Developing Mathematical Reasoning through Games of Strategy Played Against the Computer

by Jenny Houssart[1] and Claire Sams[2]

This article examines 9-11 year olds' attempts to beat the computer at a game of strategy and asks how far and under what circumstances they developed a strategic approach to the game. Extracts from videotape transcripts are analysed to reveal the components of children's particular approaches, evident even amongst children who did not develop an overall strategy for beating the computer. We analyse children's mathematical reasoning in depth and identify circumstances conducive to valid reasoning. It is argued that use of appropriate discourse conventions is a significant factor in developing valid approaches, helping children to develop appropriate linguistic tools of their own, provided that teachers use the conventions flexibly.

KEY WORDS

computer games, discussion, primary mathematics, reasoning

1. INTRODUCTION

This article examines 9-11 year olds' attempts to beat the computer at a game of strategy and asks how far and under what circumstances they developed a strategic

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approach to the game. It contributes to discussions of the circumstances under which strategic approaches are developed, a subject important to practitioners.

In the research described here, the computer takes the role of opponent in a game of strategy. The game is called ‘Lines’ and requires the players to form a line of counters in a similar way to well known games such as ‘Noughts and Crosses’, ‘Tic-Tac-Toe’ and ‘Connect Four’. The children playing the games had first been encouraged to work together following specific guidelines for classroom talk as advocated by other researchers, which will be outlined in a later section. The article’s focus is on the interactions between the children within their working groups and the decisions they collectively take. This collective problem-solving based on the use of language is known by Mercer (2000) as “interthinking.” Like Mercer, we adopt a largely social view of learning based on the links between language and thinking made by Vygotsky (1962). The principal contribution of this article is to demonstrate that when such games are deployed in particular ways, strategy games with the computer as opponent can structure learning situations to make mathematical reasoning accessible to a wider group of children than otherwise. A further contribution is to show how children’s approaches are effectively developed by guidelines for classroom discussion, but that these guidelines are most effective when used flexibly by teachers.

2. LITERATURE

Computer games and primary mathematics

Classroom teachers have necessarily to take into account the ways that children relate to computer games. Playing games is a very common way for children to use computers, particularly outside school. Computer games are also used in primary schools, particularly in mathematics, but also in other subjects.

In reviewing the history of computer use in primary schools, Fox et al (2000) begin by looking at threads which pervaded early assumptions about what computers in primary schools are for. One such thread, which the authors acknowledge is likely to have been borrowed from home computer use, is that 'computers are things you play games on' (page 6). It is therefore suggested that although games playing may have some educational benefit, many computer games related to mathematics are disappointing. Games are often drill and practice software, for example practice of number bonds, presented using cartoon contexts, attractive graphics or sound effects. The authors assert that 'some games simply waste time' (page 9).

Some writers take more positive views of computer games in the teaching of mathematics often precisely because of the way that they cut with the grain of children's extra-school activities. For example, Kafai et al (1998) point out that educational games are used by teachers for motivational reasons and that computer games draw on children's cultural disposition towards using them. Computer games and video games are a significant part of many children's culture, as outlined by Provenzo (1991), who points out that boys aged 8-11 are a large user group for video games. Gee's study (2003) of video games, including computer games, shows that this contextual promise can be realised and concludes that they can provide intricate learning experiences.

Thus, both the software and how it used are important. The issue is under what classroom circumstances computer games use is successful. Higgins (2003) offers positive case studies in discussing whether and how ICT can make mathematics teaching more effective. The software used in the case studies includes some games. He considers software qualities and concludes that it is *how* ICT is used that makes the difference. Vincent (2003), discussing computer use and primary mathematics, suggests that computer use alone is insufficient, suggesting that the right software and an appropriate open-learning environment are also required. He then provides positive examples of computer use, but points out that they demand much of teachers.

Writing in the context of primary mathematics and ICT, though not referring specifically to games, Fox et al (2000) list important skills which teachers need to develop to help children become mathematical thinkers, viz: questioning; developing mathematical vocabulary; encouraging mathematical discussion and solving problems (pages 41-42). Our argument will be that some computer games, used appropriately, can help develop these skills and hence encourage children's mathematical thinking and reasoning.

What constitutes mathematical reasoning in this context? Mason et al (1982) attach particular importance to generalising, describing it as the essence of mathematical thinking. They suggest that 'generalising starts when you sense an underlying pattern, even if you cannot articulate it' (page 9). In discussing the place of reasoning and proof in schools, Mason (2002) argues that even young children can reason using 'the discourse of 'if… then…'. The importance of generalisation has long been highlighted by mathematicians such as Polya (1954), and this tradition also underlines the significance of the process of conjecturing and the importance of being willing to modify one's conjectures. Lakatos (1976), denotes a conjecture a 'conscious guess', stressing that conjectures must be open to revision. Lampert (1990) builds on this work to suggest that it is possible to establish a classroom in which children are encouraged to offer and modify conjectures.

National educational systems attempt to shape the approaches actually adopted in practice. English curriculum documents attempt to describe the mathematical reasoning pupils should be encouraged to develop at certain ages. For children in our study this should include finding different ways of approaching problems, developing logical thinking and explaining their reasoning (DfEE and QCA 1999). Solving mathematical problems or puzzles is advocated, as is encouraging generalisation and prediction (DfEE 1999). Despite such explicit mention of reasoning in curriculum documents, inspection evidence in the years preceding the study suggests that this was not always effectively implemented. Thus, one report (OFSTED 2002) suggests a need to promote pupils' reasoning and problem solving skills, with generalising, investigating and providing reasoned arguments featuring among the examples given. A more recent report (OFSTED 2005) suggests that where teaching is at its strongest the provision of tasks requiring pupils to think and reason is a significant contributory factor. However, in other schools the development of reasoning skills is reported still to need attention and therefore the question of whether computer games can contribute to dealing with the problem is sharply posed.

Exploratory talk as a condition for learning.

