second level of inferences that the participants'

information was expected to contribute to the research questions.⁶ For example, it was at that level that I was expecting to be able to raise pointers to an understanding of the contingencies under which farm-people think about their practice. In this perspective, Pask's account of what constitutes understanding was crucial.

But the kind of understanding one constructs is not given by methodology. Methodology does not tell us what would count as a relevant domain; that is, knowledge that the researcher/teacher *assumes* she has, and uses as a mental 'map' against which she compares the responses of the informants/ pupils. So, it was exactly that 'map' which I considered to deserve investigation. Or more exactly, which needed *construction*, if it were to become functional for the conversation with farm-workers (in relation to understanding), or with pupils (in relation to "efficient" teaching). Here, Pask could offer no assistance.

3.3 METHODOLOGICAL ASPECTS OF THE EMPIRICAL STUDY

This section describes how *teachback* was used in a series of interviews about the working *practices* of agriculture as performed by small producers in the community of S. Paulo do Potengi, information being obtained mainly from *farm-workers*, indigenous *primary school teachers*, and some agents for developing agriculture in community, such as agricultural technicians and agents of the Brazilian Bank.

The discussion is divided in three parts. In section 3.3.1, the structure of the conversation is described in terms of: the participants, the teachback, and the domain. Section 3.3.2 focuses on the interviews. Section 3.3.3 proposes a framework for treating protocols.

3.3.1 The structure of the Conversation.

A total of approximately 40 hours of verbal exchanges, conducted and recorded by the researcher, constitutes the main unit which is taken as the "*Referent-Conversation*". The reasons for taking the total (in contrast with, for example, the unique short occasion of interaction between researcher with one subject) relate

⁶ This second level of inferences should not be confused with the interpretative level of analysis of data which belongs exclusively to the researcher. Inferences are a way of constructing information to be used as data.

exactly to the function that the empirical study is required to fulfil in this research, and which can be described in terms of at least three complementary aspects.

Firstly, there is the fact that the characterization of communal knowledge is proposed to be done *via* case studies *in knowledge*, in which circumstance the focus of attention is not on the individuals; rather, issues in knowledge are discussed so as "to make a case" which help us to understand and to clarify the nature of communal knowledge. It is exactly by looking at the whole set of exchanges, that such issues can be properly grasped and discussed as belonging to a discourse.

Secondly, by regarding knowledge as belonging to a discourse, we are led to think of it as existing as and being a relatively fixed reference. Regardless of the occasion on which one speaks to somebody, knowledge is there to be referred to, to be gone back into. In this case, it makes sense to interview the same individual several times. And it does not matter crucially in what order individuals are interviewed. On the contrary: it becomes important to talk to some individuals several times in a *evolving way*, until the Conversation can be seen as *stable*. In this sense, the term *interview* refers to the result of *events* of interaction of the researcher with one informant, regardless of the number of times they met or the length of their meeting(s). If farm-worker 'X' is met only once, for half an hour, this event constitutes *one* interview. A meeting of four hours distributed in five different events with farm-worker 'Y', is also one interview.

Thus, the situation can be regarded as being at the opposite extreme to, for example, the situation of a researcher who is interested in a single person's reaction to a moving picture or something of the kind; in which case one would think it unreasonable to interview him/her several times.

Finally, as far as the definition of the numbers of interviewees is concerned, the methodological attitude is not one of 'sampling' but of *selecting* subjects who are more adequately seen as *informants*. To be able to give the relevant information was then, the main criterion used for selection.

3.3.1.1 *The participants.*

Twenty four people (as in Table 3.1) were interviewed.

METHODOLOGY

TABLE 3.1: INFORMANTS

Qualification	Main Group	Additional
farm-workers	3	4
teachers	5	-
technicians	-	4
inspectors	-	4
farmers	-	2
researchers	-	2

Three farm-workers and five teachers were met each on four different events (sessions). One extra session happened with one of the farm-workers. These teachers and farm-workers composed the main group of informants. Sixteen other people were interviewed once each and make up the group of additional informants. A delineation of the informants follows in which a brief individual characterization is offered only for those in the main group.

THE TEACHERS

The five teachers, all women, were contacted first. They were all involved with the ECPC-Project, and willingly accepted collaborating with the research. One of them (teacher D.) had cooperated with my research for the master's degree; three of them (teachers E., Z. and F.) I knew from my work in the Project during 1984; and one (teacher V.) I had never met. Her name was suggested by the others as someone who could inform about agriculture, and as being a competent teacher. Except for teacher D., they had had the opportunity to implement the programme of "Agriculture" at least twice.

Teacher D. : She was the most experienced and qualified of them all, with a university degree in Pedagogy. Her participation in the Project started very early, and despite being trained to implement the science programme, she had always participated as a local coordinator and not directly as a primary teacher. In the community, she was one of two supervisors of the LOGOS-Project, a national programme intended to provide secondary school degrees to (out-of-range) 18+

year-olds, through a kind of personalized system of instruction. She had taught all the other four teachers with whom she had a good relationship. Her education can be regarded as typically urban, but rural in origin. Her parents, both peasants with no land, had decided to invest in the education of their children, keeping them away from work in agriculture. She admitted to lacking knowledge and experience in agriculture, but was able to give justifications for a peasant's practices, particularly when a 'scientific' kind of knowledge was required. She came closest to having an understanding of the researcher's questions, particularly in relation to cubação.

Teacher E. : She had worked with the Project since its beginning, teaching the third and fourth years (simultaneously) of the primary school in a rural village called Cabaço, where she had grown up in a family of peasants (small owners). She had moved to the town when she got married, but kept travelling to teach in Cabaço every day. She was not too open when in a group discussion, usually waiting to be invited to talk; but was respected by her colleagues, for her interventions were usually extremely pertinent and accurate. She could take some time to expose a problem or a doubt, but would rarely keep it to herself. She seemed to be thinking all the time during both the activities of the Project and the interviews, and was very secure in her understanding of the content to be transmitted (even when she might be wrong). She was prepared to review her ideas when challenged; but would retain them if not satisfied with the others' arguments. She had a secondary degree from the LOGOS-Project, and more recently had assumed the headpost of a primary school of the State.

Teacher Z. : She worked in the Project from the beginning, working as a primary teacher (for the third and fourth years) in a unique kind of school⁷. She had decided with E. to extend the implementation of the Project's programme to other schools, and became involved with the training of a new group of teachers. By the time this study started, she had a post of head teacher in a 'municipal' school. She had an accurate factual knowledge about agriculture, as she lived most of her life in the rural area (family with no land). Her knowledge of science was limited, but

⁷ This is the "Escola S. Francisco", maintained by donations of rich families and institutions. Located in the town, it is intended to promote the education of poor children (usually from the rural areas surrounding the town). The teachers can be considered as volunteers as they don't earn monthly salaries; but are paid with the money obtained from a Fair organized once a year (which gives them much less than a regular teacher receives).

good in certain areas. Thus, for example, she was secure about the content developed by the Project, but she became visibly less interested in the interviewing process when questions on geometry were put. She had a secondary degree from the LOGOS-Project.

Teacher F. : She also participated in the Project from the beginning. As a community leader in the village "Riacho Salgado"⁸, teacher F. was an enthusiast for new approaches to teaching, particularly those concerned with people's lives. She had a fair knowledge of agriculture, and her reports were always 'contextualized cases' (reflecting a more general situation, but expressed through concrete stories). It was easy for her to absorb the relevance-aspect from the Project and to incorporate it at the level of the pedagogic discourse. But because lacking expertise in the content itself, it was difficult for her to manage the organization of the content so as to establish links between problems and application. Particularly in the interviews, she showed how much her ability to think about science was restricted to a traditional pedagogic practice (where knowledge is treated as content to be transmitted). But if she was an example of a teacher with a superficial knowledge in science, she was also an example of a teacher who was extremely knowledgeable about both the peasants' situation and rural pedagogic practices. She had a degree from the LOGOS-Project.

Teacher V. : She started implementing the Project's programme under the supervision of teachers E. and Z., in 1985 (she did not participate in its development). She also had a secondary degree from the LOGOS-Project, and was teaching the four years (simultaneously) in a rural primary school. The school was located in a big farm (which was clearly moving towards a kind of factory farming) where she was born on a family of peasants. Having no land, her father was a 'morador' in the farm. She was an intelligent, knowledgeable teacher, with well articulated speech. On several occasions, during the interviews, she seemed anxious and unhappy with the fact of not being able to make sense of the purpose of the interview. This did not happen with the other teachers, probably because they were aware of the kind of use the Project had made of their previous reports (which, as they had recognized, were valuable for the development of the programme). However, this fact did not constitute an obstacle to the interview. Rather, her reported knowledge was accurate showing that she had, indeed, a

⁸ The land of Riacho Salgado (originally a very big farm) belongs to members of her family. Due to inheritance, it is at present divided into small properties.

strong urban influence. After the last interview, when left with the "worksheet" (see chapter 7), she was the only one to try to answer the included questions, by writing to me.

THE FARM-WORKERS

My idea was to start with a small group of farm-workers who could be considered expert farm-workers, recognized as such by people in the community; and to include others if it became necessary. I had a small list of names suggested by researchers and agricultural technicians with a wide experience in the area (who had worked with small producers). But before deciding, I talked to some people living in SPP, such as the Monsignor (Monsenhor) Expedito (priest in SPP for more than 30 years), some technicians, nuns and teachers.

Two recommended names were farm-workers S. and Ce. (recognized unanimously as 'experts'), the former living in the town and the latter in Cabaço. They had a family relationship with teachers D. and E. (father and father-in-law, respectively), which seemed an useful fact to be explored, having regard to my intention to clarify, as much as possible, how knowledge is maintained/transmitted in a peasant (family-based) society.

A third name was added; that is, of farm-worker J., who had lived all his life in big farms as a 'morador'. He was the father of teacher Z., and his choice for the main group deserves a brief comment.

At the time I had to select names of farm-workers to interview, I used the word 'expert' in a loose sense, to indicate a requirement for 'good informants'; that is, experienced and knowledgeable persons with the ability to talk about their work. Certainly, people who suggested names of 'experts' were aware of the fact that I was involved with research and thus needed informants able to produce useable accounts (which could be transformed into data). Obviously I was the most interested in having good data, but, as far as the selection of informants was concerned, there was an aspect of 'expertise' which needed not be misunderstood. Namely, that 'expert', in the sense I was using the word, indicated someone able to operate a discourse; but discourse, not as a label for a narrow set of empirically observable linguistic activities, but in its constitutive character (in the sense used by Woolgar, 1986).

Farm-worker J. was not recognized by some people to be a typical and experienced farm-worker, and a limitation concerning him was mentioned in terms of a possible lack of communicative attributes. Surely, this fact could lead to methodological difficulties, but it did not seem to constitute, necessarily, a restriction on his possibility for operating an agrarian discourse. The fact of being a 'morador-meeiro' for most of his life, allied to the fact that 'moradores-meeiros' did not appear as potential experts in my list, made me decide to include his name in the main group. My perspective was to create the possibility of covering all the probable situations in which the work in agriculture was done; and farm-worker J. seemed to represent an unclear case in respect to my understanding of 'expertise'.

Farm-worker J.S., a small owner, was also strongly indicated as a potential good informant, but I decided to talk to him after having delineated more clearly the issues to investigate in detail. He was mentioned again by the teachers during the meetings, and interviewed on one occasion at the end of the Conversation (and then considered as an additional informant). Like him, farm-worker M., working as a 'diarista', was indicated by the teachers as someone who could help to understand particular issues; and was interviewed once.

Two other farm-workers, Jo. and J.M. (father and son), were included in the group of additional informants. They worked as 'empregados' in a medium sized farm in a different 'município', and were indicated by the owner of the farm as experts in cubação. They were interviewed in one occasion, just for the researcher to have an idea of how the method was applied outside SPP (the local situation of the study).

Farm-worker Ce. (55 years-old): He was born in "Riacho da Cruz", a small village in the neighbouring 'município' of S. Pedro, where he lived up to forty years old, and then moved to "Cabaço"-SPP, where he still lives. He had been a small owner for most of his life, and possessed two tracts of land: one of his own and another from his second wife (he was widow). For this reason, F.W. Ce. had always had control over his production up to the phase of harvesting; selling the yield to intermediate entrepreneurs. He usually planted separate fields of cotton and manioc (this in small amounts as his tracts did not present a large proportion of 'arisco'), keeping interposed mixed fields of corn and beans, for subsistence and market. His tracts were located in the middle way between SPP and S. Pedro, and near to the main road in the valley of the Potengi river (they did not face the river); and within an area of priority of the EMATER's programmes of rural

development. Thus, F.W. Ce. maintained a long and straight relationship to both agencies of diffusion of technologies and financial support. He was extremely knowledgeable of the forms of those relations, and was respected by technicians of the EMATER and by his peers. He had introduced his children to work in agriculture and was widely recognized as someone who had had an influence in the "teaching" of other children. During the interviews he used a very specialized terminology for describing the tasks involved in the work of agriculture in terms of roles that people usually perform. He proved to have a very definitive idea about the character of schooling and about the function of school knowledge in promoting a distinct kind of discourse other than the agricultural one. In addition, F.W. Ce. was experienced about the mechanisms through which knowledge participates in the world of agriculture. He was the father-in-law of teacher E..

Farm-worker S. (70 years-old): He was born in the village "Olho d'água" in the neighbouring 'município' of S. Tomé. He moved with his family to a big farm in SPP (his father was a 'morador') when he was fifteen years old. He had never had land of his own, and started working on his own when he got married (as is usual among peasant people). At the age of forty years old, he moved to the main town of SPP to give conditions to his children to study. Since then, he had kept walking 18 km every day to come to the place(s) of working, establishing with the farmers a relation of 'meeiro'. At the age of sixty years old he had become sometimes 'comodatário', at others 'arrendatário'. His main crop was cotton, and because the land he planted was never 'arisco', he had rarely cultivated manioc. He had always planted a field of beans, corn and broad-beans for subsistence and market. He had his own tools of work including sometimes an ox for driving the 'capinadeira' which he usually rented (he had had one in earlier times). His experience in agriculture did not include teaching peasant children. F.W. S. was extremely conscious about the relevance of culture and knowledge which goes beyond the limits of the peasants' lives. For his children, he set himself the hard task of promoting school instruction at all costs. During the interviews he showed to have an unique mastering of intellectual reasoning, getting involved, sometimes, in a true process of 'mental lucubration'. He easily attempted to propose generalized explanations and to test them. F.W. S. was father of teacher D..

Farm-worker J. (62 years-old): He was born in "Jardim do Seridó", a 'município' located in the 'Sertão' of the state of RN. He moved to SPP with his family at the age of thirty years old; living almost his life as a 'morador/meeiro' in big farms. At the age of 59, he moved to the main town of SPP, working as a 'comodatário' just for

having a field of beans and corn for his subsistence. Being a 'morador', he had always worked under the orders of a farmer, usually for growing cotton. As a 'meeiro', he could have a small tract of his own responsibility for planting crops for subsistence in addition to cotton. His experience in the teaching of children was just part of his duty of bringing up his children as peasants. During the interviews he was worried about not being able to account for everything that goes on in agriculture; but he made a genuine effort to collaborate. So much so, that it was impossible for him just to keep the position of informant. He easily became involved in a real process of learning when trying to answer the researcher's questions (he looked for help with more 'expert' farm-people). F.W. J. was father of teacher Z..

TECHNICIANS

Four technicians were interviewed, separately, on one occasion each. Two were from the local office of the EMATER; one was from the local 'Sindicato dos trabalhadores rurais' (rural trade union); and one was from the 'Serviço de Assistência Rural' (an institution supported by the Church) who had a large experience with farm-workers in the area (this interview was not tape-recorded, but the technician wrote down on the paper the main points of the argument he wanted to make).

INSPECTORS

Four inspectors of the Bank of Brazil were interviewed once each. Three of them worked in the local branch of the Bank: the manager who authorizes loans; the director responsible for the registration of properties; and the field inspector. They were interviewed in a group meeting. A fourth inspector of the Bank, with a long experience in SPP, was interviewed individually. He had just been transferred to the North Region of the country where conflicts involving land were frequent (he left SPP the day after the interview).

FARMERS

Two farmers were interviewed once each. One big farmer was from a nearby 'município' who had a large experience with the manioc production. The other was a small farmer from SPP who was indicated by a teacher as being able to answer some questions related to cubação.

RESEARCHERS

Two researchers from the Project were interviewed once at the beginning of the study. One was the coordinator who had had a long experience with teachers in SPP, and had been responsible, together with two other researchers, for the thematic investigation for the programme of Agriculture. The second was a researcher assistant who had interviewed farm-people during the preliminary phase of the thematic investigation.

3.3.1.2 *Teachback and criteria for ensuring understanding.*

As was said earlier, *constraints* for elicitation were to be set in terms of criteria for reaching an understanding. As far as an *understanding* is evidenced by explanations at levels L^0 and L^1 , *teachback*, as a heuristic device, had to secure this condition during the interviews. Understandings were elicited by the researcher who communicated with the informants using his/her language (teaching back codified versions of the informants' accounts). Explanations were elicited verbally, which led the researcher to prompt the informants with *questions*. Some questions were more appropriate than others in this function, in such a way that it is possible to distinguish between L^0 and L^1 questions (as Johnson, 1983; and Ogborn & Johnson, 1984 have suggested).

Thus, L^0 questions would ask people how, what, when things are done or happen. Their function were to prompt explanations which specifically evidenced the existence of concepts defined in terms of procedures which bring about a representation of the concepts. So, when the researcher asked questions such as:

. "Ok. But tell me something. This means... because what I want to know is this: how do you 'square' [organize] a tract? Because you will never mark out a *field* in this way, will you?"; or

. "Suppose someone decided to build a house with this shape and you have to estimate the area? What would you do? [...] How would you do it?";

the interviewee was seen as providing information about how he would estimate the area of a shape; and the procedure he used as giving evidence for him having (or not) the concept of area in terms of units of area.

L^1 questions would address why things happen; thus prompting explanations at the level of inferences. Contrasting with L^0 questions ("how" questions), at this level we have "why" questions such as:

. "There is one thing I would like to talk with you... I have several questions about it. Because I have learned a lot, but I still have doubts. That is it: I believe that I have learned how to reckon by cubação. I think I know how to do it with a braça. Now, what I want to know is this: if you would have to teach a child how to reckon, what would you tell him; how would you do it?"

. "[the farm-worker accounts for the practice of manuring the soil for improving production] Right. You say that the manure is a kind of fortifier for soil... it makes crops grow. [...] But what is this fortifier made of? Do you know [what it is]? [...] For example, people have anaemia. They are asked to take iron. [...] So, it is a fortifier but it is said to be iron, which it that which makes people get stronger. Other people have teeth-problems. So it is said that children need to take calcium. Do you think it could be something of this kind which happens? How do you know what is that makes soil get stronger?"

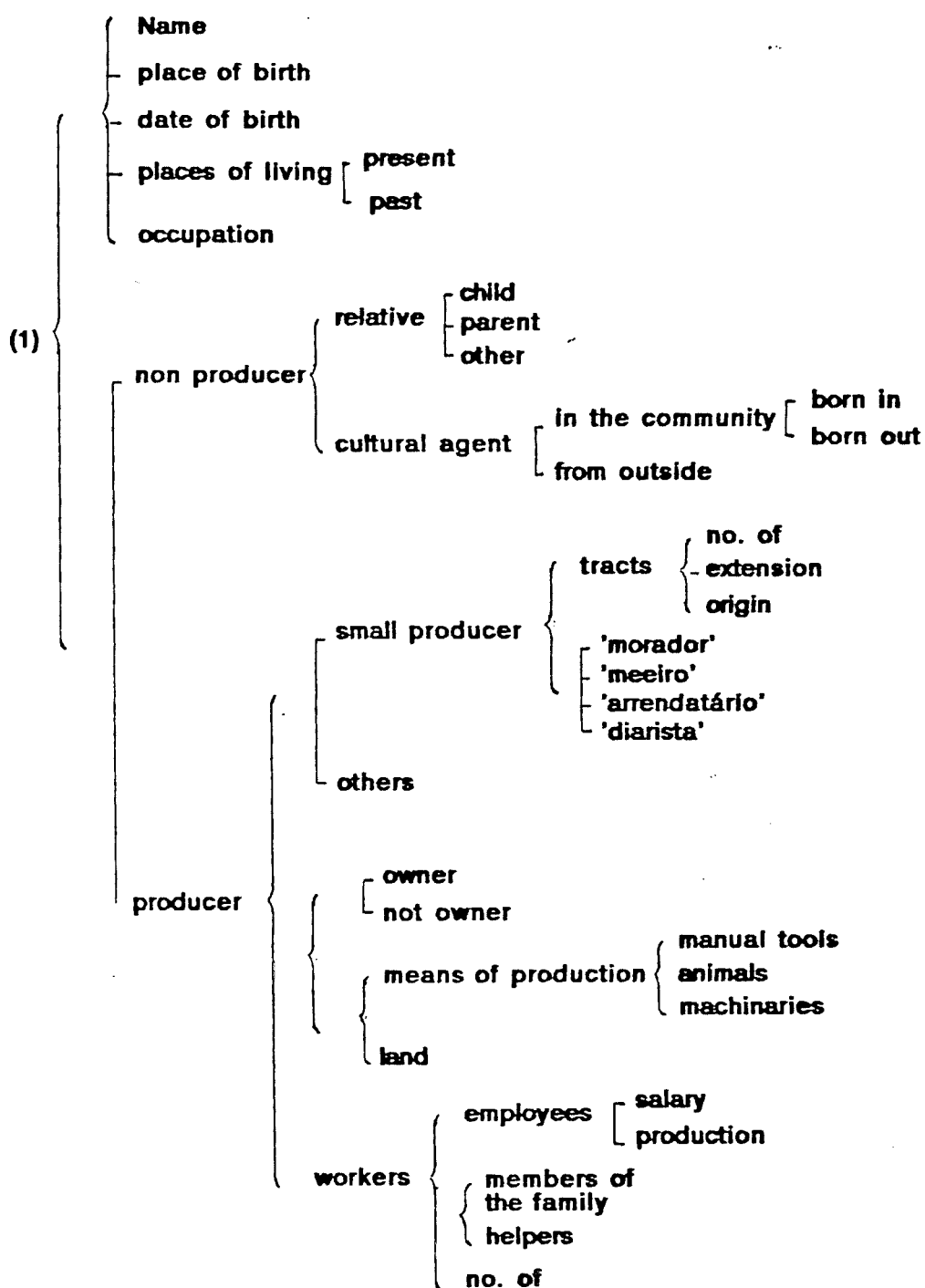
. "[the reckoning procedure that the farm-worker uses is set only for quadrangles] Let me see... I drew a shape here... with 3 sides. Because I want to know how do you do when the tract has not 4 edges." [which is a way to ask "how do you know that your procedure always work?"]

. "Do you know why it is that loosening soil and scraping up weeds make difference to the growing of plants? [...] You say that the person needs to know how to manage well the hoe. Why? Could you explain this?"

Passages of the conversation in which attempts to reach level 1 were made, are usually extended, and not always successful. Prompting informants with questions "why" does not necessarily lead to L¹ knowledge (that is, to understanding). On the other hand, some propositions not directly expressed in a form of L¹ questions (but which are set in the context of a L¹ discussion) can function as clues for L¹ explanations. Examples pervade instances in chapter 7.

3.3.1.3 *The domain.*

The initial decision of what was to be understood from interviewing people, belonged to the researcher. Emphasis was given to the process of manufacture of practical knowledge, particularly to the primeval meaning of ongoing events for and by farm-people. A congruence between the *praxis*/discourse and the social/cognitive dichotomies was presumed. Within such a perspective, the tangle of issues already offered by the ECPC-Project was organized in a *network* (Figure 3.1), constituting an initial representation of the domain to talk about.



cont.

Figure 3.1

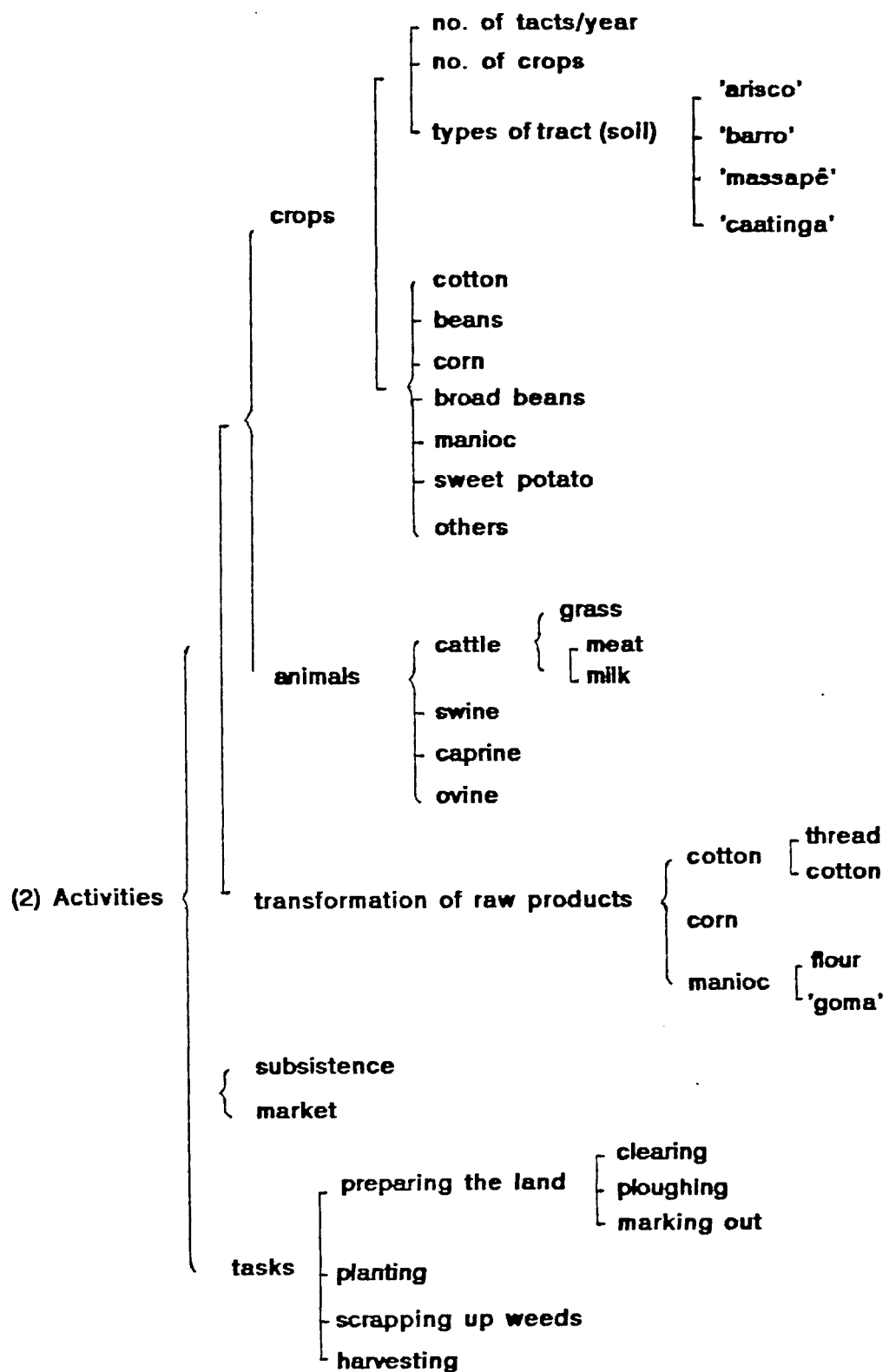


Figure 3.1

In other words, the network described information I sought as I saw it. But farm-people could, and did, end up determining *what* I could understand and *how* to explain it. For this purpose, principles of requesting (or giving) accounts at both levels L⁰ and L¹ were maintained in constructing a *domain* for conversation with people. That is, a goal of reaching understanding was set by the researcher with the possibility of incorporating the informants' contributions.

3.3.1.4 *Concluding remark.*

In structuring the conversation as mentioned above, my first aim was to understand a discourse. When we do so, informants create new explanations, and think explicitly about the taken-for-granted discourse. This gives to the researcher a possibility of a further level of analysis, which is *about* the discourse (not just of structures *within* the discourse). In other words, to attempt to understand a discourse raises questions about what a discourse is, about what it is to describe a discourse, about the nature of theory and explanation, and about the nature of commonsense. This thesis does not attempt to answer these questions directly, but some reflections on them appear in a *theoretical appendix* (Ap. 3.A).

3.3.2 The interviews.

3.3.2.1 *The starting point.*

It is generally accepted in research, that an interview meeting is a social encounter; but not to be a *natural* social meeting in the every-day life of people. It is important to remember, however, that there is a sense in which interviews can (or should) become legal social meetings for a given purpose. Interviews based on Conversation Theory seem to carry this sense, in that it is important to establish, from the very beginning, a 'contract' about the roles that participants will play (set in terms of knowledge expertise).

In the case of this research, the roles were set in terms of different kinds of expertise involved which had themselves a social character. Farm-workers were legitimate experts in agriculture; teachers in schooling and in its relation to practice; and the researcher in *interviewing*. To have researchers asking eccentric questions is something people in SPP are used to, and if there was a sense in which I was recognized by people, it was in this sense. The support given by the Monsignor of SPP saying that "she/he is from the university and wants some

interviews", is a pass for any researcher to circulate in the community as a 'member'. Thus, when farm-workers and teachers agreed to participate as informants, they took the work as a compromise they knew about.

Thus, with teachers, questions should concern issues of pedagogy and curriculum organization; also, because teachers were involved with the ECPC-Project, they were supposed to have a concern with the relationships between what is taught in schools and what people know in the community. On the other hand, with farm-workers, questions should concern work in agriculture. However, these distinctions were not to be taken as a matter of fact. They were relevant in that they related to the necessity of giving to the process of interviewing a 'correct' social character. Actually, some questions from the school perspective were addressed to farm-workers and *vice versa*. In fact, the clearest account I have in the transcripts about the distinction between school knowledge and practical knowledge, was made by a farm-worker.

But this was not the only sense in which the *social* was regarded as important. Following a Freirian point of view, I was working with the supposition that the nature of the conditions on which communal knowledge depends is social. That is, *praxis* determines the way knowledge is shaped in a specific form, at any historical moment. As far as *labour* is taken as the fundamental unit for defining such conditions, this historical moment is to be seen as located in a given period determined by relations which derive from the proper situation of productive labour. Thus, the method which could help me to understand the manner in which different modes of common-sense reasoning are formed should try to model knowledge on the closest and most fundamental form of social organization. It was within such a perspective that farm-people to be interviewed were taken to be *small producers*. For the representation of communal knowledge, the implication was that the first thing to look at would be the background features of the work in agriculture. That is, the focus should be on the actual activity of work, and on the conditions of people's performance as small producers; the central questions being "why is this activity important?"; "what does it help people to know/understand?"; "what view of the world does it help to convey?"

Initial information was collected about the informants as small producers. Sociologically, this would require information about forms of ownership, relations of work and of production in the Region of study, and related issues. Results have been given mainly in chapter 2, summarized in the network of Figure 2.9.

The discussion about *soil* was introduced:

- . with *farm-workers*, as an extension of the objective conditions of work, in terms of types of soil and kinds of crops they deal with;

- . with *teachers*, in terms of the sequence of the science programme, which also started by addressing the same issues; namely, types of soil and kinds of plants.

3.3.2.2 *Formst of the interviews.*

As explained, informants of the main group (3 farm-workers and 5 teachers) were each interviewed four times. Thus, four blocks of sessions were organized. In each block one session was conducted with each of the eight informants before proceeding to the next block, when eight subsequent sessions were carried on (two interview-sessions were conducted with one farm-worker in the fourth block; totalling 9 sessions in the last block). Material obtained from the interviews was cumulative for the researcher who aimed at constructing information out of the interviewees' responses. But as each block of eight sessions had a specific background problem to investigate, the interviews can be seen as developed in four phases (Table 3.2). 14 additional informants were interviewed during these phases, as in Table 3.3 (2 researchers had been interviewed previously).

The sessions were restricted to 45 minutes in length (on average). The length of blocks of sessions and of intervals between blocks is indicated in Table 3.4. The researcher used the intervals, for transcribing and analysing the protocols. Thus, in interval 1, taking into account the research questions already asked, topics were selected for investigation in phase 2.

Particularly, in addition to the 'sociological' data, information was elicited about:

- . Labour-energy.
- . Manioc House (industry of manioc flour).
- . Land (soil + cubação).
- . Planting.
- . Growing crops.
- . Harvesting/Storing.
- . Market exchange.
- . Production.

METHODOLOGY

TABLE 3.2: INTERVIEWS

Phase	Focus	Purpose	Sequence of sessions(*)
1	People in the community.	Situating the informants.	Z-J-D-S-E-Ce-V-F
2	Practice of agriculture.	Describing how and why things are done/happen.	Z-S-J-Ce-E-V-F-D
3	Soil, organization of the field, and cubação.	Exploring soil and making cubação problematic.	Ce-E-Z-S-D-J-V-F
4	Cubação.	Investigating cubação further.	D-E-Z-S-V-Ce-F-J-Ce

(*) Informants as identified in section 3.3.1.

TABLE 3.3: ADDITIONAL INTERVIEWS

Qualification(**)	Block			
	1 st	2 nd	3 rd	4 th
farm-workers	-	-	-	4
technicians	-	-	-	4
inspectors	-	-	-	4
farmers	-	-	1	1

(**) Two researchers were interviewed previously to the 1st block.

TABLE 3.4: PERIODS

Situation	Length (days)
block 1	8
interval 1	14
block 2	3
interval 2	28
block 2	3
interval 3	12
block 4	10
total	78

In interval 2, the results indicated that in addition to soil, cubação should be taken as a case in knowledge relevant to science and schooling. Thus, phase 3 emphasized soil and cubação, with 'the organization of the field' playing the role of linking the two cases. The analysis of information obtained in phase 3 pointed out the relevance of further investigation about cubação; which was then taken as the main issue in phase 4. The analyses in the different intervals are described in chapters 4 and 7.

3.3.3 A framework for treating protocols.

Because, as it happened, *cubação* turned out to be the central case in relation to which the characterization of communal knowledge was proposed for this thesis, I decided not to take the whole set of transcripts as the referent system for analysing what is known (the main distinctions in terms of which farm-people think about their practice). Rather, I decided to delimit the analysis to part of the material; namely starting from *cubação*.⁹

But I did not take such a decision for reasons of economy. Actually, to work with part of the material implied that, if I had information which I would want to use as data, I would probably need to establish *relations* to other information which is usually found in the context of the whole set of the transcripts. In other words, after having seen what a particular informant says, it could become relevant to ask, for example, "In what circumstances did he say that?" And to answer this question no possibility exists other than going back to the whole set of transcripts to try to retrieve what is needed. It is important to stress, however, that such a requirement imposes itself not only because I wanted to develop the field work to become knowledgeable about a given problematic, but because there was an intention in this research to use information for an application (with the additional difficulty that, about *cubação* I did not have clear research questions related to application).

Because it was in relation to *soil* that the initial research questions were posed, this theme became the complementary case to *cubação* in the attempt to characterize common knowledge relevant to science and schooling. But methodologically, *soil* could also be taken as part of the context in relation to which to discuss *cubação*. Results concerning soil, land and production have been already introduced in chapter 2, exactly because, for the reader, it would be very difficult to follow the study of the two central themes out of their proper context. Some results about *soil* come next in chapter 4 and I will relate them to the use them to the "discovery" of *cubação*. *Cubação* comes in chapters 5 to 8.

⁹ This decision does not invalidate the argument raised in page 68, that it is by looking at the whole set of exchanges that issues in knowledge can be properly grasped and discussed as belonging to a discourse. The argument still holds and the question is now to ask about the implications, for methodology, of adopting such a decision, regarded that *there is a 'whole'*.

In addition, there was the question of the different *levels of analysis* to be established (as shown in Table 3.5).

TABLE 3.5: DIMENSIONS FOR TREATING DATA

Dimension	Function	Kind of use	Example
what is known	to answer research questions	illustrative instances	accounts about soil
what was 'found'	to clarify, to explain, to propose	evidence	the nature of cubação
what can be speculated	to provide elements for 'theorizing'	'hints'	the idea of area in 'c'

These levels were defined after a difficult process of trying to construct classes from the transcripts *and* having to match them with the analytical requirements of the dimensions specified in Table 3.5. This attempt resulted in establishing four levels of analysis which were then used more systematically in the reporting of results. They are:

The level of practice (in terms of the necessity/possibility distinction): things that are necessary and done; things that people can imagine doing and that can be done; things that people can not do, but they can understand the idea; things that could be done but they are not what people do; things that are needed but people do not know how to do.

The level of expertise: what is generally known in community; what is known to a particular group of people; how it is known; who knows and for what purpose.

The level of discourse: what is said, thought or understood when one comes to discuss what is known; what is not known, not feasible to be said or thought, and not reliable to be understood.

The level of skilled performance: what people can do in terms of the relevant manipulations required by the discourse; how fluid they can be about it; whether they can see their performance in relation to other possible systems; and the extent to which this is habitual or not habitual.

Two implications followed. First, that the same text could be re-analysed at different levels; and second, that different levels would require different amounts and kinds of interpretation.

In the analysis, distinctions between levels are left implicit, except where necessary.

CHAPTER 4

SOIL AND THE 'DISCOVERY' OF CUBAÇÃO

4.1 INTRODUCTION

The *general* research questions were listed in chapter 1. This chapter discusses research questions at the level of the particular case studies of the thesis, that is, the *specific* research questions. These belong to two distinct groups: the initial questions (referring to soil) which constituted the starting point of the empirical study; and questions about cubação, which originated from the investigation itself.

The initial questions arose from an analysis of the programme of *Agriculture* as developed by the ECPC-Project, in the light of some criticisms raised by researchers while reporting the execution of the programme with pupils.¹ The focus of the researchers' problems/ difficulties was the activity called "The Transformations of Soil", which was part of a group of tasks compounding a pedagogic unit about soil.

The chapter is organized in four parts. Section 4.2 reviews the Project's pedagogic unit about soil and introduces the initial research questions. Section 4.3 summarizes the main features of the study concerning soil. Section 4.4 describes how the 'discovery' of cubação turned out to pose novel and specific problems. A conclusion is outlined in section 4.5.

¹ The report was part of an interview I conducted with two researchers involved with the implementation of the Project (particularly with the programme of *Agriculture*); in which I exposed my intention to investigate communal knowledge by asking peasant people to talk about their practice.

4.2 THE INITIAL RESEARCH QUESTIONS

4.2.1 The pedagogic unit about soil.

The unit about soil was organized around an empty Table (Table I) requiring, from the pupils, a description of the stages involved in the cultivation of soil for growing crops; and which was intended to lead to a scientific account of the events related to the peasants' practice. In very simple terms, the stress was on *work* that people *do* to soil which *transforms* it for the purpose of growing crops. Descriptions within Table I were then proposed to be made at the level of performance evoking explanations.

Stages were to be described in terms of the following factors: *tools* (what is used), *operation* (what they are used for, to do what), *performance* (how the task is done), *explanation* (why it is done), *phase* (when it is done); and were intended to represent the main classes of responses for the characterization of the *activity* "the cultivation of soil" in terms of the stages: *to break/clear, to burn, to plough, to mark out, to drill, to plant.*

Table I : The Cultivation of Soil

Stage	tools	to do what	how	why	when
to clear					
to burn					
to plough					
to mark out					
to drill					
to plant					

The classes derived from a *thematic investigation* carried out with the group of primary teachers, participants in the implementation of the programme. The researchers' supposition was, then, based on a kind of compromise between their own knowledge and the teachers' knowledge about the peasants' practice. The important fact is that, for both researchers and teachers, these classes were

acceptable as a means by which the peasants' practise could be characterized and discussed; and consequences could be analysed in terms of scientific knowledge.

Thus, for example, the class "to plough" could be described in terms of:

"to drive (operation) a 'capinadeira' (tool) so as to mix the soil already cleaned (phase); in blending the soil, one modifies the distances between grains (performance); facilitating the drilling and the planting (phase); and facilitating the penetration of air and water, necessary to the growing of plants (explanation)."

The class "to mark out" could be described as:

"to drive (operation) a 'capinadeira' (tool) so as to define the planting places of seeds in a ploughed tract of land (phase); in defining the distance between planting places, one delimits an area of land (performance); establishing a zone in which seeds can be planted (phase); and creating a repository of nutrients for each bunch (explanation)."

And so on.

One could say that the description of stages resembled a kind of representation through stories. A story is a kind of compromise between formalism and context. It has ritual aspects which are the formal parts, it follows a clear pattern; but at the same time it is specific and concrete. Thus, in the Table, *tools* and *operation* are to be seen working at the contextual level; *performance* and *explanation* pulling towards the formal level; and *phase* playing an intermediary role, as it establishes a kind of repetitive chain in regard to which causal relations can be attributed either to interpretations from commonsense or explanations from science.

It was through a discussion of the changes that each stage implies, that science was to be introduced. For example, it was in relation to a discussion of the consequences of mixing the soil (or of defining the planting places), that knowledge about the composition of soil (or about the role of nutrients in the growing of plants) would be taught. That is, it was through this *representation* (Table I) that the discussion about the cultivation of the soil was proposed, by the Project, to be conducted.

The method, following Freire, was to take the representation *coded* in the Table, as a *scene to be decoded/transduced* by both researchers and pupils in a teaching situation. Which means that the use of the Table should presuppose an attempt to

elicit pupil's understanding to be contrasted with a scientific one. But it is important to stress that to elicit pupils' conceptions in such an activity is not to ask what pupils know that can be written down in the spaces of the Table. Rather, it is to contrast the very meaning of the classes as held by the pupils with the meaning attributed to these classes by the researchers/teachers. Only in this way could one reach a discussion of the peasants' practice in which both commonsense and science could participate.

But Table I was just part of the whole unit. At a complementary level, transformations could be described in terms of what soil affords. Thus, for example, the size of the soil's particles can (could) be such that, if soil is irrigated, it will (would) afford the movement of water into the root at such and such rate.

In connection with descriptions presupposed by Table I, one could say, for example, that in marking out the planting places where seeds are sown, the farm-worker delimits an area of land (the area of the culture). When plants are cultivated, this area is taken as a repository of nutrients for the bunch, and soil is described in terms of its constituents. A description of soil as a system of particles could be written in which the functioning of these constituents in the growing of plants becomes the main focus of attention.

Thus, given certain requirements which are posed by the mechanisms of the living plant, the model of soil, described in terms of an arrangement of particles with given size, becomes enriched with other features. These turn out to be called factors of importance in the growth of plants. It is a composition of these features with the particle-model, that constitutes a basis for distinguishing\ characterizing types of soil. *Soil* is then considered to have *constituents* (which can make things happen, or not) and *properties* (which are invariant under certain transformations).

In so far as the pedagogic function of the unit is concerned, the programme made use of *processes of transformations* as a *unifying concept* to organize the content with semantic and intentional relations. Within the perspective mentioned above, *soil* is conceived of as *an entity which affords transformations through people's performances*; and *kinds of soil* and *kinds of plants* turn out to be the main variables in terms of which explanations are expected to be furnished.

The principles used in the organization of the content (and which constituted the structuring framework of the whole unit) can be seen encapsulated in the following questions.

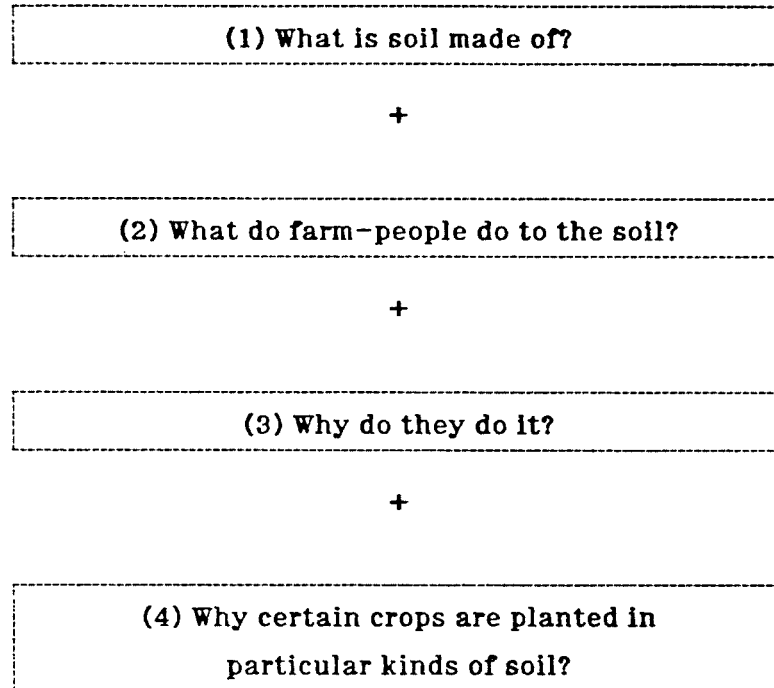


Figure 4.1: The framework used by the ECPC-Project for structuring the content about Soil

Related to question (1), the programme discussed the soil constituents, focusing on their attributes/properties. The distinction between clayey and sandy soils followed as an implication. Question (2) concentrates on how the work is done and raised a discussion about possibilities of performance for growing crops. Models of soil are the focus of question (3) and were intended to serve as a ground for the discussion of the processes at work. Question 4 poses the problem which motivated the organization of the content about soil framed on questions (1), (2) and (3).

4.2.2 The introductory problem .

The researchers' complaint was that the description involved in Table I failed, not in its parts, but as a whole. The problem seemed to be, essentially, one of *representation*; but its precise nature was not clear.

One possibility was that the Table was too 'analytic' an approach. People usually do not see their knowledge as analysed, but as triggered by the occasion². Also, the possibility that the problems had to do with overlap between the classes was considered, using evidence from teacher's notebooks. But these possibilities were dismissed as not really fundamental.

A possible diagnosis was that the Project had not sufficiently pursued a systematic investigation of people's practical reasoning about agriculture, so as to uncover features which might provide explanatory possibilities for understanding, at the level of these everyday practices, the nature and origins of pupil's accounts, seen as deriving from an agricultural discourse. The elements of the Table seemed not to make sufficient contact with that practical discourse. Hoping for clues pointing to what might be 'wrong', I decided to attempt a further exploration of everyday agricultural practice and discourse.

To try to understand the problem better, an analysis of the main contingencies (in terms of the relevant distinctions) under which farm-people think about the cultivation of the soil for growing crops seemed to be a fruitful starting point; particularly if this investigation were to be set in a perspective of discussing what we could learn about the possibilities of farm-people to operate a school/scientific discourse. For my own purposes, in addition to showing how people's understanding could be relevant in learning about soil, one valuable thing to come out of such an analysis was the possibility of investigating the nature of common-sense knowledge in its relation to science, from the community point of view.

As I said earlier in chapter 3, I was working with the supposition that the conditions on which common-sense knowledge is developed is social. Thus, cultivation should be taken in terms of the *productive work* (labour), and contingencies in terms of both social and cognitive variables.

² This seems to hold for any kind of research trying to characterize/represent people's performance/knowledge. For example, if we consider interviewing mothers about child care, we could easily make up some categories that come from the parents' discourse. Thus, when the baby cries, no doubt if we observe the mother carefully, we find considerable regularity and rules. But if we were to write out what she does, and ask her that -what she does, and to tell us why, she probably would find that it makes any sense at all; she could say for example, things such as: "I try this first, if it does not work I try that"; and so on.

If the problem motivated me to look at the process of manufacture of practical knowledge, it was not really practical actions that I needed to confront in direct observation. Rather, it was the *meaning* of ongoing events for and by farm-people.

4.3 THE STUDY CONCERNING SOIL

Large amount of results concerning soil was used in the construction of chapter 2. In this section I summarize the main aspects of the investigation about soil, focusing on the issues which turned out to be interesting for further investigation.

4.3.1 The interviews: farm-people thinking about soil.

Information about soil was elicited in the first three phases of interviewing (chapter 3, page 83). Intermediary analysis was conducted mainly in intervals 1 and 2. Complementary information was got from additional informants in phase 4.

As mentioned in chapter 3, questions addressed to farm-workers had a direct concern with the work in agriculture; while questions addressed to teachers were set in a pedagogic discussion. But the framework I used to structure the sequence of the interviews was the same for both. It was quite similar in most respects to the framework used by the Project as mentioned in section 4.2.1 (Figure 4.1); but presented a fundamental difference. While the Project allowed the discussion about production to follow as a fact to be analysed from information about science, I started by taking *production* as the primary reference for situating the material practices to be considered in connection with the analysis I wanted to develop.

For instance, instead of first distinguishing clayey and sandy soils (as the Project does), and then using this information to try to understand why manioc is always planted in arisco (for example); what I did was to consider the *land* worked by farm-workers as a condition related to the peasants' situation of having to plant certain kinds of crops (these related to the social categories of small producers), and to ask for the kinds of distinctions beneath forms of land's conceptualization. The distinctions in terms of soil (framed on science) were then to be seen *as conditions* which participate, with others, in the configuration of a *praxis* which has a more broad and complex determinant. Thus, instead of focusing on the clayey-sandy distinction, I started by trying to establish a network of the variety of ways *land* is treated/thought by people in relation to the productive work. It was in a discussion of land in terms of the main distinctions which arise out of a

social/cognitive analysis, that I have tried to equate *arisco* to sandy soil, and *barro* to clay. This accounts for the terms in which the discussion in chapter 2 is conducted.

Given the above framework and having set a particular position for using it, each block of sessions can be described.

Phase 1: People in the community

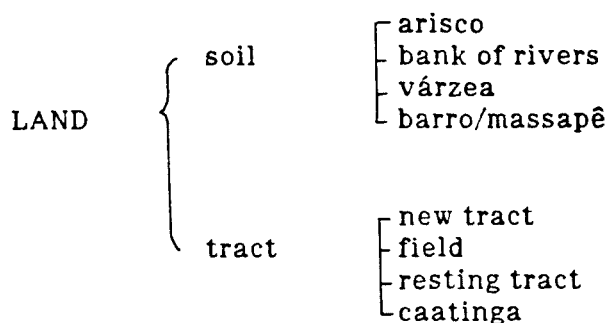
The purpose of the sessions of phase 1 was to characterize the informants as small producers. The Network in Figure 3.1 (pp. 78-79) was used to guide the selection of questions to ask. *Production* was the background issue for introducing soil as a relevant object of discussion.

In so far as teachers were *non producers*, they were invited to talk about what they knew from being a member of a peasant community, and/or about the situation of another producer (usually a relative) known to them. In addition, teachers were invited to talk about the results of the implementation of the programme of Agriculture, including the difficulties they faced in teaching the main concepts and distinctions.

'Tract' was the generic term employed by farm-people to designate land. The following types of 'tract' were recognized by all the farm-workers as existing 'categories':

arisco, barro, new tract, várzea, caatinga, massapê, barro de louça, field, resting tract.

From this group, kinds of *soil* were distinguished. *Land* was considered as the general category.



My attempt was then to clarify these 'categories' as they were mentioned by the farm-workers in the successive meetings. 'Categories' such as *caatinga*, *new tract*, *field*, and *resting tract*, which were initially included by farm-workers under the generic class 'kinds of tract', could be distinguished as denominations of tracts for the purpose of describing their use in the configuration of the farm (as described in chapter 2, page 36).

As far as the characterization of the practice of transforming the soil for cultivation was concerned, the main outcome of the discussion about land/soil conducted in phase 1, was then a set of propositions about possibilities for planting, having regard to the nature of the available soil. Thus, farm-people would say as reported in chapter 2 that:

- (a) Manioc is *only* planted in 'arisco' (a sandy soil).
- (b) Sweet potatoes are *always* planted in 'arisco' but can be planted in the *bank of rivers*.
- (c) Cotton is *always* planted in 'barro' (a clay soil) or 'várzea' (a rich clay soil), but *can* be planted in other kinds of soil.
- (d) Beans are *always* planted in 'barro' or 'arisco', but *can* be planted in the bank of rivers.
- (e) Corn *can* be planted in any kind of soil.

In summary, one could say that "*possibilities about planting*" and "*the configuration of the farm*" constituted two *situations* with respect to which *soil* could be discussed (Figure 4.2).

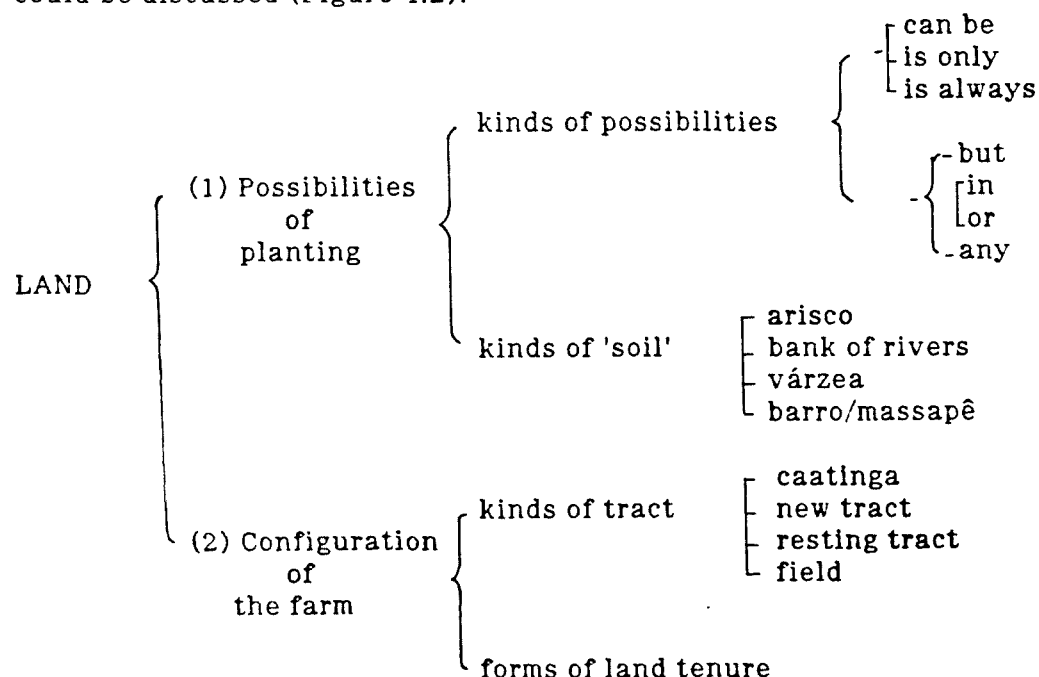


Figure 4.2

Phase 2: Practice of Agriculture

The sessions in phase 2 extended the discussion of *possibilities* for planting, focusing on the farm-workers' *performances* and *motivations*. The issues discussed were encapsulated in questions (1), (2) and (3) of the framework (Figure 4.1), and used to anchor the discussion about soil in this phase. To teach back a description of how and why things are done/happen in terms of the distinction necessity/possibility turned out to be the aim of phase 2.

Following roughly the structure of Table I, emphasis was placed on the conditions which make the soil into an entity affording the transformations presupposed by the terms of the Table. The kind of model of soil underlying the use of the Table for arriving at '*scientific explanations*' was presented to the informants in the form of an *analogy of soil* with a 'piece of cake'. The questions raised by the researcher in this respect were considered by the farm-workers as *non-sense*.³ Two examples are worth transcribing. The initials are F.W. for the farm-worker and C. for the researcher.

Instance 4.1

C. Right. Now... I want to ask you something which does not belong to agriculture; but which can help me to explain what kind of thing I want to know about. Suppose that I have a piece of cake. It is a fruit cake... it has pieces of fruit, it is soft, and so on. You look at it and there are things you can say, such as: "- well, this cake must have flour, because this is something that every cake has; also sugar. It has yeast, because it is very soft. There are pieces of fruit, and perhaps milk or water; and so on." Consider now a 'piece' of ground. If I look at it, what kinds of things could I say that the ground has? In other words, what is the soil made of? What does exist on the land?

F.W. Do you mean inside the land?

C. Yes, on the ground which you plant, where the crops stand.

F.W. Well, this is something I do not know how to answer.

C. What do you think it has... what it should have?

F.W. Now... humm... well, this is something I don't know... yes, I do not know. Now... because the land we are talking about... this land is not the land which I would refer to in terms of *mil covas*; for example 16 *mil covas*, or 20. The land we are now talking about is a land which is not equally good in all its extension. I think that this is the point you are trying to make. The land is not equal in all its extension. It may have a piece which is '*várzea*'... another piece can be '*barro*'; and it can contain '*arisco*'. Also, I could say that there are parts of '*caatinga*'. There are all qualities of land.

C. What can a kind of land have which the other doesn't have? What is it that changes?

³ The same did not happen with teachers, which were able to give -with the help of the researcher- a fair account of the model as presupposed by school science.

F.W. That it is. This is what I don't know how to answer. No, I know nothing about this. The only thing I know is that actually there is no land with the same quality in all its extension. I don't think it would exist a situation in which one could say: "- this land is uniformly homogeneous". One will find every kind.

C. Right. But, for example, if you consider a piece of land, you can say it contains sand, or it contains clay. If it is 'barro', what does it contain? More grains, more water?

F.W. Right, one can find everything. Stones and water. Yes, one can say that the land contains these things.

C. And about living things? Would them be important? What does the land contain which is important for growing crops? This is what I would like to know.

F.W. No, I don't know.

C. Tell me something. When you plant, you plant a seed. Is this right?

F.W. Yes, this is right.

C. After one year you have a plant. What could we say about the soil which could have relation to the fact of a plant going from a seed to...

F.W. This is something I cannot answer.

C. Could one think of it in the same way one would think about a child growing up? I would say that to become an adult a child has to eat.

F.W. Yes, you could.

C. What does it happen to a plant? I wonder... which kind of stuff would we find in the soil which could help...

F.W. To provide the strength that the plant has.

C. Yes, right. What is it that gives strength to the plant?

F.W. I do not understand this.

C. Have you never thought about these things?

F.W. No, never. I was never taught about these things. Because we never talk in these terms. Our life is always of doing things, doing, and doing... and in this way life goes. I have never heard: "the land has this, and this; which is what gives it strength which is necessary to this and this." No, never.

C. Ok. I understand. But tell me something. You know that some people insist that it is important to manure the soil, don't you?

F.W. Yes, this is right. They say it is a good thing, but we never do it. This is not a tradition here.

C. So, you don't know what one adds to the soil when one manures it?

F.W. No, I don't. But if you ask me things such as how long does it take for seeds to germinate, and so on, I am prepared to tell you.

Instance 4.2

[the question is similar to the one in instance 4.1]

F.W. I do not understand your question. ['pause'] It is not an easy question... because there are many kinds of land.

C. All right.

F.W. If you consider the 'arisco', you will find just sand.

