

Online Appendix

for

"Fixed vs. Flexible Pricing in a Competitive Market"

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8 Online Appendix 1

8.1 Proof of Propositions 1, 2 and 3

The proof is by induction. In what follows we show that the claims in the propositions hold in the terminal period T . Then we establish the inductive step. To start, substitute the terminal payoffs $\pi_{T+1} = u_{h,T+1} = u_{l,T+1} = 0$ into (3), (4) and (5) to obtain

$$\begin{aligned}
 U_{h,f,T} &= U_{l,f,T} = \frac{1-z_0(q_{h,f,T}+q_{l,f,T})}{q_{h,f,T}+q_{l,f,T}} (1-r_{f,T}) \\
 U_{l,b,T} &= \frac{1-z_0(q_{h,b,T}+q_{l,b,T})}{q_{h,b,T}+q_{l,b,T}} (1-r_{b,T}) \\
 U_{h,b,T} &= U_{l,b,T} + z_0(q_{h,b,T} + q_{l,b,T})(r_{b,T} - y_T + \varepsilon).
 \end{aligned}$$

A high type buyer requests negotiations if $y_T < r_{b,T} + \varepsilon$. This requires θ to be large enough, which we assume to be the case for now. The fact that $y_T < r_{b,T} + \varepsilon$ implies that $U_{h,b,T} > U_{l,b,T}$. At fixed price firms, on the other hand, we have $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{h,T} \geq \bar{U}_{l,T}$. Similarly, substituting $\pi_{T+1} = 0$ into profit functions (7) and (8) and re-arranging yields

$$\Pi_{f,T} = 1 - z_0(q_{h,f,T} + q_{l,f,T}) - q_{h,f,T}U_{h,f,T} - q_{l,f,T}U_{l,f,T}, \quad (8.1)$$

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T} + q_{l,b,T}) - q_{h,b,T}U_{h,b,T} - q_{l,b,T}U_{l,b,T} + q_{h,b,T}z_0(q_{h,b,T} + q_{l,b,T})\varepsilon. \quad (8.2)$$

Lemma 2 *In a competitive search equilibrium all flexible firms post the same list price $r_{b,T}$ and cater to high type buyers only. Similarly, fixed price firms post the same list price $r_{f,T}$, but their customer base depends on ε . If $\varepsilon \leq 0$ then they cater to both types of customers but if $\varepsilon > 0$ then they cater to low types only.*

The Lemma establishes how customer demographics would look like if a competitive search equilibrium were to exist (it does not prove existence). These results greatly facilitate the characterization of the equilibrium, which we accomplish subsequently.

Proof of Lemma 2. The proof consists of the following steps.

- Step 1. Flexible firms cannot attract both types of customers; they attract either the high types (hagglers) or the low types (non-hagglers).
- Step 2. Flexible firms attract high types only.
- Step 3. Fixed price firms cannot attract high types only; they attract either both types or just the low types.
- Step 4a. If $\varepsilon \leq 0$ then fixed price firms attract both types of customers.
- Step 4b. If $\varepsilon > 0$ then fixed price firms attract low types only.
- Step 5. All flexible firms post the same list price $r_{b,T}$ and all fixed price firms post the same list price $r_{f,T}$.

Step 1. We prove that flexible firms cannot attract both types of customers. By contradiction, suppose they do, i.e. consider a flexible firm where expected demands $q_{h,b,T}$ and $q_{l,b,T}$ are both positive. This means that $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} = \bar{U}_{l,T}$. Recall that $U_{h,b,T} > U_{l,b,T}$. It follows that $\bar{U}_{h,T} > \bar{U}_{l,T}$. The seller's profit equals to

$$\begin{aligned} \Pi_{b,T} &= 1 - z_0 (q_{h,b,T} + q_{l,b,T}) - q_{h,b,T} U_{h,b,T} - q_{l,b,T} U_{l,b,T} + q_{h,b,T} z_0 (q_{h,b,T} + q_{l,b,T}) \varepsilon \\ &= 1 - z_0 (q_{h,b,T} + q_{l,b,T}) - (q_{h,b,T} + q_{l,b,T}) U_{l,b,T} - \Delta, \end{aligned}$$

where $\Delta := q_{h,b,T} z_0 (q_{h,b,T} + q_{l,b,T}) (r_{b,T} - y_T + \varepsilon)$. Note that Δ is positive as $r_{b,T} - y_T + \varepsilon > 0$. Now suppose that this seller keeps his price intact at $r = r_{b,T}$ but changes the rule from 'flexible' to 'fixed'. We claim that the seller loses all high type customers ($q_{h,f,T} = 0$) but gains new low type customers one-for-one, so that his new expected demand $q_{l,f,T}$ equals to his previous expected demand $q_{h,b,T} + q_{l,b,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. Since $\bar{U}_{h,T} > \bar{U}_{l,T}$ there are two possibilities:

- $U_{h,f,T} = \bar{U}_{h,T}$ and therefore $U_{l,f,T} > \bar{U}_{l,T}$. This case is impossible since, $U_{l,f,T}$, by definition, cannot exceed the market utility $\bar{U}_{l,T}$.

- $U_{l,f,T} = \bar{U}_{l,T}$ and therefore $U_{h,f,T} < \bar{U}_{h,T}$. This means that $q_{l,f,T}$ is positive and satisfies $U_{l,f,T} = \bar{U}_{l,T}$ while $q_{h,f,T} = 0$ since $U_{h,f,T} < \bar{U}_{h,T}$. This scenario is possible.

Since $U_{l,f,T} = \bar{U}_{l,T}$ and $U_{l,b,T} = \bar{U}_{l,T}$ (from above) we have $U_{l,b,T} = U_{l,f,T}$. This implies that

$$\frac{1-z_0(q_{h,b,T}+q_{l,b,T})}{q_{h,b,T}+q_{l,b,T}}(1-r) = \frac{1-z_0(q_{l,f,T})}{q_{l,f,T}}(1-r)$$

and therefore $q_{l,f,T} = q_{h,b,T} + q_{l,b,T}$. So, by switching to fixed pricing, the seller indeed keeps his total demand intact. The seller now earns

$$\Pi_{f,T} = 1 - z_0(q_{l,f,T}) - q_{l,f,T}U_{l,f,T}.$$

Using the equality $q_{l,f,T} = q_{h,b,T} + q_{l,b,T}$ it is easy to show that $\Pi_{f,T} - \Pi_{b,T} = \Delta > 0$, i.e. the seller earns more than he did before; hence the initial outcome could not be an equilibrium.

Step 2. We now show that flexible firms attract high types only. Suppose the opposite is true i.e. they attract low types only (the third scenario where they attract both types is ruled out in Step 1). This means that $U_{l,b,T} = \bar{U}_{l,T}$ and $U_{h,b,T} < \bar{U}_{h,T}$ therefore $q_{l,b,T} > 0$ and $q_{h,b,T} = 0$. Recall that $U_{h,b,T} > U_{l,b,T}$. It follows that $\bar{U}_{h,T} > \bar{U}_{l,T}$. According to our conjecture high types stay away from flexible firms, so they must be shopping at fixed price firms. This means that $U_{h,f,T} = \bar{U}_{h,T}$. Recall, however, that $U_{h,f,T} = U_{l,f,T}$, which implies $U_{l,f,T} > \bar{U}_{l,T}$; a contradiction since $U_{l,f,T} \leq \bar{U}_{l,T}$ by definition.

Step 3. Suppose there is a fixed price firm that caters just to high types. This implies $U_{l,f,T} < \bar{U}_{l,T}$ and $U_{h,f,T} = \bar{U}_{h,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{h,T} < \bar{U}_{l,T}$; a contradiction since $\bar{U}_{h,T} \geq \bar{U}_{l,T}$.

Step 4a. We will show that if $\varepsilon \leq 0$ then fixed price firms attract both types of customers. The previous step established that fixed price firms serve either both types of customers or low types only. Below we rule out the second alternative.

By contradiction suppose fixed price firms indeed attract low types only, i.e. suppose that $q_{l,f,T} > 0$ and $q_{h,f,T} = 0$. This implies that $U_{h,f,T} < \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$; hence $\bar{U}_{l,T} < \bar{U}_{h,T}$. From Step 2 we know that flexible firms attract high types only, i.e.

$q_{h,b,T} > 0$ and $q_{l,b,T} = 0$. This implies that $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} < \bar{U}_{l,T}$. A fixed price firm solves

$$\max_{q_{l,f,T} \in \mathbb{R}_+} 1 - z_0(q_{l,f,T}) - q_{l,f,T} U_{l,f,T} \quad \text{s.t.} \quad U_{l,f,T} = \bar{U}_{l,T}.$$

The first order condition (FOC) implies that

$$z_0(q_{l,f,T}) = \bar{U}_{l,T} \Rightarrow \Pi_{f,T} = 1 - z_0(q_{l,f,T}) - z_1(q_{l,f,T}).$$

Similarly a flexible firm solves

$$\max_{q_{h,b,T} \in \mathbb{R}_+} 1 - z_0(q_{h,b,T}) - q_{h,b,T} U_{h,b,T} + z_1(q_{h,b,T}) \varepsilon \quad \text{s.t.} \quad U_{h,b,T} = \bar{U}_{h,T}.$$

The FOC is given by

$$z_0(q_{h,b,T}) + [z_0(q_{h,b,T}) - z_1(q_{h,b,T})] \varepsilon = \bar{U}_{h,T}.$$

Thus

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - z_1(q_{h,b,T}) + q_{h,b} z_1(q_{h,b,T}) \varepsilon.$$

Suppose $\varepsilon = 0$. Then the equal profit condition $\Pi_{b,T} = \Pi_{f,T}$ implies that $q_{l,f,T} = q_{h,b,T}$. Substituting this into the FOCs above we have $\bar{U}_{h,T} = \bar{U}_{l,T}$; a contradiction since $\bar{U}_{l,T} < \bar{U}_{h,T}$. Now Suppose $\varepsilon < 0$. The equal profit condition implies that $q_{l,f,T} < q_{h,b,T}$. To see why fix some $q_{h,b,T}$ and note that $\Pi_{f,T} > \Pi_{b,T}$ even when $q_{l,f,T} = q_{h,b,T}$ because $\varepsilon < 0$. The function $\Pi_{f,T}$ falls if $q_{l,f,T}$ decreases, so if $q_{l,f,T}$ exceeds $q_{h,b,T}$ then $\Pi_{f,T}$ further exceeds $\Pi_{b,T}$. It follows that for equal profits we must have $q_{l,f,T} < q_{h,b,T}$. Recall that $U_{h,T} > U_{l,T}$. This requires

$$\varepsilon \leq \frac{z_0(q_{l,f,T}) - z_0(q_{h,b,T})}{z_0(q_{h,b,T})(1 - q_{h,b,T})} \text{ if } 1 \leq q_{h,b,T}.$$

Hence, there are two scenarios:

- $1 > q_{h,b,T}$ and $\varepsilon > \frac{z_0(q_{l,f,T}) - z_0(q_{h,b,T})}{z_0(q_{h,b,T})(1 - q_{h,b,T})}$: Recall that $q_{l,f,T} < q_{h,b,T}$. It follows that $z_0(q_{l,f,T}) > z_0(q_{h,b,T})$ which in turn implies that $\varepsilon > 0$; a contradiction since $\varepsilon < 0$.
- $1 < q_{h,b,T}$ and $\varepsilon < \frac{z_0(q_{l,f,T}) - z_0(q_{h,b,T})}{z_0(q_{h,b,T})(1 - q_{h,b,T})}$: This case, too, produces a contradiction. To see why note that the equal profit condition $\Pi_{b,T} = \Pi_{f,T}$ implies that

$$\varepsilon = [(1 + q_{h,b,T}) z_0(q_{h,b,T}) - (1 + q_{l,f,T}) z_0(q_{l,f,T})] / q_{h,b,T}^2.$$

Substituting this into the inequality above we need

$$z_0(q_{h,b,T}) - z_0(q_{l,f,T}) > (q_{h,b,T} - q_{l,f,T}) z_0(q_{l,f,T}) (q_{h,b,T} - 1).$$

Since $q_{l,f,T} < q_{h,b,T}$ and $1 - q_{h,b,T} < 0$ the left hand side of the inequality is negative whereas the right hand side is positive; a contradiction.

Step 4b. We show that if $\varepsilon > 0$ then fixed price firms cater to low types only. Step 3 establishes that fixed price firms cannot be catering to high types only. This leaves two possibilities: either they serve both types or they serve low types only. Below we rule out the first alternative, which means that if an equilibrium exists where some sellers compete with fixed pricing, then those sellers must be catering to low types only.

