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Online Appendix: "Consumer Choice and Market Outcomes under Ambiguity in Product Quality"

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In this online appendix, we extend our model and results to incorporate (i) more than 2 products (part A) and (ii) preference of loyal customers (part B). All proofs are given in part C. Where necessary, we provide detailed expressions so that other researchers can follow each step of the extensions.

Appendix A: Extension to Multiple Product

We first show the extension of the demand curve to multiple products by first considering the simplest scenario involving 3 products and 2 customers. We then extend the demand curve involving 3 customers to demonstrate the notion of symmetric predisposition distributions for each product. Finally we will extend our logic to 3 products with continuous mass of consumer, and show the equilibrium market outcomes when 3 products are engaged in symmetric competition. This result is applicable to n products.

A.1. 3 products and 2 customers

There are three products, A, B, and C, each characterized fully by price-quality pairs (p_A, q_A), (p_B, q_B), (p_C, q_C) respectively. There are two customers, 1 and 2, each with different predisposition towards each of the three products. Customer 1's predisposition to products A, B, and C are respectively, Ω_{1A}, Ω_{1B}, and Ω_{1C}. Customer 2's predisposition to products A, B, and C are respectively, Ω_{2A}, Ω_{2B}, and Ω_{2C}. The ambiguity in the product market environment is given by $\xi > 0$. According to the multi-attribute utility model, customer $i \in \{1,2\}$ derives utility from product $j \in \{A,B,C\}$,

$$v(p_j, q_j, \Omega_{ij}, \xi) = q_j - p_j - \Omega_{ij}\xi.$$

Because we adopt the convention of subtracting the predisposition, smaller values of Ω_{ij} represent stronger predisposition. For example, $\Omega_{1A} - \Omega_{1B} < 0$ indicates that customer 1 is more favorably predisposed to product A than B. Customer $i \in \{1,2\}$ will purchase product A if and only if

$$\begin{aligned}
& q_A - p_A - \Omega_{iA}\xi > q_B - p_B - \Omega_{iB}\xi, \text{ and } q_A - p_A - \Omega_{iA}\xi > q_C - p_C - \Omega_{iC}\xi \\
\Leftrightarrow\ & (q_A - q_B) - (\Omega_{iA} - \Omega_{iB})\xi > (p_A - p_B) \text{ and } (q_A - q_C) - (\Omega_{iA} - \Omega_{iC})\xi > (p_A - p_C) \\
\Leftrightarrow\ & p_A < (q_A - q_B) + p_B - (\Omega_{iA} - \Omega_{iB})\xi \text{ and } p_A < (q_A - q_C) + p_C - (\Omega_{iA} - \Omega_{iC})\xi \\
\Leftrightarrow\ & p_A < \min\{(q_A - q_B) + p_B - (\Omega_{iA} - \Omega_{iB})\xi,\ (q_A - q_C) + p_C - (\Omega_{iA} - \Omega_{iC})\xi\}.
\end{aligned}$$

Similarly, customer $i \in \{1,2\}$ will not purchase product A if and only if

$$q_A - p_A - \Omega_{iA}\xi < q_B - p_B - \Omega_{iB}\xi, \text{ or } q_A - p_A - \Omega_{iA}\xi < q_C - p_C - \Omega_{iC}\xi$$

$$\Leftrightarrow\ (q_A - q_B) - (\Omega_{iA} - \Omega_{iB})\xi < (p_A - p_B) \text{ or } (q_A - q_C) - (\Omega_{iA} - \Omega_{iC})\xi < (p_A - p_C)$$
$$\Leftrightarrow\ p_A > (q_A - q_B) + p_B - (\Omega_{iA} - \Omega_{iB})\xi \text{ or } p_A > (q_A - q_C) + p_C - (\Omega_{iA} - \Omega_{iC})\xi$$
$$\Leftrightarrow\ p_A > \min\{(q_A - q_B) + p_B - (\Omega_{iA} - \Omega_{iB})\xi,\ (q_A - q_C) + p_C - (\Omega_{iA} - \Omega_{iC})\xi\}.$$

Writing out for each customers 1 and 2, the demand for product A, which can take values 0, 1, or 2 is given as follows.

$$D_A(p_A,p_B,p_C) = \begin{cases} 0, & \text{if } p_A > \min\{(q_A-q_B)+p_B-(\Omega_{1A}-\Omega_{1B})\xi,\ (q_A-q_C)+p_C-(\Omega_{1A}-\Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A-q_B)+p_B-(\Omega_{2A}-\Omega_{2B})\xi,\ (q_A-q_C)+p_C-(\Omega_{2A}-\Omega_{2C})\xi\}, \\ 1, & \text{if } p_A < \min\{(q_A-q_B)+p_B-(\Omega_{1A}-\Omega_{1B})\xi,\ (q_A-q_C)+p_C-(\Omega_{1A}-\Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A-q_B)+p_B-(\Omega_{2A}-\Omega_{2B})\xi,\ (q_A-q_C)+p_C-(\Omega_{2A}-\Omega_{2C})\xi\}, \\ & \text{– or –} \\ & \text{if } p_A > \min\{(q_A-q_B)+p_B-(\Omega_{1A}-\Omega_{1B})\xi,\ (q_A-q_C)+p_C-(\Omega_{1A}-\Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A < \min\{(q_A-q_B)+p_B-(\Omega_{2A}-\Omega_{2B})\xi,\ (q_A-q_C)+p_C-(\Omega_{2A}-\Omega_{2C})\xi\}, \\ 2, & \text{if } p_A < \min\{(q_A-q_B)+p_B-(\Omega_{1A}-\Omega_{1B})\xi,\ (q_A-q_C)+p_C-(\Omega_{1A}-\Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A < \min\{(q_A-q_B)+p_B-(\Omega_{2A}-\Omega_{2B})\xi,\ (q_A-q_C)+p_C-(\Omega_{2A}-\Omega_{2C})\xi\}. \end{cases}$$

If p_A is relatively low, both customers 1 and 2 would purchase product A, so $D_A = 2$. As p_A increases, the demand falls to 1. The customer that defected would either purchase product B or C, depending on that customer's predisposition levels Ω_{iB} and Ω_{iC}. Therefore, unlike the two-product case, the demand for product B is no longer 2-(demand for A) because of the existence of product C. Demand for product B and C are respectively,

$$D_B(p_A,p_B,p_C) = \begin{cases} 0, & \text{if } p_B > \min\{(q_B-q_A)+p_A-(\Omega_{1B}-\Omega_{1A})\xi,\ (q_B-q_C)+p_C-(\Omega_{1B}-\Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B-q_A)+p_A-(\Omega_{2B}-\Omega_{2A})\xi,\ (q_B-q_C)+p_C-(\Omega_{2B}-\Omega_{2C})\xi\}, \\ 1, & \text{if } p_B < \min\{(q_B-q_A)+p_A-(\Omega_{1B}-\Omega_{1A})\xi,\ (q_B-q_C)+p_C-(\Omega_{1B}-\Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B-q_A)+p_A-(\Omega_{2B}-\Omega_{2A})\xi,\ (q_B-q_C)+p_C-(\Omega_{2B}-\Omega_{2C})\xi\}, \\ & \text{– or –} \\ & \text{if } p_B > \min\{(q_B-q_A)+p_A-(\Omega_{1B}-\Omega_{1A})\xi,\ (q_B-q_C)+p_C-(\Omega_{1B}-\Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B-q_A)+p_A-(\Omega_{2B}-\Omega_{2A})\xi,\ (q_B-q_C)+p_C-(\Omega_{2B}-\Omega_{2C})\xi\}, \\ 2, & \text{if } p_B < \min\{(q_B-q_A)+p_A-(\Omega_{1B}-\Omega_{1A})\xi,\ (q_B-q_C)+p_C-(\Omega_{1B}-\Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B-q_A)+p_A-(\Omega_{2B}-\Omega_{2A})\xi,\ (q_B-q_C)+p_C-(\Omega_{2B}-\Omega_{2C})\xi\}; \end{cases}$$

$$D_C(p_A,p_B,p_C) = \begin{cases} 0, & \text{if } p_C > \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\Big\}, \text{ and} \\ & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \\ 1, & \text{if } p_C < \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\ & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \\ & \text{– or –} \\ & \text{if } p_C > \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\ & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \\ 2, & \text{if } p_C < \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\ & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}. \end{cases}$$

As expected, all demand for product is decreasing in its price. We examine the resulting demand function using a simple example next.