It has been argued that specific rules of discourse are an important condition for learning. The project analysed here builds on previous projects in primary classrooms which encouraged children to work and talk together in particular ways while using computers. Mercer's (1995) notion of *exploratory talk* is defined as talk in which partners engage critically but constructively with each other's ideas. It is contrasted with *disputational talk*, characterised by disagreement, and *cumulative talk* in which speakers build positively but uncritically on what their partner has said. In *exploratory talk*, compared to the other two types, 'knowledge is made more publicly accountable and reasoning is more visible in the talk' (Mercer 1995, page 104)

Previous projects have used the exploratory talk notion to help children develop reasoning skills, for example in science (Mercer et al 2004). Central to these projects and the current one, is agreement within a class on a set of rules for talk. The rules produced by one class of 9-10 year olds in one of the projects are listed below:

We share our ideas and listen to each other

We talk one at a time

We respect each other's opinions

We give reasons to explain our ideas

If we disagree we ask 'why?'

We try to agree in the end

(Wegerif and Dawes, 2004 page 25)

The purpose of the project on which this article was based was to use the ‘talk rules’ approach with 9-11 year olds working on mathematical computer games and to examine its utility.

3. RESEARCH QUESTIONS

In summary, whether computer games are useful in developing mathematical reasoning has been the subject of some dispute, and it appears that there is room for improvement in English classroom practice in developing reasoning skills more widely. Some researchers have advocated the use of ‘talk rules’ to maximise reasoning skills, which offer the prospect of maximising their potential.

Our research questions are therefore as follows:

How far and under what circumstances do children develop mathematical reasoning when using computer games of the strategic type examined here?
Assuming that they are at least to some extent successful, we also ask the question: How significant are rules of discourse in facilitating any such development?

4. THE PROJECT

The work considered here draws on a larger project carried out in two phases and involving nine teachers and their classes. All the schools were primary schools in England and the children were aged 9-11. There was some variation between classes, but classes generally worked on the project for between four and eight lessons each, and three lessons per class were video taped.

Prior to the project, teachers were introduced to the idea of exploratory talk and the 'talk rules' in training sessions, and were provided with suggested lesson outlines designed to introduce children to the talk rules. Each class agreed ground rules for talking together in computer lessons. In practice, the ground rules agreed resembled those listed above, establishing an ethos in which all views were sought and welcomed. It was suggested to teachers that computer lessons should begin with a reminder of the ground rules, a central talk objective for the lesson should be made explicit to the pupils and that they should model the ground rules in their own interactions with pupils.

As part of project preparation, the teachers explored the mathematics software they would use with children. Five possible programs were suggested from the SMILE Co-ordinates package, and three from the SMILE Numeracy package. The teachers decided which of these programs to use.

The program considered here is 'Lines', a program from the Co-ordinates package, essentially a computer game similar to 'Noughts and Crosses' or 'Connect Four'. The game's aim is to place four counters in a line on a grid on the computer screen before one's opponent. Counters are positioned by typing in co-ordinates and the teacher's notes accompanying the software suggest that 'Lines' offers opportunities for developing logic as well as using co-ordinates (SMILE Mathematics 2000). Children can either play the game against each other or against the computer.

In this project children were asked to play together in teams of two or three against the computer. When the game is played in this way, the computer plays a broad ‘best strategy’ but those playing it are always allowed the first move. Because of the inherent advantage, it is possible to develop a winning strategy when playing the computer as currently programmed. By strategy we here mean an *overall, reasoned and winning approach*; we do not therefore attempt to distinguish between strategy and, for example, tactics. Although children face an expert opponent it is possible for them always to win if they develop an appropriate strategy.

All lessons followed the structure then advocated for primary mathematics teaching in England (DfEE 1999), emphasising direct teaching and interactive oral work. Each lesson began with a whole class introduction, which teachers used to discuss the talk rules, to explain the specific talk objective and to talk about the ‘Lines’ game. In three of the four lessons the introduction was also used to play all or part of a game as a whole class against the computer. For the middle part of the lesson children worked in pairs or groups of three at computers and at this point the video camera focussed mainly on one computer. The lessons concluded with a whole class plenary session, which was used to evaluate both the game and the quality of the group talk. The lessons all took place in specialist computer rooms, two of which were also equipped with interactive whiteboards.

5. METHOD

Lessons of about one hour using ‘Lines’ were video taped in the four different schools with permission from the school, the teacher, the children and their parents.

The whole of each lesson was videoed by one of the researchers using at least one video camera and in some cases by using two, in order to cover all relevant angles. In each school one lesson using ‘Lines’ was taped, as well as lessons using other software. The video tapes were transcribed in their entirety and the spoken parts of the tapes were analysed. In some cases, where there was lack of clarity about what precisely was said, visual evidence was used to confirm the verbal record. The tapes also contained other useful visual information for example in terms of children pointing to relevant parts of the screen. Where discussion focused around particular games, tapes were watched serially in order to recreate the moves accurately, and these moves were checked against the game software to confirm what moves the computer made in specific situations. The software was also used to recreate the games in the form of diagrams.

Initial analysis focussed on one of the lessons (School A) in which several groups of children claimed to have developed winning strategies. From the transcript of this lesson, components of winning strategies were identified. These were then used as categories for coding the transcripts of the other three lessons. Both the identification of categories and the coding for the subject of repeated discussions between researchers and others involved in the project in order to reach some agreement.

6. FINDINGS

The Computer as Opponent

Initial analysis of one of the four transcripts by Monaghan (2004) identified statements by the children positioning the computer as a cunning adversary and suggested the idea of 'outsmarting the computer' as a theme of the data. Similar ideas occur in all four transcripts, with the idea of outsmarting the computer, frequently encouraged by teachers, appearing as a motivational factor. This suggests that the games at least had a motivational role. Our research questions are initially considered by looking at those children who claimed to develop overall winning strategies and then by examining the components of such strategies.

Winning strategies

In one of the four lessons (school A) several groups of children claimed to have developed strategies for always beating the computer. The teacher was anxious that they did not simply repeat the same winning game, though doing this is not always possible as the computer's moves vary to some extent. The teacher wished to encourage them to develop strategies which would work on all games rather than simply remembering a series of winning moves. She therefore challenged these groups to win using specified but different starting points, though she avoided starting points too near the edge of the board. In the concluding whole class discussion, three of these groups were invited to share their strategies by playing games against the computer, and all were successful.

Analysis of these games reveals several components of a winning strategy in the game. These are sometimes articulated and sometimes implicit in the moves made. It is not always clear to what extent they are deliberately developed and to what extent they are intuitive.

- Choosing a suitable starting point.
- Maintaining a balance between trying to win and blocking.
- Predicting what the computer might do (and noticing what it did).
- Planning ahead.
- Seeing beyond the particular.

