# The shape of habitable space



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**Abstract**
How is it that a representation of the state of a system – such as the axial map – can explain dynamic behaviour such as movement flows? This paper investigates the relationship between spatial configuration and behaviours that take place in time – specifically, movement. A method is presented for incorporating time systematically in a representation of spatial configuration. This is based on assuming a universal maximum walking speed for pedestrians, and it is shown that the resulting three dimensional mapping of space-time can be constructed from sections of the surface of cones. Properties of this representation are investigated and first it is shown that a uniform grid results in an approximately flat surface in space-time. All the main forms of deformation of the urban grid are found to result in 'warping' the space-time surface of the uniform grid into valleys and ridges. A method is proposed for summing space-time surfaces constructed from all root locations. Finally, the implications for space syntax theory and methodology of the space-time representation are discussed. It is concluded that one of the properties of the conventional axial map is that it internalises aspects of the temporal domain within its construction, and this may account for its explanatory success.

**Keywords**
space,time,
experience



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## Introduction

Space syntax analysis has developed a number of methods for representing and quantifying the morphology of built space. The methods start with the shape of the boundary of space and work back to a subdivision of continuous space into a discrete subset of related 'spaces' (such as axial lines or convex spaces) which then form the subject of study. In working back from the boundary the methods adopt a predominantly allocentric as opposed to an egocentric world view, and this characterises the whole theory of the social logic of space. Thus for instance the methods measure how a specific 'space' is constituted by its relations to all other 'spaces' in a system, while the theory considers an individual's identity to be constituted, not only by their own subjective view, but also by the views of all others in the social group. This approach has been borne out empirically through its ability to explain various aspects of the social functioning of buildings and urban environments, and through its ability to predict aspects of aggregate human behaviour such as average movement patterns over time. There is however considerable interest in understanding the mechanisms at the individual level that give rise to these observed regularities in aggregate behaviour. Here I propose that two factors need to be brought into the theoretical and methodological framework if we are to do this. First, the egocentric world view, which considers the morphology of space from an individual's current viewpoint. This brings with it the dimension of orientation or heading, since individuals have a forward facing field of vision. Second, the dimension of time within which individuals experience the spatial and social environment. The time dimension brings with it the issue of metric space since individuals can only move within a relatively constrained range of speeds.

**Habitable space**

62.2 In order to investigate this I define 'habitable space' as a mathematical space with the following properties: two spatial dimensions in plan (everything defined here could be extended to three spatial dimensions, but note that humans are generally constrained to move on the ground plane); a time dimension (which I have chosen to display vertically); an angle of vision (assumed to be a constant for an individual) and a heading (Figure1).

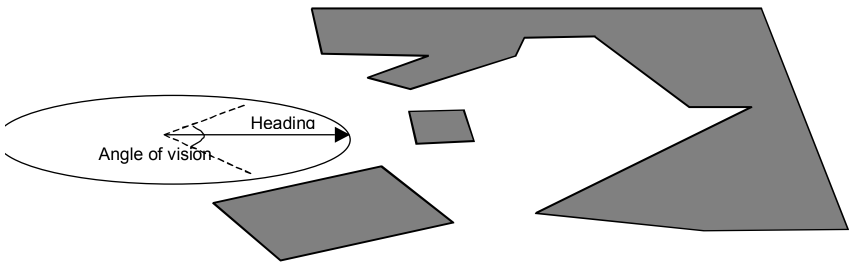


**Figure1: Orientation and angle of vision within a two spatial dimension environment define an individual's instantaneous field of view. Their maximum speed defines the locations they could reach within a given time.**

Time represented vertically gives rise to a cone of accessible space-time, thus an individual moving at their maximum speed appears as a path on the surface of the cone, if static they trace a vertical path through time with no change in the spatial components. If at a speed less than the maximum the path lies within the volume of the cone. Their heading, angle of vision and the morphology of the environment give rise to a field of view (Figure 2). This field of view defines at any instant the range of locations towards which an individual is able to move directly, including points in open space and on the boundary (for example paintings on a wall or doorways to other spatial systems). The visual field at any point also defines the other individuals with whom they are visually co-present in the environment at that time, and so with whom they can potentially interact 'face to face'.

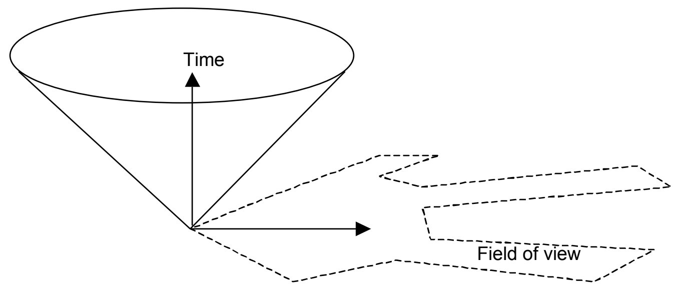


**Figure 2: Time cone and field of view**

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It should be noted that people are also thinking beings, with imaginations and memories. They have therefore a more or less well informed understanding of the spatial potential of the system outside the boundary of their current visual field. Thus their decisions on where to move next should generally not be assumed to be completely (or even mainly) dependent on what they can currently see, but must often be informed by their beliefs about the morphology of space beyond their current visual field. However, in formulating a plan to reach an objective outside the immediate visual field, an individual is constrained to move directly towards some point that is currently visible.

As an individual moves through space their position changes relative to the boundary of the environment and so does their field of view (Figure 3). In a similar fashion, as they change their gaze direction or heading so their field of view changes. At a particular instant however, the affordances of the environment including the other individuals with whom they can potentially interact are defined by the field of view. This defines the information available to the individual through vision. Movement and change of direction give rise to a landscape in this three dimensional space-time. This can perhaps best be represented as a surface made up of sections of the surface of cones. In order to illustrate this for the hypothetical building plan shown in Figure 4 these surfaces are developed from two different points of view.Figure 5 shows the surface from a relatively accessible point (A) in the centre of the plan and Figure 6 from a relatively isolated point (B). In each figure the three dimensional surface is shown from four different viewpoints.

