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Dynamic structural response of laminated glass panels to blast loading

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Abstract

This paper presents an analytical model of ‘fully-clamped’ laminated glass panel subjected to pulse-pressure loading in an air-blast. The dynamic structural response of the laminated glass panel is decoupled into two distinct phases of motion: phase I is controlled by the elastic deformation of the brittle glass plies whilst phase II is dominated by the large inelastic deformation of the Polyvinyl Butyral (PVB) interlayer. Transition between the two phases follows the large scale fragmentation of the glass plies - this will be captured through a stress-based damage criterion. Following the work of others, the PVB interlayer is idealised as a rigid-perfectly plastic material which allows its large inelastic response to be modelled within the constitutive framework of limit analysis. An interactive yield criterion is adopted to capture the simultaneous influence of bending and membrane stretch that governs plastic flow in the interlayer. Predictions by the analytical model are validated against existing experimental data and they will be shown to be in good agreement. Parametric studies were performed to elucidate the effects of total mass (per unit area) and thickness ratio on the maximum transverse deflection. The efficacy of Youngdahl’s technique on desensitising pulse shape effects is also studied.

Keywords: Laminated glass, large deformation, crack formulation, limit analysis, pulse shape

1. Introduction

The structural response of laminated glass panels to high-intensity, short-duration pulse pressure encountered in an air-blast is the subject of several recent assessments, see [1–

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3]. Typically, a laminated glass panel comprises of two annealed or tempered glass sheets interposed by a single layer of Polyvinyl Butyral (PVB) – a thermoplastic polymer membrane with good optical properties. Following large-scale fragmentation of the glass plies on both the back and front faces (also known as inner and outer glass plies, respectively, in Fig. 1), the panel is capable of retaining its structural integrity through the polymer interlayer which captures the majority of the glass fragments, whilst continuing to deform in an inelastic manner; further absorbing a significant proportion of the transmitted blast energy before the onset of interlayer or boundary failure failure. Extensive experimental work had already been conducted by others on the performance of laminated glass panels to blast loading - examples of recent works can be found in [4–10]. However, an analytical model of its large non-linear dynamic structural response, post large-scale fragmentation of its the outer and inner glass plies, is still lacking.

Nomenclature

a	half length of laminated glass panel
A_{ij}	extensional stiffness
b	half width of laminated glass panel
D	damage variable
D_{ij}	bending stiffness of a glass ply
E_g	Young's modulus of glass ply
h_o, h_c, h_i	outer glass, interlayer and inner glass thicknesses
$\bar{h} = h_i/h_c$	thickness ratio
I_{eff}	equivalent impulse per unit area
I^*	non-dimensional impulse
M	bending moment
M_0	fully plastic bending moment
M_{xx}, M_{yy}, M_{xy}	bending moment resultants
N	membrane force
N_0	fully plastic membrane force
N_{xx}, N_{yy}, N_{xy}	membrane force resultants
$p(t)$	reflected over-pressure time-history
p_{eff}	equivalent pressure
p_0	peak reflected over-pressure
F	generalised force
Q_{ij}	plane stress-reduced stiffness
t	time
t_d	positive phase
t_1	time when fracture initiates
t_2	time when plastic deformation ends
t_{mean}	mean time for equivalent loading
T	kinetic energy of one-quarter plate

w_0	transverse central deflection
$\bar{w}_m$	non-dimensional maximum deflection
W	transverse deflection field
x, y	in-plane coordinates
z	out-of-plane coordinate
α	decay coefficient
ϵ_{ij}	strain components
$\epsilon_{ij}^0, \epsilon_{ij}^1$	membrane and bending strains
$\phi(x, y)$	mode function
μ	mass per unit area of laminated glass panel
$\bar{\mu}$	mass per unit area ratio
Π	total potential (strain) energy
ρ_g, ρ_c	glass and interlayer densities
σ_0	static yield strength
σ_{11}	principle stress
σ_{ij}	stress components
σ_t	maximum tensile stress
θ	relative rotation across a straight hinge line
ξ, δ	positions of travelling plastic hinge line

26

27 The elastic deformation of laminated glass panels *before* the onset of large scale fragmenta-
 28 tion of the outer and inner glass plies had been extensively studied by others – see [3, 11–13]
 29 – where the panel is typically modelled by the von Karman’s nonlinear plate theory. For
 30 example, Wei and Dharani [11] modelled the PVB interlayer as a linear viscoelastic mate-
 31 rial and adopts a simple damage criterion – brittle fracture is assumed to initiate when the
 32 surface energy needed to break the glass ply equals to its strain energy – to predict frag-
 33 mentation of the glass plies. Their results showed a counter-intuitive structural response
 34 where the negative phase of a blast pulse could act to cause panel breakage if its positive
 35 phase is not sufficiently violent to induce damage in the first place. More recently, Linz et al.
 36 [13] extended the model of Wei and Dharani [11] by using a higher order deflection function
 37 to approximate the deformed panel profile measured experimentally by Hooper et al. [5].
 38 Their extension leads to a more accurate prediction of the transverse panel deflection and
 39 the predicted crack densities of the glass plies also matches the pattern observed in blast
 40 experiments by Hooper et al. [5]. An alternative interlayer material made of transparent
 41 glass fiber-reinforced composite was investigated by Zhu and Khanna [3]. They showed that
 42 cracking of the glass plies does not occur unless its maximum principle stress is greater than
 43 the tensile strength of the tempered glass.