Video transcript analysis shows that these issues were frequently discussed, even by those who had not yet developed overall winning strategies. They are considered in turn in the sections below. Most of the discussions arose from two simple starting questions: ‘Where shall we put the next counter?’ and ‘Why?’ These led to similar issues being discussed in all four classes. Some of the discussions below occurred when the teacher was addressing the whole class, but others arose when children were working together at the computers.

Deciding where to start

Where to start was discussed in all four lessons. In three lessons, the game was played by the whole class against the computer as an introductory activity, and teachers explicitly discussed possible starting points. In the examples below the game was played at level one which involves an eight by eight grid.

Extract 1

TEACHER: …Where do you think we should put our counter, Seth?

SETH: Four-four.

TEACHER: You think four-four. That's nicely central.

SETH: I think, because it's right in the middle and we could go up across, diagonal, loads of different ways.

(School B, whole class introduction)

In one class the starting point issue was discussed in the lesson introduction and returned to several times during its course. In this class the issue of where to start was related to whether it is an advantage to go first, with some children arguing this was not the case.

Extract 2

STUDENT: If you go first you don't know where the other player is going to go so you need to know to stop them.

TEACHER: Ah, so you think it's better to know where someone's gone first so you can stop them? Brian, you think it's better to go first? Can you tell me why?

BRIAN: You need to put it in the middle.

(Later)

TEACHER: … so we're pretty much swayed over to the fact that it's only Brian who thinks it's more advantageous, so we'll see if we can come up, see if we still agree by the end.

(School C, whole class introduction)

Extract 3 (Later in same lesson)

BRIAN: We've won one game.

TEACHER: Can I ask you a question? Did you place your counter in the middle?

STUDENTS 1/2/3: Yes.

TEACHER: Brian was the only one of you three who said you should place your counter in the middle and you two said you shouldn't place your counter in the middle. Have you changed your idea about that?

STUDENT 2: Yes.

TEACHER: Why?

STUDENT 2: Because then you can go anywhere. You can go there, there and there.

(School C, group of children working at computer, teacher interacting with group)

Later in the same lesson, the teacher suggested that winning students starting in the middle started somewhere else. In many cases, they were unable to win when they started further from the middle. In the concluding discussion, there was general agreement that it was advantageous to go first and to put the counter in the middle, though one child raised the important issue that this in itself was insufficient to win.

Extract 4

TEACHER: Why do we agree with Brian? Why is it important to put the counters in the middle… go on, Todd.

TODD: Every time we went in the middle he beat us.

TEACHER: You don't think it's necessarily successful?

TODD: No. He beat us two ways.

(School C, plenary)

Winning versus blocking

Another issue students had to consider was how to maintain the balance between trying to form a line of their own and trying to block the computer. It was more common for children to concentrate on blocking at the expense of trying to win than vice versa. In one lesson, a pair of children tended to block rather than trying to win, apparently because they had frequently lost.

Extract 5

MALCOLM: Blocking him there, yeah? We're on defensive this time, yeah?

CHRISTINE: Go here.

MALCOLM: In defensive.

CHRISTINE: Go here.

MALCOLM: Yeah. Defensive.

(School D, pair of children working at computer)

Sometimes the issue of whether to try to win or try to block was considered in whole class games.

Extract 6

TEACHER: ...Brian's saying that he thinks we shouldn't be defending, we should be trying to win and forcing the computer to stop us from winning. Put your hands up if you think we should go here now. That's, I think, the majority…

(School A, whole class introduction)

It is interesting that the class taking this decision was that in which several groups believed they could always win. This issue was also raised in another class with experience of playing this game from previous lessons.

Extract 7

TEACHER: …we have started to build on our line here and we are blocking the computer. That's very, very good. I like your thinking on that. Thank you. What do you believe, Gavin?

GAVIN: I think we should go to four-three because then the computer has a valuable, a very, very valuable decision to make. Block us or we block him or he can continue his line and we win.

(Later)

HELEN: We're able to agree with Gavin, because the first job is to get four in a row instead of blocking the computer.

TEACHER: Well done, remembering our primary objective. Good.

(School B, whole class introduction)

Sometimes, when children were working at computers they discussed blocking and proceeded to build their own line. Occasionally, they held the idea suggested above that a particular move could both block and build their own line.

Extract 8

CATHY: Yeah, but I think we should do two-six, because then we're stopping him and we can get on with our lines.

JONTY: No, but then we're thinking of stopping him instead. You can do a line.

(Later)

BRIAN: I agree because it's going to stop him, so I agree, I agree.

JONTY: And it can make us go in a line.

BRIAN: Exactly.

(School C, group of children working at computer)

The issue of whether blocking or 'offensive' strategies are optimal is objectively complex, but maintaining a balance between the two is important. This can be seen by looking at the winning game by Dave's group, shown in Figure 1. This shows that the children's first three moves, (4,4), (5,4) and (6,4), were aimed at winning rather than blocking. Their next move, (3,6), was essential to block the computer, but simultaneously became part of their eventual winning line. Similarly, their next move, (3,2), although essentially intended to block, could have contributed to another line if required, and the following move, (4,5), although essential to forming a winning line, also denied the computer the option of a horizontal line at that point.