Example A-1. For simplicity, assume symmetric qualities $q_A = q_B = q_C$. Moreover, assume that customer 1 is most favorably predisposed to A, then B, then C, i.e., $\Omega_{1A} < \Omega_{1B} < \Omega_{1C}$, while customer 2 has the reverse predisposition, $\Omega_{2C} < \Omega_{2B} < \Omega_{2A}$. For simplicity of illustration, assume that

$$\Omega_{1A} = -1 < \Omega_{1B} = 0 < \Omega_{1C} = 1, \text{ and } \Omega_{2C} = -1 < \Omega_{2B} = 0 < \Omega_{2A} = 1.$$

Then, the demand expression for A simplifies to,

$$D_A(p_A, p_B, p_C) = \begin{cases} 0, & \text{if } p_A > \min\{p_B + \xi, p_C + 2\xi\} \text{ and } p_A > \min\{p_B - \xi, p_C - 2\xi\}, \\ 1, & \text{if } p_A < \min\{p_B + \xi, p_C + 2\xi\} \text{ and } p_A > \min\{p_B - \xi, p_C - 2\xi\}, \\ & \text{– or –} \\ & \text{if } p_A > \min\{p_B + \xi, p_C + 2\xi\} \text{ and } p_A < \min\{p_B - \xi, p_C - 2\xi\}, \\ 2, & \text{if } p_A < \min\{p_B + \xi, p_C + 2\xi\} \text{ and } p_A < \min\{p_B - \xi, p_C - 2\xi\}. \end{cases}$$

$$= \begin{cases} 0, & \text{if } p_A > \min\{p_B + \xi, p_C + 2\xi\}, \\ 1, & \text{if } \min\{p_B - \xi, p_C - 2\xi\} < p_A < \min\{p_B + \xi, p_C + 2\xi\} \\ 2, & \text{if } p_A < \min\{p_B - \xi, p_C - 2\xi\}. \end{cases}$$

Similarly, the demand expression for C simplifies to,

$$D_C(p_A, p_B, p_C) = \begin{cases} 0, & \text{if } p_C > \min\{p_A - 2\xi, p_B - \xi\} \text{ and } p_C > \min\{p_A + 2\xi, p_B + \xi\}, \\ 1, & \text{if } p_C < \min\{p_A - 2\xi, p_B - \xi\} \text{ and } p_C > \min\{p_A + 2\xi, p_B + \xi\}, \\ & \text{– or –} \\ & \text{if } p_C > \min\{p_A - 2\xi, p_B - \xi\} \text{ and } p_C < \min\{p_A + 2\xi, p_B + \xi\}, \\ 2, & \text{if } p_C < \min\{p_A - 2\xi, p_B - \xi\} \text{ and } p_C < \min\{p_A + 2\xi, p_B + \xi\}. \end{cases}$$

$$= \begin{cases} 0, & \text{if } p_C > \min\{p_A + 2\xi, p_B + \xi\}, \\ 1, & \text{if } \min\{p_A - 2\xi, p_B - \xi\} < p_C < \min\{p_A + 2\xi, p_B + \xi\} \\ 2, & \text{if } p_C < \min\{p_A - 2\xi, p_B - \xi\}. \end{cases}$$

We notice that as ambiguity ξ increases, the price range for which $D_A = 1$ and $D_C = 1$ becomes larger. In other words, the demand for product A becomes price insensitive (or sticky) at $D_A = 1$, and demand for product C becomes price insensitive (or sticky) at $D_C = 1$. This is because customer 1 will stick to purchasing product A (his/her most favorably predisposed product) while customer 2 will stick to purchasing product C (his/her most favorably predisposed product). What happens to product B? The demand expression for B is,

$$D_B(p_A, p_B, p_C) = \begin{cases} 0, & \text{if } p_B > \min\{p_A - \xi, p_C + \xi\} \text{ and } p_B > \min\{p_A + \xi, p_C - \xi\}, \\ 1, & \text{if } p_B < \min\{p_A - \xi, p_C + \xi\} \text{ and } p_B > \min\{p_A + \xi, p_C - \xi\},, \\ & \text{– or –} \\ & \text{if } p_B > \min\{p_A - \xi, p_C + \xi\} \text{ and } p_B < \min\{p_A + \xi, p_C - \xi\}, \\ 2, & \text{if } p_B < \min\{p_A - \xi, p_C + \xi\} \text{ and } p_B < \min\{p_A + \xi, p_C - \xi\}. \end{cases}$$

We have three cases depending on the values of p_A and p_C.

$$\text{(i) if } p_A < p_C: \quad D_B(p_A, p_B, p_C) = \begin{cases} 0, & \text{if } p_B > \min\{p_A + \xi, p_C - \xi\}, \\ 1, & \text{if } p_A - \xi < p_B < \min\{p_A + \xi, p_C - \xi\}, \\ 2, & \text{if } p_B < p_A - \xi. \end{cases}$$

$$\text{(ii) if } p_A = p_C: \quad D_B(p_A, p_B, p_C) = \begin{cases} 0, & \text{if } p_B > p_A - \xi, \\ 1, & \text{if } p_B = p_A - \xi, \\ 2, & \text{if } p_B < p_A - \xi. \end{cases}$$

$$\text{(iii) if } p_C \leq p_A: \quad D_B(p_A, p_B, p_C) = \begin{cases} 0, & \text{if } p_B > \min\{p_A - \xi,\ p_C + \xi\}, \\ 1, & \text{if } p_C - \xi < p_B < \min\{p_A - \xi,\ p_C + \xi\}, \\ 2, & \text{if } p_B < p_C - \xi. \end{cases}$$

Without loss of generality, let's take the first case where $p_A < p_C$. As ambiguity ξ increases, the region where $D_B = 1$ increases only if $p_A + \xi < p_C - \xi$, or when $p_C >> p_A$. In other words, when price of product C is exorbitantly high, with more ambiguity, customer 2 who is most favorably predisposed to C will settle for B instead. When this is not the case, increasing ambiguity ξ decreases the region of prices p_B where $D_B = 2$ and increases the the region of prices p_B where $D_B = 0$, while maintaining the constant region of prices p_B where $D_B = 1$.

The asymmetric effect of ξ on the three demand curves occurred because while the products were symmetric in their qualities, $q_A = q_B = q_C$, they were asymmetric on the consumer predispositions. That is, only 2 products (A and C) had most favorably predisposed customers, while product B had both neutrally predisposed customers. □

We next show that having three equal quality products and three customers with predispositions equally diversified among three products results in three symmetric demand curves.

A.2. 3 products and 3 customers

There are now 3 customers, 1, 2, and 3. Extending the demand functions from the previous section to take values 0,1,2, and 3, we have:

$$D_A(p_A, p_B, p_C) = \begin{cases} 0, & \text{if } p_A > \min\{(q_A - q_B) + p_B - (\Omega_{1A} - \Omega_{1B})\xi,\ (q_A - q_C) + p_C - (\Omega_{1A} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A - q_B) + p_B - (\Omega_{2A} - \Omega_{2B})\xi,\ (q_A - q_C) + p_C - (\Omega_{2A} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A - q_B) + p_B - (\Omega_{3A} - \Omega_{3B})\xi,\ (q_A - q_C) + p_C - (\Omega_{3A} - \Omega_{3C})\xi\} \\ 1, & \text{if } p_A < \min\{(q_A - q_B) + p_B - (\Omega_{1A} - \Omega_{1B})\xi,\ (q_A - q_C) + p_C - (\Omega_{1A} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A - q_B) + p_B - (\Omega_{2A} - \Omega_{2B})\xi,\ (q_A - q_C) + p_C - (\Omega_{2A} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A - q_B) + p_B - (\Omega_{3A} - \Omega_{3B})\xi,\ (q_A - q_C) + p_C - (\Omega_{3A} - \Omega_{3C})\xi\}, \\ & \qquad - \text{or} - \\ & \text{if } p_A > \min\{(q_A - q_B) + p_B - (\Omega_{1A} - \Omega_{1B})\xi,\ (q_A - q_C) + p_C - (\Omega_{1A} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A < \min\{(q_A - q_B) + p_B - (\Omega_{2A} - \Omega_{2B})\xi,\ (q_A - q_C) + p_C - (\Omega_{2A} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A - q_B) + p_B - (\Omega_{3A} - \Omega_{3B})\xi,\ (q_A - q_C) + p_C - (\Omega_{3A} - \Omega_{3C})\xi\}, \\ & \qquad - \text{or} - \\ & \text{if } p_A > \min\{(q_A - q_B) + p_B - (\Omega_{1A} - \Omega_{1B})\xi,\ (q_A - q_C) + p_C - (\Omega_{1A} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A - q_B) + p_B - (\Omega_{2A} - \Omega_{2B})\xi,\ (q_A - q_C) + p_C - (\Omega_{2A} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_A < \min\{(q_A - q_B) + p_B - (\Omega_{3A} - \Omega_{3B})\xi,\ (q_A - q_C) + p_C - (\Omega_{3A} - \Omega_{3C})\xi\}, \\ 2, & \text{if } p_A < \min\{(q_A - q_B) + p_B - (\Omega_{1A} - \Omega_{1B})\xi,\ (q_A - q_C) + p_C - (\Omega_{1A} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A < \min\{(q_A - q_B) + p_B - (\Omega_{2A} - \Omega_{2B})\xi,\ (q_A - q_C) + p_C - (\Omega_{2A} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A - q_B) + p_B - (\Omega_{3A} - \Omega_{3B})\xi,\ (q_A - q_C) + p_C - (\Omega_{3A} - \Omega_{3C})\xi\}, \\ & \qquad - \text{or} - \\ & \text{if } p_A < \min\{(q_A - q_B) + p_B - (\Omega_{1A} - \Omega_{1B})\xi,\ (q_A - q_C) + p_C - (\Omega_{1A} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A > \min\{(q_A - q_B) + p_B - (\Omega_{2A} - \Omega_{2B})\xi,\ (q_A - q_C) + p_C - (\Omega_{2A} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_A < \min\{(q_A - q_B) + p_B - (\Omega_{3A} - \Omega_{3B})\xi,\ (q_A - q_C) + p_C - (\Omega_{3A} - \Omega_{3C})\xi\}, \\ & \qquad - \text{or} - \\ & \text{if } p_A > \min\{(q_A - q_B) + p_B - (\Omega_{1A} - \Omega_{1B})\xi,\ (q_A - q_C) + p_C - (\Omega_{1A} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A < \min\{(q_A - q_B) + p_B - (\Omega_{2A} - \Omega_{2B})\xi,\ (q_A - q_C) + p_C - (\Omega_{2A} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_A < \min\{(q_A - q_B) + p_B - (\Omega_{3A} - \Omega_{3B})\xi,\ (q_A - q_C) + p_C - (\Omega_{3A} - \Omega_{3C})\xi\}, \\ 3, & \text{if } p_A < \min\{(q_A - q_B) + p_B - (\Omega_{1A} - \Omega_{1B})\xi,\ (q_A - q_C) + p_C - (\Omega_{1A} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_A < \min\{(q_A - q_B) + p_B - (\Omega_{2A} - \Omega_{2B})\xi,\ (q_A - q_C) + p_C - (\Omega_{2A} - \Omega_{2C})\xi\} \text{ and} \\ & \quad p_A < \min\{(q_A - q_B) + p_B - (\Omega_{3A} - \Omega_{3B})\xi,\ (q_A - q_C) + p_C - (\Omega_{3A} - \Omega_{3C})\xi\}; \end{cases}$$