To start, suppose, by contradiction, that there is a fixed price seller who attracts both types of customers, i.e. suppose that $q_{h,f,T}$ and $q_{l,f,T}$ are both positive and satisfy $U_{h,f,T} = \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{l,T} = \bar{U}_{h,T}$. Letting $q_{f,T} := q_{h,f,T} + q_{l,f,T}$, a fixed price seller solves

$$\max_{q_{f,T} \in \mathbb{R}_+} \Pi_{f,T} = \max_{q_{f,T} \in \mathbb{R}_+} 1 - z_0(q_{f,T}) - q_{f,T} U_{h,f,T} \quad \text{s.t.} \quad U_{h,f,T} = \bar{U}_{h,T}.$$

After substituting the constraint into the objective function, the FOC is given by

$$z_0(q_{f,T}) = \bar{U}_{h,T} \Rightarrow \Pi_{f,T} = 1 - z_0(q_{f,T}) - z_1(q_{f,T}). \quad (8.3)$$

We argue that this seller would earn more if he were to switch to flexible pricing. Note that after such a switch he would attract high types only (Steps 1 and 2), i.e. $q_{h,b,T} > 0$ and $q_{l,b,T} = 0$. He solves

$$\max_{q_{h,b,T} \in \mathbb{R}_+} \Pi_{b,T} = \max_{q_{h,b,T} \in \mathbb{R}_+} 1 - z_0(q_{h,b,T}) - q_{h,b,T} U_{h,b,T} + z_1(q_{h,b,T}) \varepsilon \quad \text{s.t.} \quad U_{h,b,T} = \bar{U}_{h,T}.$$

The FOC is given by

$$z_0(q_{h,b,T}) + [z_0(q_{h,b,T}) - z_1(q_{h,b,T})] \varepsilon = \bar{U}_{h,T} \quad (8.4)$$

and therefore

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - z_1(q_{h,b,T}) + q_{h,b,T} z_1(q_{h,b,T}) \varepsilon.$$

We can now compare expected profits and show that the deviation is profitable, i.e. $\Pi_{b,T} > \Pi_{f,T}$.

To start note that expressions (8.3) and (8.4) together imply that

$$\varepsilon = \frac{z_0(q_{f,T}) - z_0(q_{h,b,T})}{z_0(q_{h,b,T})(1 - q_{h,b,T})}.$$

Recall that ε is positive; thus $q_{h,b,T} \neq q_{f,T}$, so we have either $q_{f,T} < q_{h,b,T}$ or $q_{f,T} > q_{h,b,T}$.

- Suppose $q_{f,T} < q_{h,b,T}$. Under this specification we have $\Pi_{f,T} < \Pi_{b,T}$. To see why, fix some $q_{h,b,T}$ and note that $\Pi_{f,T} < \Pi_{b,T}$ even when $q_{f,T} = q_{h,b,T}$. The function $\Pi_{f,T}$ decreases as $q_{f,T}$ decreases, so if $q_{f,T}$ falls below $q_{h,b,T}$ then $\Pi_{f,T}$ falls further below $\Pi_{b,T}$.
- Suppose $q_{f,T} > q_{h,b,T}$. Let $\Delta := \Pi_{b,T} - \Pi_{f,T}$. We will show that Δ is positive. Substitute ε into $\Pi_{b,T}$, and use the fact that $z_1(q) = qz_0(q)$ to obtain

$$\Delta = (q_{f,T} - q_{h,b,T})z_0(q_{f,T}) + \frac{z_0(q_{h,b,T}) - z_0(q_{f,T})}{q_{h,b,T} - 1}.$$

Since $q_{f,T} > q_{h,b,T}$ the first expression on the right hand side is positive. The inequality $q_{f,T} > q_{h,b,T}$ implies that $z_0(q_{h,b,T}) > z_0(q_{f,T})$. For ε to be positive the denominator must be negative, hence we have $q_{h,b,T} > 1$. It follows that the second expression, too, is positive. Hence Δ is positive, which means that the deviation is profitable, i.e. $\Pi_{b,T} > \Pi_{f,T}$.

Step 5. Recall from Step 3 that flexible firms cater to high types only; so, consider such a firm with price $r_{b,T}$ and expected demand $q_{h,b,T}$. From Step 4b we know that its FOC is given by

$$z_0(q_{h,b,T})[1 + \varepsilon - \varepsilon q_{h,b,T}] = \bar{U}_{h,T}$$

Solving $U_{h,b,T} = z_0(q_{h,b,T})[1 + \varepsilon - \varepsilon q_{h,b,T}]$ for the list price $r_{b,T}$ we have

$$\hat{r}_{b,T} = 1 - \frac{z_1(q_{h,b,T})(y_T - \varepsilon q_{h,b,T})}{1 - z_0(q_{h,b,T}) - z_1(q_{h,b,T})}.$$

Now consider another flexible firm with price $r'_{b,T}$ and expected demand $q'_{b,T}$. His FOC is given by

$$z_0(q'_{h,b,T})[1 + \varepsilon - \varepsilon q'_{h,b,T}] = \bar{U}_{h,T}.$$

Combining both FOCs we have $q'_{h,b,T} = q_{h,b,T}$. This, in turn, implies that $\hat{r}'_{b,T} = \hat{r}_{b,T}$ as the price function above is one-to-one. Going through similar steps one can show that fixed price firms, too,

post identical prices. This completes the proof of Lemma 2. ■

Now we can start characterizing the equilibria. There are three cases.

8.1.1 Case 1: $\varepsilon = 0$.

Per Lemma 2 if $\varepsilon = 0$ then flexible firms attract high types, i.e. we have $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} < \bar{U}_{l,T}$ and therefore $q_{h,b,T} > 0$ and $q_{l,b,T} = 0$. Substituting $\varepsilon = 0$ and $q_{l,b,T} = 0$ into (8.2) yields

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - q_{h,b,T}U_{h,b,T}.$$

The seller's problem is $\max_{q_{h,b,T} \in \mathbb{R}_+} 1 - z_0(q_{h,b,T}) - q_{h,b,T}U_{h,b,T}$ subject to $U_{h,b,T} = \bar{U}_{h,T}$. After substituting the constraint into the objective function, the first order condition (FOC) is given by $z_0(q_{h,b,T}) = \bar{U}_{h,T}$. The second order condition is trivial, hence the solution corresponds to a maximum. Substituting the FOC into $\Pi_{b,T}$ yields

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - z_1(q_{h,b,T}). \quad (8.5)$$

Now consider a fixed price seller. If $\varepsilon = 0$ then fixed price sellers attract both types of customers, i.e. $q_{h,f,T}$ and $q_{l,f,T}$ are both positive and satisfy $U_{h,f,T} = \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Since $U_{h,f,T} = U_{l,f,T}$ we have $\bar{U}_{l,T} = \bar{U}_{h,T}$. Letting $q_{f,T} := q_{h,f,T} + q_{l,f,T}$ denote the total demand, the fixed price seller solves

$$\max_{q_{f,T} \in \mathbb{R}_+} \Pi_{f,T} = \max_{q_{f,T} \in \mathbb{R}_+} 1 - z_0(q_{f,T}) - q_{f,T}U_{h,f,T} \quad \text{s.t.} \quad U_{h,f,T} = \bar{U}_{h,T}.$$

The FOC is given by $z_0(q_{f,T}) = \bar{U}_{h,T}$. The seller's profit, therefore, is equal to

$$\Pi_{f,T} = 1 - z_0(q_{f,T}) - z_1(q_{f,T}). \quad (8.6)$$

Both FOCs together imply that $q_{h,f,T} + q_{l,f,T} = q_{h,b,T}$, i.e. expected demands at a fixed and flexible firm must be identical. Substituting this equality into the feasibility conditions in (13) and using the fact that $q_{l,b,T} = 0$ one obtains

$$q_{h,b,T} = \lambda_T, \quad q_{h,f,T} = \lambda_T(\varphi_{f,T}^* - \eta_T)/\varphi_{f,T}^* \quad \text{and} \quad q_{l,f,T} = \lambda_T\eta_T/\varphi_{f,T}^*,$$

where $\varphi_{f,T}^*$ denotes the fraction of fixed price firms. Note that for any $\varphi_{f,T}^* \in [\eta_T, 1]$ expected demands $q_{h,f,T}$ and $q_{l,f,T}$ are both positive and satisfy the relationship above. This means that $\varphi_{f,T}^*$ is indeterminate, so we have a continuum of equilibria where $\varphi_{f,T}^*$ can be anywhere in between η_T and 1. Furthermore, in any given equilibrium flexible sellers and fixed price sellers have the same expected demand λ_T .

Now we can obtain equilibrium payoffs and list prices. Recall that $\bar{U}_{l,T} = \bar{U}_{h,T} = z_0(q_{h,b,T})$. Since $q_{h,b,T} = \lambda_T$ we have $u_T = z_0(\lambda_T)$. Similarly substituting $q_{h,b,T} = q_{f,T} = \lambda_T$ into (8.5) and (8.6) yields sellers' equilibrium profit $\Pi_{f,T} = \Pi_{b,T} \equiv \pi_T = 1 - z_0(\lambda_T) - z_1(\lambda_T)$. Given that $u_T = z_0(\lambda_T)$ one can obtain the equilibrium fixed price by solving $U_{h,f,T} = z_0(\lambda_T)$ for $r_{f,T}$ and the equilibrium flexible price by solving $U_{h,b,T} = z_0(\lambda_T)$ for $r_{b,T}$. We have

$$r_{f,T}^*(\lambda_T) = 1 - \frac{z_1(\lambda_T)}{1 - z_0(\lambda_T)} \quad \text{and} \quad r_{b,T}^*(\lambda_T) = 1 - \frac{z_1(\lambda_T)(1-\theta)}{1 - z_0(\lambda_T) - z_1(\lambda_T)}.$$

Finally substituting $\varepsilon = 0$ and $u_{h,T+1} = 0$ into (1) yields the equilibrium bargained price $y_T^* = 1 - \theta$. Observe that expressions for $r_{f,T}^*$, $r_{b,T}^*$, y_T^* , π_T and u_T can be obtained by substituting $u_{T+1} = \pi_{T+1} = 0$ into expressions (14), (15), (16), (17) and (18) on display in Proposition 1, confirming the validity of the Proposition for the terminal period T .

So far we assumed that high type buyers are sufficiently skilled in bargaining. Now we can put some structure behind this assumption. A buyer negotiates if $y_T \leq r_{b,T}^*$, which, after substituting for $r_{b,T}^*$ and re-arranging, is equivalent to $\theta \geq \bar{\theta}(\lambda_T)$, where $\bar{\theta}(\lambda_T) := z_1(\lambda_T) / [1 - z_0(\lambda_T)]$. So, high types negotiate if their bargaining power exceeds threshold $\bar{\theta}$ and purchase at the list price otherwise. Straightforward algebra reveals that if $\theta > \bar{\theta}(\lambda_T)$ then $r_{b,T}^*(\lambda_T) > r_{f,T}^*(\lambda_T) > y_T$, i.e. flexible firms advertise higher prices than fixed price firms.

The case $\theta < \bar{\theta}(\lambda_T)$ is trivial. Since even hagglers do not find it worthwhile to negotiate the list price, the availability of bargaining becomes immaterial and the model collapses to a fixed price setting. Technically this is equivalent to the outcome where $\varphi_t^* = 1$, i.e. where all firms trade via fixed pricing, post $r_{f,T}^*$ and serve both types of customers. The total demand at each firm equals to λ_T , whereas the equilibrium payoffs are still given by $u_T = z_0(\lambda_T)$ and $\pi_T = 1 - z_0(\lambda_T) - z_1(\lambda_T)$.

8.1.2 Case 2: $\varepsilon < 0$.

In what follows we will show that if $\varepsilon < 0$ then no firm adopts flexible pricing. The proof is by contradiction, i.e. suppose that an equilibrium exists where at least one firm adopts flexible pricing.

We will show that this firm earns less than its fixed price competitors. To start recall that if $\varepsilon < 0$ then a flexible firm attracts high types only while low types stay away (Lemma 2) i.e. $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} < \bar{U}_{l,T}$ hence $q_{h,b,T} > 0$ and $q_{l,b,T} = 0$. The flexible firm solves

$$\max_{q_{h,b,T} \in \mathbb{R}_+} 1 - z_0(q_{h,b,T}) - q_{h,b,T}U_{h,b,T} + z_1(q_{h,b,T})\varepsilon \quad \text{s.t.} \quad U_{h,b,T} = \bar{U}_{h,T} .$$

The first order condition is given by

$$z_0(q_{h,b,T}) + [z_0(q_{h,b,T}) - z_1(q_{h,b,T})]\varepsilon = \bar{U}_{h,T}.$$

It follows that

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - z_1(q_{h,b,T}) + q_{h,b,T}z_1(q_{h,b,T})\varepsilon.$$

Now consider fixed price firms. Per Lemma 2 they attract both types of customers i.e. $q_{h,f,T} > 0$ and $q_{l,f,T} > 0$ and satisfy $U_{h,f,T} = \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{l,T} = \bar{U}_{h,T}$. Letting $q_{f,T} := q_{h,f,T} + q_{l,f,T}$, a fixed price seller solves

$$\max_{q_{f,T} \in \mathbb{R}_+} 1 - z_0(q_{f,T}) - q_{f,T}U_{h,f,T} \quad \text{s.t.} \quad U_{h,f,T} = \bar{U}_{h,T}.$$

The FOC is given by $z_0(q_{f,T}) = \bar{U}_{h,T}$; therefore

$$\Pi_{f,T} = 1 - z_0(q_{f,T}) - z_1(q_{f,T}).$$

We will show $\Pi_{f,T} > \Pi_{b,T}$. First note that the FOCs together imply that

$$z_0(q_{h,b,T}) + [z_0(q_{h,b,T}) - z_1(q_{h,b,T})]\varepsilon = z_0(q_{f,T}) \Rightarrow \varepsilon = \frac{z_0(q_{f,T}) - z_0(q_{h,b,T})}{z_0(q_{h,b,T})(1 - q_{h,b,T})}.$$

The fact that $\varepsilon < 0$ implies that either we have (i) $q_{f,T} < q_{h,b,T}$ and $q_{h,b,T} > 1$ or we have (ii) $q_{f,T} > q_{h,b,T}$ and $q_{h,b,T} < 1$. Now we can compare profits. Let $\Delta \equiv \Pi_{f,T} - \Pi_{b,T}$. We will show that $\Delta > 0$. Note that

$$\begin{aligned} \Delta &= z_0(q_{h,b,T}) - z_0(q_{f,T}) + z_1(q_{h,b,T}) - z_1(q_{f,T}) - q_{h,b,T}z_1(q_{h,b,T})\varepsilon \\ &= \frac{z_0(q_{h,b,T}) - z_0(q_{f,T})}{1 - q_{h,b,T}} - z_0(q_{f,T})(q_{f,T} - q_{h,b,T}) \end{aligned}$$

The first step follows after substituting for $\Pi_{f,T}$ and $\Pi_{b,T}$ whereas the second step is obtained after substituting for ε and noting that $z_1(q) = qz_0(q)$. Observe that under condition (i) both terms of Δ are positive; hence $\Delta > 0$. Under condition (ii) the first term is positive but the second one is negative so we need a closer inspection. Fix $q_{h,b,T} < 1$ and note that Δ falls in $q_{f,T}$ under the restrictions of (ii). It follows that Δ reaches a minimum when $q_{f,T} \searrow q_{h,b,T}$ (recall that under (ii) we have $q_{f,T} > q_{h,b,T}$). Note that $\lim_{q_{f,T} \searrow q_{h,b,T}} \Delta = 0$; thus $\Delta > 0$ when (ii) holds.