44 Even though the transverse deflection of a panel in its pre-cracked régime is well-predicted
 45 by the aforesaid models, they assume an interlayer material with either a linear-elastic or
 46 viscoelastic property; consequently, they cannot be extended to model the large inelastic

deformation of the interlayer in the post-cracked régime. Whilst PVB exhibits viscoelastic characteristics under pulse-pressure loading of a long time duration [14], Wei et al. [12, 15] showed that any changes to its shear modulus is negligibly small for a blast pulse of short positive-phase duration of between 0-100 ms. Furthermore, tests carried out by Iwasaki et al. [16] and Bennison et al. [17] showed that the material behaviour of PVB is viscoelastic (rubbery) at small strain rates but it switches to nominally elasto-plastic at higher strain rates beyond 8 s^{-1} . As the material behaviour of PVB at high strain rates is often similar to that at lower temperatures [18, 19], it is common for PVB to be idealised as elasto-plastic when modelling the blast response of laminated glass panels - see the works in [4, 6, 9, 10, 20]. Under high strain rate loading, such as that encountered in an air-blast, the uniaxial yield strain of PVB ($\sim 10\%$) is negligibly small compared to its rupture strain ($\sim 200\%$) and its Young's modulus (0.53 GPa in [5] and 0.18 GPa in [7]) is also significantly lower than that of glass (72 GPa in [5] and 70 GPa in [7]) [4]. Hence, its initial elastic response can be neglected and a rigid plastic material response assumed for the PVB. The same assumption is made to develop the analytical model presented in this paper.

There are two principal mechanisms through which panel (structural) failure in the post-cracked régime [4, 7] occurs, viz. interlayer failure (as a result of interlayer rupture or cutting by the glass splinters) or boundary failure (due to partial or complete pull-out of the interlayer from the boundary). Existing experimental studies in [4, 5, 7] found that boundary failure occurs during elastic rebound (in an opposite sense to the loading direction after the maximum transverse deflection of the panel is reached); this is largely a result of the negative suction phase and will not be considered here. Instead, in this paper, we shall present an analytical model that could be used to predict the temporal evolution of transverse central deflection of a laminated glass panel due to the positive phase of the blast pulse, *before* the onset of structural failure. This work should be of interests to designers of laboratory- and industrial-scale blast resistant laminated glass panels.

The outline of this paper is as follows: Section 2 describes the development of the analytical model; in Section 3, a comparison between analytical predictions and existing experimental results is made; and Section 4 presents the results of the parametric studies to elucidate the effects of thickness and pulse shape on the deformation in laminated glass panel.

2. Model formulation

The dynamic structural response of a laminated glass panel will be modelled as two distinct phases: phase I is controlled by the elastic deformation of the brittle glass plies and phase II by the large inelastic deformation of the interlayer. The key features of the proposed model are as follows: (1) the classical von Karman nonlinear plate theory is used to model the phase I deformation of the panel; (2) a damage criterion is proposed for the onset of large scale fragmentation of the glass plies; (3) the large inelastic deformation of the interlayer is modelled within the framework of limit analysis similar to that employed in [21, 22]; and (4) an interaction yield condition is employed to capture the simultaneous influence of

membrane forces and bending moment. This model will be used to calculate the temporal evolution of the transverse panel deflection of the laminated glass panel subjected to pulse-pressure loading arising from an air-blast, and predictions will be validated against existing experimental data in Section 3.

2.1. Panel geometry and materials description

Figure 1: Schematic of laminated glass panel subjected to a uniformly distributed blast load.

Consider a fully clamped rectangular laminated glass panel subjected to an uniformly distributed blast load, shown schematically in Fig. 1. The panel consists of two glass plies that are perfectly bonded to a PVB interlayer. The glass plies and the PVB interlayer have identical total length $2a$ and width $2b$, where $a > b$. The outer glass ply (facing the blast pulse), interlayer, and inner glass ply have thickness of h_o , h_c and h_i , respectively. Reflective symmetry – both geometric and loading – exists on two planes along ($x = 0$, $-h_c/2 - h_o \leq z \leq h_c/2 + h_i$) and ($y = 0$, $-h_c/2 - h_o \leq z \leq h_c/2 + h_i$) so that only a quarter of the panel needs to be modelled.

Table 2: Material properties for the glass plies

Experiment	ρ_g (kg/m ³)	E_g (GPa)	ν_g	σ_t (MPa)
Hooper et al. [5]	2530	72	0.22	80
Zhang et al. [7]	2500	70	0.227	80

Following [4, 5], the glass plies are assumed to be linear elastic with a maximum tensile strength σ_t which is insensitive to strain-rate effects. The material properties of the glass plies (density ρ_g , Young's modulus E_g , poisson's ratio ν_g and maximum tensile stress σ_t) in the two separate experiments of [5] and [7] are listed in Table 2 - both sets of glass properties will be used to simulate the corresponding experiments in Section 3. At typical strain rates

Table 3: Material properties for the interlayer

ρ_c (kg/m ³)	σ_0 (MPa)
1100 [5, 7]	18 [7]

of between 8 s⁻¹ to 118 s⁻¹, several authors [16, 17, 23] have modelled the inelastic response of PVB as a linear strain hardening material with a static yield strength of $\sigma_0 = 11$ MPa and an ultimate strength of $\sigma_u = 27$ MPa (corresponding to a rupture strain of 200%) [4, 9, 10]. Zhang et al. [7] found that the static yield strength of PVB increases further to $\sigma_0 = 18$ MPa at the higher strain rate of 200 s⁻¹. In this study, the post-yield strain hardening of PVB is ignored for simplicity and it is idealised as a rigid-perfectly plastic material with a static yield strength of $\sigma_0 = 18$ MPa [7]. The material properties of interlayer material to be used in the analytical model is given in Table 3.