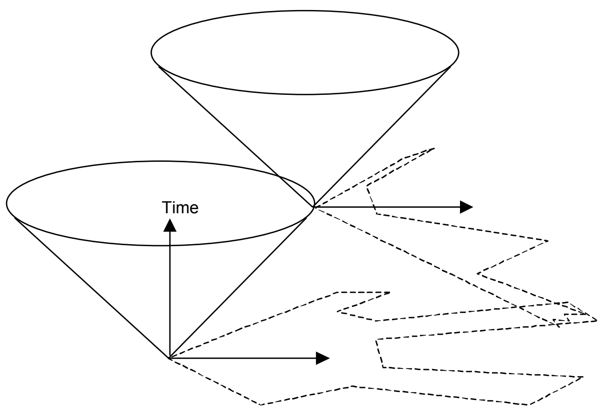



**Figure 3: Field of view changes through time with movement**

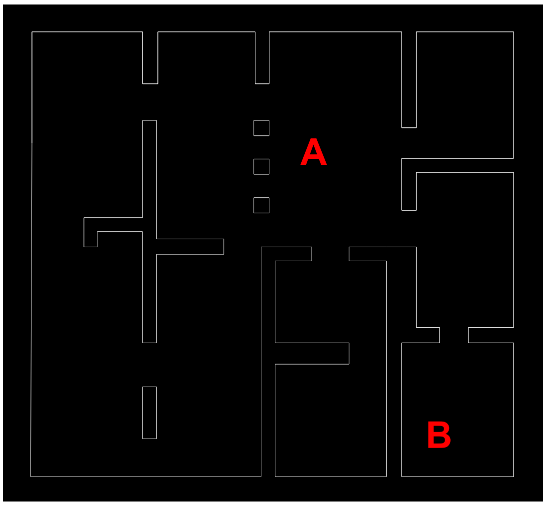
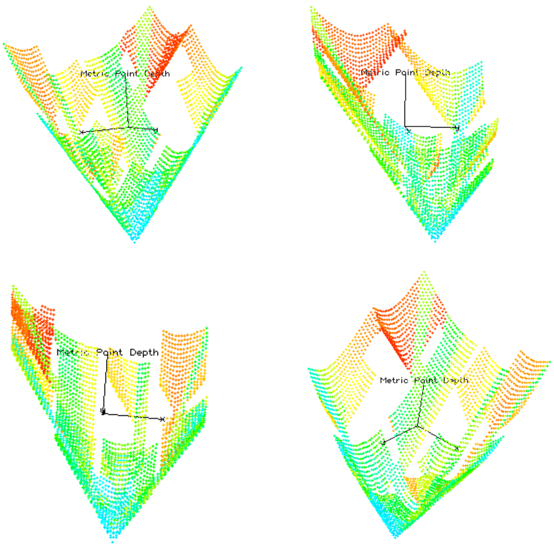



**Figure 4: Plan of a hypothetical house with points A and B marked.**





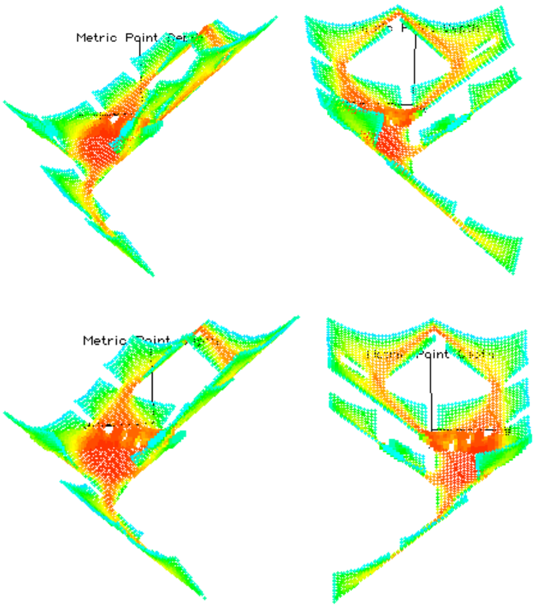



**Figure 5: Space-time cone from a central location A on plan (viewed from different viewpoints since this is a 3D surface)**

**Figure 6: Space-time cone from an isolated location B on plan**

Space-time cone representations share a characteristic of the justified graph used in conventional space syntax analysis: they show differences in spatial relations for different root locations, and so show how a root location is 'constituted' by its relations to other locations in the configuration. In contrast to the justified graph however, space-time cones directly represent the detailed geometric shape of the spatial configuration since they array every point in space vertically on the cone's surface according to its metric distance from the origin location. In this way they can be produced from the plan form without the intervening step of representing continuous space as a set of discrete 'spaces' (convex, axial, etc.). In point of fact the way these diagrams are produced does involve the subdivision of the plan form of the building into a discrete grid, however the principle of this is that the subdivision could be arbitrarily fine and so can be assumed, in the limit, to hold for a continuous representation.

Even relatively simple spatial configurations are found to be objectively different when viewed from different root locations. Figure 7 shows that a circular space is a perfect cone when considered from its centre (7a) and a pointed 'Pringle' shape considered from an edge (7b). Similar effects can be seen for a rectangular space (Figure 8) considered from its centre (a), an edge (b) and a corner (c).