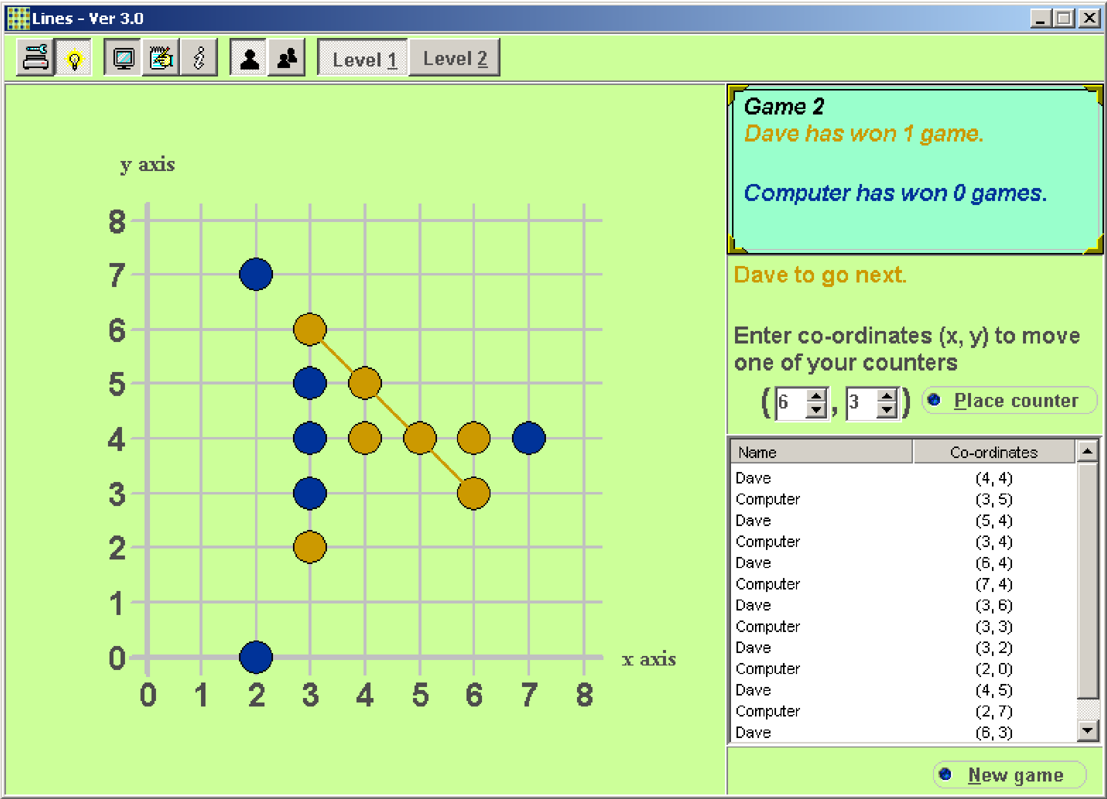

Figure 1. Dave's group win against the computer.

Predicting what the computer might do

An important part of strategy development involved predicting the computer's moves. This was raised in a whole class introduction as shown below, where the teacher reinforces a child's point.

Extract 9

TEACHER: … She's thinking what moves the computer will make and where the computer counter is placed, so she thinks four-five because she thinks that we have a chance of blocking one of the computer's ways of going.

(School C, whole class introduction)

When children were working at computers, their conversation often included predictions of the computer's next move. Sometimes, children also commented on the computer's previous move. Predictions about what the computer might do sometimes appeared to be related to children's views of the computer. Initially, some children were naïve in their views, or at least in their hopes, of what the computer might do. Similarly, they sometimes failed to see the sense of the computer's moves, as shown in the extract below.

Extract 10

MALCOLM: You know he's going to block us. If we go there we can block him. We need to block him there because we don't want to have to win a new way.

CHRISTINE: Six-seven.

MALCOLM: Ha-ha-ha. He won't do that. It's not even in line with anything.

CHRISTINE: Yeah, but look.

(School D, pair of children working at computer)

Slightly more common than naïve views were assumptions that the computer would do what the children themselves would do in the same situation. Increasingly, children appeared to assume that the computer would make the 'best' move in any given situation and was unlikely to make stupid mistakes or miss obvious opportunities. Most children seemed to accept that the computer was likely to behave in this way. In the extract below, two girls working together predict the computer's next move, and their prediction is proved correct.

Extract 11

ELLA: He'll go diagonal. We can go diagonal as well. Do you think we should go there?

ANNE: Yeah.

ELLA: Okay.

ANNE: Yes, so he did go there.

ELLA: But do you see what's happening? He can go there or there.

(School B, pair of children playing game using interactive whiteboard)

Shortly after this discussion an adult who had been listening asked the girls about their comments.

Extract 12

ADULT: What was that thing you were saying about the diagonal line?

ELLA: He was there because we started at four-five. We were at four-four in the middle. He went there and we went along because he was here. Then he went there, and there, and there, and he got a diagonal. Because he adapts.

ADULT: Right. I wonder if you could use that strategy.

ELLA: Yeah.

ADULT: To try and beat the computer.

(School B, pair of children playing game using interactive whiteboard)

In the discussions above, it seems the two girls were considering doing what the computer did, and the adult drew this point out. Although the girls introduced the idea of ‘learning from the computer’, the adult reinforced it and the girls seem to have shifted slightly from borrowing a specific type of move (a diagonal line) to borrowing the more general idea of ‘adapting’. After this discussion, the two girls began another game and won in five moves, as shown in Figure 2.

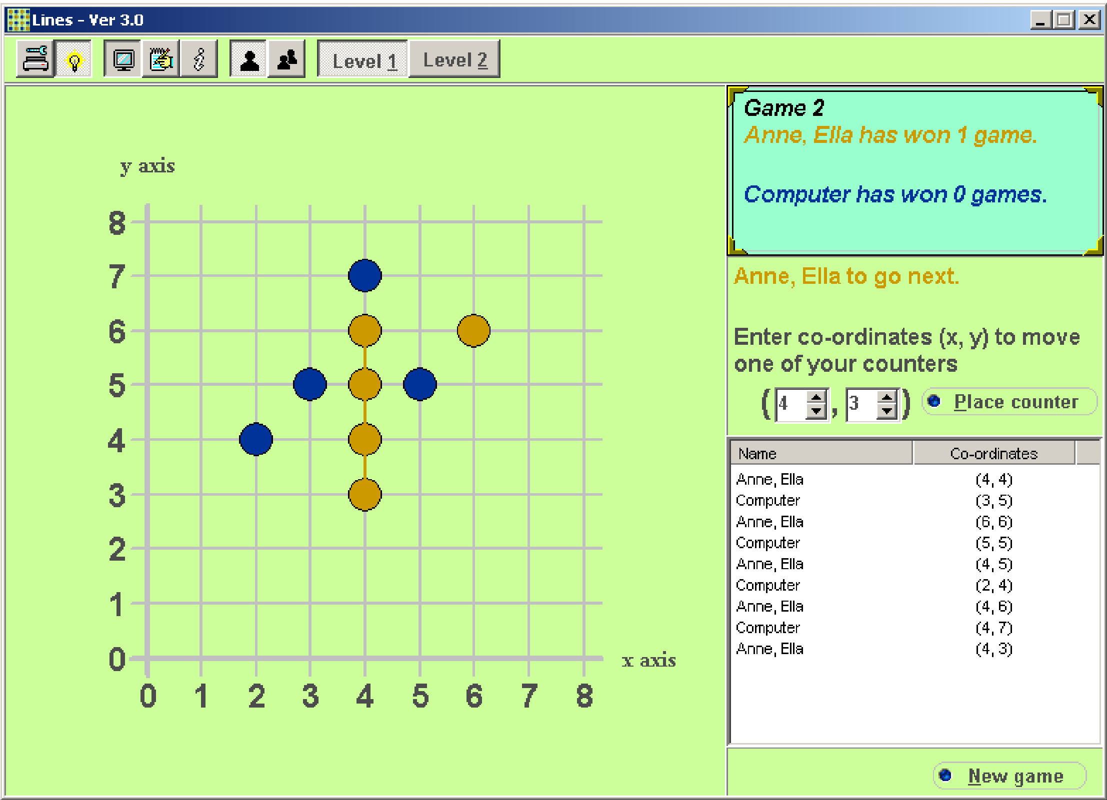

Figure 2. Anne and Ella win in five moves.