2.2. Loading

When the blast pulse interacts with a structure that is not parallel to the direction of the wave, it is reflected and reinforced. This type of pulse-pressure loading is known as the reflected pressure. The initial reflected over-pressure of a typical blast pulse in air (corresponding to an equivalent TNT of 15 kg at a 10 m standoff) – this is the difference between the reflected pressure and the ambient atmospheric pressure – increases rapidly, over a negligibly small rise time, to a peak value of p_0 . This reflected over-pressure then decays exponentially to zero over a time duration t_d , known as the ‘positive phase’. This is followed by a period of under-pressure which creates a partial vacuum – ‘negative suction phase’ – before returning to zero again [24]. Several authors [25–27] have previously shown that the negative phase plays an important role in the blast response of structures only if the peak under-pressure is comparable to its over-pressure counterpart - this is typically the case for ‘weak’ blast waves [27]. Otherwise, experimental studies by Zhang and Hao [7] and Aune et al. [28] have shown that the deformation of elasto-plastic plates is driven primarily by the positive phase of the reflected over-pressure, whilst the negative phase only affects its final configuration during elastic rebound. Since the present study is concerned only with intense blast loading (where the peak over-pressure is much greater than its under-pressure counterpart) and our objective is to model the temporal evolution of the transverse central deflection of a laminated panel before the onset of interlayer or boundary layer, it is also reasonable to neglect the contributions from the negative phase. The ‘modified Friedlander equation’ [29, 30] is used here to approximate the positive phase of the reflected over-pressure versus time history given by

$$p(t) = p_0 \left(1 - \frac{t}{t_d}\right) e^{-\frac{\alpha t}{t_d}}, \quad 0 \leq t \leq t_d \quad (1)$$

where p_0 is the peak reflected over-pressure, t_d is the duration of the positive phase and α ($\alpha \geq 0$) is the decay coefficient that determines the pulse-shape.

2.3. Equations of motion

The modal approximation technique, see [31–34], is employed here to derive the equations of motion for the laminated glass panel. Assuming that the dynamic response of the panel results in deformation that continually evolves towards a modal solution, then its velocity field $\dot{W}(x, y, t)$ may be written as a product of separate functions of space and time given by

$$\dot{W}(x, y, t) = \phi(x, y)\dot{w}_0(t) \quad (2)$$

where the partial function $\phi(x, y)$ is an admissible mode function, or mode shape, that satisfies the geometric boundary conditions and the temporal function $\dot{w}_0(t)$ is the generalised transverse velocity to be determined by the Lagrange equations of the 2nd kind. Similarly, the corresponding deflection field (W) may also be written, using the same partial functions, as follows:

$$W(x, y, t) = \phi(x, y)w_0(t). \quad (3)$$

The transverse deformation of the laminated glass panel is divided into two uncoupled phases. For simplicity, it shall be assumed that deformation of phases I and II are uncoupled; this is justified since the time period for elastic deformation prior to fragmentation of the glass plies is much shorter than the structural response time for plastic bending and stretching of the interlayer [5, 7] - this will be shown later in Section 3.

In each phase, the mode shape $\phi(x, y)$ is used to calculate total strain energy (Π) and kinetic energy (T) of the panel in terms of the generalised deflection $w_0(t)$. The corresponding equation of motion governing $w_0(t)$ in each phase will be given in the subsections below.

2.4. Phase I (pre-crack phase): $0 < t \leq t_1$

An admissible transverse velocity field for a quarter panel in phase I is [3]

$$\dot{W}(x, y, t) = \dot{w}_0(t) \sin\left[\frac{\pi(x+a)}{2a}\right] \sin\left[\frac{\pi(y+b)}{2b}\right]. \quad (4)$$

Its total kinetic energy at any given time t can be written as

$$T = \frac{1}{2}\mu \int_0^b \int_0^a \dot{W}^2(x, y, t) dx dy \quad (5)$$

where $\mu (\triangleq \rho_g h_i + \rho_c h_c + \rho_g h_o)$ is the mass per unit area of the panel; ρ_g and ρ_c are the glass and interlayer density, respectively.

We now proceed to derive expressions for the total strain energy of the panel by first establishing the constitutive equation that relate the membrane force N and bending moment M to the strains in the k th lamina, where $k = 1, 2, 3$ refers to outer glass ply, composite interlayer and inner glass ply respectively. Here, the classic plate deformation theory by von Karman is employed, where it will be assumed that the transverse normal remains perpendicular to the mid-surface after deformation. The components of stress σ in the k th lamina of a three-layer laminate give rise to the following stress resultants for membrane force N and bending moment M given by [35]

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}^{(k)} = \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}^{(k)} dz \quad (6a)$$

and

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(k)} = \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}^{(k)} z dz. \quad (6b)$$

The linear-elastic constitutive relation between the stress and strain components for the k th lamina of a three-layered laminate is [35]

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}^{(k)} = \begin{Bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{Bmatrix}^{(k)} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \quad (7)$$

where $Q_{ij}^{(k)}$ are the reduced plane-stress stiffness; $Q_{66}^{(k)} = E^{(k)}/2(1 + \nu_{(k)})$, $Q_{11}^{(k)} = E^{(k)}/(1 - \nu_{(k)}^2)$, $Q_{12}^{(k)} = \nu_{(k)} Q_{11}^{(k)}$; $E^{(k)}$ is Young's modulus and $\nu_{(k)}$ is Poisson's ratio. The strain components are written in the form of

$$\begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \epsilon_{xx}^1 \\ \epsilon_{yy}^1 \\ \gamma_{xy}^1 \end{Bmatrix} \quad (8)$$

where $(\epsilon_{xx}^0, \epsilon_{yy}^0, \gamma_{xy}^0)$ are the membrane strains and $(\epsilon_{xx}^1, \epsilon_{yy}^1, \gamma_{xy}^1)$ are the bending strains, which can be expressed according to the von Karman strain-displacement relations, respectively, as follows:

$$\epsilon_{xx}^0 = \frac{1}{2} \left[\frac{\partial W(x, y, t)}{\partial x} \right]^2, \quad \epsilon_{yy}^0 = \frac{1}{2} \left[\frac{\partial W(x, y, t)}{\partial y} \right]^2, \quad \gamma_{xy}^0 = \frac{\partial W^2(x, y, t)}{\partial x \partial y} \quad (9)$$

and

$$\epsilon_{xx}^1 = -\frac{\partial^2 W(x, y, t)}{\partial x^2}, \quad \epsilon_{yy}^1 = -\frac{\partial^2 W(x, y, t)}{\partial y^2}, \quad \gamma_{xy}^1 = \left(\frac{\partial^2 W(x, y, t)}{\partial x \partial y} \right)^2. \quad (10)$$