C. Only sand?

F.W. Basically it is a sandy land. Sand, whatever its size. If you have 'barro', you can have different slices. It can be more stony; or more soft; or a different one.

C. When... Some people say that it is important to manure the soil. When a farm-worker manures the soil, what does he add to the land?

F.W. I don't know... ['pause'] When... If someone is ill, and takes an injection, what happens? What is he injecting?

C. Right. This is the question I am asking you.

F.W. In other words, the manure can be like a medicine for the land. As when we burn... it is like a fortifier.

C. Right. You say that the manure is a kind of fortifier for the soil... it makes crops

grow. [...] But what is this fortifier made of? Do you know what it is?

F.W. No, I don't.

C. For example, people have anaemia. They are asked to take iron. [...] So, it is a fortifier but it is said to be iron, which is that which makes people get stronger. Other people have teeth-problems. So, it is said that children need to take calcium. Do you think it could be something of this kind which happens? How do you know what is that makes soil get stronger?

F.W. Well, I don't know.

C. Do you think that the intention could be similar?

F.W. Yes, it should be. If you are weak, feeling weak, you then take a fortifier. It is the same thing with the land. It is weak, it no longer produces the same amount. We say that the land gets weak. Then one adds manure to the soil and the land creates strength. From the manure, the land gets strength to make the plant grow.

C. And about the water? Is the water something...

F.W. The water... it is right. If there is no water on the soil, there is no fortifier which works.

C. Are you saying that for the manure to be a fortifier the soil must contain water?

F.W. Yes, the soil must be wet. Because if it is dry... Because... there are two kinds of fertilizers. There is one fertilizer which keeps [he uses the word 'sustains'] the wet. You can use chemical fertilizer in a dry-land that it keeps the wet. One can plant sugar-cane in a tract with chemical fertilizer. It does not matter whether it does not rain or whether the land is weak. [...] The chemical fertilizer always work. It does not need water to be wet. It is wet by its own nature. The manure needs water. With water, the manure gets strong. The chemical fertilizer is made in such a way as to give support to the land. It gives strength to the land, by itself. It is just to prepare the field and to spread fertilizer upon the soil that one will get always a green field.

C. And what happens if one plants cotton, corn or beans in a field which received chemical fertilizer?

F.W. Perhaps it will not work so well because what happens to the sugar-cane is that it covers the whole tract; and the same does not happen with the others. In this case, with the tract uncovered, the fertilizer is exposed to the sun rays and gets weak very soon.

The contrast between these two informants' knowledge of the conditions of plant growth could have been a fruitful line of investigation to pursue.

The distinction to highlight at this point is between the idea of a tract conceived of in terms of a certain number of *mil covas* (suggesting 'area' of the tract to be an exclusive property) and the idea of soil as an entity which could require different forms of treatment and thus afford different productions (suggesting the soil to have inclusive properties).⁴ As the interviews developed, the focus of the discussion shifted from *production* to *productivity*.

⁴ "A determinate property is *exclusive* if and only if each possible part of all its instances instantiates this very same property (under the same determinable).

A determinate property is *inclusive* if and only if each possible part of all its instances instantiates some other property under the same determinable." (Johansson, 1989).

Instance 4.3

F.W. It is not a good thing to have the land too wet. Last year, entire fields of beans were lost because of the water. But if we plant in the 'arisco', this will not be a big problem. There was a time in which I planted 1 *cuia* of beans and produced 100 *cuías*, which is more or less 5 kg. We did not talk in terms of kilos, which is a more recent measure... introduced around 1940, 45. We talked in terms of *cuías*, and 100 *cuías* are more or less 5 litres. It can give more, depending on the kind of beans, that is, depending on the size of the grain. In a similar way, one can say that 1 *mil covas* take 3 kg of beans.

C. Do you say 1 *mil covas* of land?

F.W. Yes. 1 *mil covas* of land take 3 kg of beans. Or more. But if one knows how to plant it will take 3 kg only.

C. 3 kg by *mil covas*. And how many seeds do you plant in one "cova"?

F.W. 4.

C. Do all the 4 seeds germinate?

F.W. Yes, they do. It happens... sometimes, one seed does not germinate. But if the beans are of a good quality, the seeds will all germinate.

C. Do you allow all the four plants to grow?

F.W. Yes, we do. Beans, corn, and broad beans. They are all planted in groups of 4. Some people keep only 1 plant. But I like to keep 4.

C. In which situation it yields more?

F.W. Well... you mean the broad beans or any one?

C. The cotton, for example.

F.W. Well, with cotton it is a bit different, because we plant more grains. We plant always 10 to 15 seeds in each "cova".

C. How many plants germinate?

F.W. The cotton has a very delicate seed. But all the grains usually germinate. The point is that we keep only four plants.

C. Why do you leave only 4?

F.W. Because if we leave all the plants, the bunch will be very crowded and it will not grow properly. It grows but does not produce.

C. And whether does one keep only one plant?

F.W. It is ok.

C. But does it yields more?

F.W. It gives more than if we keep 10 plants... in one "cova".

C. But if one compares 1 plant with four... in which situation does one get more?

F.W. It produces more when one keeps only one plant.

C. You mean that 1 plant alone yields more than one plant in a bunch of four. Is this right? Or it is one plant which gives more than the group of four?

F.W. Let us say that it can produce the same as the group of four ... Because when one keeps only one, the production is very high. If we keep 2, or 3, we would have a difference. But people usually leave 4 plants, because sometimes one plant will die.

In the discussion which followed, the researcher insisted on the comparison -in respect to production- of different organizations of the planting system. Unanimously, the informants were able to recognize that one plant would represent the best situation for production, but that they did not made any effort to reorganize the spacing between plants (that is, to change the area of the culture). Also, informants maintained that planting on a basis of bunches with 3-4 plants would represent the best arrangement for production.

An interesting remark was made by a farm-worker while commenting on the researcher's insistence in focusing on the 1-plant grid. A piece of transparent sectional pad had been used during the interview to help the researcher to represent the kinds of organization of the field suggested by the farm-worker.

Instance 4.4

F.W. Because it is very important... It is important for us to see this. Yes, it is important, indeed. Because the comrade is a farm-worker and he says: "-I know how to do the work." But he plants a very crowded bunch, which he does not prune. And the result is that he does not get anything. When we see this thing you prepared [he refers to the fields drawn on the sectional pad]... this grid... We can see that one plant will produce more... That is, to plant on the basis of 1 plant by "cova" is more productive than to keep crowded bunches. However, I still think that my way of planting is more appropriate [3-4 plants]. Because, in my way, we do not lose in any sense: neither time, nor land.

What is interesting in this comment is the reference made to the *time* invested on the land. In so far as the system of planting is at issue, the comment suggests that, in addition to the spacial arrangement and to the part-whole relation involved, attention should be addressed to how farm-people understand the *area* of planting from the perspective of production. As indicated in chapter 2, the productivity of investment demands an intensification of production. If land is available, the incorporation of new land is one natural tendency, and an *extensive* kind of development usually follows. But if land is not available, mechanisms for creating "new land" become necessary (in which case one would talk of an *intensive* kind of development). *Work* is one of such mechanisms, to which *time* is related. The question of the representation of *work* performed on the land inevitably arises, and will be taken up later in a different context, that of modelling cubação.

As far as the network in Figure 4.2 is concerned, the outcomes of phase 2 could be incorporated and represented as in Figure 4.3.

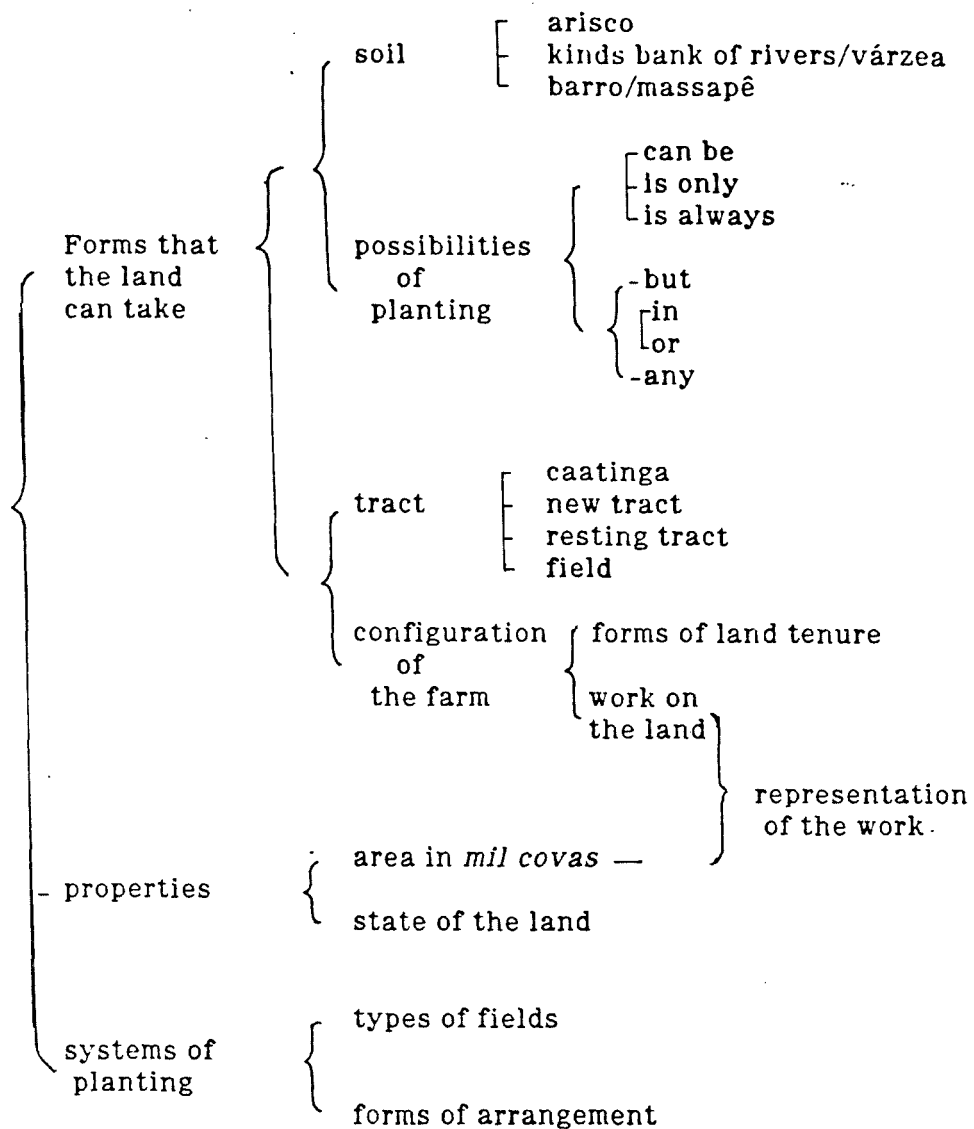


Figure 4.3

Phase 3: Soil and the organization of the field

Two groups of pairs of land attributes (opposite attributes) as in Table 4.1 were organized for discussion in session 3. They defined the state of the land in such a way that attributes in group 1 would refer to land as soil, and attributes in group 2 would be related to tracts of land. The attributes had been all mentioned by the informants in the previous two phases of interviews.

TABLE 4.1: LAND - Attributes

GROUP 1	GROUP 2
(1.a) wet : dry	(2.a) covered : uncovered
(1.b) hard : soft	(2.b) suitable : unsuitable
(1.c) tied : untied	(2.c) sustains wet : soaks up
(1.d) hot : fresh	(2.d) prospers : declines
(1.e) bad : good	(2.e) arduous : easy going
(1.f) strong : weak	(2.f) yields good : resists
(1.g) smooth : uneven	(2.g) turns weak : keeps strong
(1.h) dark : light	(2.h) easy-wet : hard-wet
(1.i) new : old	
(1.j) stony : smooth	
(1.k) healthy : ill	
(1.l) tired : vigorous	

Each type of tract was then discussed with the informants in terms of the above attributes. The idea was to take these attributes as *dimensions* which are (could be) used by farm-people in conceptualizing the kind of entity which is land; and which are (could be) used by people in evaluating the various ways in which land is designated as *soil/tract*: *arisco*, *barro*, *massape*, *várzea*, *bank of rivers*, *caatinga*, *new tract*, *field*.

Two kinds of attempts were involved. One started from a particular form in which land is named, and invited the informant to talk about it in terms of each pair of polar attributes. For example, questions were put in the form:

- . "What can you say about *arisco* in terms of its colour-state; is it more dark or more light? How much?"
- . "Would you say that *barro* is a hot soil or is it fresh? When is it fresh?"
- . "Can you say that *caatinga* is a kind of soil that turns weak or does it keep strong?"

The second attempt asked the informant to consider a given land in relation to other(s) kind(s) previously described in terms of given attributes. Thus, questions were made in the form:

- . "*Arisco* is a dark soil. Is *várzea* a dark or a light soil? Is it dark in the same way as the *arisco*?"
- . "*Massape* soaks up easily. Does a new tract soaks up or is it more like *arisco* which sustains wet underneath?"

The nature and the behaviour of soil were at issue. The following quotation from a farm-worker exemplifies the kind of answer obtained.

"*Arisco* is a dry land; as we can see. But it is wet, fresh and thus crops grow. On the other hand, *barro* is a kind of land... In this time of the year it is very wet... too much wet. But when *barro* becomes dry, it also becomes hard and tied⁵. *Arisco* is wet; but we can also say it is dry, because if it rains today, tomorrow it will not be wet. But it stays wet inside. *Barro* is different. [...] *Arisco* is soft, because if we dig it we find nothing hard. Only after one metre of depth... After one metre, it changes, and becomes hard. [...] But *arisco* is tied; it seems to contain cement. Bricklayers use it sometimes. [...] But certainly it is not like *barro de louça*. Because *barro de louça* does not untie. [...] *Arisco* is a fresh land. It is not like *várzea*, where a man can not walk, so hot it becomes. [...] Some time ago, *arisco* was considered to be a bad land. Nowadays it is regarded as a very good land. Because nobody used to spread manure upon the soil. *Arisco* is good to manure because it is smooth. So, it is considered to be very strong today."

The main outcome of phase 3 can be summarized in the following statements.

- . Land as *soil* has properties. *Arisco*, for example, in addition to *being of sand* contains some property-instances: it is wet, dark, soft, healthy, and so on. Thus, land is not soil if it does not have properties as those listed in Table 4.1 (group 1).
- . But these properties must themselves be sustained by something in order to exist. Kinds of *soil* (in terms of the constituents which give to the soil its state) play such a role.
- . A similar account could be made for land as *tract*, in which case, kinds of tract would be thought of in terms of the functioning of the tract in the configuration of the farm. Thus, *caatinga* would contain property-instances such as: to be uncovered, to decline, to turn weak, and so on.
- . In both cases (soil and tract), a property can be subsumed under another in such a way that a kind of soil can be turned into another. For example, *arisco* can be turned into *barro*; a *new tract*, into a *field*.

As far as soil is concerned, results suggested the following ways of looking at it as an entity affording transformations (Figure 4.4).⁶

⁵ The term used was "ligado". The sense is that the soil becomes "glued together" like cement.

⁶ The network describes kinds of answers and thus should not be taken as an attempt to represent categories for describing ways of looking at an entity which affords transformations.

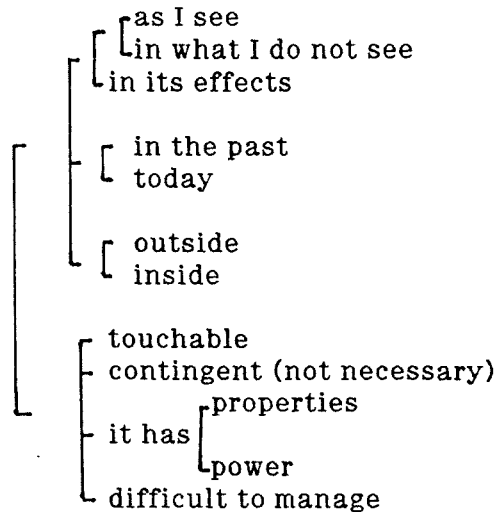


Figure 4.4

In addition to the discussion of land in terms of attributes, in phase 3 emphasis was given to the organization of the field. It was with respect to this discussion that the relevance of distinguishing between *new tracts* and *fields* arose.

The above discussion does not yet concern social variables, and these were also considered important.

4.4 THE RESEARCHER'S QUESTIONS ABOUT CUBAÇÃO.

It was as a result of the attempt to uncover further contingencies in terms of such social variables, that *cubação* was 'discovered'. As will be seen, *cubação* then replaced soil as a subject to be pursued in depth. The above discussion of soil will not, in this thesis, be taken any further, but indicates, together with the results in chapter 2, many fruitful further lines of investigation.

To have said that *cubação* was '*discovered*' denotes two things: first, that I did not know about the existence of *cubação* as a method for reckoning the numbers of '*mil covas*', the standard and traditional unit for measuring land; and second, that *cubação* could represent a given domain for conversation with people in order to characterize the nature of communal knowledge. Embracing these two senses, I would say that I did not know that *cubação* was a *discursive practice*.

A characterization of *cubação* is initiated in chapter 5, results and implications being discussed in further chapters. In this section it only remains to introduce

the kinds of questions I posed which, resulting from the 'discovery', turned out able to be treated as research questions in the rest of the empirical study.

4.4.1 Cubação in two words.

Cubação is a method for reckoning land, used by illiterate farm-people in the NE of Brazil, for more than a century. In this Region, land is expressed in units of 'mil covas' (in Portuguese, this would be translated as "a thousand of pits on the ground"; in its NE-Brazilian use, the word 'covas' -which means pits- expresses the meaning of a small elevated manloc bed)⁷. Thus, cubação embraces a procedure (typically algorithmic) for estimating the number of 'mil covas' (it will be fully described in chapters 5 and 6).

The procedure is generally used by farm-workers in various situations in agriculture, including commercial transactions with farmers, agricultural technicians and inspectors of the Brazilian Bank. It is orally learned and orally transmitted from one generation to the other, through the work of agriculture; and possesses no relationship with the pedagogic discourse which goes on in schools.

The underlying 'rationale' of the method of cubação turns out to be the same as one which has existed in different places and times in history, such as the 'acre-system', the 'Roman-system' and the 'Aztec-system' of surveying.

When algebraicized, it can be shown to estimate correctly the area of an infinite class of shapes (the general criterion for which cubação applies will be introduced in chapter 6). Belonging to such a class, one will find "four-sided" shapes whose pairs of opposite sides add equal, and whose shape-parameter is equal to $1/16$ (which is the shape parameter of the square)⁸. Within these shapes, one will find, for example, the sector of circle drawn in Figure 4.5-A, and the segment of circle in Figure 4.5-B.

⁷ The related term 'cova-de-mandioca' (pits-of-manloc) designates small clouds announcing a tempest.

⁸ The shape-parameter is defined to be the area of the shape whose perimeter is equal to 1 unit in length. Thus, the square whose perimeter is equal to 1 unit has sides equal to $1/4$, and area $1/16$ square units.

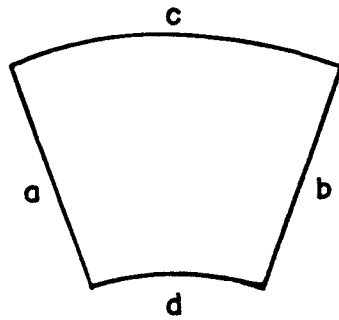


Figure 4.5-A

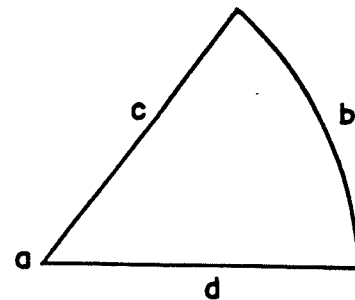


Figure 4.5-B

Cubação is *unique* in that it estimates the area of these shapes in one single application of the procedure, without making use of irrationals. It is *senseless* in that the succession of steps involved in the calculation, establishes no understandable relation to the task of calculating the area of a square-shape. It is *naïve* in that it can be shown, in its use by farm-people, to be non-accurate (sometimes over-estimating; in others under-estimating the actual value). It is *intriguing* in not containing an explanation for the meaning of the expression 'mil covas'. It is *instrumental* in offering a potential for understanding geometrical reasoning, and for understanding the power of knowledge in society.

4.4.2 Cubação: levels of elicitation and the main questions.

The method of cubação came to my knowledge during the interviews with farm-people, while I was trying to raise the main social contingencies under which they think about their practice of cultivating the soil for growing crops. As said earlier, the first meeting with each participant of the main group started by situating individuals as small producers, which necessarily requires information about forms of ownership and relations of work.

Cubação was referred to by a farm-worker (it was his first meeting in five; and the sixth in the total of forty nine meetings comprising the whole study), after being asked about how he knew how much land he owned.

In trying to define his situation as small owner, I inquired about the area of his property, as the area of the possessed land is a factor used to distinguish people

as peasant or not. During the meeting, the farm-worker said he owned "some land". Asked about how much was this "some", he answered: "more or less seven hectares". Because it is unusual for farm-people to express area in hectares, I took the "more or less" to represent an approximation to the value of the area expressed in 'mil covas' and asked directly: "how many mil covas do you own?" His answer was again: "more or less 3 'mil' and 300 'covas' for each hectare". Certainly to have more or less seven hectares of land (or fourteen, as he came to say he possessed a second tract of "more or less seven hectares") did not constitute a problem for categorizing the farm-worker as a small-owner. But the insistence in using the expression "more or less" seemed to be rather peculiar. Farm-workers could be illiterate, but in questions involving land, half a metre can lead to serious contention. So, at the end of this first meeting, after having talked about the expected issues, I insisted again with the question of the area, and asked:

"Suppose you own a tract of land. Do you know how much land do you own? That is, how many hectares do you own?"

His answer was straightforward: "Yes, I do." The dialog which followed is worth transcribing. The initials used are C. for the researcher; F.W. for the farm-worker; and T. for a teacher who, having staying quiet all the meeting in a nearby sofa, could no longer remain silent and made a remark.

C. How do you do to know how much land do you own?

F.W. I measure it.

C. Do you measure?

F.W. Yes, I do; I measure it with a '*braça*'.

C. How is this?

F.W. A *braça* is equal to ten '*palmas*' [1 palmo being the width of a spanned hand]. We call this a *braça*. Then, it is just to go on measuring.

C. But you measure what. Do you measure the boundaries?

F.W. Yes, the four sides of the tract. Then, let us say that we got a side with 40 *braças*; this gave 50; and the others 80 and 70 ... I stop here. Then I take, I add, I 'cubo'; and I know how many 'mil covas' I own.

C. How did you call this? Is it to 'cubar'?

F.W. Yes, to 'cubar' ... *cubação*.

C. How do you do it, this reckoning?

T. It is to do the proper calculation. He knows how to do it.

F.W. Yes, I do.

C. How is this calculation; please, explain it to me. How do you proceed?

F.W. I know how to do. In this case, south, it gave 40 ...

C. Do you need a piece of paper? Do you write something or it is head foremost?

F.W. Yes, I do, I do. But are you sure all this will not take too much of your time?

The farm-worker was right. From that day, in every single meeting with the participants one will find some information about *cubação*. But on that day, the

dialog ended abruptly. The description of the few steps involved in the calculation did not take more than two minutes. Also, I did not need more than that to realize that the procedure had provided an incorrect value for the area of his exemplary tract. The meeting was over; but the effort to make sense of that distinctive method had just started. What happened next in relation to cubação can be indicated as follows.

The local unproblematic level. First, it was necessary to establish that cubação was something which 'really' existed for the functioning of planting and land. As said earlier, I was aware that farm-people currently express area in number of 'mil covas', but I did not know about their method. Thus, in a first step, my attempt was to stay entirely with cubação seen as natural for the farm-worker and to conceptualize it in its unproblematicness: bringing out that which is assumed unproblematically; investigating the situations to which it applies (what it is used for). And to consider 'mil covas' as the unproblematic unit in which area is described. During this phase, it was essential to guarantee *understanding* between researcher and informants. In addition, I would have to define the possible perspectives from which to explore cubação as a domain for further investigation. Two perspectives were particularly important.

The social-researcher perspective. This was essential to clarify the proper nature of cubação as communal knowledge, and involved questions such as:

"what would be the role of cubação in the social relations? How would it be transmitted, sustained? What would be the character of expertise, in relation to cubação?"

The science-educator perspective. The perspective of schooling was an obvious one to consider, particularly in its relation to communal knowledge as the following questions exemplify:

"how do farm-people think about area? What would be the relations of common knowledge to school knowledge in this area?"

The problematic level. Contrasting with the local unproblematic level, there was the level at which the researcher, forgetting farm-workers all together, became worried about cubação, for she was an educated person who calculates in a different way. From her point of view, the method was incorrectly applied in almost any situation. Also, she could not find a clear and definitive explanation for the use of the expression 'mil covas'. Thus, the researcher was left with questions:

"how would farm-people react to different geometrical shapes? What would this tell us about their ideas about cubação and about their understanding of the concept of area?"; and

"how do we interpret 'mil covas'? How is 'mil covas' to function in relation to both planting and geometry?"

4.5 CONCLUSION

The questions about cubação listed above could, in themselves, motivate different kinds of interesting studies. This thesis could, for instance, be seen as an attempt to make of cubação an investigation in ethnomathematics.

But the problems which in fact motivated my approach to the study of cubação, are of a different origin. If assumptions are made which support the idea that social interactions construct typifications and recipes which make reality, this does not imply that I am necessarily involved in showing how such a process happens.

The problems addressed in this thesis started from a concern about schooling, framed on the ECPC-Project's attempt to implement the programme of Agriculture. It was in an attempt to understand how farm-people think about land that cubação turned out to be interesting.

The results on soil, whilst not pursued further here, do already illustrate the important differences between communal knowledge and the frame of reference one would use if approaching the topic from a scientific viewpoint. The obvious example is the rich and structured set of terms used to characterize types of soil and of tract; terms unknown to science, and (broadly speaking) to educated people. Yet to discuss agriculture with people in any other terms is not to discuss agriculture at all, in their understanding of it.

The study concerning soil has suggested that people's discourse about land presupposes an ontology embracing natural kinds; which characterizes a particular way of conceiving soil, distinct from the idea of area given in *mil covas*. As soon as one becomes involved in trying to understand cubação, it becomes a problem to try to make sense of the fundamental reasoning of farm-people which could generate their accounts about land.

Also, the study pointed out the relevance of clarifying how *work* is represented by farm-people in organizing their systems of planting. In this situation, and as far as productivity is concerned, the *area* of planting can be identified as related to the *time* spent on working on the land; but how exactly such a conceptualization works when decisions need to be made, was not clear.

In order to get a better understanding of these issues from the perspective of science education, a model of cubação should be investigated in which the notion of *area* -as we learn in schools- was to be re-thought.

CHAPTER 5

CUBAÇÃO

5.1 INTRODUCTION

This chapter attempts to contribute two essential dimensions to our understanding of cubação; namely, the discursive character of the geometry of cubação; and the social relations of ownership and work in which land inheres. Thus, it is argued that cubação is in a sense just geometry, and that an analysis of the procedure of cubação should pay attention to those principles which might determine the particular ways of solving problems of area by this method. On the other hand, it is argued that cubação is a geometry situated historically and socially, with living meaning in an actual context. It is a geometry whose uses relate to fundamentals of human social life: food, work and ownership.

Two perspectives for exposing ways of approaching questions about the status and meaning of cubação, as a *geometry*, are discussed: the present time and historical origins. As a *situated* geometry, the different ways in which cubação is involved in the material working out of social relations are considered. Starting from social forms of relating to the land, the ways in which cubação participates in negotiations within the sphere of production are analysed; followed by an account of the ways in which cubação has to confront a distinct *logic* of measurement materialized in the hectare-system.

The chapter is organized in four parts. First, a few words are necessary to introduce the method of cubação (section 5.2). Its communal character is addressed in section 5.3. Perspectives on the geometric status of cubação are used to delineate the main lines of inquiry for approaching and reporting results (section 5.4). Finally, in section 5.5, methodological and problematic issues are briefly outlined.

5.2. A LOOK AT CUBAÇÃO

Cubação is a method used by farm-workers in the NE of Brazil to determine the extent of tracts of land. For more than a century it has been routinely performed as part of the work of agriculture, wherein the organization of fields for planting requires sets of tracts to be delimited.

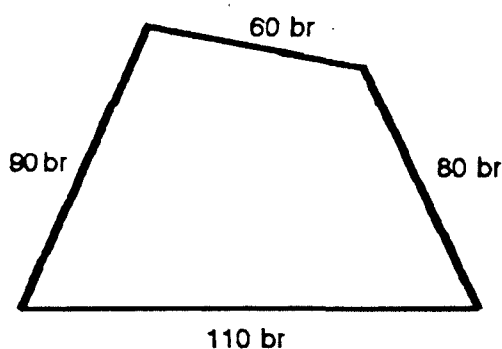
In this region, amount of land is usually expressed in numbers of *mil covas*, an old unit which has been known since colonial times to be approximately one third of a hectare, and to represent the amount of sugar cane transformed into sugar in a mill, in one day. Also, *mil covas* is known to be equivalent to a 'quadra' (square) of 625 square *braças*, 1 *braça* being approximately equal to 2.2 metres¹.

So, if we take this 'quadra' as a unit of area, tracts of land can be easily reckoned in *mil covas*: if lengths are measured in *braças* (br), one possible solution is to find the area of the tract in *square braças* and then to transform the result into *mil covas* (1 *mil covas* = 625 sq br); if lengths are given in metres, the result of the calculation can be obtained in hectares which is then converted to *mil covas* (1 ha = 3 *mil* and 305 *covas*). As relations between systems of units can be established, no question of raising discord about results is expected to arise if farm-workers keep using the traditional unit: the hectare-system functions as normative.

However, to do this is not at all to perform cubação. The method is not recognized as such if a particular procedure is not pursued. This is a procedure in which measurements of lengths are carried out in the field in *braças*; and then, by means of a unusual succession of arithmetical operations, the 'area' of the tract is obtained in *mil covas*.

Roughly speaking the sequence involves: (a) to add -two by two- the opposite sides of the tract, which in some way must be conceived of as a quadrangle; (b) to multiply the results of (a); (c) to multiply by 4; (d) to divide by 10; and finally (e) to check the results so obtained. For example, applied to the situation in Figure 5.1, cubação would give the area of the tract as 11 *mil* and 560 *covas* (or 3.5 hectares); and the following steps would have to be performed:

¹ Andrade, M. C. de (1980) *A Terra e o Homem no Nordeste* S. Paulo: Livraria Editores Ciências Humanas.



step 1: $(90 + 80)$
 step 2: $(60 + 110)$
 step 3: $[(90 + 80) (60 + 110)]$
 step 4: (28900×4)
 step 5: $(115600 / 10)$
 step 6: 11560
 step 7: checking sums
 answer: 11 mil and 560 covas

Figure 5.1

In other words, if we think of sides $a = 90$ br, $b = 60$ br, $c = 80$ br, and $d = 110$ br, the area of the tract is given by $\{[(a + c) (b + d)] 4/10\}$.

What is unusual about this procedure is that, while the first two kinds of solution do distinguish between "transforming units" and "calculating the area", (tacitly understood in terms of Euclidean procedures), cubação refers to a sequence of operations in which such a distinction is not required.

One might say that the procedure, rather than being unusual, appears senseless, as soon as we realize that the succession of steps establishes no understandable relation to the task of calculating the area of a quadrangle. The whole process does not reasonably grow out of the inner requirements of the situation in which area is to be estimated and understood; it appears to be blind to the issue of "how the area is built up structurally out of a fundamental unit", at least in just this form.²

² The situation resembles those "ugly procedures" referred by Wertheimer when discussing aspects of his theory of problem solving in relation to the problem of finding the area of a parallelogram (Wertheimer, M. *Productive Thinking*, 1959, ch. 1). He distinguishes between sensible solving, where the understanding of the subject matter does happen by grasping the relations structurally required in view of the whole; and solving by external procedures, in which case blind thinking proceeds without the realization of the inner relatedness between means and ends. For example, as far as a rectangle of sides a and b is concerned, the sensible procedure of multiplying " $a \times b$ " is contrasted with the following ugly procedure:

1. First subtract b from a
2. Square the remainder
3. Square b and subtract it from the last result
4. Square a and subtract it from the last result
5. Multiply it by -1

$$\begin{aligned}
 &a - b \\
 &\quad (a - b)^2 \\
 &(a - b)^2 - b^2 \\
 &(a - b)^2 - b^2 - a^2 \\
 &a^2 + b^2 - (a - b)^2
 \end{aligned}$$

Those of us who have been socialized into the mathematical Euclidean way of doing calculations of area have to hold as evident two things: one is a notion of what an *area* is; another is a notion that there is a *method* which is underwritten by being apparently able to be demonstrated to be the way to reach the *area*-idea you started with. Euclidean area and Euclidean method are tied together by a supposedly logical structure; and we have got to penetrate the organic whole of the issue before it makes any sense. We can not understand Euclidean area without knowing the method and we can not understand the method without the concept of area. And both can not be completely understood if we do not keep in view that what purports to hold them together is a logic. 'Area' belongs to an integrated theory. Thus, as soon as one introduces another method for estimating area, this theoretical circle is interrupted or suspended. At this moment the question immediately arises of what method is this? How does it work? What is its nature?

If we conceive of cubação as a feasible -indeed, actual and operative at the social level- method for reckoning area of land, the Euclidean circle of logic for thinking of area is interrupted and we are obliged to make a hypothesis about the method by which cubação is done. What kinds of assumptions are made? What is involved in doing that? How do we interpret results which are obtained in '*mil covas*'? In Portuguese, the word '*cova*' means 'the planting place of seeds' and '*mil*' means 'a thousand'; so, how is *mil covas* to function in relation to planting? Is there in cubação any relationship between area and the number of planting places such as *mil covas* = 1000 planting places?

If any relation is to be thought of between cubação and teaching area in school, the issue addresses, certainly, a relevant point. How would a science teacher explain it? How can one think of it in relation to what is taught about area in the primary school?

(make it positive)

6. Divide it by 2

ab

What makes this procedure ugly -says Wertheimer- "is not the great number of steps; neither the incorrectness of the operations involved (they are correct), or the lack of generality of the procedure (it is generally true), or the lack of demonstrative proofs (it can be proved geometrically). It is ugly by contrast with the sensible procedure for which the essential thing is to see the area structured in accordance with the characteristic form of the figure. Thus, "*a x b*" is not simply a multiplication of two terms, for one of them *means* the number of *squares in one row*, the other the number of *rows*. The two terms in the multiplication have different structural and functional meaning and unless this is realized the formula, even the meaning of the multiplication itself, cannot be understood."

Alternatively, one might characterize the procedure as naïve rather than senseless, as soon as the attempt to apply the procedure to different cases indicates its non-accurate nature. In almost all actual situations results are only approximations, some of them being quite sizeable. Regarding its widespread and significant use in practices and negotiations in agriculture, the question arises of how valid cubação is as a method for estimating the area of land. How can one make sense of it?

However, this fact does not seem to be problematic for farm-workers. If this is so, how would farm-workers react to being asked to do cubação on various different shapes for which it 'falls'? What would this tell us about their ideas about cubação?

5.3 CUBAÇÃO AS COMMUNAL KNOWLEDGE

At first sight it seems a trivial fact that for the purpose of establishing agreements concerning ownership, land must be delimited and reckoned. All one would need is a competent and well-reputed surveyor. People are free to negotiate land, and laws exist which regulate formal agreements. This is what a superficial approach to the question indicates; but as revealed in chapter 2, there is much more to the appropriation of the land in Brazil than this level of analysis makes evident. The structure of land holding and the actual forms of land tenure in SPP were shown to express contradictions which are to be understood as *social facts*.

It was pointed out that to try to frame in a network the intricate picture of social relations that arises in the Brazilian rural world is, at least, a hard task. Complex and controversial is the interpretation of the diversity of ways in which the agricultural production is organized within the country and made specific in each particular Region. But in so far as cubação is concerned, the social connotation that land acquires in the process of its appropriation becomes rather important. It is exactly in the realm of social relations that different perspectives of practising cubação can be distinguished and discussed. The social forms of relating to the land constitute then a background context from which cubação emerges in a significant means for analysis.

Actually, as an outcome of social interactions and negotiations under the peasant mode of production, cubação can be best characterized as a *discursive practice* in relation to which measurements of area are carried out. It is possible to distinguish two instances of practical application. One in which the system of measurement

embedded in cubação represents a necessary and sufficient condition for carrying out negotiations between farm-workers and farmers. And other in which cubação has to confront, explicitly, the methods of surveying introduced in the community *via* integrated programs for rural development (of investment and education).

While in the former case cubação can be practised with no reference to the hectare-system of measurement, in the latter it has to tolerate its influence. In this case, two levels can be distinguished. At one level, only equivalence of units are necessary. At a deeper level of thinking the procedure for estimating area is challenged and the traditional and the official practices have then to confront each other. Expressed in its reverse form, the hectare-system *cannot* ignore cubação.

But there is a unique instance of confrontation in which the *discursive* character of cubação turns out to be *silent*. This happens when children have to learn the metric-system at school. In this case, cubação is simply *ignored*.

5.3.1 The social forms of relating to the land and cubação.

In so far as one moves the focus of the analysis to the social level, instead of *forms of ownership* (as described in chapter 2) it would be more adequate to talk about *the social forms of relating to the land*, as to the variety of forms of ownership corresponds a variety of relations of work.

Actually, "*fazendeiro, arrendatário, posseiro, pequeno proprietário, morador, meeiro, sócio, comodatário, diarista, empreiteiro, and empregado*" (as defined in the network of Figure 2.9, page 46), should be more adequately understood as *social categories* which relate to one another from the perspective of the social division of work, and in which case land is essential to a person's social definition. In other words, these are social categories which define people in relation to land.

In this perspective, land and man constitute a unique and inseparable reality, in which there is, necessarily, at least one tract of land for each peasant. The challenge these people face is to know what tracts they 'own', what possibilities for living these tracts represent, and under what conditions people will hold them.

It is exactly at the concrete level at which these questions are sorted out – namely, the personal level of establishing agreements and relations of work – that cubação

relates to the appropriation of the land; how exactly, is the point I will attempt to clarify.

There are five categories of *pequenos produtores* which are specially fundamental for this analysis: *arrendatário*, *meeiro*, *comodatário*, *diarista* and *pequeno proprietário*. In the first place, they are important because taken together, they represent a major category which includes a vast contingent of people who have -in different degrees- been expropriated from their means of production (namely, tools and land). It is to this category of people that cubação belongs.

Secondly, in so far as my interest is to describe how exactly cubação relates to the appropriation of the land, these five categories emerge as satisfactory for establishing all the variations in which cubação has been practised. Thus, the other categories can be referred back to these variations, and the analysis can be made more general.

Basically, such variations can be brought out by trying to answer two questions:

- . "how are agreements established in each case?"; and
- . "in what ways is cubação involved in them?"

A brief description of the five categories follows, from which these questions will be discussed.

5.3.2 The five basic categories and the requirements for reckoning land.

Without tools and land, the *diarista* can only sell his labour-force in days of work. He does so mainly to the *fazendeiros*. But at certain critical periods -like preparing the land or cropping- he will also have his work contracted by all the others. There are two ways in which the *diarista* can be paid: in terms of *diárias*, when the payment is made by the number of work-days, independently of the work done; or in terms of an *empreitada*, when the *diarista* will receive payment, by having completed a given task which requires several days. In both cases, prices are fixed on the basis of the amount and the physical cost of work required in one day. These amounts can vary with the kind of soil, the kind of crops, the type of activity and the system of planting, in particular to the way in which the field is to be organized. Both the *diarista* and the owner know *a priori* what patterns can be expected from different combinations of such conditions. It is when they make an offhand appraisal of the costs for particular patterns, that cubação becomes

relevant to the discussion. For that, to each pattern of task, they will immediately associate, as given, a possible number of *mil covas*/day. It is the price of a 'mil covas'/day that is crucial to the agreement. Also, but differently, cubação is involved when the agreement requires a particular extension of the tract to be worked, which usually happens in the case of *empreitada*. In this case, the worker must be able to reckon the total number of *mil covas* then worked.

On the other hand, *meeiros*, *arrendatários* and *comodatários* have in the land their 'natural place' of work from where they extract most of their families' subsistence. For that, part of their tracts will be allotted for planting the *lavoura* which usually is composed of the traditional cereals for home consumption: corn, beans and manioc. They know their standard requirements for food (which are usually far from any standard recommendation of a health organization), and they refer to them in terms of numbers of *mil covas* to be cultivated. In other words, amount of food is related to production and expressed in *mil covas*.

But as they pay for the land, *meeiros* and *arrendatários* will have most of their land used for making a profit. Again, the agreements on the amount to be paid—either in money or in production—takes place by defining a given number of *mil covas* as a fixed reference. The boundaries of the tract to be used are established in *braças*, in such a way as to enclose a suitable area for raising sufficient crops both for paying a rent (money or crops) and for cereals for their own consumption. As the system of planting is imposed by the owner, a given system of work is consequently established, which is usually reckoned by the number of *mil covas* related to the expected production; for example, a farm-worker would say: "I have a tract of 3 *mil covas* of corn and a tract of 5 *mil covas* of cotton". And these are not two different pieces of area, but two systems of crops which are planted over the same tract, in two different 'grids'. It is each possible composition of grids that defines the system of work.

As the *arrendatário* establishes a legal ownership, he must register the contract in a Register-office. For that, he needs to provide details about the boundaries and the area of the tract. Cubação is the system he knows and it is by transforming *mil covas* to hectares that he will be able to provide this information.

To delimit the tract is also a requirement to be satisfied by the *comodatário*, whose agreement requires him to give the tract back after one or two years; not virgin, but prepared for planting.

The *pequeno proprietário* owns the land but he is not the same as a big owner (*fazendeiro*). His production is family-based in the same way as the three former categories. Also, he is not the same *pequeno proprietário* of the old times: his dependency on money has increased and thus also his debts. His family suffers the same degree of hardship and is exposed to the same intense regime of work as those of *meeiros*, *arrendatários* and *comodatários*. In the same way, he depends on having food and on making a profit. And so, he is involved with several tasks which require cubação: in adopting a system of work, in establishing boundaries, in estimating production, in predicting amount of labour, in reckoning land. Sometimes, his tract is so small that he has to rent a tract for making profit. However, because he is a legal owner, he must both register (cadastre) the tract in a Register-office and pay tax. Once more, cubação comes up to discussion: for the cadastre, he needs to state the limits and area of the tract; for estimating the tax, he needs to be able to give the percentage of planted and unplanted area.

5.3.3 How cubação is involved.

Trying to summarize, I would say that there are nine main *tasks* -proper to small producers- in which cubação can be present. They can be described as:

(1) *To fix the amount of rent.* Letting/renting negotiations are established on the basis of an estimated production. In this case, the number of *mil covas* not only indicates the area of the tract, but provides a reference for establishing the amount to be paid.

(2) *To adopt a given system of work.* A system of work refers to the combination of different grids of cultures (crops) which exist in a given tract. Allied to the size and characteristics of the land, the degree of work involved in a given system is an important factor to be considered by farm-workers and farmers in most kinds of negotiations. The description of a given system of work is usually made in terms of the number of *mil covas* for each crop. For example, in a tract of 16 *mil covas* (in area), a farm-worker can plant 16 *mil covas* of corn (2601 plants); or 16 *mil covas* of corn (2601 plants) plus 11 '*mil*' and 920 '*covas*' of beans (7600 plants); or 16 *mil covas* of beans (10201 plants). Each possibility makes a different system of work.

(3) *To describe boundaries.* Boundaries are required to be described in the metric system. They include both information about perimeter and area. For farm-workers this information is always obtained *via* a transformation of units from *mil covas* to ha and from *br* to metres.

(4) *To establish boundaries.* Tracts should be delimited for planting, which means that a given area should be physically embraced. The sides of the tract are established in order to guarantee the expected number of *mil covas*.

(5) *To estimate the percentage of planted and unplanted land.* Again, this is information which should be given in the metric system. *Mil covas* is the actual estimated answer which is then converted into hectares.

(6) *To define the amount of crops to share.* Sharing is one of the oldest forms of labour payment in agriculture. Old also is the way of referring to the amount of crops to be shared: *mil covas*.

(7) *To predict amount of labour.* Amount of labour is always an important factor to be considered in agriculture. Conditions of work can change, market rules can change, but *mil covas* are still the reference-entity for talking about it.

(8) *To reckon worked land.* Worked land must always be reckoned. It can describe the area occupied by the crops, or the amount of labour spent, or the amount of harvest. *Cubação* is the method used and the result in *mil covas* is the answer in all these cases.

(9) *To estimate production.* Amount of production does not refer only to the amount of harvest. Production has a stronger meaning, including also the historical conditions in which the harvest is obtained. *Mil covas* is also used to denote such a meaning.

As some of these tasks are part of agreements, it is possible to think of them as existing also in relation to the owners or contractors. In Table 5.1 they are addressed for each category.

TABLE 5.1: TASKS

Cubação is required	fazend.	arrend.	posseiro	peq.prop.	morador	meieiro	sócio	diarista	empreit.	empreg.
To fix the amount of rent	X	X	-	X	-	-	-	-	-	-
To adopt a system of work	X	X	X	X	X	X	X	X	-	X
To describe boundaries	X	X	-	X	-	-	-	-	-	-
To establish boundaries	X	X	X	X	X	X	X	X	X	X
To estimate cover, uncover, land	X	-	-	X	-	-	-	-	-	-
To define amount of crop to share	X	-	-	X	X	X	X	-	-	-
To predict amount of labour	X	X	X	X	-	-	-	X	X	X
To reckon worked land	X	X	X	X	X	X	-	X	X	X
To estimate production	X	X	X	X	X	X	X	X	X	X

5.3.4 Internal and external relations.

So far I have been describing two kinds of social relations in which cubação participates: one internal, related to those practices concerning agreements between farm-workers themselves; and another, external, in which relations are imposed by the owners or by institutions of public services (such as the Register Office or the 'INCRA': Instituto Nacional de Colonização e Reforma Agrária). Cubação not merely mediates interpersonal relations between equals, but also has existence in relationships with agents of external culture. While in the former case its use does not require equivalence to any other system of measurement, in the latter a transformation of units and a comparison to 'Euclidean procedures' is rendered necessary.

This latter case -in which cubação needs to be contrasted with a different system of measurement- is also present when farm-workers have a relationship with the Brazilian Bank for getting agricultural mortgages, or with EMATER for receiving technical advice. Despite being distinct in administrative terms, in the community these two institutions articulate efforts towards the development of production.

For the Brazilian Bank, the farm-worker has basically to provide information about the amount of land which is to be invested in. As he has to provide this information in hectares, a transformation of units is required. Also, he has to negotiate with the bank the exact extent of his land in cases in which problems arise. This can happen before the credit is decided -particularly if it is the first attempt to get credit or if a large tract is involved- or after, if the harvest is not enough to cover the debt (as is well known, the available credit is based on a forecast production which is supposed to render sufficient outcomes for paying the debt). In this case, the negotiation involves the confrontation of two different systems of measuring area.

In a less problematic context, the same confrontation happens when the farm-worker interacts with the agents of the EMATER. The adoption of modern techniques is synonymous with the growth of production through intensive development in which productivity is intended to be achieved by increasing production per unit area (which can happen by increasing the response of soil/plants; by mechanizing tasks; and/or by incorporating/ assembling land). The *hectare* is the official system through which the agents of the EMATER 'think' and develop their programmes.

5.3.5 The confrontation of systems: metric versus cubação.

It is fundamental to the nature of cubação that its pursuit contrasts with the metric system. Cubação can constitute an autonomous system, but its existence within agricultural production cannot avoid demarcation disputes between cultured and indigenous methods of measuring land. Thus, the first thing a farm-worker will state very clearly, is that there exist two kinds of 'cubação': one in *braças* and other in *metres*; a distinction which shows that farm-people have indeed a sense of where the limits of their method lie.

Farm-workers insist that they do not know how to do cubação in *metres*. But there is a subtle aspect to be considered about what it is taken to be 'the metre-system'. Asked about the area of a tract for building a house, or about the volume of water reservoirs, farm-workers will make use of traditional geometrical formulas of squares and quadrangles (deriving the formula of a triangle from the rectangle). However, these metric procedures used by farm-workers are never referred to as being the procedures used by technicians and inspectors when they measure in hectares. It seems that, for farm-people, the hectare-system is not fundamentally different from cubação. What is different are the methods of measuring and the way in which results are expressed. While in cubação it is not possible to establish a correspondence between the number of 'mil covas' and planting places, in the hectare-system such a correspondence naturally follows.

This does not mean that farm-people are unable to make sense, for a given tract, of the number of 'covas' obtained by cubação as if they were planting places. Freed from the square grid which is imposed by the practise of planting *fields*, farm-workers prove to be extremely skilful in 'tessellating' *new tracts*, in order to attribute them a certain number of plants (see chapter 2, p. 36, for the distinction between *fields* and *new tracts*).

But there is a geometrical shape which seems to be unattained by cubação in any imaginable sense: the circle. Circular shapes are beyond the scope, not of cubação, but of the farm-people's expertise. It is not simply recognized as a non-prescribed shape by cubação (as a counterpart to the hectare system), but it is recognized to be out of the scope of masters in measuring land (which can include experts in the hectare system). This does not mean that circular forms do not occur in measuring land, but that the skilful performances it requires are what constitute a real metric system in *opposition* to cubação.

5.3.6 Expertise in cubação.

When speaking of cubação, every person in SPP has something to tell us. Generally speaking, people will say that this is a traditional method performed by farm-people in measuring land; that it makes reference to the four Cardinal points; that there is a '4' to be considered in the calculation; or that it is a way of knowing the number of 'mil covas'. These are prevalent aspects of the knowledge of cubação in the community. But they make nobody an expert, if the person does not know how to use the method in actual cases.

To be an expert in cubação is, then, to be able to operate the method in actual situations such as those described in section 5.3.3. Some cases are easy to solve; others more difficult. Those involving quadrangle-tracts are usually unproblematic in that they represent cases to which the method is supposed to apply. These, any expert in cubação knows how to do. Also, in respect to quadrangles, any expert in the hectare system knows whether a farm-worker falls or not in a particular solution.

Recurrent situations for which contentions arise require a more experienced expert: one who can have a more refined control over the conditions to which the method applies; and thus, can arrive at similar results obtained by the hectare-system. Farm-workers who are frequently called to mediate transactions, and who have the status of community leaders, usually solve disagreements by exercising their authority; which does not hold only among their peers, but among technicians and inspectors of the bank as well.

5.4 APPROACHING AND REPORTING CUBAÇÃO

There are two lines of thought which are particularly interesting for exposing ways of approaching questions about the status and meaning of cubação, as proposed as issues in section 5.2. One can see cubação against a historical background in which parallels are established between both ancient formulations of mathematics and greek geometry; and address a perspective of alternatives. The other line looks at cubação as an autonomous system without alternatives, as it is at present as an actual functional element of daily discourse.

It is basically from these two lines of inquiry -which are now briefly described- that I shall propose the main perspectives for reporting results.

5.4.1 Cubação as an autonomous system.

I start by focusing on the practical discursive culture of cubação. More precisely, I ask the question of *what is its discursive character?* For that, I turn to the conversation-protocols from which evidence can be obtained about how cubação exists as a simply functioning element of what happens nowadays. I have selected two long quotations: the first refers to a discussion in which the farm-worker was teaching the researcher what cubação is about [quotation (a)]; the second comes from the farm-worker's attempt to explain how he would teach cubação to a child [quotation (b)]. Other short quotations are introduced to complement some of the arguments.

Quotation (a)

C. = researcher S. = farm-worker

...

C. There is another thing I would like to know. You said that everybody here uses cubação. How do you do it, what is it?

S. Cubação... well, it is in this way... cubação... There is cubação in metres, which is something that I do not understand very well because I have never practised it, I have never tried to learn about it. In metres, you know? Now, one metre... I know that 1 ha are 400 square braças. In other words, 100 braças... No, each 'aceiro' [side of the tract] being 100 metres... here, this square: 100, 100, 100 and 100 m, this square gives 1 ha.

C. Did you call it 400 metres? Square metres?

S. 400, square. If each 'aceiro' has 100 meters, it is 400, isn't it?

C. OK.

S. 400 square metres: 100, 100, 100 and 100. This is one ha. And one ha gives you 3 'mil' and 220 'covas'.

C. 3220.

S. 'Mil covas', you know?

C. 'Mil covas'?

S. ...mil covas, gives 1 ha. Now, about cubação. This is something I know. I know what it is, because I know, indeed: I was correctly taught. So, one square... 400 square metres, you already know that they make 1 ha. Well, now about cubação in braças... well, this is something from an early period. This, everybody understands.

C. This is something that everybody understands.

S. Yes. Because... let's say that I measured in ha. OK, this is right. Now let's think of cubação in braças.

C. Do you want to write?

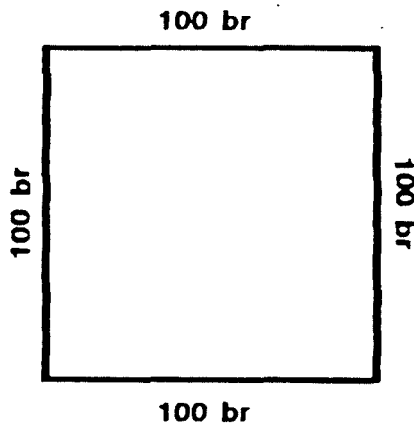
S. No, it is not necessary.

C. It is not necessary.

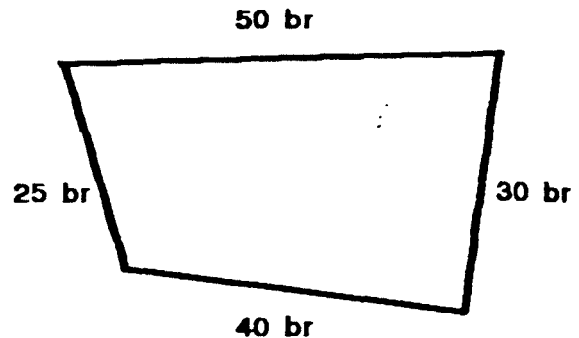
S. Let's imagine that I have worked an area of land of about 15 mil covas. I don't know how much I have worked, all right? Thus I will measure it. Let's say that I have 100 braças here, 100 here... I will try to do very simple so as you can understand clearly. [The tract is represented in Figure 5.2-A]

C. All right.

S. 100 braças in each 'aceiro' make 400 square braças; is that right?



5.2-A



5.2-B

Figure 5.2: Representations of the tracts referred to by farm-workers S. (5.2-A) and Ce. (5.2-B) during the conversations. They were not used or drawn in explicating cubação.

C. OK.

S. These 400 square *braças* give you 16 mil covas, you know. 16 mil covas. Now, how is it that one does cubação... because that is what you want to know about, isn't it?

C. Yes, how is it that one makes cubação.

S. Well. 'Nascente' [East] ...if we have a piece of paper, I would like to write now.

C. Here it is. And that works like a pencil.

S. Any piece of paper is appropriate. Well, let's say that we have the 'nascente' here: 100 *braças*. [The farm-worker writes down the number 100. He does not attempt to draw a picture of the tract in any moment of the conversation]

C. OK, all right.

S. 'Poente' [West], 100 *braças*. Now we add them: the result is 200 *braças*, isn't? Now, North, another 100; and South, another 100, isn't? Then they make the same 200. Then I multiply this 200 here by that 200 there. I know that cubação in *braças* is in this way. Then, we multiply, we add... multiply by a 4. [... interruption for finding his glasses] Well, here we are.

C. Yes, we have 200 and 200.

S. 200 and 200. Then we multiply [... calculation = 200×200].

C. All right.

S. Yes. Well, the result is correct until now. I don't need to check it, it is correct.

C. OK.

S. But I will do it. Because this is something that I will always do [checking sums].

C. OK.

S. Now we multiply. We take this 4. And it will be this 4 that will solve our problem... Didn't I tell you that it would result 16 mil covas?

C. Yes.

S. Very well. Then, you will 'see' it. Look: [... calculation: multiplying by 4]. Now, we take out this [last zero]... and we finish. We have 16 mil covas. Cubação in 'braças', is to do in this way.

C. It means that we have 100 *braças* here...

S. Yes. And then we have 400 square *braças*. Very well, this is 16 mil covas. And if we do it in this way, the result is correct. You can even do the small squares of

'*mil covas*', that the result is correct. Thus, I was...

C. What is a square of '*mil covas*'?

S. Sorry?

C. Did you say "a square of '*mil covas*'"?

S. Yes. 25. 25 *braças*... square... 25 makes 100 *square braças*.

C. Will you do it in "25 by 25"?

S. No. Because we apply cubação and we know the result. We can have... as it was shown to be correct here, we can use it... suppose we have only 95 here. There we have 80. You can do cubação, and you will get the number of *mil covas*. An exact result is this [the 16 *mil covas*].

Quotation (b)

C. = researcher Ce. = farm-worker

...

C. There is one thing I would like to talk with you... I have several questions about it. Because I have learned a lot, but I still have doubts. That is it: I believe that I have learned how to reckon by cubação. I think I know how to do it with a *braça*. Now, what I want to know is this: if you would have to teach a child how to reckon, what would you tell him; how would you do it?

Ce. A child...

C. Yes, what is it, how to do it.

Ce. Well, I would take a piece of paper and would start doing ...

C. Right, here it is [a piece of paper].

Ce. ... I would start doing the sketch of the calculation, wouldn't it?

C. But what would you tell him, about what you would be doing? You would start making him to perform the calculation...

Ce. Yes, I would say: -my son, let's try some calculations here... Lets reckon this tract. Suppose I have contracted ... somebody has contracted work to be done in a tract of land which is ... 50 *braças* South and the North is 40.

C. OK.

Ce. 5 and 4 make 9. It makes 90 *braças*. Then I will take the 'poente' [West]... I will express it in my way, all right?

C. All right.

Ce. 'Poente' gives 30 and 'nascente' [East] gives 25.

C. Humm, humm.

Ce. 25... So, here it makes 55 *braças*, doesn't it? Now, let's put [write] the 55 under the 90. [...calculations = multiplication].

C. Ok.

Ce. ... And then, to see... Here, I am not 'seeing' how many '*covas*' I have got. So, I must introduce the four sides [... multiplying by 4].

C. OK.

Ce. Thus, here it is indicating ... it is showing... here it is showing 1 *mil covas*...

C. Is it necessary to take the zero out?

Ce. Yes, you must 'kill' this zero [the last digit]. One *mil covas*... 1980 '*covas*'.

C. All right. You mean... you would teach him to do these operations.

Ce. Yes. [.....]

C. Mr. Ce., I would like to know something: you have only learned to reckon in *braças*, haven't you?

Ce. Yes.