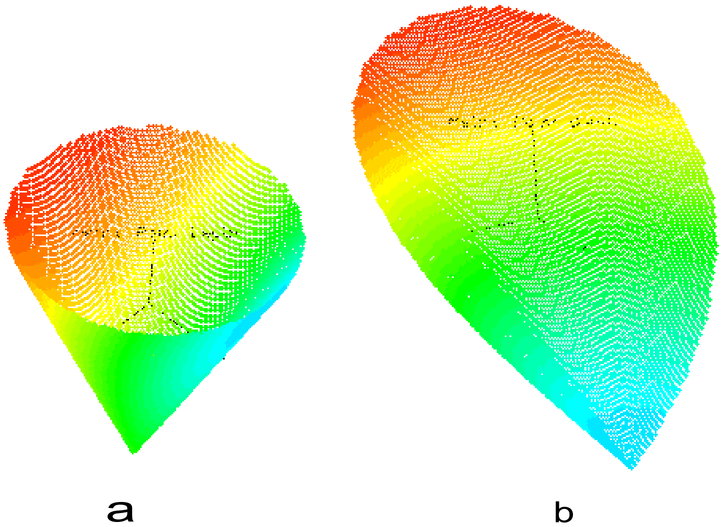



**Figure 7: A circular space viewed from its centre (a), and from its edge (b).**



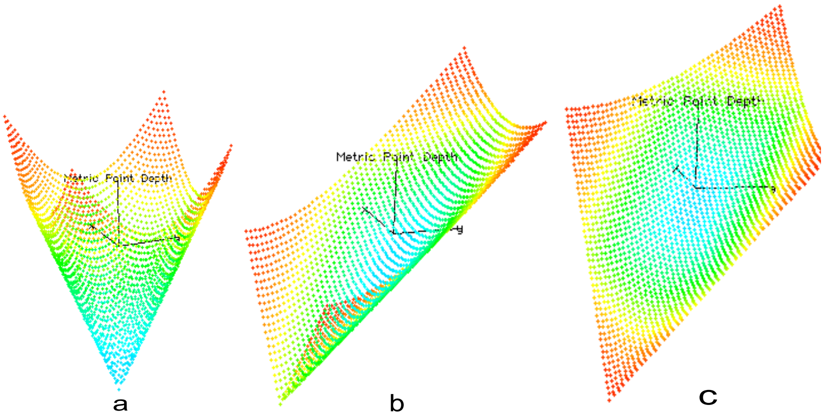



**Figure 8: A rectangular space viewed from its centre (a), an edge (b) and a corner (c).**

Slightly more complex spaces such as an L-shaped room show an important property of the representation. An L-shape, when considered from its elbow is visible in its entirety, and so is all laid out on the surface of a single space-time cone – there is a direct path from the apex of the cone to any point in the L-shaped space along a single heading (Figure 9a). However, when considered from one leg of the L, the other leg is around a corner, and will involve a move to the elbow of the L, a change of heading and a second move to any destination point. This means that the space-time representation from this point of view is made up of patches of the surface of two cones (Figure 9b). The first with its apex at the root location, the second with its apex at the inner corner of the L-shape's elbow, with the two conical surfaces glued together along the line extending through open space from the root location tangential to the inner corner.

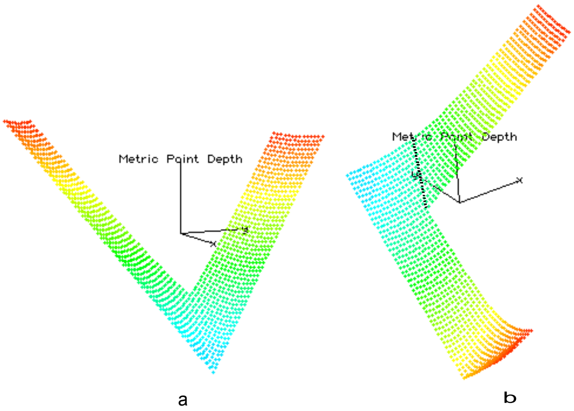



**Figure 9: An L-shaped space viewed from its elbow (a) and an arm (b).**

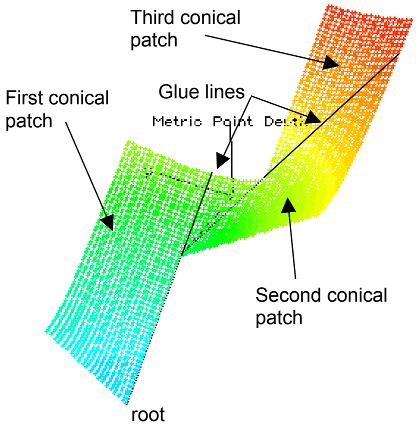



**Figure 10: A surface from one leg of a Z-shaped space is composed of a series of conical patches.**



If the system is further complicated by introducing another corner to make a Z-shaped space we find that a third cone surface patch must be glued beyond the second internal corner, this time along a line extending from the apex of the previous conical surface, tangential to the second vertex (Figure 10). This method of gluing conic patches together along their straight-line edges guarantees that the surface is smooth and continuous. New cones are located at convex vertices on the boundary where a change of heading is required if all points in space are to be reached along minimum time paths. It is a property of cones with their apex at a vertex that any path tangential to the vertex maps onto a straight line on the cone surface.

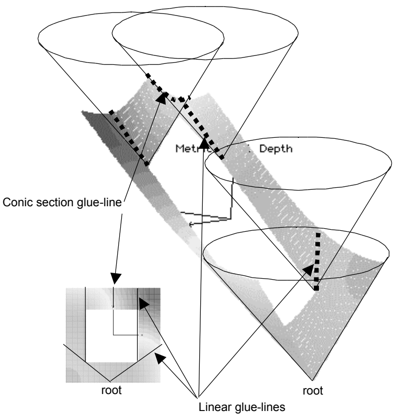



**Figure 11: A general space-time surface is composed of a patchwork of part surfaces of cones, 'glued' together either on straight line edges extending from convex vertices on the boundary, or at conic sections where two cones intersect.**

It is immediately apparent that second and subsequent 'glue lines' will be on the same alignment as the extension past vertices of lines in the all line axial map, and substantially the same as Peponis e-partitions (although with some minor definitional differences. Peponis et al., 1997).

There is a second type of condition that is regularly found in the construction of these patchwork surfaces. Where an island of built form produces a circuit in space, the surface meets and joins itself on the opposite side of the island. In this situation the cones meet at a conic section. In general, space-time surfaces are made up of a patchwork of sections of cones glued together in these two ways (Figure 11).