$$D_B(p_A, p_B, p_C) = \begin{cases} 0, & \text{if } p_B > \min\{(q_B - q_A) + p_A - (\Omega_{1B} - \Omega_{1A})\xi,\ (q_B - q_C) + p_C - (\Omega_{1B} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B - q_A) + p_A - (\Omega_{2B} - \Omega_{2A})\xi,\ (q_B - q_C) + p_C - (\Omega_{2B} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B - q_A) + p_A - (\Omega_{3B} - \Omega_{3A})\xi,\ (q_B - q_C) + p_C - (\Omega_{3B} - \Omega_{3C})\xi\}, \\ 1, & \text{if } p_B < \min\{(q_B - q_A) + p_A - (\Omega_{1B} - \Omega_{1A})\xi,\ (q_B - q_C) + p_C - (\Omega_{1B} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B - q_A) + p_A - (\Omega_{2B} - \Omega_{2A})\xi,\ (q_B - q_C) + p_C - (\Omega_{2B} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B - q_A) + p_A - (\Omega_{3B} - \Omega_{3A})\xi,\ (q_B - q_C) + p_C - (\Omega_{3B} - \Omega_{3C})\xi\}, \\ & \qquad - \text{or} - \\ & \text{if } p_B > \min\{(q_B - q_A) + p_A - (\Omega_{1B} - \Omega_{1A})\xi,\ (q_B - q_C) + p_C - (\Omega_{1B} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B - q_A) + p_A - (\Omega_{2B} - \Omega_{2A})\xi,\ (q_B - q_C) + p_C - (\Omega_{2B} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B - q_A) + p_A - (\Omega_{3B} - \Omega_{3A})\xi,\ (q_B - q_C) + p_C - (\Omega_{3B} - \Omega_{3C})\xi\}, \\ & \qquad - \text{or} - \\ & \text{if } p_B > \min\{(q_B - q_A) + p_A - (\Omega_{1B} - \Omega_{1A})\xi,\ (q_B - q_C) + p_C - (\Omega_{1B} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B - q_A) + p_A - (\Omega_{2B} - \Omega_{2A})\xi,\ (q_B - q_C) + p_C - (\Omega_{2B} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B - q_A) + p_A - (\Omega_{3B} - \Omega_{3A})\xi,\ (q_B - q_C) + p_C - (\Omega_{3B} - \Omega_{3C})\xi\}, \\ 2, & \text{if } p_B < \min\{(q_B - q_A) + p_A - (\Omega_{1B} - \Omega_{1A})\xi,\ (q_B - q_C) + p_C - (\Omega_{1B} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B - q_A) + p_A - (\Omega_{2B} - \Omega_{2A})\xi,\ (q_B - q_C) + p_C - (\Omega_{2B} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B - q_A) + p_A - (\Omega_{3B} - \Omega_{3A})\xi,\ (q_B - q_C) + p_C - (\Omega_{3B} - \Omega_{3C})\xi\}, \\ & \qquad - \text{or} - \\ & \text{if } p_B < \min\{(q_B - q_A) + p_A - (\Omega_{1B} - \Omega_{1A})\xi,\ (q_B - q_C) + p_C - (\Omega_{1B} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B > \min\{(q_B - q_A) + p_A - (\Omega_{2B} - \Omega_{2A})\xi,\ (q_B - q_C) + p_C - (\Omega_{2B} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B - q_A) + p_A - (\Omega_{3B} - \Omega_{3A})\xi,\ (q_B - q_C) + p_C - (\Omega_{3B} - \Omega_{3C})\xi\}, \\ & \qquad - \text{or} - \\ & \text{if } p_B > \min\{(q_B - q_A) + p_A - (\Omega_{1B} - \Omega_{1A})\xi,\ (q_B - q_C) + p_C - (\Omega_{1B} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B - q_A) + p_A - (\Omega_{2B} - \Omega_{2A})\xi,\ (q_B - q_C) + p_C - (\Omega_{2B} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B - q_A) + p_A - (\Omega_{3B} - \Omega_{3A})\xi,\ (q_B - q_C) + p_C - (\Omega_{3B} - \Omega_{3C})\xi\}, \\ 3, & \text{if } p_B < \min\{(q_B - q_A) + p_A - (\Omega_{1B} - \Omega_{1A})\xi,\ (q_B - q_C) + p_C - (\Omega_{1B} - \Omega_{1C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B - q_A) + p_A - (\Omega_{2B} - \Omega_{2A})\xi,\ (q_B - q_C) + p_C - (\Omega_{2B} - \Omega_{2C})\xi\}, \text{ and} \\ & \quad p_B < \min\{(q_B - q_A) + p_A - (\Omega_{3B} - \Omega_{3A})\xi,\ (q_B - q_C) + p_C - (\Omega_{3B} - \Omega_{3C})\xi\}; \end{cases}$$

$$
D_C(p_A,p_B,p_C) = \begin{cases}
0, & \text{if } p_C > \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\
 & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \text{ and} \\
 & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{3C}-\Omega_{3A})\xi,\ (q_C-q_B)+p_B-(\Omega_{3C}-\Omega_{3B})\xi\} \\
1, & \text{if } p_C < \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\
 & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \text{ and} \\
 & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{3C}-\Omega_{3A})\xi,\ (q_C-q_B)+p_B-(\Omega_{3C}-\Omega_{3B})\xi\} \\
 & \qquad -\text{ or }- \\
 & \text{if } p_C > \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\
 & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \text{ and} \\
 & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{3C}-\Omega_{3A})\xi,\ (q_C-q_B)+p_B-(\Omega_{3C}-\Omega_{3B})\xi\} \\
 & \qquad -\text{ or }- \\
 & \text{if } p_C > \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\
 & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \text{ and} \\
 & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{3C}-\Omega_{3A})\xi,\ (q_C-q_B)+p_B-(\Omega_{3C}-\Omega_{3B})\xi\} \\
2, & \text{if } p_C < \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\Big\}, \text{ and} \\
 & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \text{ and} \\
 & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{3C}-\Omega_{3A})\xi,\ (q_C-q_B)+p_B-(\Omega_{3C}-\Omega_{3B})\xi\} \\
 & \qquad -\text{ or }- \\
 & \text{if } p_C < \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\
 & \quad p_C > \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \text{ and} \\
 & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{3C}-\Omega_{3A})\xi,\ (q_C-q_B)+p_B-(\Omega_{3C}-\Omega_{3B})\xi\} \\
 & \qquad -\text{ or }- \\
 & \text{if } p_C > \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\
 & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\}, \text{ and} \\
 & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{3C}-\Omega_{3A})\xi,\ (q_C-q_B)+p_B-(\Omega_{3C}-\Omega_{3B})\xi\} \\
3, & \text{if } p_C < \min\{(q_C-q_A)+p_A-(\Omega_{1C}-\Omega_{1A})\xi,\ (q_C-q_B)+p_B-(\Omega_{1C}-\Omega_{1B})\xi\}, \text{ and} \\
 & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{2C}-\Omega_{2A})\xi,\ (q_C-q_B)+p_B-(\Omega_{2C}-\Omega_{2B})\xi\} \text{ and} \\
 & \quad p_C < \min\{(q_C-q_A)+p_A-(\Omega_{3C}-\Omega_{3A})\xi,\ (q_C-q_B)+p_B-(\Omega_{3C}-\Omega_{3B})\xi\}.
\end{cases}
$$

EXAMPLE A-2. For simplicity, we take the products to be symmetric in quality, i.e., $q_A = q_B = q_C$.