The inequality $\Delta > 0$ implies that if fixed and flexible sellers compete in the same market then fixed price sellers earn more than flexible sellers; so there cannot be an equilibrium where flexible pricing is adopted by any firm. The implication is that if $\varepsilon < 0$ then the only possible outcome is where all sellers trade via fixed pricing, which we have already characterized in Case 1.

8.1.3 Case 3: $\varepsilon > 0$.

Per Lemma 2 if $\varepsilon > 0$ then flexible stores attract high types only i.e. $q_{h,b,T} > 0$ and $q_{l,b,T} = 0$ satisfying $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} < \bar{U}_{l,T}$. Substitute $q_{l,b,T} = 0$ into the expression of $\Pi_{b,T}$ to obtain

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - q_{h,b,T}U_{h,b,T} + z_1(q_{h,b,T})\varepsilon.$$

The seller's problem is

$$\max_{q_{h,b,T} \in \mathbb{R}_+} 1 - z_0(q_{h,b,T}) - q_{h,b,T}U_{h,b,T} + z_1(q_{h,b,T})\varepsilon \quad \text{s.t.} \quad U_{h,b,T} = \bar{U}_{h,T}.$$

The FOC is given by

$$z_0(q_{h,b,T}) + [z_0(q_{h,b,T}) - z_1(q_{h,b,T})]\varepsilon = \bar{U}_{h,T}. \quad (8.7)$$

The second order condition is satisfied if

$$-z_0(q_{h,b,T}) - z_0(q_{h,b,T})[2 - q_{h,b,T}]\varepsilon < 0. \quad (8.8)$$

If $q_{h,b,T} \leq 2$ then the inequality is satisfied irrespective of ε . If $q_{h,b,T} > 2$ then we need $\varepsilon < 1/(q_{h,b,T} - 2)$. The right hand side is positive. Since ε is assumed to be positive but small, the inequality is satisfied; hence the the solution of the FOC yields a maximum.

Substituting (8.7) into $\Pi_{b,T}$ yields

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - z_1(q_{h,b,T}) + q_{h,b,T}z_1(q_{h,b,T})\varepsilon. \quad (8.9)$$

Now consider fixed price sellers. They attract low types only (Lemma 2), i.e. $q_{h,f,T} = 0$ and $q_{l,f,T} > 0$ satisfying $U_{h,f,T} < \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{h,T} > \bar{U}_{l,T}$. Substituting $q_{h,f,T} = 0$ into the expression of $\Pi_{f,T}$ yields

$$\Pi_{f,T} = 1 - z_0(q_{l,f,T}) - q_{l,f,T}U_{l,f,T}. \quad (8.10)$$

The seller solves

$$1 - z_0(q_{l,f,T}) - q_{l,f,T}U_{l,f,T} \text{ s.t. } U_{l,f,T} = \bar{U}_{h,T}.$$

The FOC implies

$$z_0(q_{l,f,T}) = \bar{U}_{l,T} \text{ and therefore } \Pi_{f,T} = 1 - z_0(q_{l,f,T}) - z_1(q_{l,f,T}). \quad (8.11)$$

Recall that $\varphi_{f,T}$ denotes the fraction of sellers who compete with fixed pricing. Substituting $q_{h,f,T} = q_{l,b,T} = 0$ into the feasibility conditions in (13) yields

$$q_{l,f,T} = \eta_T \lambda_T / \varphi_{f,T} \text{ and } q_{h,b,T} = (1 - \eta_T) \lambda_T / (1 - \varphi_{f,T}).$$

We will show that there exists a unique $\varphi_{f,T}^* \in (0, \eta_T)$ satisfying the equal profit condition $\Pi_{f,T} = \Pi_{b,T}$, proving the equilibrium exists and it is unique. Let $\Delta(\varphi_{f,T}) \equiv \Pi_{b,T} - \Pi_{f,T}$. Combining (8.9) and (8.11) it is easy to show that

$$\Delta(\varphi_{f,T}) = z_0(q_{l,f,T}^*) + z_1(q_{l,f,T}^*) - z_0(q_{h,b,T}^*) - z_1(q_{h,b,T}^*) + q_{h,b,T}^* z_1(q_{h,b,T}^*) \varepsilon.$$

Note that Δ rises in $q_{l,b,T}$, which in turn rises in $\varphi_{f,T}$, and that Δ falls in $q_{l,f,T}$, which in turn falls in $\varphi_{f,T}$. It follows that $d\Delta/d\varphi_{f,T} > 0$. Furthermore note that $\Delta(\eta_T) > 0$, whereas $\Delta(0) < 0$ if ε is small. To see why, note that $\Delta(0) = -z_0(q) - z_1(q) + qz_1(q)\varepsilon$, thus $\Delta(0) < 0$ if

$$\varepsilon < (1 + q)/q^2, \text{ where } q \equiv (1 - \eta_T) \lambda_T. \quad (8.12)$$

The expression on the right hand side is positive. Since ε is assumed to be positive but sufficiently small, the inequality is satisfied. The Intermediate Value Theorem implies that there exists a unique $\varphi_{f,T}^* \in (0, \eta_T)$ satisfying $\Delta(\varphi_{f,T}^*) = 0$. Since $\varphi_{f,T}^* < \eta_T$ we have $q_{h,b,T}^* < \lambda_T < q_{l,f,T}^*$ i.e. fixed price firms are more crowded than flexible firms.

The equilibrium payoffs are immediate from the first order conditions (8.7) and (8.11). We have

$$u_{h,T} = z_0(q_{h,b,T}^*) + [z_0(q_{h,b,T}^*) - z_1(q_{h,b,T}^*)] \varepsilon, \quad u_{l,T} = z_0(q_{l,f,T}^*), \quad \pi_T = 1 - z_0(q_{l,f,T}^*) - z_1(q_{l,f,T}^*).$$

Given that high and low type buyers earn, respectively, $u_{h,T}$ and $u_{l,T}$ one can obtain the equilibrium flexible price by solving $U_{h,b,T} = u_{h,T}$ for $r_{b,T}$ and the equilibrium fixed price by solving $U_{l,f,T} = u_{l,T}$ for $r_{f,T}$. We have

$$r_{b,T}^* = 1 - \frac{z_1(q_{h,b,t}^*)(1-\theta)}{1-z_0(q_{h,b,t}^*)-z_1(q_{h,b,t}^*)} \left[1 + \varepsilon - \frac{q_{h,b,t}^* \varepsilon}{1-\theta} \right] \quad \text{and} \quad r_{f,T}^* = 1 - \frac{z_1(q_{l,f,t}^*)}{1-z_0(q_{l,f,t}^*)}.$$

Finally substituting $\pi_{T+1} = u_{h,T+1} = 0$ into (1) yields the bargained price $y_T^* = (1-\theta)(1+\varepsilon)$. Observe that expressions for $r_{f,T}^*$, $r_{b,T}^*$, y_T^* , π_T , $u_{h,T}$ and $u_{l,T}$ can be obtained by substituting $u_{h,T+1} = u_{l,T+1} = \pi_{T+1} = 0$ into (20), (21), (22), (23), (24) and (25) in Proposition 3, confirming the validity of the Proposition for the terminal period T .

A high type buyer negotiates if $y_T^* \leq r_{b,T}^* + \varepsilon$. After substituting for $r_{b,T}^*$ and y_T^* and re-arranging this condition is equivalent to

$$\theta \geq \tilde{\theta}_T \equiv \frac{z_1(q_{h,b,T}^*)}{1-z_0(q_{h,b,T}^*)} - \frac{\varepsilon z_1(q_{h,b,T}^*) q_{h,b,T}^*}{(1+\varepsilon)[1-z_0(q_{h,b,T}^*)]}.$$

If $\theta < \tilde{\theta}_T$ then even hagglers do not find it worthwhile to negotiate the list price. The availability of bargaining becomes immaterial and the model collapses to a fixed price setting which was characterized earlier in Case 1.

This completes the proof of the terminal period T . Going through a similar analytical process one can establish the inductive step as well. As the analysis is largely the same the inductive step is relegated to the Online Appendix 2.

8.2 Other Proofs

Proof of Proposition 4. In what follows we prove that $d\pi_t/d\varepsilon > 0$ and $du_{l,t}/d\varepsilon < 0$, where π_t is given by (23) and $u_{l,t}$ is given by (25). The proof is by induction, where we start with the terminal period T . Substituting the terminal payoffs $u_{l,T+1} = \pi_{T+1} = 0$ into (23) and (25) yields

$$\pi_T = 1 - z_0(q_{l,f,T}^*) - z_1(q_{l,f,T}^*) \quad \text{and} \quad u_{l,T} = z_0(q_{l,f,T}^*),$$

and therefore

$$\frac{d\pi_T}{d\varepsilon} = z_1(q_{l,f,T}^*) \frac{dq_{l,f,T}^*}{d\varepsilon} \quad \text{and} \quad \frac{du_{l,T}}{d\varepsilon} = -z_0(q_{l,f,T}^*) \frac{dq_{l,f,T}^*}{d\varepsilon}.$$

Our goal is to show that the first derivative is positive and the second one is negative. Notice that both relationships hold if $dq_{l,f,T}^*/d\varepsilon > 0$, so below we establish that this is indeed the case. Let $\Delta_T \equiv \Pi_{b,T} - \Pi_{f,T}$, where $\Pi_{b,T}$ is given by (8.9) and $\Pi_{f,T}$ is given by (8.10), and note that the expected demand $q_{l,f,T}^*$ satisfies $\Delta_T = 0$. By the implicit function theorem we have

$$\frac{dq_{l,f,T}^*}{d\varepsilon} = -\frac{\partial \Delta_T / \partial \varepsilon}{\partial \Delta_T / \partial q_{l,f,T}^*}.$$

Note that Δ_T rises in ε and falls in $q_{l,f,T}^*$. It follows that $dq_{l,f,T}^*/d\varepsilon > 0$. This proves the claim for period T . Now for the inductive step suppose that $d\pi_{t+1}/d\varepsilon > 0$ and $du_{l,t+1}/d\varepsilon < 0$. We will show that $d\pi_t/d\varepsilon > 0$ and $du_{l,t}/d\varepsilon < 0$. Notice that

$$\begin{aligned} \frac{d\pi_t}{d\varepsilon} = & -\frac{du_{l,t+1}}{d\varepsilon} \beta \left[1 - z_0(q_{l,f,t}^*) - z_1(q_{l,f,t}^*) \right] + \frac{d\pi_{t+1}}{d\varepsilon} \beta [z_0(q_{l,f,t}^*) + z_1(q_{l,f,t}^*)] \\ & + (1 - \beta u_{l,t+1} - \beta \pi_{t+1}) z_1(q_{l,f,t}^*) \frac{dq_{l,f,t}^*}{d\varepsilon} \end{aligned}$$

The first line is positive due to the inductive step. Hence, in order to establish $d\pi_t/d\varepsilon > 0$ it suffices to show that $dq_{l,f,t}^*/d\varepsilon > 0$. Let $\Delta_t \equiv \Pi_{b,t} - \Pi_{f,t}$, where $\Pi_{b,t}$ is given by (9.9) and $\Pi_{f,t}$ is given by (9.11), and note that $q_{l,f,t}^*$ satisfies $\Delta_t = 0$. By the implicit function theorem we have

$$\frac{dq_{l,f,t}^*}{d\varepsilon} = -\frac{\partial \Delta_t / \partial \varepsilon}{\partial \Delta_t / \partial q_{l,f,t}^*}.$$

Note that Δ_t rises in ε and falls in $q_{l,f,t}^*$; thus $dq_{l,f,t}^*/d\varepsilon > 0$. This proves the claim $d\pi_t/d\varepsilon > 0$. The other claim can be proved by going through similar steps. ■

Proof of Proposition 5. Consider Eq-PS first. Along this equilibrium path the expected demand at any store at time $t-1$ is equal to λ_{t-1} , so each seller trades with probability $1 - z_0(\lambda_{t-1})$. The law of large numbers implies that $s_{t-1}(1 - z_0(\lambda_{t-1}))$ sellers trade and exit the market. Each transaction involves one seller and one buyer, so the total number of buyers who trade and exit is also $s_{t-1}(1 - z_0(\lambda_{t-1}))$. The number of sellers present in period t is, then, $s_t = s_t^{new} + s_{t-1}z_0(\lambda_{t-1})$, whereas the number of buyers is $b_t = b_t^{new} + b_{t-1} - s_{t-1}(1 - z_0(\lambda_{t-1}))$.