For a simple open space without obstructions the space-time surface is a cone, however, as soon as obstructions are created by varying the morphology of the boundary or adding islands, the way the cone patches are joined together changes the overall shape of the surface. Figure 12 shows that the shape of a simple uniform

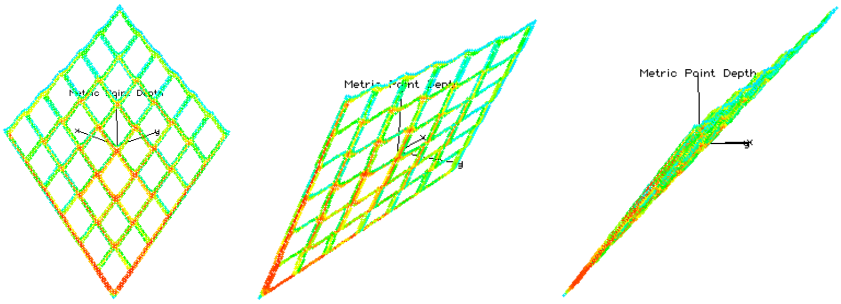


**Figure 12: The space-time surface for a grid from its corner is nearly a flat plane**

grid, viewed from its corner approximates a flat plane. The deviation of the whole surface from flatness appears to be due to the relative width of the 'streets' and the effect this has in allowing minimum time paths to cross streets on a diagonal – the narrower the streets, the flatter the resulting plane. This flattening of the space-time surface by a simple grid results from the constraint the grid places on possible headings for movement. From its extreme corner only two headings are possible – say north or west. When one has been selected the next decision that a moving person must make is whether to continue ahead, or whether to turn onto an orthogonal street. Only two headings are possible and the result is a planar surface. From the centre of the grid there are four possible headings, and this results in the space-time surface approximating an inverted four-sided pyramid, with each face of the pyramid still approximating a flat surface (Figure 13).



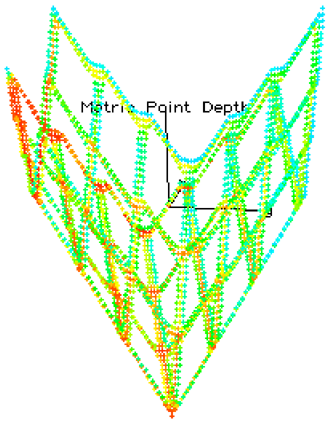
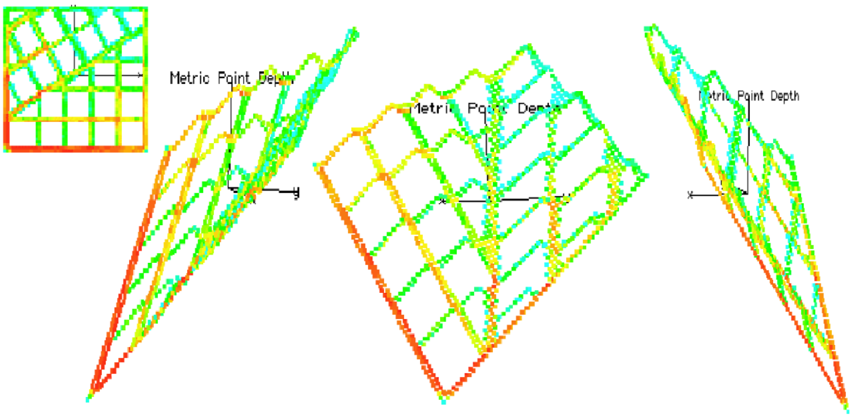



**Figure 13: The uniform grid from its centre approximates a four-sided pyramid rather than a cone.**

Where two grids of different orientations collide, the result is to fold the planar space-time surface along the joining alignment (Figure 14). Looking in detail at the form of the surface we can see that the effect of the fold is to reduce the height of the surface (its time dimension) from where it would have been if it were a uniform grid and therefore flat.

The reduction in height of the surface can be analysed as follows. The space-time surface of the uniform orthogonal grid composed of relatively narrow streets viewed from its corner, shown in Figure 12, is constructed by joining a series of L-shaped strips on space-time cones (the smallest cones in Figure 15). Each pair of strips map onto orthogonal paths on each cone, but as each is joined together they compose a nearly flat grid. However, if the space had been completely open and devoid of building 'islands' the surface would have been described by the surface of the larger cone. These results show that the deformation of a grid results in bends in

Figure 14: The effect of diagonal grid juxtaposition is to create a fold in the planar space-time surface.




The flattening of the grid is a result of the precise way in which cones come together. Two cones must be joined
62.8 when a building island forces a path to deviate from a straight line. In Figure 16 the surfaces of the two smaller cones are aligned only where they both touch the inner surface of the larger containing cone. The path shown as a dark line reaches the top of the lowest small cone, and then as it turns to move around the corner of a building it must move onto a new conical surface on the upper smaller cone. The change of heading involves turning around the time axis of the upper cone before continuing to move on a new heading up its surface. However, this change of path from the first to the second cone surface involves moving off the surface of the larger containing cone and into its interior space. The larger cone is of course merely the extension into the future of the original cone surface, and so a change of heading on the path in space involves a move into that cone's interior. It is for this reason that the surface formed by the 'gluing together' of cones surfaces for a uniform grid lies within the larger cone in Figure 15.