Notice that each customer $i \in \{1,2,3\}$ has three predisposition quantities Ω_{iA}, Ω_{iB}, and Ω_{iC}, whose 3 pair-wise differences determines their purchase choice. We assume the following inequality:

$$\Omega_{1A} = -1 < \Omega_{1B} = 0 < \Omega_{1C} = 1,$$

$$\Omega_{2B} = -1 < \Omega_{2C} = 0 < \Omega_{2A} = 1,$$

$$\Omega_{3C} = -1 < \Omega_{3A} = 0 < \Omega_{3B} = 1.$$

In other words, customer 1 is most favorably predisposed to A, followed by B, then C. Customer 2 is most favorably predisposed to B, followed by C, then A. Customer 3 is most favorably predisposed to C, followed by A, then B. This is symmetric in the sense that each product has exactly 1 most favorably predisposed, and also exactly 1 second- and exactly 1 least favorably predisposed customers. Moreover, the magnitude of the difference in predispositions are also identical,

$$
\begin{array}{lll}
\Omega_{1A}-\Omega_{1B} = -1\,; & \Omega_{2A}-\Omega_{2B} = 2\,; & \Omega_{3A}-\Omega_{3B} = -1\,; \\
\Omega_{1A}-\Omega_{1C} = -2\,; & \Omega_{2A}-\Omega_{2C} = 1\,; & \Omega_{3A}-\Omega_{3C} = 1\,; \\
\Omega_{1B}-\Omega_{1C} = -1\,; & \Omega_{2B}-\Omega_{2C} = -1\,; & \Omega_{3B}-\Omega_{3C} = 2\,.
\end{array}
$$

Plugging in these values, the demand expressions above simplify to

$$D_A(p_A,p_B,p_C) = \begin{cases} 0, & \text{if } p_A > \min\{p_B+\xi,\ p_C+2\xi\},\ p_A > \min\{p_B-2\xi,\ p_C-\xi\},\ p_A > \min\{p_B+\xi,\ p_C-\xi\} \\ 1, & \text{if } p_A < \min\{p_B+\xi,\ p_C+2\xi\},\ p_A > \min\{p_B-2\xi,\ p_C-\xi\},\ p_A > \min\{p_B+\xi,\ p_C-\xi\}, \\ & \quad - \text{ or } - \\ & \text{if } p_A > \min\{p_B+\xi,\ p_C+2\xi\},\ p_A < \min\{p_B-2\xi,\ p_C-\xi\},\ p_A > \min\{p_B+\xi,\ p_C-\xi\}, \\ & \quad - \text{ or } - \\ & \text{if } p_A > \min\{p_B+\xi,\ p_C+2\xi\},\ p_A > \min\{p_B-2\xi,\ p_C-\xi\},\ p_A < \min\{p_B+\xi,\ p_C-\xi\}, \\ 2, & \text{if } p_A < \min\{p_B+\xi,\ p_C+2\xi\},\ p_A < \min\{p_B-2\xi,\ p_C-\xi\},\ p_A > \min\{p_B+\xi,\ p_C-\xi\}, \\ & \quad - \text{ or } - \\ & \text{if } p_A < \min\{p_B+\xi,\ p_C+2\xi\},\ p_A > \min\{p_B-2\xi,\ p_C-\xi\},\ p_A < \min\{p_B+\xi,\ p_C-\xi\}, \\ & \quad - \text{ or } - \\ & \text{if } p_A > \min\{p_B+\xi,\ p_C+2\xi\},\ p_A < \min\{p_B-2\xi,\ p_C-\xi\},\ p_A < \min\{p_B+\xi,\ p_C-\xi\}, \\ 3, & \text{if } p_A < \min\{p_B+\xi,\ p_C+2\xi\},\ p_A < \min\{p_B-2\xi,\ p_C-\xi\}\ p_A < \min\{p_B+\xi,\ p_C-\xi\} \end{cases}$$

$$D_B(p_A,p_B,p_C) = \begin{cases} 0, & \text{if } p_B > \min\{p_A-\xi,\ p_C+\xi\},\ p_B > \min\{p_A+2\xi,\ p_C+\xi\},\ p_B > \min\{p_A-\xi,\ p_C-2\xi\}, \\ 1, & \text{if } p_B < \min\{p_A-\xi,\ p_C+\xi\},\ p_B > \min\{p_A+2\xi,\ p_C+\xi\},\ p_B > \min\{p_A-\xi,\ p_C-2\xi\}, \\ & \quad - \text{ or } - \\ & \text{if } p_B > \min\{p_A-\xi,\ p_C+\xi\},\ p_B < \min\{p_A+2\xi,\ p_C+\xi\},\ p_B > \min\{p_A-\xi,\ p_C-2\xi\}, \\ & \quad - \text{ or } - \\ & \text{if } p_B > \min\{p_A-\xi,\ p_C+\xi\},\ p_B > \min\{p_A+2\xi,\ p_C+\xi\},\ p_B < \min\{p_A-\xi,\ p_C-2\xi\}, \\ 2, & \text{if } p_B < \min\{p_A-\xi,\ p_C+\xi\},\ p_B < \min\{p_A+2\xi,\ p_C+\xi\},\ p_B > \min\left\{p_A-\xi,\ p_C-2\xi\},\right. \\ & \quad - \text{ or } - \\ & \text{if } p_B < \min\{p_A-\xi,\ p_C+\xi\},\ p_B > \min\{p_A+2\xi,\ p_C+\xi\},\ p_B < \min\{p_A-\xi,\ p_C-2\xi\}, \\ & \quad - \text{ or } - \\ & \text{if } p_B > \min\{p_A-\xi,\ p_C+\xi\},\ p_B < \min\{p_A+2\xi,\ p_C+\xi\},\ p_B < \min\{p_A-\xi,\ p_C-2\xi\}, \\ 3, & \text{if } p_B < \min\{p_A-\xi,\ p_C+\xi\},\ p_B < \min\{p_A+2\xi,\ p_C+\xi\},\ p_B < \min\{p_A-\xi,\ p_C-2\xi\}; \end{cases}$$

$$D_C(p_A,p_B,p_C) = \begin{cases} 0, & \text{if } p_C > \min\{p_A-2\xi,\ p_B-\xi\},\ p_C > \min\{p_A+\xi,\ p_B-\xi\}, p_C > \min\{p_A+\xi,\ p_B+2\xi\} \\ 1, & \text{if } p_C < \min\{p_A-2\xi,\ p_B-\xi\},\ p_C > \min\{p_A+\xi,\ p_B-\xi\},\ p_C > \min\{p_A+\xi,\ p_B+2\xi\} \\ & \quad - \text{ or } - \\ & \text{if } p_C > \min\{p_A-2\xi,\ p_B-\xi\},\ p_C < \min\{p_A+\xi,\ p_B-\xi\},\ p_C > \min\{p_A+\xi,\ p_B+2\xi\} \\ & \quad - \text{ or } - \\ & \text{if } p_C > \min\{p_A-2\xi,\ p_B-\xi\},\ p_C > \min\{p_A+\xi,\ p_B-\xi\},\ p_C < \min\{p_A+\xi,\ p_B+2\xi\} \\ 2, & \text{if } p_C < \min\{p_A-2\xi,\ p_B-\xi\},\ p_C < \min\{p_A+\xi,\ p_B-\xi\},\ p_C > \min\{p_A+\xi,\ p_B+2\xi\} \\ & \quad - \text{ or } - \\ & \text{if } p_C < \min\{p_A-2\xi,\ p_B-\xi\},\ p_C > \min\{p_A+\xi,\ p_B-\xi\},\ p_C < \min\{p_A+\xi,\ p_B+2\xi\} \\ & \quad - \text{ or } - \\ & \text{if } p_C > \min\{p_A-2\xi,\ p_B-\xi\},\ p_C < \min\{p_A+\xi,\ p_B-\xi\},\ p_C < \min\{p_A+\xi,\ p_B+2\xi\} \\ 3, & \text{if } p_C < \min\{p_A-2\xi,\ p_B-\xi\},\ p_C < \min\{p_A+\xi,\ p_B-\xi\}\ p_C < \min\{p_A+\xi,\ p_B+2\xi\}. \end{cases}$$

Further simplifications result in the following symmetric demand expressions:

$$D_A(p_A,p_B,p_C) = \begin{cases} 0, & \text{if } \ p_A > \min\{p_B+\xi,\ p_C+2\xi\}, \\ 1, & \text{if } \ \min\{p_B+\xi,\ p_C-\xi\} < p_A < \min\{p_B+\xi,\ p_C+2\xi\}, \\ 2, & \text{if } \ \min\{p_B-2\xi,\ p_C-\xi\} < p_A < \min\{p_B+\xi,\ p_C-\xi\}, \\ 3, & \text{if } \ p_A < \min\{p_B-2\xi,\ p_C-\xi\}. \end{cases}$$

$$D_B(p_A,p_B,p_C) = \begin{cases} 0, & \text{if } \ p_B > \min\{p_A+2\xi,\ p_C+\xi\}, \\ 1, & \text{if } \ \min\{p_A-\xi,\ p_C+\xi\} < p_B < \min\{p_A+2\xi,\ p_C+\xi\}, \\ 2, & \text{if } \ \min\{p_A-\xi,\ p_C-2\xi\} < p_B < \min\{p_A-\xi,\ p_C+\xi\}, \\ 3, & \text{if } \ p_B < \min\{p_A-\xi,\ p_C-2\xi\}. \end{cases}$$

$$D_C(p_A,p_B,p_C) = \begin{cases} 0, & \text{if } \ p_C > \min\{p_A+\xi,\ p_B+2\xi\}, \\ 1, & \text{if } \ \min\{p_A+\xi,\ p_B-\xi\} < p_C < \min\{p_A+\xi,\ p_B+2\xi\}, \\ 2, & \text{if } \ \min\{p_A-2\xi,\ p_B-\xi\} < p_C < \min\{p_A+\xi,\ p_B-\xi\}, \\ 3, & \text{if } \ p_C < \min\{p_A-2\xi,\ p_B-\xi\}. \end{cases}$$

From these expressions, we see that if the prices were set so that $p_A = p_B = p_C$, each product will get 1 customer who is most favorably predisposed, and that customer is less likely to switch to another product when ambiguity ξ increases. This is illustrated in Figure A-1. □

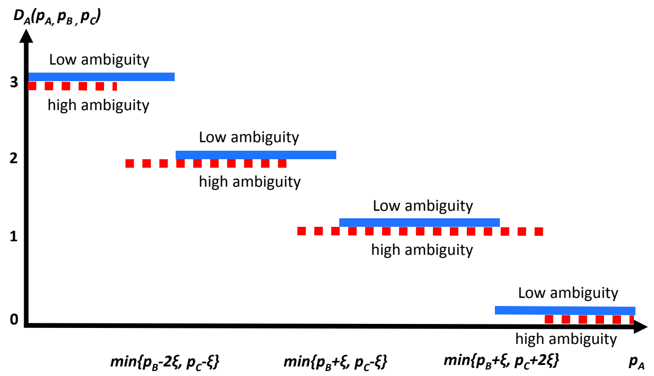


Figure A-1 Illustration of demand curve $D_A(p_A,p_B,p_C)$ when $p_B = p_C$. When $\min\{p_B+\xi,\ p_C-\xi\} < p_A < \min\{p_B+\xi,\ p_C+2\xi\}$, demand is less responsive to price changes for higher levels of ambiguity ξ.