Substituting Eqs. 7 and 8 into Eq. 6a, and re-arranging, leads to the constitutive equation that relates the membrane force and bending moment resultants to the strain in the k th lamina given by

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}^{(k)} = \begin{Bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{11} & 0 \\ 0 & 0 & A_{66} \end{Bmatrix}^{(k)} \begin{Bmatrix} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} \quad (11a)$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}^{(k)} = \begin{Bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{11} & 0 \\ 0 & 0 & D_{66} \end{Bmatrix}^{(k)} \begin{Bmatrix} \epsilon_{xx}^1 \\ \epsilon_{yy}^1 \\ \gamma_{xy}^1 \end{Bmatrix} \quad (11b)$$

where $A_{ij}^{(k)}$ and $D_{ij}^{(k)}$ is, respectively, the extensional and bending stiffness, defined in terms of the lamina stiffness $Q_{ij}^{(k)}$ as follows:

$$(A_{ij}, D_{ij})^{(k)} = \int_{z_k}^{z_{k+1}} Q_{ij}^{(k)}(1, z^2) dz. \quad (12)$$

In general, the interlayer is too soft to provide any meaningful flexural resistance in the pre-crack phase [14]; hence, its plane stress-reduced stiffness is set to zero, i.e. $Q_{ij}^{(k=2)} = 0$.

Last, the total strain energy of a quarter of the k th laminate is obtained by combining the strain energy associated to bending and membrane as follows:

$$\begin{aligned} \Pi^{(k)} = \Pi_b^{(k)} + \Pi_m^{(k)} = & \frac{1}{2} \int_0^b \int_0^a [M_{xx}^{(k)} \epsilon_{xx}^1 + M_{yy}^{(k)} \epsilon_{yy}^1 + M_{xy}^{(k)} \gamma_{xy}^1] dz dx dy \\ & + \frac{1}{2} \int_0^b \int_0^a [N_{xx}^{(k)} \epsilon_{xx}^0 + N_{yy}^{(k)} \epsilon_{yy}^0 + N_{xy}^{(k)} \gamma_{xy}^0] dz dx dy. \end{aligned} \quad (13)$$

Given that the generalised force is

$$F = p(t) \int_0^b \int_0^a \phi(x, y) dx dy \quad (14)$$

and the *Lagrangian* is

$$\mathcal{L} = T + \sum_{k=1}^3 \Pi^{(k)}, \quad (15)$$

the equation of motion governing $w_0(t)$ is found by substituting Eqs. 5, 13, 14 and 15 into the well-known Lagrange equation of the 2nd kind given by

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{w}_0} \right) + \frac{\partial \mathcal{L}}{\partial w_0} = F. \quad (16)$$

Phase I terminates when fracture of the inner glass ply occurs at time $t = t_1$, which is determined by a damage criterion to be introduced next.

2.5. Damage criterion

Large-scale fragmentation of the glass ply leads to a complete loss of its load carrying capacity (i.e. all its stress components reduce to zero). Following [5], this is assumed to occur when the following criterion is met

$$D = \frac{\sigma_m}{\sigma_t} = 1 \quad (17)$$

where D is a state variable that increases monotonically with the maximum principle stress σ_m and σ_t is the maximum tensile strength of the glass ply.

It will be shown later in Section 3 that the maximum principle stress always occurs at the centre of inner glass ply so that

$$\sigma_m = \sigma_{11}^{(3)}(x = 0, y = 0, z = h_c + h_i/2) = (\sigma_{xx}^{(3)} + \sigma_{yy}^{(3)})/2 + \sqrt{(\sigma_{xx}^{(3)} - \sigma_{yy}^{(3)})^2/4 + (\sigma_{xy}^{(3)})^2} \quad (18)$$

and this is the same as the analytical predictions of [3, 12].

2.6. Phase II (post-crack phase): $t_1 < t \leq t_2$

Figure 2: Deformation mechanisms for one-quarter of the plate [21].

The phase II analysis follows closely the formulation of ductile plates given by [21]. In phase II, both glass plies no longer possess any load carrying capacity even though the fragmented glass remains glued to the PVB interlayer; hence its corresponding total strain energy (see Eq. 13) becomes $\Pi^{(1)} = \Pi^{(3)} = 0$. In our model, the laminated glass panel is idealised as a rigid-perfectly plastic plate with a mass per unit area of $\mu (\triangleq \rho_g h_o + \rho_c h_c + \rho_g h_i)$ and thickness h_c . For simplicity, it is assumed that all the glass splinters remain attached to the interlayer following large-scale fragmentation of the glass plies and therefore the total

mass of the panel between the two phases are unchanged. An interactive yield criterion to account for the combined effects of bending moment M and membrane force N given by [36]

$$\frac{|M|}{M_0} + \frac{N^2}{N_0^2} - 1 = 0 \quad (19)$$

is employed here, where $M_0 = \sigma_0 H^2/4$ and $N_0 = \sigma_0 H$ are the fully plastic bending moment and in-plane membrane force, respectively; σ_0 is the static yield strength. It is convenient to recast Eq. 19 in a non-dimensional form as follows:

$$|\bar{M}| + \bar{N}^2 - 1 = 0 \quad (20)$$

where $\bar{M} = M/M_0$ and $\bar{N} = N/N_0$ are the non-dimensional fully plastic generalised stresses.