Although the computer plays what could loosely be designated an ‘expert’ strategy, its pre-programming creates an element of predictability. Some children exploited this and developed parsimonious winning strategies requiring few moves, though the same strategy might be less successful against another 'expert' player. An example is the game played by Ryan's group, who seemed to have worked out from experience the best place to put their second counter. When asked by the teacher to start in a different place, this group adapted their moves and still won. Comparison of their game (see Figure 3) with that of Anne and Ella shows similarity in the computer's moves despite using a slightly different starting point. A few children appeared to understand this similarity and used it to predict the computer's probable moves.

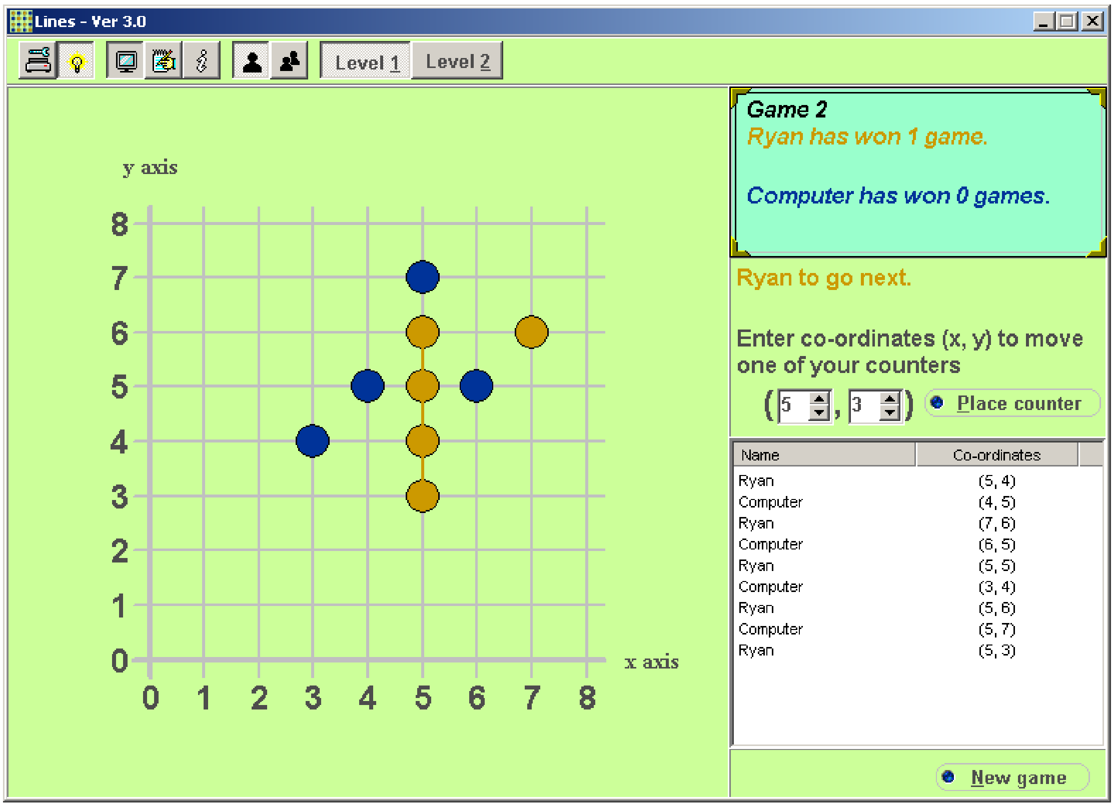

Figure 3. Ryan's group win in 5 moves.

Planning ahead

To win, children needed to plan ahead by thinking several moves ahead and usually having more than one option. Alternative moves were sometimes linked to the computer's predicted moves. The first incident below considers alternative moves. The second incident raises a similar issue but also shows language development to discuss particular moves, in this case two-way option and three-way option. This idea is further explored in the subsequent section.

Extract 13

CHRISTINE: What about…

MALCOLM: We've won. We can either go there or there. We've won it.

(School D, pair of children working at computer)

Extract 14

JOHN: If we go there…

CHARLIE: If we can go there then we can go there and we still have a two-way option.

TEACHER: It's important to maintain your two-way option, isn't it?

JOHN: But if we go there he might block us.

CHARLIE: Yeah, but we could have a three-way option.

(School B, group of children working at computer, teacher interacting with group)

Extract 15

TEACHER: Can some people explain to me why they enjoyed that game? What did you like about it? Go on, Julian.

JULIAN: Because you had to sort of plan where you were going to go. You couldn't just be 'put it there, put it there, put it there, put it there.' The computer, you might be on one route but the computer might be on a different one, blocks you off but he's still got a route going up and you've stopped your route…

(School D, plenary)

The above discussion provides a good description of the importance of planning in developing a winning strategy. In common with previous discussions, it involves forward planning. Some children took the important step of working backwards from a winning position effectively, and could therefore confidently say that winning would follow a certain move. This is illustrated in the discussion below, taken from a whole class introduction. The game position at the time of the discussion is shown in Figure 4. Discussion starts with whether to block or try to win but Darren subsequently convinces the class that they can win from this position. He was correct and they won after two further moves [i.e. (4,5) then (4,3)].

Extract 16

TEACHER: Hang on, hang on, we've got to think about this one.

STUDENT: We need to block it.

STUDENT: Two-three.

TEACHER: Alright, two-three, you think. Go on, Darren.

DARREN: If you go four-five, yeah…

TEACHER: Hang on a minute. Four-five, yeah?