In the example above, the three customers' predispositions from each firm's viewpoint, are distributed as follows (maintaining consistent order of comparison, i.e., A compares with B then C; B compares with C then A; C compares with A then B):

$$\begin{array}{llll} & \text{customer 1} & \text{customer 2} & \text{customer 3} \\ \text{Firm A:} \ (\Omega_A-\Omega_B,\ \Omega_A-\Omega_C) \in \Big\{(-1,-2), & (2,1), & (-1,1)\Big\}, \\ \text{Firm B:} \ (\Omega_B-\Omega_C,\ \Omega_B-\Omega_A) \in \Big\{(-1,1), & (-1,-2), & (2,1)\Big\}, \\ \text{Firm C:} \ (\Omega_C-\Omega_A,\ \Omega_C-\Omega_B) \in \Big\{(2,1), & (-1,1), & (-1,-2)\Big\}. \end{array}$$

In other words, the 3 customers' relative preferences are distributed in an identical manner for all three firm's viewpoint. In other words, the distribution are identical. We next extend this notion of symmetry for continuous mass of consumer population.

A.3. Extension to continuous mass of consumer population

Recall that customer i will purchase product A (and not B or C) if and only if

$$q_A - p_A - \Omega_{i,A}\xi > q_B - p_B - \Omega_{i,B}\xi, \quad \text{and} \quad q_A - p_A - \Omega_{i,A}\xi > q_C - p_C - \Omega_{i,C}\xi.$$

Equivalently, if and only if

$$\Omega_{AB} \triangleq (\Omega_{i,A} - \Omega_{i,B}) < \frac{(q_A - q_B) - (p_A - p_B)}{\xi} \quad \text{and} \quad \Omega_{AC} \triangleq (\Omega_{i,A} - \Omega_{i,C}) < \frac{(q_A - q_C) - (p_A - p_C)}{\xi}.$$

Let us normalize the demand to 1, and have $h_A(\Omega_{AB}, \Omega_{AC})$ denote the probability mass of consumers from firm A's point of view. Similarly, $h_B(\Omega_{BC}, \Omega_{BA})$ and $h_C(\Omega_{CA}, \Omega_{CB})$ denote the probability mass of consumers from firm B and C's perspective respectively.

All consumers i's that satisfy both inequalities above will purchase product A. Thus, the demand for product A is

$$\begin{aligned} D_A(p_A, p_B, p_C) &= \int_{-\infty}^{\frac{(q_A-q_B)-(p_A-p_B)}{\xi}} \int_{-\infty}^{\frac{(q_A-q_C)-(p_A-p_C)}{\xi}} h_A(\Omega_{AB}, \Omega_{AC}) d\Omega_{AB} d\Omega_{AC} \\ &= H_A\left(\frac{(q_A - q_B) - (p_A - p_B)}{\xi}, \frac{(q_A - q_C) - (p_A - p_C)}{\xi}\right). \end{aligned}$$

This is visualized in Figure A-2. Similarly, we have

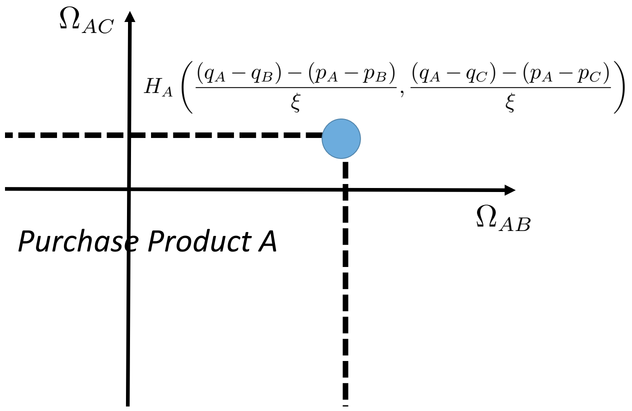


Figure A-2 **Visual illustration of demand curve $D_A(p_A, p_B, p_C)$. Note that as p_A increases, the region shrinks both horizontally and vertically. Thus, we will see later that when we take derivative with respect to p_A it will involve the sum of two partial derivatives.**

$$\begin{aligned} D_B(p_A, p_B, p_C) &= H_B\left(\frac{(q_B - q_C) - (p_B - p_C)}{\xi}, \frac{(q_B - q_A) - (p_B - p_A)}{\xi}\right), \\ D_C(p_A, p_B, p_C) &= H_C\left(\frac{(q_C - q_A) - (p_C - p_A)}{\xi}, \frac{(q_C - q_B) - (p_C - p_B)}{\xi}\right). \end{aligned}$$

We next present the equilibrium result for the special case of symmetric competition, where all firms have equal qualities and no firm has a predisposition advantage (i.e., they are similarly situated products).

Corollary A.1 **(Symmetric Competition)**. *Suppose that $q_A = q_B = q_C$, and $h_A(\Omega_{AB}, \Omega_{AC})$, $h_B(\Omega_{BC}, \Omega_{BA})$, and $h_C(\Omega_{CA}, \Omega_{CB})$ are identically distributed. Then*

$$p_A^* = p_B^* = p_C^* = \frac{\xi}{3h(0,0)}, \quad D_A^* = D_B^* = D_C^* = \frac{1}{3}.$$

The expression is easily generalized to n firms, where for each firm j, $p_j^* = \frac{\xi}{nh(0)}$, $D_j^* = \frac{1}{n}$.

Appendix B: Presence of Loyal Customers: Asymmetric Case

To make intuition precise for the asymmetric case, we suppose that predispositions are uniformly distributed in $\Omega \in [-x+K, x+K]$. Using the uniform distribution for H, we have

$$D_A(p_A,p_B) = \begin{cases} \ell_A + (1-\ell_A-\ell_B)\left[\left(\frac{(q_A-q_B)-(p_A-p_B)}{2x\xi}\right)+\left(\frac{x-K}{2x}\right)\right] & \text{if } p_A \le \bar{p}_A, \\ 0+(1-\ell_A-\ell_B)\left[\left(\frac{(q_A-q_B)-(p_A-p_B)}{2x\xi}\right)+\left(\frac{x-K}{2x}\right)\right] & \text{if } p_A > \bar{p}_A. \end{cases}$$

$$D_B(p_A,p_B) = \begin{cases} \ell_B + (1-\ell_A-\ell_B)\left[1-\left(\frac{(q_A-q_B)-(p_A-p_B)}{2x\xi}\right)-\left(\frac{x-K}{2x}\right)\right] & \text{if } p_B \le \bar{p}_B, \\ 0+(1-\ell_A-\ell_B)\left[1-\left(\frac{(q_A-q_B)-(p_A-p_B)}{2x\xi}\right)-\left(\frac{x-K}{2x}\right)\right] & \text{if } p_B > \bar{p}_B. \end{cases}$$

Let, $p_A^{0*} \equiv \frac{\Delta Q}{3} + \left(x - \frac{K}{3}\right)\xi$ and $p_B^{0*} \equiv -\frac{\Delta Q}{3} + \left(x + \frac{K}{3}\right)\xi$ denote the equilibrium prices without loyal customers (i.e., when $l_A = l_B = 0$) as in Corollary 2. Provided that the loyal customers have higher thresholds than the equilibrium market price, we have the following equilibrium prices with loyal customers.

Proposition B.1. *Suppose that $h(\Omega) = 1/2x$, with $\Omega \in [-x+K,\, x+K]$ for some K and some $x > 0$, and that the degree of ambiguity $\xi > 0$. If $\bar{p}_A > p_A^{0*}$ and $\bar{p}_B > p_B^{0*}$, then*

(i) if $\bar{p}_A \le 2\bar{p}_B + \Delta Q - \left(x\left[1+\frac{2l_B}{1-l_A-l_B}\right]+K\right)\xi$ & $\bar{p}_B \le 2\bar{p}_A - \Delta Q - \left(x\left[1+\frac{2l_A}{1-l_A-l_B}\right]-K\right)\xi$,

$$(p_A^*,p_B^*) = \left(\left(x\left[\frac{1}{3}+\frac{2}{3}\left(\frac{1+l_A}{1-l_A-l_B}\right)\right]-\frac{K}{3}\right)\xi + \frac{\Delta Q}{3},\ \left(x\left[\frac{1}{3}+\frac{2}{3}\left(\frac{1+l_B}{1-l_A-l_B}\right)\right]+\frac{K}{3}\right)\xi - \frac{\Delta Q}{3} \right);$$

(ii) if $\bar{p}_A \le 2\bar{p}_B + \Delta Q - \left(x\left[1+\frac{2l_B}{1-l_A-l_B}\right]+K\right)\xi$ & $\bar{p}_B > 2\bar{p}_A - \Delta Q - \left(x\left[1+\frac{2l_A}{1-l_A-l_B}\right]-K\right)\xi$,

$$(p_A^*,p_B^*) = \left(\bar{p}_A,\ \left(x\left[1+\frac{2l_B}{1-l_A-l_B}\right]+K\right)\frac{\xi}{2} - \frac{\Delta Q - \bar{p}_A}{2} \right);$$

