THE LANGUAGE DIMENSION OF MATHEMATICS TEACHING

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How do children develop their understanding of mathematical vocabulary?

Teachers often use informal, everyday language in mathematics lessons before or alongside technical mathematical vocabulary. Although this can help children grasp the meaning of different words and phrases, you will find that a structured approach to the teaching and learning of vocabulary is essential if children are to . . . begin using the correct mathematical terminology . . .

Some children may start school with a good understanding of mathematical words when used informally, either in English or their home language. Find out the extent of their mathematical vocabulary and the depth of their understanding, and build on this.

You need to plan the introduction of new words in a suitable context, for example, with relevant real objects, mathematical apparatus, pictures and/or diagrams. Explain their meanings carefully and rehearse them several times . . . Encourage their use . . . through your questioning. You can help sort out any ambiguities or misconceptions . . .

The final stages are learning to read and write new mathematical vocabulary in a range of circumstances, ultimately spelling the relevant words correctly.

(*NNS Mathematical Vocabulary Book*, DfEE, 1999, p.2)

How do you teach the mathematical meaning of a word like dimension? Do you give a definition from a textbook or a dictionary? Do you use a few 'real' shapes or draw diagrams on the blackboard? Or do you allow children to 'pick it up' through hearing you use the word and eventually using it themselves?

The NNS vocabulary book [1] offers some advice on how children develop their understanding of mathematical vocabulary. The booklet advocates a 'structured approach to the teaching and learning of vocabulary', suggests 'explaining meanings carefully', and invites you to 'sort out any ambiguities or misconceptions your pupils may have'. (An edited extract from the booklet is shown in the box, below). What do you think?

Applied linguists study the many aspects of how language is used. A discussion with applied linguists could help think about children's learning and use of mathematical language. For this reason, at a recent mathematics education conference, we organised a discussion group on language and mathematics education. Two of us are applied linguists (Constant and Brian) and two of us work in mathematics education (Richard and Candia). The group met at the British Congress of Mathematics Education meeting held in June last year, and involved teachers, advisers, researchers and others involved in mathematics education. Our discussions were stimulated by the NNS advice referred to above. We also looked at a transcript from a Y5 lesson in which the class work on the concept of *dimension* [2]. An extract is shown in the box on the right (in the transcript, **bold** shows extra emphasis, round brackets ( ) show where the recording is hard to transcribe and square brackets [show where two people are talking at once.

In this article we each think about an issue that the discussion group led us to consider. Our purpose is not to summarise the discussion which took place, but to capture how our own thinking was taken forward. We would be delighted to hear from the other participants who were present.

Candia

The main issue that arises for me is the way in which the booklet represents the nature of mathematical language. The title of the booklet (***Mathematical vocabulary***), its format (lists of words) and the emphasis on 'vocabulary', 'terminology' and 'words', sideline other aspects of mathematical language. Knowing words is obviously crucial, but it is not enough for communicating mathematically. Learners also have to be able to make sense of the special ways in which the words are put together to make mathematical meaning.

Discussion of the *dimensions* transcript helped me to achieve a clearer focus on this issue. In the guidance, teachers are advised 'to explain meanings carefully and 'to sort out ambiguities', so I considered what that might mean in the context of the *dimensions* lesson. In the lesson the talk of the children and the teacher constructs a multi-faceted notion of dimension. This includes:

- the idea of 2D as 'flat' and 3D as 'solid'
- listing dimensions (breadth, length, height, etc.) invoking an implicit 'two-ness' or 'three-ness'
- a notion of 3D involving something 'extra' when compared to 2D
- diagrammatic representations of 2D (a square) and 3D (a 2D isometric drawing of a cube)
- imagining what might be meant by one and even zero dimensional objects.

All of these aspects of the meanings of *two-dimensional* and *three-dimensional* seemed relevant, valid and at some points mathematically sophisticated, though often incomplete or ambiguous. Yet at no point during the lesson did it seem possible or even appropriate to explain or to remove all ambiguities from the ways in which the words were being used.

As advised by the NNS, I turned to a mathematical dictionary [3] and found the following definition for *dimension*,

> *the number of measures needed to give the place of any point in a given space, the number of coordinates needed to define a point in it.*

It seems unlikely that such a formal definition is accessible to Y5 children or very useful to their teacher. I would also question whether it captures the richness of the mathematical thinking about dimensions that the children and teacher were engaged in during the lesson. Moreover, even this definition is not entirely unambiguous, as the nature of the 'given space' is left open. For example, the question of whether a circle is one-dimensional or two-dimensional is not immediately resolvable. This is not a weakness in the definition but a characteristic of the mathematical concept itself.

Listing vocabulary may be helpful in drawing teachers' attention to some aspects of mathematical language that children need to be able to use. The danger is that by isolating words in this way, the complexity of mathematical language and of mathematical meanings is misrepresented.

Constant

As an applied linguist I found the NNS advice a complex web of ideas, values and beliefs that reflect a particular social and educational point of view. I will mention two issues which interested me.