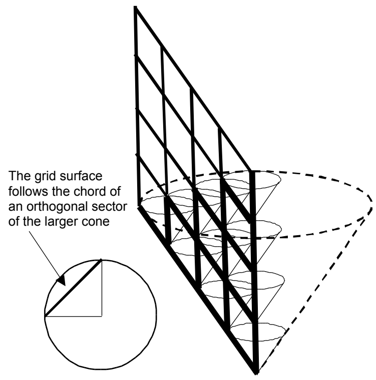



**Figure 15: The construction of a 'flat' grid surface out of sections of smaller space-time cones.**

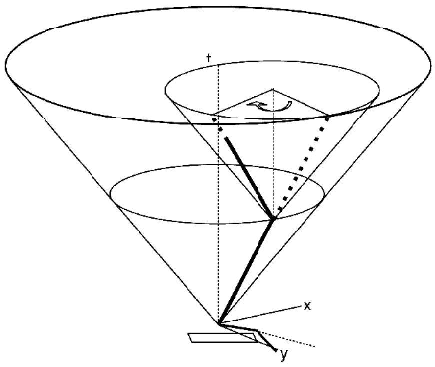



**Figure 16: The effect of a path turning around a building is to move the trajectory away from the surface of the original space-time cone and towards its interior.**

When two grids on different alignments are brought together the result is to deform the surface of the flat grid towards that of the larger cone by allowing certain paths to be constructed on sectors of the smaller cones that are less than orthogonal (and so with shallower chords). This accounts for the bend in the grid surface apparent in Figure 14.

Another form of grid deformation is grid intensification through block subdivision. A simple example of this (due to Hillier) is reproduced as a space-time surface viewed from one corner of the grid in Figure 17. The grid intensification can be seen to warp the grid (reducing its height in the time dimension).

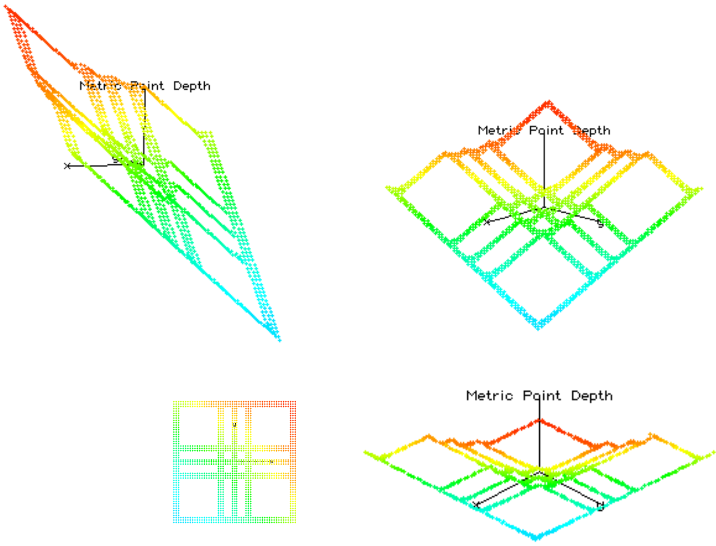



**Figure 17: Grid intensification through reduction in block size results in a two-dimensional 'warp' in the space-time grid surface.one edge.**



Viewed from the centre of one edge of the grid, the warp is seen to produce a curve in the area beyond the central intensification area (Figure 18). This is generated by the reduction in the size of the island blocks while the street width is maintained allowing diagonalised routes to be taken through the central area of the grid. A similar effect is seen when the grid deformation takes the form of widening a street alignment to form a boulevard (Figure 19 a & b), or of forming an open square (Figure 19 c & d). In all these cases, allowing diagonal segments on paths through the system introduces a cone shaped warp into the surface of the grid, and effectively reduces the time taken to reach points beyond the deformation.

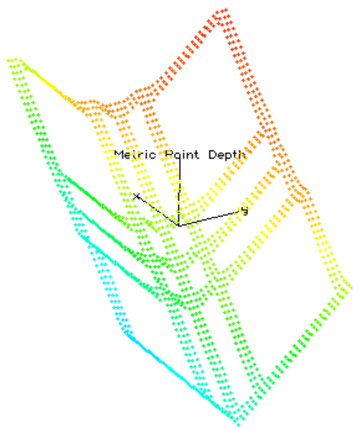



**Figure 18: The central grid intensification leads to a curved warp in the space-time grid surface viewed from the centre of**

Figure 20 shows that an opposite effect is obtained by interruption of the grid through increasing the size of one of the building islands. In this case rather than a valley being formed in the surface of the grid, a ridge appears.

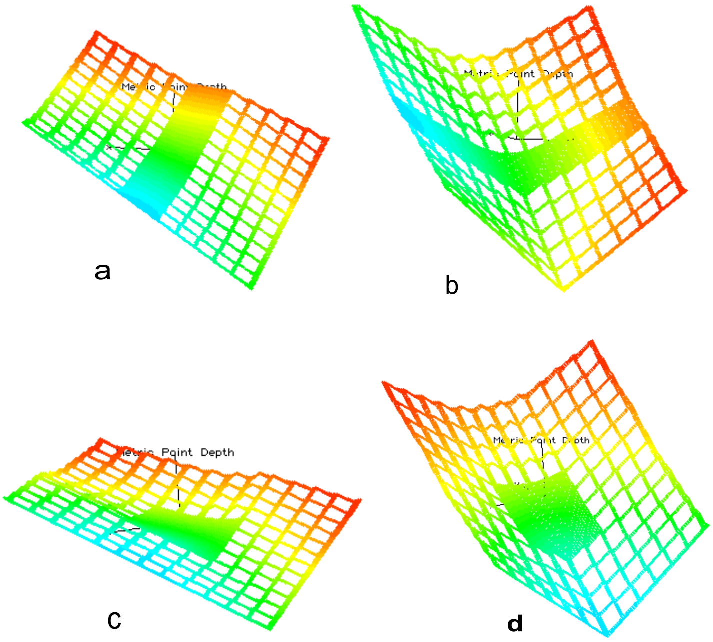



**Figure 19: Effects of open space on warping the space-time surface of the grid.**

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These results show that the deformation of a grid results in bends in its space-time surface towards or away from the surface of the containing space-time cone. Integrating features such as open spaces, diagonal elements or intensification of the grid through block subdivision warp the surface towards the containing cone, whilst interruptions in the grid move it towards the cone interior. These deformations move the surface up or down the time dimension, although clearly they can never fall ‘below’ the surface of the containing cone.