(iii) if $\bar{p}_A > 2\bar{p}_B + \Delta Q - \left(x\left(1+\frac{2l_B}{1-l_A-l_B}\right)+K\right)\xi$ & $\bar{p}_B \le 2\bar{p}_A - \Delta Q - \left(x\left[1+\frac{2l_A}{1-l_A-l_B}\right]-K\right)\xi$,

$$(p_A^*,p_B^*) = \left(\left(x\left[1+\frac{2l_A}{1-l_A-l_B}\right]-K\right)\frac{\xi}{2} + \frac{\Delta Q + \bar{p}_B}{2},\ \bar{p}_B \right);$$

(iv) if $\bar{p}_A > 2\bar{p}_B + \Delta Q - \left(x\left[1+\frac{2l_B}{1-l_A-l_B}\right]+K\right)\xi$ & $\bar{p}_B > 2\bar{p}_A - \Delta Q - \left(x\left[1+\frac{2l_A}{1-l_A-l_B}\right]-K\right)\xi$,

$$(p_A^*,p_B^*) = (\bar{p}_A,\bar{p}_B).$$

We notice that having loyal customers has the effect of scaling the value of x. From the expressions, we can also see that the equilibrium prices (p_A^*,p_B^*) increases from (p_A^{0*},p_B^{0*}) to $(\bar{p}_A,\bar{p}_B)$ as l_A, l_B, or both increase. Moreover, for any level of loyal customers (l_A,l_B), (p_A^*,p_B^*) is nondecreasing in ambiguity ξ.

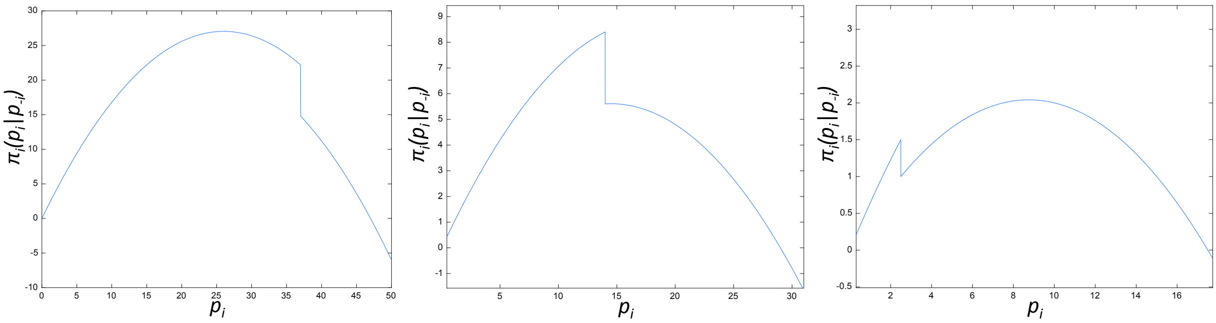
Appendix C: Proofs

Proof of Corollary A.1. We will first derive the best response prices for each firms, and then the equilibrium prices. Firms will, by changing prices, maximize $p_A D_A(p_A,p_B,p_C) = p_A H_A\left(\frac{-(p_A-p_B)}{\xi}, \frac{-(p_A-p_C)}{\xi}\right)$. By taking first-order conditions, the profit maximizing price p_A is one that satisfies the expression,

$$\frac{\partial\left\{p_A H_A\left(\frac{-(p_A-p_B)}{\xi},\frac{-(p_A-p_C)}{\xi}\right)\right\}}{\partial p_A} = H_A\left(\frac{-(p_A-p_B)}{\xi},\frac{-(p_A-p_C)}{\xi}\right) + \frac{p_A\partial\left\{H_A\left(\frac{-(p_A-p_B)}{\xi},\frac{-(p_A-p_C)}{\xi}\right)\right\}}{\partial p_A} = 0$$

$$\Leftrightarrow -\frac{1}{p_A} = \frac{\frac{d}{dp_A}\left\{H_A\left(\frac{-(p_A-p_B)}{\xi},\frac{-(p_A-p_C)}{\xi}\right)\right\}}{H_A\left(\frac{-(p_A-p_B)}{\xi},\frac{-(p_A-p_C)}{\xi}\right)}.$$

Figure C-1 Three profit curves that result in three different expressions for optimal (profit-maximizing) prices.



Similarly, we have $-\frac{1}{p_B} = \frac{\frac{d}{dp_B}\left\{H_B\left(\frac{-(p_B-p_C)}{\xi},\frac{-(p_B-p_A)}{\xi}\right)\right\}}{H_B\left(\frac{-(p_B-p_C)}{\xi},\frac{-(p_B-p_A)}{\xi}\right)}$, and $-\frac{1}{p_C} = \frac{\frac{d}{dp_C}\left\{H_C\left(\frac{-(p_C-p_A)}{\xi},\frac{-(p_C-p_B)}{\xi}\right)\right\}}{H_C\left(\frac{-(p_C-p_A)}{\xi},\frac{-(p_C-p_B)}{\xi}\right)}$.

The best responses of three firms result in three equations and three unknowns. The solution to this system of equations correspond to the unique equilibrium prices, which we show next. Notice first that because $H_A = H_B = H_C$, and by symmetry, $p_A = p_B = p_C$. So the three equations become:

$$-\frac{1}{p_A} = \frac{-h_A(0,0)/\xi}{H_A(0,0)}, \quad -\frac{1}{p_B} = \frac{-h_B(0,0)/\xi}{H_B(0,0)}, \quad \text{and} -\frac{1}{p_C} = \frac{-h_C(0,0)/\xi}{H_C(0,0)}.$$

Since $-h_A(0,0)/\xi = -h_B(0,0)/\xi = -h_C(0,0)/\xi$, it follows that $\frac{H_A(0,0)}{p_A} = \frac{H_B(0,0)}{p_B} = \frac{H_C(0,0)}{p_C}$.

We next solve for H_A. Utilizing the fact that $H_C = 1 - H_A - H_B$, we have (i) $\frac{H_A}{p_A} = \frac{H_B}{p_B}$, and (ii) $\frac{H_B}{p_B} = \frac{1-H_A-H_B}{p_C}$. Solving for expression (ii), we have $H_B\left(\frac{1}{p_B}+\frac{1}{p_C}\right) = \frac{1-H_A}{p_C} \Leftrightarrow H_B\left(\frac{p_B+p_C}{p_B}\right) = (1-H_A) \Leftrightarrow H_B = \frac{p_B}{p_B+p_C}(1-H_A)$. Substituting this expression back into (i), we get

$$\frac{H_A}{p_A} = \frac{1-H_A}{p_B+p_C} \Leftrightarrow H_A(p_B+p_C) = p_A - p_A H_A \Leftrightarrow H_A = \frac{p_A}{p_A+p_B+p_C}.$$

Similarly, we have $H_B = \frac{p_B}{p_A+p_B+p_C}$, and$H_C = \frac{p_C}{p_A+p_B+p_C}$. Since, $p_A = p_B = p_C$ in equilibrium, we have that $H_A = H_B = H_C = \frac{1}{3}$. To find the expressions for the prices, we revisit the first-order equations and plug in the values for H_i, and we have $p_A = p_B = p_C = \frac{\xi}{3h(0,0)}$. □

Proof of Proposition B.1. We first begin by deriving the following expressions for the best-response prices. Given price p_B (resp. p_A) of firm B (resp. firm A), the best response prices of firm A (resp. firm B) $p_A^*(p_B)$ ($p_B^*(p_A)$) is given by ($\Delta Q = q_A - q_B$),

$$p_A^*(p_B) = \begin{cases} \frac{x\xi l_A}{1-l_A-l_B} + \frac{\xi(x-K)}{2} + \frac{\Delta Q + p_B}{2}, & p_B < 2\bar{p}_A - \Delta Q - (x-K)\xi - \frac{2x\xi l_A}{1-l_A-l_B}, \\ \bar{p}_A, & p_B \in 2\bar{p}_A - \Delta Q - (x-K)\xi + \left[-\frac{2x\xi l_A}{1-l_A-l_B}, 2\sqrt{\frac{2x\xi l_A \bar{p}_A}{1-l_A-l_B}}\right], \\ \frac{\xi(x-K)}{2} + \frac{\Delta Q + p_B}{2}, & p_B > 2\bar{p}_A - \Delta Q - (x-K)\xi + 2\sqrt{\frac{2x\xi l_A \bar{p}_A}{1-l_A-l_B}} \end{cases}$$

$$p_B^*(p_A) = \begin{cases} \frac{x\xi l_B}{1-l_A-l_B} + \frac{\xi(x+K)}{2} + \frac{p_A - \Delta Q}{2}, & p_A < 2\bar{p}_B + \Delta Q - (x+K)\xi - \frac{2x\xi l_B}{1-l_A-l_B}, \\ \bar{p}_B, & p_A \in 2\bar{p}_B + \Delta Q - (x+K)\xi + \left[-\frac{2x\xi l_B}{1-l_A-l_B}, 2\sqrt{\frac{2x\xi l_B \bar{p}_B}{1-l_A-l_B}}\right], \\ \frac{\xi(x+K)}{2} + \frac{p_A - \Delta Q}{2}, & p_A > 2\bar{p}_B + \Delta Q - (x+K)\xi + 2\sqrt{\frac{2x\xi l_B \bar{p}_B}{1-l_A-l_B}} \end{cases}$$

A discontinuous drop occurs when the price exceeds the willingness-to-pay, $\bar{p}_i$, of the loyal customers. This can occur in three different points (see Figure C-1), leading to three different expressions for the optimal price. After finding the expressions for the optimal prices in each case, we identify the conditions for each case.