Now turn to the proportions of hagglers and non hagglers. In period $t-1$ the total demand at any fixed price firm equals to λ_{t-1} of which $\lambda_{t-1}\eta_{t-1}/\varphi_{f,t-1}^*$ are non-hagglers and $\lambda_{t-1}(\varphi_{f,t-1}^* -$

$\eta_{t-1})/\varphi_{f,t-1}^*$ are hagglers (Proposition 1). Since buyers are equally likely to be selected at the point of transaction, the probability that the purchasing customer is going to be a low type equals to $\eta_{t-1}/\varphi_{f,t-1}^*$. There are $\varphi_{f,t-1}^*s_{t-1}$ fixed price firms present in the market, each seller trades with probability $1 - z_0(\lambda_{t-1})$ and each transaction involves one buyer and one seller; so, the number of non-haggler customers who trade and exit equals to

$$\varphi_{f,t-1}^*s_{t-1} \times (1 - z_0(\lambda_{t-1})) \times \frac{\eta_{t-1}}{\varphi_{f,t-1}^*} = \eta_{t-1}s_{t-1}(1 - z_0(\lambda_{t-1})).$$

Remaining buyers move to period t . The number of non-hagglers present in period t , given by $\eta_t b_t$, equals to

$$\eta_t b_t = b_t^{new} \eta_t^{new} + b_{t-1} \eta_{t-1} - \eta_{t-1} s_{t-1} (1 - z_0(\lambda_t)).$$

It follows that η_t is given by expression (27), on display in Proposition 5. This completes the discussion on Eq-PS. Along Eq-FP, as in Eq-PS, the expected demand at any store at time $t - 1$ is equal to λ_{t-1} so b_t and s_t evolve as in (26). The proportion of hagglers, too, evolves as in (27), but this is rather irrelevant because along Eq-FP buyers do not negotiate anyway.

Now consider the final scenario, Eq-FS, where non hagglers shop at fixed price stores and hagglers shop at flexible stores. The number of fixed price sellers trading and exiting the market at time $t - 1$ is equal to $s_{t-1} \varphi_{f,t-1}^* (1 - z_0(q_{f,t-1}^*)) \equiv l_{t-1}$ whereas the number flexible sellers trading and exiting the market is equal to $s_{t-1} (1 - \varphi_{f,t-1}^*) (1 - z_0(q_{h,t-1}^*)) \equiv h_{t-1}$. Each transaction involves one buyer and one seller; thus $s_t = s_{t-1} - (l_{t-1} + h_{t-1}) + s_t^{new}$ and $b_t = b_{t-1} - (l_{t-1} + h_{t-1}) + b_t^{new}$. Finally note that there are $b_{t-1} \eta_{t-1}$ non-hagglers in the market at $t-1$, of which l_{t-1} exit the market while the rest move to period t . Therefore $\eta_t = [b_{t-1} \eta_{t-1} - l_{t-1} + \eta_t^{new} b_t^{new}] / b_t$. This completes the proof. ■

Proof of Remark 6. If $\varepsilon \leq 0$ then $r_{f,t}^*$, π_t and u_t are given by (14), (17) and (18). Letting $x_t \equiv 1 - \beta u_t - \beta \pi_t$ these expressions can be re-written as follows:

$$\begin{aligned} \pi_t &= 1 - \beta u_{t+1} - [z_0(\lambda_t) + z_1(\lambda_t)] x_{t+1} \\ u_t &= \beta u_{t+1} + z_0(\lambda_t) x_{t+1} \\ r_{f,t}^* &= 1 - \beta u_{t+1} - x_{t+1} \frac{z_1(\lambda_t)}{1 - z_0(\lambda_t)} \end{aligned}$$

Letting $\Delta r_{f,t}^* \equiv r_{f,t}^* - r_{f,t-1}^*$ and noting that $x_t = 1 - \beta + z_1(\lambda_t)\beta x_{t+1}$ we have

$$\Delta r_{f,t}^* = (1 - \beta) \left[\frac{z_1(\lambda_{t-1})}{1 - z_0(\lambda_{t-1})} - \beta u_{t+1} \right] + x_{t+1} \left[\frac{\beta z_1(\lambda_t) z_1(\lambda_{t-1})}{1 - z_0(\lambda_{t-1})} + \beta z_0(\lambda_t) - \frac{z_1(\lambda_t)}{1 - z_0(\lambda_t)} \right].$$

Our goal is to show that $\lim_{\beta \rightarrow 1} \Delta r_{f,t}^* = 0$. It is clear that if $\beta \rightarrow 1$ then the first term, which is a multiplicative of $1 - \beta$, will vanish; however the second term, which is a multiplicative of x_{t+1} needs some inspection. The equation $x_t = 1 - \beta + z_1(\lambda_t)\beta x_{t+1}$ pins down the relationship between x_t and x_{t+1} . Iteration on t yields

$$x_{t+1} = (1 - \beta) \times \left[1 + \sum_{i=1}^{s-1} \beta^i \prod_{j=1}^i z_1(\lambda_{t+j}) \right] + \underbrace{\beta^s \prod_{j=1}^s z_1(\lambda_{t+j})}_{O(s)} \times x_{t+1+s},$$

where $s \in \mathbb{N}_+$ is an arbitrary integer. The terms $z_1(\lambda_{t+j})$ are all strictly less than 1. Since T is large, one can pick s large enough to ensure that $O(s) \approx 0$; hence

$$x_{t+1} \approx (1 - \beta) \times \left[1 + \sum_{i=1}^{s-1} \beta^i \prod_{j=1}^i z_1(\lambda_{t+j}) \right].$$

Consequently we have $\lim_{\beta \rightarrow 1} x_{t+1} = 0$; and therefore $\lim_{\beta \rightarrow 1} \Delta r_{f,t}^* = 0$. This completes the proof for $\Delta r_{f,t}^*$. The remaining cases pertaining Δy_t^* and $\Delta r_{b,t}^*$ can be proved similarly. ■

9 Online Appendix 2

9.1 Inductive Step

Our goal in this section is to establish that the claims in Propositions 1, 2 and 3 hold true in period t , assuming they are true in period $t + 1$. We start by re-arranging the expected payoffs for buyers and sellers. Noting that $\sum_{n=0}^{\infty} \frac{z_n(q)}{n+1} = \frac{1-z_0(q)}{q}$, the expression for $U_{i,f,t}$, given by (3), can be re-written as

$$U_{i,f,t} = \frac{1-z_0(q_{h,f,t}+q_{l,f,t})}{q_{h,f,t}+q_{l,f,t}} (1 - r_{f,t} - \beta u_{i,t+1}) + \beta u_{i,t+1}. \quad (9.1)$$

Similarly we have

$$\begin{aligned} U_{l,b,t} &= \frac{1-z_0(q_{h,b,t}+q_{l,b,t})}{q_{h,b,t}+q_{l,b,t}} (1 - r_{b,t} - \beta u_{l,t+1}) + \beta u_{l,t+1} \\ U_{h,b,t} &= \frac{1-z_0(q_{h,b,t}+q_{l,b,t})}{q_{h,b,t}+q_{l,b,t}} (1 - r_{b,t} - \beta u_{h,t+1}) + z_0 (q_{h,b,t} + q_{l,b,t}) (r_{b,t} + \varepsilon - y_t) + \beta u_{h,t+1} \end{aligned} \quad (9.2)$$

Note that

$$U_{h,b,t} = U_{l,b,t} + z_0 (q_{h,b,t} + q_{l,b,t}) (r_{b,t} - y_t + \varepsilon) + \left[1 - \frac{1-z_0(q_{h,b,t}+q_{l,b,t})}{q_{h,b,t}+q_{l,b,t}} \right] \beta (u_{h,t+1} - u_{l,t+1}). \quad (9.3)$$

Using these expressions we can now rewrite $\Pi_{f,t}$ and $\Pi_{b,t}$. Equation (9.1) implies that

$$[1 - z_0 (q_{h,f,t} + q_{l,f,t})] r_{f,t} = [1 - z_0 (q_{h,f,t} + q_{l,f,t})] (1 - \beta u_{i,t+1}) + \beta u_{i,t+1} - (q_{h,f,t} + q_{l,f,t}) U_{i,f,t}$$

Substituting this relationship into (7) yields

$$\begin{aligned} \Pi_{f,t} &= 1 - z_0 (q_{h,f,t} + q_{l,f,t}) (1 - \beta \pi_{t+1}) - (q_{h,f,t} + q_{l,f,t}) (U_{i,f,t} - \beta u_{i,t+1}) \\ &\quad - [1 - z_0 (q_{h,f,t} + q_{l,f,t})] \beta u_{i,t+1}. \end{aligned} \quad (9.4)$$

Similarly combining (9.2), (9.3) with (8) yields

$$\begin{aligned} \Pi_{b,t} &= 1 - \beta u_{h,t+1} - z_0 (q_{h,b,t} + q_{l,b,t}) (1 - \beta \pi_{t+1} - \beta u_{h,t+1}) - q_{h,b,t} (U_{h,b,t} - \beta u_{h,t+1}) \\ &\quad - q_{l,b,t} (U_{l,b,t} - \beta u_{l,t+1}) + q_{h,b,t} z_0 (q_{h,b,t} + q_{l,b,t}) \varepsilon + \frac{1-z_0(q_{h,b,t}+q_{l,b,t})}{q_{h,b,t}+q_{l,b,t}} q_{l,b,t} \beta (u_{h,t+1} - u_{l,t+1}) \end{aligned} \quad (9.5)$$

We can now start characterizing the equilibria. There are three cases: $\varepsilon = 0$, $\varepsilon < 0$ and $\varepsilon > 0$.

9.1.1 Case 1: $\varepsilon = 0$.

Per the inductive assumption we have $u_{h,t+1} = u_{l,t+1} = u_{t+1}$. Substituting $u_{h,t+1} = u_{t+1}$ into (1) yields the expression for the bargained price y_t^* , which is on display in Proposition 1 (equation (16)). For now we assume that $y_t^* \leq r_{b,t}$, which requires θ to be sufficiently large. Furthermore we conjecture that players prefer to transact immediately rather than waiting (verified below).

One can show that flexible firms post the same list price $r_{b,t}$ and cater to high types while fixed price firms post the same list price $r_{f,t}$ and cater to both types if $\varepsilon \leq 0$ and cater to low types if $\varepsilon > 0$. In other words, Lemma 2, which was valid in the terminal period T , is also valid in period t . The proof is almost identical to the proof of Lemma 2; hence it is skipped here.

Since $u_{h,t+1} = u_{l,t+1}$ we have $U_{h,f,t} = U_{l,f,t}$. In addition $U_{h,b,t} > U_{l,b,t}$ since $r_{b,t} > y_t$. Now consider a flexible firm. Since flexible firms attract high types only we have $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$ and $U_{l,b,t}(r_{b,t}) < \bar{U}_{l,t}$, and thus $q_{h,b,t} > 0$ and $q_{l,b,t} = 0$. Substituting these into (9.5) we have

$$\Pi_{b,t} = 1 - \beta u_{t+1} - z_0(q_{h,b,t})(1 - \beta\pi_{t+1} - \beta u_{t+1}) - q_{h,b,t}(U_{h,b,t} - \beta u_{h,t+1})$$

A flexible firm solves $\max_{q_{h,b,t} \in \mathbb{R}_+} \Pi_{b,t}$ s.t. $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$. The FOC is given by

$$z_0(q_{h,b,t})[1 - \beta u_{t+1} - \beta\pi_{t+1}] = \bar{U}_{h,t} - \beta u_{t+1}. \quad (9.6)$$

The SOC is trivial, hence the solution to the FOC yields a maximum.

Fixed price firms attract both types of customers, i.e. $U_{h,f,t}(r_{f,t}) = \bar{U}_{h,t}$ and $U_{l,f,t}(r_{f,t}) = \bar{U}_{l,t}$ thus $q_{h,f,t} > 0$ and $q_{l,f,t} > 0$. Note that $u_{h,t+1} = u_{l,t+1} = u_{t+1}$ and that $U_{h,f,t} = U_{l,f,t}$. Thus $\Pi_{f,t}$, given by the expression in (9.4), becomes

$$\Pi_{f,t} = 1 - \beta u_{t+1} - z_0(q_{h,f,t} + q_{l,f,t})(1 - \beta\pi_{t+1} - \beta u_{t+1}) - (q_{h,f,t} + q_{l,f,t})(U_{h,f,t} - \beta u_{t+1})$$

A fixed price firms solves $\max_{q_{h,f,t}, q_{l,f,t} \in \mathbb{R}_+^2} \Pi_{f,t}$ s.t. $U_{h,f,t}(r_{f,t}) = \bar{U}_{h,t}$ and $U_{l,f,t}(r_{f,t}) = \bar{U}_{l,t}$. (It appears that the seller faces two separate constraints, one for high types and one for low types. Recall, however, that $U_{h,f,t} = U_{l,f,t}$, which, in turn, implies that $\bar{U}_{l,t} = \bar{U}_{h,t}$; thus both constraints are identical.) The FOC implies that

$$z_0(q_{h,f,t} + q_{l,f,t})[1 - \beta u_{t+1} - \beta\pi_{t+1}] = \bar{U}_{h,t} - \beta u_{t+1}. \quad (9.7)$$

FOCs (9.6) and (9.7) together imply that $q_{h,f,t} + q_{l,f,t} = q_{h,b,t}$, i.e. expected demands at all firms, fixed or flexible, should be identical. Substitute $q_{l,b,t} = 0$ into the feasibility constraint (13) and use the fact that $q_{h,f,t} + q_{l,f,t} = q_{h,b,t}$ to obtain

$$q_{h,b,t} = \lambda_t, \quad q_{h,f,t} = \lambda_t(\varphi_{f,t}^* - \eta_t)/\varphi_{f,t}^* \quad \text{and} \quad q_{l,f,t} = \lambda_t\eta_t/\varphi_{f,t}^*.$$

Note that, $\varphi_{f,t}^*$ is indeterminate and can take any value within $[\eta_t, 1]$; hence, there is a continuum of equilibria where any fraction $\varphi_{f,t}^* \geq \eta_t$ of sellers compete via fixed pricing while the rest compete via flexible pricing. In addition, note that in any equilibrium the total expected demand at each firm equals to λ_t .