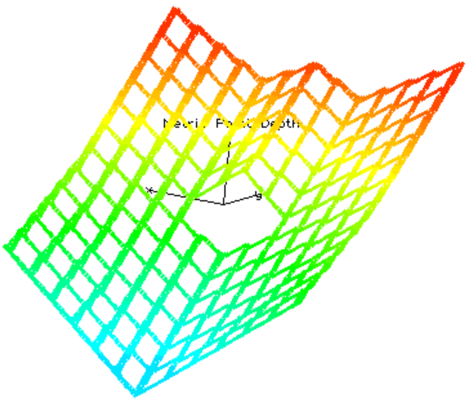


**Figure 20: The effect of interruption of the grid is to form a ridge in the space-time surface.**

When the analysis is applied to real urban systems we find all of these different forms of deformation of the grid are present. Figure 21 shows an analysis of an area of central London with the root taken in the centre of the analysed area. The form of the surface approximates the inverted pyramid for an ideal grid but with a number of clear ridges and valleys in the surface produced by grid deformation.

When analysed from a number of different root locations, it becomes clear that certain features in the urban landscape regularly become valleys in the resulting surface, whilst others are more consistently raised into ridges. These features correlate with areas which are systematically integrated into, or segregated from, their surroundings.

So far each of these features has been presented from one viewpoint at a time. Thus a grid appears as a flat surface when considered from one corner (Figure 12), or an inverted pyramid from its centre (Figure 13). Each of these representations can be seen as a geometric counterpart of the justified graph in which the shape of space is transformed to represent how it might appear (or be experienced through time and space) from one starting point.

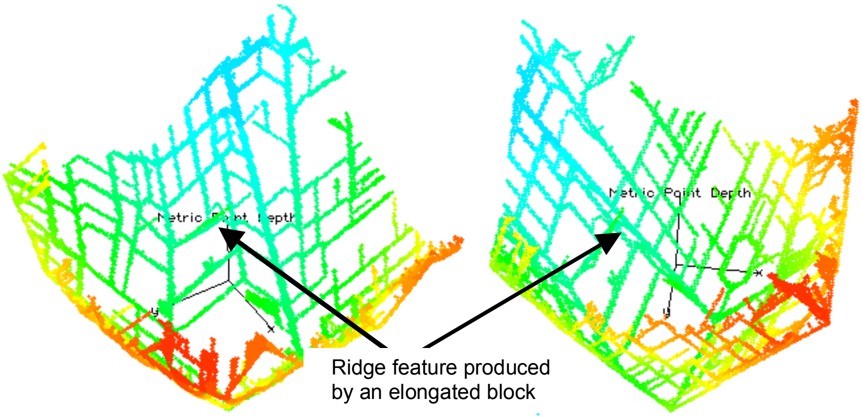



**Figure 21: The Charlotte Street area in West London viewing the conical surface from two different angles.**



Clearly, one direction in which this analysis could now progress is to integrate these representations from all points of view. In order to do this we need to add together space-time surfaces considered from all possible root locations. There are a number of possible ways of performing this kind of addition. One approach is to add the differences between travelling directly up the surface of the space-time cone from each root location to each destination location (assuming that direct 'as the crow flies' flight were possible this would give a Euclidean distance), and the true metric distance on shortest paths through the open space system (that is going 'around the houses').The difference between the total Euclidean distance and the total metric distance to all points in the open space of the system provides a measure of the distance into the time cone's interior (and thus the reduction in speed) to get between any two points. The total metric depth and the total Euclidean depth for two areas of the City of London are shown in Figure 22. As would be expected total Euclidean depth is a parabolic surface, however there are clear local deformations in the total metric depth surface. The difference between these two surfaces is shown in Figure 23. This shows a number of distinct foci of minimum difference which pick out areas of grid intensification, and the meeting points of diagonals in the grids.

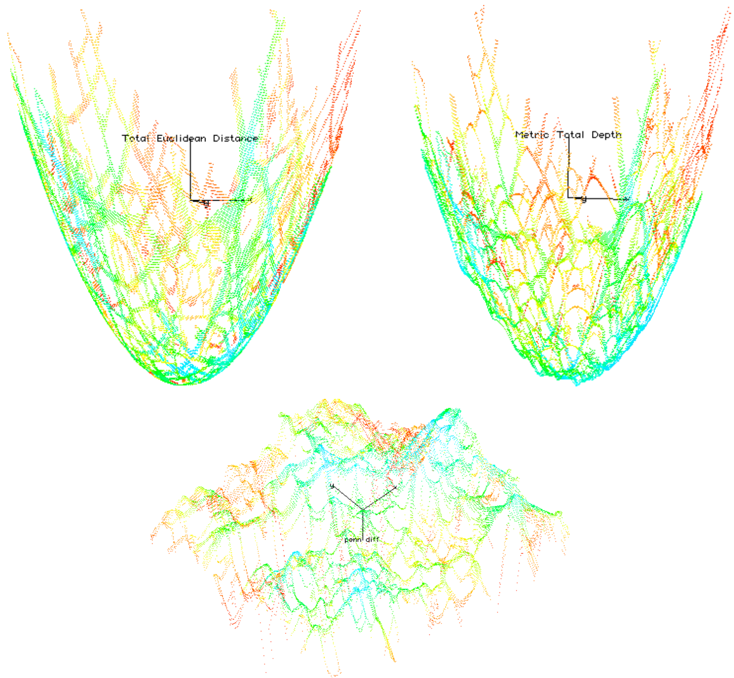
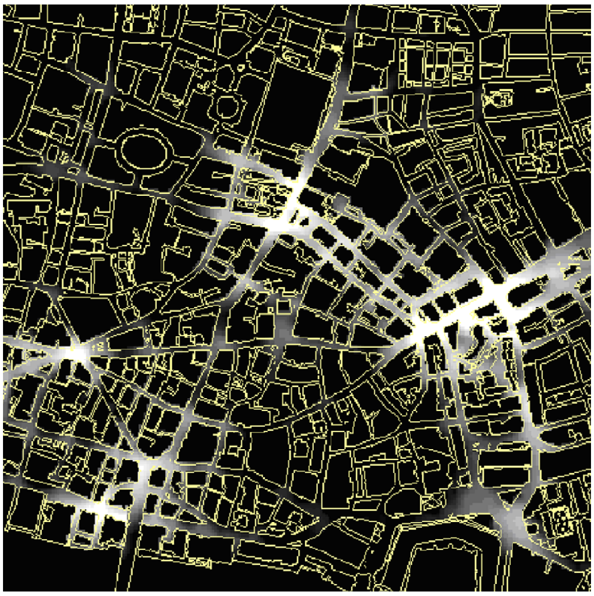