C. Here, does everybody use it?

Ce. Here, people only use to do it in *braças*.

C. Yes, in *braças*. Why do you think that this method of reckoning is correct... why?

Ce. In *braças*?

C. Yes.

Ce. I am not sure if I understand you. I think in these terms: I imagine that when we talk in '*covas*'... 25... '*covas*'... which is 1 *mil covas*... Let's say 25 *braças* ... Because this is 1 *mil covas*. One *braça* ... we put here to produce 1 *mil covas*. It must have [he refers to the sides of the tract] 25 here, 25 here, 25 here and 25 here, in *braças*. When we reckon it, it gives you 1 *mil covas*.

From these extracts, what interests me is to raise possible ways of thinking about the discursive character of cubação. In relation to this there are other questions which I also asked empirically, such as: what kind of knowledge is cubação; who owns it; who uses it; for what purposes (these were discussed in section 5.3). In this section, I shall restrict myself in situating those aspects which are relevant only for setting the main perspectives of analysis of cubação as a *geometry*.

First of all, it is possible to argue that cubação exists as something which is explicable –it is clear and well defined– and not problematic. In this sense, the researcher's task is to understand it and not to question it. There is evidence in the above quotations that both people regard cubação as certainly correct. They are prepared to tell and to explain it to the researcher. If the researcher has difficulties in understanding it, this is a problem which does not pertain to cubação. On the other hand, if difficulties arise for any person to explain it, this also has nothing to do with cubação. He or she is simply not an expert in it. Easily, and spontaneously throughout the conversations, people try to establish the 'limits of expertise'.

"...Now, about cubação. This is something I know. I know what it is, because I know indeed: I was correctly taught. [...] Very well, this is 16 *mil covas*. And if we do in this way, the result is correct. You can even do the small squares of 1 *mil covas*, that the result is correct." (Farm-worker)

"... Counting is a very precious thing. I am not entirely literate, and so my method of reckoning is this one [cubação]. [...] ...and it always proved to be correct. It is accepted whatever the situation. [...] The EMATER's technician approves it. Let's say, someone, somewhere, contracted an 'empreitada' and he does not know how to do cubação. So, to be secure about the owner's counting he comes to me and I reckon it. After the negotiation is completed I usually ask him: –did it work well? –Yes, it did." (Farm-worker)

"...He [the father-in-law] knows how to do cubação. [...] I have learned with him. But I have only a vague idea. He takes North and South, and West and East. And then he multiplies by 4." (Primary teacher)

"...I do not have any practice. I have never cared for knowing about it. Now, my father... my brothers, all them have practice. They are quite skilful... much more than me, who is a literate person. I have not paid too much attention to it until recently... only because now I need it (to inform the researcher)." (Primary teacher)

It is necessary to point out that expert-people with respect to cubação are not expert by reasons of possessing a specialized academic culture. Actually, a typical person who will not know about it is a university lecturer; while a typical person who will, is an illiterate farm-worker.

In a second approach to the quotations a and b one can say that both farm-workers S. and Ce., despite explaining the method in terms of a particular numerical example, do actually propose idealized cases. This suggests that cubação has a general character and is not tied to a specific context. Exemplary tracts of land are simply part of what can be identified as a typical way of describing the method. The explication, in both cases, is in its essence, procedural. It involves a sequence of general rules for arriving at a correct algebraic relation. Specifically, both farm-workers refer to the square of side 25 'br' as 'giving' 1 *mil covas*. In addition, farm-worker S. thinks of transformations between systems of units. As he explains, in the same way as that in which a tract of '400 sq m' (10000 m²) gives 1 hectare (by the metric system), a tract of '400 sq br' (10000 br²) gives 16 *mil covas* (by cubação). And he knows that 1 ha \approx 3 'mil' and 220 'covas' (also he knows that 1 br \approx 2.2 m). Thus, cubação can be seen as a set of procedural rules for reckoning the area of a square whose area has an algebraic relation with other answers to the same question. Particularly, the standard unit square in cubação can be regarded as the '100 sq br' square (each side = 25 br).

At this point, however, it is necessary to clarify what it could mean for a farm-worker *to arrive at a correct algebraic relation*. It is interesting to notice that those who practise cubação insist that they do not know the metres-method of reckoning. To know the procedure and to be able to express the above relationships do not help him either (a) to start from a tract measured in metres so as to get the result in hectares; or (b) to get the number of hectares from a result given in *mil covas*.³

"...Ah, in metres I don't know... Because I have never studied this way of reckoning. The EMATER's technicians are very good in this kind of thing. They come to make a 'cisterna' [reservoir] and say: -let's do it with a number of metres by a number of metres, and with a given depth... And then they give the result in litres. This technique I do not own. I never studied it."
(Farm-worker)

³ The conversation-protocols contain several examples to support the argument that if farm-workers are not able to perform this task it is not because they lack mathematical abilities such as cross-multiplication. Evidence about this will be suggested in later chapters.

"...In metres I don't know. No. I only know how to do it in *braças*. [...] I have never become interested in it, no. I have never needed it as... Because nobody asks me to do it. And actually I have never tried to learn, you see? Now, in *braças*. I have learned. I know that in metres it could be easier, but I don't know it. Now, I know that 1 *mil covas*... 1 ha makes 3 '*mil*' and 220 '*covas*'... I know that it gives 10 thousand... it gives 10 thousand square meters, you know? Now, 1 *mil covas*, in metres, I don't know." (Farm-worker)

So, there are kinds of relations between *mil covas* and other quantities which are just given; they constitute what somebody else says and are to be taken for granted as correct. In principle people imagine that there should be a way to corroborate these relations, but they don't do it or they don't know it. In this sense, "to arrive at a *correct* algebraic relation" is -for farm-workers- a criterion much more socially than algebraically established, in the sense that the result to which they arrive at should 'match' an expected value suggested by reference to the *social use* they make of the result.

There is certainly the possibility of analysing cubação as an algebraic formulation, but even in this case the term '*algebraic*' should be carefully used in its complete mathematical sense, specially if the focus is on the farm-workers' practice. If any algebraic formulation was to represent the procedure of cubação, not only should transformations of units be easily performed in both directions (as going from hectares to *mil covas* and *vice-versa*), but the problem of obtaining the linear dimensions of a tract from the area given in *mil covas* should also exist and be solved as a natural one. This does not seem to be the case, as one can see from the following extract:

C. = researcher S. = farm-worker

...

C. If you have a square of 32 *mil covas*, how about their sides? How are they?

S. Well... in this case, 32 *mil covas*, isn't it? This is a thing that you can get only by reckoning... If you have the tract which someone can reckon and then he can get this result.

C. Is it possible to do the reverse? Someone says: -look, this is a tract with 32 *mil covas*; how long are their sides... if it is like a square?

S. If it is a square... I am not sure. Now, if they are 32 *mil covas*, being in two... 200 by 100 it is correct. Now, as a square, I don't know...

C. Humm. 200 by 100. Are you saying that it would be like... one square of a 100, plus a square of a 100?

S. Yes, it is like that, indeed.

C. Why?

S. Because in this case it will give 32. It is correct. To reckon this one, I don't know how many *braças* it can result. You are asking me like this: in one single square. In this case I don't know... One can eventually find... because sometimes one can reckon and find the correct [expected] answer."

In this example, the question that the researcher is asking is how to solve an equation, particularly by going backwards. The farm-worker's answer is that one should go forwards and then one can get the result. The number of *mil covas* is a result. And we get it by applying the procedure of cubação.

The example is significant because indicates that in actual terms cubação involves a dimension of skilled performance. In relation to other relevant definitions of skills -such as arithmetic or understanding different measures- the question arises of what calculational competence these people actually have. Several examples can be found in the protocols which inform us about the ability of illiterate people to calculate and which presumably is to be conceived of as belonging to a set of academic competences; namely, competences which are usually proper to the school's pedagogic practice.

Certainly, the characterization of people's mathematical performance can be seen as an important element in planning, choosing the curriculum and communicating. Particularly, it can be of great value for the schooling of peasant children.

But there are at least two other levels to which this discussion is relevant, if the stress is upon schooling in relation to communal knowledge. First, so far as cubação involves school's attributable skills, it turns into a case in which such skills are developed without any valuation, especially from the perspective of the school. The question arises of "how is it that mathematical skills are valued at all, both in schools and in everyday life?"

Secondly, this is a discussion which looks out towards the level at which farm-workers can establish a suitable discourse about measurement with different agents of the external culture in the community, such as the agricultural technician, the inspector of the Brazilian Bank and the primary school teacher. It is in the confrontation of their practices that the relationships between systems of measurement and mathematical skills can become problematic or not. It is in this context that both the dimension of skilful performance and the 'algebraic' character of cubação (as indicated above) are to be adequately understood.

To look at cubação as skilled performance brings out the problematic character of trying to describe the 'tacit'. Knowledge which is fundamentally tacit may include far more than we can tell.

In this respect it is worth remembering that the analysis of a skilful performance in terms of its constituent elements remains always incomplete. Indeed, as Polanyi tells us:

"... the identification of the constituent motions of a skill tends to paralyse its performance. Only by turning our attention away from the particulars and towards their joint purpose, can we restore to the isolated motions the qualities required for achieving their purpose."

(Polanyi, 1969, p. 126)

Therefore, if we are to give some credit to Polanyi's words, it would be more adequate to talk about representing or describing cubação in terms of trying to 'grasp' a subject or an art. In this way, 'grasping', as a peculiar combination of skilful *doing* and *knowing*, attempts to account for both *mastering* a skill and *understanding* a comprehensive object or situation.

So far as the strict analogy between knowledge and skill is concerned, the task of trying to grasp cubação should include the possibility of seeing something hidden that may yet be accessible. According to Polanyi, this is to be seen in relation to the attempt of a skilled person to solve problems. As he says:

"Agog with his problem, the inventor speculates on the possibilities offered by the field of experience, and by his sustained efforts to solve his problem brings about the emergence of its solution. (...) To see a problem is to see something hidden that may yet be accessible. The knowledge of a problem is, therefore, like the knowing of unspecifiabes, a knowing of more than you can tell. But our awareness of unspecifiable things, whether of particulars or of the coherence of particulars, is intensified here to an exciting intimation of their hidden presence. It is an engrossing possession of incipient knowledge which passionately strives to validate itself. Such is the heuristic power of a problem."

(Polanyi, 1969, p. 131-132)

Within this perspective, the analysis of cubação in terms of a procedure should also pay attention to those principles which might determine the particular ways of solving problems of area by this method. In relation to this analysis, a distinction should be made between trying to get information about the actual possibilities of applying the method in the routine of every day life (how they do it), and trying to understand people's reasoning (why they do it). Both are to be investigated within the artificial situation created by the researcher.

If we go back to the description given by farm-worker Ce. in quotation (b), p. 127, it is possible to see in operation a set of mnemonic principles for procedures, such

as "to kill the last zero so as to 'see' the number of *mil covas* or "to multiply by 4 because there are four sides". So, as far as the characterization of cubação can be conceived of in terms of such principles, there is a set of rules which potentially determine a field of conditions for the procedure to be applied. "To have four sides", for example, seems to constitute one such necessary condition and, in this sense, it can be said to constitute a contingent factor in the farm-worker's performance in solving problems of area for different shapes. On the other hand, it is also possible to suggest that an alternative background of ideas may be discerned, which would account for the way problems are solved by farm-workers and which differs from those derived from a Euclidean framework.

As rules for performing measurement and arithmetic operations, the farm-workers' accounts resemble more an algorithm for posing and solving problems of area involving general quadrangle-tracts. In this sense we can say that, contrasting with the '*algebraic*' perspective, there is a perspective which stresses the '*algorithmic*' character of cubação. In the former perspective (algebraic) there are present both a relation to other systems of measuring and an idea of conversion; in the latter (algorithmic), the method can be seen in terms of both a mnemonic set of rules and a natural core of ideas.

The '*algebraic*' and the '*algorithmic*' constitute, then, two complementary perspectives which set up a starting point for investigating the discursive character of cubação and the possibility of its formalization. Together, these perspectives make up a basis for discussing, particularly: (a) the relative valuation of school-knowledge and communal-knowledge; (b) the confrontation of discourses about measurement which are meaningful at the social level; and (c) questions of application to schooling.

5.4.2 The historical perspective.

There are two historical perspectives on the nature of calculation which also help us to approach questions about the nature of cubação. They have a basis in the contrast between both '*naïve geometry*' and '*rhetorical algebra*' in the one hand, and '*greek geometry*' on the other. '*Naïve geometry*' is used by Gray (1979) to refer to a kind of naïve formulation of mathematics in which numbers are represented by geometrical segments, say lines, squares, rectangles or cubes; and which had existed in early developments of geometry. In his words:

"To represent a number as a line one took a fixed, but arbitrary, unit length and repeated it as often as necessary; representations of square numbers in terms of a unit square proceeded similarly. The method was traditional in Babylonian and Egyptian mathematics, and was referred to by Plato as being common in Greek mathematics. The early, but not the late, work of the Pythagoreans was cast in such a form."

(Gray, 1979, p. 12)

'Rhetorical algebra' is identified with the Babylonian formulation of mathematics in which two characteristics are present: a good number system and a rhetorical formulation of mathematical problems. Gray characterizes it as follows:

"Essentially, rhetorical algebra is a set of procedures expressed in words and illustrated with numerical examples for solving certain problems: finding solutions to equations, calculating areas and volumes. BM13901, a tablet containing 24 similar problems, starts as follows:

I have added the area and the side of my square: 45. Take 1, divide it in two: 30, and multiply: $30 \times 30 = 15$. Add 15 and 45: 1, the square of 1. Subtract the 30 (which you had multiplied by itself) from the 1. You have 30, the side of the square.

Since all numbers have been expressed as parts of 60, we should express the original equation as $x^2 + x = 3/4$. The coefficient of x is 1; halve that and square it ($1/2^2 = 1/4$. Add $1/4$ and $3/4$ (and form $x^2 + x + 1/4 = 3/4 + 1/4$). Both sides are squares; take square roots [$(x + 1/2)^2 = 1$]. Therefore $x + 1/2 = 1$. Subtract the half from both sides; $x = 1/2$.]"

(Gray, 1979, p. 3)

A complementary account of the form in which Babylonian algebraic problems are presented can be found in Neugebauer (1969). He says:

"From actually computed examples it becomes obvious that it was the general procedure, not the numerical result, which was important. If accidentally a factor has a value 1 the multiplication by 1 will be explicitly performed, obviously because this step is necessary in the general case. Similarly we find regularly a general explanation of the procedure. Where we would write $x + y$ the text would say "5 and 3, the sum of length and width". Indeed it is often possible to transform these examples directly into our symbolism simply by replacing the ideograms which were used for "length", "width", "add", "multiply" by our letters and symbols. The accompanying numbers are hardly more than a convenient guide to illustrate the underlying general process. Thus it is substantially incorrect if one denies the use of a "general formula" to Babylonian algebra. The sequences of closely related problems and the general rules running parallel with the numerical solution form de facto an instrument closely approaching a purely algebraic operation. Of course, the fact remains that the steps to a consciously algebraic notation was never made."

(Neugebauer, 1969, p. 43)

In accounting for these two 'kinds' of mathematics, Gray contrasts their characteristics with 'greek geometry'. In relation to 'rhetorical algebra' he says:

"...a procedure expressed verbally is not a formula, it cannot be manipulated into equivalent forms or checked against another intended to solve the same problem. For these reasons rhetorical algebra is without proofs and can accommodate different and incompatible answers. [...] The disadvantages of rhetorical algebra are that it is difficult to think in it for an extended period, that it is non-explanatory, and that it even contains contradictory estimates of areas and volumes. In response, the Greeks formulated geometry, and intended using it to attain proofs and propositions. [...] Certainly one appeal of geometry is that it treats existing things clearly, mathematically existing that is, but that is if anything better. Geometry then becomes an analysis of (true) reality, and the deductive method an inquiry into the world."

(Gray, 1979, p. 12)

Gray does not state the issue at this point, but it seems correct to suppose that "contradictory estimates of areas and volumes" also account for non-accurate results. In a previous argument we can find reference to the Greeks' interest in questions of "rigour and logical validity" (p. 1). Yet, in another place he says:

"There is only one way out of the profusion of contradictory and non-explanatory results in rhetorical algebra and that is to find a way of making coherent sense of the results -at least those which are right. I believe that it is in attempting to do that that the Greeks were led to geometry, not for its own sake but as a method of proof."

(Gray, 1979, p. 3)

In relation to 'naïve geometry', the contrast with 'greek geometry' is presented in terms of the movement from procedures to proof, in which case one would speak of theorems rather than results, a theorem being a result for which there is a proof.

It is beyond of the scope of the present investigation to discuss 'greek geometry' or issues such as the deductive method, the proof of theorems and so on. Nonetheless, from Gray's accounts arise some relevant ideas. One of them is the normative character attributed to 'greek geometry' which makes possible to give a meaningful account not only of the results obtained in the more preliminary 'kinds' of mathematics but also a meaningful account of both 'naïve geometry' and 'rhetorical algebra' themselves. This suggests, for example, a perspective of alternatives for approaching cubação. And Euclidean geometry -as it is taught in schools- seems to be the natural choice for such a contrast. In addition, the contrast poses the question of how to consider the discursive character of 'geometries' for which the rules of formal logic do not seem to apply.

Another idea is the characterization of these two mathematics in terms of a 'procedure' which has an 'algebraic-concern' -if we take into account Neugebauer's words- but which can not simply be replaced or represented by a formula; if it is so replaced, what is left? Also, the non-explanatory character of a Babylonian-like procedure poses certainly the question of how would one conceive of talking about 'understanding' (how and why) in relation to a description of cubação given in that form?

In summary, for a researcher aiming to formalise or specify the nature of cubação these constitute important issues. Summing up the previous discussion, they suggest two different but complementary ways of describing it, which are now indicated.

5.4.3 Perspectives for describing cubação.

Two different approaches are suggested in describing results: one more abstract in which the algebraic formulation of the metres-procedure constitutes a basis for discussing cubação as a procedure for reckoning the standard square; and another more practical in which cubação is to be understood as a kind of problem-solving activity involving measurement and arithmetic skills.

In other words, there are two perspectives in approaching cubação which must be kept distinct in reporting results. As a *reckoning procedure*, cubação is to be more adequately contrasted with Euclidean Geometry, when the question of *validity* of the method for reckoning land becomes important, particularly for a researcher trying both to make sense of the method and to interpret the result given in *mil covas*. As an *algorithm* for solving problems, it is more appropriate to focus on the task of *practising the procedure*, when the stress should be on the *skilled performance* of the farm-workers.

5.5 RESULTS, METHODOLOGICAL AND PROBLEMATIC ISSUES

As discussed earlier, my account of cubação is based on a empirical study developed with adults in a rural community in the NE of Brazil, where the method has been practised for more than a century. As a matter of fact, no written account is available about cubação; and so no better possibility manifests itself to the researcher other than to ask about it directly and openly of those who practise or those who are aware of its application. In this perspective, "to know about

cubação" emerges as a process of "*knowing with people*"; and in this sense, the description of results should be conceived of as an account of the researcher's task of coming to understand what people are doing and thinking.

In this respect one point deserves attention. While the term *mil côvas* is generally known among rural and urban people – "educated" or not – the same can not be said in relation to the procedure. Only those who have grown up in rural areas or those who relate with peasants in the community (like priests, teachers, farmers, agrarian specialists or inspectors of the Brazilian Bank) recognize it as part of a distinct and intelligible method for reckoning land. Also, not everybody understands the procedure to the same extent. In this way, "to know about cubação with people" evolves to a process of "*knowing with different kinds of expert at different levels*".

Questions arise of how expertise can be understood and characterized in this case? In what sense can we formalize cubação? If so – how? Can it be algebraicized? Does that change it? In which way should it be reported? What is it to speak about dimensions of knowledge in cubação?

Methodologically, "knowing about cubação" presents itself as a question of *eliciting and representing information* and it is in this way that the task of producing data is conceived. A peculiar style of 'interviewing' people was used which can be best seen and characterized as a particular procedure demanded from the requirements of the general 'methodological paradigm' proposed in Chapter 3. As was said before, this paradigm has its foundations in some ideas of Pask's Conversation Theory.

As far as a characterization of cubação is concerned, the paradigm suggests two things; which can be seen in a discussion of (a) levels of understanding, and (b) kinds of postures towards understanding.

Levels of understanding.

One thing that the paradigm suggests is that there are two levels – in the quotations from the conversation protocols – of what it means to find out about cubação and which should be distinguished. There is a superficial level at which one wishes to claim to understand what the informants are telling us, as they understand it. In this case the farm-workers would in principle agree with the

researcher's codified version of what was informed, if an attempt at explaining to them in reverse were made.

For example, if we go back to the farm-workers' account in quotations (a) and (b) -pages 125 to 128- it is possible to say that, at the superficial level, farm-worker S. [quotation (a)] would in principle be ready to accept my description of his account of cubação, if I were to repeat it as an appropriate specific sequence of operations for obtaining the number of *mil covas*. In the same way, farm-worker Ce. would be probably willing to accept my account of what is involved in teaching cubação to a child if I were to describe, as he does, the performance of a valid specific sequence of measurement and arithmetic operations which will end up with the number of *mil covas*.

But also, there is a deeper interpretative level at which one wishes to claim that the knowledge of cubação is of a certain kind -or has a certain character. The systematization of cubação in terms of an 'algebraic' and an 'algorithmic' perspectives have emerged partially from an attempt to get at this level. For this, what the informants say is relevant, but not necessarily of immediate relevance. There is no point here in talking about teaching back the researcher's interpretations in the same way as was argued at the superficial level, in which case the attempt is to 'codify' what the farm-workers are saying. The quality of the effort required by the farm-workers to accept their accounts as representative of this deeper level would be different: probably, if that were to be the case, they would have to enter more deeply and formally into the researcher's interests.

For example, farm-worker S. would have to start revealing cubação as a set of procedural rules for reckoning the area of a square whose area has an algebraic relation with other answers to the same question. Farm-worker Ce. would have to teach cubação as a problem-solving activity, in which case a given sequence of rules-of-thumb would apply to any irregular quadrangle.

As far as the reporting of results is concerned, the description of cubação which follows in later chapters touch upon more than what would count as 'knowledge' about cubação as treated at the superficial level of codification. Also present, as an important dimension, are those requirements (methodological and fundamental) for cubação to be given to "*connaissance*", and which belongs to a more interpretative level of discussion.

Kinds of posture towards understanding.

A second thing that the paradigm remind us of is that informants and researcher have indeed two different kinds of posture towards *understanding*.

The farm-workers I talked to devoted much of their time teaching me about cubação. This happened because, as a researcher, I had decided to make of cubação a further problem of research. That is, after having established with some degree of confidence the way cubação functions, I was left with two main questions which I took over during the remaining of the Conversation:

- . "How would farm-people react to different geometrical shapes for which cubação fails? What would this tell us about their ideas about cubação and about their understanding of the concept of area?"
- . "How do we interpret 'mil covas'? How is 'mil covas' to function in relation to both planting and geometry?"

Despite constituting the fundamental motivation for most of what was discussed with farm-workers and teachers, these questions were not addressed directly to them. They were told, on different occasions and in different ways that these were some of the main worries of the researcher; they were invited to 'think with' the researcher about them; but they were not charged any responsibility for solving them. Apart from some questions raised by the teachers concerning the notion of *area*, farm-workers did not take them as problematic at all, for they did not exist as such in their practice.

Also, it is possible to say that the puzzling questions posed by the researcher were seen, by the informants, as "part of a more general inquietude about a practice not fully comprehended; with a bit of patience and time the researcher would be able to grasp the whole thing, and those questions would probably disappear".

Comments such as:

- "It is very easy.";
- "Do you understand?";
- "You can come and talk as and when you wish.";
- "Can you see this?";
- "I will give you a methodology for you to understand better.";
- "You did not ask but I will tell you something."; or even
- "I feel sorry about not being able to teach you about 'X'.";

were frequently made during the conversation.

Thus, if on the one hand the researcher had a justified interest in the identification of different stages within the Conversation, on the other, informants and teachers never abandoned their position of *informants* about their practice, their situation, their experience, their thinking, their motives. This is important to realize because, methodologically, it has some implications.

For instance, it indicates that the farm-workers' accounts needed to be 'transformed' by the researcher into useful information for the purpose of clarifying the researcher's questions, *during the proper Conversation and not only later.*

In addition, when I decided to take only part of the protocols to localize a case in knowledge -namely, cubação- I did not mean to keep the analysis restricted only to what was said about cubação, but also to clarify who said it, when it was said, in what context; and this required going back to those parts of the protocols where, sometimes, *there was no direct reference to cubação.* That is, to make sense of some of the farm-workers accounts, I had to *infer* what kinds of conditions could be taken as accounting for the meaning of their sayings and which were present in the discussion; and this required me to search for information about other issues, such as the informants' understandings about the characteristics of soil, tracts, and about the layout of the farms. Also, I was led to make inferences about the situation of farm-people within the system of production.

For example, one of the most crucial distinctions which needed to be made during the interviews was that between *fields* and *new tracts*. For farm-workers, a *field* is a fixed entity, in that it is marked out and delimited by a pre-established grid. No small producer can alter the distance between planting places (these compose a kind of square-grid of unit equal to 1 square braça). Also, the form of a field can vary, but what counts as area is still the number of plants spread over the tract. In addition, it is the owner who defines what this number of pits will be. The farm-worker is completely blocked in his potentiality of *thinking*. *Possibilities* are not allowed in a field.

On the other hand, a *new tract* opens the possibility of farm-workers to think beyond their practice. Tracts can have any shape; one can plant as much as one likes; rows can be arranged in any order; and so on.

The relevance of such a distinction for establishing real conditions for involving farm-people in a kind of abstract thinking, is enormous. It reveals an important qualification for the world of *praxis* to become a world of supposals about reality. In this respect, it is interesting to conclude with a remark from Oakeshott (1985; 1st ed. 1933):

"To suppose (as also to imagine or to deny) is to assert something categorically. To say what would be, or what might be, or what may be, is to say something of what it is. And no judgement whatever can avoid this implicit reference to reality. But what is important here is to understand that, whatever a supposal asserts about reality, it never asserts what is supposed. To say what would be, is to say something, but not that thing, of what is. And consequently a world of supposals is a world of judgements which have some reference to reality, but not the reference represented by their explicit character. Unless we know more about reality than what is explicit in this world of judgements, we know nothing."

(Oakeshott, 1985, p. 216)

CHAPTER 6

THE GEOMETRY OF CUBAÇÃO SEEN FROM THE PERSPECTIVE OF EUCLIDEAN GEOMETRY

6.1 THE PROCEDURE IN ITS ESSENCE

When interpreted literally cubação means "the construction of a cube", though the same verb 'cubar' is used elsewhere in the loose sense of 'estimating'. In the context of agriculture in S. Paulo do Potengi the word both refers to the act of measuring land and indicates -by means of the expression "*mil covas*"- how much land exists in a given tract. As explained by a farmer:

"cubação is a procedure for reckoning area, in order to 'see' the number of *mil covas*".

6.1.1 A typical description.

Essentially, the procedure of cubação refers to a special class of tracts: those which are square-shaped. As described by the farm-workers, it is always expressed in words and illustrated with numerical examples. A typical case would be:

"In the field, you have identified the position of your tract in terms of local features (rivers, roads, etc.) and its extent decided by identifying four edges, labeled West, East, North and South. You have measured each edge by means of a *braça*: suppose each is 25 *braças*. You have a '100-square'.

On the paper, write down the four '25s'. Add the 25-West and its opposite 25-East: 50; and the 25-North and its opposite 25-South: 50. Multiply the results: 2500. As you have four edges, multiply the latter result by 4: 10000. Ignore the last digit: 1000. Read the result as '1 *mil covas*'. You have 1 *mil covas*, the area of your '100-square'."

A *braça* is a wooden rod made by the farm-worker to be 10 *palmas* long; a *palm* being a unit of length based on the width of an expanded hand. Apart from representing an instrument of measuring, a *braça* designates the basic unit of length of the system of cubação. 1 *braça* = 2.2 metres. The unit of area called *mil covas* can be 'converted' to the metric system by means of the relation 1 ha = 3305 *covas*.

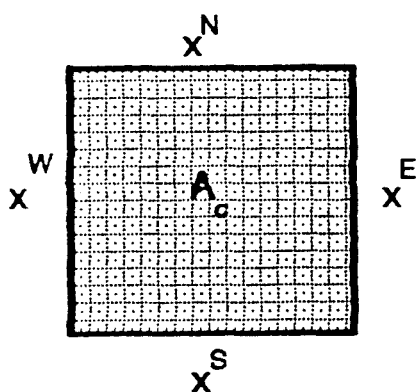
6.1.2. A starting point to think of the question of validity.

If the validity of cubação is to be 'tested' as a procedure for measuring area, two issues deserve attention: its adequacy for the purpose at hand and, by extension, the accuracy of the results produced.

One possible starting point for thinking about these questions is to try to check the procedure against another intended to solve the same problem. *Euclidean Geometry* (EG) serves the purpose particularly because it is the kind of geometry we still learn at the basic levels of schooling, the geometry most of us -educated people- use to visualize the physical universe. If any relation is to be intended between common sense knowledge of ordinary people and Science taught in schools, it is exactly EG which would be appropriate for the job at hand. Namely, for the *adequacy-accuracy* analysis, EG functions as *normative*.

6.1.3. Transducing the typical description.

The initial requirement for such an analysis is to 'transduce' the procedure as it is currently expressed by the farm-workers into an algebraic formulation capable of comparison with Euclidean procedures. Two steps are involved: the first involves a single re-expression of the rhetoric into a mathematical formulation and would look as follows:



$$A_c = [(x^W + x^E) (x^N + x^S)] 4/10 \quad (1)$$

with

x^W, x^E, x^N, x^S in *braças* (br)

and

A_c in *mil covas*.

Figure 6.1

Thus, in describing the procedure we would say that there is a 4x-square as represented in Figure 6.1, whose area (A_c) can be expressed by the formula (1). We could call that a *typical representation*.

The second step involves trying 'forget' that formula (1) contains meaning and focus only in the algebra. By doing that we would re-write (1) as:

$$A_c = [(2x) (2x)] 4/10 , \quad (2)$$

and the result as:

$$A_c = 1.6 x^2 \text{ covas} . \quad (3)$$

This latter formulation constitutes a transduced representation of the rhetorical description.

The point of distinguishing these two steps is that if on the one hand it is legitimate to view cubação from the perspective of algebra or EG, on the other it is important not to forget that this is not merely a "re-expression of the same idea", but is to *transduce* to a different *frame of reference* which contains its own system of norms and rules, not shared by the original. This argument will be more fully discussed in the analysis of the procedure as it is practised by the farm-workers, when the actual medium of expression and the system of 'motivations' will appear as two fundamental requirements.

6.1.4. Cases of exact estimation.

The ordinary procedure for estimating the area of a square (let us say in a Euclidean method, A_E) gives the result:

$$A_E = x^2 \text{ br}^2 . \quad (4)$$

Therefore, by comparing (3) and (4) it is possible to conceive of the 1.6 factor as a conversion factor from br^2 to covas. We establish:

$$1 \text{ br}^2 = 1.6 \text{ covas}$$

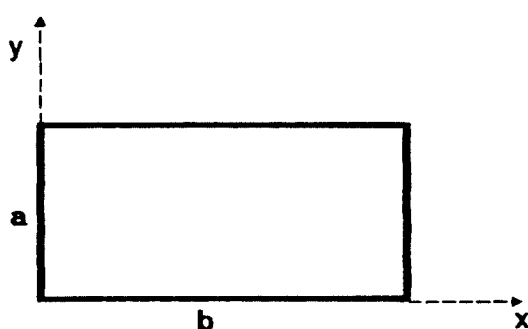
for which we would say that A_c can be normalized to Euclidean area units.

In doing so, we impose that the algebra of the *cubação-procedure* and the algebra of the *Euclidean-procedure* stay exactly in the same way. Thus we would claim that cubação gives a correct estimation of area for square tracts.

On the basis of the above equivalence it is possible to extend the domain of 'application' of the procedure to *Regular Rectangles* (RR) and *Regular Angular Rectangles* (RAR).

(a) RR

A RR is any ordinary rectangle with straight edges and four right angles and can be located by rectangular coordinates (x,y) as in Figure 6.2.



The Euclidean area is:

$$\int_0^b x \, dx \int_0^a y \, dy = a b . \quad (5)$$

The area by cubação (A_c) is:

$$[(2a) (2b)] 4/10 .$$

Figure 6.2

Multiplying and dividing by 4:

$$A_c = 1.6 [(2a) (2b)/4] .$$

As we have assumed $1 \text{ br}^2 = 1.6 \text{ covas}$, we can normalize A_c to Euclidean area units resulting:

$$A_c = [(2a) (2b)]/4 , \quad (6)$$

which is exactly the Euclidean area ($a b$) in (5).

(b) RAR

A RAR is a surface shape like that in Figure 6.3-A and can be located by polar coordinates (r, θ) [Figure 6.3-B].

Two opposite edges are straight and equally long, the other two being arcs of circumference specified by angular relations.

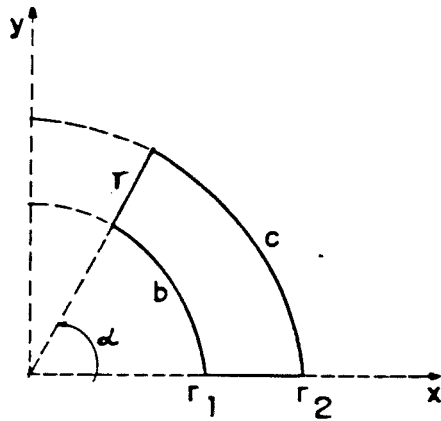


Figure 6.3-A

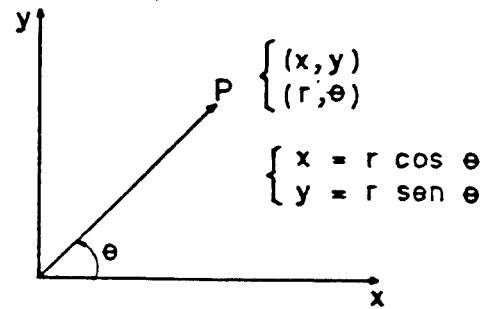


Figure 6.3-B

The Euclidean area (A_E) of a RAR is:

$$\int_0^\alpha d\theta \int_{r_1}^{r_2} r dr = (\alpha/2) (r_2^2 - r_1^2) .$$

As:

$$r = r_2 - r_1 ; b = \alpha r_2 ; c = \alpha r_1 ,$$

we can write:

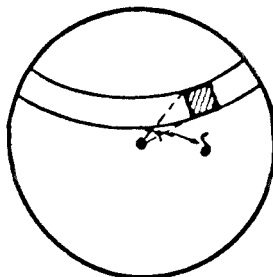
$$A_E = r (c + b)/2 . \quad (7)$$

The normalized area by cubação (A_c) is:

$$A_c = [(2r) (c + b)]/4 ,$$

which is exactly the same area A_E in (7).¹

1



A similar calculation as the one suggested for the two dimensional case (RAR, Figure 6.3-A) can be generalized to the surface of the sphere. For that case, it can be shown that the area of the strip in the figure, as calculated by cubação, is the same - as a first approximation - as the one calculated by using conventional Euclidean methods. The corrections are of order δ^2 , where δ is the difference between the two polar angles which define the delimiters of the strip.

6.1.5. Adequacy of the procedure for estimating the area of tracts.

6.1.5.1. Reckoning area by cubação.

Having regard to the traditional procedure of measuring area in Euclidean geometry (see Appendix 6.A), the *reckoning procedure* of cubação (in the 'transduced' perspective, not in terms of what farm-workers do) can be conceived as an appropriate formula for calculating the area of two basic *unit regions*: the RR and the RAR. The advantage of cubação lies in the fact that a range of different shapes can be used (squares, RRs, RARs, circles, semi-circles, or fractions of a circle) which are reckoned by *the same and unique simple formula*.

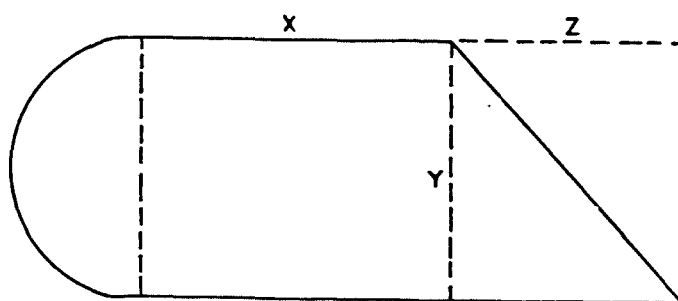


Figure 6.4

For example, the area of the shape represented in Figure 6.4 can be easily obtained by dissecting it in a semi-circle, a rectangle and a triangle. Their areas can be obtained by means of the same *formula*, applied respectively to a semi-circle of radius $y/2$, a rectangle of sides x and y , and to a rectangle of sides y and z . Particularly, the area of a quadrant can be obtained by formula (7) by making $b=\emptyset$, $r=y$ and $c=[ny]/4$.

The total area will be:

$$A_{total} = [A_{s-c}] + [A_R^{(xy)}] + 1/2 [A_R^{(yz)}] ,$$

which is:

$$A_{total} = [\pi/2] y^2 + [x + (z/2)] y .$$

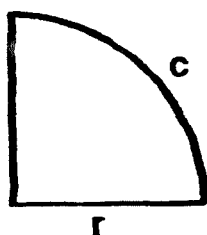


Figure 6.5

In addition, regions bounded by curves can be more adequately covered by an Angular Triangle than by means of a triangle. The area of a circle can be easily derived from the area of a quadrant $A_q = [r c]/2$ [Figure 6.5]

By adding the four pieces, we come to the whole area of the circle (with circumference C):

$$A_c = [r C]/2 ,$$

which is correctly the Euclidean area:

$$A_E = \pi r^2 .$$

In this way, irregular and bounded regions can be measured by means of slightly distorted unit regions RR and RAR. In addition, cubação, to first order of small quantities, also gives the correct area of Irregular Rectangles (IR) and Irregular Angular Rectangles (IAR).

(a) IR

An IR is any shape possessing at least one side making small angle with a suitable oriented RR. Consider the distortion made by a small extension dy to one side of a RR [Figure 6.6].

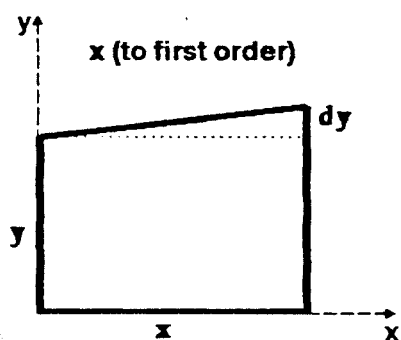


Figure 6.6

The Euclidean area is:

$$xy + 1/2 [x dy] .$$

The normalized area by cubação is:

$$(2x) (2y + dy)/4 ,$$

which is

$$xy + 1/2 [x dy] .$$

The same argument applies to extension or contraction of any side.

(b) IAR

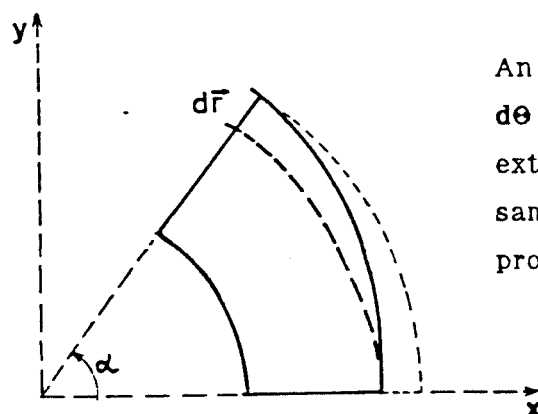


Figure 6.7

An IAR is any RAR submitted to dr or $d\theta$ variation. In Figure 6.7 a small extension dr was applied in which the same kind of argument can be produced as for the IR.

In other words, cubação is insensitive to small dilatations of sides, which contributes to the production of more accurate results in the case of general irregular regions.

In summary, we can say that, in its essence, cubação:

- . contains one simple procedure stated in a general form for quadrangles, which
- . can be understood as a 'formula' for estimating standard unit regions;
- . generates a good -adequate and accurate- estimate of area; and
- . is extensible to shapes (for example, circles) for which at first sight it is not well adapted.

6.2 THE QUESTION OF 'ACCURACY' IN RELATION TO THE ACTUAL PRACTICE

In section 6.1, the argumentation for cubação as a good estimate of area had a basis described from a formal point of view, in which 'practical criteria' were stressed. However, the perspective did not claim any practical purpose other than to measure the area of a general region in its own right. Actually, in order to account for its inner nature, the context and the functionality of the method was put aside and cubação was treated -in a sense detached- as a 'reckoning-like procedure'. In the discussion, *accuracy* was brought as an important issue in relation to which the *validity* of cubação was analysed.

Not quite the same is looking at cubação acting as a 'surveying-like method'. A method of surveying participates in a system of external relationships referred to a system of social rules. Thus, farm-workers use the method not because it actually represents an accurate procedure for the purpose of reckoning tracts of land; but for reasons which are to be set out in relation to the forms of distributing, getting access and using land. Namely, there is another level of accounting for *validity* which is better regarded from a social perspective in which cubação is to be thought of in relation to the agricultural mode of production.

This latter view, which emerges from the discussion in chapter 5, does not concern us in this chapter. What seems necessary at this point, is to qualify how the procedure is used by the farm-workers in practice. Two issues are relevant to be addressed:

- (a) the ways in which actual shapes are reckoned; and
- (b) the problematic character of the farm-worker's practice, regarded the results so obtained.

From this discussion, arguments arise which point to the non-accurate nature of cubação. The implications of such a fact are raised in Chapter 7. Euclidean Geometry still represents the perspective from which the analysis is carried out.

6.2.1. Reckoning actual shapes.

Despite essentially referring to squares, the procedure of cubação is applied by the farm-workers to solve quadrangles in general. Also, the same procedure is used to solve all sorts of shapes in one single process. That is, first there is a "transformation" of the actual shape into a quadrangle, and then the procedure is applied. The various kinds of specific solutions which were given by the informants will be presented in Chapter 7. A comparison with the kinds of solutions expected from Euclidean procedures will also be discussed in Chapter 7.

In this chapter, I will only try to exemplify instances which suggest that for farm-workers cubação can properly be applied to any shape, and discuss the more obvious and problematic implications which follow from this fact.

Basically, the fact which emerges from looking at the protocols is twofold: on the one hand, there is the argument that any shape can exist in actual situations; on the other, there is the argument that cubação applies whatever the shape. Taken together, these arguments establish a basis for the researcher to discuss the problematic character of their practice.

It is appropriate in this case, to let the farm-workers talk for themselves. I will show a sequence of extracts from the protocols which exemplify the arguments. They refer to the discussion of different aspects of solving problems for general shapes and come from the conversation with different farm-workers. A brief comment will be made in the end of the examples.

C. = Researcher FW. = Farm-worker

Example 1

[...]

C. OK. All right. I will show you some shapes and ask you some questions. The first thing I want to know is this: are there shapes like these, which are not regular quadrangles, whose sides are not regular?

FW. Yes, there are. This one [a triangle] is what "we" call 'flat-iron' ('ferro de engomar'). That one is no more a flat-iron because it has this side... But this one is.

Example 2

[...]

C. Might the tract have always four 'aceiros' (sides)?

FW. Yes, it might.

C. And about this one... Suppose you have a tract with this shape [triangle]. Does it happen to exist a tract with this shape?

FW. It does. This is what "they" call a triangle.

Example 3

[...]

C. If the tract is like this now. I don't know if there are tracts with this shape... [it is a quadrangle; one side is a semi-circumference and two others make a very obtuse angle]

FW. It has, it has.

Example 4

[...]

C. Let's try another shape, because this one is not very helpful. For example, this one. Suppose that this side is straight... You would do this side with... [tract with more than four sides, concave/convex]

FW. Yes, I would take this 'braça' here... But I have here this corner... and there I have that one... Now... It becomes more difficult...

C. From what point to what point would you measure. How would you do it? This is what I want to know. Or you don't measure along this line... Or you do something different.

FW. Yes, this case is more difficult... In this case...

C. Perhaps this is not an actual case...

FW. It is not so frequent, but it may happen... And then, it must have a way to do it.

Example 5

[...]

FW. The person who taught me, taught me to do cubação for a square... Nobody tells how to do for shapes such as these.

C. Perhaps actual tracts are not like these...

FW. Yes, but sometimes it happens... Because when you go to delimit a tract, the usual practice is to try to get one which is regular... But it may happen. I never did it for a tract like this [triangle].

Example 6

[...]

C. Now, if you have a tract with five sides. How do you do it? Are there tracts with five 'aceiros'?

FW. Yes, there are... perhaps it would happen...

C. How would you do it?

FW. I don't know... In this case... For the case of a triangle I could find a way to start... a starting point... But in this case...

C. Yes, you have pointed out an answer. At least you set the problem clearly. And this is a lot, for a beginning.

FW. This one I don't know... I cannot even try to think of a possible answer. Because...

C. Why not?

FW. Because I have never seen a tract like this.

C. You never saw. Humm...

FW. No, never. Because the person has to start measuring... For example, if one measures in this way... [he continues proposing a solution]

Example 7

[...]

C. Another question. How would you do it if you had a tract like this [triangle].

FW. With three 'aceiros'? It has only three 'aceiros' ...

C. Yes.

FW. I do it in this way

[he gives a solution]

C. So, it is not a problem to reckon a tract with three sides.

FW. No, it is not.

C. And when it has five sides? I want to understand every thing... Look, this one has five sides.

FW. Well, it is a little more complex, because it has five...

C. Humm... How do you solve it?

FW. You can do it in this way.

[he proposes a straightforward solution]

FW. The solution is not so different.

C. Yes, Ok, all right. But if you have this one [a circle].

FW. What? A round shape?

C. Well, I am not sure if there are cases like this...

FW. Yes, I have never done one like this. But I think that it could be in this way... [he continues proposing a solution].

From these examples, it is possible to say that farm-workers accept the fact that (a) actual tracts can have -and sometimes do have- any shape; (b) the general procedure is set out for quadrangles; (c) but the possibility should exist in principle to solve any shape by this method.

Arguments about (b) and (c) can also be seen when farm-workers are giving a description of the procedure. In this situation there are usually two kinds of tracts to which the farm-workers refer to in the conversations. One is a typical square

(a 100-square or a 400-square); the other is a irregular quadrangle. The quotations (a) and (b) which were presented in Chapter 5, pages 125 to 128, exemplify the point. Farm-workers always make reference to a given tract when accounting for the method. Remarks such as "the sides of the tract can be different"; or, "it is not necessary to have equal sides" are usually made.

To visualize the problematic character of such a practice is not difficult. The method -which was shown to be accurate for RRs, RARs, IRs and IARs- fails in a number of circumstances, the trapezium and the 'square-circle' being examples of this. The former represents a case of an over-estimation while the latter a case of under-estimation. Follow the examples.

Example 1: The trapezium

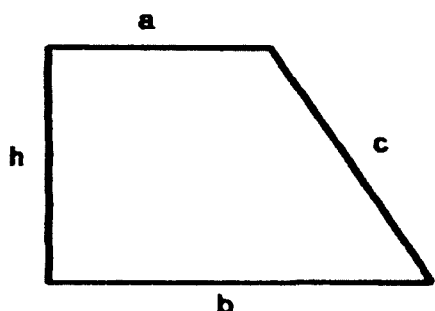


Figure 6.8

Consider the trapezium in Figure 6.8. The Euclidean area (A_E) is:

$$[(a + b)/2] h . \quad (8)$$

The area by cubação (A_c) is:

$$[(a + b) (h + c)] 4/10 .$$

Normalizing cubação to Euclidean area units:

$$A_c \rightarrow E = [(a + b)/2] [(h + c)/2] . \quad (9)$$

By comparing (8) and (9) we realize that:

$$[(h + c)/2] > h$$

(as c is always greater than h for a trapezium),

which implies that:

$$A_c > A_E .$$

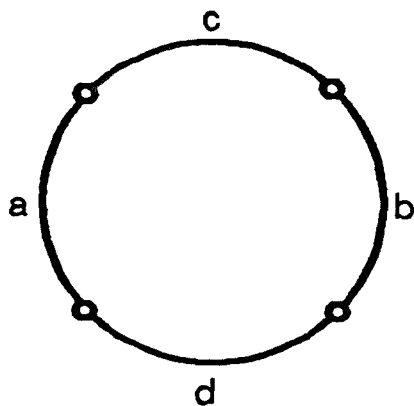
The same is valid whatever the trapezium, as for any one different from the regular, the pair of opposite sides $(h + c)$ will be greater.

Example 2: The 'square-circle'

Consider the case of a 'square-circle' as in Figure 6.9, where:

$$a = b = c = d = (\pi r)/2 .$$

The Euclidean area (A_E) is:



$$\pi r^2 . \quad (10)$$

The area by cubação (A_c) normalized to Euclidean units is:

$$(\pi r^2) \pi/4 . \quad (11)$$

Figure 6.9

By comparing (10) and (11) it can be seen that:

$$A_c < A_E \quad (\text{as } \pi < 4).$$

What these two cases indicate is that results by cubação - *in the way the method is used by the farm-workers* - can be less or greater than the actual value. How much they would represent a good estimation or depart far from the Euclidean answer is a question lacking a conclusive and definitive answer.

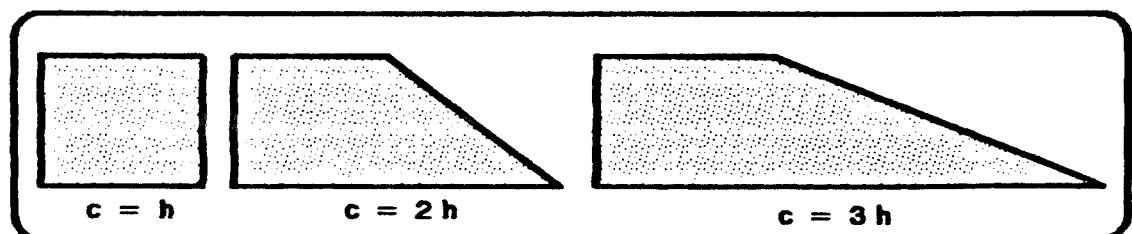


Figure 6.10

For the 'square-circle' in example 2, cubação under-estimates the real value by 20%. In example 1, the difference will depend on the relation between h and c , being more accurate for situations in which $c \rightarrow h$. When $c = h$ we have a rectangle as a particular case and the result is exact. For $c = 2h$ cubação will over-estimate by 50%. For $c = 3h$, by 100% [Figure 6.10].

An interesting trapezium is represented in Figure 6.11 (trapezium AECD).

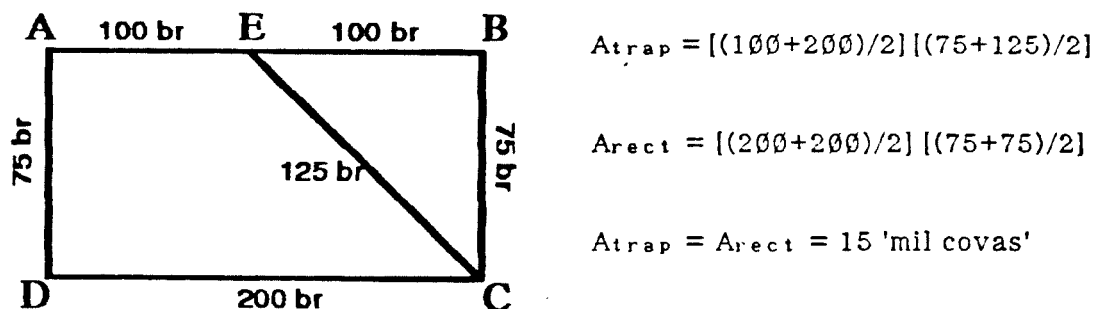


Figure 6.11

By cubação its area is over-estimated by 33% and has the same value as the area of the rectangle ABCD (15 *mil covas*). In this case, the question arises of "what sense can we make of the triangle EBC?"

In summary, the fact that the farm-workers possess in their hands an efficient and appropriate procedure for the purpose of reckoning tracts of land does not guarantee its 'correct' application, within the perspective of Euclidean Geometry. Despite essentially referring to squares, the procedure of cubação is applied by the farm-workers to solve quadrangles in general.

Therefore, when we look at the way in which cubação is practised, we are led to think of it as an inappropriate procedure for working out the area of a tract of land. But this fact does not seem problematic for farm-workers. The implications which follow from this fact will be discussed in the following chapter. But before that still remains to discuss two questions:

- . "how to formulate the general condition for which cubação correctly estimates the area?" and
- . "what is the general form of the shape for which cubação correctly indicates the area?"

6.3 PRELIMINARY ELEMENTS FOR A GEOMETRY OF CUBAÇÃO

The general procedure Γ of cubação, stated in a general form for quadrangles (Figure 6.12), contains two basic factors Φ and \S :

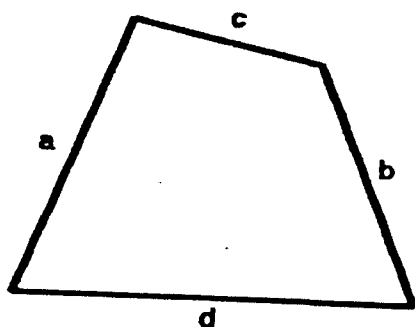


Figure 6.12

$$\S = [(a + b)/2] [(c + d)/2] \quad (12)$$

and

$$\Phi = 1.6 ,$$

related in the following way:

$$\Gamma = \Phi \S .$$

Our problem is to characterize a class of quadrangles which are correctly estimated by $\Gamma = \Phi \S$, having in mind the practice of planting by cubação. Namely, to analyze the criteria of applicability of the procedure in those cases which help us to understand the farm-workers' practice. The intention is to derive some elements of a geometry of cubação which can function as a background reference to discuss the farm-workers' ideas about area and about cubação.

6.3.1 Criterion of applicability and the general form of the shape in cubação.

We have seen in section 6.2 that the basic algorithm for obtaining the area of a quadrangle does not guarantee that a correct answer is always worked out, as it is insufficient to determine the proportion of the figure itself. There is no restriction relating $(a + b)/2$, $(c + d)/2$ and Φ which accounts for the kind of shape one is actually estimating.

Thus, as far as cubação is concerned, what is required is to re-express the procedure Γ in a way as to 'fix' the shape while the computation is conducted. That is, to formulate the procedure in a way which -we know- does apply to a given class of shapes; a class of shapes which has a concern with the reality of practising cubação.

Consider, for example, four rods of lengths $a = 8$ br, $b = 12$ br, $c = 14$ br and $d = 10$ br, joined at their ends. The quadrilateral which they would form could have a variety of shapes. Four possible shapes are illustrated in Figure 6.13. These

shapes are quite different, and if the rods are bolted at the corners so that the angles are not held rigid, any of these quadrilateral shapes could be deformed to take the shape of one of the others. In addition, by cubação, all of them would have the same 'area'.

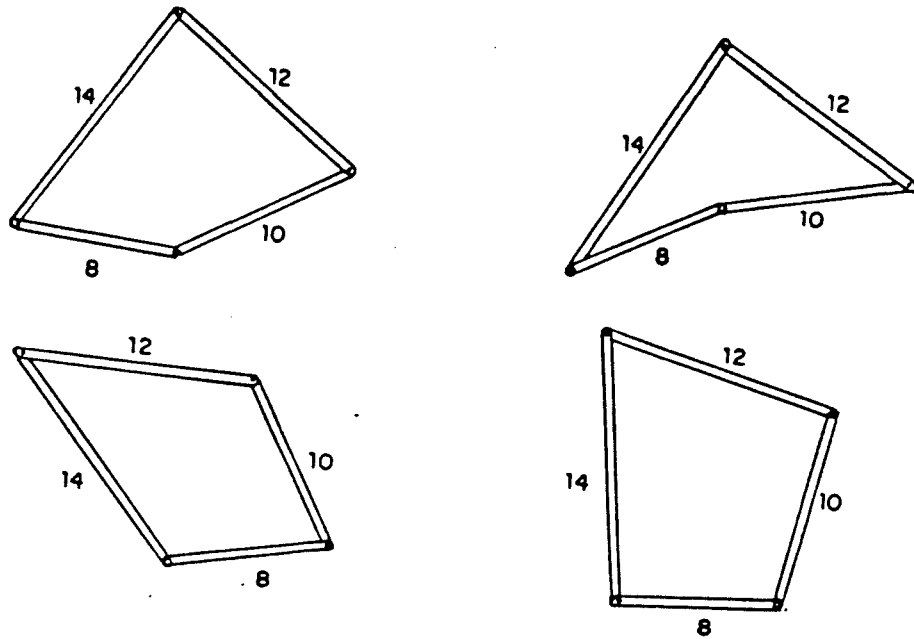


Figure 6.13

A good example of this lack of rigid shape of quadrilaterals is trellis work. As Lang explains:

"A length of trellising may be bought as a compact piece, with the slats touching one another and no open spaces between them. It may then be opened out to form the familiar pattern of squares or it could be opened a little less to give a diamond trellis. When such a piece of trellis is in place, firmly screwed to supporting posts, it will become the rigid support for climbing plants which we are used to seeing in gardens. The rigidity, however, is imposed on the trellis by the supporting posts and is not inherent in the criss-cross structure itself."

(Lang, 1960, p. 185)

In fact, every many-sided framework will behave in the same way if jointed at the corners. The only simple intrinsically rigid shape made out of strips of which the corner bolts do not lock the angles is the triangle: three given lengths can have one, and only one, possible shape. If we take the quadrilateral already discussed,

we will see that of all the possible shapes which could be made with four rods of lengths 8, 10, 12 and 14 br, one of them will be such that the distance from the intersection of the 8 and the 10 rods to the intersection of the 12 and 14 rods is exactly 16 br. If the 16-rod is bolted across this diagonal, the quadrilateral is

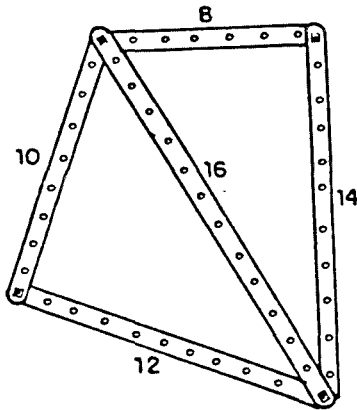


Figure 6.14

then equivalent to two triangles and it is quite rigid (Figure 6.14) This geometrical fact finds practical application in structural work in that it constitutes a criterion for determining uniquely the shape and size of a quadrangle via the rigidity of a triangle. It is also because a triangle can without ambiguity be completely defined by three of its elements that a network of triangles obtained by scale drawing gives an accurate mapping of a whole area of land measured by surveyors (method of triangulation).