DARREN: Then we go there, then there. It goes either side and you then get one behind and we've won.

(School A, whole class introduction)

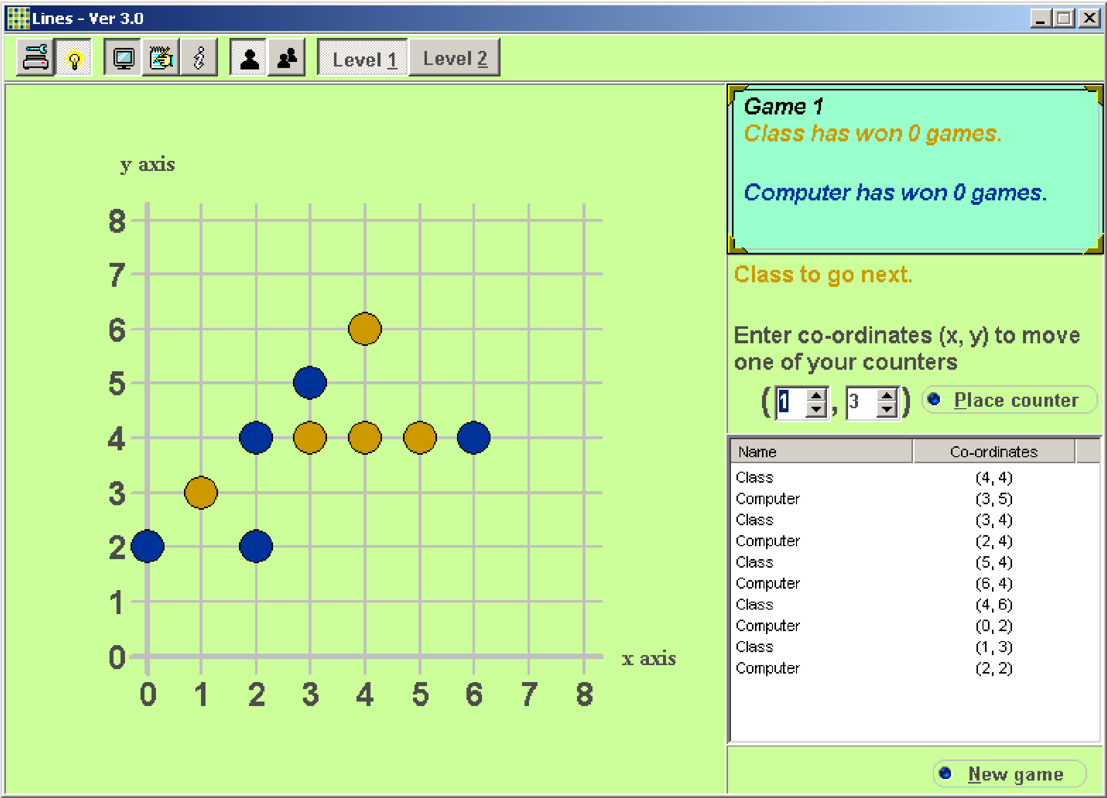

Figure 4. Class game against computer at start of Extract 16.

Seeing beyond the particular

Children were also helped by starting to see beyond particular moves and thus being able to draw general messages from their experience. Some teachers encouraged this though it proved quite difficult for children. One teacher tried to talk to the class about 'a situation like this' as shown below but the children appeared to find it difficult to grasp or to give examples of such situations. Another teacher tried to encourage children to develop general strategies rather than seeing each game as separate, as illustrated in the second incident below.

Extract 17

TEACHER: …now, from your experience of your programme, what do you think we should do? Hopefully we'll be a little bit more unanimous. Lilly?

LILLY: Four-two.

TEACHER: You think four-two. Did you come across a situation like this?

LILLY: Er, no.

TEACHER: No. Gavin, you noticed a situation like this, didn't you? What did you notice?

(School B, plenary)

Extract 18

TEACHER: …and that's really good because you're not actually going to the game each time trying to win a new game, it's you who has a set strategy and that's how chess players play as well. They have set strategies to help them win the game and that's what you're going along the lines of, actually thinking ahead. Okay. Not just thinking about this move and the next move, but knowing how you can win. Some of you, I know, have written down for me the strategies that you used. We probably won't have time to listen to everybody's, okay, and I do think there's one group I will ask at the end who I think have come up with a strategy for winning wherever they start within reason, some of it kind of in the middle.

(School A, plenary)

Although the children did not discuss generalities in the same way as teachers, their conversations suggest that they were beginning to recognise similar moves or situations. This is evident in the informal language that was generated to describe certain moves or strategies.

One strategy generating such language was forming a winning line ‘at an angle’ rather than on a forty-five degree diagonal. Although such lines were introduced to children when the game was explained, they rarely tried to form such lines, though the computer often won in this way. In the first extract below, these are referred to as ‘ones with gaps.’ In the second incident, drawn from a different class, a similar line is known as a ‘big one.’

Extract 19

MALCOLM: We couldn’t really figure out how to do the ones with gaps. So we kept trying to do them in straight lines and diagonals instead of what you said a minute ago, two down and one across.

(School D, plenary)

Extract 20

JONTY: So he’s won. There’s a big one. I get him. I think we should do a small one. If we start here, we should start a really big line.

(School C, group of children working at computer)

There were several pauses in Jonty's comment, and he initially seemed surprised that the computer had won. The diagram for this group (Figure 5) shows the computer's winning line, which Jonty calls a 'big one'. The diagram also shows that if the possibility of a 'big one' is ignored, Jonty's group appear to be in a winning position before the computer's final move, hence their surprise.