Within the constitutive framework of limit analysis, travelling plastic hinge lines develop along the principal stress-moment directions [22]. These hinge lines propagate from the vicinity of the plate boundary (but not at the boundary) towards the centre of the plate [36]. Based on the locations of the travelling hinge lines, the deformation of the panel can be divided into two distinct mechanisms as shown schematically in Fig. 2: (1) when the plastic hinge lines – EF and EH – propagate towards its centre; and (2) when the plastic hinge line EF reaches $y = 0$ and remains there. Note that plastic hinge line EF (see Fig. 2a) always reaches the centre of the panel first because $b < a$ [21]; and then plastic hinge E (see Fig. 2b) continues towards the centre until the motion of the hinge ceases (at a finite distance $a - \xi$ from the centre), leaving a stationary deformation mechanism.

For the deformation mechanism shown in Fig. 2a, the transverse velocity profiles in zone ①, ② and ③ of the panel are

$$\dot{W}(x, y, t) = \dot{w}_0(t) \frac{a - x}{\xi(t)}, \quad \text{Zone ①} \quad (21a)$$

$$\dot{W}(x, y, t) = \dot{w}_0(t) \frac{b - y}{\delta(t)}, \quad \text{Zone ②} \quad (21b)$$

$$\dot{W}(x, y, t) = \dot{w}_0(t), \quad \text{Zone ③} \quad (21c)$$

where $0 < \xi(t) \leq a$ and $0 < \delta(t) \leq b$. When the travelling plastic hinge line EF reaches $y = 0$, the transverse velocity profile for the mechanism shown in Fig. 2b becomes

$$\dot{W}(x, y, t) = \dot{w}_0(t) \frac{a - x}{\xi(t)}, \quad \text{Zone ①} \quad (22a)$$

$$\dot{W}(x, y, t) = \dot{w}_0(t) \frac{b - y}{b}, \quad \text{Zone ②.} \quad (22b)$$

Following Yu and Chen [21], the panel is assumed to have the following sequence of deformation. First, the central portion of the panel (circumscribed by the hinge lines FEH in

Fig. 2a) travels with a constant transverse acceleration until the end of the positive phase (of the blast pulse); hence, the motion of the panel can be written as

$$\ddot{w}_0(t) = p(t)/\mu, \quad \text{if } t < t_d. \quad (23)$$

Over the same time period, plastic hinge lines EF and EH propagate towards the centre. The non-dimensional position of the travelling plastic hinge lines are given by

$$\bar{\xi} = \xi/a = \int_0^t \dot{\xi} dt \quad \text{and} \quad \bar{\delta} = \delta/b = \int_0^t \dot{\delta} dt \quad (24)$$

where $\dot{\xi}$ and $\dot{\delta}$ are the non-dimensional velocity of the travelling plastic hinge line as follows [21]:

$$\dot{\xi} = \frac{96f_1 - (2 - \bar{\delta})\bar{\xi}^2\bar{p}}{\bar{I}(4 - 3\bar{\delta})\bar{\xi}} \quad (25)$$

and

$$\dot{\delta} = \frac{96\gamma^2 f_2 - (2 - \bar{\xi})\bar{\delta}^2\bar{p}}{\bar{I}(4 - 3\bar{\xi})\bar{\delta}} \quad (26)$$

where $\gamma = a/b$ is the aspect ratio, $\bar{p} = 4p(t)a^2/M_p$ is the non-dimensional pressure and $\bar{I} = \int_0^t \bar{p} dt$ is the non-dimensional impulse. The non-dimensional parameters f_1 and f_2 can be written as a function of the non-dimensional transverse panel deflection ($\bar{w}_0 = w_0(t)/h_c$) as follows:

$$f_1 = 1 + (1.5 - \bar{\delta})\bar{w}_0^2 \quad (27a)$$

$$f_2 = 1 + (1.5 - \bar{\xi})\bar{w}_0^2 \quad (27b)$$

if $\bar{w}_0 < 1$, or

$$f_1 = 0.5 + (4 - 2\bar{\delta})\bar{w}_0 \quad (28a)$$

$$f_2 = 0.5 + (4 - 2\bar{\xi})\bar{w}_0 \quad (28b)$$

if $\bar{w}_0 \geq 1$.

Second, the central portion of the panel (zone ③ in Fig. 2a) travels at a constant transverse velocity, i.e.

$$\ddot{w}_0(t) = 0 \quad (29)$$

until the travelling plastic hinge line EF reaches $y = 0$ when $\delta = b$. Henceforth, the deformation mechanism switches from that shown in Fig. 2a to 2b.

Finally, the panel decelerates in accordance to the plastic strain energy dissipation on the plastic hinge lines given by Eq. 16. The total strain energy is dissipated through bending and membrane stretching of the interlayer given by

$$\begin{aligned} \Pi = \Pi_b^{(2)} + \Pi_m^{(2)} = & \int_0^b [M_A \theta_1] dy + \int_0^a [M_A \theta_2] dx \\ & + \int_{a-\xi(t)}^a [M_B + N w_0(t) \frac{a-x}{4\xi(t)}] \theta_3 dx / \sin \varphi + [a - \xi(t)] (2\theta_1) (M_B + N w_0(t)/4) \end{aligned} \quad (30)$$

where $\theta_1 = w_0(t)/\xi(t)$ and $\theta_2 = w_0(t)/b$ are the relative rotations at the outer fully clamped boundary of the plastic zones ① and ②; $\theta_3 = w_0(t)/\sin \varphi b$ is the change in rotation across the inclined plastic hinges which lie at the intersections of plastic zone ① and ②; M_A and M_B are the bending moments at the support (along hinge lines AC and CD) and in the interior of panel (along plastic hinge lines CE , EF and EH), respectively. Note that the parameters M_A , N , M_B and $\xi(t)$ are the unknowns in Eq. 30. They can be recast in a non-dimensional form of $\bar{M}_A$, $\bar{N}$, $\bar{M}_B$ and $\bar{\xi}$, respectively; and computed using the procedure outlined below.