Now we can characterize prices. Combining the FOC (9.6) with indifference constraint $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$ yields

$$z_0(\lambda_t)[1 - \beta u_{t+1} - \beta \pi_{t+1}] = U_{h,b,t}(r_{b,t}) - \beta u_{t+1},$$

where $U_{h,b,t}$ is given by (9.2). Solving this equality for $r_{b,t}$ yields expression (15), on display in Proposition 1. Similarly the FOC (9.7) along with $U_{h,f,t}(r_{f,t}) = \bar{U}_{h,t}$ implies

$$z_0(\lambda_t)[1 - \beta u_{t+1} - \beta \pi_{t+1}] = U_{h,f,t}(r_{f,t}) - \beta u_{t+1},$$

where $U_{h,f,t}$ is given by (9.1). Solving this equality for $r_{f,t}$ yields expression (14), on display in Proposition 1. High type buyers negotiate if $r_{b,t}^* \geq y_t^*$, which, after substituting for $r_{b,t}^*$ and y_t^* , is equivalent to $\theta \geq \bar{\theta}_t \equiv z_1(\lambda_t)/[1 - z_0(\lambda_t)]$. Given the expressions for $r_{f,t}^*$ and $r_{b,t}^*$ one can verify that the equilibrium payoffs π_t and u_t are indeed as in Proposition 1 (equations (17) and (18)). In addition note that if $\theta > \bar{\theta}_t$ then $r_{b,t}^* > r_{f,t}^* > y_t$.

If $\theta < \bar{\theta}_t$ then $r_{b,t}^* < y_t$; thus no bargaining takes place as the list price $r_{b,t}^*$ falls below the bargained price y_t^* . In this parameter region the model collapses to a fixed-price setting where $\varphi_t^* = 1$, i.e. where all sellers trade via fixed pricing and post $r_{f,t}^*$ serving both types of customers. The equilibrium demand at each firm is λ_t and the expected payoffs for buyers and sellers remain the same as in (17) and (18).

Transact Now or Wait? The inequality in prices raises the issue of whether players should keep searching for better deals. Below we prove that they are better off trading immediately instead of waiting. There are two cases: (i) $\theta \geq \bar{\theta}_t$ and (ii) $\theta < \bar{\theta}_t$.

Eq-PS: If $\theta \geq \bar{\theta}_t$ then fixed and flexible stores coexist in the same market and prices satisfy

$r_{b,t}^* > r_{f,t}^* > y_t^*$. The worst case scenario for a buyer is buying at the highest price $r_{b,t}^*$ whereas the worst case scenario for a seller is selling at the lowest price y_t^* . If players transact at these prices then they clearly would transact at more favorable prices.

Consider a buyer who contemplates trading at $r_{b,t}^*$. He purchases if $1 - r_{b,t}^* > \beta u_{t+1}$, i.e. if the immediate surplus is greater than the present value of search in the next period. After substituting for $r_{b,t}^*$ the inequality is satisfied if $1 - \beta u_{t+1} - \beta \pi_{t+1} > 0$. One can verify that the expression on the left hand side is positive. To see why use the expressions for u_t and π_t to obtain $x_t = 1 - \beta + \beta z_1(\lambda_t) x_{t+1}$, where $x_t \equiv 1 - \beta(u_t + \pi_t)$. We want to show that x_t is positive for all $t = 1, 2, \dots, T$. Note that if $x_{t+1} > 0$ then $x_t > 0$. Substituting the terminal conditions $u_{T+1} = \pi_{T+1} = 0$ yields $x_T = 1$, which, of course, is positive. Hence x_t is positive for all $t < T$. Since the expression is positive, the buyer is better off purchasing at $r_{b,t}^*$ rather than waiting. Since the buyer is willing to transact in this worst case scenario, it is clear that he is ready to transact at lower prices $r_{f,t}^*$ and y_t^* as well.

Now consider a seller. The worst case scenario for him is to sell at y_t^* . He agrees to transact if $y_t^* > \beta \pi_{t+1}$, which, after substituting for y_t^* , is equivalent to $1 - \beta u_{t+1} - \beta \pi_{t+1} > 0$. We know this inequality holds, so the seller, too, wishes to sell instead of walking away. Since he is willing to sell at y_t^* , it is clear that he is ready to sell at higher prices $r_{f,t}^*$ and $r_{b,t}^*$ as well.

Eq-FP: If $\theta \geq \bar{\theta}_t$ then all sellers compete via fixed pricing and post $r_{f,t}^*$. A buyer transacts if $1 - r_{f,t}^* > \beta u_{t+1}$, which after substituting for $r_{f,t}^*$ is equivalent to

$$(1 - \beta u_{t+1} - \beta \pi_{t+1}) \times z_1(\lambda_t) / [1 - z_0(\lambda_t)] > 0.$$

Since the term $1 - \beta u_{t+1} - \beta \pi_{t+1}$ is positive the inequality holds. Similarly the seller transacts if $r_{f,t}^* > \beta \pi_{t+1}$, which is equivalent to

$$(1 - \beta u_{t+1} - \beta \pi_{t+1}) \times [1 - z_1(\lambda_t) / [1 - z_0(\lambda_t)]] > 0$$

Both expressions inside the parentheses are positive hence the inequality holds.

9.1.2 Case 2: $\varepsilon < 0$.

As in the terminal period, we will show that if $\varepsilon < 0$ then there cannot be an equilibrium where firms adopt flexible pricing. The proof is by contradiction, i.e. suppose that there is an equilibrium where a firm adopts flexible pricing. We will show that this firm earns less than its fixed price

competitors. Recall that if $\varepsilon < 0$ then a flexible firm attracts high types only while low types stay away i.e. $U_{h,b,t} = \bar{U}_{h,t}$ and $U_{l,b,t} < \bar{U}_{l,t}$ hence $q_{h,b,t} > 0$ and $q_{l,b,t} = 0$. Substituting $q_{l,b,t} = 0$ along with the fact that $u_{h,t+1} = u_{l,t+1} = u_{t+1}$ (inductive step) into expression (9.5) we have

$$\Pi_{b,t} = 1 - \beta u_{t+1} - z_0(q_{h,b,t})(1 - \beta\pi_{t+1} - \beta u_{t+1}) - q_{h,b,t}(U_{h,b,t} - \beta u_{t+1}) + z_1(q_{h,b,t})\varepsilon$$

A flexible firm solves $\max_{q_{h,b,t} \in \mathbb{R}_+} \Pi_{b,t}$ s.t. $U_{h,b,t} = \bar{U}_{h,t}$. The FOC is given by

$$z_0(q_{h,b,t})(1 - \beta\pi_{t+1} - \beta u_{t+1}) + [z_0(q_{h,b,t}) - z_1(q_{h,b,t})]\varepsilon = \bar{U}_{h,t} - \beta u_{t+1}.$$

The second order condition is trivial since $\varepsilon < 0$. It follows that

$$\Pi_{b,t} = 1 - \beta u_{t+1} - (z_0(q_{h,b,t}) + z_1(q_{h,b,t}))(1 - \beta\pi_{t+1} - \beta u_{t+1}) + q_{h,b,t}z_1(q_{h,b,t})\varepsilon$$

Now consider fixed price firms. They attract both types of customers i.e. $q_{h,f,t} > 0$ and $q_{l,f,t} > 0$ and satisfy $U_{h,f,t} = \bar{U}_{h,t}$ and $U_{l,f,t} = \bar{U}_{l,t}$. Since $u_{h,t+1} = u_{l,t+1} = u_{t+1}$ we have $U_{h,f,t} = U_{l,f,t}$; and therefore $\bar{U}_{l,t} = \bar{U}_{h,t}$. It follows that $\Pi_{f,t}$, given by (9.4), becomes

$$\Pi_{f,t} = 1 - \beta u_{t+1} - z_0(q_{h,f,t} + q_{l,f,t})(1 - \beta\pi_{t+1} - \beta u_{t+1}) - (q_{h,f,t} + q_{l,f,t})(U_{h,f,t} - \beta u_{t+1}),$$

Letting $q_{f,t} \equiv q_{h,f,t} + q_{l,f,t}$, a fixed price firm solves $\max_{q_{f,t} \in \mathbb{R}_+} \Pi_{f,t}$ s.t. $U_{h,f,t}(r_{f,t}) = \bar{U}_{h,t}$. The FOC implies that

$$z_0(q_{f,t})(1 - \beta u_{t+1} - \beta\pi_{t+1}) = \bar{U}_{h,t} - \beta u_{t+1}.$$

Hence

$$\Pi_{f,t} = 1 - \beta u_{t+1} - (z_0(q_{f,t}) + z_1(q_{f,t}))(1 - \beta\pi_{t+1} - \beta u_{t+1}).$$

We will show $\Pi_{f,t} > \Pi_{b,t}$. First note that the FOCs together imply that

$$\varepsilon = \frac{z_0(q_{f,t}) - z_0(q_{h,b,t})}{z_0(q_{h,b,t})(1 - q_{h,b,t})} (1 - \beta u_{t+1} - \beta\pi_{t+1}).$$

Observe that $1 - \beta u_{t+1} - \beta\pi_{t+1}$ is positive; thus the inequality $\varepsilon < 0$ implies that either we have

(i) $q_{f,t} < q_{h,b,t}$ and $q_{h,b,t} > 1$ or we have (ii) $q_{f,t} > q_{h,b,t}$ and $q_{h,b,t} < 1$.

Now, let $\Delta \equiv \Pi_{f,t} - \Pi_{b,t}$. We will show that $\Delta > 0$. Note that

$$\begin{aligned}\Delta &= [z_0(q_{h,b,t}) - z_0(q_{f,t}) + z_1(q_{h,b,t}) - z_1(q_{f,t})] (1 - \beta u_{t+1} - \beta \pi_{t+1}) - q_{h,b,t} z_1(q_{h,b,t}) \varepsilon \\ &= \left\{ \frac{z_0(q_{h,b,t}) - z_0(q_{f,t})}{1 - q_{h,b,t}} - z_0(q_{f,t}) (q_{f,t} - q_{h,b,t}) \right\} (1 - \beta u_{t+1} - \beta \pi_{t+1})\end{aligned}$$

The first step follows after substituting for $\Pi_{f,t}$ and $\Pi_{b,t}$ whereas the second step is obtained after substituting for ε and noting that $z_1(q) = qz_0(q)$. The term $1 - \beta u_{t+1} - \beta \pi_{t+1}$ is positive; thus focus on the expression inside the curly brackets (call it Ω). Under condition (i) both terms of Ω are positive; hence $\Delta > 0$. Under condition (ii) the first term of Ω is positive but the second one is negative so it needs a closer inspection. Fix $q_{h,b,t} < 1$ and note that Ω falls in $q_{f,t}$ under the restrictions of (ii). It follows that Ω reaches a minimum when $q_{f,t} \searrow q_{h,b,t}$ (recall that under (ii) we have $q_{f,t} > q_{h,b,t}$). Note that $\lim_{q_{f,t} \searrow q_{h,b,t}} \Omega = 0$. Hence $\Omega > 0$ and therefore $\Delta > 0$ in the region $q_{f,t} > q_{h,b,t}$.

The fact that $\Delta > 0$ implies that fixed price sellers earn more than flexible sellers; hence there cannot be an equilibrium where flexible pricing is adopted. The implication is that if $\varepsilon < 0$ then the only possible outcome is the one where all sellers adopt fixed pricing (Eq-FP), which we have already characterized in Case 1.

9.1.3 Case 3: $\varepsilon > 0$.

If $\varepsilon > 0$ then flexible firms cater to high types only i.e. $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$ and $U_{l,b,t}(r_{b,t}) < \bar{U}_{l,t}$ thus $q_{h,b,t} > 0$ and $q_{l,b,t} = 0$. Substitute $q_{l,b,t} = 0$ into $\Pi_{b,t}$, given by (9.5), and use the fact that $z_1(q) = qz_0(q)$ to obtain

$$\Pi_{b,t} = 1 - \beta u_{h,t+1} - z_0(q_{h,b,t}) (1 - \beta u_{h,t+1} - \beta \pi_{t+1}) - q_{h,b,t} (U_{h,b,t} - \beta u_{h,t+1}) + z_1(q_{h,b,t}) \varepsilon$$

A flexible firm's problem is $\max_{q_{h,b,t} \in \mathbb{R}_+} \Pi_{b,t}$ s.t. $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$. The FOC is given by

$$z_0(q_{h,b,t}) (1 - \beta u_{h,t+1} - \beta \pi_{t+1}) + [z_0(q_{h,b,t}) - z_1(q_{h,b,t})] \varepsilon = \bar{U}_{h,b,t} - \beta u_{h,t+1} \quad (9.8)$$

The second order condition is satisfied if

$$-z_0(q_{h,b,t}) [1 - \beta u_{h,t+1} - \beta \pi_{t+1}] - \varepsilon [2z_0(q_{h,b,t}) - z_1(q_{h,b,t})] < 0.$$

If $2z_0(q_{h,b,t}) > z_1(q_{h,b,t})$, i.e. if $2 > q_{h,b,t}$ then the inequality is satisfied irrespective of ε . If $2 < q_{h,b,t}$ then we need $\varepsilon < (1 - \beta u_{h,t+1} - \beta \pi_{t+1}) / (q_{h,b,t} - 2)$. The expression on the right hand side is positive. Since ε is assumed to be positive but sufficiently small the inequality is satisfied; hence the SOC holds.