But if cubação shares lack of rigidity with a 'free' trellis, *rigidity*, in the sense applied to the triangle, is not the best criterion for regarding the procedure as appropriate in estimating areas. With cubação, *rigidity* is more 'flexible'. That is, in addition to the possibility of 'opening' and 'closing' the trellis we are allowed to move the corner bolts in order to fix them at other positions, such as, for example, the one for which each sum of opposite sides is equal to the semi-perimeter. Thus, in terms of the perimeter P , the procedure $\Gamma = \Phi \S$ can be written as:

$$\Gamma = \Phi \{[(P/2)/2] [(P/2)/2]\} ,$$

or

$$\Gamma = (1.6/16) P^2 ,$$

or

$$\Gamma = 0.1 P^2 \tag{13}$$

What is interesting about this new formulation is that it expresses the area of the quadrangle in terms of the perimeter squared. In such a case, it is known from

geometry, that the factor of proportionality, here represented by the value 0.1 covas, is a parameter which measures the area of figure of respective shape having *perimeter of unit length* (1 braça), and thus it is characteristic of this shape. For this reason, it is called "shape parameter", and will be represented here by $k_n 'x'$, where n indicates the kind of shape and 'x' the system of measurement under consideration.

Formula (13) can then be written in its general form as:

$$\Gamma = k_n 'c' P^2 \quad (14)$$

where

$k_n 'c' = 1.6 k_n$; and where k_n is the shape parameter of the square in units of br^2 , and 1.6 is the factor Φ for converting br^2 in 'mil covas'.

This geometrical fact brings about two important implications.

(a) First implication.

Any figure whose shape parameter is known can be estimated by cubação simply by multiplying its value by the perimeter squared.

In this way, the perimeter constitutes area by means of the shape parameter which imposes 'rigidity' to the shape of the figure. Thus, it is not acceptable, for example, that quadrangles (a) and (b) in Figure 6.15 have the same area only because they both have four equal sides. One needs to search for their shape parameters which, in this case, are different. For quadrangle (a):

$$k_{(a)} 'c' = 1.6 (1/16) = 0.1 \text{ covas} ,$$

and for quadrangle (b):

$$k_{(b)} 'c' = 1.6 (\sqrt{3}/32) \approx 0.0866 \text{ covas} .$$

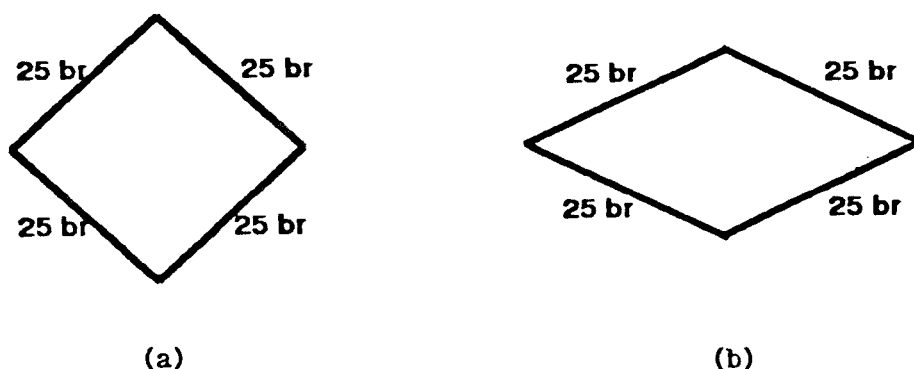


Figure 6.15

While quadrangle (a) has 1000 covas, quadrangle (b) has only 866 covas.

The following examples of application of formula (14) can be useful in further discussions; so it is worth anticipating their relevance.

Example 1: A rectangle of sides 1 br x 10 br, whose shape parameter is ≈ 0.0330 covas (see Table 6.1), has area:

$$\begin{aligned}\Gamma_{\text{rect.}} &= [(1.6)(1/48.4)] \times 22^2 = \\ &= (1.6 \times 10) \text{ covas.}\end{aligned}$$

If we change the system of units from br to chain, the shape parameter for a rectangle of perimeter 1 chain is $(1/48.4)$ sq chains ($k_{\text{rect 'acre'}} = [1/48.4] \text{ acre}$). As it is known (see chapter 8), the procedure Γ for obtaining the area of a quadrangle tract of land, in acres, is $\Gamma = 0.1 \text{ §}$. Thus $k_{\text{sq 'acre'}} = 0.00625 \text{ acre}$; and the area of the corresponding rectangle in the acre-system is given by (0.1×10) , which is equal to 1 acre. This is the definition of an acre: the area of a rectangle of 1 ch x 10 ch.

Example 2: The area of a circle expressed in terms of the shape parameter will be:

$$\Gamma_{\text{circle}} = (1.6) [(1/4\pi)C^2] ,$$

which is correct when normalized to Euclidean area units. We have already seen – page 148 – that $A_c = (r C)/2$ (where r is the radius and C is the circumference). The question arises of how to use these two relations (namely, A_c and Γ_{circle}) in thinking about area.

Table 6.1: SHAPE PARAMETERS FOR CUBAÇÃO

Shape	Φ	$k_{\text{shape}} (br^2)$	$k_{\text{shape}}' C^2$ (covas)
rectangle (a/10) x (a)	1.6	1/48.4	≈ 0.0330
equilateral triangle	1.6	$\sqrt{3}/36$	≈ 0.0769
rectangle (a/2) x (a)	1.6	1/18	≈ 0.0888
square	1.6	1/16	$= 0.1000$
regular hexagon	1.6	$\sqrt{3}/24$	≈ 0.1155
circle	1.6	1/4 π	≈ 0.1273

Historically, these relations participated in ancient mathematics such as the Egyptian, Chinese and Babylonian. Their functioning in the construction of mathematical thinking has been a matter of investigation in the history and philosophy of mathematics. And this seems to be a fruitful source for the analyst who has to take decisions about teaching geometry.

For example, historians would suggest that expression $A_c = (r C)/2$ seems to have come first (as 'natural'): its rules (which were 'lost' in Babylonia), remain clearly in favour throughout history, always attached to the procedure of finding the area of a quadrilateral as the average of one pair of opposite sides times the average of the other. Expression $\Gamma = (1/4\pi) C^2$, usually related to the problem of the construction of a square equal to the circle in area (that is, to the computation of Γ), is seen to be "the transform of a relation standing close to intuition" (Seidenberg, 1973, p. 185). The relation to which Seidenberg refers to as "standing close to intuition" is exactly A_c ; and the transformation is such, that an estimation of π is required (in the ancient civilizations just mentioned, $\pi = 3$, and $\Gamma = (1/12) C^2$). It is important, however, to make it clear, that what interests me is not to propose that farm-workers' knowledge is similar to ancient mathematical knowledge; or to suggest that teaching should 'repeat' history.

From the point of view of research into communal knowledge, what is needed is to clarify the types of skill performance which are required (or not) in the use of these relations (which would involve reversibility, incommensurability, etc.), and the infra-logic relevant to such a use (the relation part/whole, for example). To decide how to manipulate this information in planning teaching is a different question; and presupposes other aspects such as why one would teach those relations, for what purpose, and at what levels.

(b) Second implication.

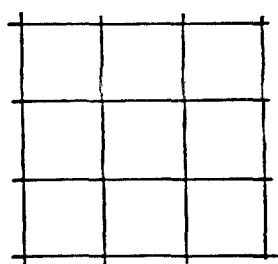
Consider again formula (13):

$$\Gamma = 0.1 P^2$$

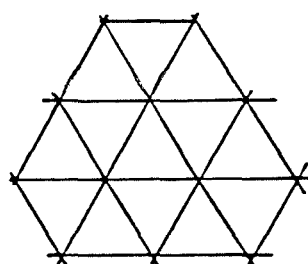
obtained from the general procedure $\Gamma = \Phi S$, by imposing that the sum of opposite sides of a quadrangle must be equal to the semi-perimeter. In doing so, we realized that the factor 0.1 could be taken as a characteristic parameter of a particular class of shapes: the squares. Recalling the definition of 1 'mil covas' as the area of a 100-square, and considering the practice of marking out the fields in the form of a regular {4,4} tessellation², 1 *braça* apart, the square can be, indeed, regarded as a *typical region*.

However, the shape of a typical region is far from unique. Any "square" would serve, provided it has four sides, each pair of opposite sides equal to the semi-perimeter, and the same area. All "squares" belonging to such a class would have $k_{sq} \cdot c = 0.1$ covas; the 'square'-fundamental region having area equal to 0.1 covas (perimeter = 1 *braça*).

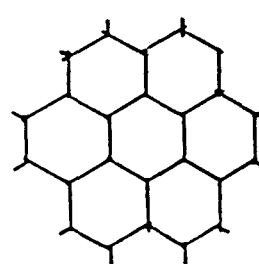
² The term *tessellation* is used for any arrangement of polygons fitting together so as to cover the whole plane without overlapping. A tessellation is said to be *regular* if it has regular faces and a regular vertex figure at each vertex. A regular tessellation is indicated by $\{p, q\}$, which represents a set of p -gons, q at each vertex, fitting together side by side to cover the whole plane simply and without gaps, such as the {4,4}, {3,6} and {6,3}:



{4,4}



{3,6}



{6,3}

But "what forms of quadrangle, with equal sum of opposite sides, have the same shape parameter (area for perimeter = 1 unit of length) as the square?" If we take the sides of these quadrangles as straight segments, the answer is "it is the square itself" (it is really unique). But if we permit the sides to be appropriately distorted, we will see that some other forms of "square" can be found. To this class belong all those shapes obtained by fixing one straight side and sweeping it through the plane in a way such as to keep constant the perpendicular distance between the opposite sides traced by the ends of the straight side, rotations being permitted (it was shown in section 6.1.4, that cubação correctly estimates the area of a RAR).

Some examples are drawn in Figure 6.16 (which includes the "triangle" as a particular case for which one side becomes zero). They are "squares" because they have the same shape parameter of the square, and the sum of opposite sides is the same for both pairs.

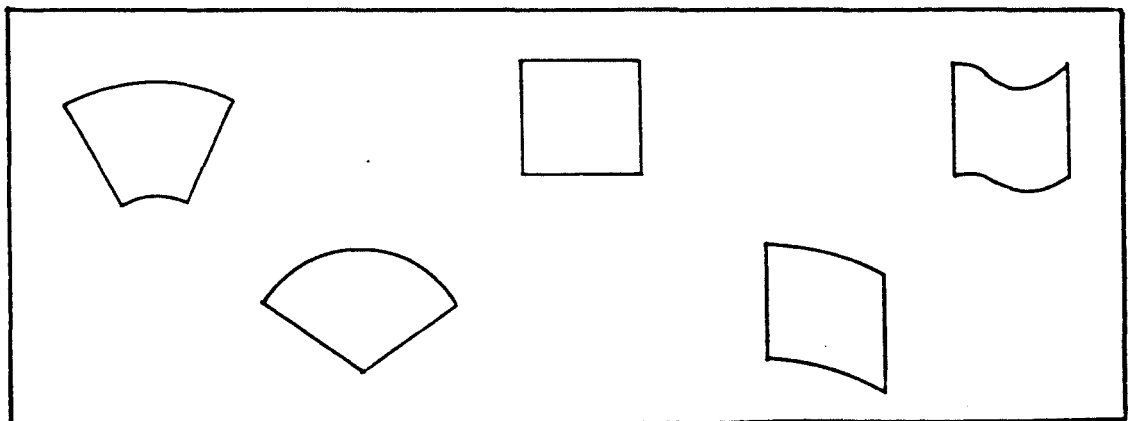
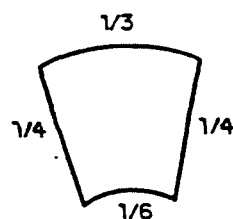


Figure 6.16

One fundamental RAR for which the area is equal to the shape parameter is represented in Figure 6.17. It has perimeter equal to 1 br and sides equal to fractions of unity. Applying cubação:



$$A_{RAR}'c' = [(1/3 + 1/6) (1/4 + 1/4)] 4/10$$

$$A_{RAR}'c' = [(1/2) (1/2)] 4/10$$

$$A_{RAR}'c' = 0.1 \text{ covas .}$$

Figure 6.17

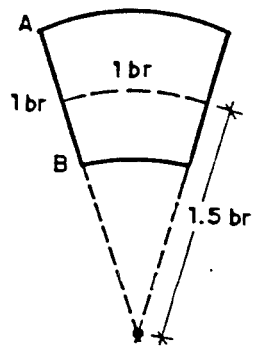


Figure 6.18

Such a fundamental RAR was obtained by *reducing* the RAR in Figure 6.18 (obtaining by pulling a *braça* AB perpendicular to its present direction in such a way that the central point moves 1 *braça* in a circumference of radius 1.5 *braças*) to its similar RAR of perimeter 1 *braça*.

In a similar way it is possible to define, for all the other "squares", the adequate proportions of sides for which the shape parameter = 0.1 covas. If we were to consider, for example a field of land marked by successive RARs (as defined in Figure 6.17) with one plant inside each cell, two similar "squares" would be immediately defined (they are represented in bold, in the 'field', in Figure 6.19).

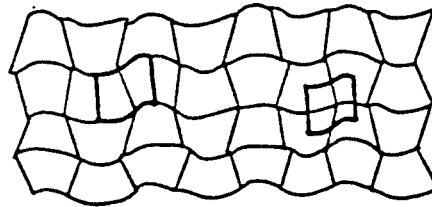


Figure 6.19

From the point of view of their areas, all the four "fields" in Figure 6.20 are equivalent: all of them are composed of cells with the same amount of 'covas' (cells with perimeter = 1 *braça*). That is, every possible fundamental region, whether we choose a square or any other shape, has *the same area* as the typical 4-square. For, inside a sufficiently large circle, the number of planting places (cells) is equal to the number of replicas of any fundamental region; thus every possible shape has for its area the same fraction of the area of the circle (the shape can be a single cell of a sum of cells).

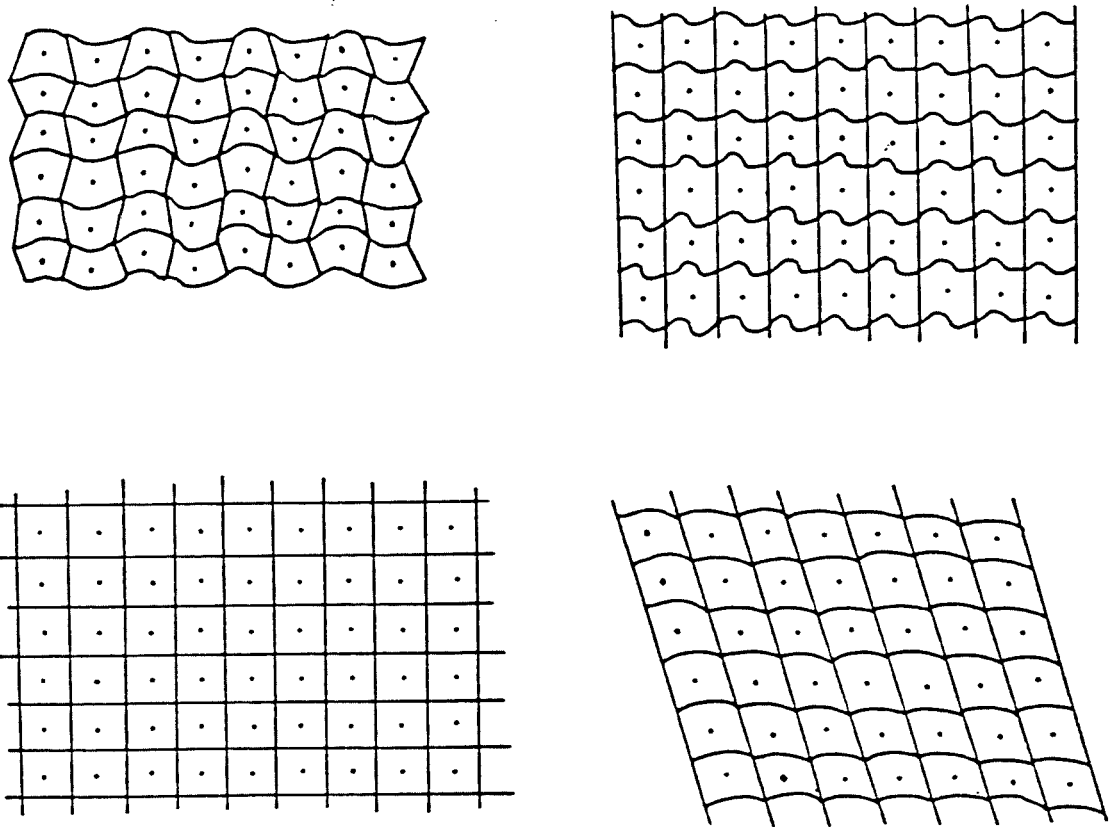


Figure 6.20

It is an interesting fact that in the same way that a regular $\{3,6\}$ can be constructed out of a $\{4,4\}$ trellis by closing it appropriately, the top right field in Figure 6.20 can be partly closed to generate a 'regular' $\{3,6\}$ tessellation of 'triangles', with one side properly distorted (see Figure 6.21).

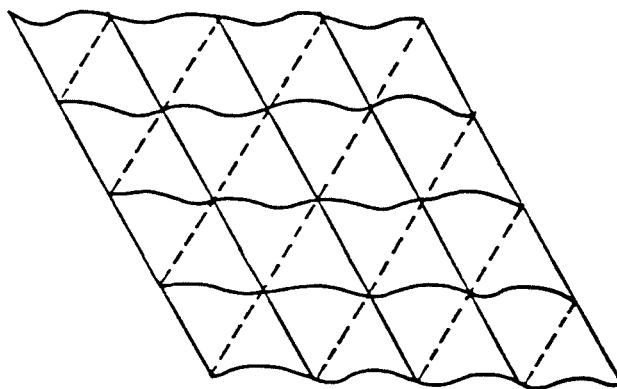


Figure 6.21

It is also interesting that, if we consider this $\{3,6\}$ field of 'equilateral triangles', the $\{6,3\}$ dual³ tessellation of 'regular hexagons' can also be constructed ('hexagons' will have two pairs of opposite sides congruent but not straight, as in Figure 6.22). Obviously, both the 'triangle' and the 'hexagon' will have different areas (related to their shape parameters). The relevant implication is that the planting places in a $\{p,q\}$ regular field constitute the vertices of the dual $\{q,p\}$, whether both of them are regular or 'regular'.

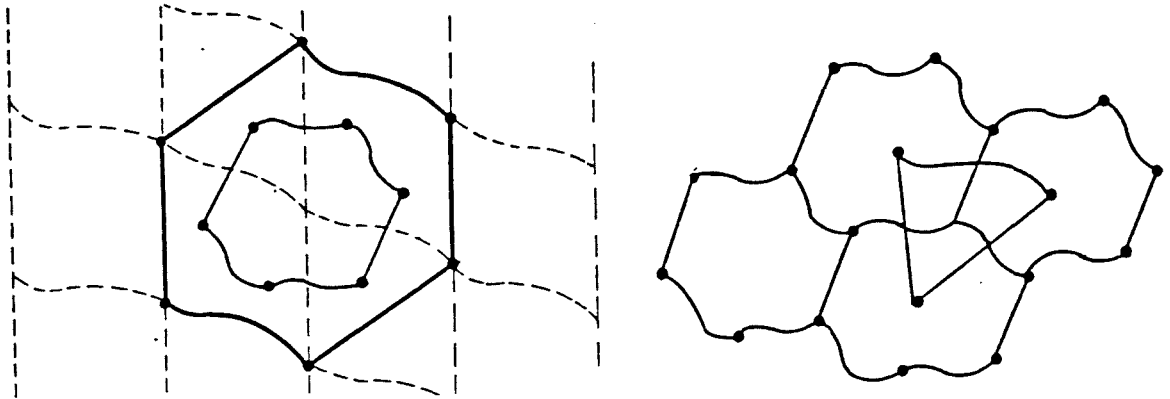


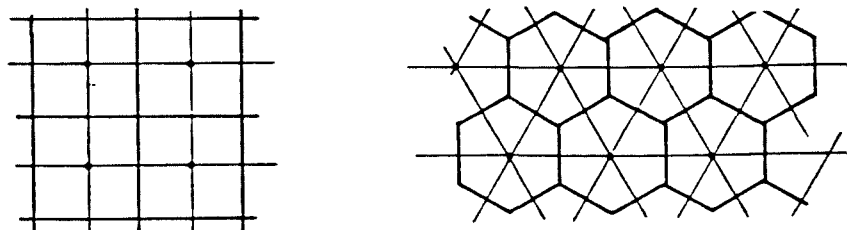
Figure 6.22

The implication which arises in connection with the above argument (which is not difficult to prove) can be proposed as follows.

Any shape similar to a 'square'-fundamental region or to the sum of 'square'-fundamental regions is correctly estimated by cubação.

Some situations to which this proposition applies are represented in Figure 6.23. Consider, for example, the left-hand-side shape, obtained by the addition of a square ($A_{sq} = r^2$), a quadrant ($A_q = (r c)/2$), and a rectangle ($A_{rect} = r b$). The total area (A_T) is:

³ The *dual* of $\{p, q\}$ is the tessellation whose edges are the perpendicular bisectors of the edges of $\{q, p\}$. Thus, the dual of $\{p, q\}$ is $\{q, p\}$, and vice versa; the vertices of either are the centres of the faces of the other.



$$A_T = r^2 + (r c)/2 + (r b) = r^2 + r (b + c/2) .$$

As a quadrangle with pairs of opposite sides given by $[r \text{ \& } r]$ and $[(r + b) \text{ \& } (r + c + b)]$, the total area is obtained:

$$A_T = [(r + r)/2] [(2r + 2b + c)/2] ,$$

which is equal to:

$$A_T = r^2 + r (b + c/2) .$$

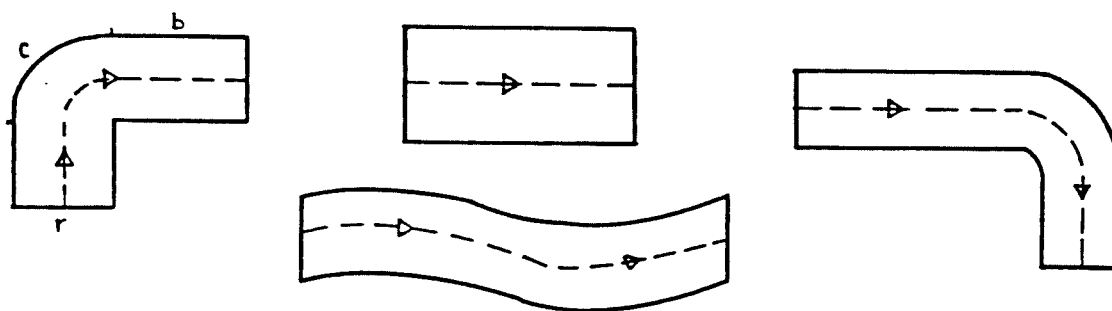


Figure 6.23

6.3.2 Conclusion.

Thus, as far as the farm-workers' practice is concerned, the question of "what is the general form of the shape for which cubação correctly indicates the area?" can be answered as follows (Figure 6.24):

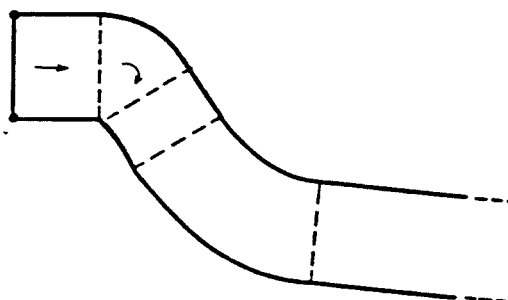


Figure 6.24

Cubação correctly gives the area of any tract marked out by the ends of a stick moved perpendicular to its present direction, and/or rotated. A wheeled plough, rake, or seed drill is such an instrument.

It follows that any tract marked out by ploughing successive strips will also be correctly estimated. The question arises of how much the traditional definitions of units of areas in terms of practical attributes try, exactly, to guarantee a correct application of the generic procedure presupposed by the system of measurement.

Consider for example the definitions of some units of area used in surveying in different places and times.

Acre. One *acre* is defined as the area of a 4-pole-strip which is 1 *furlong* long. *Furlong* comes from "*furrow long*" and is generally suggested to be "the distance oxen could pull a plough before having to pause for breath".

Iugerum. One *iugerum* is defined as a rectangle of (120 x 240) *Roman feet* or (1 x 2) *actus*. The *actus* (120 *Roman feet*) constituted the basic unit of length in the Roman period, and literally meant "a driving", "the distance which oxen pulling a plough were driven before turning".

Feddan. The name *feddan* is applied (in Palestine) both to a unit of livestock in the *fellah's* farm and to the area which can be worked by that unit in a fixed period of time. In ordinary usage, a *feddan* means "a piece of ground which can be tilled, i.e. ploughed and sown, with a yoke of oxen in the space of one day".

The question makes sense when we realize that knowledge of surveying is characteristically algorithmic: it involves a set of procedures for isolating problems and solving them, a set of assumptions and permissible deductions, a way of thinking about things in which what is 'correct' about results is taken for granted rather than explicitly demonstrated. When procedures such as these are transmitted, they act as a check upon the body of transmitted facts, allowing them to be re-derived or excluded if no proof can be found (Gray, 1979).

CHAPTER 7

FARM-PEOPLE THINKING ABOUT CUBAÇÃO

7.1 INTRODUCTION

This chapter attempts to contribute to the construction of a possible answer to the problematic questions raised by the researcher in relation to cubação; namely:

"how would farm-people react to different geometrical shapes? What would this tell us about their ideas about cubação and about their understanding of the concept of area?"; and

"how do we interpret 'mil covas'? How is 'mil covas' to function in relation to both planting and geometry?"

The chapter reports direct results from the Conversation with farm-workers and teachers when I tried to make cubação problematic in two respects: in respect of geometry (section 7.2) and in respect of the meaning of the result given in 'mil covas' (section 7.3). Some material from transcripts is used to provide illustrative instances for the argumentation.

7.2 CUBAÇÃO MADE IN SOME DEGREE PROBLEMATIC TO FARM-PEOPLE IN RESPECT OF GEOMETRY

7.2.1 The sense in which cubação was problematic to the researcher.

As I said in chapter 4, cubação came to my knowledge during the first set of interviews. From the beginning, it was obvious that the procedure was inadequate for estimating the area of those exemplary tracts used by farm-workers while explaining how to perform cubação. My attempt was then to try to *understand* the method better; working at the local unproblematic level, and seeing cubação in its unproblematicness. This, I did during the second phase of interviewing (as defined in chapter 3, page 83), when both farm-workers and teachers were invited -or thought it to be necessary- to explain and to apply cubação to concrete examples.

In looking at the transcripts of this set of interviews, it was inevitable for the researcher to analyse some of the informants' accounts in terms of her personal

geometrical knowledge. And the reason was clear: in terms of the researcher's knowledge, these accounts did not make good sense. The researcher could 'understand' the direct accounts offered by the informants, but what they were saying was not sufficient for reconstructing explanations within the framework of Euclidean Geometry.¹ Accounts such as:

"it is not necessary to have a quadrangle with equal sides";
 "west must always be added to east, and north to south";
 "this tract has 200 *braças* (to designate the area of the tract);"
 "one multiplies by 4 because there are four sides";
 "one multiplies by 4 to 'see' the number of *mil covas*";
 "one should ignore the last digit to show the result in *mil covas*";
 "the expression *mil covas* does not designate the number of "*covas*"
 (plants or pits)";

did not inform about what the farm-workers and teachers were saying when they systematically kept using those expressions. They seemed to contain features of a content which marked out geometrical *explanations* in cubação, but it was not clear what such a content could be (what to say of their features?). Also, it was impossible to get this information from the protocols I had up to that point.

In a first approximation it was reasonable to suppose that selections of geometrical knowledge about *area* from the common experience of applying the method to actual situations would be involved. At the level of social practices it seemed plausible that this notion could be connected with the division of tracts, with the strip-tesselation of the fields, and with estimation for work to be done on the land. Euclidean area involves some notion of 'disassembling and reassembling'; also, this idea of area rests at bottom on the possibility of tessellating the plane.

Accordingly, it would not be wrong to assume that a cluster of common sense schemes would be in operation. But could one suppose more refined categorizations of the phenomenon which would require supplementary schemes? That is, could the concept of area be part of some cognitive entity underlying a mode of 'theorizing', typical of cubação?

The point is that, farm-workers give answers that the researcher can codify; but "*do they understand area as she does?*". If the farm-workers were thinking of area

¹ What happened during this stage of the interviewing process could be described in terms of positions (b) and (c) as characterized in Appendix 3.A. It was not easy for the researcher, trying to make sense of people's accounts, to 'see' the informants 'following' the process of understanding.

in the Euclidean sense, then cubação would look a strange way to work out such a thing. But farm-workers are thinking like a person planting things or buying land and they do not question the method.

To try to answer this, the researcher was obliged to make hypotheses about both how the method is performed *and* about area (method and area are tied together by a logical structure). Given that there are cases (some remarkable) to which cubação correctly applies, it would be difficult to hold that the method was definitely wrong; in which case it would be the notion of *area* that needed challenging.

But would it be plausible to talk about a kind of cognitive foundation of cubação in which the concept of area could be taken in some respect as *developed* from cubação to school/Euclidean geometry (or *vice-versa*)? If this were the case, what would be involved in such a change?

These were then the *general* questions I had about cubação which made it an interesting case for further investigation. But they were *not* questions to be answered straightforwardly. What they indicated was that more information was necessary about "*why were farm-people saying those things?*" And this would require, not direct information about cubação, but information about how farm-people think about cubação.

Thus, making the assumption that the farm-workers I was interviewing were representative (in social terms) of those experts who could come to do it correctly (even if differently), my attempt was to try to clarify, from the perspective of Euclidean Geometry, "*what do people do which makes them experts?*" Because there are many people in the community who do cubação and succeed: they are called 'the experts' in cubação (a fact that came to my knowledge during the second set of interviews). It was at this point, that it became necessary to make assumptions about the role of farm-workers in a discursive community. The exact characterization of cubação as a *discursive practice* remained to be made, but its roots were planted in that very moment. Also, it was at that time that the answerable research questions about cubação were more clearly set. The network in Appendix 7.A shows the kinds of issues I had considered up to that stage about cubação (which I used in chapters 2 and 5).

7.2.2 Farm-people's reactions to different geometrical shapes: some raw results.

The discussion in this section is organized around four points:

- (a) The stages of interviewing.
- (b) What the informants did.
- (c) Features of the informants' responses.
- (d) Summary of the relevant ideas.

(a) *The stages of interviewing.*

To try to understand why it is that farm-people succeed in reckoning actual shapes, leads one immediately to ask "*how would farm-people explicate what happens when we present both different shapes and different solutions?*" In showing both 'shapes for which cubação fails' and 'alternative answers to the same problems', questions such as:

"does the situations (shapes) arise; does cubação apply to those shapes; can we adapt it or not; does it work; will people struggle to make it right; will they repair; how will farm-people deal with discrepancies; which one will they think it is right (judgements of likeness and differences); what would count as plausible explanations?

could be discussed and a deeper understanding of the firmness of assumptions, and of the fluidity of might be obtained. Thus, by looking at what people do, and at how far they go with their responses, it would be possible to draw inferences which could help us to answer the research questions. What was expected was that the questions posed by the researcher to the informants could make cubação problematic, such as to provide pointers to a speculation about the concept of area in cubação. There was no expectation about coming to construct a conclusive answer.

The investigation which followed was set in two stages, the second following from an analysis of the first (they correspond to the third and fourth phases of interviewing; see chapter 3, page 83).

(a.1) STAGE ONE

Four shapes as in Figure 7.1 were presented to all the informants of the main group (and to an additional farmer), one at a time: an irregular quadrangle; a triangle; an irregular pentagon; and a circle. Additional shapes were introduced in different stages of the interview, with the intention to raise problems and to introduce contradictions (not necessarily all the additional shapes were shown to all the informants). These shapes are drawn in Figure 7.2.

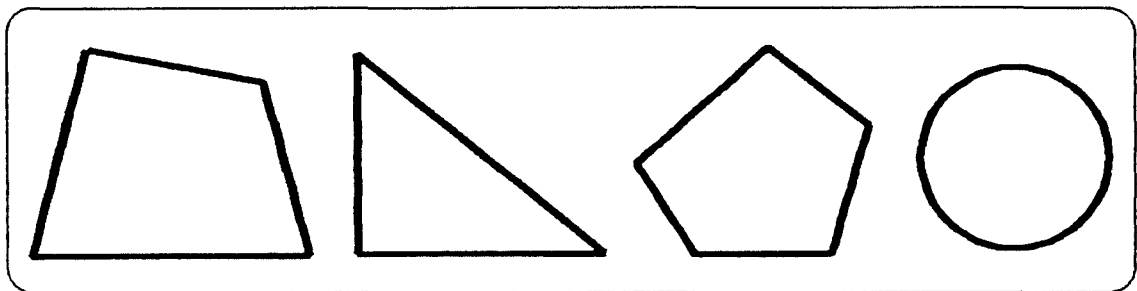


Figure 7.1

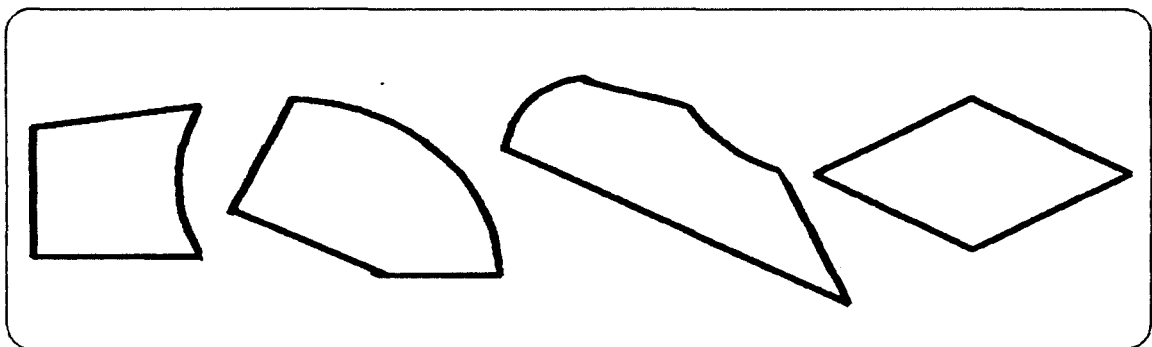


Figure 7.2

All the informants were asked to solve the problems by cubação, including the teachers. These were also asked to solve the shapes by Euclidean geometrical procedures.

(a.2) STAGE TWO

The solutions given by the informants in *stage one* were analysed in their differences and similarities. A worksheet (which is included in Appendix 7.B) was prepared to guide the discussion with the main group of informants. It comprised

a summary of the informants' responses in respect to three shapes:

- . the *quadrangle* (which is the relevant shape for describing the method), which was presented with three different formulations of the method;
- . the *triangle* with a description of seven procedures for solving it; and
- . the irregular *pentagon*, with three alternative procedures.

The descriptions represented the researcher's version of what the informants were saying (thus, a codified version). Each description, for which a shape-transformation was involved, was accompanied by a pictorial representation (which is something farm-people never use in doing or explicating cubação). A group of questions was raised for each shape such as (see Appendix 7-B for details):

- . Are the various methods/procedures different?
- . Would you expect these procedures to give the same number of *mil covas*?
- . Is it possible to know? How do you come to a conclusion?
- . Which one do you believe to be correct? What reasons do you have to justify your answer?
- . Why are certain procedures not correct?

Both farm-workers and teachers were asked to try to answer the above questions. In addition, teachers were invited to discuss some geometrical aspects such as congruence, similarity, assembling and re-assembling of shapes, relations between linear and area units of measurements, and so on. It was presupposed that such a discussion was essential to an understanding of both cubação and about the possibilities of farm-people to think about cubação from the perspective of the school geometry.

A set of cardboard geometrical shapes and pieces of transparent sectional pad were used to assist the teachers in their attempt to answer the above questions from the position of someone who is able to think in a Euclidean way (the results from stage one had indicated that this would not occur 'naturally').

During stage two, parallel interviews were conducted with additional informants to clarify specific points. Some of these points were related to the discussion which was going on with the main group; but some were required to complement information about the local unproblematic level of analysis.

(b) What the informants did.

(b.1) STAGE ONE

The following happened in the first stage of the investigation:

(b.1.1) all the farm-workers and the farmer offered at least one solution to the quadrangle, to the triangle, and to the pentagon (some offered more than one solution);

(b.1.2) one farm-worker (S.) and the farmer (J.M.) offered a similar solution to the circle, which was proposed by the farmer as a general solution for any shape;²

(b.1.3) one teacher (D.) was able to solve the problems by means of Euclidean procedures; the others made some attempts but came to the conclusion that they had "forgotten" how to do it;

(b.1.4) the teachers were not able to solve shapes by cubação; but tried to describe features of solutions given by farm-workers; most of the time was used to raise problematic aspects in relation to both cubação and school geometry; one teacher (D.) tried to solve some shapes by cubação, and her solutions were similar to the farm-workers'.

(b.2) STAGE TWO

What happened in stage two can be summarized as follows.

(b.2.1) All the informants tried to answer most of the questions raised in the worksheet, during the interviewing time. But no informant gave conclusive answers.

(b.2.2) The farm-workers tried to reaffirm their own solutions in terms of what they believed to be correct, and were not interested in checking the results for the different procedures. Being asked about "how can we know if a particular solution is correct" and "why certain results are not correct", they inquired about some elements of others' solutions which they tried to assimilate. In trying to accommodate their own solutions to the solutions in which these elements were present, two different things happened. One farm-worker (J.) kept tied to the algorithm and did not make any significant attempt to investigate the plausibility of his own solutions. He insisted with the fact that he was not an expert in cubação. Two farm-workers (S. and Ce.) kept the rationale of the reasoning presupposed by cubação and offered similar accounts to sustain their solutions as plausible (their solutions were initially different and during the discussion they

² The circle was not included in the worksheet which was used in stage two. The idea was to try to keep only those shapes for which all the three farm-workers had proposed a solution (thus, farm-workers would be able to recognize, among solutions, some of their own). But because the solution given to the circle had a generic character, it was applied to the other three shapes (quadrangle, triangle and pentagon) as cases, and these solutions were included in the worksheet.

evoked the generic solution given to the circle). But their posture in relation to the issue was different. While farm-worker Ce. had justified his answer in terms of his 'expertise' in cubação, farm-worker S. had insisted on the necessity of proving and testing his arguments.

(b.2.3) To compare the results obtained from different procedures was considered to be important by all the teachers; and they spent some time in doing the calculations. Some solutions presented in the worksheet were chosen as correct; but in doing so, they tried to distinguish between what they knew would be acceptable as solutions within cubação and what they accepted as a personal solution. In relation to accepting one of the farm-workers' solutions as of their own, only one teacher (E.) used as a criterion 'that which is near to the way one thinks in school'. The others referred to aspects such as 'tradition' or 'to be accepted by a community of people'.

(b.2.4) In discussing aspects of Euclidean Geometry, teachers not only tried to account for cubação in school terms, as they tried to do the reverse; that is, they sought seeing features of cubação in the school algorithms. They did not know how to do it, but raised several pertinent questions; getting involved in a true process of trying to understand systems of measurement in both global and local levels.

All the informants asked to keep the worksheet after the interview; and I asked them to let me know about their ideas in case they had a more definitive answer about any item. One teacher (V.) outlined some written ideas after the meeting and sent them to me. Another teacher (E.) contacted me personally to tell about some of her conclusions (this meeting was not tape recorded but I took notes while she was talking).

(c) Features of the informants' responses.

In looking at the informants' responses, there are two distinctions which are worth considering. One concerns the stage in which responses were provided. Stage two evolved from stage one, and most of what comes in stage two becomes clear when this fact is taken into account. The other concerns the group of informants (farm-workers or teachers) in that different perspectives were set for these two groups in discussing similar and related issues.

In stage one, while farm-workers were invited to solve shapes by cubação (how and why), teachers were asked to do the same but at a different level, that is, as people who had at their disposal an additional analytical instrument (school geometry). In stage two, while farm-workers were invited to justify solutions within the system of cubação, teachers were asked to account for their plausibility (and this required, for example, making inferences about fundamental features of both cubação and Euclidean Geometry).

The organization of this item considers these distinctions and is organized in two parts.

- (c.1) Stage one
 - . The farm-workers
 - . The teachers
- (c.2) Stage two
 - . The farm-workers
 - . The teachers

(c.1) STAGE ONE

THE FARM-WORKERS

Farm-workers were not at all reluctant to provide solutions for the different shapes. In some occasions, they recognized that they had never reckoned such shapes; a fact which did not keep them from contributing with a personal solution.³ In discussing the various shapes for which cubação would fail, different levels of conditionality for existence, actuality and possibility were constructed; which shows that, indeed, in reasoning about area, farm-people can go further than the immediate experience.

(c.1.1) About the kinds of solutions.

Most of the solutions given by the farm-workers can be described in terms of an *adaptation of the traditional algorithm in cubação to a sequence of steps* intended to *transform the shapes* into a square; in order to set the 'square condition' for applying the method (in that it presupposes a tract with four edges). This was so, except for one kind of solution given to the triangle, in which case the informants *modified the square-condition* to keep both the *steps and the shape unchanged*. These two kinds of solution are examined in (c.1.2) and (c.1.3) respectively, where instances from the transcripts are provided.

³ There was only one special case (namely, the circle) in which the informants' attempt was preceded by an explicit assertion that "this is something for the 'mestre' (bricklayer), in that it requires adopting the metric system. As I said in chapter 5, p. 123, skilful performances required in solving the circle characterize what is generally taken by farm-people, to be a real metric system in opposition to cubação.

(c.1.2) About the transformation of shapes.

To have four edges specified, is an essential condition for applying cubação to a tract. Without four edges, it is impossible to talk about North, South, West and East; and there would be no reason for the peculiar step that multiplies by four the product of the sum of the opposite sides. As one farm-worker said:

"It is necessary to multiply by 4 because there are four edges; and the method is set for 'squares'."

Thus, the solution of trying, first, to transform non-squares into congruent 'square-shapes', would be in principle an acceptable idea. But let us look at how farm-workers did that. In general terms, cases of transformations are identified in the protocols in the form of *compensations between sides, under perimeter invariance*. 'Compensations' are of the following kinds.

Appending sides: Appending involves the jointing of two sides to make one (which was used for the pentagon). In the words of a farm-worker, this kind of compensation was expressed as follows.

Instance 7.1

C. How do you solve this [a pentagon]?

F.W. It can be in this way. One can make this round. If this is 50 and this is 30, then one can write down 80, multiply these and then one adds these two. And then one takes this and... [He speaks very quickly and it is difficult to follow his idea].

C. Please, wait a moment. I want to write down what you are saying; because I want to understand clearly what you are saying ... it is very important and I want to learn it. Now, this is 50 ... then, which one do you take?

F.W. I take this one [he shows an opposite side].

C. Let us say it has 20.

F.W. Then we add them. Let us say they are West and East. So, these two are North and South; and we add them to multiply by the previous result.

C. All right, we multiply. But what do we do of this side?

F.W. Well, this one It was used. Because initially we had made this [he shows one corner] round ... you know? Don't you understand what I am saying? Look, we measure all around these two sides ... all them, in one goal. We start here and go: 1, 2, 3, 4, 5, 6, ... and so on. Then we get 50.

C. All right, I understand now. It is the same to say that, first, we had to add two sides.

F.W. Yes.

C. Ok. Actually, it would be like to do this first. Look ... [near the pentagon I draw a quadrangle, trying to represent the transformation involved].

F.W. It is exactly in this way.

C. Then I take...

F.W. Yes, this is right, it is very good [he keeps looking at the drawing]. This one [he shows the quadrangle] ... it is here ... in this tract [the pentagon].

C. Then, when you start, you measure all this line... Then you apply cubação.

F.W. Yes. But it is West and East, ok? It is always West and East; and North and South. It is very easy.

Equilibration of the shape: The equilibration rule is a generic rule which transforms the shape into its isoperimetric square. In the transcripts, it arises in one of two forms: or it is explicitly stated, or it can be presupposed by the solution involved. Instances are found in the protocols.

Instance 7.2

F. ... Look, I will give you a technique, a direction for you to follow. You will understand it better.

C. All right.

F. It is in this way. You have four edges ... they are not equal ... You think that the result will not be exact [he uses the word 'positive']. You believe it will be an approximation, don't you?

C. Yes.

F. Then, you add the four edges; and then you divide by four to get equal sides. They all have the same size. Do you understand?

C. You add the 4 ...

F. Yes, you add the 4, you see ... the four edges. When you finish you divide the result by 4 ... to equalize the tract.

C. Ah!

F. Do you understand?

C. Yes, I do.

F. Yes, you add the four edges and the result you divide by four. Then you get an exact result ... Is it clear now?

C. Yes, it is.

F. It gives an exact amount. You add the opposite sides, multiply the results, multiply by 4, and take the last digit out. It is 'cubado'.

C. Could we solve a tract such as this one [a circle] in this way?

F. Yes, we could. We can. You measure it all. You start here ... you fix a point. Then you measure the whole boundary. When you come back to the starting point, you have a given number of *braças*. It is just to divide by 4.

Instance 7.3

C. Yes, Ok, all right. But if you have this one [a circle]?

F.W. What? A round shape?

C. Well, I am not sure if there are cases like this ...

F.W. Yes, I have never done one like this. But I think it could be in this way.

C. How would you do?

F.W. I think that we could go walking around the whole shape ... and then we divide it in two parts ... and we multiply. Then we consider the 4 ... It would be in this way... it can only be in this way. Is that right? Suppose that we measured all along the round shape and that the result was 100 *braças*.

C. 100 *braças*. Ok.

F.W. Then we multiply 50 by 50. So, we would write ... let us say ...

C. 50 by 50.

F.W. Yes ... But this calculation is deceptive. Because we can have a small tract [it seems that he is thinking in the perimeter] and the result will be bigger than what it should [it seems that he thinks of the square]. Because this is a round shape.

Because of its general character, the *equilibration* rule was considered to apply to any shape and was included as a possibility for the three shapes in the worksheet (as both method and procedure). About its application to the triangle, the farmer expressed an idea as follows.

Instance 7.4

F. [...] with 3 edges... suppose you do not want to apply cubação to 3 edges directly, but to apply it to 4. What you can do then is to make four sides out of three. The only thing you need to do is to add all the 3 edges...the number of *braças* they have...and the result you divide by 4 ...you equalize the sides.

C. All right.

F. So you get four equal sides. Then it is just to take them in pairs and to multiply the results ... and to multiply by 4 ... It is finished. You have squared the tract. [...]

F. Thus, if you are not sure if the result will be exact ['positive'], what you can do is to add all the edges... This one [he shows one edge] will give you more because it is curve ... when you come to measure it, it will give more than the others ... than you add them all... the four edges. Then you divide by 4. That is, you equalized them. The four edges now have each the same size. Do you understand?

C. Yes, I do.

F. Because what lacks in one edge, the other complements, grows...doesn't it?

Algebraic compensation: The rule (which was used for triangles), says to apply cubação as if the shape were a quadrangle. Thus, somehow, the farm-worker has to consider the *existence* of a fourth side (whether it is zero, or has an insignificant length); which should not be taken as a change in perimeter. This would correspond to the *breaking up* of one side performed algebraically.

Instance 7.5

C. [...] Another question. How would you do it if you have a tract like this [triangle].

F.W. With three edges? It has only three edges ...

C. Yes.

F.W. I do it in this way ... I do the same thing.

C. Suppose that the edges are of length 50, 30 and 30. Let us suppose.

F.W. You want to know how I 'cubo', don't you?

C. Yes, how do you apply cubação.

F.W. I do it in this way. 30 I add to ... what are the lengths? Is it 50 here?

C. Yes, 50 here and these two are 30 and 30.

F.W. 30 ... 50 ... There is another one here, with 30 ... Well, they are 80. I do this. Then I take this 30 and I write it down here. I had written 80 here ... Then I multiply these two. After that, I write down that 4, which are the 4 edges. You see ... here it does not matter if the tract has 3 edges. I do it in this way because in this way it will provide the correct number of 'mil covas'.

C. Thus, you add two of them.

F.W. Yes, two.

C. Now, do you choose which sides you add?

F.W. Any two. [...]

Instance 7.6

- C. This is one thing I want to know. When the tract has 3 edges, how do you do it?
 F.W. I add 1 *braça* here. 1 or 2 ...
 C. Let me see ... I drew a shape here ... with 3 sides. ... Because I want to know how do you do when the tract has not 4 edges.
 C. You have to add at least 1 *braça*.
 C. Has the tract to have always 4 edges?
 F.W. It has.
 C. For example, suppose we have this tract [I show the drawing].
 F.W. You put 2 *braças* here [in one corner]. And then you consider the number of *braças* that each edge has. [...]
 C. All right. You introduce only a small piece.
 F.W. Yes. Suppose this one has 35 [the opposite to the small edge].
 C. All right. Then you add these two and those two.
 F.W. Yes. It gives 37 *braças*, doesn't it? If I consider the small one.
 C. Yes. OK. Does this mean that this corner can never be empty?
 F.W. Yes, never. Because if it is completely empty, you can not do *cubação*.

Because all the three kinds of compensation presuppose *interchange* of pieces between sides, one could say, more generally, that they are particular cases of a *breaking/making up* procedure. Thus, when one side is *broken up*, another is *made up*.

(c.1.3) About the modification of the 'square-condition'.

The modification of the 'square-condition' happened only in relation to the triangle. It was proposed by a farmer, but considered by a farm-worker as a possibility about which he was not sure (later, it was also referred to by the teachers as a procedure used by some farm-workers).

Instance 7.7

- C. ... My problem is this. When your tract is not a quadrangle ... it does not have four edges...such as this one [a triangle]. What do you do?
 F. You add two edges and multiply the result by one. All right?
 C. I add two ...
 F. Yes, and multiply by one.
 C. 'One'... is it this one [showing one edge]?
 F. Yes. And then you multiply by 3.
 C. By 3?
 F. Yes. And then you take this digit out. It is finished. Do you understand?

Instance 7.8

- C. Suppose you have a tract like this ... with three edges. How do we do to do *cubação*? Does it happen to have tracts such as this?
 F.W. It happens. Now, I do not know if it is right as I imagine ... this one, this, and this one [he shows the edges]... There is one which has no length.

C. Then you would take this and this ...
 F.W. And this other alone.
 C. Then you would multiply the previous result by this one ...
 F.W. Yes, by this one.
 C. Would you multiply it by 4?
 F.W. Well, this is exactly... Yes. Just a moment ... I am not sure that it gives the right result with 4... But it should give. Because... In the end, it does not matter if here they are two or only one. Where is that piece of paper?
 C. It is here.
 F.W. Let us make an experience. Please, do it, just for us to see ...
 [he asks me to calculate the same shape with 4 and 3. The difference is significant and he gets confused about which result is correct].
 F.W. I am inclined to believe that the right thing to do is to multiply by 3.
 C. So, you think ...
 F.W. That this one, with 3 is correct.
 C. OK. But why do you think so?
 F.W. Because we do not actually have one side...I mean, one edge. There is only...it is... I could do, following this side...but if I could at least put 1 *braça* here [he shows one corner], it would be right. Yes, I am not able to explain.

From the extracts, the condition is modified from 4 to 3 sides. And the steps and the triangular shape are not altered. Considered in its relation to the transformation of shapes, this change suggests three things. Firstly, that this is not a case of algebraic compensation in that the triangle is not taken as if it were a square; it is a triangle, and it can be reckoned as such.

Secondly, it suggests the possibility of the existence of a constitutive rule for area (expressed in 'mil covas') which is taken by reference to the perimeter. This 'rule' would say that each side provides a given fraction of the area which the perimeter embraces; and it would account, for example, for the meaning of the farm-workers' explanations that "we multiply by 4 because there are four edges"; or "we multiply by 3 because there are three edges". Thirdly, to coexist with the *breaking/making up* procedure, this rule would have to contain an account of how to compute fractions when sides are created or appended, *under both area and perimeter invariance* (which is something not immediately conceived in the perspective of Euclidean Geometry).

THE TEACHERS

(c.1.4) About the kinds of solutions.

Teachers were asked about both how to solve shapes by 'school geometry' (that is, as they knew) and by *cubação*. Their solutions can be described with respect to these two situations.

Solutions by 'school geometry'. The answers given by the teachers were straightforward. The right ones (given by one single teacher) were proposed in terms of the traditional procedures we learn in the primary school: using adequate formulas for some regular shapes, added to the principles of congruence, assembling/disassembling of shapes, conservation of area, etc..

Among the wrong ones, there were two kinds: to add the edges of the shape (that is, area = perimeter) as in instances 7.9 and 7.10; and to apply the same steps as in cubação (instance 7.11).

Instance 7.9

- C. Consider a square whose side is 1 metre: 1m, 1m, 1m and 1m. Now. What is its area? What do you do to calculate the area ... in the metre-system?
 T. I add the four sides.
 C. You add the four sides. Humm. Thus, you say that it has an area of 4 metres.
 T. I think I am ... It is 4 '*squared-metres*', isn't it?
 C. Is it 4 metres in this way ... '*squared*' [I show the perimeter]?
 T. Yes, it is.

Instance 7.10

- C. Ok. Let us start. Consider this shape [a square]. What is the area of this shape, if it has 4 metres here and 4-here?
 T. In this case it is 16 metres, isn't it? [she follows the boundaries with a finger].
 C. 16 ... what? 16 metres?
 T. Yes.
 C. Do you use the word 'metres' to indicate the area?
 T. Yes, I do.
 C. In the case of cubação they use the word 'covas'; is that right?
 T. Yes. Because they refer to a big tract. But in this case, if it is a 'cisterna' (a small reservoir) or a house ... one will use metres.
 C. All right, one uses metres. But the answer ... if I ask you what is the area of this square here, will you reply it is 16 metres?
 T. Yes, I will.

Instance 7.11

- C. Now, suppose you have to calculate the area of this shape, using the metres procedure ... all right? Suppose someone decided to build a house with this shape, and you have to estimate the area. What would you do? Suppose it has sides equal to 6 m, 4 m, 3.5 m, and 5 m. How would you do it?
 T. I think I would use the same procedure as in cubação, but using metres. But the calculation I would do in a similar way as if I were reckoning ... if I were applying cubação to a tract of land.
 C. In the same way as if it were a tract of land?
 T. Yes. It would be similar to the case of cubação in *braças*.
 C. Humm. All right. But you would try ... instead of saying 5 metres, would you try to find how many *braças* there are in 5 metres, or not?
 T. No, I would try to do it directly, using metres.
 C. Would you do the calculation with the same numbers I gave you in metres?

T. Yes, because the place had been measured in metres.

C. Humm. All right. And about your answer? Would you give it also in metres?

T. Yes ... in this case we can not give the answer in squared-metres, can we? I would give it in ... in cubic metres.

It is interesting to notice that in the former two solutions, units of area were given in terms of *squared-metres or metres* (for area = perimeter); while in the latter, in *cubic metres* (in the case of area = cubação).

Solutions by cubação. When asked to do cubação, the teachers made it clear that this was something not habitual for them. But they agreed to learn the method, given that cubação was effective in practical terms and that it could constitute a relevant topic for teaching. To learn the algorithm was not a problem. But to try to make sense of the expression 'mil covas', for someone who was not able to work out properly the notion of 'units of area', became a major obstacle for discussing cases for which the method would fail (such as the triangle, the pentagon and the circle). Thus, it was not a surprise that the only teacher who was able to carry the discussion further, had demonstrated some expertise in the use of the notions of Euclidean area and Euclidean methods.

But the personal solutions given by this teacher were not very different from those presented by farm-workers (except for one step in reckoning the circle). However, by contrast with the discussion with farm-workers, the dialogue with this teacher was enriched with legality-aspects. Aspects which did not belong to the field of application of Euclidean methods, but to cubação itself. Three passages from the conversation with this teacher follow in instances 7.12, 7.13, and 7.14.

Instance 7.12

C. [... after having checked that the teacher had grasped how to apply cubação to a quadrangle] All right. Now ... what I want to know is this. How would you apply cubação to a triangle? If you were using this procedure?

T. I think it is not possible.

C. Don't you have any idea?

T. No. I think it is not possible. In this case, it is not possible to do it. Well, I don't know.

C. You believe it is not possible. Why?

T. Because the procedure says one should take North and South, West and East, doesn't it? These measures are prescribed in this way. How can I do it? I don't know.

C. This is what I want to know.

T. I think that it is not possible.

C. You think it is not possible.

T. Yes. Because a tract, I think, it has to have always North, South, West and East.

C. Humm. So, if you have a tract like this [triangle] located in the field, what does

it happen?

T. Right. This one will be zero. I would take this one as zero.

C. So, you would take one edge with length zero. This is something I would like to know.

T. If this one is zero... then I would take the others and would do it as usual.

C. All right.

T. I would take these two, then these other two... I would multiply the results, and then the result I would multiply by 4 ... I would do it in the same way.

Instance 7.13

C. Now, suppose we have a tract with 5 edges. What would we do?

T. Well, I would try this ... I would make one side here [she shows the extreme points of two consecutive sides].

C. This one ... Does it mean that you would draw a straight line between these two points?

T. Yes ... But ... is this line equal to the addition of these two edges? ... I think that I would add the two sides. That is, I would draw one edge equal to the addition of these two. I would add so as to make ...

C. ... one side.

T. One side.

C. All right, ok.

T. I would add, not drawing this straight line between these two points, but trying to fit one edge, equal to the addition of these two edges. Then, the tract would have four edges. Is that right?

C. Ok. Then you would take two edges out of five, and would transform them into one edge. Then, you would proceed with the calculation using this edge.

T. Yes. And then I would take the opposite edges ...

C. And apply cubação as you know. All right.

Instance 7.14

C. Now, have you got any idea about how to do cubação in this case [a circle]?

T. You mean to do cubação in the way my father does?

C. Yes. Doing cubação in *braças*.

T. I understand ... in *braças*.

C. Suppose someone only knows how to do cubação in *braças*. All right? Then, he faces the problem of having to reckon a tract such as this. What does he do? This is what I want to know. He only knows how to do cubação in *braças*.

T. Well, he is in trouble. Because ... if the tract is round ... how can he do? I don't know ... Be sincere ... how would you do ... a circle?

C. I don't know.

T. I don't know either ... I don't know. Because ... Cristina, there are no 'distances of nothing'...no 'distances in *braças*'. How would he consider 'measures' to add?