The plastic hinge E propagates towards point F at a non-dimensional velocity of

$$\dot{\bar{\xi}} = \frac{24M_P[f_1(4 - 3\bar{\xi}) - \gamma^2 f_2 \bar{\xi}^2]}{a^2 \mu \dot{w}_0 \bar{\xi} (4 - 3\bar{\xi})} \quad (31)$$

until the cessation of its motion ($\dot{\bar{\xi}}(t) = 0$). This occurs at a finite distance from point F and results in a permanent deformation mechanism. Following [21, 37], it is assumed that the membrane force $\bar{N}$ is identical throughout the plastic hinge lines. According to Eq. 19, the yield condition at the support (i.e. hinge lines AC and CD) and in the interior of panel (i.e. plastic hinge lines CE and EF) are, respectively,

$$\bar{M}_A = 1 \quad (32)$$

and

$$\bar{M}_B + \bar{N}^2 = 1, \quad \text{if } \bar{M}_B > 0 \quad (33a)$$

$$\bar{N} = 1, \quad \text{if } \bar{M}_B = 0. \quad (33b)$$

For fully clamped plates [21, 22],

$$\bar{N} = \bar{w}_0, \quad \bar{M}_B = 1 - \bar{N}^2, \quad \text{if } \bar{w}_0 < 1 \quad (34a)$$

$$\bar{N} = 1, \quad \bar{M}_B = 0, \quad \text{if } \bar{w}_0 \geq 1. \quad (34b)$$

Substituting Eqs. 30, 31, 34 into Eq. 16 gives the equation of motion governing $w_0(t)$. Phase II ends when the motion of the interlayer ceases, i.e.

$$\dot{w}_0(t) = 0. \quad (35)$$

Figure 3: Flow-chart on the numerical implementation of the analytical model.

2.7. Numerical implementation

A flow-chart on the numerical implementation of the analytical model is shown in Fig. 3. The corresponding equation of motion is solved numerically using the fourth-order Runge-Kutta method.

3. Validations

Analytical predictions by the model will now be validated against two independent sets of experimental data from the literature by Hooper et al. [5] and Zhang and Hao [7]. In both sets of experiments, the PVB panels tested have identical dimensions of $1.5 (2a) \times 1.2 (2a) \times 6.76 \times 10^{-3}$ m where $h_o = h_i = 3 \times 10^{-3}$ m and $h_c = 0.76 \times 10^{-3}$ m. Material properties for

Table 4: Equivalent mass of TNT and standoff distance in Hooper et al. [5] and Zhang and Hao [7]. Also shown are the corresponding loading parameters that were used in the current model for the different test case.

Test no.	Equivalent TNT (kg)	Standoff distance (m)	p_0 (KPa)	t_d (ms)	α
1	15	10	180	9.0	1.0
2	30	16	152	12.0	0.9
3	30	14	172	8.0	1.0
4	10	10	123	9.5	1.0
5	10	9	165	9.3	1.2
6	10	12.3	100	11.0	0.8

Data for tests 1-3 are from [5]; 4-6 are from [7].

the glass ply, the PVB interlayer and the loading parameters (in the respective experiments) are given in Tables 2, 3 and 4 respectively.

Figure 4 compares the analytical predictions and existing experimental results in the literature: tests no. 1-3 were by Hooper et al. [5] and tests no. 4-6 were by Zhang et al. [7]. The black curves plot the predicted reflected over-pressure time history $p(t)$ whilst those in blue the transverse central deflection time-history $w_0(t)$. In spite of the simplifications made in the formulation of the analytical model, there is a broad general agreement between the temporal evolution of the transverse central deflection w_0 – for both the pre- and post-crack phases – with both sets of experimental data. It is worth emphasising, again, that the current model does not take into account interlayer (in test-4 in [7]) or boundary (in tests 1-3 in [5] and test 5 in [7]) failures. In general, predictions by the analytical model agree well with experimental data before the onset of boundary failure. The predicted time-histories for the transverse panel deflection (see test no. 1-3), and the maximum transverse deflection, are also in excellent agreement with the FE predictions by Hooper et al. [5] where boundary and interlayer failure were also neglected. Intuitively, it is not entirely unexpected that boundary failure (tests no. 1-3 in [5]) may occur before the maximum transverse deflection is reached if the blast load is sufficiently intense. Test no. 5 by [7] is an example of a ‘counter-intuitive’ boundary failure where the window panel is ‘sucked’ outwards by the negative blast pressure during elastic rebound. Table 5 compares the predicted maximum transverse deflections of the glass panels – at either the point of cessation of motion or before boundary failure occurs – to their experimental counterparts. In all cases, the error is less than 5%.

Figure 5 compares the predicted deflection profiles (taken horizontally through the panel at $x = 0$ – corresponding to tests no. 1-3 in Fig. 4) – against the finite element results by Hooper et al. [5]. In the early stages of deformation ($t \leq 8$ ms), the predicted deflection profile is in excellent agreement with its numerical counterpart, apart from slight discrepancies

Figure 4: Comparison of analytical predictions with the experimental results by (a) Hooper et al. [5] and (b) Zhang and Hao [7]. — current analytical predictions; - - - experimental results; -.- numerical predictions by Hooper et al. [5]

near the boundaries, for all three tests. The predicted profiles begin to depart significantly from its numerical counterpart after the end of the positive phase. This is expected since the analytical model ignores the negative phase loading whose effect is to lower the velocity

Figure 5: Comparison of predicted deflection profiles taken horizontally at the centre of the panel at $x = 0$ against the finite element results by Hooper et al. [5]. t_d gives the time at the end the positive phase. — current analytical predictions; -.- numerical predictions [5]

of the plastic hinge line travelling towards the centre of the panel at $y = 0$, leading to a smaller central flat region. Notwithstanding, the current analytical model – based on a travelling plastic hinge line concept – successfully predicts a relatively flat central region whose area decreases in size with increasing panel deflection: this agrees with both the numerical predictions and experimental measurements made by Hooper et al. [5].