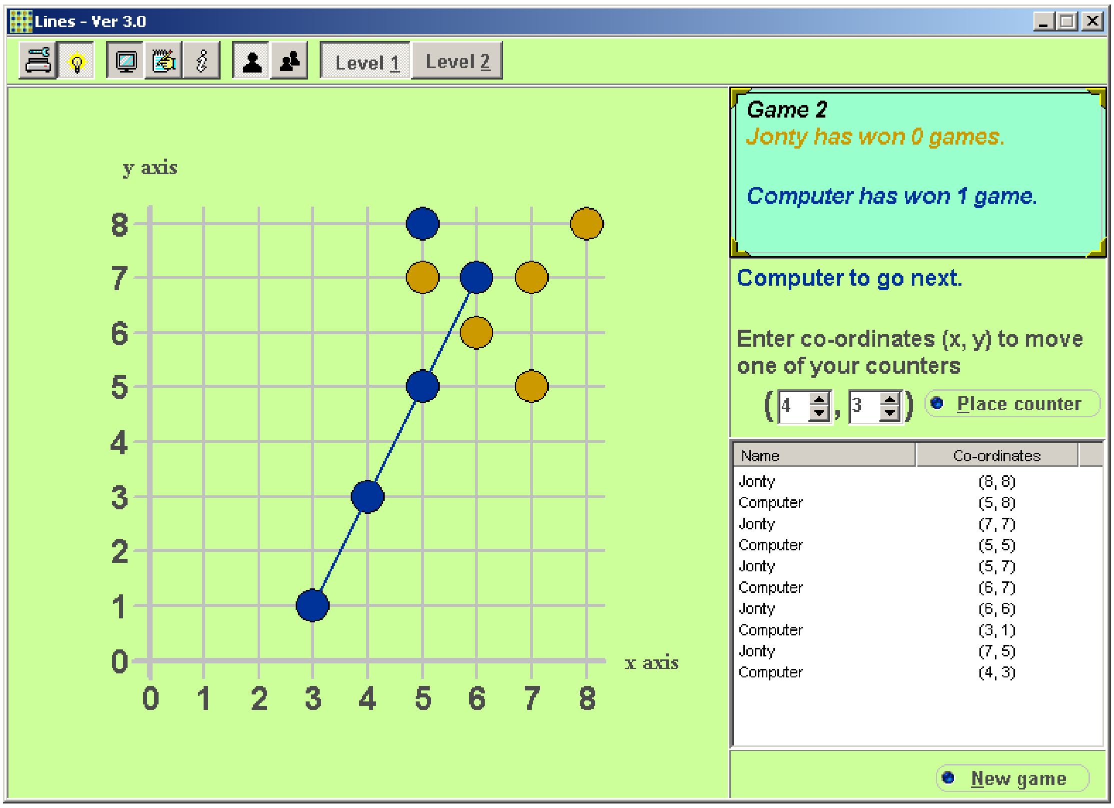

Figure 5. Computer wins against Jonty's group with a 'big one'

In both examples above, children have invented terminology to describe a particular way of winning. They are transcending using the terminology to describe what has happened, and are combining it with the idea of learning from the computer.

Another informal phrase occurred in the last section where discussion occurred about two-way and three-way options. In another school children developed the idea of the two-way trick. The conversation below occurred in a plenary when a group of children demonstrated their winning strategy on the harder level of the game, using a 10 by 10 grid.

Extract 21

STUDENT: Four-five and then they go two-four. Then we go seven-seven.

STUDENT: Have we still won?

STUDENT: Yeah, we've got a two-way trick.

(School A, plenary)

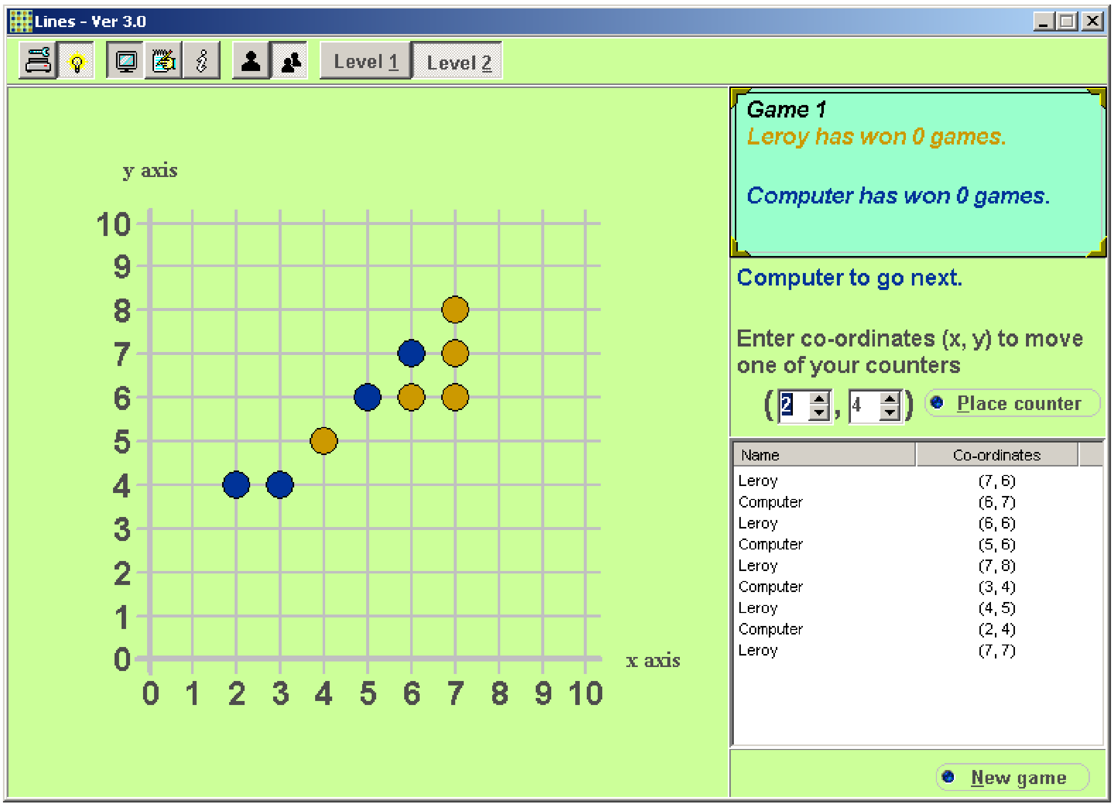

Figure 6. Leroy's group have a 'two-way trick'.

The diagram for the game at this point (Figure 6) illustrates the student's meaning when he claims to have a two-way trick. As the children have three counters in a line and a space at either end they must win because the computer can block one end, but they win by going to the other end. In addition to recognising two-way tricks, some children in this class also made the significant move of attempting to set up a two-way trick to win, thus articulating part of a winning strategy.

Extract 22

BRIAN: What do we do now? Try and get a two-way trick?