It follows that

$$\Pi_{b,t} = 1 - \beta u_{h,t+1} - [z_0(q_{h,b,t}) + z_1(q_{h,b,t})] (1 - \beta u_{h,t+1} - \beta \pi_{t+1}) + q_{h,b,t} z_1(q_{h,b,t}) \varepsilon \quad (9.9)$$

Now consider fixed price sellers. Recall that they attract low types only, i.e. $U_{h,f,t} < \bar{U}_{h,t}$ and $U_{l,f,t} = \bar{U}_{l,t}$; hence $q_{h,f,t} = 0$ and $q_{l,f,t} > 0$. Substituting $q_{h,f,t} = 0$ into $\Pi_{f,t}$, given by (9.4), yields

$$\Pi_{f,t} = 1 - \beta u_{l,t+1} - z_0(q_{l,f,t}) [1 - \beta u_{l,t+1} - \beta \pi_{t+1}] - q_{l,f,t} (U_{l,f,t} - \beta u_{l,t+1})$$

The seller solves $\max_{q_{l,f,t} \in \mathbb{R}_+} \Pi_{f,t}$ s.t. $U_{l,f,t}(r_{b,t}) = \bar{U}_{l,t}$. The FOC is given by

$$z_0(q_{l,f,t}) [1 - \beta u_{l,t+1} - \beta \pi_{t+1}] = \bar{U}_{l,t} - \beta u_{l,t+1}. \quad (9.10)$$

The SOC is trivial; hence the solution corresponds to a maximum. It follows that

$$\Pi_{f,t} = 1 - \beta u_{l,t+1} - [z_0(q_{l,f,t}) + z_1(q_{l,f,t})] (1 - \beta u_{l,t+1} - \beta \pi_{t+1}) \quad (9.11)$$

Recall that $\varphi_{f,t}$ denotes the fraction of sellers who compete with fixed pricing. Substituting $q_{h,f,t} = q_{l,b,t} = 0$ into the feasibility conditions in (13) yields

$$q_{l,f,t} = \eta_t \lambda_t / \varphi_{f,t} \quad \text{and} \quad q_{h,b,t} = (1 - \eta_t) \lambda_t / (1 - \varphi_{f,t}).$$

We will show that there exists a unique $\varphi_{f,t}^* \in (0, \eta_t)$ satisfying the equal profit condition $\Pi_{f,t} = \Pi_{b,t}$, proving the equilibrium exists and it is unique. Let $\Delta(\varphi_{f,t}) \equiv \Pi_{b,t} - \Pi_{f,t}$ and note that Δ rises in $q_{h,b,t}$, which in turn rises in $\varphi_{f,t}$, and that Δ falls in $q_{l,f,t}$, which in turn falls in $\varphi_{f,t}$. It follows that $d\Delta/d\varphi_{f,t} > 0$. Furthermore note that $\Delta(\eta_t) > 0$ as $u_{h,t+1} > u_{l,t+1}$ (from the inductive step) whereas $\Delta(0) < 0$ if ε is small. To see why, note that $\Delta(0) < 0$ if $\varepsilon < \bar{\varepsilon}$, where

$$\bar{\varepsilon} \equiv \frac{\beta(u_{h,t+1} - u_{l,t+1})}{qz_1(q)} + \frac{1+q}{q^2} [1 - \beta u_{h,t+1} - \beta \pi_{t+1}], \quad \text{and} \quad q \equiv (1 - \eta_t) \lambda_t. \quad (9.12)$$

The expression for $\bar{\varepsilon}$ is positive. Since ε is positive but sufficiently small the inequality $\varepsilon < \bar{\varepsilon}$ holds. Since $\Delta(0) < 0$, $\Delta(\eta_t) > 0$ and Δ is rising in $\varphi_{f,t}$, by the Intermediate Value Theorem there exists a unique $\varphi_{f,t}^* \in (0, \eta_t)$ satisfying $\Delta(\varphi_{f,t}^*) = 0$. Since $\varphi_{f,t}^* < \eta_t$ we have $q_{h,b,t}^* < \lambda_t < q_{l,f,t}^*$ i.e. fixed price firms are more crowded than flexible firms.

Now we can obtain equilibrium prices and payoffs. Substituting $q_{l,b,t}^* = 0$ into the expression for $U_{h,b,t}$, given by (9.2), yields

$$U_{h,b,t} = \frac{1 - z_0(q_{h,b,t}^*)}{q_{h,b,t}^*} (1 - r_{b,t} - \beta u_{h,t+1}) + z_0(q_{h,b,t}^*) (r_{b,t} + \varepsilon - y_t) + \beta u_{h,t+1}.$$

Solving $U_{h,b,t} = \bar{U}_{h,t}$, where $\bar{U}_{h,t}$ is given by (9.8), for $r_{b,t}$ yields the expression for $r_{b,t}^*$, which is on display in Proposition 3 (equation ((21))). Similarly substituting $q_{h,f,t}^* = 0$ into $U_{l,f,t}$, given by (9.2), and solving the equation $U_{l,f,t} = \bar{U}_{l,t}$, where $\bar{U}_{l,t}$ is given by (9.10), for $r_{f,t}$ yields the expression for $r_{f,t}^*$ (equation (20)). Equilibrium payoffs π_t , $u_{h,t}$ and $u_{l,t}$ are immediate from the first order conditions (9.8) and (9.10). Finally the equilibrium bargained price y_t^* is obtained by substituting $u_{h,t+1}$ into (1).

High type buyers bargain if $y_t^* \leq r_{b,t}^* + \varepsilon$. After substituting for $r_{b,t}^*$ and y_t^* and re-arranging this condition is equivalent to

$$\theta \geq \tilde{\theta}_t \equiv \frac{z_1(q_{h,b,t}^*)}{1 - z_0(q_{h,b,t}^*)} - \frac{\varepsilon z_1(q_{h,b,t}^*) q_{h,b,t}^*}{[1 - z_0(q_{h,b,t}^*)][1 - \beta u_{h,t+1} - \beta \pi_{t+1} + \varepsilon]}.$$

If $\theta < \tilde{\theta}_t$ then even high types would not opt for bargaining; thus the availability of bargaining becomes immaterial and the model collapses to a fixed price setting, characterized earlier (Eq-FP).

Transact Now or Wait? We have already established that players are better off trading immediately along Eq-FP (see Case 1 above). What remains to be done is to establish this claim for the other outcome, i.e. Eq-FS. Along this equilibrium high types shop at flexible stores and low types shop at fixed price stores. Start with flexible stores. The worst case scenario for a high type buyer is to purchase at $r_{b,t}^*$ (the alternative is buying at the bargained price y_t^* , which is less than $r_{b,t}^*$). The buyer purchases if $1 - r_{b,t}^* > \beta u_{h,t+1}$. After substituting for $r_{b,t}^*$ the condition is equivalent to $\varepsilon < Q$, where

$$Q \equiv (1 - \beta u_{h,t+1} - \beta \pi_{t+1}) \frac{1 - \theta}{q_{h,b,t}^* - 1 + \theta}.$$

Notice that Q is positive. To see why, note that the numerator is positive, but the sign of the denominator, $q_{h,b,t}^* - 1 + \theta$, needs inspection. Recall that along Eq-FS we have $\theta \geq \tilde{\theta}_t$ and note

that $\tilde{\theta}_t \geq z_1(q_{h,b,t}^*)/[1 - z_0(q_{h,b,t}^*)]$. The expression $q_{h,b,t}^* - 1 + \theta$ is increasing in θ ; thus in order to show that it is positive it suffices to show that $q_{h,b,t}^* - 1 + z_1(q_{h,b,t}^*)/[1 - z_0(q_{h,b,t}^*)] > 0$. It is easy to verify that this inequality holds true for all values of $q_{h,b,t}^*$, which means that $q_{h,b,t}^* - 1 + \theta$ is also positive; thus Q is positive. Since ε is positive but small the inequality $\varepsilon < Q$ holds; hence the buyer is better off purchasing instead of waiting. (One can show that $Q > \bar{\varepsilon}$, where $\bar{\varepsilon}$ is given by (9.12); thus $\varepsilon < Q$ as long as $\varepsilon < \bar{\varepsilon}$.)

Now consider the seller, whose worst case scenario is selling at y_t^* . The seller agrees to trade if $y_t^* > \beta\pi_{t+1}$. Substituting for y_t^* the condition is equivalent to $(1 - \theta)[1 - \beta u_{h,t+1} - \beta\pi_{t+1} + \varepsilon] > 0$. Both expressions are positive; thus the inequality holds.

Now consider a fixed price firms, where low types shop. A low type buyer purchases if $1 - r_{f,t}^* > \beta u_{l,t+1}$. After substituting for $r_{f,t}^*$ the condition is equivalent to

$$(1 - \beta u_{l,t+1} - \beta\pi_{t+1}) \times z_1(q_{l,f,t}^*)/[1 - z_0(q_{l,f,t}^*)] > 0.$$

Since $1 - \beta u_{l,t+1} - \beta\pi_{t+1} > 0$ the inequality holds. Similarly the seller trades if $r_{f,t}^* > \beta\pi_{t+1}$, i.e. if

$$(1 - \beta u_{l,t+1} - \beta\pi_{t+1}) \times [1 - z_1(q_{l,f,t}^*)/[1 - z_0(q_{l,f,t}^*)]] > 0$$

Expressions inside the brackets are positive; hence the inequality holds. This completes the proof.

■

9.2 Model with $N \geq 2$ Types of Buyers

In the main text buyers are divided into two types according to their bargaining abilities. Here we consider a setting with N types, where type 1 buyers are the least skilled in bargaining ("non-hagglers") and type N buyers are the most skilled. Our goal is to check if the results in the main text remain robust to this variation. As this exercise is a robustness check, rather than a full blown analysis, we focus on the one shot game with $\varepsilon = 0$ and then elaborate on what would happen if $\varepsilon < 0$ or $\varepsilon > 0$.

The Outcome of Bargaining. Letting $\theta_i \in [0, 1)$ denote the bargaining power of type $i = 1, 2, \dots, N$ buyers, we fix $\theta_1 = 0$ and assume that negotiation skills increase in i , that is $\theta_{i+1} > \theta_i$. As in the benchmark, bargaining may ensue only if there is a single customer at the store. If two or more customers are present then the item is necessarily sold at the list price. Furthermore we assume that a buyer's negotiation skill manifests itself at the bargaining table, i.e. once negotiations start the seller can tell how skilled his customer is and correctly identify the parameter θ_i . Notice that identifying who is the most/least skilled among multiple customers is not an issue as the item is sold at the posted price under that contingency. Consider the negotiation process between a seller and a type i buyer. The bargained price y_i can be found as the solution to the following maximization problem:

$$\max_{y_i \in [0, 1]} (1 - y_i)^{\theta_i} y_i^{1 - \theta_i}$$

The solution yields $y_i = 1 - \theta_i$. Since $\theta_{i+1} > \theta_i$ we have $y_{i+1} < y_i$, i.e. higher types bargain lower prices. Since $\theta_1 = 0$, type 1 never bargains. We assume that θ_2 is sufficiently large to ensure that $r_b \geq y_2$, i.e. type 2 buyers are skilled enough to obtain a lower price than the posted price. (Otherwise the model collapses to a setting with $N - 1$ types, where type 1 and type 2 buyers are the non-hagglers.) Clearly, if type 2 is skilled enough to ask for bargaining then the higher types ($3, 4, \dots, N$) are more than capable of doing so.

Expected Payoffs. Let $q_{i,m}$ denote the expected demand consisting of type i buyers at a store trading via rule m and let

$$q_m \equiv \sum_{i=1}^N q_{i,m}, \quad \text{where } m = f, b \text{ and } i = 1, 2, \dots, N$$

denote the total demand at that store. It follows that the expected utility of a type i buyer at a

fixed price store is given by

$$U_{i,f} = \frac{1 - z_0(q_f)}{q_f} (1 - r_f), \text{ for } i = 1, 2, \dots, N.$$

At a flexible store, on the other hand, we have

$$\begin{aligned} U_{1,b} &= \frac{1 - z_0(q_b)}{q_b} (1 - r_b) \quad \text{and} \\ U_{i,b} &= z_0(q_b) (1 - y_i) + \sum_{n=1}^{\infty} \frac{z_n(q_b)}{n+1} (1 - r_b), \quad i = 2, 3, \dots, N. \end{aligned}$$

The first line is the expected utility of a type 1 buyer (they never negotiate), whereas the second line is the expected utility of a type i buyer, who would negotiate if he is the sole customer at the store. These expressions are similar to their counterparts in the baseline model and can be interpreted similarly. Basic algebra reveals that

$$U_{i,b} = U_{1,b} + z_0(q_b) (r_b - y_i) \quad \text{and} \quad U_{i+1,b} = U_{i,b} + z_0(q_b) (y_i - y_{i+1}) \quad \text{for } i = 2, 3, \dots, N \quad (9.13)$$

Since $r_b > y_2$ and $y_i > y_{i+1}$ we have $U_{i+1,b} > U_{i,b}$. Now turn to sellers. A fixed price seller expects to earn

$$\Pi_f = [1 - z_0(q_f)] r_f.$$

The expression for Π_f is the same as its counterpart in the benchmark model; however flexible sellers' expected profit is slightly more cumbersome, because they face the prospect of meeting all types of customers and each type negotiates a different price. We have

$$\Pi_b = \sum_{i=2}^N \prod_{j=1, j \neq i}^N z_0(q_{j,b}) z_1(q_{i,b}) y_i + \left[\prod_{j=2}^N z_0(q_{j,b}) z_1(q_{1,b}) + \sum_{n=2}^{\infty} z_n(q_b) \right] r_b$$

To understand the first term note that with probability $\prod_{j=1, j \neq i}^N z_0(q_{j,b}) z_1(q_{i,b})$ the seller gets exactly one type i customer, in which case he charges the bargained price y_i (recall that the seller can identify the type of the customer during the negotiation process). To account for all types, the expression needs to be summed over all i , but the summation starts from $i = 2$ because type 1 customers never negotiate. The second expression inside the brackets represent the probability of getting exactly one type 1 customer or getting more than one customer, regardless of the type.