Firstly, the strong emphasis on specialist mathematical vocabulary raises an interesting issue about what counts as mathematics in the mathematics curriculum. As the guidance implicitly acknowledges, it is possible in primary school to talk mathematics in 'informal', everyday language. The argument here, however, appears to be that this kind of talk *is not* mathematics until formal vocabulary is used. This view may be predicated on the assumption that

'Dimensions'

7	T:	What's the difference then between two dimensional and three dimensional. W tells us it's **flat** that's fine. Are there anything else to say. F.
8	Student F:	Um a (three dimensional shape) has breadth, length and height.
9	T:	Well done. This would be a two dimensional shape (*draws a square*) (...) and a three dimensional shape will have an extra dimension. That would be a **solid** shape (draws a cube) okay G.
		...
36	Student V:	And a sphere is three dimensional
37	T:	And a sphere is three dimensional. What would be a one dimensional circle then?
38	Student A:	(...) a line (*shrugs*)
39	T:	Just a diameter (*points to diameter from before*). Yes J
40	Student J:	(mm a two dimensional is flatter ...)
41	T:	Yep flat. Look. (*picks up a plastic circle from a set*) I don't like these (...) coz they look like three dimensional don't they. They're thick but they're not meant to be, they're meant to be two dimensional. Okay, they're flat shapes (*picks up a square*)
42	Student:	A cylinder
43	T:	Yeah that's a cylinder (*laughs, waves circle*) (and that's a)
44	Student:	a cuboid
45	T	cuboid (*waves square*). But it's not meant to be it's meant to be flat. Yes K.
46	Student K:	There's no such thing as a one dimensional shape coz a line is kind of like a rectangle filled in
47	T	Yeah. What just a line? (*points to board*)
48	K	Yeah
49	T	Like a- what like [(...) (*gestures thinness*)
50	K	[a rectangle filled in
51	T	(*Giggles*) Very clever. Like a dot (*draws dot*) oops (*erases, does again*) like that. It's interesting isn't it. Yes H?
52	Student H:	(...) sometimes things made out of paper's um um two dimensional
53	T	Yeah
54	Student H:	(...) has just a tiny tiny **tiny** (*gestures thinness*)
55	Student	Very thin
56	T	So you've got to draw it on paper so it's going to have certain thickness (*gestures thinness*).

Some pupils, for a variety of reasons, may access new information and specialist language through reading as effectively as through speaking or listening.

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primary school pupils need to be prepared for the more formal mathematics content and language which they will encounter in secondary school and beyond. This assumption raises the question of the relationship between formal language and mathematical content in a wider sense. In the transcript of the Y5 lesson there are several instances of highly relevant contributions which may at first glance appear to be expressed in non-mathematical language. For example, when exploring the meaning of one (or indeed two) dimension:

54 Student H: (...) has just a tiny tiny **tiny** (*gestures thinness*)
55 Student Very thin

These utterances arguably represent a moment of talking, thinking and learning about mathematics. If we accept this to be the case, then there is a need to pay attention to this kind of seemingly non-specialist mathematical language.

Secondly, the NNS advice seems to suggest that mathematics learning takes place in and through a sequence from spoken to written language. This sequence clearly applies to very young children. In the primary years, however, it is *not* the case that all pupils necessarily:

- develop mathematical language in a fixed sequence of spoken language first and written language last
- rely solely on mathematical vocabulary to read and write mathematics texts.

As children's reading and writing develop, their understanding of written language can enhance speaking and listening. Some pupils, for a variety of reasons, may access new information and specialist language through reading as effectively as through speaking or listening. The many pupils with overseas educational backgrounds who are learning to use English as an additional language may actually find classroom spoken language more difficult to access than written text in a subject such as mathematics.

The NNS advice seems to assume that reading and writing in mathematics activities will follow once mathematical vocabulary has been taught and rehearsed. Reading and writing are far more complex processes which require, among other things, a knowledge of conventions and curriculum expectations.

Richard

The structured approach suggested by the NNS guidance makes teaching mathematical language seem a straightforward process of explaining meanings and 'sorting out' ambiguities. During the discussion group at the BCME conference, I became intrigued by the notion of 'sorting out' ambiguities. Are ambiguities a problem in mathematics teaching and learning?

Some participants linked the issue of ambiguity to the role of formal definitions. Perhaps a clear definition can remove any ambiguity. So where do definitions come from? Dictionary definitions, like the one Candia quoted, are generally derived from how words are *used*. The idea of students *using* language is largely missing from the NNS perspective, which places the onus firmly on the teacher to define, use, draw attention to and encourage the use of words through questioning. Yet from a language point of view, meaning arises through using words to mean things.