C. Yes. Where does one measure? That is the trouble. What does one measure?

T. One measures the land ...

C. Ok. But one measures ... when someone measures a tract he measures the boundaries.

T. Yes.

C. He goes on measuring and at the same time, somehow, counting... reckoning the four sides. He measures along this line [showing the perimeter].

T. Yes, that is the place where he goes on measuring ... But it is round ... And he has to measure a straight distance ... from here to here, for example [she shows two opposite points on the circumference], and from here to here.

C. You mean the diameters in cross?

T. Yes, the diameters.

C. Two diameters?

T. Two diameters. Because in dividing by four one gets only two diameters. But then I don't know how to calculate.

(c.1.5) About problematic aspects.

The major problematic aspects raised by the teachers were addressed to a clarification of the meaning of *mil covas*; and questions were mainly concerned with the notion of area.

"Do we reckon *area* when we use cubação in *mil covas*?"

"We *read* area in terms of a number of units of area: is *mil covas* a unit of area?"

"What does it mean to *reckon* the area of a tract?"

"Why not *express* area in square braças?"

These and other related questions will be discussed in some detail in section 7.3. For this reason, instances from protocols will be given in that discussion. They are, however, important to be mentioned here, in that part of the discussion with teachers which followed in stage two was explicitly organized to explore them.

(c.2) STAGE TWO

THE FARM-WORKERS

What farm-workers had tried to do in stage two was briefly described in item (b.2.2), page 175. Here I shall indicate the general features of their answers, showing how farm-workers had reacted to different shapes and solutions. The passages are lengthy, but illustrative of three different kinds of '*expertise*' in relation to cubação. The intention is not to propose a typology of expertise, but to expose cases in which, clearly, different levels of *understanding* are involved.

(c.2.1) The lay expert.

A *lay expert* is an expert who knows how to do cubação only in those cases prescribed by the method. In other words, he knows the method; he can think of a personal solution when conditions change (how he would do); but he is not able to explain why his answer can be correct (or not). Attempts to reach understanding lead to knowledge at L⁰.

Instance 7.15

[... after having presented the different procedures for the triangle, the farm-worker focuses on procedures 6 and 7]

F.W. Yes ... I don't know. As I understand ... each one has a way to do cubação ... Because these are two different modes. This one divided this edge in two. But what can be this '2'? The other added [2 *braças*] ... that is, he augmented. He augmented ... but why someone divides by 2? Because if he had divided by 2, without adding these 2. Because this side, if it is broken... in half ... this 100 ...

C. Humm. Right.

F.W. It is this one added to this one, isn't it?

C. Yes. And then he takes these two, and these two [the pairs of opposite sides].

F.W. Yes. 80 and 50; and 50 ...

C. 50 and 60.

F.W. But then the person gets ... Some people do it in this way ... Let us say that this is correct.

C. Right. Now, my problem is exactly to say whether it is correct or not. When do we know?

F.W. Yes, I don't know. When do we know... Because... if both are correct?

C. I made the calculations. Do you want to see the results? Just a moment ... Here they are. This one, dividing 100 *braças* in two, gave 5720 'covas'. This one, adding 2 *braças*, the result was 4664 'covas'.

F.W. Yes, that is ...

C. And there is this other case, which multiplies by 3. Remember, we have discussed this one.

F.W. Yes, I remember.

C. Well, in this case, the result is 4200.

F.W. Are there those 2 *braças* in this case?

C. Well, it says one should add these two edges, multiply by this other, and then multiply by 3.

F.W. For example, this one divides in 3 sides. So, how would it be possible to add 2 *braças*?

C. All the others ... one takes 4 sides.

F.W. And about this one, which he adds 2 here?

C. He adds 2 and multiplies by 4.

F.W. In this case ... was the result correct?

C. No... well, it is because... how do I know which one is correct?

F.W. Yes ... because in this case ... we have to try to find someone who can tell us if it is correct or not. I have no condition to answer this.

C. That is what I want to know. Which one is ...

F.W. I don't know ...

C. But if you had to believe in one. Which one do you think is correct?

F.W. I believe is this one. I make my mind that is this one... in 3. That is, the tract broken in three. Because there are two small sides. I can even break this edge in two; that I think it will be more correct than adding 2 *braças*.

C. So, you believe that this can be also correct?

F.W. Yes, because it was squared. He started from 4 sides, but this edge ...

C. It has the same 100 *braças*.

F.W. Yes. It has the same 100 *braças*. Exactly. But in that other case, why would someone add 2 *braças*?

C. He adds just to have condition to perform the calculation.

F.W. But it did not give a correct result, did it?

C. All the results were different.

F.W. Yes, you really need someone more experienced in cubação. To know which one is correct.

[... after the discussion had finished]

C. You can have the worksheet, and look at it another time.

F.W. Well. I am really sorry about not being able to teach you precisely ...

C. No, you were very helpful.

F.W. ... which mode of doing cubação is correct. Because, when a person teaches something ... because I can teach you about this [the quadrangle]. This one, I know how to do.

C. You mean, with four edges.

F.W. With four edges. The natural one. Now, with 3 edges ... the explanation I got is more or less as I told you. Now, the others ... the round shape, shapes with sharp ends, and so on ... these I don't know.

(c.2.2) The community-leader expert.

The *community-leader expert* is more experienced than the *lay expert*, in that he knows how to control the conditions and limits of application of the method besides those cases prescribed by the method. Thus, he knows the method; he can propose personal solutions when conditions change (how to do); and he is able to think of reasons for a given answer to be plausibly correct (explain why). But when the reconstruction of explanations are difficult to establish, he appeals to social motives. In this case, we have knowledge at L¹, but including social rules, which from the point of view of Euclidean geometry, are *outside* the discourse.

Instance 7.16

[... after the farm-worker was presented with the three methods for the quadrangle; the focus being on the equilibration rule]

F.W. I know. I understand ... This can be a good way to do.

C. Do you think it can be more correct than ...

F.W. I never used it, to tell you this.

C. You never did it.

F.W. Actually, I had never seen a person doing cubação in this way. I do it as perhaps this comrade does. He tries, and it works.

C. Yes, he is from the main town of SPP. He says that he does in this way. This is a way I understood cubação can be done. But he does it also in this other way [the traditional]. Thus, I don't know ... What do you think ... if you have to say which one is correct? What could we do? What would you do?

F.W. Am I supposed to know?

C. If one wants to know if the result is correct ...

F.W. Wait a moment ... [he keeps silent for a moment]. It is because ... in the way I do cubação, the tract can have different sides ... one shorter, other longer ... and my cubação is correct. Even if I do not equalize the sides. So, it is to resolve this doubt that you are asking me if ... for us to know ...

C. If you would have to decide if this method is correct or not; what would you do? How would you compare it with your method? In which way?

F.W. ... [pause for one minute] ...

C. Would you do the calculation, or not? Would you try to do it?

F.W. I am not sure if I would do it ... no.

C. You think it would not give ... Humm ... Because I was trying to do the calculation to see if the results were different or not. The sums ...

F.W. Were you? And what did you get ... in this way [equilibration] ... was the result

bigger than my result?

C. That one ... it gave a little less. The traditional way yield more. And the third method here in the worksheet gave much less ... in this way, taking ...

F.W. Dividing?

C. No. Taking the two shorter ones and the two longer ones. Instead of taking the opposite sides in cross.

F.W. Humm ... ***[pause]*** Yes ... Counting is a very precious thing.

C. Humm.

F.W. I am not entirely literate, and so, my method of reckoning is this one [cubação].

C. Is this one.

F.W. Yes ... and it always proved to be correct. It is accepted whatever the situation.

C. Right. And if the technician from the EMATER comes ...

F.W. The EMATER's technician approves it. Let us say, someone, somewhere, contracted an 'empreitada' and he does not know how to do cubação. So, to be secure about the owner's counting he comes to me and I reckon it. After the negotiation is completed I usually ask him: -did it work well? -Yes, it did.

C. So, people frequently call you to do cubação?

F.W. Yes, they call me. And I always do cubação in this way. There, they also do it in this way.

C. Right.

F.W. But here it was also approved, wasn't it? In the way I do it.

C. Yes, it was. Now, what I was trying to understand is how could we do to compare methods, to know if one is wrong or right. Do you understand? Because ... when I come to try to reckon a triangle ... Do you remember we had talked about the triangle, don't you? A triangle such as this, which had measured 60, 80 and 100 braças. Then ...

F.W. And about the others? How did they solve it?

C. I found seven different ways of doing it.

F.W. Seven? Different?

C. Yes. Which I don't know if they are different or not. Do you understand? And I was trying to see if I could find a way to compare to what extent they are similar or different.

F.W. What did they say?

C. Look ... I will show you what they are...

In the discussion which follows, the various procedures for the triangle are described; calculations are made; and results, all different, are exposed to the farm-worker. What is relevant in this discussion is to look at how the farm-worker tries to think about the plausibility of some results. He makes two attempts.

Attempt one. Initially, he tries to establish the conditions for which the procedure demanded by the method gives a correct result; and he does so by reference to a tract that he knows that cubação predicts a correct result. Then, he tries to apply the same reasoning to the tract in question.

Instance 7.17

F.W. So, did you write down the different results?

C. Yes, I made the calculation for all the cases. But I don't know the reason for the

differences. Do we have any way to say which one is correct?

F.W. Yes ...

C. All right? This is what ...

F.W. We know that 4 'mil covas' is the result of 4 edges of 50 *braças* each; don't we?

C. Yes. This is right.

F.W. Then we could ... because, in this case we have 200 *braças*.

C. Humm.

F.W. The whole boundary. Then one multiplies [he does not use the verb to add] because one knows that each edge has 50 *braças*. Then, suppose we can say how many *braças* our tract has, if we turn it round.

C. Ah, you are saying this: 100, plus 80, plus 60 *braças*.

F.W. Exactly. Let us compare.

C. It gives ... let us try.

F.W. If this one is ...

C. 240 ... 240 *braças*.

F.W. So, it will give more than 4 'mil covas'.

C. It will give more? But how much more? How can we do?

F.W. It exceeded in 40 *braças*; is that right?

C. 40. Yes.

F.W. It passed in 40 *braças*. ***[pause]*** These 40, we need to know how many 'mil covas' it embraces.

C. How many 'mil covas'?

F.W. How many covas.

C. But is it 40 in this way: 10, 10, 10 and 10?

F.W. Yes. And then we multiply these.

C. You multiply ...?

F.W. 10 with ... No, 20 times 20.

C. 20 times 20, 400. And then by 4?

F.W. Yes.

C. So, it gives 160 [taking the last digit out of 1600].

F.W. Yes, 160 'covas'.

C. Thus, the result will be 4 'mil' and ...?

F.W. 160 'covas'.

C. 4160 'covas'... this tract.

F.W. Yes. This would be correct.

Attempt two. In carrying the discussion further, and being compelled to compare results, the farm-worker is led to the conclusion that the above result is different (smaller) than his own solution, which prescribes the addition of 2 *braças* to one corner. He insists on keeping the result of his procedure as the correct one, on the basis of his authority as expert. His effort is then concentrated in reconstructing the above solution, so as to increase the number of 'mil covas' attached to the remaining length of 40 *braças*. Starting from the difference in the result (that is, 500 'covas'), he tries to construct a square-shape, whose area is equal to 500 'covas'. His answer is a quadrangle with pairs of opposite sides given by 12 & 13; and 25 & 25 *braças*. His problem is then to know how 500 'covas' could be attached to the remaining 40 *braças*? His second attempt, to establish what result was plausibly correct, arises from this discussion, when he tries to 'eliminate' the troublesome 40 *braças*.

Instance 7.18

C. And now, how do we know which one is correct?

F.W. The correct one is the one I do.

C. Right.

F.W. My result is correct. We can try to make a comparison. Let us make a triangle with 200 *braças*.

C. Humm. A triangle with 200 *braças*.

F.W. Let us make a triangle with 200 *braças*. What can we do to construct a triangle with 200?

C. Let us say ... 50 here, 70 here, and 80 *braças* here.

F.W. Does it give 200?

C. Yes, because 80 and 70 makes 150; and 150 and 50 makes 200.

F.W. Yes, right. Now, let us divide it to construct the 4 edges. If the result gives 4 'mil covas' ...

C. So, you divide by ... now, you want to do cubação in which way?

F.W. In the way that comrade explained. He said he would add the 3 edges ...

C. Yes, he would add them and would divide by 4. So, 200 divided by four gives 50. And how many covas there are in a square of 50 *braças*?

F.W. 4 'mil covas'.

C. Now, let us try to do cubação in your way, adding 2 *braças*. Where do we put these 2 *braças*?

F.W. Here.

C. Right. So, we get 80 plus 70 ... is that right? 150. Now, 150 times 52. I will do the calculation [calculation]. It results 3120.

F.W. 3120.

C. And we were expecting to get ...?

F.W. 4 'mil covas'.

C. Yes. 4.

F.W. ...[pause]... Yes. This triangle does not give the same as the square. It has the *braças* because the edges are very long. But it does not embrace...

C. Yes, it does not have the area. Right. So, you think that this way is correct [adding two *braças*]?

F.W. Yes. This is correct.

C. In the end, this method of adding the edges ...

F.W. Yes, it is not approved. Because, I do cubação in this way since I was a child.

(c.2.3) The 'intellectual' expert.

What distinguishes the 'intellectual' expert from the *community-leader expert* is the fact that, in reconstructing explanations, the former tries to make use of 'mathematical' reasons (correct or not), and not social motives (as the latter). Their postures towards cubação are different.

Instance 7.19

[... after the presentation of the three methods for the quadrangle]

F.W. I am attached only to this method [the traditional]. I was accustomed to this way of doing cubação; and I never changed. I only do it in this way. Now, if someone wants to do it differently, he can. There is nothing very wrong with the fact that a person does it using another method. But I do it in this way.

C. Is it?

F.W. Yes.

C. Because I did the calculation in these three ways. And the results were different. One gives 5280 'covas'; another 5040; and another 5290.

F.W. You mean that there are three modes.

C. They were three ways. Now, do you think that these other two are wrong?

F.W. No, I would not say that. Because there are times ... it is because I don't want to say that they are wrong ... No. What I can say is that my way is this one. Now, about the others I don't know if they are correct. My way to do cubação is correct ... Because if we do it in this way ... Suppose ... look, it is very easy. Let us say that we have 50 *braças* in each side. We will have 100 times 100.

C. All right.

F.W. Yes, 100 ... and this. Yes, it has to result in 4 'mil covas', whether we want or not.

C. Is it?

F.W. Yes. Because ... [calculation] Yes. It really gives 4 'mil covas'. So, this way is right. Now, if we take the 4 'mil covas' ... this ... ***[pause]*** In other case as well ... 25, 25, 25 and 25. Yes, all the other cases are similar [he thinks in all other cases of squares]. So I can also take the 200 and divide it by 4, because it will give 50 for each edge ... Right ... it will give 4 'mil covas' as well.

C. So, if I divide by 4, will it give the same result?

F.W. Yes, it will give the same result.

C. I understand. Right. Yes ... and about this one, when we consider the triangle?

F.W. Well, I think that these two modes will be correct also in this case. In my way, this side counts for two.

C. Right.

F.W. Then, the person thinks it is not an exact result. But it is not exact as the other one ... in that we say that the methods work as in the case of one 'mil covas'. Now. This gives a basis for people to negotiate 'empreitadas' ... Because, here, if one plants in an adequate way, one can plant ...

C. One thousand?

F.W. 2 thousand, 3 thousand ... It does not matter how much we plant. Now, one 'mil covas' is something which is given ... from a very old time. And we use it in our transactions.

The discussion about the triangle settles this line of reasoning. That is, correct procedures for the square can be applied to other shapes; and one chooses the one which is more practical. It does not matter if the result is not exactly correct; and a suggestion is made that this is something to be resolved by the metric system. *"The metre-system allows one to see the squares, and in this way one can count them and compare"*. The reasoning is extended to the pentagon.

Instance 7.20

C. Right. I was saying that in relation to the '5-edges', three procedures were proposed. [... description of the procedures].

F.W. Humm. If I would do it, I would do it as follows: suppose one edge has 50 *braças* [he changes the lengths of the 5 edges I had in the worksheet].

C. Right.

F.W. The other has 60 ... They are five edges ... other has 25; other 38. The small one has ... let us say, 12.

C. Ok.

F.W. I add them all and I divide ... to follow the other farm-worker. The result I

- divide by 4. [...] I think that in this case, this way is right. The result is 100 and ...?
- C. 185.
- F.W. 185 divided by 4. Is that right?
- C. Divided by 4 ... yes, right.
- F.W. [calculation with some difficulty; the researcher helps]
- C. So, they are 4 sides of 46 ... do you understand?
- F.W. Yes ... 4 sides of 46 ... Yes, that is. Then ***[pause]***.
- C. Then, we would add 46 and 46 ...
- F.W. Now ... yes.
- C. 46 and 46 ...
- F.W. [he checks the calculation] Yes, you are right.
- C. Humm.
- F.W. Now, there is one *braça* here ... where do we put this *braça*?
- C. Humm.
- F.W. It is only then that the result can be given in *mil covas*. That is, there is this remainder here ... where can we put it? This *braça* ... So, here we have 180 and ...
- C. So, you come back to the tract again ...
- F.W. Sorry?
- C. You put 184 back, but you actually have 185 to distribute.
- F.W. Well, here they are added ... it is solved ... it is everything here. It is 4 times 46, isn't it?
- C. Humm.
- F.W. This is the result but there is 1 *braça* out.
- C. Humm.
- F.W. Because here we have 184 ... I will do the calculations to see what happens. [the farm-worker tries some calculation but get confused; he asks to try to solve this *cubação* on his own, and passes over the conversation to his son -a technician- who was waiting to be interviewed; he comes back half an hour later]
- F.W. Look, I was wrong about this ... it was right here. We did 50, 60, 25, 38 and 12, didn't we?
- C. Yes, we did.
- F.W. Then, it gave 185; which, divided by 4, attributes 46 to each edge. Is that right?
- C. Yes.
- F.W. 46, 46, and 46. Now, there is 1 *braça* here that I put it here.
- C. You put it here [one side becomes 47]. Right.
- F.W. Look, this tract will not give 4 'mil covas'. Can you see this?
- C. Yes, yes.
- F.W. It will not give 4 'mil covas', but it could have happened. It is everything proved here. Look here.
- C. You set the proofs. Right. Yes ... I understand now. All right. Yes ...
- F.W. So, that is how it is. Now, you can ask the question again that ... you can show how other people do it. Do you have any other case with you?
- C. Yes, but not with these numbers. I have with different lengths.
- F.W. Humm. Because I would like to know if one is like this, exact...
- C. I can do it right now. Because I have a description of the procedures.

The solution by another procedure provides a different result, and the farm-worker returns to expose his 'proofs'; that is, the reasons he has for accounting for the correctness of his procedure. His attempt is, to some extent, similar to the one provided by the community-leader expert exemplified above, in that he has to explain how a given number of 'mil covas' ($4000 - 3422 = 578$ 'covas') can be

attached to a given length in *braças* ($2000 - 185 = 15$ *braças*). But his perspective is different. Starting from a situation which he knows can be correctly solved by the traditional method of *cubação*, he tries to find a way to go from his result (3422 'covas') to the nearest correct result (4000 'covas'), by enlarging the shape (adding up the 15 *braças*). But if his intention is clever (in seeking to construct a rule of enlargement), his performance does not go further than showing that the area of a 200-square is 4 'mil covas'.

THE TEACHERS

The discussion with teachers in stage two can be described in terms of two inter-related enquiries: one concerned with school geometry and other with *cubação*. In relation to the former, the teachers showed some improvement. The effort of 'retrieving' from their 'memory' the basic facts treated in school geometry, which were not 'remembered' during stage one, changed the quality of the discussion which followed in stage two in respect to the managing of procedures for estimating area. But teachers still had difficulties in accounting for possibilities of thinking about *cubação*.

The teachers' attitude in this discussion conveys an impression of teachers who have an Euclidean posture, but whose reasoning is tied/constrained to the logic of *cubação*.

In trying to answer the questions proposed in the worksheet, the teachers' immediate reaction was to compare results; and to try to find a criterion to say what it is that makes a given solution correct (which is typically Euclidean and contrasts with the farm-workers' reaction). But to abandon some previous ideas (such as "perimeter makes area"), or to reaffirm others (such as "area is conserved when pieces are reassembled"), they had to make an effort (Euclidean in nature) to link propositions that could be taken for granted also within the *cubação*-system.

Adhering themselves to some taken for granted ideas, they tried to suggest possibilities of solutions without any necessary concern with what goes on in actuality. Examples show the nature of the teachers' accounts, and suggest that possibilities had indeed, and primarily, a logical concern.

(c.2.4) Cubação and the conservation of area.

Teachers became stricken by the fact that cubação does not predict a correct result for equivalent but not congruent shapes. Area must be conserved when submitted to a rearrangement of pieces. On the other hand, cubação must be right. At some time during the conversation, these two 'principles' were challenged. Exposed to the non-conservative character of cubação, teachers tried to adopt one of two alternatives to account for discrepancies, which, ultimately, suggest the necessity of a constraint to make cubação work properly.

Cubação must accommodate different modes of reckoning. This solution suggests a differentiation in the way of applying the method. That is, cubação needs to be used differently in order to guarantee the same result in cases of equivalent but not congruent shapes. There is no explicit concern about what these alternative ways of using cubação could be, or mean.

Instance 7.21

[... after having cut and rearranged a rectangle, the teacher is asked about the two areas (original and transformed)]

C. If I ask you now, to compare the areas.

T. They will be different, but I don't know about how can we do it.

C. Do you think that in this case the area of this shape will be different from the previous one [a congruent shape to the original one is used]? The area.

T. Well, cubação will probably be different. Now, the conditions indicate that the area will be the same. But the application of cubação will be different.

C. You think that if I use cubação, there will be a difference?

T. Yes, but only in the way one does cubação. Because the area will be the same.

C. The area will be the same ... You say, the mode of doing cubação will be different, but the number of 'covas' will be the same.

T. Yes.

C. Right. Why do you think so?

T. Because ... everything here indicates that there is the same quantity of land.

C. Humm. Ok.

T. There must exist the same amount of land. The land is here in a different manner.

C. But originally, it is the same land.

Cubação depends on a shape parameter. A second account of how to deal with the apparent non-conservative property of cubação suggests that the formulation of cubação must be understood as being a function of the form of the shape.

Instance 7.22

C. Right. Thus, one thing that this discussion does help us to understand is this. There are certain things that we can do to a shape, such as to divide and rearrange

their pieces, which do not alter the area. Is that right?

T. Right.

C. Which means that sometimes ... Suppose that I give you this shape to reckon. One possible way of doing it is to recompose the figure so as ...

T. So as to get easier shapes.

C. Easier for you to do the calculation.

T. But this is something I have thought about. But each time I tried it, the result was different [she talks about cubação].

C. Right. So, this poses you a question. When you consider these two shapes and try to solve them by cubação, the results are different. Is this right?

T. Yes. So, there must exist one way of doing it which accounts for each kind of area. That is appropriate to each form.

C. Ok. What you are suggesting is this. Perhaps, what farm-workers use is one same method applied to different shapes, but which must contain some factor proper to each shape. Without which it is not possible to do cubação.

T. Yes, it must be this.

Unfortunately, at that stage, no inference could be made about a plausible relation to be established between the procedure of cubação and kinds of shapes, other than to recognize that, the procedure is correct for 'square-shapes'.

(c.2.5) The procedure of cubação in the metric system.

The idea of using the procedure of cubação (the algorithm) for reckoning area in the metric system was present in the teachers' argumentation since stage one. One example was given in instance 7.11, page 183. But it is in stage two, that the possibility of the existence of a procedure of cubação with an equivalent function (that is, to provide what could be meant by 'mil covas') to the metric system, starts becoming more clearly conclusive. Such a possibility was not directly pointed out, but could be inferred from extracts of the protocols, such as the following one.

Instance 7.23

T. [...] Because I have tried to solve these shapes in 'thousand' different ways. And each time ... For example, the triangle. If I do it in metres I have: base times height, divided by two. The result is 3. If lengths are given in *braças*, the result is 3 *square braças*. If I do it by 'squaring' it, using that same technique, I will get 14 'covas'. In another case, the result being 36, in a different way, I will get 104 'covas'.

C. Humm. First, what you need to know is that 1 *braça* = 2.2 metres, and ...

T. No, but without having to transform from metres to *braças*. Suppose we have the sides given in *braças*. We can not use the same technique we use in metres because the results will be different.

C. Yes. You can use the same technique you use in metres for the area of a triangle, but the area will be expressed in *square braças*.

T. Yes, this is right. You find the result in *braças*. But I want to know in 'mil covas'. The method which provides the result in 'mil covas' ... what does it provide in the case of metres?

(d) *Summary of the relevant ideas.*

The arguments in this section have been indicative rather than absolute, aiming at revealing, tentatively, some fundamental propositions which could be taken as proper to the way of thinking about area in cubação. A summary of the relevant propositions is presented, without any attempt to categorization. The intention is simply to organize them for further consideration.

<p>P1</p>	<p>It should be possible to conceive of a general procedure which has as a particular instance the case of cubação applied to a quadrangle.</p>
<p>P2</p>	<p>Equivalent but not congruent shapes presuppose different algorithms for estimating their areas in one single process.</p>
<p>P3</p>	<p>Cubação lacks a kind of parameter which is characteristic of the shape, without which it is not possible to accept that one unique procedure can apply to any shape.</p>
<p>P4</p>	<p>There must exist 'a procedure of cubação' for the metric system whose description does not involve changes of units between systems.</p>
<p>P5</p>	<p>Any shape can exist. Some are known to people by experience. Others are not. In existing, these shapes have to have a given number of <i>mil covas</i> attached to it. This is so, even if one has never seen those shapes.</p>

FARM-PEOPLE THINKING

P6	Edges can be compensated without prejudice to the total area.
P7	The perimeter in actual tracts is itself a criterion as to their area.
P8	To each edge of a polygon, one can attach (make correspond) a fraction of the total area embraced by the perimeter.
P9	When one side is <i>broken up</i> , part of its attached area is <i>given</i> to the <i>made up</i> side (which <i>receives</i> it); in such a way that both area and perimeter remain unchanged.

7.3 CUBAÇÃO MADE IN SOME DEGREE PROBLEMATIC TO FARM-PEOPLE IN RESPECT OF THE MEANING OF THE RESULT

Any serious attempt to get at the farm-people's understanding of the notion of area can not ignore a discussion of the expression '*mil covas*'. Method and area can be regarded as tied by a logical structure, but there is no active logical structure outside the system of measurement. Systems of measurement are related to the processes of producing its results; and the field of mnemonic rules (algorithms) which guarantees the possibility of *transmission* of these systems is linked to the forms of logic that govern geometry.

Euclidean Geometry seems to accommodate a variety of systems of measurement, particularly those related to surveying. Thus, *mil covas* can be converted into hectares; hectares into acres; acres into square miles; square miles into *centuria*; *jugera* into hectares; hectares into *aroura*⁴; and so on. In such a way that, in calculating area in any of these systems, it is always possible to make use of

⁴ *Aroura* is a Greek word which literally means 'arable land'. As a Ptolomaic unit of area it denoted 100 cubits square (Dilke, 1971, pp. 27-28).

Euclidean procedures and to express the result in terms of a multiple of a given number of units of area.

However, the system of cubação not only does not underwrite those procedures (as I showed in section 7.2), but seems to denounce, in the way results obtained in *mil covas* are justified by farm-people, the idea of "area expressed in terms of units of area".

Classically, in mathematics teaching, a teacher may happen to mention that if we measure the area of a field, we know how many plants we can cultivate there; and by having an adequate arrangement of crops we can improve efficiency. However, in talking to farm-people, they insist that there is no correspondence between the area expressed in *mil covas* and the number of planting places/holes/pits on the ground (that is, the number of "covas").

At first sight, this seems a reasonable answer. Being native in a peasant community, farm-people would be expected to tell us more than that. Probably, an experienced farm-worker would fundamentally relate the result of cubação given in *mil covas*, not to the number of "covas" - the planting places of seeds; but to the amount of production (food) he would need to eat, or to the amount of money he would have to buy goods. Ultimately, for a farm-worker, to reckon the number of *mil covas* is not at all to measure the area of a table or even the area of his house. *Mil covas* could be conceptualized as 'an amount of something'; but any attempt of measuring the area of a tract for the purpose of defining the possible amount of pits would be to reverse the essential order of the natural way of thinking of area, in which case the important problem would be to know what his family will do, how much they will eat and how much money they will get.

However, at the same time at which they insist about the non-correspondence fact and recognize that *mil covas* is something which mediates transactions, they also insist that *mil covas* is a measurement of the area of a tract; that is, farm-people's accounts have also a geometrical concern, even when they say that they do not understand what an area is.

It was this concern that I took over for further clarification. At this point, the interest is in the arguments raised by farm-people during the conversation.

7.3.1 Farm-people's arguments about the meaning of 'mil covas'.

Both, farm-workers and teachers were asked about the meaning of *mil covas* during the second set of interviews, and their replies were unanimous and categorical: the expression does not represent the number of planting places.

The immediate reaction of the researcher was to try to establish whether this assertion could be substantiated with reference to known practices. To clear up the way for the investigation of a deeper interpretation of the notion of area in relation to *cubação*, this seemed to be the first thing to try. My attempt was, then, to enquire about a historical motivation for the relation between the 100-square tract and the result of 1 *mil covas*. More precisely, I asked whether *mil covas* could be seen as a representation of an actual field which had existed in some possible past and which could account for the present unit of measurement called *mil covas*. As I did not get very far with this discussion (and I had supposed this would not be trivial to establish), I decided to postpone it to subsequent meetings. I did not include any explicit group of related questions in the third and fourth blocks of sessions. If the previous debate had prompted the informants in any problematic sense, opportunities were expected to arise in examining other points related to the notion of area.

To re-start the discussion about the meaning of *mil covas* in the following meetings, was the informants' initiative, not mine. As a result, three lines of reasoning (typically Euclidean) developed; which indicate that farm-people know, or are able to think about *area* from a Euclidean perspective. One line took over the problem of displaying 1 thousand of "covas" in a 100-square of 1 *mil covas*. The other established the possibility of getting the result in *mil covas* from counting the number of "covas" on the ground. And a third line established the distinction between "*the area of the tract*" on the one hand, and "*the area of the culture*" on the other.

Instances showing the informants' argumentation are organized in four groups. First, it is necessary to exemplify how the expression *mil covas* was seen as unrelated to the number of "covas" on the ground. Then, instances of the three 'lines of reasoning' as mentioned above are given separately. The initials used are: Tec.= technician, T.= Teacher, F.W.= Farm-worker, F.= Farmer, and C.= Researcher.

(a) *The relation between mil covas and "covas" on the ground.*

Instance 7.24

[The teacher was trying to clarify the distinction between area and perimeter with respect to cubação]

C. This is a doubt I had since the beginning. Which is, to be able to understand what the expression *mil covas* really means.

T. Yes. This is the case.

C. One thing I tried, was to see whether the word *covas* in this expression, means holes/pits.

T. No, this is certainly not. Because one can dig as many pits as one wants. There can be plentiful or sparse pits.

C. Right. What I think we are led to accept is that *mil covas* is really a measure of area, but without seeing it in terms of a number of pits on the ground ...

Instance 7.25

[After the farmer had stated the *equilibration rule*]

F. Because what lacks in one edge, the other complements, grows...doesn't it? Then, during the plant-phase ... when you plant, it is ok. You can even count the '*covas*'. It results 'positive'. Do you understand?

C. But wait a moment. When you count the *covas* ... For example, the result of this calculation in *braças* you express in *mil covas*. Is that right?

F. Yes, in *mil covas*.

C. But this is not the number of "*covas*" dug in the tract ... or it is?

F. No, it is not. They say it is not. Now, it will depend on the technology of the planting phase. It depends on the distances between plants. You can plant more or less.

Instance 7.26

F.W. Things happen in this way. It is precisely as if ... yes, there are different modes [of doing cubação]. Now, there is this calculation ... Because, another day you asked me about *mil covas* ... what the devil is this thing of *mil covas*? Didn't you? So, I asked a friend: -who did invent it? He answered that he does not know. That this is something which belongs to the ordinance of the land, the place. Because in the South, they measure in '*alqueires*'. In some places they use '*a 50*'. In others, *mil covas*. It is a kind of law that people get attached to ... it is something which comes from nobody who invented it. It is a thing invented by the statute of the place. So, this thing of *mil covas* is an invention which was invented somehow, but we do not know who did it. Thus, he did the calculation, but I did not understand very well [the calculation aimed at finding the number of "*covas*" in 1 *mil covas*].

Instance 7.27

[The researcher interviews a Tec.. At the end of the session, a T. and a F.W. join the discussion]

T. I know. I agree with you [the Tec.]. If you divide [the area of the tract] by the spacing of the culture, you get the number of "*covas*". Now. What happens to the farm-workers is that *mil covas* is considered to be an area ... a given piece of land which is there; it is 'that' tract. Now, about this I agree: if you measure with the metre and so on, you can get the number of "*covas*". It happens to be the same.

F.W. Yes, right.

T. In *mil covas*, which is an area, it can even coincide. So, I agree in this sense.

Tec. It is this that I think to be different. The difference lies exactly at this point. [...] Why is it that his method is not correct? Because we have ... a certain quantity of squares very well marked ... Then we know: it is just to multiply. But why is it that they do not need to multiply in this sense? This is the problem.

C. You [the Tec.] ask where that area 'is'. And you [the F.W.] ask him back where the area 'is'. But I guess I can understand the question ...

F.W. About area I know nothing. [...] What I say is that *mil covas* is a way we have to orientate our transactions: to rent a tract, to buy or sell it, and so on.

C. Right.

F.W. Because you know... When we go buying sugar, do we say: - we want some sugar? No, we do not buy *some* sugar.

C. We have to say ...?

F.W. One kilo, two kilos, three. It is the same thing here. We must have some basis. Now, this has nothing to do with the number of "*covas*" we can plant there. We can plant in the way we want, as much as we want.

C. Right.

F.W. Things are as I was telling him [Tec.] before. One hectare is 10 thousand cubic metres, is that right?

Tec. Yes, square.

F.W. Ok, square is ok. Now this was a cubação, wasn't it? The result was 10 thousand metres, wasn't it? Now, in this case, there are actually 10 thousand of plants in such a tract.

C. Right.

F.W. In this case the result is exact.

C. Yes, It seems that there is no doubt. You [F.W.] know what this thing of area is about. [...]

F.W. If I have 100 *mil covas* I know how to derive the area in hectares. But if I have 100 hectares, I don't know how many *mil covas* there are in the tract.

It is clear from people's accounts that *mil covas* and "*covas*" on the ground, despite being each an *amount*, are not entities of the same nature. While the number of "*covas*" can be equated to *area*, the number of *mil covas* would be better characterized as an entity socially defined such as the amount of *work* invested on the land (*work* is here privileged from a process of inference which takes into account the discussion in chapters 2, 4 and 6). Also, while area in hectares can be obtained from the number of *mil covas*, the work on the land can not be easily derived from the number of "*covas*" on the ground.

(b) *One thousand of "covas" in one mil covas.*

Instance 7.28

F.W. Is the tape recorder turned on?

C. Yes. I have just turned it on.

F.W. Yes, because in our last meeting you asked me if this thing of *mil covas* ... '25-square-braças' as I told you ... if this was one thousand of "*covas*", exactly.

C. Yes. If it has one thousand of "*covas*", one thousand of pits.

F.W. Yes. It has.

C. Has it?

F.W. It has.

C. Ah, let me see. Because I want to know about this.

F.W. Now, every row with 40 "covas". [...] Because I did it right. I attributed exactly one thousand of "covas". [...] 25 times 40.

C. Each row with 40.

F.W. Yes.

[the researcher tries to draw the pits on a grid, but the farm-worker prefers to work on his own draft which contains just one row of 40 points].

F.W. Now, about the distance between "covas". It is... 1 m and 39 cm.

C. 1 m and 39 cm.

F.W. The distance from one "cova" to the other.

C. Here, in one row. [...] Now, and about the width?

F.W. The width ... it is 10 *palmas*, which is 1 *braça*.

C. Which is equal to 2 m and 20 cm. [...] Right, ok.

F.W. Right. Now, this would be for the case of planting corn, you understand? Now, if the person wants to plant beans in the middle, he can. Let us say, a row of beans.

C. Here? [...] Together?

F.W. Yes. Now, one can plant beans as one wishes.

C. As one wishes?

F.W. Yes, it can be. Because it can be 1 metre, or less. This is also true for the cotton. Because the cotton ... being a *new tract* ... in a *new tract* one always plant cotton within a distance less than 1 metre. It is less ... about 3 *palmas*. Thus, one gets more.

C. Do you mean that this happens only in a *new tract*?

F.W. It is only in a *new tract*. Yes. If one wants, one can plant more dense. Because then ...

C. It is all right. Ok. But tell me something. This means ... because what I want to know is this: how do you 'square' a tract? Because you will never mark out a *field* in this way, will you? You always ... when you settle a square grid, you always make this distance equal to the other. Is that right?

F.W. Yes, right.

C. You will always use the '*capinadeira*' ... which leaves a square grid behind.

F.W. Well, it is right. Now ... because actually, the tract that we have been talking about, is a *new tract*.

C. Humm. A *new tract*.

F.W. Not a *field*. Because the field contains small fixed roods. This does not happen in the other case, where one can adjust more or less the distances. This is what I did ... One has to divide.

C. To make fit one thousand of "covas".

F.W. Yes. To make them fit. And the result was this. Yes, precisely. Now, one still has to plant. And one can plant cotton or beans. Or both if one wants, inserting alternate rows between the corn-rows.

C. Ok. I understand. Ah! Right. Now I start to understand more clearly.

F.W. Is it?

C. Right. Ok.

F.W. This one is a tract ...

C. I took note about this. I made a copy of your tract ... it is much better now. For now, we don't need the square grid.

This extract suggests (summing up the previous inferences) that, given a certain amount of *mil covas*, a tract can be delimited which represents the amount of work performed according to a given practice (the historian would tell us, for example, that one *mil covas* is a tract which produces sugar cane to be processed into sugar

in one day; in a similar way, an *acre* is the distance oxen could pull a plough before having to pause for breath). The delimited tract can then be described in terms of the number of "*covas*"; that is, the *area* of the tract can be measured; in such a way that a unit of measurement is then defined (in the case of *cubação* one could think of the '100-square' as defining 1 *mil covas*; in a similar way in that a rectangle of (1 x 10) chains would define 1 *acre* in the *acre-system*).

(c) *The result in mil covas from the number of "covas".*

Instance 7.29

[this conversation followed from a discussion where the farm-worker had exposed his difficulties in trying to understand some explanations given by a more experienced farm-worker]

C. Let us see what it is possible to understand. For example, we know that 1 *mil covas* is the measure of a tract 25 by 25 *braças*. But this is something you already knew. What you was then able to see is that in one such *mil covas*, if you consider the actual way a *field* is planted today, that is, squared metre by metre, what you then say is that there will exist more than one thousand "*covas*" planted in 1 *mil covas*. Is that right?

F.W. Yes. It will give more than one thousand of "*covas*".

C. So, one plants more than one thousand of "*covas*" in 1 *mil covas* today.

F.W. Yes, it gives 3 thousand and ...

C. 3025 "*covas*" [55 x 55 = 3025].

F.W. Yes, in 1 *mil covas* one can fit this amount of "*covas*".

C. Right. Another thing: you realized that, for tracts one does not know ... What he did was to divide the tract in such a way to get near to a shape that you can calculate. Or ... Then ... in this other case ... [...] Ok. Here he repeated what you did just now: 3025 ... Ok. This is possible to know.

F.W. Now, about this, what he told me is that ... he would divide 'the half by the half' ...

C. Ok ...

F.W. I am not sure about what he did here.

C. Yes ... 55 by 55. It is because there are 55 rows ... [25 *braças* = 55 m].

F.W. In this case, counted 1 by 1 ... if the tract is marked out.

C. Right, if it has been marked out. You can count row by row.

F.W. So this [what C. says] accounts for the calculation ...

C. Yes, but when you do so, you do not get any more the result in *mil covas*. What you get is the number of planted "*covas*". You do not get the number of *mil covas*.

F.W. Save that the comrade knows beforehand, that 1 *mil covas* has this amount here. Then, he can divide, count... And if the final amount does not correspond to 1 *mil covas*... then he will have to modify here [the 1 *mil covas*]. For example, if he counts 2250 ... [pause] ... [...] In this same tract... in a tract such as this ...

C. You do not get 3 thousand and so plants ...

F.W. No, it did not result 3 thousand plants, which then means that he has less than 1 *mil covas*. Then, starting from this relation, he can know the result ... Whether the result is 800 *covas* or 900 *covas*.

C. Humm... All right.

F.W. Do you understand what I am saying?

C. Right. You are saying this: You know the number of "*covas*" existing inside 1 *mil covas*. If you plant and the result is this [showing 2250 "*covas*"], you then ask in which size of *mil covas* you had planted. Is this right?

F.W. Yes. What I want to know is the number of "covas" before I get the *mil covas*. Because 1 *mil covas* is this amount here [3025]. Now, distributing 2250 plants, how many covas do I get? If it is a square, for example.

C. [calculation] The result is 742 covas.

F.W. Precisely, this is what happens. Because if the comrade knows ... yes, he knows that 1 *mil covas* has 3025. Then he measures, he counts everything ... Then ... yes, precisely ...

C. What you are saying is that one possible way to estimate the size of the tract in *mil covas* is to count the number of rows ...

F.W. The rows, yes.

C. And to count how many "covas" there are in these rows, and then to compare.

F.W. To compare, right. This is exactly what you did right now.

C. Ok. Did your friend tell you all this; or it is you who proposes ...?

F.W. No, I am calculating in this way. Because he did it for the case of 1 complete *mil covas*.

C. Right. And now, you are trying to see how can you use ...

F.W. I am trying to know what can the comrade do if he has a smaller or bigger tract than the one which has 1 *mil covas*. So, he can measure, count, and know the number of "covas" inside the tract. Then, he divides by this... to know the result in *mil covas*. Because 1 *mil covas* is this square, isn't it? [...]

C. And if the tract is bigger ...?

F.W. It can happen and he divides by 3025. [...] He did not explain this. But now I thought ... because I had the idea [he uses the expression 'to remember'] to mark out a small or big tract.

C. You ... I don't know if you can see it, but what I can say about what you are doing, is that you are using a procedure proper to the metres-systems [...] in which you find the area measuring in metres and counting the number of plants. You are doing cubação in metres, and using it to derive the result in *mil covas*. It is right. Your method is right.

This extract suggests that cubação (with the result given in units of *mil covas* which is taken as a representation of the amount of work invested on the land) can be regarded as similar to a procedure that -in fact- estimate the area of a tract (with the result given in *braças squared*, 'normalized' to *mil covas*).

(d) *The "area of the tract" and the "area of the culture"*.

Instance 7.30

Tec. My way of doing it [cubação] is this. I have already found the area of the tract. Then, I divide it by 625 ... 625 or by the area of the culture which will be planted in that tract. That is, the way in which it will be planted: if the culture will be in squares ... Because there are cultures which one plants, for example, 2 by 2. Or rectangular, or triangular ones. The triangular ... If one plants a triangular culture, following the area of a triangular tract, it is different.

C. Right. I am understanding. What you are saying is this... what you are trying to say is ... If I plant in this way [in a square grid] the number of "covas" will be different than if I plant in this way [a triangular grid].

Tec. No, no!

C. So, let us re-start. Explain me everything again.

Tec. Look, the tract is ... suppose it is a triangular tract. So, this is the area of the tract. It is here, the area.

C. Right. Humm.

Tec. If I want to plant a squared area ... let us say, 2 by 2 ... Then I will have several small squares of 2 to plant.

C. Ok.

Tec. Then, I divide this area ... this one [the area of the tract]. Suppose that I have the area of the triangle. Then I divide this result by the area of the culture ... the area of ... the spacing I will use in arranging the culture. If it is planted 2 by 2, this area will be, then, 4. So, it is the area of the tract divided by 4. [...] Now, if you divide in triangles. Let us suppose ... you will plant in triangles [...] Then, you find the area of this small triangle.

C. Humm.

Tec. And divide the triangle by ...

C. The large by the small triangle.

Tec. Right ... yes, it is the same thing.

C. Ok. I understood.

Tec. But there is still another thing. Consider this case: "How many covas of 'gerimum' (pumpkin) a tract of 2500 ha can admit, using a quincunx⁵ with a distance of 5 metres." The quincunx ... it is the triangle. Then, we see the culture. It is planted in each 5 metres. Each 5 metres has one plant.

C. Humm, I understand.

Tec. It does not matter the direction we consider, plants lay 5 metres apart.

C. Right. Would it be correct to say that it is a triangle with sides equal to 5 metres?

Tec. Exactly. They have each 5 metres.

C. Ok. So, you would throw the small triangles inside the tract. Would it be like drawing triangles in this way? With one plant at each corner?

Tec. Exactly, each laying 5 metres from the other.

C. It would look like a grid. Is this right? In this way? [I draw a {3.6} tessellation].

Tec. Humm.

C. You must correct me if I am wrong, because I don't know these things. All right?

Tec. Then ... We find the area. It is S over d^2 times the factor. But I don't know what this factor means.

C. What factor is this?

Tec. 1.155.

C. What is it?

Tec. It is a factor... it is like n , which is 3.14 and just this.

C. It is equal to ...

Tec. 1.155.

C. 1.155?

Tec. Just this.

C. It is a factor.

Tec. Yes, a factor.

C. Which you call n , and which is equal to ...? It is S over ...

[the remaining of this extract from the conversation only clarifies the terms of the expression $N = (S/d^2) n$, where N = number of "covas", $n = 1.155$, S = area of the tract, and d = spacing between plants in the triangular grid. No further information is given about this relation which clarifies the meaning of n ; a meaning for n is suggested by the researcher during the stage of interpretation of results].

⁵ "Quincunx n . (Arrangement of) 5 objects set so that 4 are at corners of square or rectangle and the other at its centre, esp. as basis of arrangement in planting trees." (The Oxford Illustrated Dictionary, 1981).

What is interesting about the technician's account is the possibility of thinking of the "*area of the culture*" from the perspective of cubação. As he says, his formula should account for the area of the tract divided by the "area of the culture". From the discussion in chapter 4, this "area" seemed to relate to the farm-people's understanding of the productivity of their work. It is reasonable then to assume that the above formula potentially gives us a measuring procedure for *mil covas* which can be seen as accounting for a measure of both the *work* performed on the land and the *area* of the shape of the tract. To speculate further about this is, however, to change to a different level of analysis. This task, I leave to the next chapter; when the possibility of formulating a model of cubação (in which *mil covas* is taken to express the amount of work performed on the land) is raised.

CHAPTER 8

THE NOTION OF AREA

8.1 INTRODUCTION

This chapter makes interpretations of the results described in chapter 7. It is divided in four parts. The first part, section 8.2, discusses issues related to the *logics of geometries*. Section 8.3 suggests elements for a *model of cubação*. Part three (section 8.4) proposes a slightly different manner of 'modelling' the notion of *area*; one which is regarded as more germane for preserving the logical reasoning related to the criteria of applicability of *cubação* as addressed in chapter 6 (section 6.3). Finally, section 8.5 concludes by raising aspects which would require a more careful examination if the intention were to discuss implications for pedagogy and curriculum development.

More abstractly, this chapter tries to place *common knowledge* in a discussion of historical and fundamental aspects of human reasoning.

8.2 INTRODUCTION TO THE LOGICS OF GEOMETRIES

I want to start this chapter with a brief comment on two aspects:

- (a) the conservation of both area and perimeter under shape transformation; and
- (b) the idea of *area* being modelled on something different than a number of units of area which is conserved under shape transformation of assembling/reassembling.

The intention is to clarify the perspective from which a model for school geometry in consonance with *cubação* can be developed. In so far as the "conservation principle" and "the idea of area" help us to recognize what can be correct about the reasoning in *cubação*, they address a perspective for dealing with the *logic* of *cubação*. However, in so far as the principles of *cubação* express standards not usual to school geometry, it would be more adequate to speak of *logics* (in the plural).

The conservation principle.

The "transformation of shapes" appeared as a ready artifice by means of which farm-people think about those cases in which cubação fails. The demand posed by cubação for a tract to have 4 sides, seemed to account for such an artifice when the shape is not a quadrangle; which leads one to see the transformation-manoeuvre as a 'natural' kind of solution. But as far as an Euclidean account of cubação is concerned, nothing requires that, in trying to find a way out for these transformations, propositions such as P6, P7, P8 and P9 in chapter 7 (pp. 197-198), should necessarily follow. As transformations are performed under perimeter invariance, these propositions suggest that, at the bottom of the farm-people's reasoning, there is the idea that different shapes having the same perimeter have the same area. That this idea can be considered as following 'naturally' from cubação is not in any obvious sense trivial. Actually, it seems to be unnecessary, not to say naïve or incorrect.

Comparison of the areas of figures having equal bounding perimeters has been the focus of some old isoperimetric problems¹; from which we learn, for example, that the conversion of any irregular quadrilateral into a regular one of equal perimeter is necessarily accompanied by an increase in area. Also, for regular polygons, it is known that the circle is greater than any polygon with the same perimeter. In addition, the idea contained in P7 has been considered by mathematicians as a misconception among non-mathematicians².

¹ For example, we are told by Heath (in his introduction to the thirteen Books Euclid's Elements, 1956, vol. 1, p. 26, Dover) that, in commenting on Pappus, Proclus says: "The subject of isoperimetric figure was a favourite one with Pappus, who wrote a recension of Zenodorus's treatise on the subject. Now, on I. 35 Proclus speaks about the paradox of parallelograms having equal area (between the same parallels) though the two sides between the parallels may be of any length, adding that of parallelograms with equal perimeter the rectangle is greatest if the base be given, and the square greatest if the base be not given etc. He returns to the subject on I. 37 about triangles. [...] Lastly, the "four-sided triangle", called by Zenodorus the "hollow-angled", is mentioned in the notes on I. Def. 24-29 and I. 21." (footnotes containing references were not included). Pappus's commentary was about the 4th century A.C.; and Zenodorus's treatise was about the 2nd century B.C.)

² For example, Heath's commentary to the Euclid's *Elements* (*idem*, pp. 332-333) says: "Proclus had evidently remarked again in the missing passage that, in the case of both parallelograms and triangles between the same parallels, the two sides which stretch from one parallel to the other may increase in length to any extent, while the area remains the same. Thus the *perimeter* in parallelograms or triangles is of itself no criterion as to their area. Misconception on this subject was rife among non-mathematicians; and Proclus (p. 403, 5 sqq.) tells us (I) of

The idea of the impossibility of having different isoperimetric shapes with the same area is so strong in us, that it seems that the only pertinent question to follow is "how inaccurate are the results obtained by farm-people in proceeding those transformations?" (that is, about how much they fall?). This position, which I took initially with respect to cubação, I was obliged to suspend (for analytical reasons) if any use were to be made of the information arising out of the protocols. To be able to continue, the only solution was to change the question to something like:

"In what way can sides be compensated without prejudice?"

"In what way is the perimeter in geometrical shapes of itself a criterion for their area?"

"What kind of *breaking/making up* rule does preserve unchanged both perimeter and area?"

That is, instead of focusing on regular shapes and seeing what happens to the area when shapes change under perimeter invariance (a posture more germane to the problems in school geometry), what I had to do was to ask for the conditions (shapes and rule) under which both area and perimeter do not change. But when and why should one ask this question in school geometry?³

The notion of area modelled for use in school geometry.

In Euclidean Geometry there are two distinct ways in which two quadrangles may be related to each other: they may be the same shape and size (congruent) or the

describers of countries (...) who drew conclusions regarding the size of cities from their perimeters, and (2) of certain members of communistic societies in his own time who cheated their fellow members by giving them land of greater perimeter but less area than they took themselves, so that, on the one hand, they got a reputation for greater honesty, while on the other, they took more than their share of produce. Cantor (*Gesch. d. Math.* I, p. 172) quotes several remarks of ancient authors which show the prevalence of the same misconception. Thus, Thucydides estimates the size of Sicily according to the time required for circumnavigating it. About 130 B.C. Polybius said that there were people who could not understand that camps of the same periphery might have different capacities. Quintilian has a similar remark, and Cantor thinks he may have had in his mind the calculation of Pliny, who compares the size of different parts of the earth by adding their length to their breadth." Cantor (who I guess to be Moritz for reasons of the cited *Gesch. d. Math.*) lived in the transition 'nineteenth-to-the-twentieth' century.

³ It is important to make clear that I did not raise this question to clarify something which is not generally 'known' with respect to geometry. The necessity of formulating the question in this way relates to the intention of the research in posing questions to the information given by farm-people so as to discuss problems of application to schooling.

same shape only, when they are said to be similar. They are *congruent* if they can be made to coincide exactly with one another, in which case they are sometimes said to be equal (or equivalent) in all respects. Two *similar* quadrangles are copies in the sense of enlargement and reduction copies.

The method of establishing both coincidence and facsimile is, at least in principle, the movement of one quadrangle until (a) it is placed exactly upon the other in the case of 'coincidence'; (b) it produces a scale copy (enlarges or 'shrinks') in the case of 'facsimile'. In Euclidean Geometry, motion does not squash things.

Also, a given construction can be performed anywhere in space with the same results each time. Quadrangles are said to have 'images'. Thus, in Euclidean Geometry, a quadrangle and its image under an isometry are considered to be geometrically equivalent (congruent). If a group of isometries is enlarged to include changes of scale, we have the group of similarities of the Euclidean plane.

This latter formulation of the problem largely suppresses question of rigid-body motion in favour of the concepts of congruence and similarity, added to the ability to make certain constructions arbitrarily in space. Rather than discuss movement directly, school geometry prefers to work with parallel lines ("the tracks along which a translation is performed", to use Gray's words, 1979, p. 30), angles and length ratio.

When one measures the *area* of a quadrangle in space, this shape can be seen as representing the map of a physical object. We can imagine it lying in a (x,y) plane and we might agree that its size was adequately measured by approximating the surface with flat squares and measuring it in the way we would probably use to measure the surface with a square grid. Since this method of measurement reduces to measuring lots of squares and adding, it is independent of the choices of axes x , y . This is as it should be, for the area of a quadrangle is a property of the surface itself and not of the coordinates with which we might happen to describe it. We can similarly use the invariance of the rectangle-area to measure shapes in space.

Accordingly, it can be assumed that *space* in which quadrangles (and their areas) are constructed and exist is homogeneous (any one point resembles any other), isotropic (it has no preferred direction), and absolute (in the sense of requiring an absolute base of reference against which all distances or sizes have equal absolute

measurements; and in the sense of being a totality to which no attribution is made of functions or physical states). Geometrically this corresponds to a system of Cartesian coordinates, to which all locations, sizes or movements in a three-dimensional space can be related. Reflected in this assumption, space can be spontaneously conceived as a self-contained entity, infinite or finite, an empty vehicle, ready and having the capacity to be filled with things; space can make things happen but nothing acts upon it. An entity which is 'there', and which is experienced as an always-present and self-sufficient given.

If this idea of space can have a pedagogic function in teaching geometry or mechanics, it certainly does not represent the concluding view one wants to convey. There is much more to space than this notion contains. For example, *space* would have to accommodate definitions given in terms of the extension of material bodies or fields bordering on each other (as a landscape composed of natural elements). The measurable distances within such a web of different elements are aspects of physical space. Beyond that, it is the mutual influences of material things that determine the space between them: distances can be described by the amount of light energy that reaches an object from a light source, or by the strength of the gravitational attraction exerted by one body upon another, or by the time it takes for one thing to travel to the next.

This perspective would require, for example, accepting the idea that space is in no way given by itself, but occurs only in the presence of perceived things. Although space, once it is established, is experienced as given, the experience is generated only through the interrelation of objects. Space turns out to be considered creation of existing objects, and some geometrical properties of space can be treated as shapes (in the same way that some physical properties can be treated as 'fields of force', for example).

The important here are two things. *First*, it is important to recognize that whatever framework we adopt, the *reality* of space is not denied. The idea of the shape of space itself can be expressed in terms of its intrinsic geometry⁴; in terms of which *spaces* will differ. Thus, squaring with the 'absolute-view', space can be a concrete particular (a kind of substance). On the other hand, according to the 'relative-view', space is real, but its reality is exactly that of the material system of which it is a property.

⁴ In this respect see Harré (1986), chapter 6.

Second, and as far as the above two views can be seen as related to Euclidean and non-Euclidean geometries, *area* has one unique definition. As Greenberg tells us:

"What then does 'area' mean in hyperbolic geometry? We can certainly say intuitively that it is a way of assigning to every triangle a certain positive number called its area, and we want this area function to have the following properties:

1. Invariance under congruence. Congruent triangles have the same area.
2. Additivity. If a triangle T is split into two triangles T_1 and T_2 by a segment joining a vertex to a point of the opposite side, then the area of T is the sum of the areas of T_1 and T_2 ."

(Greenberg, 1973, p.265)

This is precisely how area is defined in Euclidean Geometry. The problematic point raised by Greenberg with respect to the above definition concerns the pertinence of the whole system of *measuring* area on the basis of square units (rectangles do not exist in hyperbolic geometry). The problem is then to know how to calculate it. The answer, which was given by Gauss in 1794, includes a formula for the area of the triangle which is proportional to the *defect* (the difference between 180° and the angle sum of the triangle, which, in hyperbolic geometry, is less than 180°).

In summary, as far as school geometry is concerned, there is no apparent necessity to search for a formulation of the notion of area different from that which defines a geometrical quantity which is invariant under congruence. Its 'reality' can be modelled on conservation, whether the shape is taken as an object in space or as a property of an abstract/construed space.

But with respect to *cubação*, the *reality* aspect does not seem to be so unproblematic. *Mil covas* is not identified as a measure of area which is real in the sense mentioned above for school geometry. But farm-workers do not deny that *mil covas* are 'seen', for example. On the other hand they have an adequate grasp of the reality of the space in which results are obtained in the metric system. Thus, in respect to which 'real world' would their propositions constitute an acceptable discourse? How to *represent* 'mil covas'? How to imagine the 'existence' of 'mil covas' as something which can be 'experienced' within both geometry and a system of measuring land?