(school A, group of children working at computers)

7. DISCUSSION

Identifying the mathematics

The 'Lines' program clearly offered children the opportunity to practise using co-ordinates, and the motivation to do so without making errors. This practice was provided as children tackled the more complex task of trying to beat the computer. This is consistent with the approach advocated by Hewitt (1996) who says that practice of routine tasks can be subordinated to more complex tasks. The program offered opportunities to use mathematical language, for example 'horizontal, vertical, diagonal'. There were also occasions where precision in the use of mathematical language was important, for example when discussing 'directions' or 'diagonals'. However our focus here will be on how using the program in the way described offered opportunities for mathematical reasoning.

Prediction, considered by English curriculum documents (DfEE 1999) a constituent of mathematical reasoning, was central to developing a winning strategy. Prediction was generally related to what the computer might do, together with the consideration of alternatives. The use of phrases of the 'if … then …', type identified by Mason (2002) as part of mathematical reasoning's discourse, were common in discussing what might happen.

To develop winning strategies, children needed to see beyond particular moves or games and to develop general ideas about the type of moves that could help them and the sort of situations that might develop. It became clear that children were doing this as informal language developed to describe certain situations. A prime example was the phrase 'two-way trick' to describe a winning situation. The development of this and similar phrases signified that some children were starting to appreciate the game's underlying generalities. In addition, development of these phrases offered children a further tool to discuss games with each other, but could also develop their thinking further to realising that a two-way trick was required to win against an expert opponent, and hence to start trying to set up two way tricks. This resembles Sfard's (1991) process of 'condensation', whereby individuals become increasingly capable of thinking about something as a whole rather than delving into details, with a name or label usually introduced as part of the process. Sfard argues that as a result of this, making comparisons and generalising is rendered considerably easier.

The role of the computer

The literature suggests that successful use of computers depends both on the software and how it is used. Although many computer games have limited use, the fact that this is a game of strategy rather than a drill and practice game is important. One key factor in its use was asking children to play together against the computer rather than against each other. One consequence of this was that children were motivated to try to beat the computer. Another important factor is that because the computer was playing an ‘expert’ strategy, the children were compelled to develop sophisticated strategies themselves to have a prospect of winning as they rapidly realised that there was little hope of the computer not taking winning opportunities if offered. Thus it can be argued that the computer had a role in encouraging children to develop components of an expert strategy, such as prediction.

Increasingly children based predictions on experience of what the computer had done in past games, as shown in extracts 10 and 11. The computer also modelled winning strategies, so that some children learned possible ways of winning from it, as shown in extracts 11 and 12. Thus, having the computer as an opponent meant, crucially, that children had to develop ‘expert’ strategies to win and that such strategies were demonstrated for them.

Other factors assisting mathematical development

Another factor in the way the computer was used in this project was the encouragement to develop *exploratory talk* via the talk rules and the modelling of such talk by the teachers. In particular, the apparently simple question ‘Where should we put our counter?' followed by 'Why do you think that?' enabled many children to reason together in ways that led them to develop winning strategies which they might not have been able to achieve without being taught how to reason together effectively. This finding is consistent with those of related projects (Mercer at al 1999, 2004). Much of the talk in the current project was between teacher and children in the whole class parts of the lesson, due to changing teaching styles in primary mathematics in England. It is therefore relevant to consider teachers' roles in helping children to develop their reasoning.

Initial analysis of one transcript, which is reported elsewhere (Monaghan 2004), highlighted the teacher's crucial role in modelling appropriate language and behaviour. The extracts quoted make it clear that the teachers abide by the talk rules in encouraging children to explain their ideas and give reasons, while trying to establish a climate in which all ideas are listened to, with agreement as an eventual goal. Teachers sometimes did this in a neutral way, for example when inviting children to vote to reach a decision. On other occasions, teachers slightly *adapted* the talk rules to influence events. Thus although all ideas from children were accepted and considered, some were reinforced by teachers as particularly useful or important. Occasionally the reinforcement was subtle as in extract 1, 'That's nicely central', and sometimes more direct as in extract 7 'Well done, remembering our primary objective. Good.' They thereby channelled children towards more fruitful approaches.

Sometimes teachers expanded on children's points, as in extract 9 where the teacher mentions predicting the computer's moves. A variation exists in extract 12 where the adult encourages children to expand on an important point they have raised. Sometimes the teachers reminded children of points discussed previously, as in the discussion about starting points in extracts 2 and 3. Another important teacher role (seen in extracts 17 and 18) was to encourage children to think beyond particular situations and develop strategies which would always work. Thus, teachers channelled learners' thinking, expanded on it, reminded them of previous relevant ideas and encouraged generalisation.

The way teachers adapted the talk rules in order to emphasise certain points while still maintaining discussion of alternatives is consistent with other researchers' findings. Sherin (2002) identified a similar process, described as filtering, in a classroom where the teacher was attempting to establish a mathematical discourse community.

In sum, clear evidence of mathematical reasoning occurred. In particular children conjectured, and modified and justified conjectures, practices important to mathematicians but not always present in primary mathematics classrooms. This was encouraged by a combination of factors: a game of strategy with the computer as opponent, teaching children how to collaborate effectively using exploratory talk and appropriate teacher behaviours to adapt the 'talk rules'. Computer use was moreover beneficial since all children were able to engage in such reasoning because the game provided an accessible starting point not requiring specialist mathematical knowledge on the part of children or teachers.

8. CONCLUSION

The research questions may therefore be answered in the following way. Positive accounts of computers' utility in the primary classroom are supported by our analysis, provided that certain conditions are met. A precondition for children demonstrating and developing mathematical reasoning is clearly that the program itself encourages it, and this was the case here. There was clear evidence of mathematical reasoning at a level unusual in many primary classrooms. Indeed, the computer itself modelled winning strategies for children, allowing them to learn directly from it. Children developed winning strategies through collectively creating an appropriate language ('two-way trick'). This reflected use of the game by groups of children playing against the computer rather than against each other. Teachers' modelling of exploratory talk was helpful in developing this language, and it was important that on occasion teachers modified the rules in order to help children move forward. Thus, the rules of discourse were on occasions constructively adapted by teachers rather than applied rigidly.

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ACKNOWLEDGEMENT

This work arises from the 'Thinking together with SMILE project', which was based at The Open University, directed by Rupert Wegerif and funded by Nuffield. Thanks are due to the teachers and children who took part in the project. The authors would also like to thank members of the project team, the Centre for Mathematics Education and SMILE Mathematics who have assisted with this article.

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