In the *dimensions* transcript, the teacher sometimes behaves in NNS ways. At one point, for example, she rehearses vocabulary for parts of a circle: circumference, diameter, radius. At other moments, however, the class engages in an intriguing exploration of the meaning of dimension, through working on what can be said with the word and done with the concept. The discussion is full of ambiguities, some of which are made explicit, such as when the teacher observes that her plastic shapes are supposed to be 2D but are in fact 3D. A student extends this idea, commenting that there cannot be a 'real' version of a 1D shape, since a line on paper must always be 2D. There is an underlying ambiguity in this discussion concerning the relationship between mathematical concepts and their representation, something that arises at all levels of mathematics. We can draw a 'circle', for example, and even though it can never be truly circular we call it a circle anyway. Indeed one of the features of mathematical talk is this regular use of a particular, imperfect exemplar to stand for a whole class of objects. The drawn circle stands for all circles that can be imagined. The plastic shape stands for all 2D squares, even though it is really three dimensional. This way of talking and thinking is as much a part of mathematics as particular concepts like *dimension*. It is not enough for students to hear a few examples of a word being used or to be given a formal definition. They need to explore the concepts involved, push at the limits of definitions, change them, and most of all, make the meaning their own as they learn to talk mathematically. In the dimensions transcript, the richest mathematics is taking place not when vocabulary is being rehearsed, but when words and concepts are being explored. Arguably it is at these times that students' mathematical language is also developing.

Brian

The 'dimensions' discussion brings out some of the complex features of language. The teacher, for example, has implicitly recognised a key feature of language – the establishment of social relations and of rights to the floor. Within this framework, the students are able to explore, question, attempt solutions and then listen as others propose alternatives. Language is used as a resource rather than as a fixed set of rules. The 'rules' model of language, however, is privileged by the NNS document with its concern for 'correct terminology', 'spelling correctly' and 'sorting out ambiguities or misconceptions'. As the other contributors point out above, ambiguity may be endemic to much mathematical reasoning and can be a productive and important part of children's exploration. To align ambiguity with 'misconceptions' already gives the wrong message and to suggest that it can be eradicated sets up problems for the development of students' knowledge. To suggest, further, that this is the function of 'language' is to misconceive the actual uses and richness of language as a resource that the 'dimensions' lesson illustrates.

The 'dimensions' discussion also shows how children can call upon *multi-modal* resources [4] to explore the ambiguity embedded in the concept of 'dimension'. By drawing shapes in the air, touching and feeling plastic objects and pointing to drawings on the board, children complement their language use by using other modalities – seeing, gesturing, touching. This is possible because the context provided by the teacher is one of exploration and of interactive social and linguistic relations. She facilitates their participation not only by open questions (line 37), but by statements that students can follow up:

41	T:	Okay, they're flat shapes (*picks up a square*)
42	Student:	A cylinder

She also notices children wanting to contribute and gives them space to do so, in this case eliciting the most complex statement of the extract: "There's no such thing as a one dimensional shape coz a line is kind of like a rectangle filled in" (line 47). Again she reinforces this complex comment, "yeah", and immediately follows up with a further question and gesture that arise from the interchange including the child as a full participant: "What just a line? (*points to board*)".

The rest of the interchange develops the concepts further but also establishes a comfortable social relationship, through, for example, giggling, gesturing, questioning, affirming, and admitting mistakes ("oops").

This interaction bears very little relation to the NNS statement on vocabulary. A 'structured approach' does not necessarily demand 'correct terminology or 'sorting out ambiguities or misconceptions' nor the categorical advice of the NNS text with its shoulds, needs and directives. Instead ambiguity is fore-grounded and recognised, multimodality is invoked, not just vocabulary, and there is an implicit awareness of the interactive and social nature of language in use.

Discussion

The above discussion distinguishes between the rather simplistic view of language implicit in the NNS document and the rich and complex ways in which applied linguists approach language as a social activity. The simplistic view frequently uses 'language' to refer to 'vocabulary', as illustrated above. Likewise the concept of 'definition' is frequently taken to require a precise meaning for a particular word. The broader view taken by applied linguists looks beyond vocabulary to consider other dimensions of language, including:

- seeing language as a process rather than as a fixed entity and as a resource rather than as a set of rules
- exploring the social role of language, such as in establishing relationships
- recognising how language use is woven in with other means of communication and meaning making, such as gestures and symbols.

The teacher and students in the lesson extract work with these dimensions of language throughout their discussion.

These ideas raise many questions for our practice as mathematics teachers. We conclude, therefore, with an invitation to reflect. How do we use spoken and written language in our teaching? How do our students use language in our lessons? How do we use the non-verbal aspects of language, the visual or the tactile? How does the social aspect of language use relate to mathematics teaching and learning in our classrooms? And what opportunities do we offer students so that they make mathematical meaning their own?

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Information about the British Congress of Mathematics Education can be found at www.bcme.org.uk

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References

1 NNS ***Mathematical vocabulary book***, DfEE, 1999.
2 The transcript comes from research by Richard Barwell (see 'Whose words?' MT 178) which is funded by ESRC grant number. R00429934027.
3 K. Selkirk, ***Longman mathematics handbook***. Harlow, Longman, 1990.
4 G. Kress & T. van Leeuwen, ***Multimodal discourse: the modes of contemporary communication***. London, Arnold, 2001.