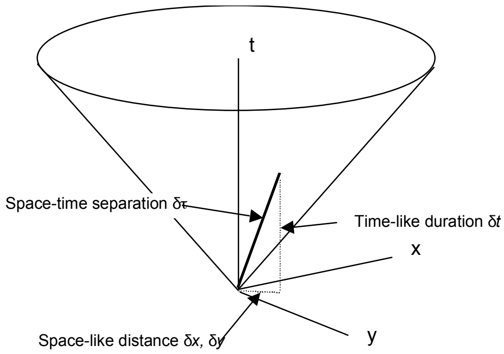

**Figure 22: Total Euclidean distance for the open space in the city of London (left) and total metric depth (right) and their difference (below) plotted as 3D surfaces.**

Figure 23: A measure of the difference between total Euclidean and total metric depth plotted on a grey scale with white is least difference.



62.12 This measure can perhaps be best understood in terms of making a distinction between measurements of separation in space, those in time, and those in terms of both (Figure 24.).

Figure 24: Distance, duration and space-time separation (or rapidity) mapped onto the space-time cone.




Where a path in space passes around an obstacle it inevitably involves a change of direction. This involves moving from one conical patch to another on the space-time surface, and so will entail a move into the main cone's interior. Space-time separation will thus begin to differ from the actual length of the trip, just as the actual distance travelled will differ from the Euclidean 'as the crow flies' distance moved (Figure 25.).

This is of course one of the primary effects of built morphology on the configuration of space and its use by people: to move around an environment you must go around the houses, and this takes time. The second main effect is that the morphology of the environment defines a local visual field, and so defines the area from which one can derive visual information and within which one can potentially be considered visually co-present with others.

Although the metric space through which the observer moves is continuous, the set of fields of view needed to completely apprehend the whole spatial system is discrete, as are the number of straight line segments on a path needed to traverse allthese fields of view. The discrete nature of space, considered in terms of the number of point locations one needs to visit in order to directly see all other locations, is of particular social significance. It is the discretisation of visual space that allows individuals to be co-present with one another, and which brings moving observers into each other's field of view as well as that of the static occupant. Space syntax has developed a number of methods of observation of spatial behaviour and space use that can now be interpreted through the space-time representation. Figure 26 illustrates two of the main observation methods in terms of observation of 'events' in space-time.



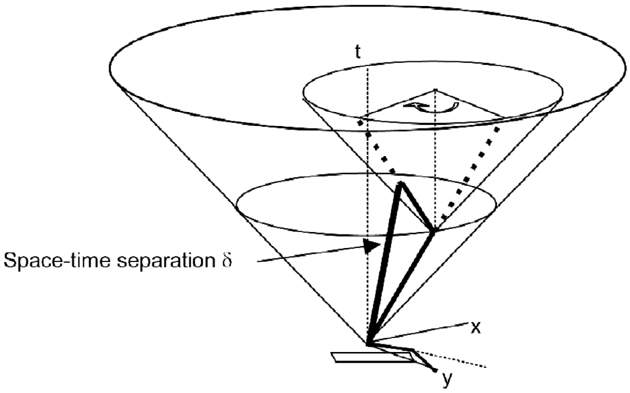



**Figure 25: Distance, duration and space-time separation for a trip involving moving around an obstruction.**

Snapshot observations are performed by defining an area on plan and then observing the number, location and behaviour of all those present within that area at some instant in time. A photograph is sufficient to capture this information so long as one can determine from people's posture and gait whether they are moving or static. Clearly, this form of observation maps onto a horizontal (spatial) plane in the representation. The observation area is an area in space, and both moving and static individuals map onto points in that space (although movement direction is often represented by an arrow). Gate observations are commonly used to observe movement flows in urban areas. In these observations the observer stands on one side of a street and defines a notional line (or 'gate') crossing the street. A count is made of all those crossing the gate in either direction during a specified observation period (say 5 minutes). Although the counts may be further subdivided to record direction of

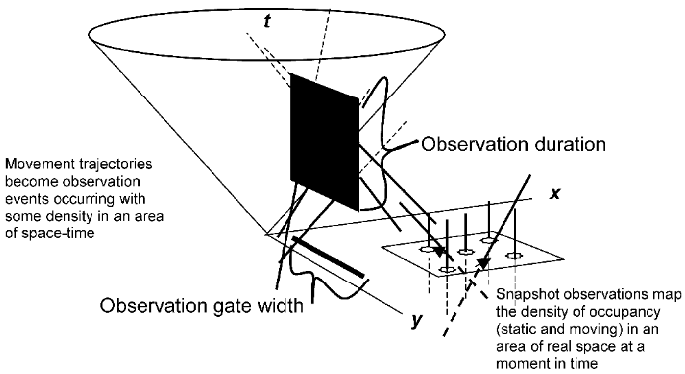



**Figure 26: Snapshot observations map onto an instant of time in an area of space, whilst gate observations map onto an area in space-time with some extension in space and some duration in time.**

movement or to record movement on opposite sides of a the street separately, the principle is the same. In this case it is clear that observations are not made in an area of space, but on a line in space, and not at an instant in time, but over an interval of some duration. This clearly maps onto an area in space-time (on a vertical plane in our representation), rather than in space alone, with movement flows crossing it to provide 'instantaneous' events. Static presence is not captured by gate observations.