First, we take the first order condition of $\pi_A^\ell(p_A, p_B) \equiv p_A D_A(p_A, p_B)$ for the expression when $p_A \le \bar{p}_A$. This corresponds to the case when the loyal customers have high willingness-to-pay so that the firm can maximize assuming that it will retain the loyal customers. For this case, we have

$$p_A D_A(p_A,p_B) = p_A l_A + p_A(1-l_A-l_B)\left(\frac{\Delta Q/\xi+(x-K)+p_B/\xi}{2x}-\frac{p_A}{2x\xi}\right).$$

Taking the first-order conditions, we have

$$\begin{aligned}\frac{\partial p_A D_A(p_A,p_B)}{\partial p_A}=0 &\Leftrightarrow l_A+(1-l_A-l_B)\left(\frac{\Delta Q/\xi+(x-K)+p_B/\xi}{2x}-\frac{p_A}{2x\xi}\right)-\frac{(p_A(1-l_A-l_B)}{2x\xi}=0\\ &\Leftrightarrow l_A+(1-l_A-l_B)\left(\frac{\Delta Q/\xi+(x-K)+p_B/\xi}{2x}-\frac{p_A}{2x\xi}\right)=\frac{(p_A(1-l_A-l_B)}{2x\xi}\\ &\Leftrightarrow l_A+(1-l_A-l_B)\left(\frac{\Delta Q/\xi+(x-K)+p_B/\xi}{2x}\right)=\frac{(p_A(1-l_A-l_B)}{x\xi}\\ &\Leftrightarrow p_A^{l*}=\frac{x\xi l_A}{1-l_A-l_B}+\frac{\Delta Q+p_B}{2}+\frac{(x-K)\xi}{2}.\end{aligned}$$

This condition when $p_A^*=p_A^{l*}$ occurs when,

$$p_A\le\bar{p}_A\Leftrightarrow\frac{x\xi l_A}{1-l_A-l_B}+\frac{\Delta Q+p_B}{2}+\frac{(x-K)\xi}{2}\le\bar{p}_A\Leftrightarrow p_B\le 2\bar{p}_A-\Delta Q-(x-K)\xi-\frac{2x\xi l_A}{1-l_A-l_B}.$$

Similarly, the first order conditions of $\pi_A^0(p_A,p_B)\equiv p_A D_A(p_A,p_B)$ for the expression when $p_A>\bar{p}_A$, corresponding to the case when all the loyal customers have low willingness-to-pay and the firm can maximize assuming that it will not retain the loyal customers. This is,

$$p_A^{0*}=\frac{\Delta Q+p_B}{2}+\frac{(x-K)\xi}{2}.$$

The condition when $p_A^*=p_A^{0*}$ occurs when $\bar{p}_A$ is small, specifically when $\pi_A^0(p_A^{0*},p_B)>\pi_A^l(\bar{p}_A,p_B)$. We first evaluate $\pi_A^0(p_A^{0*},p_B)$ and then $\pi_A^l(\bar{p}_A,p_B)$.

$$\begin{aligned}\pi_A^0(p_A^{0*},p_B)&=p_A^{0*}(1-l_A-l_B)\left[\frac{1}{2x}\left(\frac{\Delta Q+p_B-p_A^{0*}}{\xi}\right)+\left(\frac{x-K}{2x}\right)\right]\\ &=p_A^{0*}(1-l_A-l_B)\left[\frac{1}{2x\xi}\left(\Delta Q+p_B-\left(\frac{\Delta Q+p_B}{2}+\frac{(x-K)\xi}{2}\right)\right)+\left(\frac{x-K}{2x}\right)\right]\\ &=p_A^{0*}(1-l_A-l_B)\left[\frac{1}{2x\xi}\left(\frac{\Delta Q+p_B}{2}\right)-\left(\frac{x-K}{4x}\right)+\left(\frac{x-K}{2x}\right)\right]\\ &=p_A^{0*}\frac{(1-l_A-l_B)}{2x\xi}\left[\frac{\Delta Q+p_B}{2}+\frac{(x-K)\xi}{2}\right]=\frac{(1-l_A-l_B)}{2x\xi}(p_A^{0*})^2;\\ \pi_A^l(\bar{p}_A,p_B)&=\bar{p}_A l_A+\bar{p}_A(1-l_A-l_B)\left(\frac{\Delta Q+p_B}{2x\xi}+\frac{(x-K)\xi}{2x\xi}-\frac{\bar{p}_A}{2x\xi}\right)\\ &=\bar{p}_A l_A+\bar{p}_A\frac{1-l_A-l_B}{x\xi}\left(\frac{\Delta Q+p_B}{2}+\frac{(x-K)\xi}{2}-\frac{\bar{p}_A}{2}\right)=\bar{p}_A l_A+\bar{p}_A\frac{1-l_A-l_B}{x\xi}\left(p_A^{0*}-\frac{\bar{p}_A}{2}\right).\end{aligned}$$

We have that,

$$\begin{aligned}\pi_A^0(p_A^{0*},p_B)=\pi_A^l(\bar{p}_A,p_B)&\Leftrightarrow\frac{(1-l_A-l_B)}{2x\xi}(p_A^{0*})^2=\bar{p}_A l_A+\bar{p}_A\frac{1-l_A-l_B}{x\xi}\left(p_A^{0*}-\frac{\bar{p}_A}{2}\right)\\ &\Leftrightarrow\bar{p}_A l_A+\bar{p}_A\frac{1-l_A-l_B}{x\xi}\left(p_A^{0*}-\frac{\bar{p}_A}{2}\right)-\frac{(1-l_A-l_B)}{2x\xi}(p_A^{0*})^2=0\\ &\Leftrightarrow\bar{p}_A^2\left(\frac{1-l_A-l_B}{2x\xi}\right)-\bar{p}_A\left(l_A+\left(\frac{1-l_A-l_B}{x\xi}\right)p_A^{0*}\right)+\left(\frac{1-l_A-l_B}{2x\xi}\right)(p_A^{0*})^2=0\\ &\Leftrightarrow\bar{p}_A^2-\bar{p}_A\left(\frac{2l_Ax\xi}{1-l_A-l_B}+2p_A^{0*}\right)+(p_A^{0*})^2=0\end{aligned}$$

Applying the quadratic equation, we have

$$\bar{p}_A=\frac{x\xi l_A}{1-l_A-l_B}+p_A^{0*}-\frac{1}{2}\sqrt{\left(\frac{2l_Ax\xi}{1-l_A-l_B}+2p_A^{0*}\right)^2-4(p_A^{0*})^2}=p_A^{l*}-\sqrt{(p_A^{l*})^2-(p_A^{0*})^2}$$

We want to find p_B that makes this equality hold. So,

$$\bar{p}_A = p_A^{l*} - \sqrt{(p_A^{l*})^2 - (p_A^{0*})^2} \Leftrightarrow \sqrt{(p_A^{l*})^2 - (p_A^{0*})^2} = p_A^{l*} - \bar{p}_A \Leftrightarrow (p_A^{l*})^2 - (p_A^{0*})^2 = (p_A^{l*} - \bar{p}_A)^2$$
$$\Leftrightarrow -(p_A^{0*})^2 = -2p_A^{l*}\bar{p}_A + \bar{p}_A^2 \Leftrightarrow (p_A^{0*})^2 - 2p_A^{l*}\bar{p}_A + \bar{p}_A^2 = 0 \Leftrightarrow (p_A^{0*})^2 - 2\left(\frac{x\xi l_A}{1-l_A-l_B} + p_A^{0*}\right)\bar{p}_A + \bar{p}_A^2 = 0$$
$$\Leftrightarrow (p_A^{0*})^2 - 2\bar{p}_A p_A^{0*} + \left[\bar{p}_A^2 - \left(\frac{2x\xi l_A}{1-l_A-l_B}\right)\bar{p}_A\right] = 0.$$

Applying the quadratic equation with respect to p_A^{0*}, we have

$$p_A^{0*} = \frac{1}{2}\left(2\bar{p}_A + \sqrt{4\bar{p}_A^2 - 4\left[\bar{p}_A^2 - \left(\frac{2x\xi l_A}{1-l_A-l_B}\right)\bar{p}_A\right]}\right) = \bar{p}_A + \sqrt{\left(\frac{2x\xi l_A}{1-l_A-l_B}\right)\bar{p}_A}.$$

Since $p_A^{0*} = \frac{\Delta Q}{2} + \frac{(x-K)\xi}{2} + \frac{p_B}{2}$, solving for p_B we have

$$\frac{\Delta Q}{2} + \frac{(x-K)\xi}{2} + \frac{p_B}{2} = \bar{p}_A + \sqrt{\left(\frac{2x\xi l_A}{1-l_A-l_B}\right)\bar{p}_A} \Leftrightarrow p_B = 2\bar{p}_A - \Delta Q - (x-K)\xi + 2\sqrt{\left(\frac{2x\xi l_A}{1-l_A-l_B}\right)\bar{p}_A}.$$

Thus, when $p_B > 2\bar{p}_A - \Delta Q - (x-K)\xi + 2\sqrt{\left(\frac{2x\xi l_A}{1-l_A-l_B}\right)\bar{p}_A}$, firm A is better off setting price as if loyal segment does not exist.