8.3 A MODEL OF CUBAÇÃO

In chapter 5, I argued that two essential dimensions to our understanding of *cubação* were the *discursive* character of the geometry of *cubação*, and the *social*

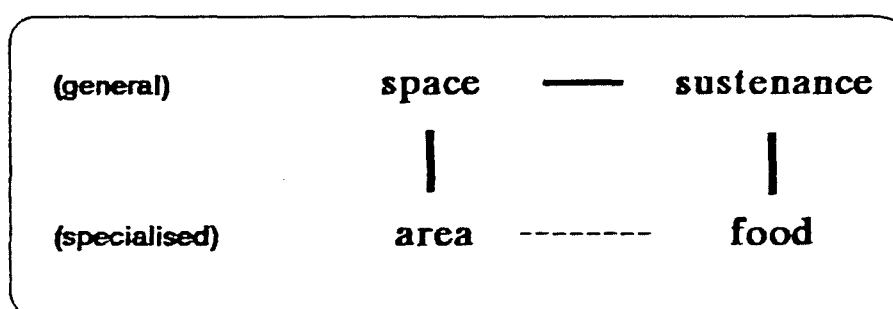
relations of ownership and work in which land inheres. In other words, it was asserted that cubação is a *geometry*, but it is a historically/socially *situated* geometry. In this section I will begin to speculate about a model of cubação having regard to the results described in chapter 7.

The purpose of the model is to clarify aspects which can help to assign real content to the geometry of cubação; and so, to afford meaning to the farm-people's explanations about area measurements. The motivation which suggests such a task returns to the beginning of this study, when, starting with soil, I expressed my interest in an analysis of contingencies under which people think about their practice.

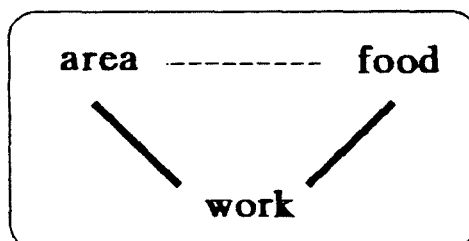
But differently from soil, for which *science* provides an objective reference for locating both the meaning and the reality of things, events and practices (through a discussion of *technology*); the *discursive practice* which is cubação, represents an autonomous kind of formation which (as I hope to have suggested) has crossed the '*threshold of scientificity*' (though not the '*threshold of formalization*')⁵; and so, exempts science of its functionality in respect of the quality of living. This is a peculiar way to say that, in relation to cubação, science offers no possibility of judgement externally from commonsense.

⁵ These expressions come from Foucault and have a specific meaning which is important to clarify. As he says: "It is possible to describe several distinct emergences of a discursive formation. The moment at which a discursive practice achieves individuality and autonomy, the moment therefore at which a single system for the formation of statements is put into operation, or the moment at which this system is transformed, might be called the *threshold of positivity*. When in the operation of a discursive formation a group of statements is articulated, claims to validate (even unsuccessfully) norms of verification and coherence, and when it exercises a dominant function (as a model, a critique, or a verification) over knowledge, we will say that the discursive formation crosses a *threshold of epistemologization*. When the epistemological figure thus outlined obeys a number of formal criteria, when its statements comply not only with archaeological rules of formation, but also with certain laws for the construction of propositions, we will say that it has crossed a *threshold of scientificity*. And when this scientific discourse is able, in turn, to define the axioms necessary to it, the elements that it uses, the propositional structures that are legitimate to it, and the transformations that it accepts, when it is thus able, taking itself as a starting-point, to deploy the formal edifice that it constitutes, we will say that it has crossed the *threshold of formalization*. [...] Their [the threshold] chronology, in fact, is neither regular nor homogeneous. The discursive formations do not cross them at regular intervals, or at the same time, thus dividing up the history of human knowledge (*connaissances*) into different ages [...]. They are, in fact, events whose dispersion is not evolutive: their unique order is one of the characteristics of each discursive formation." (Foucault, 1986, pp.186-187).

For this reason, I was led to look more deeply at the features of the 'perceptible experience' of farm-people and to try to clarify to which concepts they refer; that is, the *existence of mil covas* was set for investigation, also, as a problem of *representation*. Results from the empirical study suggested an interpretation for *mil covas* related to the *productive work* spent/invested in cultivating the land for planting for subsistence. Having regard to the *geometry* of cubação, this perspective leads one to look at the *area* obtained by the procedure of cubação as the *fundamental entity* from which geometry itself should be derived. Thus, it would be adequate to suggest a model of cubação starting from *area* as a *primitive*. Suppose we say that the primitive elements are:



Each of these elements is an *amount*. Production introduces another amount; namely, *work*.



But *work* performed on the land can be split into *effort x duration* (not assembled; these are not primitives). That is, if dW is taken as the 'work' done on the land by a farm-worker who cultivates a given area with a stick moved perpendicular to its present direction and/or rotated (displacement equal dx , and duration equal to dt), we have

$$dW = P dt ,$$

where P = effort, is the time rate of doing work on the land (power). Work can be split in another way, as in

$$dW = F dx ,$$

where F is the force. This is how the concept 'work' is developed in physics. Putting the two together,

$$Fdx = dW = P dt ,$$

and

$$P = F v ,$$

where v is dx/dt , the speed of movement.

On the land, *effort* is representable as the width of cultivation; *duration* of the effort by the length (Figure 8.1). So, *area* (dW) is broken into a product ($a \times b$) or [$r \times (c/2)$] (Figure 8.2). *Food* broadly is *not*.

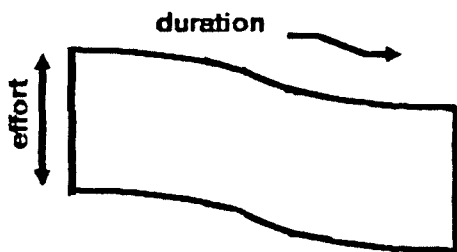


Figure 8.1

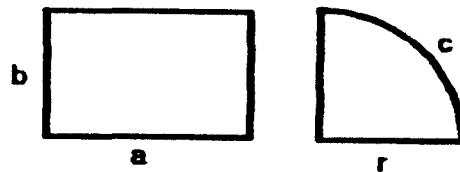


Figure 8.2

In summary, *area* expressed in *mil covas* can be regarded as a measure of *work* performed in given circumstances. But contrasting with work (*labour*) which, for example, requires '*reposition*' from one day to the next, the *area* given in *mil covas* is there to be measured over and over again (independent of time, place, agent), always providing the same result. Yet, *food* is produced only if conditions exist; including not only those related to the possibilities of transforming the soil for cultivation, but those related to effort.⁶

In so far as the *geometry* of *cubação* is concerned, the question arises of "what are the implications for school geometry of taking *area* as a *primitive*?" The answer, which would have a value for the understanding of problems of differentiation and change between commonsense and science, will not, however, be provided in this thesis. What is needed at this point is to speculate about the *possibility* of

⁶ To deepen the discussion of labour within economy would be certainly interesting. For the ECPC-Project it would be essential. But to extend the model of *cubação* to incorporate fundamental issues from economy is a difficult task to be adequately performed in this thesis. Here, the main concern is with the elements of the *geometry* of *cubação* which can help us to understand the *reality* aspect of *mil covas*.

constructing a geometrical discourse which takes into account the logic of the geometry of cubação as it was sketched in chapter 6; and which, at the same time, incorporates to the discourse those ideas which transpose from commonsense what can be accepted as teaching about geometry. Inserted within a pedagogic discourse, such a geometry would evoke accounts (a) proper to cubação; and (b) analysable within the framework of Euclidean geometry.

For example, having regard to the elementary level of reconstructing geometry in the primary school, area could be taken as a *unity* (primitive), and "ploughed fields" drawn in a cardboard, would allow pupils to identify area=work (easily confirmed by weighting the obtained "fields"). A comparison of areas in terms of weight-units could be made (as weight food grows), in such a way that equal areas would be produced by the same width, moved a given "pull-length" (analysable in terms of the average of two edges). The procedure of cubação, added to a system of units, could then be used as an introduction to traditional school geometry. An analytic account of cubação which seeks the reconstruction of didactic entities within a pedagogic geometrical discourse is provided in the following section.

8.4 RECONSTRUCTING DIDACTIC ENTITIES FOR SCHOOL GEOMETRY

From an analysis of the farm-workers' accounts in chapter 7, we are told that the only figures which could be said to *exist* were those obtained in a *procedural manner*, and that the truth-statements made by farm-workers while justifying such a procedure were intended to guarantee the existence of certain basic forms such as sides, perimeters and squares. Statements are part of people's discursive practice, but the conception I am calling *existence* was used to denote a 'demand' about which the researcher had some *reservations* (for example, about "a line having an area", or "an area being invariant under perimeter congruence"). In the context of the geometry of cubação, these "reservations" seem to fade away; which leads one to suppose that an alternative model for school geometry can be investigated.

Thus, to attempt to investigate cubação as a didactic entity for geometry, 'compatible' with cubação, I will try to make sense of the farm-workers' accounts and practices, but I will be obliged to *depart* from their discourse in the sense that I will do more than just adopt their ideas. To start, I will have to consider cubação as a system of surveying; and to model the idea of area in a slightly different way than we traditionally do in teaching geometry.

8.4.1 Cubação as a method of surveying.

The *acre-system*, adopted in Britain and United States, is one of the most important systems of surveying. The procedure (Γ) for reckoning area in acres⁷ is generally stated for a quadrangle of opposite sides (a & b), (c & d) (measured in *chains*), in terms of factors:

$$\S = [(a + b)/2] [(c + d)/2]$$

and

$$\Phi = 1/10 ,$$

related in the following way:

$$\Gamma = \Phi \S .$$

Applied to a rectangle of sides (1 x 10) chains, it gives 1 acre. It is with respect to such a rectangle that the basic unit of the *acre-system* is defined. In a similar way, *cubação* or any other surveying method can be formulated. The requirement is to fix adequately, having regard to the units of length (x) and area (A) proper to each system. Thus, for the *cubação-system* (x = *braças*; A = *mil covas*), $\Phi_{c'} = 16/10$; for the *metre-system* (x = metres; A = square metres), $\Phi_m = 1$; for the *hectare-system* (x = metres; A = hectares), $\Phi_{ha} = 1/10000$.

For example, if we apply (Γ) to a tract of side 100 units in length, we will have:

100 *braças* -> procedure of *cubação* -> result in *covas* = 16000 covas

100 *metres* -> procedure of metres -> result in *sq m* = 10000 m²

100 *chains* -> procedure of acres -> result in *acres* = 1000 acres

100 *metres* -> procedure of hectares -> result in *hectares* = 1 hectare

For the analysis which follows, the appropriateness of the "surveying formulation" relates to three facts:

(a) it represents an intermediary formulation between the typical representation of the procedure of *cubação* (expression 1, chapter 6, p. 143), and the algebraic formulation of Euclidean procedures (presupposed for use in the *metre-system*, which I am taking as characteristic of the school geometry);

(b) the rule expressed in \S played an important role in the development of mathematics in nearly all ancient civilizations (see Appendix 8.A for a picture of how this rule appears in different situations); and

⁷ An account of procedure Γ for the *acre-system* can be found in Usill (1898); also in McEntyre (1978).

(c) it provides an adequate formulation for deriving the alternative formulation of the procedure in terms of the shape parameter (k) and the perimeter (P); namely $\Gamma = \Phi k_{\text{shape}} P^2$ (which was used for setting the preliminary elements for a geometry of cubação -chapter 6, section 6.3).

In addition, in considering cubação as a method of surveying, the analysis can be made more general; and the notion of area in cubação can be extended to other systems.

8.4.2 The notion of area in cubação.

I have already suggested in chapter 6 that the area in *mil covas* can be expressed in terms of the addition of 'square-fundamental' regions; that is, as the addition of regions with area equal to the *shape parameter*. The problem now, is to know how to interpret the shape parameter in measuring area. It is this question which leads us to formulate the notion of area in a slightly different way. I will restrict the analysis to cubação, and -later- discuss the implications for the other systems.

Instead of thinking of a figure covered by a given number of units of area (square-shaped), we will think of:

(a) a *perimeter* P embracing a given figure (instead of an area covering a bounded region);

(b) the *area* $A'_{c'}$ of the figure defined in terms of the square of the perimeter:

$$A'_{c'} = k_n' c' P^2$$

(instead of in terms of the square of any other linear dimension such as, for example, the side and/or diagonal for polygons);

(c) the *shape parameter* $k_n' c' = \Phi' c' \cdot k_n$ of such a figure in units of *mil covas* (instead of the unit of area, given by a square of side one). Since k_n measures the area of figure of the respective shape having *perimeter of unit length*, it is characteristic of this shape;

(d) the number of shape parameters $N = P^2$ (instead of the number of units of area which is the area itself).

The key distinction is that the area (A) of a given figure, when defined in terms of the perimeter and the shape parameter, is not equal to the number of fundamental units which measure the whole region (number of *shape parameters*,

P^2), because this number does not represent a magnitude of the same kind of the whole (the number of *cells*, which is the area). Consider as an example the area $A'c'$ of a 100-square (side = 25 *braças*) given by:

$$A'c' = k_4'c' \cdot 100^2$$

where $k_4'c' = \Phi'c'$, k_4 is equal to 0.1 covas.

Thus, in order to "see" the number of *mil covas* (as an *Euclidean area* in terms of the number of cells) we must multiply the result of P^2 (the number of *fundamental squares* which will cover the square = 10000 *shape parameters*) by 0.1 (the area of each fundamental square). This is the same to say that we must divide the result of P^2 by 10; or, as the farm-worker would say, "to ignore the last digit". That is,

$$A = 1000 \text{ covas} = 1 \text{ mil covas} ,$$

or

$$A = 1000 \text{ cells} .$$

In other words, the perimeter alone does not structure the shape or size of the figure, which means that, in order to "see" an area, one needs, somehow, to "fix" the perimeter around a given shape. This is exactly the role of the shape parameter in the above formula. It gives *form* to the cells which compose the area and to the area itself. Thus, in the case of cubação, each cell would have the area equal to 10/16 br^2 , in such a way in that the area of the tract, expressed in br^2 , would be equal to 625 br^2 .

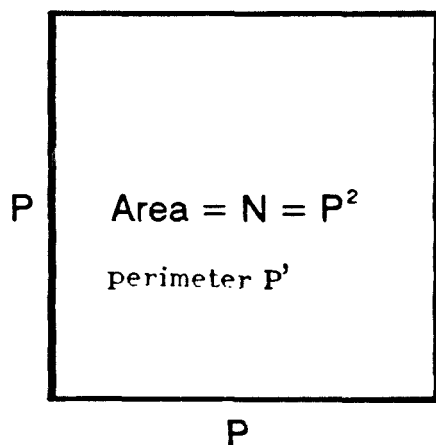


Figure 8.3

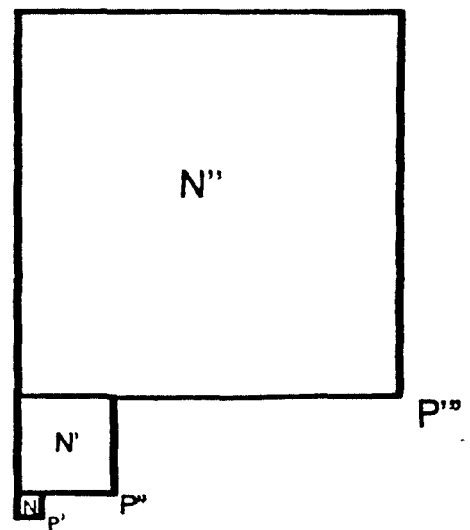


Figure 8.4

The new fact here is that P^2 is equated to the number of 'square-fundamental' regions of area 0.1 covas, N . In so far as $N = P^2$, it turns out that N can be seen as the number of units of area of a square of side P (Figure 8.3). This square has perimeter $P' = 4P$, with area $k_4 P'^2$. Thus, $N' = 16P^2$. If N' is now taken as the area of perimeter $P'' = 4P'$ (see now Figure 8.4), N'' will be equal to $16P'^2$. This process of deriving new squares can continue indefinitely; its reverse being also true. We can represent it as in Figure 8.4.

8.4.3 Farm-people's ideas in the context of the geometry of cubação.

(a) *Perimeter makes area.*

One striking consequence that such a process implies, and which can be of considerable relevance in attributing meaning to people's accounts in the context of the geometry of cubação, is that *sides* can be regarded *as if* they contained a given area. That is, there is a sense in which a segment b (such as the side of a polygon) can be regarded as a perimeter which embraces an area. In the case of a square-shape, each side b would have attached to it an area $A_{c \cdot b}$ equal to the area of a square of perimeter b in units of *mil covas*; which -as it happens- would then be equal to $1/16$ of the total area. As there are four sides, when we multiply $A_{c \cdot b}$ (the area of one side) by 4, we get the number of *mil covas* with which this side contributes to the whole area. In this way, the area of a square can be written in terms of the addition of its sides' contribution. The point is not that one can attach $1/4$ of the area to each of the four sides (this is trivial); but the fact that what 'determines' this amount is the possibility of looking at each side as an actual area, of shape similar to the 'area-mother', but which requires 'weighting'.

What is interesting about this formulation is that it also holds for any regular polygon of n sides, each of length b u (Figure 8.5); one which has perimeter $P = nb$ u and area $A_n = k_n P^2 u^2$. The only condition is, then, that the area attached to each side needs to be reckoned as if b were the perimeter of a similar polygon. If we call the area of this similar polygon A_n^* , the contribution of each side will be nA_n^* . Thus,

$$A_n = n^2 A_n^* .$$

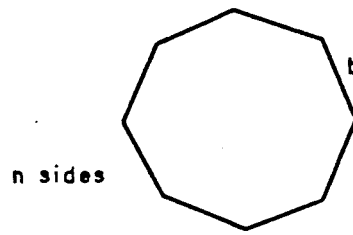


Figure 8.5

This can be easily verified by making $b^* = b/n$ in the formula

$$A_n^* = 1/4 [nb^{*2} \cotan(\pi/n)] ,$$

and comparing with

$$A_n = 1/4 [nb^2 \cotan(\pi/n)] .$$

When $nb^* = 1$, $P^{*2} = b = 1$ and $A_n^* |_{b=1}$ turns out to be equal to k_n :

$$A_n^* |_{b=1} = k_n = (1/4n) [\cotan(\pi/n)] .$$

Thus, the area $A_n |_{b=1}$ turns out to be $n^2 k_n$; which means that the number of *shape parameters* (P^2) is equal to n^2 , the number of *cells*. And if we add several $A_n |_{b=1}$, we can see the area in terms of the "area of the tract". Thus, the definition of the unit of area as the square of side one unit length, turns out to be a particular case for which one can *reckon* "area" by counting the "number of units".

(b) The transformation rule.

We can now speculate about how isoperimetric transformations can be correctly performed in cubação. Consider a square of side x and perimeter $4x$ being transformed into a rectangle of sides $(x - a)$ and $(x + a)$ as in Figure 8.6.

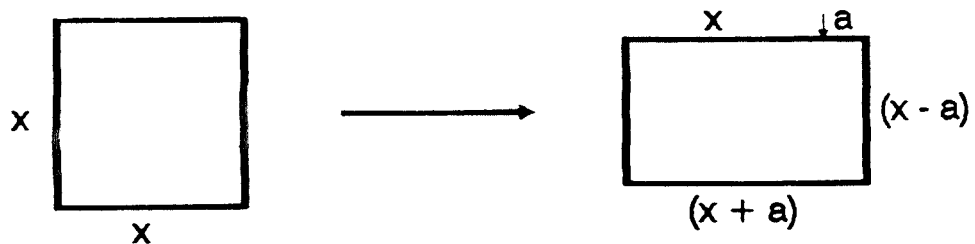


Figure 8.6

Having in mind the constitutive rule $A_n = \sum n A_n^*$, each *segment* (with two ending points) can be said to contribute with the product of the inverse of the unitary perimeter fraction which it represents, times A_n^* . For a square, each side represents 1/4 of the unitary perimeter; thus, contributing to the whole area with $4A_n^*$. The same rule, applied to the rectangle, will consider six contributions (C) produced by three kinds of segment: x , $(x - a)$ and a . Each will contribute twice with values given by:

$$C_x = 4 A_x = 4 k_{rect} x^2 ,$$

$$C_{(x-a)} = [4x/(x - a)] A_{(x-a)} = 4x k_{rect} (x - a) ,$$

$$C_a = [4x/a] A_a = 4x k_{rect} a ,$$

where

$$k_{rect} = [1/(4x)^2] [(x - a) (x + a)] .$$

So, the total area of the rectangle will be:

$$A_{rect} = 2 [C_x + C_{(x-a)} + C_a] ,$$

which is exactly

$$A_{rect} = x^2 - a^2 .$$

Thus, for example, if we have $x = 5$ *braças* and $a = 3$ *braças*,

$$A_{sq} = 25 \text{ br}^2 = 40 \text{ covas} ,$$

and

$$A_{rect} = 16 \text{ br}^2 = 25.6 \text{ covas} .$$

Conservation of both area and perimeter holds only when we can talk of the same shapes; namely, shapes with the same shape parameter k (as in the example of Figure 8.7). In this case, $A_{sq} = A_{sector} = 25 \text{ br}^2 = 40 \text{ covas}$.



Figure 8.7

(c) *The accumulation factor.*

It is typical in school geometry to conceive area in terms of units of area. Accordingly, if we know the area of a tract of land we may know how many plants we can cultivate there. The taken for granted assumptions we need to make are: (a) that the field is regularly arranged; and (b) that we know the area of the basic unit cell of the network so obtained (that is, the "area of the culture"). School geometry looks at the amount of plants on the field.

As far as agriculture is concerned, this formulation can be used for thinking about the most economical utilization of the tract; so it is not a surprise to find educated farmers or technicians using it. However, it is interesting to see that, in trying to work out the number of "covas", technicians also use the logic relevant to cubação which replaces the 'number' of planting places by what can be called the 'accumulation factor' (starting from a defined amount of elementary cells, cubação delimits certain arrangements of cells defined by convenient limiting values; which are then used in measuring area).

Consider, for example, the problem posed by a technician during the interviews (reproduced in Instance 7.30, chapter 7, p. 205).

"How many "covas" of 'gerimum' (pumpkin) a tract of 2500 ha can admit, using a quincunx with distances between plants of 5 metres."

The solution he gave was

$$N = S/(d^2) n . \quad (4)$$

He explained that this expression accounted for "the area of the tract (S) divided by the area of the culture" (A_{cult}); that is,

$$N = S/A_{cult} . \quad (5)$$

Also, n was a factor equal to 1.155 and d was the spacing between plants in a triangular grid. The suppositions to be made in this case are (a) that the 'area of the culture' is given in terms of the shape parameter of the hexagon (k_6); and (b) that the 'area of the culture' is the cell of the {6,3} tessellation which circumscribes each pit planted in a {3,6} tessellation, with $d = 5$ metres (Figure 8.8).

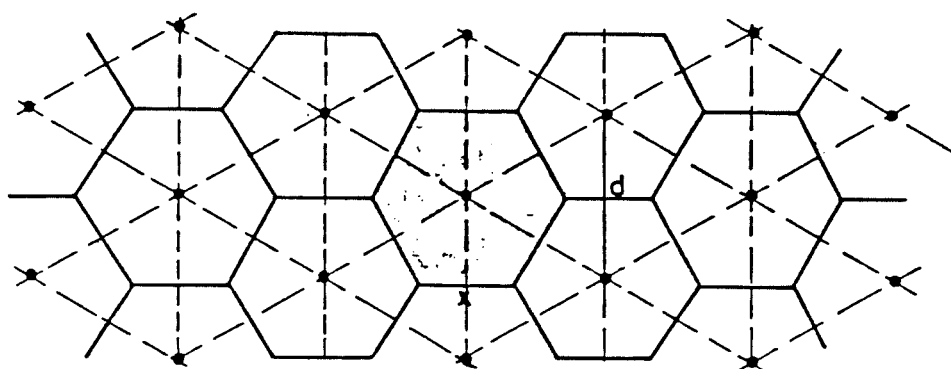


Figure 8.8

Thus,

$$A_{\text{cell}} = k_6 (6x)^2 .$$

Side x can be written in terms of d , in such a way that:

$$x^2 = d^2/3 ;$$

consequently

$$A_{\text{cell}} = 12 k_6 d^2 .$$

Replacing A_{cell} in expression (5),

$$N = S/(12 k_6 d^2) . \quad (6)$$

Multiplying and dividing expression (6) by $16k_6$ we get

$$N = [S (16k_6)]/[(192k_6^2) d^2] .$$

As $k_6 = \sqrt{3}/24$,

$$N = (1.155 S)/d^2 ,$$

which is exactly (4).

What is interesting about this formulation is that the 'factor' n in formula (4) can be seen as $16k_6$. Thus, we can think of N as the result obtained by a procedure Γ' similar to $\Gamma = \Phi \S$; one in which

$$\Gamma' = 16k_6 's q \blacksquare' A s q \blacksquare ; \quad (7)$$

where $A_{sq\ m}$ is the area of the tract in numbers of square units (S/d^2), each with area $16k_u$. This is the same to say that

$$\Gamma' = (k_n/k_4) A_{sq\ m} ,$$

or that

$$\Gamma' = 10k_n 'c' A_{sq\ br} = (k_n 'c' / k_4 'c') A_{sq\ br} . \quad (8)$$

In a similar way, $16k_u$ can be written in terms of the shape parameter with units given in other systems of measurement. Thus, for the *acre-system*,

$$\Gamma' = 160k_n 'acre' A_{sq\ ch} = (k_n 'acre' / k_4 'acre') A_{sq\ ch} . \quad (9)$$

For the *hectare-system*,

$$\Gamma' = 160000k_n 'ha' A_{sq\ m} = (k_n 'ha' / k_4 'ha') A_{sq\ m} . \quad (10)$$

When the shape is a square, $\Gamma' = A_{sq\ u}$, whatever the system of measurement. For example, a square of side 100 units of length (u), would have area:

$$\Gamma' = A_{sq\ u} = 10000 u^2 .$$

Thus, having regard to the four above mentioned systems, we would have expressions (7), (8), (9) and (10), all providing the same result:

$$\begin{aligned} 16k_4 'sq\ m' A_{sq\ m} &= 10k_4 'c' A_{sq\ br} = 160k_4 'acre' A_{sq\ ch} = \\ &= 160000k_4 'ha' A_{sq\ m} = 10000 u^2 . \end{aligned}$$

This is the same to say that:

$$16k_4 'sq\ m' = 10k_4 'c' = 160k_4 'acre' = 160000k_4 'ha' = 1 u^2 ;$$

or that the factors in bold (16, 10, 160, and 160000) represent exactly the number of *shape parameters* (of area k_4 'system' each) composing the area which is taken as the unit area of each system of measurement.

In summary, the reckoning in cubação fulfils the task of both agriculture/surveying and geometry. It does so with such intelligent simplicity of invention that would not be wrong to say that cubação can be counted among the very few methods that survive untouched by *cultural* change in a similar way as the *acre-system* has survived. The example of cubação may stand here for the many historical ways in which the role of measuring land is conceived through ages.

Simple or complex, each system meets essentially similar tasks by displaying the variety of attitudes man brings to the challenges of his existence.

8.5 CONCLUDING REMARKS.

Obviously, a short account reflecting on the possibility of a new 'model' for school geometry raises far more questions than it could answer. To discuss implications for pedagogy and curriculum development, a more detailed investigation of issues would be required in at least three domains.

The formalization of the geometry of cubação. My account of cubação which arises out of the farm-people's thinking makes unusual propositions for school geometry. For example, it says that the *fundamental entity* is an area. A line is an area. From a geometrical point of view, a point or a line are the fundamental things. Area is a composite. But from the point of cubação, area is a primitive thing. So, shapes happen to have four sides, or a certain number of edges; and these are related to the area. My argument is that farm-people are at least correct in seeing sides as perimeters involving an area, whether or not the use they make of such a principle is appropriate. However, the discussion of how this idea can be incorporated at the level of an operative 'geometry' is far from conclusive. A more formal kind of representation would be required; one which could combine the *logical* component of the *discursive practice* which is cubação, with the *control* component which would tell how the rules of formation could be used (which would include that which can be said, or not, within the geometry of cubação). Thus, questions about the nature and on the use of knowledge would have to be more carefully examined.

The nature and use of mathematical thinking. When I used the expression *a model of cubação* I had in mind two distinct meanings. One related to the *formalization* of the geometry of cubação as mentioned above, in which what is being modelled is a *discourse*. But I was also concerned with the meaning of the *work-model* as a real or imagined number of *mil covas* represented as a delimited *area of land* which could have a geometrical concern. The function of the model was to fill out my understanding in two directions. First, it had the purpose of enabling certain inferences about the meaning of farm-people's accounts which it did not look possible to make just from their explanations. Second, the purpose was to enable an extension of the knowledge of cubação as a *mathematical* kind of knowledge. Thus, if the reader is acquainted with studies of the history of mathematics (particularly of geometry and algebra in ancient civilizations), perhaps he/she had

had the opportunity to recognize in some artefacts I have used, similar types of skilled performances which are required in solving characteristic problems involving area, such as the construction of a square equal in area to a given rectangle; the estimation of the area of a circle before incommensurability was 'discovered'; or some geometrical constructions involving either the use of bricks (as in altar constructions), or the *application of areas* (which is largely recognized as a geometric solution of quadratic equations).

It is not my purpose to offer any mathematical account of the types of skilled performance which might underlie the treatment of these problems in history; but rather to point to the necessity in getting to this level of analysis, if any attempt is to be made to develop the model to account for cubação as a *mathematical phenomenon*. As far as refinement of the model is concerned, we are brought to consider the 'reality' issue. As I had already indicated in chapter 1 (page 21), *reality* is to be seen as an *attribute of representation*, not as facts. As communal knowledge, cubação refers to a culture and can be seen as a system of representation with a proper style; its understanding requiring, somehow, a re-understanding of what school or ancient geometries have that cubação does not have. The literature contains plenty of material for such a kind of analysis. Particularly, the writings of Heath (1956), Needham (1959), van der Waerden (1963, 1983), Neugebauer (1969), Seidenberg (1962, 1973, 1978, 1984), Szabó (1978), Pottage (1983), Gray (1979), Fauvel & Gray (1987); provide a useful preliminary collection of references.

The characterization of systems of measurements. It is a matter of fact that cubação is a successful and operative method of reckoning land within the production system of agriculture in Brazil. It has been practised among peasants in the North East since colonial times, and has survived official attempts to introduce the hectare-system in the Region. It is currently used and can be seen as embedded in the practices of surveying, sustained by their necessities and rules, which practices themselves bear a similar relation to structures of ownership, labour and exchange. As an entity which participates in external relations, cubação can then be regarded – and partially explained – as the outcome of social interactions and negotiations under a particular condition of existence: the peasantry mode of production in a capitalist society. A 'picture' of this story was outlined in chapter 5.

But this is not the only perspective from which this remarkable method can be conceptualized. As a *discursive practice*, cubação can be distinguished from the expression of power relationships related to the system of social rules, in which case its inner nature would be best characterized as constituting a *mode of discourse about measurement*. Thus, it is possible to see in cubação an underlying *system of measurement* which is general in the same sense as the hectares-system or the acres-system are for reckoning area. Its investigation would demand two complementary efforts: on the one hand, it would be important to clarify how and why any system of measurement can be regarded as 'generic' and 'universal' when detached from the context of practices which constitute their actual 'motive' of operation. On the other hand, it would be necessary to imagine how the same system can participate in practices which are related to different modes of discourse about measurement. Systems exist which suit this purpose such as the Egyptian, the Chinese, the Roman, the Aztec, and the Acre systems.

The relevance of such an analysis to school geometry relates to the fact that, underlying the methods of solving problems of area by Euclidean methods, there is the metric-system of units which usually is taken for granted, and in this sense 'ignored'. When one says for example that the area of a rectangle is given by the product of its sides, it is concomitantly presupposed that the result is given in square units of lengths, the same unit of length being used to measure the sides of the rectangle. It is also immediately supposed that the unit of area is a square having a side of one unit of length. And nobody asks why we measure lengths and areas at school; or raises questions about the appropriateness and correctness of Euclidean procedures. Actually, it is not usual to look at the teaching of geometry, algebra or arithmetic as *practices* belonging to a *pedagogic discourse*.

In summary, these domains show that the *plausibility* of the model can be tested in more than one perspective if further work is carried on. For the purpose of this study, what is relevant is that results already obtained provide good reasons not only for questioning the way geometry is taught in schools, but for proposing a new way of constructing the reality of space which can account for both the way cubação and school geometry are performed. In other words, results provide good reasons for making of the study of geometry in schools an extension of commonsense. They suggest that there is something useful in the formal speculations, in the relationships. If there were no connection of cubação with anything else, then we would ask: "Should we teach about cubação in the school?"

And the question is then in a sense a practical social question and involves power relationships in a given place, in at given time, and nothing else.

But if cubação can be understood as related to deep mathematical ideas, with a long history, related to the origins of the concept of area, this makes a difference to cubação as a '*didactic entity*'. Curriculum development usually *constructs* entities for transposing from *science* or *mathematics* which can be understood as teaching about models and about the nature of space or objects. So, if we ask "why is a geometry course obsessed with *area* in terms of units of area, but not mathematicians", the reason could be addressed in terms of the well formed didactic object that the area represents.

To accept that the notion of area is a construction has, then, some implications. It offers a different kind of potential for its educational interest and for the applications that one could find for it. It offers a possible relation of methods of measuring "here, now in Brazil", with the origins of the whole idea of measuring area. It helps us not to think in terms of "there is knowledge which we efficiently pass across"; but that "knowledge has structures of its own which are there (in the curriculum) for didactic reasons". Thus, in teaching geometry, we can decide to make of the *shape parameter* a *didactic construction*; and to use the *portion-mass* instead of the *place-area* relationship⁸. As far as the relation $\Gamma = k_L P^2$ is concerned, it can be interpreted as introducing a new *conserved* quantity, generically called *mil covas*; which is independent of the system of measurement. Whatever it is, the *mil covas* concept expressed by Γ , represents an amount of work performed on the land. These ideas certainly exist as entities in science and can be reconstituted as didactic constructions. In this way, we move away from the notion that thinking about teaching is simply taking as given what we intend to do, and taking the structure of knowledge as unique and definitive.

To conclude, I would say that by trying to understand cubação -and not just by saying how it is done- *history* itself may be made problematic. At the same time,

⁸ *Portion-mass* and *place-area* are types of meronymic relations. One example of the former is "A yard is a part of a mile." One example of the latter is "The baseline is part of a tennis court." (see Winston, Chaffin and Hermann, 1987). As these authors say: "Like the members of collections, places are not parts by virtue of any functional contribution to the whole. Like the mass-portion relation, the area-place is homeomeric: every place within an area is similar to every other and the whole area in that all are areas. Unlike portions of masses, however, place cannot be separated from the areas of which they are a part." (p. 426).

by making history problematic, the question of cubação as a didactic construction can be transformed and potentially sets surveying methods in a historical mode of reasoning about area. Then other questions arise. For example, "Is the nature of the content actually appropriate for the primary level of schooling?" It may even not be. It could be that cubação is best adapted to the history of mathematics. Or: "What is it proper to teach about cubação at a given level of the primary school?"

To that extent, then, there are distinct discourses that can be seen as about *area* and it is relevant to contrast them in their similarities and differences, if the attempt is to find a way of understanding the potential of cubação in teaching in the primary school. Particularly for Science Education in the North East of Brazil, it affords an insight into questions about *possibilities* for peasant students of learning another system of measurement and of relating this to cubação. More fundamentally, it affords an insight into the *reasons* to relate a new system to cubação and of the *value* of any other system of measurement to them.

CHAPTER 9

CONCLUSIONS

9.1 OVERVIEW

This work has attempted the task of analysing *communal knowledge* in relation to schooled knowledge (aims and perspectives were introduced in chapter 1). At an abstract level, the thesis attempted to look outwards all the time to some very general questions concerning four broad issues:

- (1) the nature of communal knowledge and its valuation;
- (2) the relation between common-sense knowledge to formalized public knowledge;
- (3) the problems of elicitation and representation of people's tacit understanding;
- (4) the relation between practical discourse and school-scientific discourse.

The account of the specific communal knowledge described in the thesis was based on an empirical study with adults in a rural community in Brazil (S. Paulo do Potengi). The community was rather fully described in chapter 2; where I have also tried to set out an understanding of the theme *Agriculture* for making of it a general case in knowledge. Thus, in trying to characterize communal knowledge, themes related to agriculture were taken as case studies in knowledge relevant to science and schooling. They were soil and cubação. Through them, communal knowledge was looked as an entity:

- . of a large historical scale;
- . on a small social scale;
- . in relation to practical activities;
- . in relation to relations of power and ownership;

and seeing it as implicated in the whole fabric of living, being and knowing.

In practical terms, I started with some questions related to the application of the ECPC-Project' programme of *Agriculture*, which characterized a problem of *representation* located at the level of implementation of the science programme with pupils. The intention was to get a better grasp on how the farm-people's experiences affect the process of understanding presupposed by the use of the Project's tasks concerning soil.

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Data was qualitative, and the methodological inquiry was conceived as a process of confronting problems and information. Information was elicited from people within a perspective in which the researcher did not know exactly what she wanted to find out. First, it was accepted that practical reasoning has a large tacit component which is not well captured by a formalization. Second, there was the question of the discovery of areas of knowledge which were not predicted.

A goal of trying to reach *understanding* was set, and *teachback*, in the sense proposed by Pask's Conversation Theory, was used as a heuristic process for eliciting knowledge. As a result, informants created new explanations, and thought explicitly about the taken-for-granted discourse. This gave to the researcher a possibility of understanding discourse.

A total of approximately 40 hours of verbal exchanges, conducted and recorded by the researcher, constituted the main unit which was taken as the "Referent-Conversation". 24 people were interviewed. 3 farm-workers and 5 teachers (the main group) were met each on 4 different sessions. 16 other people were interviewed once each (the group of additional informants). Sessions were restricted to 45 minutes in length.

In analysing protocols, four levels were defined for treating data: the level of practice, the level of expertise, the level of discourse, and the level of skilled performance (methodological issues were reported in chapter 3).

Results were initially about soil and concerned social relations and conceptions of soil and land (the origin of results is described in chapter 4, and a large amount is used in chapter 2). Then, an in-depth study of *cubação* was carried on, in which the very 'existence' of *cubação* needed to be conceptualized. Perspectives for describing the knowledge of *cubação* had to be defined (chapter 5), and results were about: the method; *cubação* as communal knowledge; the formalization of the knowledge of *cubação* as a mathematical kind of knowledge; historical relations; and the work-model.

Results also concerned a level of analysis in which the discussion of soil and *cubação* reflected the complex and more abstract issues mentioned at the beginning.

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9.2 RESULTS

9.2.1 Concerning soil.

The framework used to structure the initial sequence of interviews was in most respects similar to the framework used by the Project for structuring the pedagogic unit about soil. It contained two elements: Table I and the scheme in Figure 9.1. While in Table I the stress was on *work* that people *do* to soil which *transforms* it for the purpose of growing crops, the scheme of Figure 9.1 suggested an organization of the content which makes use of the idea of "*processes of transformations*" as a unifying concept; and which presupposes soil to be an entity which affords transformations through people's performance.

Table I : The Cultivation of Soil

Stage	tools	to do what	how	why	when
to clear					
to burn					
to plough					
to mark out					
to drill					
to plant					

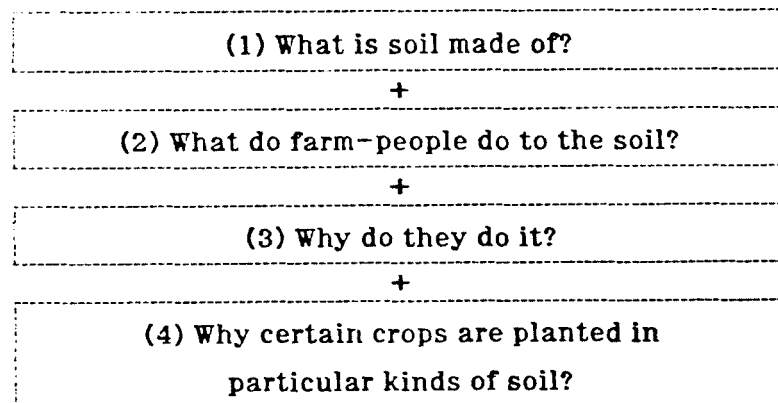


Figure 9.1

CONCLUSIONS

Taking production as a background reference for discussion, a network to represent the variety of ways land is conceived of by farm-people was established (see chapter 4); and the results framed the terms in which soil was placed and analysed in the context of production in agriculture (chapter 2). At a more 'cognitive' level, results concerning soil suggested that people's discourse about land presupposes an ontology embracing natural kinds; which characterizes a particular way of conceiving soil, distinct from the idea of area given in *mil covas*.

Also, the study pointed to the importance of clarifying how *work* is represented by farm-people in organizing their systems of planting. This question was taken later in the context of modelling cubação.

9.2.2 Concerning cubação.

The method of cubação, known to farm-workers as a way of reckoning the number of *mil covas*, says that if we think of a quadrangle of sides *a*, *b*, *c*, and *d* (Figure 9.2), the area is given by $\frac{(a + c)(b + d)}{4}$ (the method was fully described in chapters 5 and 6).

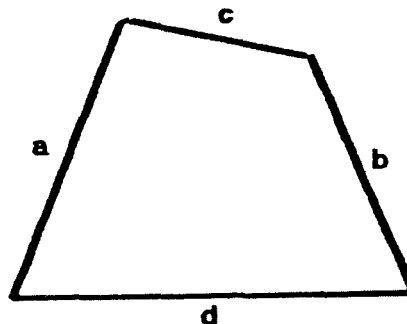


Figure 9.2

The procedure is generally used by farm-workers in various situations in agriculture, when a diversity of shapes (usually irregular ones) need be reckoned. These situations include commercial transactions with farmers, agricultural technicians and inspectors of the Brazilian Bank. It is orally learned and orally transmitted from one generation to the next, through the work of agriculture. Some prevalent aspects of cubação are known to every ordinary person in the community, but they make nobody an expert. To be an expert, means to be able to operate the reckoning in actual situations, some of which are difficult to solve (an account of farm-people thinking while solving problems is given in chapter 7). Recurrent situations for which contentions arise require a more experienced expert: one who can have a more refined control over the conditions to which the method applies.

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It is possible to distinguish two instances of practical application. One in which the system of measurement embedded in cubação represents a necessary and sufficient condition for carrying out negotiations between farm-workers and farmers. And other in which cubação has to confront, explicitly, the methods of surveying introduced in the community via integrated programs for rural development. The hectare-system cannot ignore cubação and functions as normative in the latter case. But no reference is intrinsically necessary to be made to the hectare-system in the former instance of application.

As an outcome of social interactions and negotiations under the peasant mode of production, cubação can be best characterized as a *discursive practice* in relation to which measurements of area are carried out. However, cubação possesses no relationship with the pedagogic discourse which goes on in schools.

In so far as the formulation of cubação is concerned, four kinds were seen to deserve interest (see chapter 6):

- (a) *the typical representation*, which re-expresses the rhetoric into a mathematical kind of formulation (Figure 9.3-A);
- (b) *the transduced representation*, which focus only on the algebra, within the frame of reference of Euclidean geometry (Figure 9.3-B);
- (c) *the general procedure*, which accounts for the formulation, not only of cubação, but of other systems of measurement. (Figure 9.3-C).
- (d) *the formulation in terms of the shape parameter*, which, in the context of the discursive practice which is cubação, demands an alternative formulation of the notion of area (Figure 9.3-D).

Having regard to formulations (b) and (d) mentioned above, the area of a circle (radius r and circumference C) can be written, respectively, as:

$$A_c = [r C]/2 \text{ , and } \Gamma = [1/4\pi] C^2 \text{ .}$$

When algebraicized, the simple procedure of cubação, stated for quadrangles, can be used as a formula for estimating standard units region which generates a good estimate of area; and which is extensible to shapes for which at first sight it is not well adapted.

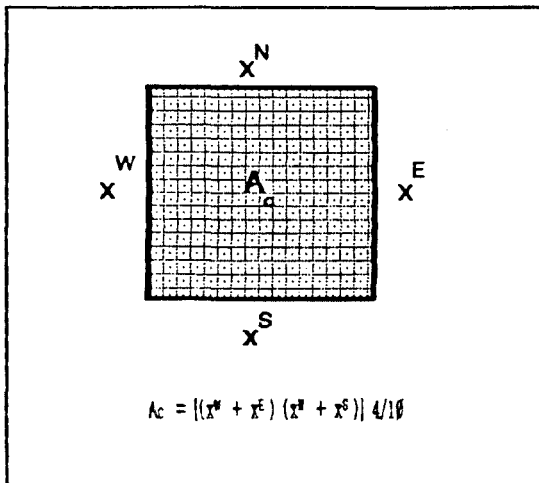


Figure 9.3-A

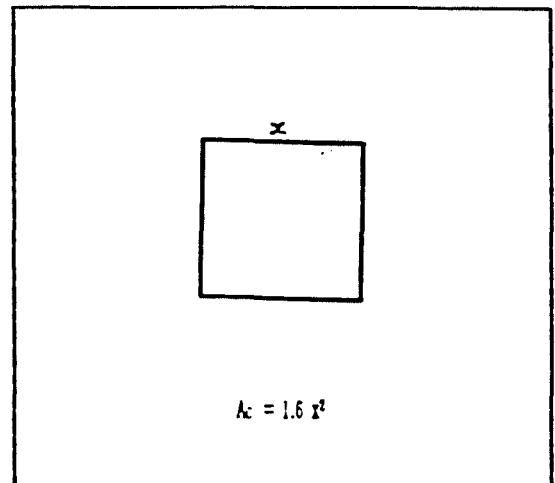


Figure 9.3-B

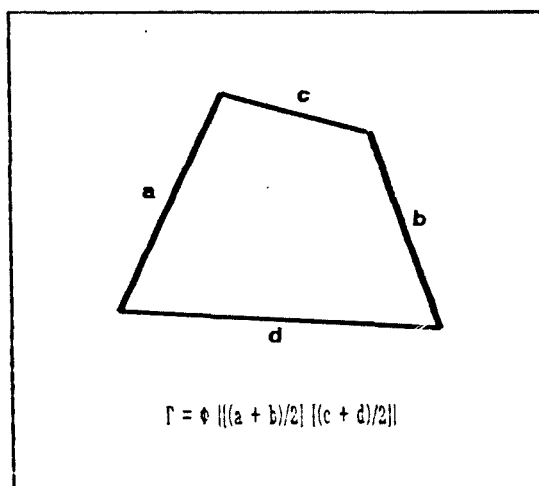


Figure 9.3-C

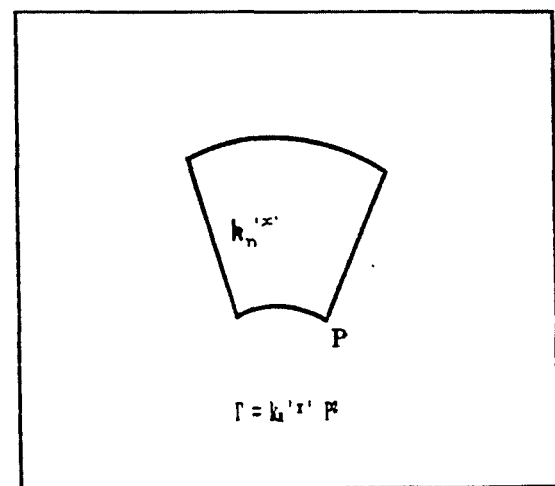


Figure 9.3-D

Cubação correctly gives the area of any shape constructed by the ends of a straight segment moved perpendicular to its present direction and/or rotated (Figure 9.4). Among these shapes, we can define "square-shapes", as a class which includes any "four-sided" shape, whose shape parameter is equal to 1/16 (which is the shape parameter of the square). The "three-sided" segment of circle in Figure 9.5 belongs to such a class ($a = \emptyset$; and $b = c + d$).

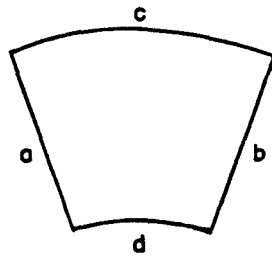


Figure 9.4

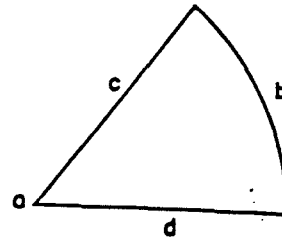


Figure 9.5

Since cubação can be formulated as a surveying method, it becomes interesting to compare it with other systems of measuring land. The result will show that the underlying 'rationale' of the method turns out to be the same as one which has existed in different places and times in history, such as the 'acre-system', the 'Roman-system' and the 'Aztec-system' of surveying. In so far as geometry is generally supposed to have its origin in measuring land, the question arises of what we can learn about more fundamental structures of reasoning about *area*, by both looking at actual/historical surveying systems and looking at the history of mathematics. From the former, it appears that methods of surveying are generally formulated in terms of the procedure ($\Gamma = \Phi \S$).

From the latter, we find that the rule expressed in § played an important role in the development of mathematics in nearly all ancient civilizations, remaining always attached to the procedure of finding the area of a quadrilateral as the average of one pair of opposite sides times the average of the others. Particularly, with respect to the problem of computing Γ for a circle [$\Gamma = (1/4\pi) C^2$], historians would suggest that expression $Ac = (r C)/2$ seems to have come first. Also, there is a suggestion that Ac refers to a relation which stands close to intuition/experience.

In so far as the farm-people's discourse about cubação was concerned, there was the problem of the formalization of the geometry of cubação. The question of assigning real content to such a geometry arose, and led to an interpretation of *mil covas* related to the *productive work* spent/invested in cultivating the land for planting for subsistence. Such a perspective led the researcher to look at the *area* obtained by the procedure of cubação as the *fundamental entity* from which geometry itself should be derived. A model of cubação, starting from *area* as a *primitive*, was proposed. The model regards *production*, *space (geometry)*, and *sustenance* as introducing the fundamental elements, representable each in terms of an "amount". They are, respectively, *work*, *area*, and *food* (Figure 9.6).

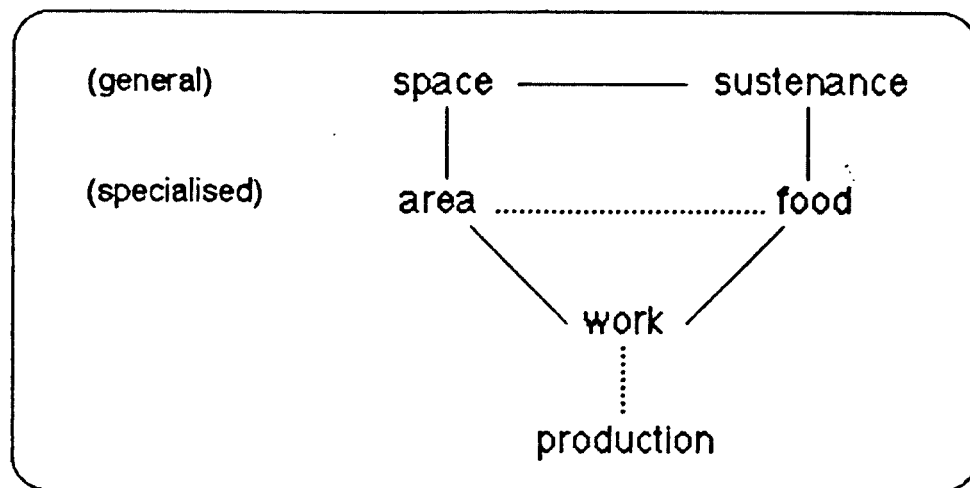


Figure 9.6

But *work* (dW) performed on the land can be split into *effort* \times *duration* (not assembled: these are not primitives). That is, dW can be taken as the 'work' done on the land by a farm-worker who cultivates a given area. On the land, *effort* is representable as the width of cultivation; *duration* of the effort by the length (Figure 9.7), and area can be seen as their product.

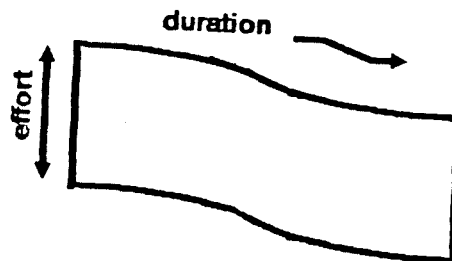


Figure 9.7

9.3 DISCUSSION

Research in Science Education has very largely treated knowledge from an essentially individual point of view. In this thesis, however, *knowledge* was regarded as a *social* entity realised in individual discursive action. Knowing becomes being a participant in a *discourse*.

Two 'forms of knowledge' had received attention. One is science and mathematics. The other is supposed to exist as knowledge supporting most human regularities in thought, feelings and behaviour, and which I have called *commonsense*. In this

CONCLUSIONS

work, both are defined in relation to a *discursive community* and thus, are supposed to be found locally in human praxis. As formalized public knowledge, science is transmitted through history as an abstract result of human inquiry. *Commonsense* is best seen in relation to *communal knowledge*.

The relevant community was a community of Brazilian peasants, and agriculture was taken as a general case in knowledge. *Land* has been privileged through the discussion of *soil* and *cubação*.

The analysis offered in this thesis was intended to help with problems which arise in characterizing the relations between commonsense knowledge and science, such as those of differentiation, development and contextuality. Such problems were particularly acute in the present study, in which case a characterization of farm-workers' and teachers' understandings were intended to be made in relation to formalized/structured bodies of knowledge.

To approach the topic of *soil*, for example, science offers us a *model*. It proposes the idea of *cycle* to think about growing plants; it proposes the idea of *conservation* to qualify and to estimate degrees of changes; it proposes the idea of *control* over events. On the other hand, in the everyday life of agriculture we will find not *soil* but *land* (*arisco, barro, massapê, ...*); not *cycle*, but a *temporal sequence* of events (preparing the land, planting, keep growing, harvesting, selling, eating) which repeat independent of man's free will and *out of his control*.

In addition, it was argued that science, discussed from the point of view of everyday practices, is fundamentally a discussion through an understanding of technology. Thus, initially, no assumption was made about ordinary subjects being inducted into a scientific discourse. But the necessity of making assumptions about the role of farm-workers in a discursive community became imperative, when *cubação* was taken as a discursive practice. A parallel was established between the functioning community of expert farm-workers and the discursive scientific community.

This work has argued that there is the possibility of formalization of communal knowledge. This formalization is not, however, a matter of categorizing people's explanations. Looked as being a 'logic' at the level of discourse (not at the level of assertion), communal knowledge was treated in terms of a formal-knowledge-based kind of representation. Such a formalization accounted in some way for

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people's explanations, but these did not constitute the object of formalization of communal knowledge. The treatment given to the protocols in chapter 7 exemplifies what is suggested here.

Considered in relation to communal knowledge, the discussion of commonsense was seen as not pure, detached from other considerations such as power, social relations and *social-historical* change. These become part of its meaning and definition.

The case of cubação illustrated here has shown clearly the importance of the social analysis for the interpretation of the information obtained from data. Without it, the contribution of this research to the investigation of farm-people's thinking would probably have been a demonstration of the distance between the worlds of agriculture and science, added to a suggestion of the farm-people's cognitive difficulties and misunderstandings. But no speculation would have arisen about how their experiences/knowledge could be considered in understanding difficulties of *representing* knowledge as presupposed by school science.

While the study of soil was marked by a more *cognitive* concern, it also required an analysis of social forms of land tenure for differentiating conditions of transforming the soil for growing crops.

In adopting a Freirean point of view, cubação and soil represented cases in knowledge to be conceived of as making problematic aspects of people's living. Accordingly, *teaching* was to be understood as both supportive of the existing community and subversive of aspects of social structure. In this perspective, the confrontation of communal knowledge with other kinds of formalized knowledge becomes inevitable, and the question arises of how science can be looked problematic as a body of knowledge which contains in itself *problems to be searched* and not bits of information to be transmitted.

9.4 IMPLICATIONS

One argument raised in the thesis was that communal knowledge can be supposed to have a large tacit component; and, as such, it does have structuring rules which are not consciously available to those who are regarded as operating within them. Commonsense relates to knowledge at the level of this 'fundamental structure'.

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The attempt to formalize communal knowledge had, then, to face the methodological problem of inferring tacit structures from interviewing data. Cubação has constituted the central topic with respect to which such a task was carried out. For the problem of dealing with deeper regularities presupposed by tacit reasoning, alternatives are available in science education in terms of the different approaches for looking at commonsense (a summary of perspectives was given in chapter 1). These would ultimately define what could count as the fundamental *structure for describing/analysing/proposing/discussing common-sense reasoning*.

This research has insisted on the necessity to getting at the historical level of analysis. It was suggested that a fruitful line of inquiry would be to clarify the types of skilled performance which are required (or not) in the use of the fundamental-structural relations, and the infra-logic relevant to such a use.

The position accepted in this thesis regards commonsense interesting when viewed as a resource out of which we manufacture formalized public knowledge. This is certainly not an exclusive implication of the approach adopted here, but seems to arise from perspectives recognized to be '*structuralist*.' The position adopted in this thesis is structuralist when it regards knowledge as discourse; but other senses of 'structuralism' can be used in research.

To regard people's knowledge as belonging to a discourse does not mean that practical discourse can be always characterized as a discursive practice (it seems that soil can not). This raises the question of how much more like cubação is waiting to be found. To have presented cubação as being 'discovered' shows that this is not a trivial question to answer. The knowledge of cubação demanded a "construction" to be built from more fundamental features of the 'perceptible experience' of farm-people; which reinforces the idea that any attempt to getting at new discoveries must contain an intention to grasp on how people's experience affect the process of understanding events/facts/entities in the world.

For schooling, the existence of communal knowledge not known to school implies that a confrontation has to take place. The case of soil stands here for the many possibilities we have for confronting knowledge and power, when we would say that knowledge is used -indeed- to differentiate people. The results on soil, did illustrate some important differences between communal knowledge and the frame of reference based on science. On the other hand, cubação represented a suitable case leading to a re-formulation of the traditional approach to school knowledge.

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APPENDICES

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APPENDIX 1.A

THE ORIGINAL MOTIVATION OF THE RESEARCH

1. INTRODUCTION

The research on which this work is based arose from a concern about schooling framed around the results of the intervention by a Science Project in Brazil (ECPC-Project)¹. This project has produced substantial analyses in four domains of activity:

- (a) development and implementation of curriculum for primary science (based on *community problems*);
- (b) teachers' in-service training;
- (c) undergraduate research-students training; and
- (d) development of research within a specific perspective in science education which has been called in Brazil *an organic approach to problematizing-teaching*². Studies of *knowledge* have special interest for the Project, particularly those concerning *science* and *commonsense*.

Basically, what the project tries to establish is a programme of investigation which can have both practical and theoretical interest for science education. There are three levels at which questions can be located:

- (1) The *fundamental level* at which the discussion is focused (a) on the view taken by the Project of Brazilian educational problems and of the possibilities for their solution; (b) on the view taken of issues such as "the cognition of reality", "the

¹ The "Ensino de Ciências a partir de Problemas da Comunidade" (ECPC)-Project has been implemented by the Department of Education of the Federal University of Rio Grande do Norte since 1983; and received financial support from CAPES (Coordenação de Aperfeiçoamento de Pessoal de Ensino Superior), an agency of the Brazilian Ministry of Education.