Table 6 compares the predicted time t_1 at which the glass plies are assumed to completely fragment (this corresponds to $D = 1$ in our model) to the ones measured in experiments and in FE analyses performed by others. The predicted time corresponding to $D = 1$ for test-1 and test-3 is 2.5 ms and 2.5 ms, respectively; they compare well to the 3.0 and 2.0 ms measured by Hooper et al. [5] for large-scale fragmentation of the glass plies and with the FE predictions of 2.5 ms and 1.75 ms by Linz et al. [13]. To provide further insights into the location of incipient fracture in the glass plies, Figure 6 plots the predicted stress distribution in the outer and inner glass plies in phase I. The principle stress was found

Table 5: Maximum transverse central deflection either at the cessation of panel motion or before boundary failure occurs.

Test no.	Present (mm)	Experiment (mm)	Error
1	169.7	166.9	1.7%
2	205.9	200.0	3.0%
3	161.2	156.6	2.9%
4	289.2	277.3	4.4%
5	338.0	330.2	2.3%
6	254.3	263.4	3.4%

Data for tests 1-3 are from [5]; 4-6 are from [7].

to be greatest at the centre of the inner glass ply (at $x = 0$, $y = 0$, $z = h_i + h_c/2$) - this also agrees with analytical predictions by Zhu and Khanna [3] and Wei and Dharani [12]. Furthermore, the fact that the maximum principle stress develops first on the inner, rather than the outer (which is a state of compression where $\sigma_{11} < 0$, as shown in Fig. 6), glass ply also agrees with the experimental observations made in [5, 7, 8]. These experimental studies have found that cracks first appear on the inner glass ply which is primarily in a tensile stress state. Although our current model ignores the formation of cracks, and its propagation, in the glass plies by assuming that both glass plies experience large-scale fragmentation when $D = 1$, the excellent agreement between model predictions and experimental data (see Fig. 4) suggests that the simple damage criterion adopted in Eq. 17, whilst approximate, is a reasonable one.

The predicted time for cessation of panel motion (t_2) - given in Table 6 - is also in good agreement with existing experimental data by [7] for tests no. 4-6. Unsurprisingly, the discrepancies for t_2 in tests no. 1-3 is because the current model does not include boundary or interlayer failure. Notwithstanding, there is a broad general agreement between the predicted t_2 and FE results by [5] where boundary failure is similarly neglected. We note further that the duration of phase II ($t_2 - t_1$) is significantly longer than that of phase I (t_1): this justifies the decoupling of the two phases of motion assumed in the formulation of our model in Section 2.

4. Parametric studies

The analytical model developed here will be used to evaluate the sensitivity of the predicted maximum transverse deflection of a laminated glass panel to its overall thickness and the pulse-shape. The maximum transverse panel deflection, its thickness and mass per unit area

Figure 6: Predicted principle stress (in MPa) contour plots on the surface of (a) outer glass ply and (b) inner glass ply for a typical panel tested in [5] with a size of $1.5(2a) \text{ m} \times 1.2(2b) \text{ m} \times 7.52 (h_o + h_c + h_i) \times 10^{-3} \text{ m}$ where $h_o = h_i = 3$ and $h_c = 1.52$. The material properties and loading parameters (corresponding to Test no. 3 by Hooper et al. [5]) are given in Tables 2 and 4, respectively. Plots are shown for $t = 1.4$, $t = 1.9$ and $t = 2.4$ ms where $D < 1$ (note that $D = 1$ occurs at $t_1 = 2.5$ ms).

329 can be non-dimensionalised as follows:

$$\bar{w}_m = \max_{0 \leq t \leq t_2} [w_0(t)/H], \quad \bar{h} = \frac{h_i}{h_c} \quad \text{and} \quad \bar{\mu} = \frac{\mu}{\mu_0} \quad (36)$$

Table 6: Time corresponding to fragmentation of glass plies (t_1) and to the cessation of motion (t_2).

Test No.	Present		Experiment		FE	
	t_1 (ms)	t_2 (ms)	t_1 (ms)	t_2 (ms)	t_1 (ms)	t_2 (ms)
1	2.5	15.8	3.0 [5]	18.1 [5]	2.5 [13]	15.7 [5]
2	2.7	16.2	-	12.3 [5]	-	16.1 [5]
3	2.5	16.1	2.0 [5]	11.3 [5]	1.75 [13]	15.5 [5]
4	2.7	16.7	-	15.9 [7]	-	-
5	2.4	14.1	-	16.2 [7]	-	-
6	3.0	14.7	-	16.7 [7]	-	-

Data for tests 1-3 are from [5]; 4-6 are from [7].

where $\mu_0 = 16.9 \text{ kg/m}^2$ is the reference mass per unit area (this is equal to that of the laminated glass panels in the experiments of Hooper et al. [5]). The non-dimensional impulse is given by

$$I^* = \frac{\hat{I}}{\mu \sqrt{\sigma_c / \rho_c}} \quad (37)$$

where $\hat{I} = \int_0^{t_d} p(t)dt$ is the impulse per unit area transmitted by the pulse-pressure loading. For the parametric study, we used the following loading parameters (corresponding to test no. 3): a positive phase duration of $t_d = 8 \text{ ms}$ and decay coefficient of $\alpha = 1$. All the laminated glass panels has length of $2a = 1.5 \text{ m}$ and width of $2b = 1.2 \text{ m}$; the inner glass and outer glass plies have identical thickness of $h_i = h_o$.