In either case the seller charges the posted price r_b . Noting that $\prod_{j=1}^N z_0(q_{j,b}) = z_0(q_b)$ and that $xz_0(x) = z_1(x)$ one can show that

$$\Pi_m = 1 - z_0(q_m) - \sum_{i=1}^N q_{i,m} U_{i,m}, \text{ where } m = f, b.$$

Now we can state the main result of this section.

Proposition 7 *If $\theta_N \geq \bar{\theta} \equiv z_1(\lambda) / [1 - z_0(\lambda)]$ then there exists a continuum of equilibria, where an indeterminate fraction $\varphi^* \geq \max\{\eta_1, \eta_2, \dots, \eta_N\}$ of sellers trade via fixed pricing and remaining sellers trade via flexible pricing. The equilibria are characterized by partial segmentation: Everyone but type N customers shop exclusively at fixed price firms whereas type N customers shop anywhere. The expected demand at each store equals to λ . Fixed and flexible price sellers post, respectively*

$$r_f^*(\lambda) = 1 - \frac{z_1(\lambda)}{1 - z_0(\lambda)} \quad \text{and} \quad r_b^*(\lambda) = 1 - \frac{z_1(\lambda)(1 - \theta_N)}{1 - z_0(\lambda) - z_1(\lambda)}$$

The equilibria are payoff-equivalent: in any realized equilibrium sellers and buyers earn $\pi = 1 - z_0(\lambda) - z_1(\lambda)$ and $u = z_0(\lambda)$ no matter which rule sellers compete with and no matter which seller's rule buyers join in. If $\theta_N < \bar{\theta}$, i.e. if type N customers are not skilled enough in negotiations then the availability of flexible pricing becomes immaterial and fixed pricing emerges as the unique equilibrium.

The proposition largely resembles its counterpart in the main text (Proposition 1), which indicates that the results remain rather robust. The key insight in here is that competition among sellers dictates bargaining deals to be designated for the most skilled type, which is why in equilibrium only the most skilled negotiators hunt for bargaining deals and everyone else shops at fixed price venues. An outcome where a firm attracts two different types of customers fails to exist, because along that scenario the lower type ends up with a lower market utility, which is incompatible with profit maximization under competition. An outcome where a firm caters exclusively to a lesser type fails to exist for similar reasons.

In what follows we prove the proposition. Steps 1, 2 and 3, reminiscent of Lemma 2 in the main text, establish how customer demographics pan out along a competitive search equilibrium. We, then, characterize the equilibrium.

- Step 1. A flexible store cannot attract two (or more) different types of customers at the same

time. It must be attracting a single type only.

We will show that the store cannot attract two different types at the same time. The fact that it cannot attract more than two types is a corollary. To start, suppose, by contradiction, a flexible store attracts types k and $k + 1$, i.e. suppose that $q_{k,b}$ and $q_{k+1,b}$ are both positive whereas $q_{i,b} = 0$ for all $i \neq k, k + 1$. The fact that $q_{k,b}$ and $q_{k+1,b}$ are both positive implies that $U_{k,b} = \bar{U}_k$ and $U_{k+1,b} = \bar{U}_{k+1}$. Recall that $U_{k+1,b} > U_{k,b}$. It follows that $\bar{U}_{k+1} > \bar{U}_k$. In addition the fact that $q_{k+j} = 0$, where $j \geq 2$, implies that $U_{k+j,b} < \bar{U}_{k+j}$. Since $U_{k+j,b} > U_{k,b} = \bar{U}_k$ we have $\bar{U}_{k+j} > \bar{U}_k$. In words all types who are better negotiators than type k must have a higher market utility than type k . The seller's profit equals to

$$\begin{aligned}\Pi_b &= 1 - z_0 (q_{k,b} + q_{k+1,b}) - q_{k,b}U_{k,b} - q_{k+1,b}U_{k+1,b} \\ &= 1 - z_0 (q_{k,b} + q_{k+1,b}) - (q_{k,b} + q_{k+1,b})U_{k,b} - \Delta,\end{aligned}$$

where $\Delta := q_{k+1,b}z_0 (q_{k,b} + q_{k+1,b}) (y_k - y_{k+1}) > 0$. The second line follows from (9.13) and note that Δ is positive because $y_k > y_{k+1}$.

Below we show that if this seller switches from flexible pricing to fixed pricing and provides his customers with market utility \bar{U}_k then he could keep his expected demand intact yet he would earn higher profits, rendering the above outcome a non-equilibrium. To start, note that if the seller switches to fixed pricing then all buyers, regardless of their bargaining ability, earn the same expected payoff

$$U_f = \frac{1 - z_0 (q_f)}{q_f} (1 - r_f)$$

at his firm. If the seller provides customers with market utility \bar{U}_k then types $k + 1$ and above will not visit that store because $\bar{U}_{k+j} > \bar{U}_k$ for all $j \geq 1$ (see above). It follows that the seller will be visited by types k or below. The fact that the seller provides his customers with market utility \bar{U}_k implies that $U_f = \bar{U}_k$. Recall that $U_{k,b} = \bar{U}_k$. It follows $U_f = U_{k,b}$, i.e

$$\Delta = \frac{1 - z_0 (q_f)}{q_f} (1 - r_f) - \frac{1 - z_0 (q_b)}{q_b} (1 - r_b) - z_0 (q_b) (r_b - y_k) = 0.$$

Fix r_b and q_b and note that, per the Intermediate Value Theorem, there exists a unique $\hat{r}_f \in (0, r_b)$ ensuring that $q_f = q_b$ while satisfying $\Delta = 0$. In words if the seller posts \hat{r}_f then he can provide his customers with market utility \bar{U}_k while keeping his expected demand intact. Recall that his prior expected demand was q_b ; by posting \hat{r}_f the seller ensures that his new expected demand q_f is the

same as q_b .

The seller's expected profit under fixed pricing is equal to $\Pi_f = 1 - z_0(q_f) - q_f U_f$. Since $q_b = q_f$ it is easy to show that $\Pi_f - \Pi_b = \Delta > 0$, i.e. the seller earns higher profits than he did before; hence the initial outcome could not be an equilibrium. This completes the proof. ■

Step 1 establishes that a flexible store can only attract a single type. This raises the question of whether a flexible store attracts, say, type k while another flexible store attracts type $k + 1$. I.e. whether a separating equilibrium where different flexible stores attract different types could exist. Below we rule out this possibility.

- Step 2. There cannot be an outcome where different flexible stores attract different types of customers. All flexible stores must attract the same type.

Consider two flexible stores, say store A and B . Suppose store A attracts type k only store B attracts type $k + 1$ only (Step 1 ruled out the possibility of a store attracting more than one type). So for store A we have $q_k^A > 0$ and $q_i^A = 0$ for all $i \neq k$ and for store B we have $q_{k+1}^B > 0$ and $q_i^B = 0$ for all $i \neq k + 1$.

Note that type $k + 1$ could shop at store A and obtain a better deal than type k as they are more skilled, but the fact that they stay away from store A indicates that their market utility is higher, i.e. $\bar{U}_{k+1} > \bar{U}_k$. Technically at store A we have $U_{k,b}^A = \bar{U}_k$. The fact that $q_{k+1}^A = 0$ indicates that $U_{k+1,b}^A < \bar{U}_{k+1}$. Recall that $U_{k+1,b}^A > U_{k,b}^A$. It follows that $\bar{U}_{k+1} > \bar{U}_k$.

Store A solves

$$\max_{q_{k,b}^A \in \mathbb{R}_+} \Pi_b^A = \max_{q_{k,b}^A \in \mathbb{R}_+} 1 - z_0(q_{k,b}^A) - q_{k,b}^A U_{k,b}^A \text{ s.t. } U_{k,b}^A = \bar{U}_k$$

The FOC implies $z_0(q_{k,b}^A) = \bar{U}_k$; hence

$$\Pi_b^A = 1 - z_0(q_{k,b}^A) - z_1(q_{k,b}^A).$$

Store B 's problem is similar, thus

$$\Pi_b^B = 1 - z_0(q_{k+1,b}^B) - z_1(q_{k+1,b}^B).$$

Stores must earn equal profits; thus $\Pi_b^A = \Pi_b^B$. This implies that $q_{k,b}^A = q_{k+1,b}^B$, which in turn implies that $\bar{U}_k = \bar{U}_{k+1}$; a contradiction. ■

- Step 3. Flexible stores must be attracting type N only.

Suppose they attract some other type, say type $k < N$. The fact that type k buyers visit flexible stores while type N buyers stay away indicates that $U_{N,b} < \bar{U}_N$ and $U_{k,b} = \bar{U}_k$. Recall that $U_{N,b} > U_{k,b}$, thus $\bar{U}_N > \bar{U}_k$. Since type N buyers stay away from flexible firms, they must be shopping at fixed price firms. This means that $U_{N,f} = \bar{U}_N$. Recall however that $U_{i,f}$ is the same for all i , thus $U_{k,f} = U_{N,f}$. It follows that $U_{k,f} > \bar{U}_k$; a contradiction since by definition $U_{k,f}$ cannot exceed the market utility \bar{U}_k . ■

Characterization of Equilibrium. Flexible stores attract no one but type N , i.e. $q_{N,b} > 0$ and $q_{i,b} = 0$ for all $i \neq N$. It follows that

$$\Pi_b = 1 - z_0(q_{N,b}) - q_{N,b}U_{N,b}$$

A flexible seller solves

$$\max_{q_{N,b} \in \mathbb{R}_+} 1 - z_0(q_{N,b}) - q_{N,b}U_{N,b} \quad \text{s.t.} \quad U_{N,b} = \bar{U}_N$$

The FOC implies $z_0(q_{N,b}) = \bar{U}_N$; hence

$$\Pi_b = 1 - z_0(q_{N,b}) - z_1(q_{N,b}).$$

The fact that flexible stores attract no one but type N indicates that types $1, 2, \dots, N-1$ must be shopping at fixed price stores. So, let $q_f = \sum_{i=1}^N q_{i,f}$ denote the total demand of a fixed price store consisting of type $1, 2, \dots, N-1$, and possibly of type N , customers. The fixed price seller solves

$$\max_{q_f \in \mathbb{R}_+} 1 - z_0(q_f) - q_f U_f \quad \text{s.t.} \quad U_f = \bar{U},$$

where \bar{U} is a generic level or market utility (as it turns out this will be equal to \bar{U}_N). The FOC is given by $z_0(q_f) = \bar{U}$. The seller's profit, therefore, is equal to

$$\Pi_f = 1 - z_0(q_f) - z_1(q_f).$$

Both sellers must earn equal profits; i.e. $\Pi_b = \Pi_f$. This indicates that $q_{N,b} = q_f$, i.e. expected demands at fixed and flexible stores must be identical. This means that $\bar{U} = \bar{U}_N$, indicating that

all buyers must earn the same market utility and that type N , too, may shop at fixed price stores i.e. $q_{N,f}$ may indeed be positive. Letting φ_f denote the fraction of fixed price sellers and η_i the fraction of type i buyers in the market, with $\sum_{i=1}^N \eta_i = 1$, we have

$$\varphi_f q_{i,f} + (1 - \varphi_f) q_{i,b} = \lambda \eta_i \text{ for } i = 1, 2, \dots, N.$$

The feasibility condition is similar to its counterpart in the main text (compare with (13)). Noting that $q_{i,b} = 0$ for $i < N$ we have

$$\underbrace{\varphi_f \sum_{i=1}^N q_{i,f}}_{\equiv q_f} + (1 - \varphi_f) q_{N,b} = \lambda \sum_{i=1}^N \eta_i = \lambda.$$

Recall that $q_f = q_{N,b}$; hence

$$q_{N,b}^* = \lambda, \quad q_{N,f}^* = \frac{\lambda(\eta_N - 1 + \varphi_f^*)}{\varphi_f^*}, \quad q_{i,f}^* = \frac{\lambda \eta_i}{\varphi_f^*} \text{ for } i < N.$$

Note that for any $\varphi_f^* \geq \max\{\eta_1, \eta_2, \dots, \eta_N\} \equiv \bar{\eta}$ expected demands $q_{i,f}^*$ are positive and satisfy the relationship above. This means that φ_f^* is indeterminate and we have a continuum of equilibria where $\varphi_f^* \in [\bar{\eta}, 1]$. Note that if $\varphi_f^* \geq \bar{\eta}$ then $\sum_{i=1}^N q_{i,f}^* = q_{N,b}^* = \lambda$, i.e. in any given equilibrium flexible sellers and fixed price sellers have the same expected demand λ . To complete the proof we need to pin down the equilibrium payoffs and prices; but this is a rather mechanic task and it can be accomplished by going through the steps outlined in the proof of Proposition 1; hence it is skipped here. ■

What if $\varepsilon \neq 0$? First, if $\varepsilon < 0$ then, as in the benchmark, no seller would offer flexible pricing. To see why, notice that if $\varepsilon = 0$ then sellers are indifferent between fixed and flexible pricing. If, however, ε falls below zero then this indifference would no longer hold because the negative ε would filter into flexible sellers' profits causing them to earn less than fixed price stores. Sellers can avoid this negative effect by switching to fixed pricing. This claim can be proved by repeating the steps in the proof of Proposition 2 because the key in that proof is the fact that a negative ε hurts flexible sellers' profits, which would remain true irrespective of whether there are two or N types of customers.