It is of course data with a temporal dimension of this sort which is considered to be in some sense dynamic, and it is the correlation between spatial configuration and movement found using syntax analysis – in particular the measure of axial integration and movement flows measured using gate observations – that appears at first sight to be paradoxical. How might it be possible for a measure of static configuration to correlate with dynamic behaviour? It is in the nature of paradoxes that they find their basis in paradigm viewpoints. In this case, the paradigm appears to be that dynamics must be associated with forces or 'prime movers'. The common assumption, perhaps owing its origins to the Newtonian thesis developed by H.C. Carey, the American sociologist in the mid 19th century (Carey, H. C., 1858), but developed most fully by urban geographers since the 1950's, is that some form of attraction field must account for human movement flows. This assumption holds that there must be some primary cause of observed flows, usually attributed to the location of attractor land uses or urban functions. Space syntax findings however appear to run counter to this. Correlations are observed between movement flows and measures of the geometry of spatial configuration alone, with land use distributions appearing to be of only secondary importance.

By synthesising a representation of time within a representation of spatial configuration is now possible to see a resolution to this paradox. This comes in two parts. First, the axial representation itself contains (and is contained by) the space-time surface representation. Second, the measure of integration applied to the axial graph (and which provides the main correlate of movement flows) is equivalent to a time expansion of the graph. In this sense I will argue, the primary representations and measures used by syntax analysis are already dynamic, or at least as dynamic as the temporal phenomena they attempt to explain.

Let us consider the nature of the space-time surfaces developed in the earlier part of this paper. The assumption of a maximum universal speed for pedestrian movement allows one to map space and time together in the form of a surface composed in a simple way from sections of the surface of cones. A single cone surface consists of radial lines emanating from the apex and running in all directions, upwards in time. For a given visual field (a point isovist) the boundary occurs at some distance in plan and some height in time away from the apex, but all space that can be seen is directly accessible in a straight line and can be arrived at by a straight line route lying on the surface of the cone. These straight lines on the cone surface represent possible trajectories both in space and in time. Now, it is clear that on the boundary of the visual field in any configuration that is not trivial (that is not visible in its entirety from one viewpoint) there will be some vertices of the bounding objects (building form) where one can see only one face of the vertex from the original viewpoint. The line starting at the cone's apex will pass tangentially past these vertices until it hits some more distant boundary. The portion of this line beyond the vertex it passes tangentially forms part of the boundary of the visual field, but this is a part of the surface that has the particular property of forming a 'gluing' line between cone surfaces in our composite space-time surface.



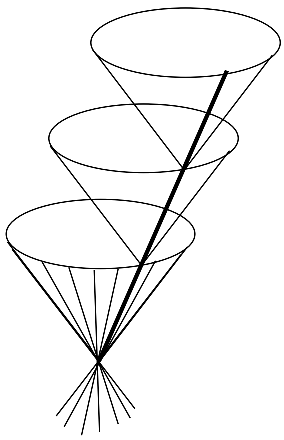


**Figure 27: Space-time cones as vector bundles with the axial line passing as a tangent to more than one vertex. The axial line has extension in both space and time dimensions.**

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One important property of these surfaces is that as the origin location (the apex of the cone) moves relative to the boundary the gluing line moves smoothly (one can imagine the tangent to the vertex pivoting around the vertex). In mathematical terms one can say that the spatial configuration gives rise to a bounded 2-dimensional manifold (a surface defined by overlapping patches – topological discs - with a well defined mapping from one to the next). In this case the patches are topological half discs defined by the half-planes defined by straight line segments of the boundary (with an orientation to define inside and outside faces), and the overlaps arise at convex vertices on the boundary where the half planes overlap (Fomenko, A. T., 1993). It can be seen directly that gluing lines associated with a cone patch at a boundary vertex occur outside the zone of overlap between half planes. It can also be seen that certain gluing lines will be tangential to more than one vertex. It turns out that all axial lines in the axial map run along the latter (Figure 27). What this means is that if one looks carefully at any space-time surface the axial map is always maintained as a set of straight line elements on that surface. Further, these straight line elements are maintained for all different origin locations generating the surface, and in this sense are stable irrespective of origin beyond the first gluing line. I hypothesise that the tangent bundle for the 2-dimensional manifold fully describes the axial map, and this in turn forms a smooth manifold of dimension 3. This object incorporates the additional dimension that allows it to form representation of both spatial configuration and the dynamic component of time.

Perhaps a more intuitively accessible way of saying this is that if one is looking for a representation to correlate with a density function composed of events mapped onto an area of the space-time (vertical) plane in Figure 26 then it must be composed of elements that have spatial extension and be at least linear in the spatial plane otherwise these elements will not 'pass through' and so be attributable to the plane containing the density function.

62.16 It is not just the axial map which correlates with observed movement flows, but the specific measure of mean depth in the axial map. However, mean depth from a node in a graph is directly related to the time expansion of that graph. The time expansion is performed by arranging every node in a graph as a set of dots in a row at time $t = 1$, and then linking to those that are directly accessible at $t = 2$, and so on until all nodes have been reached from each node in turn. Clearly the number of time steps this takes for each node depends on its total depth from all other nodes, and so is directly related to mean depth and thus the measure of integration. Taken together then, it seems perhaps somewhat less surprising that axial integration is found to correlate with movement flows, when both its representational basis and its metric allow a direct temporal interpretation.

There is an irony in this argument, however. It is impossible to overlook the parallels between relativistic physics' incorporation of space and time into Minkowski space-time, and its reinterpretation of the theory of gravity in terms of a geometric formulation, and that proposed here. Perhaps the attractor fields assumed to be present in 'gravity' models of urban systems could best be reinterpreted in terms of just such a geometrical representation of the constraints imposed by the physical geometry of space in terms of movement in both space and time.

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