4.1. Effects of total panel thickness

Figure 7a plots the variation of the non-dimensional maximum deflection $\bar{w}_m$ as a function of I^* for a range of thickness ratio between $2 \leq \bar{h} \leq 5$. To ensure that the figure is plotted for panels of identical mass per unit area, its thickness ratio $\bar{h}$ is varied by choosing the glass ply thickness h_i and interlayer thickness h_c to be $\bar{h}\mu_0/(2\rho_g\bar{h}+\rho_c)$ and $\mu_0/(2\rho_g\bar{h}+\rho_c)$ respectively. In this manner, the mass per unit area ($\mu_0 = 16.9 \text{ kg/m}^2$) of a panel is identical for different thickness ratio $\bar{h}$. For a given I^* , increasing the thickness ratio $\bar{h}$ leads to a monotonic decay of $\bar{w}_m$. This is because a higher $\bar{h}$ is tantamount to a thinner PVB interlayer; hence, there is less material to dissipate the energy. Consequently, it would take a longer time for the motion of the panel to cease and this, in turn, results in a greater transverse central deflection. Figure 7b plots the variation of $\bar{w}_m$ versus I^* for different mass per unit ratio $\bar{\mu}$. The monotonic reduction of $\bar{w}_m$ with higher $\bar{\mu}$ is largely attributed to the greater structural resistance offered by the thicker interlayer during its phase II motion. This leads to a shorter response time and a consequential reduction in its non-dimensional maximum deflection $\bar{w}_m$.

Figure 7: Variation of non-dimensional maximum transverse central deflection $\bar{w}_m$ against non-dimensional impulse I^* for laminated glass panels of different total thickness: (a) different thickness ratio $\bar{h}$ at a constant mass per unit area of $\mu_0 = 16.9 \text{ kg/m}^2$ and (b) different mass per unit area ratio $\bar{\mu}$ at a constant thickness ratio of $\bar{h} = 2$.

Figure 8: Youngdahl's equivalence parameters for a general pulse: (a) definitions for I_{eff} and t_{mean} ; (b) an equivalent rectangular pulse.

4.2. Effects of pulse shape

The analytical model developed here assumes an exponentially-decaying pressure pulse - see Eq. 1. It is, therefore, of interests to ask whether the assumed pulse shape affects the final deformation of a laminated glass panel. We will establish whether the approach of Youngdahl [38, 39] - see schematic in Fig. 8 - may be used to eliminate the effects of pulse shape when calculating the dynamic response of a panel. Youngdahl [38, 39] proposed that the resulting structural response due to a general pulse pressure could be approximated by a rectangular pulse I_{eff} with an effective load p_{eff} and pulse duration of $2t_{\text{mean}}$ given by

$$I_{\text{eff}} = \int_{t_y}^{t_2} p(t)dt, \quad p_{\text{eff}} = \frac{I_{\text{eff}}}{2t_{\text{mean}}}, \quad t_{\text{mean}} = \frac{1}{I_{\text{eff}}} \int_{t_y}^{t_2} tp(t)dt \quad (38)$$

where $p(t)$ is the actual pressure pulse, t_2 is time at the end of phase II motion (see section 2), t_y corresponds to the time when the plastic deformation starts and t_{mean} is the centroid of the effective pressure pulse in Fig. 8a. For monotonic-decaying pressure pulse, then $t_y = 0$.

Figure 9 gives a contour map of the predicted non-dimensional maximum panel deflections for an exponentially-decaying pressure pulse to its corresponding Youngdahl's equivalent (Eq. 38) in $\bar{h}$ - $\bar{\mu}$ space for practical ranges of $2 \leq \bar{h} \leq 5$ and $1 \leq \bar{\mu} \leq 4$. The laminated glass panel is subjected to non-dimensional impulse of $I^* = 0.3$. The results show that Youngdahl's approach gives an excellent approximation, with a slight under prediction, of the transverse central deflection of laminated glass panel under blast loading. This agrees with the previous findings by Yuan et al. [33] where it was found that Youngdahl's approach can successfully eliminate the dependence of the mid-span deflection of elasto-plastic beams to pulse shape for monotonically decaying blast pulse. Figure 9 also shows that decreasing $\bar{h}$ or increasing $\bar{\mu}$ leads to monotonic reduction in the transverse deflection of a panel - this is consistent with Fig. 7.

Figure 9: Contour plot of non-dimensional maximum deflection $\bar{w}_m$ and their corresponding Youngdahl's equivalent with $I^* = 0.3$. — analytical predictions by actual blast loading; - - - analytical predictions by Youngdahl's equivalent rectangular pulse.

5. Conclusions

An analytical model has been developed in this paper to analyse the dynamic structural response of laminated glass panels to blast loadings. The structural response of the laminated glass panel was separated into two uncoupled sequential phases: phase I is the elastic deformation of glass plies and phase II is the large plastic deformation of polymeric interlayer. Predictions by the analytical model were shown to be in good agreement with two independent sets of experimental data, and with the results of FE analysis, obtained from the literature. Parametric studies reveal that laminated glass panel with thick interlayer and thin glass plies tends to have small transverse central deflection. For an exponentially-decaying blast pulse, it was shown that Youngdahl's approach can successfully be applied to eliminate the dependence of the transverse panel deflection to pulse shape. The analytical framework developed here lays the foundation for investigating the structural failure of laminated glass panels. Efforts to include interlayer and boundary failures are the subject of an on-going investigation, the results of which will be reported elsewhere.

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