Finally there are cases where it is beneficial for firm to charge a price $p_A = \bar{p}_A$ so as to retain the loyal customers. In this case, the firm is better off maximizing the benefit from all the loyal customers rather than optimizing the profit without the loyal segment of consumers. This occurs when p_B is in between these two thresholds. Thus, we have the best response expression for $p_A^*(p_B)$. The best response expression for $p_B^*(p_A)$ can be found in a similar manner.

Given the best response curves, the equilibrium prices (p_A^*, p_B^*) will occur at the fixed point. There are four possible fixed points that can occur, as shown in the next Figure. The curve containing slope 1/2 denotes the best response of firm B given price p_A. The curve containing slope 2 denotes the best response of firm A given price p_B.

The four cases occur, according to when the infliction occurs in the best response functions, i.e., based on whether or not

$$\bar{p}_A \lesseqgtr 2\bar{p}_B + \Delta Q - \left(x\left[1 + \frac{2l_B}{1-l_A-l_B}\right] + K\right)\xi, \text{ and } \bar{p}_B \lesseqgtr 2\bar{p}_A - \Delta Q - \left(x\left[1 + \frac{2l_A}{1-l_A-l_B}\right] - K\right)\xi.$$

(i) If $\bar{p}_A \leq 2\bar{p}_B + \Delta Q - \left(x\left[1 + \frac{2l_B}{1-l_A-l_B}\right] + K\right)\xi$ and $\bar{p}_B \leq 2\bar{p}_A - \Delta Q - \left(x\left[1 + \frac{2l_A}{1-l_A-l_B}\right] - K\right)\xi$, then the crossing occurs when the line with slope 1/2 intersects the line with slope 2 (upper left panel of the Figure C-2). Solving for the intersecting point,

$$\begin{aligned}
p_A &= \frac{x\xi l_A}{1-l_A-l_B} + \frac{\xi(x-K)}{2} + \frac{\Delta Q + p_B}{2} \\
&= \frac{x\xi l_A}{1-l_A-l_B} + \frac{\xi(x-K)}{2} + \frac{\Delta Q}{2} + \frac{1}{2}\left(\frac{x\xi l_B}{1-l_A-l_B} + \frac{\xi(x+K)}{2} - \frac{\Delta Q}{2} + \frac{p_A}{2}\right), \\
\frac{3}{4}p_A &= \frac{x\xi}{1-l_A-l_B}\left(l_A + \frac{l_B}{2}\right) + \frac{3}{4}\xi x - \frac{3}{4}K + \frac{\Delta Q}{4}, \\
p_A &= \left(\frac{2x\xi}{1-l_A-l_B}\right)\left(\frac{2}{3}l_A + \frac{1}{3}l_B\right) + \left(x - \frac{K}{3}\right)\xi + \frac{\Delta Q}{3} = \left(x\left[\frac{1}{3} + \frac{2}{3}\left(\frac{1+l_A}{1-l_A-l_B}\right)\right] - \frac{K}{3}\right)\xi + \frac{\Delta Q}{3}.
\end{aligned}$$

Plugging in the expression for p_A into the expression for best response $p_B(p_A)$, we have

$$p_B = \frac{x\xi l_B}{1-l_A-l_B} + \frac{\xi(x+K)}{2} - \frac{\Delta Q}{2} + \frac{p_A}{2}$$

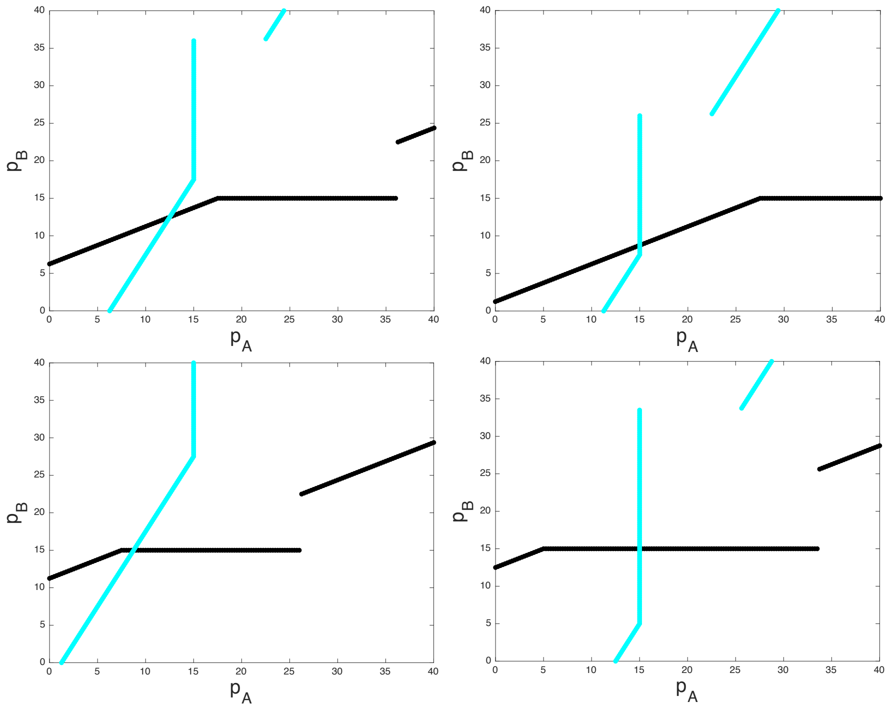


Figure C-2 **Four possible fixed points. Parameters:** $\bar{p}_A = \bar{p}_B = 15$**,** $l_A = l_B = 0.15$**,** $x = 5$**,** $\Delta Q = 0$**;** $\xi = 3.5$ **(upper left),** $\xi = 1.75$ **(rest);** $K = 0$ **(upper left and lower right),** $\Delta K = -5$ **(lower left),** $K = 5$ **(upper right).**

$$= \frac{x\xi l_B}{1-l_A-l_B} + \frac{\xi}{2}(x+K) - \frac{\Delta Q}{2} + \frac{1}{2}\left(\left(\frac{2x\xi}{1-l_A-l_B}\right)\left(\frac{2}{3}l_A + \frac{1}{3}l_B\right) + \left(x - \frac{K}{3}\right)\xi + \frac{\Delta Q}{3}\right)$$
$$= \frac{x\xi}{1-l_A-l_B}\left(\frac{4}{3}l_B + \frac{2}{3}l_A\right) + x\xi + \frac{\xi K}{2} - \frac{\xi K}{6} - \Delta Q\left(\frac{1}{2} - \frac{1}{6}\right)$$
$$= \left(\frac{2x\xi}{1-l_A-l_B}\right)\left(\frac{2}{3}l_B + \frac{1}{3}l_A\right) + \left(x + \frac{K}{3}\right)\xi - \frac{\Delta Q}{3} = \left(x\left[\frac{1}{3} + \frac{2}{3}\left(\frac{1+l_B}{1-l_A-l_B}\right)\right] + \frac{K}{3}\right)\xi - \frac{\Delta Q}{3}.$$

(ii) If $\bar{p}_A \le 2\bar{p}_B + \Delta Q - \left(x\left[1 + \frac{2l_B}{1-l_A-l_B}\right] + K\right)\xi$ and $\bar{p}_B > 2\bar{p}_A - \Delta Q - \left(x\left[1 + \frac{2l_A}{1-l_A-l_B}\right] - K\right)\xi$, then the crossing occurs when the line with slope 1/2 intersects the vertical line (upper right panel of the Figure C-2). Clearly, $p_A^* = \bar{p}_A$, and

$$p_B^* = p_B(\bar{p}_A) = \frac{x\xi l_B}{1-l_A-l_B} + \frac{\xi}{2}(x+K) - \frac{\Delta Q}{2} + \frac{\bar{p}_A}{2} = \left(x\left[1 + \frac{2l_B}{1-l_A-l_B}\right] + K\right)\frac{\xi}{2} - \frac{\Delta Q - \bar{p}_A}{2}.$$

(iii) If $\bar{p}_A > 2\bar{p}_B + \Delta Q - \left(x\left[1 + \frac{2l_B}{1-l_A-l_B}\right] + K\right)\xi$ and $\bar{p}_B \le 2\bar{p}_A - \Delta Q - \left(x\left[1 + \frac{2l_A}{1-l_A-l_B}\right] - K\right)\xi$, then the crossing occurs when the line with slope 2 intersects the horizontal line (lower left panel of the Figure C-2). Clearly, $p_B^* = \bar{p}_B$, and

$$p_A^* = p_A(\bar{p}_B) = \frac{x\xi l_A}{1-l_A-l_B} + \frac{\xi}{2}(x-K) + \frac{\Delta Q}{2} + \frac{\bar{p}_B}{2} = \left(x\left[1 + \frac{2l_A}{1-l_A-l_B}\right] - K\right)\frac{\xi}{2} + \frac{\Delta Q + \bar{p}_B}{2}.$$

(iv) Finally, if $\bar{p}_A > 2\bar{p}_B + \Delta Q - \left(x\left[1 + \frac{2l_B}{1-l_A-l_B}\right] + K\right)\xi$ and $\bar{p}_B > 2\bar{p}_A - \Delta Q - \left(x\left[1 + \frac{2l_A}{1-l_A-l_B}\right] - K\right)\xi$, then the crossing occurs when the vertical line intersects the horizontal line (lower right panel of the Figure C-2). In this case, $p_A^* = \bar{p}_A$ and $p_B^* = \bar{p}_B$. $\square$