² Pernambuco, M. e Vargas, F. (1985) *Abordagens Sociológicas do Ensino de Ciências* VI Simpósio de Ensino de Física, Universidade Federal Fluminense, Niterói: Sociedade Brasileira de Física.

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structure of constructed bodies of knowledge (science and commonsense)", "structures of thinking"; and (c) on the view taken about schooling and science education.

(2) The *pedagogical level* at which the Project provides a rationale for planning and implementing science curriculum, and at which results from (1) are used for immediate practical action (that is, mainly to inform decisions).

(3) The *analytical/critical level* at which a critique is made both to generate/illuminate topics for research and to re-think aspects of the two previous levels: in other words, to make problematic aspects of educational practice in science.

To describe in detail the Project's programme of investigation is a task which would certainly deserve a thesis by itself.

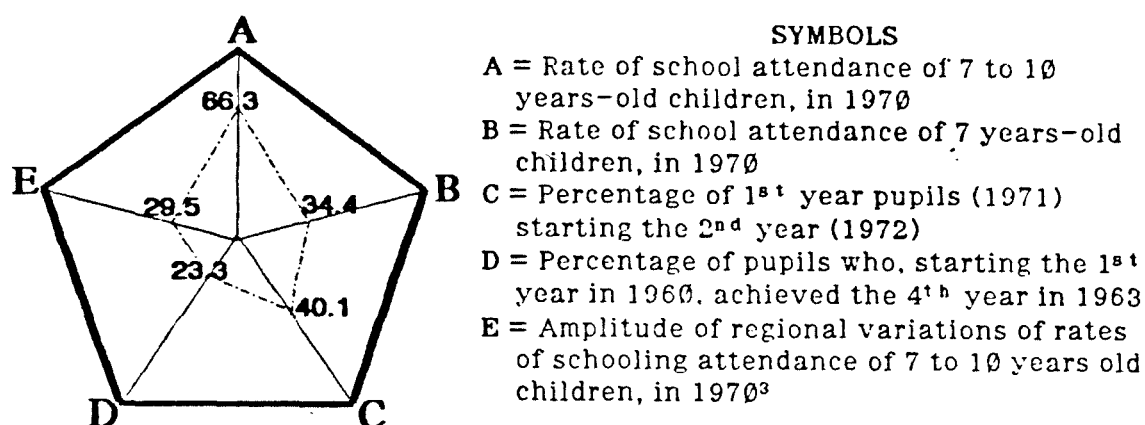
I have selected certain aspects and organized them around the discussion of points which address issues relevant to the *conceptualization* of the study.

2. ASPECTS OF EDUCATION IN BRAZIL AND THE ECPC-PROJECT

2.1. The selectivity phenomenon.

Compulsory and free primary education is a quite recent norm implemented by government policies in Brazil. The four first years of schooling were prescribed as compulsory by the National Constitution of 1946, and were subsequently expanded to eight years by the Constitution of 1969. Additional specifications of two further laws (Law nº 4.024/61 and Law nº 5.692/71) made it clear that these eight years should correspond to children aged between seven and fourteen.

But as soon as one looks at the empirical reality of primary education in Brazil, the meaning of the word 'compulsory' becomes rather un-idiomatic. The low degree of achievement of educational policies concerning the growth of primary schooling is well documented in the Brazilian literature. Figure 1 gives an example in which the facts are displayed in different ways.

Figure 1⁴

The unacceptable fact revealed by Figure 1 is called in education *the selectivity phenomenon* to which children are submitted in elementary school in Brazil. It expresses the high degree of 'drop out' ('fracasso escolar') among working-class children⁵ and has been a motif of a long and controversial debate in the educational field⁶. As is generally known in Brazil, working-class children (who

³ What Cunha calls 'amplitude' (E) is the complement of the difference between the higher (91.8) and lower (21.3) percentage rates of schooling attendance of 7 to 10 years-old, in 1970 (that is $E = 100 - |91.8 - 21.3|$); these rates correspond to the maximum and minimum values of rates taken from a Table of rates for all the 26 Federal units. In the present case, the higher rate is given for the then Guanabara State (which has been incorporated to the Rio de Janeiro State), and the lower to Territory of Acre (at present, State of Acre). The complement value (29.5%) is used in order to make the origin of scale E coincide with the origins of the other scales (that is, the center point).

⁴ From Cunha. (1980) *Educação e Desenvolvimento Social no Brasil* Rio de Janeiro: Francisco Alves, p. 144.

⁵ Mello. G. N. de (1982) *Magistério de 1^o grau: Da competência técnica ao compromisso político* S. Paulo: Autores Associados: Cortez.

⁶ Cunha (1980) *idem*. is one case within this debate. In trying to account for such a low degree of schooling, he provides a good indication of how complex it is to understand 'reality'. For example, he contrasts figures from 1964 and 1970, from which the information emerges that the percentage of 7-year-old children attending the first year had actually diminished (from 41.1% to 34.4%). To try to make sense of the unequal attendance to schooling, he has to look at and cross a huge amount of data from different sources and about different educational situations. In addition, he has to follow and trace the rate of progression of all children during several years, in which case a distinction should be made between the correct and expected progression on the one hand and the uncharacteristic progression on the other. In these attempts, children's access to, and time spent in the actual system of schooling is discussed in terms of social determinants and the strength of *selectivity* is shown to lie on working-class children.

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constitute the greater contingent of children) start attending school later in their lives. It is also known that drop out is high among them. The same child can both return several times to the same year or return further to a later year of schooling. For the majority of children, there is little direct correspondence between age and year of schooling.

To discuss the process of selectivity to which children are submitted during schooling is not trivial. Any serious analysis cannot eschew questions such as:

- . "Why is it that working-class families send their children to school later, even when places are available?"
- . "What are the actual conditions for learning which are offered to these children?"
- . "What are the inner-school mechanisms through which they fail?"
- . "Who are those children?"
- . "What possibilities do they have to succeed at school?"

These and other related questions have been central to the definition of a large range of studies in the Brazilian educational field in the last two decades.

2.2 Perspectives.

So far, I have emphasized the inappropriate character of the term *compulsory* to designate the actual situation of elementary education in Brazil. Indeed, there are a number of complicating factors which limit children's access to schools, some of which derive from changes related to social, economical and political policies implemented by successive Governments since colonial times.

It is important to stress, however, that the present situation is not at all the same as in the early part of this century, when, for example, less than 20 percent of children in school-age were actually attending the primary school, and more than 70 percent of the population was illiterate. In 1985, 23 million children were considered to be attending the elementary levels of schooling, and illiterates were estimated to be around 25 percent⁷.

The situation has changed because the nation has changed. As a result of demands from segments of Brazilian civil society, the government has been compelled to promote policies which have contributed to increase considerably the number and distribution of schools. This expansion -which has created better conditions for

⁷ Paiva, V. (1985) "Que Política Educacional Queremos?" in: *Educação & Sociedade*, ano VII, 21 S. Paulo: Cortez: Cedes.

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the working-class to get access to school- has happened at the expense of derisory salaries for teachers, the employment of non-qualified teachers; and through the provision of poor levels of teaching expressed mainly by the alienation of the programmes with respect to the reality of this socially differentiated contingent of children.

The 'new' school, freely offered to the population, is widely seen as actually serving the purpose of reinforcing social inequalities. Several educational movements have suggested the necessity of investigating the mechanisms of curriculum development offered in these eight years of schooling. In particular, questions are raised about the definition of what would be an adequate programme content and how to train competent teachers to deal with this new reality.

One of the most influential orientations supporting educational research in the last two decades, arises from an intention of a group of educators to show the mediating role performed by school education regarded the inner contradictions proper to capitalism. They point out the relevance of formal regular education as an instrument of social, political and cultural emancipation of people, and attempt to analyse the strength of inner-school factors in the determination of drop out and selectivity phenomena⁸. Their claims have a basis in two main arguments which can be summarized in the following remarks:

(1) The school is an inseparable part of the totality of the social, and so shows internally the same relations of sustenance and reproduction which are characteristic of that totality. As such, the problem of *selectivity* is situated in terms of economical determinants, as economy is the determinant of the totality of the social. To act at the level of schooling is also to act at the level of the social

⁸ The position taken by this group of educators can be seen described in detail in the following studies: Saviani, D. (1980) *Educação: do senso comum à consciência filosófica* S. Paulo: Cortez; Autores Associados; (1983) "Tendências e correntes da educação brasileira" in: Mendes, D. *Filosofia da Educação Brasileira* Rio de Janeiro: Civilização Brasileira; (1983) *Escola e Democracia* S. Paulo: Cortez: Autores Associados; Cury, C. R. J. (1985) *Educação e Contradição* S. Paulo: Cortez: Autores Associados; (1979) "Categorias possíveis para uma aproximação do fenômeno educativo" in: *Educação e Sociedade* nº 2, S. Paulo: Cortez e Moraes; Cedes; Mello, G. N. de (1982) *Magistério de 1º grau: da competência técnica ao compromisso político* S. Paulo: Cortez: Autores Associados; (1979) "Fatores intra-escolares como mecanismos de seletividade social no ensino de 1º grau" *Revista Educação e Sociedade* nº 2, S. Paulo: Cortez e Moraes; Cedes; Rosenberg, L. (1984) *Educação e desigualdade social* S. Paulo: Loyola; Brandão, Z. et al. (1983) *Evasão e repetência no Brasil: a escola em questão* Rio de Janeiro: Achiamé.

from which the school can not be disentangled. The essential implication is: those (teachers, staff and researchers) who want to make the school less selective and elitist than it is at present, have work to do in the school itself.

(2) Taking the above argument as a starting point, it becomes imperative to clarify what the possibilities for action are. Two main complementary strands are suggested which can be called the *characterization* and the *action* strands.

To *characterize* what is going on in schools becomes an important task not in its contemplative sense, but in the perspective of knowing what the mechanisms are through which more general economical determinants are made specific (that is, their power is reinforced or attenuated) within schooling⁹ and how they operate. The curriculum, the content, both the actions and representations of teachers, and the criteria of assesment, are, for example, located in the inner field of the educational system. They *mediate selectivity* and, as such, have a political character. As they are at present, schooling conditions constitute powerful mechanisms of selection and so require characterization in the perspective mentioned above.

But as *mediation* between economical determinants and the social destiny of children, these conditions must be looked at as part of a 'becoming-plan' which is intended to establish a new posture in teaching/learning. That is, the way in which the school is supposed to operate should be adequate to the characteristics of the working-class children. This does not mean keeping children restricted to a 'working-class view of the world', nor to deny a place for teaching and learning of subject matters of high level of generality and abstraction (such as science or philosophy, for example). On the contrary. The position is first of all of 'respect' and puts forward the view that such a fact should be taken into account for a more productive and profitable teaching. At present, one of the strongest educational movements in Brazil proposes a '*pedagogy of contents*' ('*pedagogia dos conteúdos*') as its front line flag¹⁰.

⁹ Thus, the initial assumption about the power of these determinants is understood as a necessary but not sufficient condition. It helps to situate the phenomenon of *selectivity* but does not explain how it is actually carried into effect.

¹⁰ Libâneo, J. C. (1986) "Os conteúdos escolares e sua dimensão crítico-social", (mimeo.); (1983) "Tendências pedagógicas na prática escolar" *Revista da Ande*, ano 3, nº 6.

To act at the level of the different modalities of pedagogic work so as to offer a better schooling, is, then, the second way in which the phenomenon of selectivity can be tackled. It constitutes an independent strand from *characterization* in the sense that it would be wrong to think that there is a definitive and unique answer to the questions of characterization from which a pedagogical practice can be uniquely derived. Certainly, results from characterization can suggest necessary implications for action. Some of them already exist and are considered as assumptions in almost any kind of proposal.

The competence of teachers is widely recognized among educators to be one of the most crucial links which needs to be improved if the school system is to be changed. The teacher is a fundamental and significant part of the functioning conditions of schooling. As such, he has been studied as a result of the action of external factors: his social origins, his educational background, his professional qualification, his age, sex, and so on. The teacher is first of all, an object of investigation.

On the other hand, as an inner-school condition, he/she can be distinguished from other conditions such as the curriculum or the teaching materials which do not think on their own. The teacher himself thinks, in addition to being an object of thinking and investigation. Studies¹¹ claim that what teachers both do and say, and the way in which they interact with students (which can be externally observed), are, in some sense related to the nature of their representations about the school, the children, the subject matter and their own role as teachers. These *representations* are, then, an integral part of their *practice*: and provide some knowledge of this practice in the same way that the observation of teachers' performance in the classroom does.

It has become widely accepted in the educational field, that any attempt to train more competent teachers should include, somehow, the possibility of working at the level of their representations.

¹¹ For references see, for example, Mello, G. N. de (1982) *idem*.

2.3 The ECPC-Project.

The ECPC-Project aims to tackle some aspects of these questions within a systematic and reflexive perspective of work based on a praxis of trying to modulate the science content taught in schools to the reality of the particular communities to which education is addressed. It starts from the belief that education -without being the determining element in processes of social change- is an important component which participates in social transformations. At the level of schooling, it is assumed that teaching can become an effective instrument of liberty since it facilitates both the comprehension of reality and the possibility of acting upon reality so as to 'transform' it.

The view taken by the Project of reality and of the adaptation of the science curriculum to this reality is informed mainly by Paulo Freire's theory of education (see section 3). His influence can be seen present in all the three levels referred to above in section 1 (fundamental, pedagogical, and analytical/critical).

One central feature of the Project's body of work is the recommendation and use of *community problems* as the starting point for curriculum design. The Project stresses that an analysis of the principles of the organization of social life of the group of people for whom education is intended, should be in some sense always involved in the definition and development of the science programme content. In such a perspective, following Freire's pedagogy, the Project works from *generating themes*. These *themes* are defined on the basis of an analysis of *contradictions* which are to be seen as present in *relations of production* and which make *problematic aspects of the local culture*. Particularly, these themes are used as a way of unfolding the science content so as to expose reality to a re-examination. Permeating this process is *dialogue*, an essential instrument in introducing the science content.¹²

As far as the development of the primary science curriculum is concerned, four different Programmes have been implemented by the Project. They are:

- (a) Drought, Water and Diseases (1983 until now)
- (b) Agriculture (1984 until now)

¹² It is not easy to state clearly in few words the full meaning and implications of such a perspective. I hope that the information reported on the next sections of this chapter, added to what follows in chapter 2, can give a sense of what is compactly expressed in this paragraph.

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- (c) Human Habitat (1985 until now)
- (d) Seismology and Earthquakes (1987 until now)

A fifth Programme -(e) Mining- is now in the analytic stage.

Programmes (a), (b) and (d) have been proposed for the 3rd and 4th years of the elementary school in a rural community, which would correspond to 9 and 10 years-old children if the actual situation were to express what is established in law. As will be shown later in this appendix (section 4) this is not the case (the median age is around 13 years-old, but has a large spread, from 9 to 23 years-old). Programme (e) is also addressed to a similar situation.

Programme (c) has been proposed for the first four years of the elementary school, for 7 to 10 years-old children, in an urban area. In this case, the age variation between the formal/legal and actual situations is not so big.

Studies within the ECPC-Project have concerned *common-sense knowledge*. Thus, the urban Programme has been concerned with an understanding of how structures of knowledge, from a psychological perspective, evolve with age. At present, in connection with *Human Habitat*, researchers investigate how the idea of *Life* changes, with 7-10 years-old children.

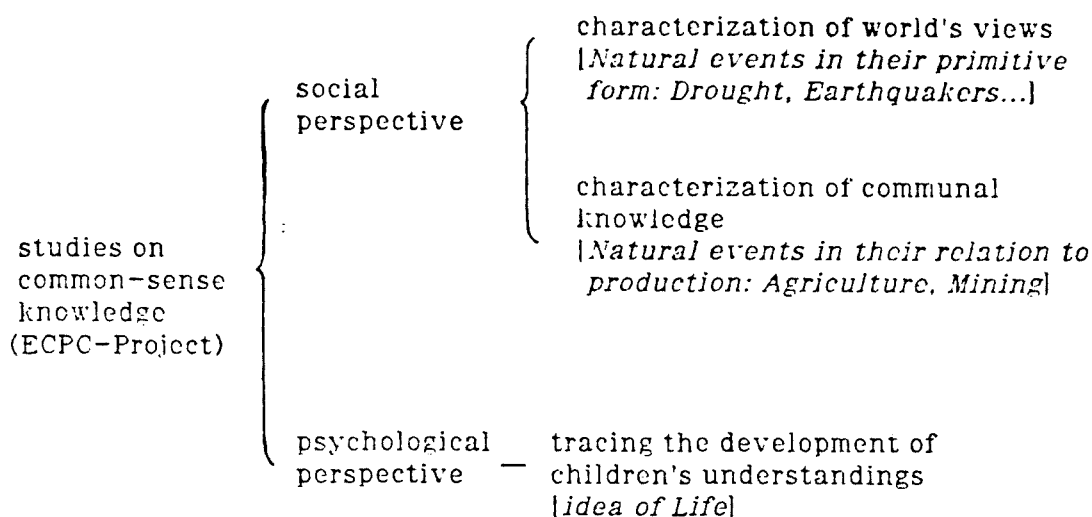
Studies related to rural Programmes emphasise common-sense knowledge from the perspective of its social construction. *Drought, Water and Diseases* and *Seismology and Earthquakes*, have themes which refer to events which affect in an important way life in the community but which are in a very clear way out of man's control. They are highly dependent on '*something else*' (God's wishes, Nature, ...) whose nature is not immediately understood by people.

To some extent, this is also true for *Agriculture* and *Mining*. But in these cases the themes are more concerned with events related to production, and which in principle can have their importance and existence 'determined' by men.¹³

¹³ Actually, it is in relation to the falling of cotton production in the Region of study (production which could be improved by means of new technology and redistribution of land), that *Mining* arises as an alternative activity for peasants. It is in an analogous sense that *Mining* becomes a complementary theme to *Agriculture*.

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In the former case the focus is on the characterization of people's *views of the world* about given phenomena, and concerns *models of explanation*, in the latter the emphasis is on *communal knowledge* in the sense of knowledge given by reference to historical relations of production.



It constitutes part of the Project's policy to involve primary teachers in all stages of curriculum development. Teachers implementing rural Programmes are mostly unqualified (which means that some of them have not yet completed a secondary qualification in Primary Teaching). The Project concentrates effort on their training as teachers, and issues of research are more concerned with teaching and curriculum development. On the other hand, teachers involved with the urban Programme have already, or are trying to get, a university degree, and issues of research are usually set at a more speculative level, when compared to those in the rural area.

It is important to stress, however, that the distinctions mentioned above try to safeguard both a respect for the actual conditions/possibilities of work and the intentions/aims of the researches. At the level of educational practice all these concerns are integrated and play a part.

3. ON FREIRE'S IDEAS

3.1 A summary of the main concepts in Freire's Pedagogy.

Paulo Freire is a philosopher of education whose thoughts and work have had, in Brazil, a profound impact not only in the field of education but also in the overall

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struggle for national development. His method for teaching illiterates in the North East Region –developed in the early sixties– was considered of great efficiency. Furthermore, it advocated that adults, in learning to write and read by such a method, come to a new awareness of selfhood and begin to look critically at the social situation in which they find themselves. In becoming aware of their reality, people often take the initiative in acting to transform the society that has denied them this opportunity of participation. In this sense, education is basically seen as a process intended to prepare the student to *participate*, not only within the immediate social environment represented by the school/community, but also in more general social changes. In other words, embedded in Freire's conceptualization there is the idea of an education which gives an instrument for the Brazilian people to participate in the historical challenges of a society in transition. As such, education emerges as a subversive force¹⁴.

Freire's work challenges the dominant narrative view of education, declaring that educational action is not an act of depositing words, in which the students are depositories and the teacher is the depositor. Rather, that liberating education consists in acts of cognition. He provides two useful insights for researchers and teachers which are synthesized in the concepts of *dialogue* and *problematizing*.

Dialogue is a process of communication which Freire counterposes to cultural invasion. Southgate and Randall summarize very clearly the contrast:

"Cultural invasion is the imposition of values, belief systems, ideology, cultural norms and practices of an imperialist culture on those it has colonized and oppressed. Its basis is an unequal relationship. Its object is social, economic and political control. The opposite process, dialogue, is based on equality in relationship (which has to have a real, material base), mutual respect, and understanding. Freire's concepts of dialogue and invasion are as applicable at the individual and small-group level as at the level of institutions and societies."

(Southgate and Randall, 1981, p. 53)

At the level of schooling one major aspect addressed by Freire is that the dialogical character of education does not begin when the teacher meets the students in a pedagogical situation, but rather when the teacher asks himself *what the dialogue with the students will be about*. The specific activity in which this question is resolved is of central importance in Freire's theory. It is known as

¹⁴ For a deeper understanding of these ideas and of the concepts which follow, see Freire, P. (1972); Southgate, J. and Randall, R. (1981); Tandon, R. (1981); Randall, R. and Southgate, J. (1981).

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thematic investigation and poses a radical perspective for planning the curriculum when compared to traditional ones. In such a perspective, the programme content is not derived by simply unfolding scientific knowledge in its logical component topics. As expressed by Freire:

"It is to the reality which mediates men, and to the perception of that reality held by educators and people, that we must go to find the programme content of education."

(Freire, 1972, p. 69)

For Freire, the object of the investigation of *meaningful themes* is not entities which constitute reality. Rather, it is the *thought-language* that men use to refer to reality. That is, the levels at which men perceive that reality and which represent their *view of the world*. In this respect, his understanding of "men situated in the world" evokes Wittgenstein's words that the world is composed of facts, not of things¹⁵.

The *investigation of meaningful themes*—the complex of their generative themes—is what, for Freire, inaugurates the dialogue of education. The methodology of that investigation must be also *dialogical*, providing the opportunity both to discover generative themes and to stimulate people's awareness in regard to these themes. To investigate the generative themes is then, to investigate man's *thinking about reality* and man's *action upon reality*, which is his *praxis*.

Since such an investigation is to serve as a basis for developing an educational programme in which teachers and students combine their *cognitions of the same object*, the search for knowledge—itself—must be based on reciprocity of action. As expressed by Freire:

"Thematic investigation, which occurs in the realm of the human, cannot be reduced to a mechanical act. As a process of search, of knowledge, and thus of creation, it requires the investigators to discover the interpretation of problems, in the linking of the meaningful themes. The investigation will be most educational when it is most critical, and most critical when it avoids the narrow outlines of partial or 'focalized' views of reality, and sticks to the comprehension of total reality. Thus, the process of searching for the meaningful thematics should include a concern for the links between themes, a concern to pose these themes as problems, and a concern for their historical-cultural context."

(Freire, 1972, p. 80)

¹⁵ "The world is the totality of facts, not of things." Wittgenstein, L. (1961) *Tractatus Logico-Philosophicus* London: Routledge & Kegan Paul, p. 5.

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Problematizing becomes then one central concept in Freire's theory. By making problematic both *reality* and *people's perception of reality*, themes can be unfolded so as to constitute *problems to be researched* and not bits of information to be transmitted. It is in this sense that his conception of education is called *problem-posing education*.

More fundamentally, these themes are to contain in them the possibility of changing people's consciousness. About this Freire says:

"In the event, however, that men perceive reality as dense, impenetrable, and enveloping, it is indispensable to proceed with the investigation by means of abstraction. This method does not involve reducing the concrete to the abstract (which would negate its dialectical nature), but rather maintaining both elements as opposites which interrelate dialectically in the act of reflection."

(Freire, 1972, p. 77)

In Freire's theory, this movement of thought can be seen exemplified in the analysis of a concrete, existential 'coded' situation. The *coding* of an existential situation is the *representation* of that situation, showing some of its constituent elements in interaction. *Coding* works towards an abstraction but requires a permanent movement from the abstract to the concrete. *Decoding* is the critical analysis of the coded situation. It is mainly by reference to the concepts of codification and decodification that Freire's ideas point to a particular methodology for teaching in the classroom. At the level of implementing actual practices, these concepts can be considered in Freire's work as sufficiently well formulated so as to constitute a good methodological ground for action. Several educational experiences exist from which his ideas can be seen in operation with success¹⁶.

¹⁶ In addition to the original educational experiences which took place in Brazil up to 1964 and from which the *method Paulo Freire* has emerged, more recent practices in which Freire's ideas play an important part can be seen described in: Wertheim, J. e Bordenave, J. D. (ed.) (1981) *Educação Rural no Terceiro Mundo: Experiências e Novas Alternativas* Rio de Janeiro: Paz e Terra; Brandão, C. R. (org.) (1981) *Pesquisa Participante* S. Paulo: Brasiliense; Brandão, C. R. (org.) (1984) *Repensando a Pesquisa Participante* S. Paulo: Brasiliense; Duarte, N. (1986) *O Ensino de Matemática na Educação de Adultos* S. Paulo: Cortez; Autores Associados; Oliveira, B. (1981) *Relatório das atividades do projeto de alfabetização de funcionários da UFSCar - junho '80 a julho '81* S. Carlos: UFSCar (off-set); Pernambuco, M. (1981) *Ensino de Ciências a partir de Problemas da Comunidade* Dissertação de Mestrado. S. Paulo: IFUSP/FEUSP; Angotti, J. A. (1982) *Solução Alternativa para a Formação de Professores de Ciências* Dissertação de Mestrado. S. Paulo: IFUSP/FEUSP; Delizoicov, D. (1982) *Concepção Problematizadora para o Ensino de Ciências na Educação Formal* Dissertação de Mestrado. S. Paulo: IFUSP/FEUSP.

3.2 Problematic issues concerning Freire's Pedagogy and knowledge.

There are two aspects which deserve attention. The first asks about possible perspectives for looking at science within a problem-posing kind of education. The second raises questions about the nature of people's representations within the same approach.

Perspectives for looking at science within a problem-posing education.

Freire's method -which was originally conceived for illiterate adults in a non-regular kind of schooling- presupposes content being unfolded by reference to *meaningful themes*. It is the thematic investigation that indicates the '*necessary content*'. Science, in this perspective, becomes *knowledge* to be integrated with other kinds of knowledge such as language, mathematics, history, geography and so on. Also, in this perspective, the movement concrete/abstract becomes a question to be solved at the methodological level, and this is one perspective in which the concepts of **codification** and **decodification** apply.

But as far as these two concepts are concerned, there is another level at which this dialectical movement of thought between concrete and abstract can be located for analysis. It refers to the level of discussion in which *science*, as knowledge, can be seen as a *codified representation of reality*. This discussion is previous to any thematic investigation and relates to the kind of understanding one can construct about the *nature* of science.

In Freire's work, it is possible to recognize an effort to make explicit both a conceptualization of society and a conceptualization of education. As an important component of his arguments there is always present a philosophical understanding of man in his relation with the world. In connection with his philosophy one can see expressed a given way of *understanding science*. But Freire's theory does not contain an *explicit formulation* on which to base an analysis of scientific knowledge itself. This, he leaves open to further investigations, and it constitutes a challenge to be taken up by specialists in the particular subject matter.

The point I am trying to make is twofold. *First*, it says that Freire's methodology presupposes a *certain* 'epistemological view' of science; and so, the question of '*what specific role does science play in a problem-posing education*' is to be answered in two respects: one methodological and one epistemological.

This is important because if one does not approach his methodology with a compatible understanding of science, one will face difficulties in making sense of the methodology itself; and in having clear what constitute possibilities and limitations to any actual implementation of the method.

For example, those who try to apply Freire's method within the constraints of teaching a specific subject matter in the regular system of education, are usually faced with this question. It is frequently suggested that in trying to teach a specific subject matter by Freire's method we have to face a restriction—in nature and scope—in the setting of themes to be developed. As a science teacher, for example, I will be concerned with questions which can be answered by science and not by history, or by geography or by theories about language¹⁷. If one approaches science with glasses provided by Freire, one gets a twofold result: a more adequate treatment of the scientific themes *and* a more effective and concrete integration of these themes with those of other areas; for what is at issue in the latter case is a problem *about* knowledge which *is* science, and not of *the* specific knowledge which *constitutes* science.

Second, if an understanding of science is to be developed at some length, hints can be found in Freire's own thought. This would certainly imply going back to certain original works which informed Freire's position: in particular to K. Kosik's *Dialectic of the Concrete* (1976), which seems to have played an important role. The ECPC-Project has made an attempt to develop such an understanding but this effort has been directed mainly to the planning and implementation of the science curriculum.¹⁸

Questioning about the nature of people's representations.

A second point in Freire's work remains open and refers to the nature of consciousness in the understanding of reality. Freire's idea about this issue makes

¹⁷ For some researchers and teachers, this sometimes becomes a source of disbelief in the method. In my view, those who think so do not trust, actually, in the knowledge (science) they have to transmit. Thus, the disagreement should not be located in methodological grounds.

¹⁸ From the perspective of the philosophy of science, an attempt to grasp characteristics of Freire's understanding about science can be found in Delizoicov, D. (1987) *Thomas Kuhn e o Processo de Codificação-Problematização-Descodificação* S. Paulo: IFUSP (mimeog.)

reference to Lucien Goldman's notions of 'real consciousness' (*consciência real*) and 'possible maximum consciousness' (*consciência máxima possível*). Goldman elaborates these notions from a marxist point of view, which means that what is in focus is *consciousness of class*¹⁹. The problem of the consciousness of class, when understood from a marxist perspective, is always related to a notion of *class*; and so, to the *representations* that people can have of totality not as individuals, but as *members of a given class*. But Freire does not establish explicitly such a parameter. Some commentators²⁰ tend to interpret this missing remark in terms of a reductionism of the levels of consciousness to an individual perception, not a social one.

To prove that this is not the case would take pages of argumentation in favour of the social approach. This is not my intention. It is worth mentioning, first because it relates to the way in which the ECPC-Project comes to articulate its proposal for planning and implementing the science curriculum from an analysis of contradictions at the level of relations of production (in which the category of *class* fits smoothly with an *economic* analysis) and using a Freirean framework in which the category of class does not seem to be so crucial. Secondly, because to

¹⁹ It seems worthwhile to quote the glosses (offered by W. Q. Boelhower, publisher) of three terms used by Goldman in *Luckács and Heidegger: Towards a New Philosophy* (1977) London: Routledge & Kegan Paul. They are:

Possible Consciousness

"The maximum adequation to reality possible by the collective consciousness of a class (keeping in mind it might never realize it) without it being led to abandon its significant structuration. It is the field, calculated by the researcher, within which the possible responses of a class can vary without there being an essential modification of its collective consciousness in its orientation toward a global structuration of society. This is a critical category of *History and Class Consciousness* and is linked with the categories real consciousness and objective possibility. Luckács uses it to explain the relation between the individual subject on the level of social class and the limits of his social praxis."

Real Consciousness

"The term given by Luckács to the rich and multiple content, the immediate empirical state, of the individual consciousness making up a class or group which is more or less coherent in its tendencies, depending upon the historical self-awareness of the individuals and the conditions for this self-awareness. It is the complement of possible consciousness."

Objective Possibility

"The external situation of a class which limits its field of possibility with regard to thought and action. The mental structures of a class also circumscribe its theoretico-practical field of possibility. The objective possibility of a class determines its possible consciousness and inversely, according to Luckács. The two are inseparable."

²⁰ For example, Manfredi, S. (1978) *Política: Educação Popular* S. Paulo: Editora Símbolo.

recognize the adequacy of the social perspective does not imply a denial of the need for and relevance of studies concerned with individual perception.

4. SCHOOL, WORK AND COMMUNITY

The research on *common-sense knowledge* on which this work is based arose from an interest in *communal knowledge* and a commitment to the particular kind of investigation developed by the ECPC-Project: *the organic approach to problematizing-teaching*. Applied to communal knowledge, this approach necessarily requires the focus to be on knowledge which is somehow common to a group of people whose material life is cast in historical relations of work.

By the time I started this study, the Project was implementing the science Programme of *Agriculture* in a rural community 80 km from Natal (the capital city of the State of Rio Grande do Norte) called São Paulo do Potengi (SPP), which functions as a nucleus for economic development in the micro-Region called *Agreste Potiguar*²¹. It was appropriate then to circumscribe the empirical field of the research to this situation. That is, to take S. Paulo do Potengi as *community* and the actual implementation of the science curriculum of *Agriculture* for the 3rd and 4th years of schooling as the experimental educational reality for contrasting *communal-practical knowledge* to *school-scientific knowledge*.

Community was then narrowed to refer to the 14 thousand citizens of S. Paulo do Potengi who live basically from agriculture and cattle-breeding. The bulk of the population is composed of peasant farm-workers who cultivate the land for *subsistence* (home consumption and surplus production of food). In the same way, the actual development of the Project's curriculum would constitute an empirical scenario for discussing issues in the relation between *communal knowledge* and *school knowledge*. And this would mean considering as protagonists the actual group of teachers and students of the 3rd and 4th years involved in the Project. It is important to make clear, however, that to delimit an empirical situation as above does not necessarily imply setting the discussion of communal knowledge only at a concrete level. As the nucleus of production of the *Agreste Potiguar*, the

²¹ *Agreste Potiguar* is one of the 10 microregions in which the State of Rio Grande do Norte is divided. It embraces 21 *municípios*, S. Paulo do Potengi being the largest and most important economically. For a more complete description of the Region see Brasil. IBGE/SUDENE (1973) *Região Programa do Agreste Potiguar* Rio de Janeiro: FIEGE.

agricultural community of SPP is part of a nation-wide economic movement of development which both contrasts and articulates with the *industrial* world. Also, in national terms, the educational system of SPP can be regarded as typically *rural* in contrast with the *urban* system of education.

From the variety of ways one could look at the *agricultural/rural* situation, there are two sorts of aspects which have relevant implications for this research. They can be seen in a discussion of the term '*community as a social entity*', and in an analysis of the relationships between *schooling and labour*.

4.1 Community as a social entity.

When the ECPC-Project tries to understand the socio-economic reality of a community such as SPP, the characterization of the variety of social categories and the nature of the relationships between them become one central issue for analysis. In SPP, social categories are represented mainly by *small producers* (*pequenos produtores*). As a social class, small producers constitute a broad economic category which can be split into specific sub-categories distinguishable in terms of forms of payment or forms of land possession. Social categories are then seen from the perspective of social classes proper to a peasant mode of production.

For the ECPC-Project this is how the analysis -at a more theoretical level- starts. But to approach the level of the actual subjects of the community, the Project has to go far beyond the abstract category of *production* to include specific and historical aspects of the processes of *production, circulation and consumption* as manifest in SPP. In such an attempt, a theoretical perspective is required to account for the social and economical contradictions of the capitalist development in Brazil, in particular for those social formations which are not typically capitalist, and which -in a superficial approach- are seen as survivals of earlier modes of production (such as black slavery, '*peonagem*', peasant production, forms of ground-rent in kind or labour, etc.).

For the ECPC-Project, *community* means more than a category of class. The Project regards it as designating, also, a collective victim of oppression which results from the capitalist development. In this respect, the Project's view squares with the

understanding proclaimed by Martins²² when discussing the new social actors of the rights of all people (*novos sujeitos dos direitos dos povos*) which he calls *community*.

4.2 Schooling and labour.

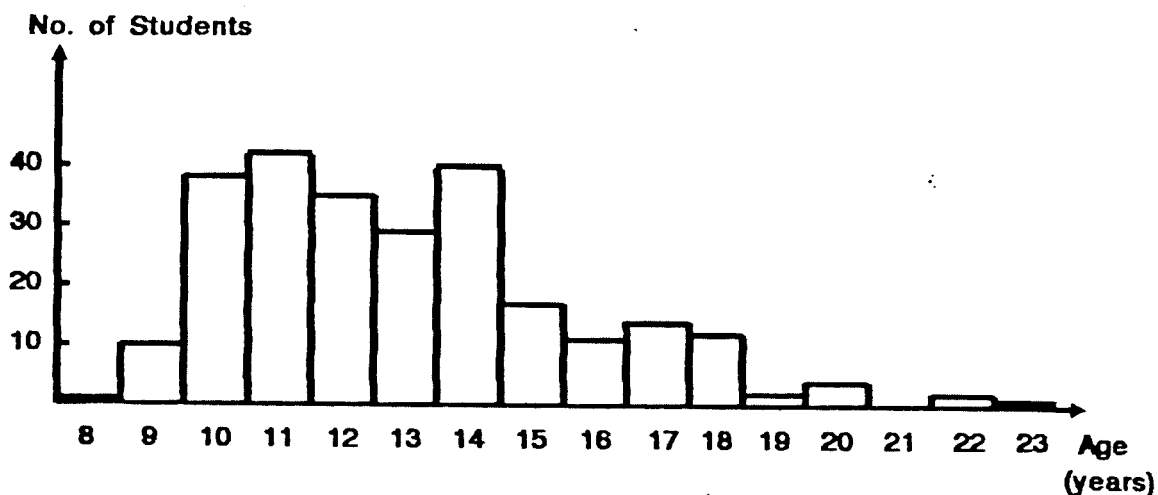
Two peculiarities of the educational context in which the programme of *Agriculture* is realized in SPP can be seen expressed in Figure 2. First, in this Figure, an age-histogram is shown for children from the 3rd and 4th years of schooling, without distinction between them. The graph is intended to represent the situation in which teaching actually occurs. That is, a given teacher has to run programmes²³ for two different class-groups, in the same classroom and at the same time.

In this situation, as far as the study of language and mathematics is concerned, teachers try to coordinate efforts to run two classes in parallel. But a more flexible treatment is given in cases such as science or social studies, as the usual programmes do not contain in themselves, teachers usually suggest, strong reasons which can account for the need for a logical/sequential organization of syllabuses, which will then mark out a clear distinction between classes.

So, as far as science classes are concerned, when one asks teachers about a pupils' characteristic such as age, they will not usually respond by distinguishing the 3rd from the 4th year of schooling.

²² Martins, J. de S. (1984) "Os novos sujeitos das lutas sociais, dos direitos e da política no Brasil rural", in: *A Militarização da Questão Agrária no Brasil (Terra e poder: o problema da terra na crise política)* Petrópolis: Vozes (my translation of the above quotation to English).

²³ During the first four years of primary schooling in Brazil, one teacher is responsible for all subject matter. Usually, he/she organizes the timetable so as to cover four traditional areas of knowledge: language, mathematics, science and social studies. Religion, arts and physical education complement the activities at a second level of relevance. The responsibility for deciding the content of these programmes belongs to the Secretary of Education of the State, who, following suggestions made by the Ministry of Education for the whole country, establishes a common core of content. The responsibility for defining and implementing the actual programmes pertains to the maintenance institutions which support the primary schools (the Secretary of Education of the State, or the Secretary of Education of the *Município*, or private institutions), but their programmes should be approved by the State. As far as the 3rd and 4th years-science-programme is concerned, water, diseases, soil, and plants are topics of the core content.



Distribution of 283 students from the 3rd and 4th grades of primary school (SPP, 1984), per age.

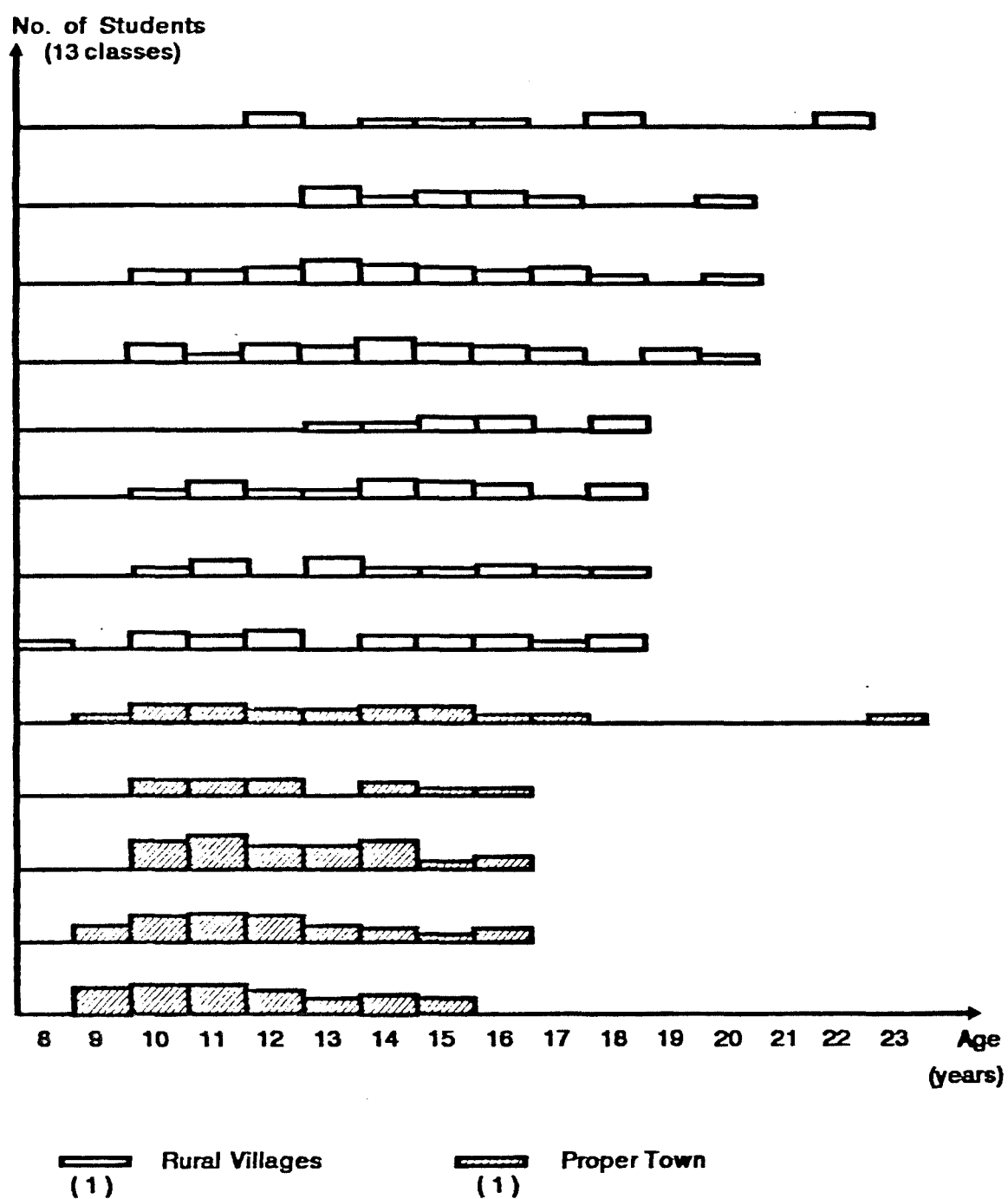
Figure 2

The second peculiarity has to do with the actual age distribution of 'children' who attend these two years of schooling. As shown in Figure 2, the difference between the actual median age and the expected ages corresponding to the 3rd and 4th years (9 and 10 years-old) is substantial. The problematic is augmented when one realizes that this kind of distribution resembles exactly what is going on in each single classroom. Figure 3 shows the age distribution for the 13 classrooms from which data presented in Figure 2 were derived.

Many studies²⁴ suggest that the main reason accounting for this situation is the relation between schooling and labour which is imposed on children in the rural world. The period from 8 to 10 years-old is not only a time of introduction to schooling, but to productive work as well. As can be seen in Figure 3, the effect is more marked in rural villages, compared with the town.

²⁴ The literature is vast in this respect. References can be found in Cadernos CEDES nº 11 (1984) *Educação: a encruzilhada do ensino rural* S. Paulo: Cortez Editora; and Grzybowski, C. et al. (1981) *Meio Rural e Educação: Notas de seminários* Rio de Janeiro: Achiamé/ANPed/CNPq.

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Distribution of students from the 3rd and 4th grades of primary school (SPP, 1984), in 13 classes, per age.

Figure 3

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These facts, however, are not just isolated peculiarities of the education of rural children²⁵. They are also a result of an imposition of existential conditions and representations which make up a 'rural' way of living. From such a perspective, *schooling* is understood as *equivalent to labour*²⁶. *Equivalent* means that, from the perspective of farm-people, both *schooling* and *labour* have comparable values.

This situation contrasts in an important way with urban education. As is generally accepted, socialization and learning are functions which, in the rural area, belong to the unit of work-production-consumption, basically represented by the family. In urban areas, these functions are taken on by the school without any relation to work. If any relation exists it lies a long way ahead and looks forward to professional careers: those in which no manual skills are necessary and which will be attained by a small and selected group of citizens.

In the *urban school*, to follow the higher stages of schooling beyond primary education or to enter the working market, are alternatives which set for children the social limits of their future biographies. The meaning and value of what they have learned (or not learned) can be seen as a function of their class condition (which defines the boundaries and relevance of what was taught), and schooling can be seen as an institutional means to the realization of ends which are ideologically dominant in society. The teachers -and their discourse- are then seen as mediators within this process.

In the *rural world*, however, the relation between schooling and labour can not be understood strictly in this way. As stressed by Martins, work has a social value which supports the kind of valuation a rural population attributes to schooling. Work and schooling are *equivalent*. Independently of the fundamental features which distinguish (or originate) the various social categories in the rural context, schooling is seen as work in itself; and in this sense, the *relation* between schooling and labour is suggested, by Martins, to assume a general character in the peasant's life: independent of who you are, or who you will be, from the point of view of class.

²⁵ For example, an analogous situation could be described for peripheral zones of urban areas.

²⁶ This notion of *equivalence* between *schooling* and *work* in the rural world is proposed and discussed by Martins, J. de S. (1981) "A valorização da escola e do trabalho no mundo rural", in Wertheim, J. & Bordenave, J. *Educação Rural no Terceiro Mundo* Rio de Janeiro: Paz e Terra.

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But to the extent to which the rural school should be seen as articulated to an urban way of living, it is appropriate to say that there is a relation between the kind of valuation attributed to schooling in a given historical situation, and the degree of development of productive labour, taken from a spectrum marked by two extreme stages: one conforming to an economy of subsistence and the other to a market economy.

For the ECPC-Project, these reflections become relevant in two ways: first, they point to the necessity to clarify how the community values schooling, having regard to the present status of the peasant mode of production. The Project suggests for SPP a stage of evident conflict between the logic of reproduction of farm-workers as small producers and the logic of the market economy, which can be transposed to the level of school valuation in terms of two distinct expectations. Linked to the old traditions and values, schooling would be accepted and valued because of being equivalent to an accustomed form of hard work. To read and write modestly and to perform simple calculations are attainments still to be achieved by many adults in this community. Parents try to keep their children in school for this purpose, no matter what their successive failures. From a modern perspective, however, the equivalence to labour would presume a training in certain abstractions which are fundamental within an urban society (of discourse, laws, rules, etc.). Teenagers would see the school as a place for acquiring habits consonant with dominant representations which belong to the relevant society. (ECPC/UFRN/CAPES/PADCT Report, 1985).

Secondly, the reflections stress once more the crucial role played by teachers. That is, in rural areas, teachers are protagonists in a deep conflict which is situated at the level of socialization: their role is to act as if they were in an urban scenario - whose function is to guarantee the socialization of children- but by being actually in a situation where children are socialized through work in agriculture.

In other words, work in urban schools is thought of as an object of study and not as a practical activity. By trying to bring this perspective to the rural world, the conflict of cultures and perspectives is inevitable. It is not difficult to foresee this conflict transposed to the level of science teaching: for example, while books and teachers talk about how to classify soil by their constituents and properties, children learn how to do it by their functional character for planting. Or again, while the *rationale of controlling variables* added to the *rationale of productivity for profits* tell them that a monoculture should be developed, their lives say: -

"use a variety of crops".

5. AGRICULTURE AS A THEME

Although, initially, the Project appears to be concerned with the problem of teaching traditional issues in Agriculture such as soil, plants, and animals, this problem is embedded in and stimulated by a wider question of the relationships between knowledge and social practices in four ways.

(1) The dominance of (a) major themes and questions from the local, empirical problem of social organization of work on the land, and of production; and of (b) questions which evoke the use of resources for thinking proper to the scientists' attempt to construct 'rationality' (processes and content).

Thus, in addition to topics such as:

- . *soil* (kinds; properties; erosion; transformations),
- . *plants* (parts; water flow; photosynthesis; developmental phases; growing),
- . *fertilizer* (composition; effects),
- . *pests* (classification; cycle of life; extermination),

the programme approaches themes related to:

- . *agriculture in community* (kinds of crops; annual cycles of production; cultivation of soil; relations between kinds of crops and types of soil),
- . *phases of work* (tools; what is done; how, why and when it is done),
- . *production* (volume; cost price; time of work; loan; interest; tax; wage; surplus-value; profit),
- . *trade and market* (storage; drainage; consumption; mediators; external relations to community),
- . *subsistence* (food; energy 'recovery'; habits; commodities; changes).

Also, questions are posed for which scientific ways of reasoning can help to construct an answer, such as:

- . "What does it mean to regard agriculture as the main economic occupation of people in this community; and to regard cotton as the basic economic product?" [interpretation of tables and graphs of percentage]
- . "What kinds of change do the actual practice of transforming the soil for planting imply? What are the consequences?" [models for soil; explanations for consequences of actions]
- . "Why certain kinds of crops are planted in particular types of soil?" [models for soil; mechanisms of growing plants]
- . "Why do people say that it is necessary to give up production of cotton for three years in order to exterminate the "bicudo"²⁷?" [behaviour of systems of reproduction; function of cycles of life]

²⁷ "Bicudo" (scientific name: *Anthonomus grandis Boheman*) is a kind of beetle.

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- . "How can we know how much profit a small owner will lose in a year of drought?" [scientific procedure for dealing with data; explanations in terms of conservation; control over nature]
- . "Why should monoculture replace crop varieties?" [rationality of work; productivity]

(2) An attempt is made to integrate resources for thinking from science (which will be introduced by the teacher) with resources for thinking from everyday life (which constitute a context of reference for pupils' thinking).

This integration is attempted by a three-stage pedagogic strategy: (a) study of reality; (b) scientific study; and (c) application of knowledge. What these stages try to guarantee is an approach to knowledge in which: *first*, the elements of the reality of everyday life are set out in terms of both questions/themes from a previous analysis ('redução temática') and pupils' understanding of the issues involved in these questions; *second*, the resources for thinking and making sense of things and events – *from science* – are introduced; and *third*, an integration of both elements of everyday life and of scientific resources is proposed in terms of an attempt to solve problems of application.

(3) An attempt is made to develop both the generality of knowledge at an abstract level and specificity at the contextual level. One example would be the attempt to set out questions related to problems and practices in the community in such a way as to bring out those traditional topics which will need to be discussed at an abstract level if an understanding in science is intended, but which also will have to be made functional for an understanding of problems and practices. For example:

- . "What is 'massapê' made of?" [composition and properties of 'massapê'] [soil composition and properties]
- . "Why is manioc always planted in 'arisco'?" [composition and properties of 'arisco' + mechanisms of water flow in the manioc] [soil composition and properties + mechanisms of water flow in plants]
- . "Why traditional practices of exterminating pests do not work for the 'bicudo'?" [cycle of life of the 'bicudo'] [cycle of life of pests]
- . "If 'bicudos' feed only on cotton, why do we find them always close to the manioc roots?" [relevance of temperature in the 'bicudo' ecosystem] [relationship between environment and life cycle].
- . "Why does one plant cotton, corn and beans just after the 'first rain'?" [role of water in germination] [role of water in the germination of those 3 crops]

At the same time at which they bring about explanations which require a scientific account of how things are, happen or function, these questions also evoke a kind of discussion in which contextual knowledge plays a part.

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(4) Entailed in (2) and (3) are attempts to clarify the logical and empirical status of the *unifying concepts* ('conceitos unificadores') in terms of which the Programme is structured. In planning a sequence of topics for teaching, *unifying concepts* function as the most general criteria for choosing what can be interpretable within science and which will make up the Programme in accord with the theme previously defined. They constitute pedagogic entities which point to the appropriate *logical* structure of the content to be tackled by the Programme.

In the case of *Agriculture*, the concept of "*processes of transformation*" has been taken as the basic *unifying concept*, and the following set of transformations has informed the planning of the actual programme.

Organic Material (humus from the decay of dead bodies of organisms)	=====>	Minerals (soil)
Minerals (soil)	=====>	Organic Material (plants)
Organic Material (food. production)	=====>	Commodities (obtained via trade. or direct exchange with crops)

The Programme, despite taking on issues related to the whole set of transformations, is mainly organized around the sequence:

Minerals	=====>	Organic Material	=====>	Organic Material
(soil)		(plants)		(food. production)

as it is this sequence which bears an analogy with the cycle of production (see chapter 2) in relation to which the actual work in agriculture is organized.

The Programme follows the sequence:

Agriculture in S. Paulo do Potengi ==> Soil ==>
 ==> Cultivation of soil ==> Plants ==>
 ==> Growing plants ==> Harvesting ==> Production

Thus, as soon as a set of transformations is shown to be essential for reason of the nature of the theme to be unfolded, a sequence of sub-themes is organized so as to compose the basic programme of issues.

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As a *unifying concept*, *processes of transformation* has been used in relation to other themes such as *Drought*, *Water and Diseases* and *Home Habitat*, and has shown to be, potentially, a powerful pedagogic entity (Pernambuco *et al.*, 1987a). Other concepts have been also investigated by the Project. In relation to this latter theme, for example, a group of four *unifying concepts* is used:

Processes of Transformation of raw material and artifacts, as well as living things, in space and time.

Cycles and Regularities of matter in transformation in space and time, at natural and artificial grounds (introduction of the notion of conservation).

Energy as an agent of transformations, a thing which allows one to differentiate objects in space, to distinguish *before* and *after*.

Scales to capture events of familiar size and extension, looking forward to a possible extrapolation to the *micro* and to the *macrocosmos*, and to both the instantaneous and the faraway (back in the past or forward to the future).

For the Project, the extent to which all these concepts would apply to Agriculture has become one issue for investigation: but at the time I started this study, only the first was formally used.

6. SUMMARY

This appendix has considered some background elements, pertinent to the conceptualization of the present research. The work developed by the ECPC-Project in Brazil has been taken as the main source of analysis, emphasis being placed on the development of a science curriculum for rural areas. More concretely, the implementation of a science programme based on *Agriculture* in a community of the North East Region will be considered as a context for discussing issues related to knowledge and schooling. In particular I have indicated the importance of studies in common-sense knowledge for the Project, and have suggested some reasons why a *social* perspective would be appropriate in dictating the kind of framework for discussing both knowledge and schooling.

I have also indicated my interest in the investigation of *communal knowledge*, taking as an empirical context the community of peasants of S. Paulo do Potengi, where the Project implements the science programme. In trying to characterize communal knowledge, themes related to *Agriculture* will be taken as case studies in knowledge relevant to science and schooling. They are *cubação* (a method of measuring areas of land) and the cultivation of *soil* for planting.

APPENDIX 2.A

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(3) INFORMATION FROM THE INTERVIEWS

APPENDIX 2.B

THE OCCUPATION OF THE REGION OF S. PAULO DO POTENGI

The conquest of the 'Agreste' started very slowly at the end of the XVI Century. At that time, the 'Zona da Mata' was almost already settled by the Portuguese and sugar production experienced its initial expansion. But it was only after the settlement of the 'Sertão', that the 'Agreste' flourished as a potentially developing region.

Thus, if we look briefly at the 'Zona da Mata' and the 'Sertão's' background we can see where some of the 'Agreste's' characteristics come from.

1. Production and labour in the early colonial period: 'Zona da Mata' and 'Sertão'.

The 'Zona da Mata' was from the beginning a very suitable region for growing sugar cane, and the Portuguese settled large sugar plantations in the area surrounding Olinda, Recife and Salvador. In addition to sugar-cane, but on a smaller scale, other kinds of crop were also cultivated on the seacoast: cotton, manioc, corn, different kinds of beans, vegetables, fruits, rice and coconut.

Sugar mills were concentrated in the hands of large rich families ('grandes senhores') and constituted whole economic units. These families worked the land partially by using the indigenous Indian population; but mainly by using imported African slave labour. But they were not responsible for cultivating all their land: several parcels of field were rented by small farmers (free citizens) who were obliged to use the 'grandes senhores's' mills for transforming the cane into sugar.

Since the beginning, cattle-breeding was also an important economic activity supporting sugar production in the 'Zona da Mata'. A great number of oxen and horses were required: the sugar mills were usually kept in motion by animal traction; also, the transport of raw cane from the fields to the factory, and the transport of sugar from the factory to the sea port, were performed by bullock cart.

Horses and cattle were, however, never raised near the sugar mills. Without wire to fence their fields, the owners were accustomed to breed their cattle far away from Salvador, Recife and Olinda (the main centres). Also, the small independent cattle-raisers had to find their places in distant areas.

When the region was invaded by the Dutch (from 1635 to 1654), this contingent of cattle-raisers were compelled to move more and more to the interior areas, killing or driving away most of the Indian tribes which lived in the territory. Contrasting with the narrow and complex 'Agreste', the extensive areas of the 'Sertão' offered a safe and salubrious ground for cattle-breeding. Thus, at the same time as that in which the Indians were expelled, the re-settlement of the region took place. Cattle farms spread abroad through the whole 'Sertão', expanding in parallel to sugar production. Big farms were kept in the hands of the 'grandes senhores' who also possessed control over sugar production.

The 'vaqueiro' was the main protagonist of this process of settlement. He was responsible for managing the big 'fazendas', as the owners usually lived in the 'Zona da Mata' and rarely came to the 'Sertão'. He could be a single employee of the land owner or he could rent tracts for his own use. Whatever the case, he received one out of each four head of cattle born on the farm. His life was not easy and he was involved in several tasks: breeding, keeping and domesticating the cattle; driving the cattle on long journeys to the seacoast market; constructing and maintaining the infra-structure for both his family and for the cattle (especially places for storing water); and producing the basic foods for subsistence.

On a larger farm, there was a team of 'vaqueiros', each taking on a specific task. In other cases, in addition to the 'vaqueiros', there were helpers who were responsible for auxiliary tasks, and also for holding the farm in the 'vaqueiro's' absence. All these people received a place for living and a small tract for planting crops for subsistence. In some situations, they were requested to pay for these facilities by giving part of their production to the land owner or to the 'vaqueiro'. Being dwellers in the farms, these people were called 'moradores' (living-in people, a category of free-citizens which already existed in the 'Zona da Mata'), who contrasted in the 'Sertão' with the 'vaqueiros', for these always worked outside their living place.

After the expulsion of the Dutch, Brazilian sugar production still experienced variations of prosperity, particularly in the second half of the XVIII Century,

when the European colonizing countries involved themselves in war and the sugar trade with the Antilles became impaired. However, if sugar cane production could be sustained, even if sometimes modestly, the decline of cattle-breeding could not be avoided when a intense drought devastated the region in the early 1790's, and as a result changes were introduced in sugar production which removed the need for cattle to work in the mills. Interested in raising productivity, some owners introduced steam plants at the beginning of the XIX Century; also, new and more efficient systems of operation were implemented.

2. The settlement of the 'Agreste'.

Despite bordering on the 'Zona da Mata'; near to the sugar centre; and presenting favorable weather conditions; the 'Agreste' was not settled in the early times. The penetration of its land started during the Dutch occupation and continued afterwards. After the extermination of the remaining Indian tribes, the government donated large blocks of land (about 10000 hectares each), where cattle could be raised. The wet places had already been appropriated for planting for subsistence; and because the region had difficult access, this condition guaranteed such places the possibility of providing themselves with the necessary food and independent life. In these places, a large number of small producers started growing 'lavouras' (basically, corn, beans and manioc) in addition to sugar cane and cotton, two economically important crops which had alternate domination during the second half of the XVIII Century.

It was in the beginning of the XIX Century that cotton created a true agrarian revolution in the 'Agreste'. Existing on a non-significant scale since the initial time of colonization, cotton became economically important for several reasons: the growth of the population and the consequent need for cotton clothes; the invention of the steam engine and the consequent industrialization of the manufacture of clothes; the authorization of international commercial transactions with countries other than Portugal (in particular with England), in 1808; and international events which had isolated some other countries which were able to produce a better quality of cotton (Southern USA).

It is possible to say that from 1750 until 1940, cotton was the only product from the North East which was able to confront the power of sugar cane, in terms of the dispute for land and arms. For the small farmers, cotton appears as a more 'demotic' culture, as it opens the possibility of planting, in the same tract, both cotton and

'lavoura'. This is not possible in the case of sugar cane.

The less expensive and less urgent industrialization of cotton, assisted the emergence of traders who treated cotton before passing it on to the market. In this sense, the production of cotton has contributed to the development of an urban life in the 'Agreste', differing from what had happened with sugar cane in the 'Zona da Mata', and with cattle in the 'Sertão'.

Particularly, the geographical area of SPP (which is located in the 'Agreste Potiguar') was settled in the latter decades of the XIX Century, at the time shown in Table 1 as the sub-cycle of cotton and sugar-cane (Table 1 gives the general characteristics of settlement of the Rio Grande do Norte in relation to the economic cycles). The stagnation of the cattle cycle, allied to climatic components (the drought of the 1970's, for example), had compelled a great number of families from different regions where cattle were bred (especially from the 'Sericó'), to search for new places and new possibilities of living. The Potengi River offered better climatic conditions and good possibilities for finding nearby markets (as it is located near Natal). Along the river, families started constituting villages whose function was to organize commercial interchanges between regional farmers, some of whom had also migrated from other parts of the State.

Together with a contingent of big farmers, small families, 'moradores', and 'vaqueiros' had settled in the region, constituting an initial class of earners. In addition to the agricultural economy, SPP acquired significance by offering a good resting-place for cattle and 'vaqueiros' in their long journeys, when crossing the region in search of profitable markets.

If the settlement of these families in the 'Agreste' had the above factors as determinants, the constitution of a 'povoado' or 'município' usually had as a loose origin, a dispute between families concerning the political and economic domination of the region. SPP was not an exception: the process of its foundation was motivated basically by a dispute between two rich families of traders: Urbano de Araújo and Pinheiro Borges, the former being identified as the founder. This happened in 1912, and the 'município' was constituted by the Central Government as such in 1938.

APPENDIX 2.B

Characteristics of RN (settlement)	Period	Cycle of NE economy
Occupation of the seaside coast. Fortifications for defense of villages; and activities of subsistence.	XVI Century	Pseudo cycle of wood.
'Lavouras' of manioc; primitive fishing. Few villages in the seacoast and river valleys (Potengi, Apodi, Trairi); religious and administr. activities.	First half of XVII Century	Economy of subsistence phase.
More intensive occupation of the oriental seacoast. Scattered occupation of Agreste (bank of river Apodi).	Second half of XVIII Century Second half of XIX Century	Cycle of cattle.
Sugar: occupation of river valleys (Cunhaú, Jacú and Trairi). Cotton: spreading all over the Estate (Seridó, Agreste and oriental seacoast).	Second half of XIX Century	Sub-cycle of sugar and cotton (International Market).
Intensive occupation of Centre-North and Agreste.	First half of XX Century	Cycle of cotton (National Market).
Intensification of industry, commerc. and basic activities. Insertion of agriculture in the market economy. Urbanization.	Second half of XX Century Present days	SUDENE phase.

Table 1 (ECPC - Project)

APPENDIX 2.C

LIST OF PROPERTIES AND THEIR AREAS

POTENGI-VALLEY (CEPA, 1984)

TRACT Nº.	DECLARED AREA (ha)	MEASURED AREA (PLANIMETRY) (ha)
01	64.0	63.8
02	98.0	98.2
03	58.0	86.0
04	480.0	322.2
05	20.0	19.5
06	14.5	4.8
07	11.0	78.6
08	66.0	10.4
09	13.0	11.4
10	36.0	45.6
11	132.0	111.6
12	108.0	28.0
13	51.0	32.6
14	14.7	15.8
15	14.5	17.2
16	13.8	19.0
17	13.8	15.8
18	24.0	34.8
19	13.8	14.4
20	14.5	19.0
21	36.0	6.0
22	1.0	1.0
23	82.0	6.8
24	167.0	61.6
25	233.0	8.7
26	94.0	74.0
27	8.0	8.8
28	8.0	12.6
29	40.0	34.8
30	16.0	25.0
31	32.0	26.2
32	14.0	9.4
33	24.0	24.4
34	9.0	9.6
35	13.5	14.0
36	38.0	26.0
37	21.0	20.0

APPENDIX 3.A

THEORETICAL APPENDIX UNDERSTANDING AND EXPLANATIONS

FORMS OF REACHING UNDERSTANDING

It was argued in chapter 3 that to regard knowledge as belonging to a discourse was methodologically important. I have suggested, accordingly, that knowledge of this kind can be characterized as a process of conversation with participant members of the discursive community; and that such a process would not change the discourse or the way people operate it.

But it is important to stress that this does not mean that nothing happens to those who are submitted to a conversation. In the same way that different things can happen to knowledge, at the individual level different things can happen to people who are led to think about the discourse that they operate. To a researcher seeking an application to schooling, these thoughts turn out to be extremely relevant, in that they can provide information about the process through which people *construct explanations*.

In other words, the analysis of communal knowledge would be incomplete if, after having submitted informants to a process of elicitation of their 'thinking', one jumped to the level of analysis which selects what is positively relevant to the interpretation of results, without asking in more detail for general features of what happened.

But what are these 'things' that can happen to knowledge and to people? In order to make this question more amenable to a clear empirically-based answer, let us reformulate it in terms of the methodological framework adopted for elicitation. Thus the question becomes: "If 'conversation with understanding' were to take effect, how would farm-people react to being invited to make argument/explanatory points (as characterized in Schank *et al.* (1982))?"

This question makes sense, particularly if one imagines the possibility of having a conversation in which the questions asked have no directive component/effect.

In such a conversation the informant would 'learn' how to interact with the researcher so as to provide accounts which could be analysed in terms of explanations, but whether or not he did so would be unrelated to the necessary process for reaching an understanding. The participant would provide good information, knowledge would be seen as transposed, but whether or not the informant tried to provide explanations would be idiosyncratic, dependent only on individual 'whim' (for example).

From an analysis of the transcripts of the interviews it is possible to identify three broad positions in respect to understanding, arranged in roughly ascending order of troublesomeness.

(a) The informant uses the process created by setting criteria of understanding (he follows the process of explanation). This seems to happen when the process of elicitation does little more than make systematic explanations which are unproblematic; and for this reason, replies are direct and clear cut. For the researcher, understanding seems to flow smoothly, involving processes such as codification or direct interpretation. No serious problems concerning agreements in teaching back arise. Misunderstandings are usually due to an inadequate construction of levels L^0 and L^1 (or because the researcher lacks what is needed in setting those levels or because the researcher has a very strong argument which identifies explanations L^1 as wrong).

(b) The informant can not easily find a way to follow the process (it is difficult to make him use the process of explanation). Analogies are used to facilitate communication. Understanding seems to involve recording, explicitly, the problem into a symbol system. Or because the informant uses a different one (when one would talk about transduction); or because explanations belong to the realm of the tacit, private or implicit knowledge (when the case will require formulation or explication as proposed by Bliss & Ogborn - 1987). Arguments at level L^0 can occur but not easily at level L^1 .

(c) The informant does not seem to care about the process (it is useless for aid in constructing explanation). In this case, if understanding is involved, cognitive operations will appear to be performed on "mental models" which can be accepted -when understanding takes place- as analogic representations of the settings presupposed by the informant. In this case, it would be more appropriate to denote what is happening to the informant as excogitation (it is difficult for the researcher to readily make sense of the informant's explanations). In other words, in looking at protocols, it would not be trivial to say what exactly the informant does, or to define whether the informant were able to achieve level L^1 . Also, the researcher would not be able to easily set the consistency of the basic argumentation made by the informant. In this case, understanding would require a kind of 'mapping'; in which a distinction between models would be necessary at the level of their structure, and questions of consistency, validity and truth would have to become problematic.

It is necessary to clarify, however, that in setting 'degrees of uselessness' of the researcher's criteria for reaching understanding, it does not follow that explanations are constructed as if the informant were not asked to work out loud in a restricted fashion. The argument is that the circumstances in which people generate explanations (and I include in these circumstances the kind of discourse/knowledge they operate) often affect the kind of explanation which is given in a variety of ways, many of them quite subtle. And that this, in turn, affect the formalization of understanding constructed by the researcher-analyst, particularly in defining forms of knowledge transposition (different kinds and levels of inference can be made from the context). The aspect to clarify refers to the character of the process of explanation behind these forms of transpositions, in that it becomes relevant for treating and analysing data.

THE PROCESS OF EXPLANATION

Different ideas concerning *explanations* are brought up at appropriate places in the thesis. My attempt will be to draw them together to show how the framework for examining protocols has incorporated what was considered to be methodologically essential about explanations. I will make a list of the main senses and comment on them at the end.¹

(a) Theories explain and provide the content of explanations (and so, enable us to give causal explanations).

(b) Explanations are discourses. One way of classifying discourses as explanatory is by reference to content.

(c) An explanatory system is required to formulate theories descriptive of the mechanisms productive of the items revealed in the analysis. Structures of generative or causal mechanisms have this function.

(d) Explaining is a speech act which makes use of a discourse which, in its literal meaning, makes reference to beings which are not capable, often, of being observed (there is a presumption that we can think beyond given experience to the imagined and hidden process that could produce observed patterns, the presumption that we can think in depth).

(e) If explanation is to capture reality, it requires a kind of historical-substantive content of ideas and theories in which experience participates.

¹ Reference will not be made to explanation as an element of the description of stages representing the main operants for characterizing practices. This sense of explanation was incorporated within the methodological framework based on Pask, and is present, indirectly, in items (j), (k), and (l).

(f) Explanations that people give when accounting for events or phenomena are not to be treated in terms of a category structure in which common-sense ideas are characterized by simple sets of defining features that are singly necessary and jointly sufficient to determine category membership.

(g) Explanations have an important role in the formalization of common-sense knowledge but do not constitute the object of formalization of commonsense.

(h) Common-sense explanations are not necessarily consensual.

(i) Explanations of commonsense are contradictory.

(j) Explaining is considered to be an extremely complex phenomenon involving several facets and features, strategies and conditions for applying knowledge.

(k) Explanations are seen as relevant to the task of 'modelling' the informant's level of understanding (explaining how and explaining why).

(l) Understanding an explanation at the level of the informant's accounts means to make inferences and to try to make sense of what the informant says by reference to the researcher's 'model'. Understanding an explanation at the level of inferences is to try to match their functioning (of inferences) as elements of the researcher's 'model'.

In the light of the above items, there are some aspects on which to comment. First, it is worth recalling that, in Science, theories have the function of explaining happenings and they do so by describing the mechanism which produces them. Theories give grounds for explanations to become elements of the description of mechanisms we believe might really exist in the world; and different theories do the work in different ways.²

Related to the variety of scientific explanations one finds in theorizing, are those questions that have to do with the nature, purpose and distinguishing marks of scientific and other kinds of explanations such as those belonging to commonsense. In focussing on the formalization of common knowledge in its relation to science, the question arises of "What counts as a plausible explanation of commonsense to be taken as data?" Two aspects are worth considering. One says that there is no need to make the formalization of commonsense conform to scientific knowledge, since scientific modes of discourse (according to the commonsense-referred

² The cases of theories discussed by Harré (1985) are illuminating in this respect. He cites, for example, the theory of motion (from mechanics), and the virus theory of poliomyelitis (from medicine). While the virus theory explains (through the concept of virus) what is described in the syndrome or course of the disease, the laws of mechanics are descriptive and do not attempt, in that science, to account for mechanisms of motion, of why the laws of impact, of momentum conservation and so on are what they are.

perspective) are unimportant for the formalization of commonsense (the opposite does not hold). What is required, is an adaptation of some basic ideas about the formalization of commonsense to the scientific point of view that *explanations are elements of a discourse*.

Theoretically, to recognize explanations as belonging to a discourse is to accept that in a communicative act, explanations are intelligible within the principles of that discourse. In this respect, there is the argument related to (a), (b), (c) and (d), that analysis of explanations should look at the process whereby different modes of explanations are constructed as discourses.

Accordingly, it was expected in this research that the informants would be able to account for how things are done, for which a corresponding practical discourse exists among people. It was also expected that the same would not happen for explanations at the level of why, for reasons and explanations are not part of their practical discourse. While direct accounts would be probably given for descriptions at the level of *how*, accounts at the level of *why* would require elicitation to be construed actively by both researcher and informants through teach back devices.

The second aspect relates to the fact that, as far as protocols are to be taken as a relevant setting, propositions (f) and (g) suggest to treat such a scene as a *resource for explanations* instead of focussing on explanations as directly and independently given (explanations could constitute a 'second-order-content', but these would still belong to a level of inferences). Particularly, and as far as the analysis of people's accounts is concerned, arguments (f), (g), (h) and (i), together with (j), (k) and (l), suggest that explanations are not to be seen as a set of empirically observable units which can be picked up directly from the protocols. Whether because there are different inferences one can draw from protocols; whether because a single account does not constitute an appropriate 'text'; or because accounts of explanations are contradictory and not necessarily consensual. This suggests that explanations, as elements of a discourse, are subjected to multiple interpretations.

But it is important to have clear that, in a discourse, a given explanation does not have an unlimited number of possible interpretations; it may well have more than one possible interpretation, but it will be constrained by the discourse to a limited set. In so far as explanations are involved in understanding, the recipient of explanations will most likely construct an inference which allows interpretation

within this set and respond appropriately for that interpretation; the informant can then inspect that response to see which interpretation it relies on. The researcher who has at his/her disposal many such explanations and responses (collected from a selection of protocols) can begin to see what the set of interpretation is, what kinds of responses are produced for each interpretation, and how apparent misinterpretations and contradictions are handled. On the basis of these data, the researcher can reasonably infer about the informant's explanatory account in a particular passage. Of course, the researcher can never be completely sure that she/he has correctly identified the informants' understandings; but since the informants themselves can never 'be sure' of their understandings - which surely does not keep them from believing that they understand one another - this hardly seems a major setback for analysis.

It is worth noticing that in the above exposition, explanation-senses were used in two main contexts: the context of a discussion of theorizing and the context of producing data. Therefore, it would not be wrong to suppose that two concepts of explanations could be at work. But there is a proposition in the above list that was not mentioned - namely, item (e) - and which, in my view, shows that along both dimensions (theorizing and producing explanations to be taken as data) there is a concept of explanation which means the same (is univocal). In touching on the 'reality' aspect, item (e) suggests that there is one category of things about which the understanding of what counts as an explanation is qualified by people (whether informants or researchers). To try to establish what such qualified understandings are about, is, for me, an *a fortiori* attempt; one which would spring from an analysis of the protocols. In this respect, the question arises of "What are the dimensions in which explanations are required?"; which is a question to be answered in a twofold attempt: one of looking at the protocols and asking "What is the content of accounts?" (the qualification of the understanding from informants); and another of clarifying "What dimensions are relevant in relation to which this content can be taken as informing the research questions?" (the qualification of understanding from the researcher). In this manner, levels of analysis could be defined in such a way that the relevant accounts could be somehow classified and used as data for the purpose of 'theorizing'.

MEASURING AREA IN EUCLIDEAN GEOMETRY

The term area both refers to the measure of a region –which is defined to be the interior of a closed curve– and designates the amount of surface 'existing' in that region. Any standard unit can be used as a *unit of area*: we could say that the area of this page is equal to the area which could be covered by one hundred

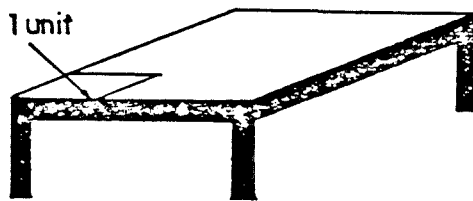


Figure 1

postage stamps. Namely, finding the area of a figure –for example, the table top in Figure 1– involves comparing its surface–region with that of a given *unit region*.

The notion that enables us to define the area of any region in terms of a standard unit region is the notion of *equivalence*. Two regions in a plane are said to be equivalent if they can be dissected into a finite number of pieces that are congruent in pairs (by translations or by half–turns). Thus, two polygonal regions are equivalent if they can be derived from each other by dissection and rearrangement.

For example, in Figure 2, the triangle ABC is equivalent to the rectangle ABED, since the parts CFJ and CFI of the former are congruent to the parts ADJ and BEI of the latter. Superposing two different dissections, it is possible to see that this kind of equivalence, which is reflexive and symmetric, is also transitive; two polygons that are equivalent to the same polygon are equivalent to each other.

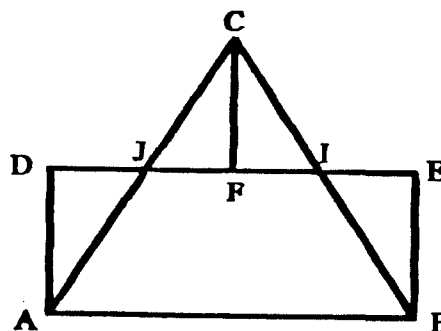


Figure 2

Based on that, we can define the area of any polygon in such a way that *equivalent polygons have the same area*, and when two polygons are stuck together without overlap to make a larger polygon, the areas are added. In summary, we can *compute* the area of a given polygon in terms of a standard polygon as unit of measurement, by dissecting it into standard polygon units *and* adding the pieces.

In Euclidean Geometry the unit of area is the *square of side one*. The unit of length is arbitrary so that if we measure area in square metres, we get a different number than if we measure in square feet (but the latter number is always proportional to the former; in the given case, the proportionality factor is $= 3,28^2$). In other words, *since we agree on a given unit of length*, the square of side "1-unit-long" represents the suitable shape to be considered as *the most fundamental unit region*.

Standard unit polygons can be defined by reference to the unit-square, making use of the procedure of placing a square-grid upon the polygon and estimating/counting the number of units that fit on it (which can be performed by means of a formula). The simplest and most useful shapes to be used as unit are the triangle and the rectangle, but nothing restricts the use of other shapes (though it is helpful if the shape can tessellate the plane). In solving problems, the appropriateness of a particular shape depends a lot on the form of the figure to be measured, particularly on its boundary. The requirement is that the figure should be completely covered by a number of units that fit on it. In that sense, a circle would certainly represent an inadequate choice for covering the table top in Figure 2. If appropriateness includes practical reasons, we would say, in addition, that the number of units should be small as possible, trying to reduce the problem to the linear measurements of the sides of the region; in which case, the complexity of the formula for reckoning the value of the unit region should be considered. In the case of Figure , we could just measure sides *a* and *b* and use the formula $a \times b$. If we have, for example, a hexagonal-figure, some possibilities exist for covering it with one unique unit shape, three of them being represented in Figures 3-A, 3-B and 3-C. Respectively, they require the computational formulas proper to a triangle, a trapezium and a hexagon.

Regions bounded by curves can be treated similarly, by regarding them as limiting cases of polygonal regions.

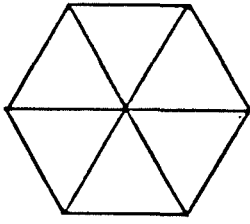


Figure 3-A

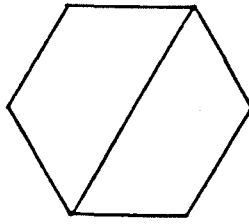


Figure 3-B

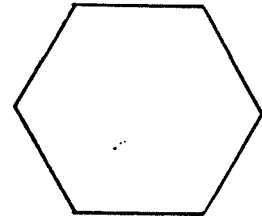


Figure 3-C

Depending on the region to be reckoned, it can be more appropriate to compute the area not directly by means of a standard unit region, but by dissecting it into a number of different -but traditionally known- shapes and adding the pieces. In this case, we are actually subdividing the initial problem into several similar problems of computing the area by a standard unit as mentioned above. For example, the region in Figure 4-A can be divided into a triangle and a semi-circle, while the region in Figure 4-B can be easily reckoned in terms of a semi-circle and a semi-ellipse.

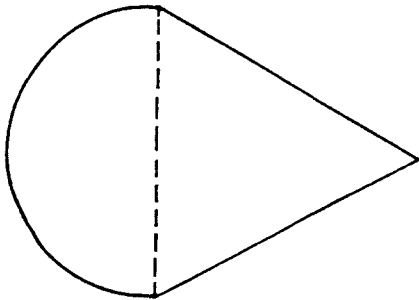


Figure 4-A

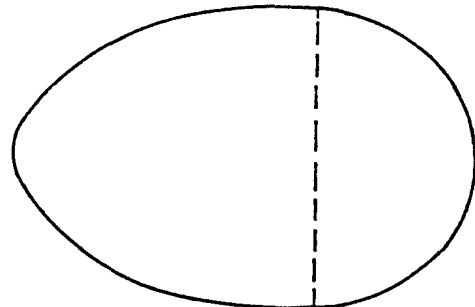
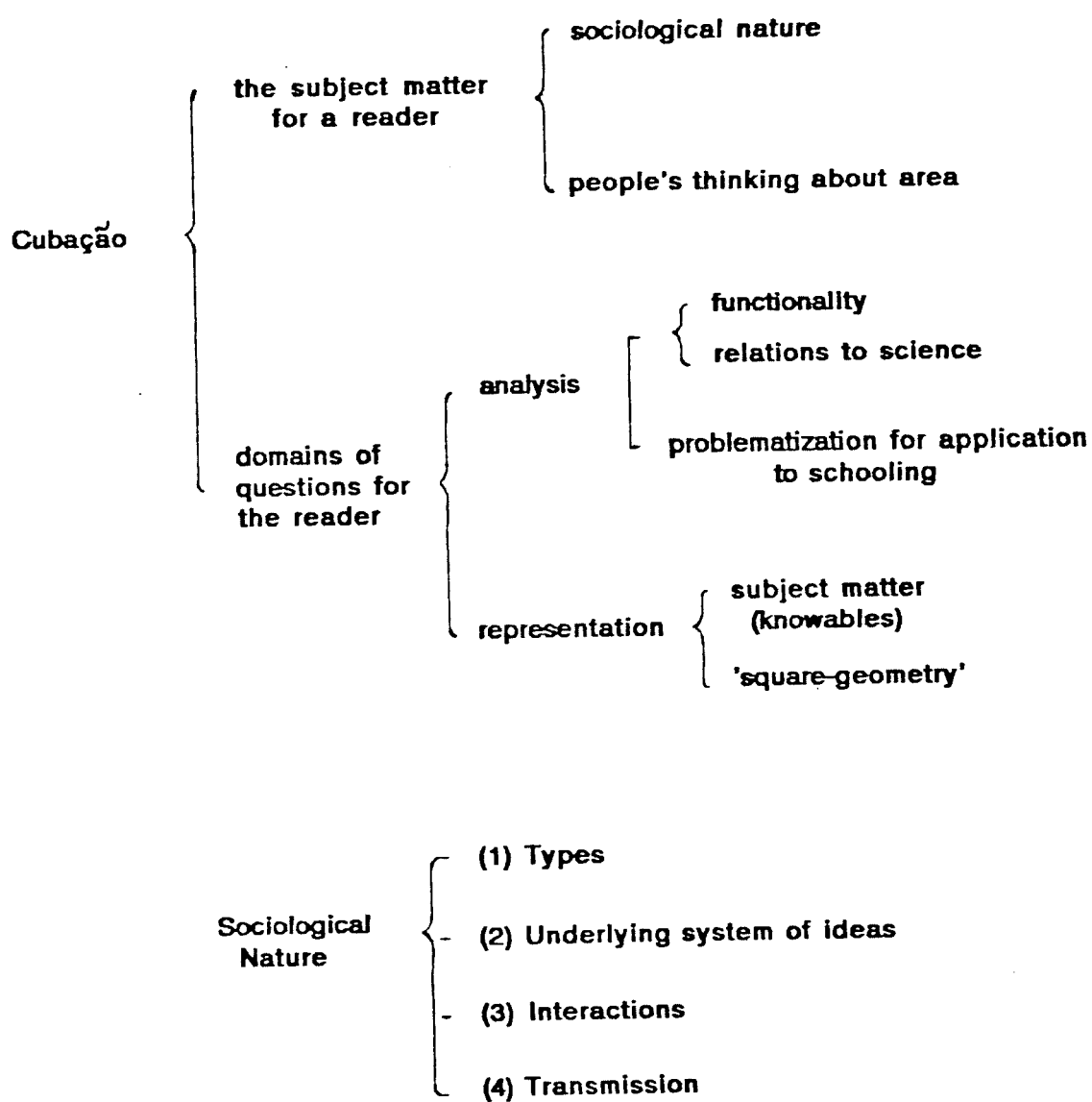
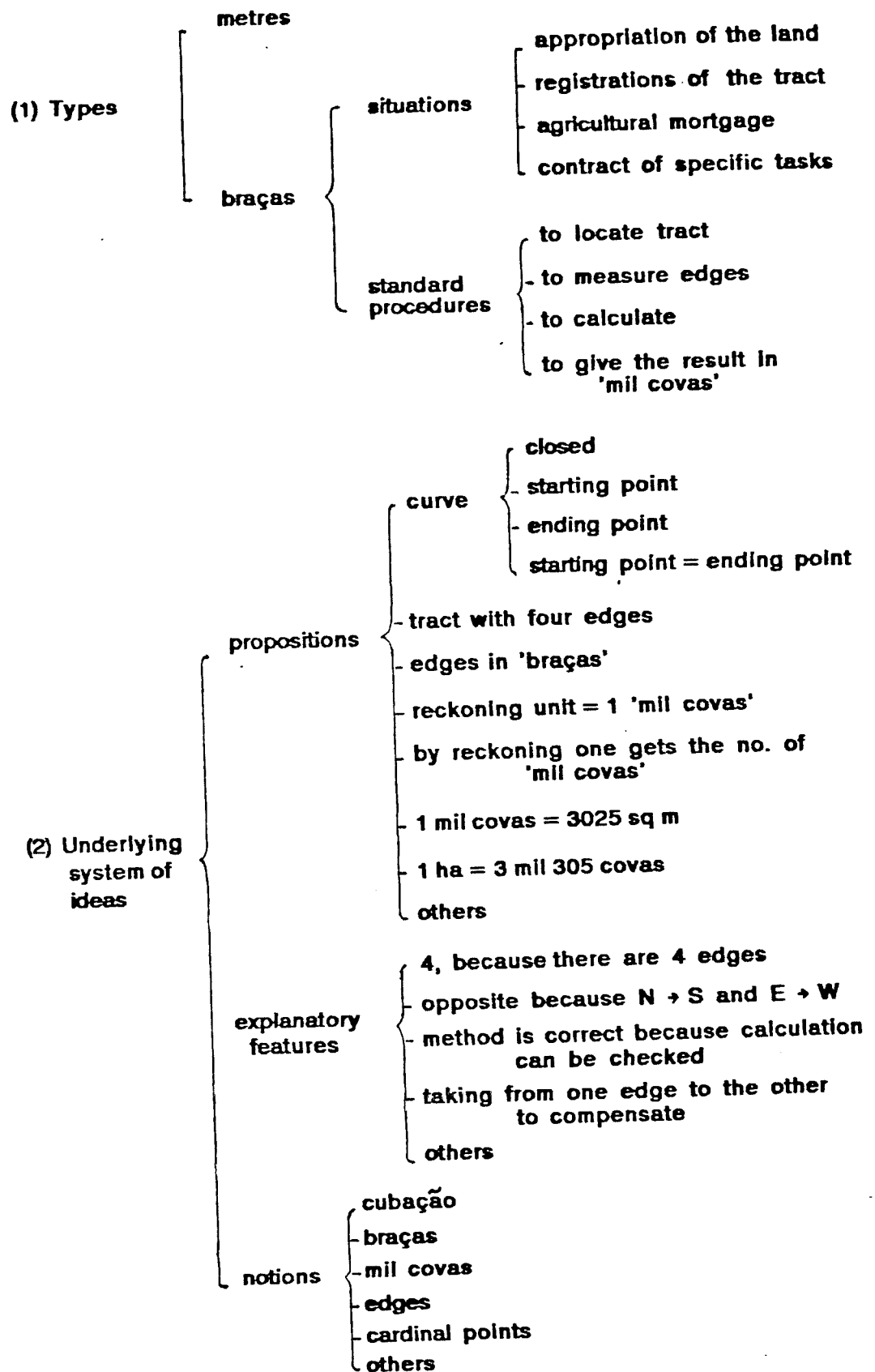
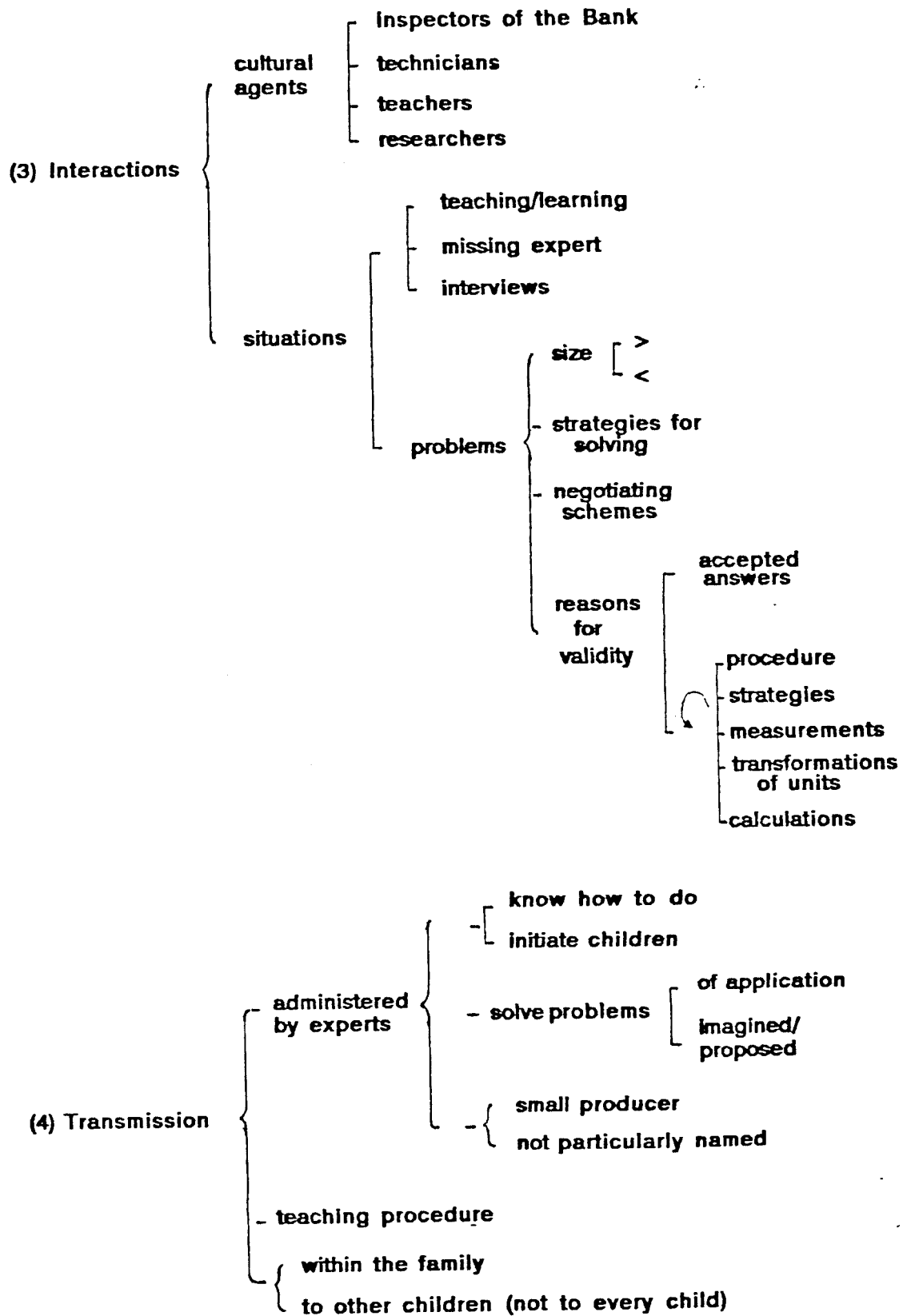


Figure 4-B

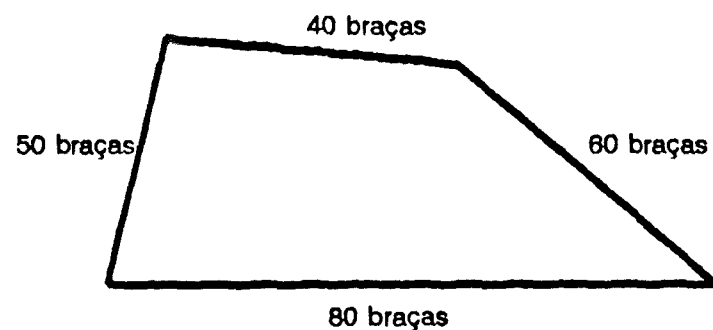
ISSUES ABOUT CUBAÇÃO







CUBAÇÃO



Three methods of reckoning tracts of land in braças are presented. You are asked:

- (1) Are the three methods the same or are they different? Why?
- (2) Are there situations in which they can be the same, and situations in which they can be different? Give examples.
- (3) Which one do you consider to be correct? In which one do you believe? Why?

METHOD 1

- (a) To add up the opposit sides:
"North and South", and
"East and West";
- (b) to multiply the results;
- (c) to multiply the last result by 4;
- (d) to ignore the last digit to the right;
- (e) the result shows the number of
'mil covas'.

METHOD 2

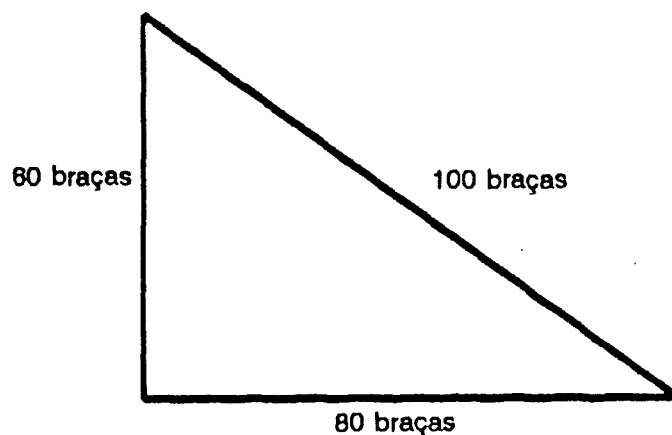
- (a) To add up the two smaller sides;
to add up the two larger sides.
- (b) to multiply the results;
- (c) to multiply the last result by 4;
- (d) to ignore the last digit to the right;
- (e) the final result shows the number of
'mil covas'.

METHOD 3

- (a) To add up the four sides;
- (b) to divide the last result by 4, to
obtain a square of equal sides;
- (c) to add up sides two by two;
- (d) to multiply the results;
- (e) to ignore the last digit to the right;
- (f) the final result shows the number of
'mil covas'.

CUBAÇAO applied to a triangular tract of land

Seven procedures for reckoning a triangular tract of land in braças are presented. These procedures can be applied, for example, to a tract such as the one represented below. You are asked :



- (1) Would you expect these procedures to give the same number of 'mil covas'? Explain how do you come to a conclusion.
- (2) Would it be possible to think of more than one way to come to know if these seven procedures are the same (or not)? Which ones?
- (3) If it happens that the procedures give different results, which one would be correct? Why?
- (4) If this is the case, why is it that certain procedures are not correct?

PROCEDURE 1

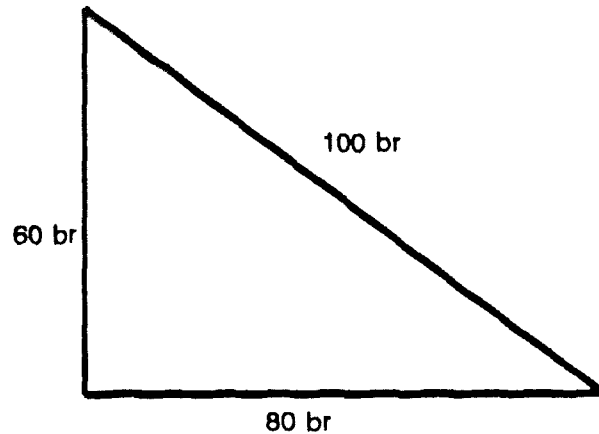
- (a) To add up any two sides;
- (b) to multiply the result by the length of the third side;
- (c) to multiply the result by 4;
- (d) to ignore the last digit to the right;
- (e) the final result shows the number of 'mil covas'.

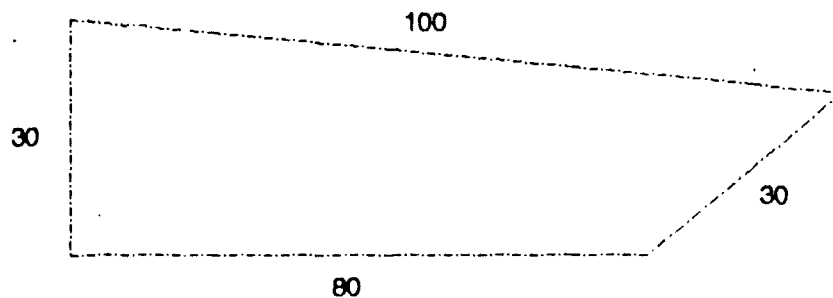
PROCEDURE 2

- (a) To add up the two larger sides;
- (b) to multiply the result by the length of the third side;
- (c) to multiply the result by 4;
- (d) to ignore the last digit to the right;
- (e) the result shows the number of 'mil covas'.

PROCEDURE 3

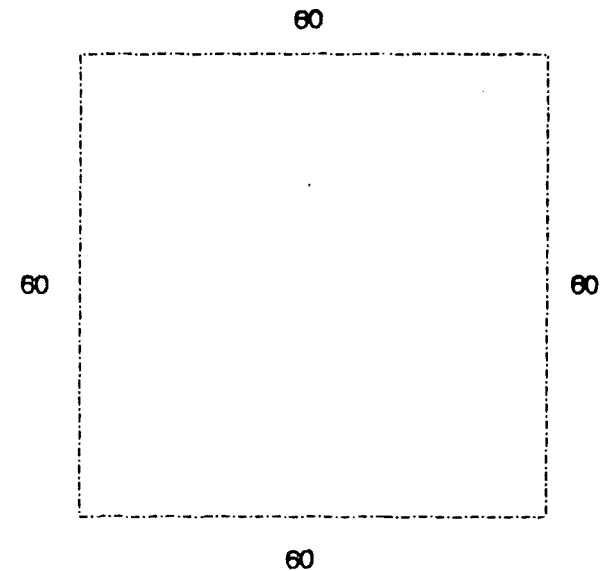
- (a) To add up any two sides;
- (b) to multiply the result by the length of the third side;
- (c) to multiply the result by 3;
- (d) to ignore the last digit to the right;
- (e) the result shows the number of 'mil covas'.





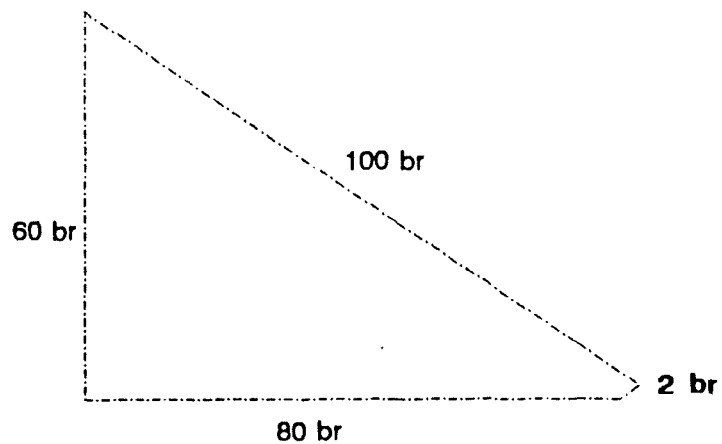
PROCEDURE 4

- (a) To add up the two larger sides;
- (b) to divide the third side by 2, to undo the triangle;
- (c) once the transformation is performed, to proceed according to method 2.



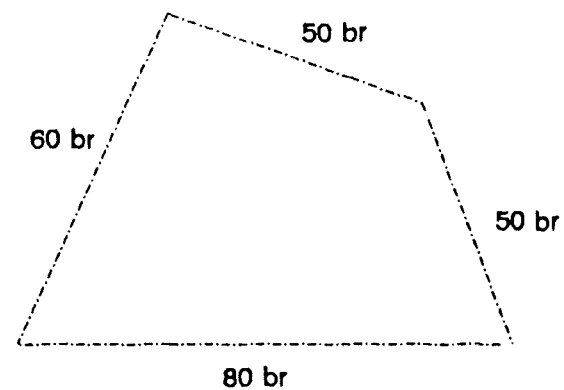
PROCEDURE 5

- (a) To add up the three sides;
- (b) to divide the result by 4;
- (c) to construct a square of side-length equal to the result of (b);
- (d) to proceed according to method 1.



PROCEDURE 6

- (a) To add up the two larger sides;
- (b) to add 1 or 2 bracas to the opposit corner of the third side;
- (c) to reckon according to method 1.

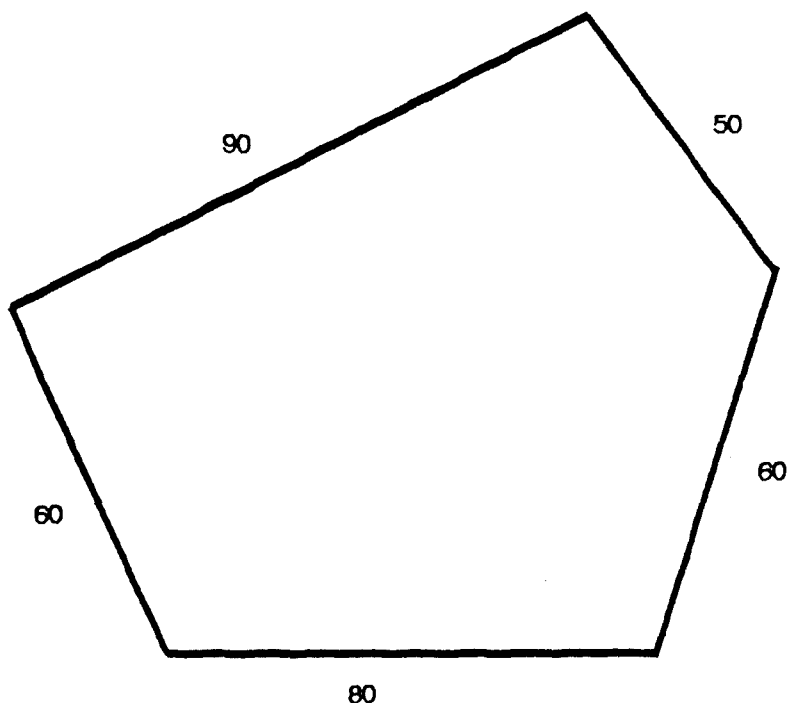


PROCEDURE 7

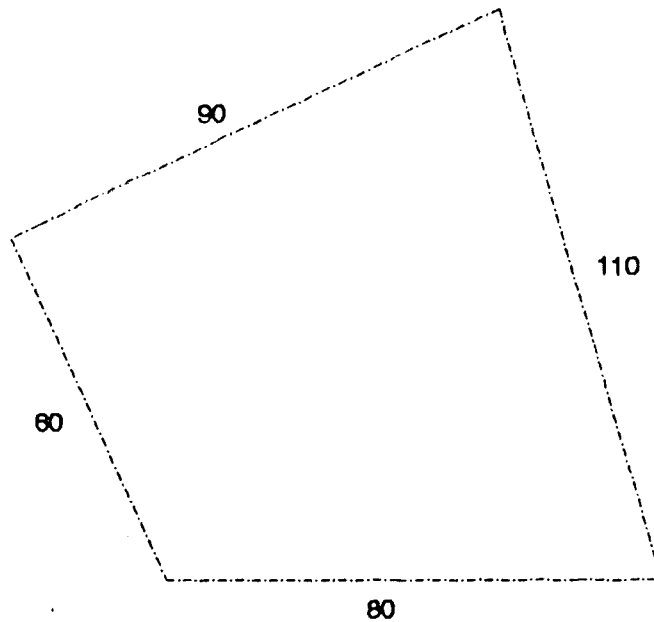
- (a) To take the larger side and to divide it up into two equal sides, to construct a tract with four sides;
- (b) to reckon according to method 1.

CUBAÇAO applied to a tract of land with five sides

Three procedures for reckoning a tract of land with five sides are presented. These procedures can be applied, for example, to the following tract. You are asked :

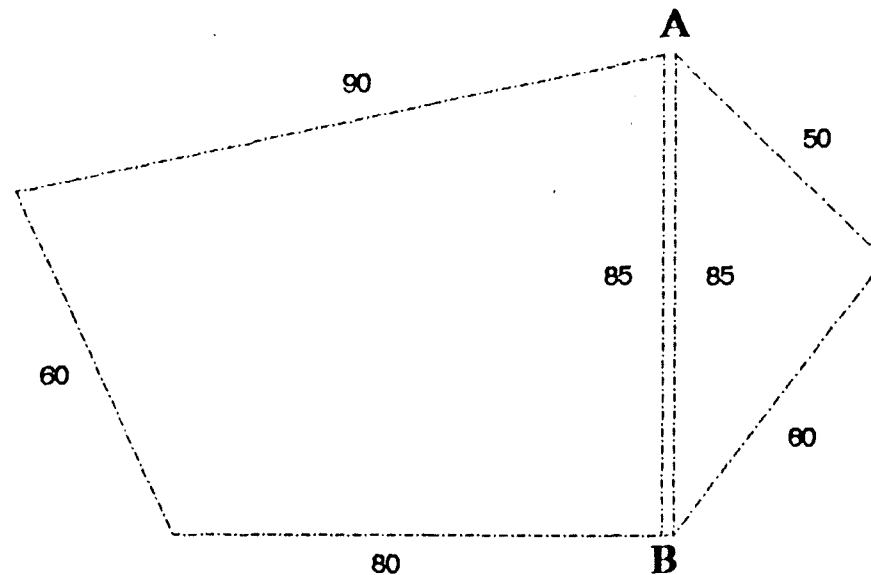


- (a) Shall the three procedures give as a result the same number of 'mil covas'? Is it possible to know?
- (b) Which one do you 'believe' to be correct? What reasons do you have to justify your answer?
- (c) If the results from different procedures are different, would be the differences significant, if you are reckoning an actual tract?



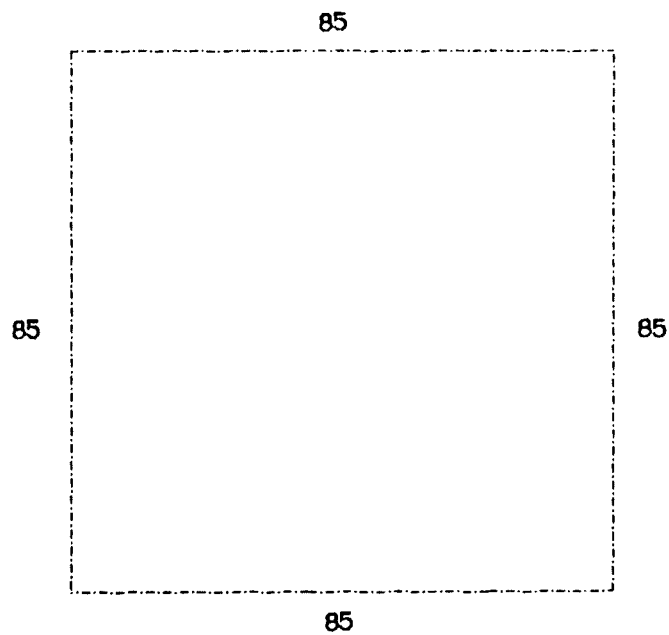
PROCEDURE 1

- (a) To add up the two smaller sides, to construct a tract with four sides;
- (b) to reckon according to method 1.



PROCEDURE 2

- (a) To draw a divisory line from corner A to corner B, to get two tracts: one quadrilateral and one triangle;
- (b) to reckon each tract separately;
- (c) to add up the results. The final result shows the number of 'mil covas'.



PROCEDURE 3

- (a) To add up all the five sides;
- (b) to divide the result by 4, to
obtain the length of one side;
- (c) to construct a tract with four
equal sides;
- (d) to reckon according to method 1.

APPENDIX 8.A

EXPRESSION § IN THE HISTORY OF MATHEMATICS

Examples of application of the rule § to an arbitrary quadrangle (to multiply the averages of opposite sides) can be found in different sources. Some of them are quoted accompanied by the author's comment on their status.

EGYPT

Example 1 (Boyer, 1968, p. 18).

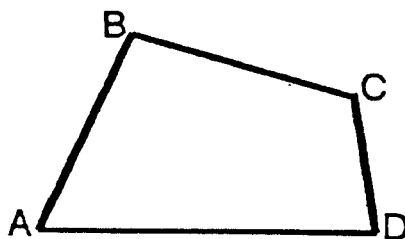
"A serious deficiency in their [the Egyptians] geometry was the lack of a clear-cut distinction between relationships that are exact and those that are approximations only. A surviving deed from Edfu, dating from a period some 1500 years after Ahmes, gives examples of triangles, trapezoids, rectangles, and more general quadrilaterals; the rule for finding the area of the general quadrilateral is to take the product of the arithmetic means of the opposite sides. Inaccurate though the rule is, the author of the deed deduced from it a corollary - that the area of a triangle is half the sum of two sides multiplied by half the third side. This is a striking instance of the search for relationships among geometric figures, as well as an early use of the zero concept as a replacement for a magnitude in geometry."

Example 2 (Eves, 1969, p. 40).

"In later Egyptian sources the incorrect formula $K = (a + c)(b + d)/4$ is used for finding the area of an arbitrary quadrilateral with successive sides of lengths a, b, c, d ."

Example 3 (Dilke, 1971, p. 30).

"The method of calculating irregular quadrilaterals in Egypt was rough and ready: it was to multiply the averages of opposite sides. Thus, in the quadrilateral below, a rough area would be obtained by the formula $1/2(AB + CD) \times 1/2(AD + BC)$. In the case of a convex quadrilateral this always results in an over-estimate, which as Déléage points out benefited the treasury."



BABYLONIAN MATHEMATICS

Example 4 (Boyer, 1968, p. 42).

"Measurement was the keynote of algebraic geometry in the Mesopotamian valley, but a major flaw, as in Egyptian geometry, was that the distinction between exact and approximate measures was not made clear. The area of a quadrilateral was found by taking the product of the arithmetic means of the pairs of opposite sides, with no warning that this is in most cases only a crude approximation."

ANCIENT ROME

Example 5 (Dilke, 1971, pp. 15-17 and p. 30).

"The method [as described in example 3], however, persisted, and we find it in the Corpus^(†)."

(†) *Agrimensores*, 'measurers of land', were the land surveyors of ancient Rome. But they not only measured it: they laid it out with more careful planning and more accuracy than in any country at any time until the late eighteenth century. [...]

A regular training of surveyors was organized under the Empire. It included cosmology and astronomy, the geometry of areas, orientation, sighting and levelling, a knowledge of land law and of the status of different types of land, as well as the techniques of centuriation, boundary definition, allocation of land, mapping and recording. What training there was for military and architectural surveyors was, it seems, separately organized. Under the late Empire the whole structure became more bureaucratic, and the *agrimensores* rose in status. They became also judges or arbitrators in cases where land law was involved.

The Corpus Agrimensorum is a collection of surveyors' manuals which has come down to us in often corrupt and fragmentary texts. They are preserved in manuscripts of which the most important, at Wolfenbüttel and in the Vatican, date from the sixth and ninth centuries. [...] The earliest technical writer in the Corpus is Sextus Julius Frontinus, governor of Britain probably from AD 74 to 78, author of works on strategy and on Rome's water supply."

MEDIEVAL MATHEMATICS

Example 6 (Lauand, 1986, pp. 102-105).

"13. *Triangular field problem* (pseudo-Alcuino): One side of a triangular field measures 30 *perticas*; the other also measures 30, and the frontage 18. Tell, if you can, how many *aripenos* has the field?

A. The two 30-sides sum 60, which halved results 30, which is then multiplied by 9 (which is half of 18), which gives 270 (which is the result in *square perticas*). To express the area in *aripenos*, divide by 144 etc...

14. *Circular field problem*: How many *aripenos* does a circular field of circumference 400 *perticas* have?

A. One fourth of 400 is 100; 100 multiplied by 100 gives 10000, which is the area. To express it in *aripenos*, divide by 144 etc...

[...] *Problems 13 and 14* show the mathematical deficiencies of the time."

HINDU MATHEMATICS

Example 7 (Boyer, 1968, pp. 232-233).

"During the sixth century, shortly after the composition of the *Siddhantas* [systems of astronomy], there lived two Hindu mathematicians who are known to have written books on the same type of material. The older, and more important, of the two was Aryabhata, whose best known work, written in 499 and entitled *Aryabhatiya*, is a slim volume, written in verse, covering astronomy and mathematics. The names of several Hindu mathematicians before this time are known, but nothing of their work has been preserved beyond a few fragments. In this respect, then, the position of the *Aryabhatiya* of Aryabhata in India is somewhat akin to that of the *Elements* of Euclid in Greece some eight centuries before. Both are summaries of earlier developments, compiled by a single author. There are, however, more striking differences than similarities between the two works. The *Elements* is well-ordered synthesis of pure mathematics with a high degree of abstraction, a clear logical structure, and an obvious pedagogical inclination; the *Aryabhatiya* is a brief descriptive work, in 123 metrical stanzas, intended to supplement rules of calculation used in astronomy and mensurational mathematics, with no feeling for logic or deductive methodology. About a third of the book is work on *ganitapada* or mathematics. This section opens with the names of the powers of ten up to the tenth place and then proceeds to give instructions for square and cube roots of integers. Rules of mensuration follow, about half of which are erroneous. [...] In the calculation of areas of quadrilaterals, correct and incorrect rules appear side by side. The area of a trapezoid is expressed as half of the sum of the parallel sides multiplied by the perpendicular between them; but then follows the incomprehensible assertion that the area of any plane figure is found by determining two sides and multiplying them."

Example 8 (Boyer, 1968, pp. 241-243).

"We have placed the work of Aryabhata around the year of 500, but the date is doubtful since there were two mathematicians named Aryabhata and we cannot with certainty ascribe results to our Aryabhata, the elder. Hindu mathematics presents more historical problems than does the Greek mathematics, for Indian authors referred to the predecessors infrequently, and they exhibited surprising independence in mathematical approach. Thus it is that Brahmagupta, who lived in the Central India somewhat more than a century after Aryabhata, has little in common with his predecessor, who had lived in eastern India. Brahmagupta mentions two values of π -the "practical value" 3 and the "neat value" 10- but not the more accurate value of Aryabhata; in the trigonometry of his best-known work, the *Brahmasphuta Siddhanta*, he adopted a radius of 3270 instead of Aryabhata's 3438. In one respect he does resemble his predecessor -in the juxtaposition of good and

bad results. He found the "gross" area of an isosceles triangle by multiplying half the base by one of the equal sides; for the scalene triangle with base fourteen and sides thirteen and fifteen he found the "gross area" by multiplying half the base by the arithmetic mean of the other sides. In finding the "exact" area he utilized the Archimedean-Heronian formula. [...] As a rule for the "gross" area of a quadrilateral Brahmagupta gave the pre-Hellenic formula, the product of the arithmetic means of the opposite sides. For the quadrilateral with sides $a = 25$, $b = 25$, $c = 25$, $d = 39$, for example, he found the "gross" area of 800. [...] Among them [quadrilaterals whose sides, diagonals, and areas are all rational] was the quadrilateral with sides $a = 52$, $b = 25$, $c = 39$, $d = 60$, and diagonals 63 and 56. Brahmagupta gave the "gross" area of $1933\frac{3}{4}$, despite the fact that his formula provides the exact area, 1764, in this case."

Example 9 (van der Waerden, 1983, p. 207).

"The first half-verse of Aryabhatiya II 7 reads in the translation of Clark:

Half of the circumference multiplied by half of the diameter is the area of the circle.

This is the well known rule

$$A = \frac{1}{2} C \cdot \frac{1}{2} d$$

which is also found in Greek and Chinese sources."

CHINESE MATHEMATICS

Example 10 (Seidenberg, 1973, pp. 180-181).

Book I [of the *Chinese Nine Books*] starts with *the area of a rectangular field*. Problem 1 reads: "Now one has a field; it is 15 steps wide and 16 steps long. The question is: How large is the field?" The answer (= 1 Mou) is given; and a second problem of a similar kind is posed and the answer is given. Then the general rule is stated. With a couple of minor exceptions, this is the format used throughout the work. [...] This is different, in the main, from the Babylonian procedure, where the problems are stated and *worked out*, but the general rule is not given (though there are a few instances of general statements). Book 1 continues with arithmetical problems (addition, subtraction, etc. of fractions), returning to geometry with problem 25 which asks for *the area of a triangle*. Then comes the *trapezoid*. With problem 31 we come to *the circle*: "Now one has a round field; the circumference is 30 steps, the diameter 10 steps. The question is: How large is the field. The answer says: 75 Pu." Clearly the ratio of the circumference to diameter is taken to be 3, though curiously no problem requires this knowledge, and throughout superfluous information is supplied. The value 3 is typically Babylonian, but the Babylonian scribe needs to know this in working his problems. The rule (in the *Nine Books*) for *the area of a circle* is to multiply one-half the circumference by one-half the diameter; three further rules are given, the third of which says to square the circumference and divide by 12 -this is the Babylonian procedure."

MISSING

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