If $\varepsilon > 0$ then we expect Proposition 3 to go through with the above caveat—that flexible stores attract type N customers and that everyone else shops at fixed price stores. To establish this claim

one needs to prove Steps that are analogous to Step 1, 2 and 3 above. A close look at their proofs reveals that the key factor driving the results is the inequality $U_{i+1,b} > U_{i,b}$ —the fact that higher types earn more at a bargaining store than lower types. With $\varepsilon > 0$ expected utilities $U_{i,b}$ would have different closed form expressions, but, nevertheless the inequality $U_{i+1,b} > U_{i,b}$ would remain as the parameter ε is orthogonal to the bargaining ability θ_i . As such, the claims in Steps 1, 2 and 3 would go through even if $\varepsilon > 0$. Once customer demographics are settled (that flexible stores attract type N customers and fixed price stores attract everyone else), the characterization of the equilibrium can, then, be accomplished by virtually repeating the same steps as in the proof of Proposition 3.

9.3 Second Round Matching

In the main text buyers who are unable to get an offer from, a firm or firms who are unable to receive a customer need to wait until the next trading period before they can try again. Here we study a variation where unmatched players may be costlessly re-matched with trading counterparts before moving to the next period—a process to which we refer as *second round matching*. In what follows we reconstruct the equilibria under this modification and show that the results of the benchmark model remain unchanged, subject to a modification in outside options. Since this exercise is a robustness check rather than a full blown analysis, we analyze the case $\varepsilon = 0$ in detail and then elaborate on what would happen if $\varepsilon < 0$ or $\varepsilon > 0$.

We assume that in each trading period two rounds of meetings take place. The first one is the matching process in the benchmark model. At the end of this round, inevitably, some buyers and sellers remain unmatched, so these players costlessly enter into a second round, where they are randomly matched with one another. One can specify a number of ways on how this may work, but to keep things simple and tractable we remain agnostic about the matching process, and simply assume that each buyer, regardless of his type, gets to trade with probability $\omega_{B,t}$ whereas each seller, regardless of whether he was fixed or flexible with the list price, gets to trade with probability $\omega_{S,t}$. The key observation is that, even in the second round players are not guaranteed to trade, i.e. the matching function may assign multiple buyers to a seller, in which case some buyers will be unable to buy, or it may assign no buyers to a seller, in which case the seller will have no choice but to wait for the next period. For now we take $\omega_{B,t}$ and $\omega_{S,t}$ as given, but at the end of this section we show how they might be tied to the fundamentals of the model, for example, via a standard urn-ball matching function.

Another issue that needs to be addressed is how the transaction in the second round is settled. This can be done in a number of ways, e.g. a fifty-fifty split, trading at the initially posted price and so on. Again, we remain agnostic about this mechanism, and instead assume that after a transaction in the second round the seller obtains payoff $p_t \in [\underline{p}_t, \bar{p}_t]$ and the buyer obtains $1 - p_t$. For now we take the boundaries of p_t as given but subsequently they will be pinned down endogenously.

Proposition 8 *Fix some $p_t \in [\beta\pi_{t+1}, 1 - \beta u_{t+1}]$. If $\theta > \bar{\theta}_t$ then there exists a continuum of equilibria, where an indeterminate fraction $\varphi_{f,t}^* \geq \eta_t$ of sellers trade via fixed pricing and remaining*

sellers trade via flexible pricing. Sellers post

$$\begin{aligned} r_{f,t}^* &= 1 - \mu_{B,t} - \frac{z_1(\lambda_t)}{1 - z_0(\lambda_t)} (1 - \mu_{B,t} - \mu_{S,t}) \\ r_{b,t}^* &= 1 - \mu_{B,t} - \frac{z_1(\lambda_t)(1 - \theta)}{1 - z_0(\lambda_t) - z_1(\lambda_t)} (1 - \mu_{B,t} - \mu_{S,t}) \end{aligned}$$

where

$$\mu_{B,t} = \omega_{B,t}(1 - p_t) + (1 - \omega_{B,t})\beta u_{t+1} \quad \text{and} \quad \mu_{S,t} = \omega_{S,t}p_t + (1 - \omega_{S,t})\beta\pi_{t+1}$$

In case negotiations ensue transaction occurs at price

$$y_t^* = 1 - \mu_{B,t} - \theta (1 - \mu_{B,t} - \mu_{S,t})$$

The expected demand at each store equals to λ_t , however the equilibria are characterized by partial segmentation of customers: non-hagglers shop exclusively at fixed price firms whereas hagglers shop anywhere. In any equilibrium sellers and buyers earn

$$\begin{aligned} \pi_t &= 1 - \mu_{B,t} - [z_0(\lambda_t) + z_1(\lambda_t)] (1 - \mu_{B,t} - \mu_{S,t}) \\ u_t &= z_0(\lambda_t) [1 - \mu_{B,t} - \mu_{S,t}] + \mu_{B,t}. \end{aligned}$$

If $\theta < \bar{\theta}_t$ then fixed pricing emerges as the unique equilibrium: all sellers post $r_{f,t}^$ and serve both types of customers. The total demand at each firm equals to λ_t and the equilibrium payoffs remain the same as above.*

In the main text a buyer's outside option is βu_{t+1} , which is the present value of his expected payoff in the next period. With the prospect of second round meetings, his outside option is $\mu_{B,t} = \omega_{B,t}(1 - p_t) + (1 - \omega_{B,t})\beta u_{t+1}$, which is a weighted average: with probability $\omega_{B,t}$ the buyer gets to trade in the second round and obtains $1 - p_t$ and with the complementary probability $1 - \omega_{B,t}$ he is unable to trade even in the second round, so he walks away with βu_{t+1} . Sellers' outside option $\mu_{S,t}$ can be interpreted similarly. A comparison between this proposition and its counterpart in the main text, Proposition 1, reveals that they are virtually identical if one updates the outside options with their current form in here, which indicates that the results remain robust.

The second round meeting gives customers and firms another chance to transact without incurring additional costs, as such, it diminishes trade frictions and improves everyone's outside options (one can show that $\mu_{B,t} > \beta u_{t+1}$ and $\mu_{S,t} > \beta\pi_{t+1}$). This effect is similar to raising the discount

factor in the benchmark model. Indeed in the benchmark model βu_{t+1} and $\beta \pi_{t+1}$ can be improved simultaneously by raising β , which lowers waiting costs for everyone and renders trade frictions less biting.

An important question is whether players would like to trade immediately rather than waiting. Although we address this issue more technically in the proof of the proposition, the answer is yes—both in the first round as well as in the second round players are better off transacting whenever they have an opportunity to do so. Recall that in the second round buyers get payoff $1 - p_t$ and sellers get p_t . The fact that $p_t \in [\beta \pi_{t+1}, 1 - \beta u_{t+1}]$ ensures that both the firms and their customers are willing to trade during the second round meetings instead of waiting for the next period. If p_t falls outside these boundaries then either the firm or the customer will walk away, rendering second round meetings immaterial and causing the model to collapse to its version in the main text. The crucial question is, then, whether players would want to transact in the first round instead of waiting for the second round. The answer is, still yes. Trade frictions may be lessened by the prospect of second round meetings, but they are not completely wiped out as no one is guaranteed to a sure trade, and therefore players are better off trading immediately instead of waiting. It is worth pointing out that the prospect of second round meetings filters into the equilibrium objects, i.e. the prices and payoffs are determined taking into consideration the the new version of outside options, which convinces buyers and sellers to trade at those prices instead of waiting.

As mentioned above, the analysis is based on the case $\varepsilon = 0$; however given the results so far we can speculate on what would happen if $\varepsilon > 0$ or $\varepsilon < 0$. A detailed comparison between the proof of Proposition 8 and the proof of its counterpart in the main text, Proposition 1, reveals that both proofs follow virtually identical steps if one replaces the outside options in the benchmark with their current form. The parameter ε is orthogonal to the determination of outside options, as such we expect Propositions 2 and 3, which correspond to cases $\varepsilon < 0$ and $\varepsilon > 0$, to go through in similar fashion.

Proof of Proposition 8. The proof is by induction; however the analysis of the terminal period is quite similar to the analysis of the inductive step; hence skipping it we directly analyze the inductive step pertaining period t .

Bargaining. The Nash product in this version of the model is given by

$$\max_{y_t \in [0,1]} (1 - y_t - \mu_{B,t})^\theta (y_t - \mu_{S,t})^{1-\theta}.$$

The solution yields

$$y_t = 1 - \mu_{B,t} - \theta (1 - \mu_{B,t} - \mu_{S,t}).$$

We assume that $y_t < r_{b,t}$, which requires θ to be sufficiently large, i.e. hagglers have sufficient bargaining power to negotiate the list price.

Expected payoff. We construct an equilibrium under the conjecture that non hagglers shop at fixed price stores whereas hagglers shop at both types of stores. Furthermore we conjecture that players transact immediately instead of waiting. We will verify both of these conjectures once we pin down equilibrium prices and payoffs. Along our conjecture, the expected utility of a high type buyer, who shops at a best offer store, is given by

$$U_{h,b,t} = z_0(q_{h,b,t})(1 - y_t) + \frac{1 - z_0(q_{h,b,t}) - z_1(q_{h,b,t})}{q_{h,b,t}}(1 - r_{b,t}) + \frac{q_{h,b,t} - 1 + z_0(q_{h,b,t})}{q_{h,b,t}}\mu_{B,t}. \quad (9.14)$$

With probability $z_0(q_{h,b,t})$ the buyer is alone at the store and purchases the item through negotiations at price y_t . With probability $z_n(q_{h,b,t})$ he encounters $n = 1, 2, \dots$ other buyers, and his probability of being able to buy is

$$\sum_{n=1}^{\infty} \frac{z_n(q_{h,b,t})}{n+1} = \frac{1 - z_0(q_{h,b,t}) - z_1(q_{h,b,t})}{q_{h,b,t}}.$$

If he manages to purchase, then he pays the list price $r_{b,t}$. Finally with the complementary probability he is unable to buy in the first round, so he obtains $\mu_{B,t}$. A flexible seller's profit is given by

$$\Pi_{b,t} = z_1(q_{h,b,t})y_t + [1 - z_0(q_{h,b,t}) - z_1(q_{h,b,t})]r_{b,t} + z_0(q_{h,b,t})\mu_{S,t}$$

If there is a single customer then the transaction occurs at price y_t , if there are more than one customer then the transaction occurs at $r_{b,t}$ and if the seller does not get a customer then he obtains $\mu_{S,t}$. Given the expression for $U_{h,b,t}$ we can rewrite the profit function as follows

$$\Pi_{b,t} = 1 - z_0(q_{h,b,t}) - q_{h,b,t}U_{h,b,t} + [q_{h,b,t} - 1 + z_0(q_{h,b,t})]\mu_{B,t} + z_0(q_{h,b,t})\mu_{S,t}.$$

Now consider a fixed price store. Letting $q_{f,t} \equiv q_{h,f,t} + q_{l,f,t}$ denote the total expected demand,

both types of buyers obtain the same expected utility at the fixed price store, where

$$U_{h,f,t} = U_{l,f,t} \equiv U_{f,t} = \frac{1 - z_0(q_{f,t})}{q_{f,t}} (1 - r_{f,t}) + \frac{q_{f,t} - 1 + z_0(q_{f,t})}{q_{f,t}} \mu_{B,t}. \quad (9.15)$$

The expression is similar to $U_{h,b,t}$ except for the fact that the transaction occurs at the fixed price $r_{f,t}$ even if there is a single customer at the store. A fixed price seller's profit is equal to

$$\Pi_{f,t} = [1 - z_0(q_{f,t})] r_{f,t} + z_0(q_{f,t}) \mu_{S,t},$$

which can be rewritten as

$$\Pi_{f,t} = 1 - z_0(q_{f,t}) - q_{f,t} U_{f,t} + [q_{f,t} - 1 + z_0(q_{f,t})] \mu_{B,t} + z_0(q_{f,t}) \mu_{S,t}.$$

Characterization of the Equilibrium. Recall that non hagglers shop at fixed price stores whereas hagglers shop at both types of stores. This means that $U_{l,f,t} = \bar{U}_{l,t}$ and $U_{h,f,t} = U_{h,b,t} = \bar{U}_{h,t}$. Since $U_{h,f,t} = U_{l,f,t}$ we have $\bar{U}_{h,t} = \bar{U}_{l,t} \equiv \bar{U}_t$. A flexible seller maximizes $\Pi_{b,t}$ subject to $U_{h,b,t} = \bar{U}_t$. Substituting the constraint into the objective function, the first order condition is given by

$$z_0(q_{h,b,t}) - \bar{U}_t + [1 - z_0(q_{h,b,t})] \mu_{B,t} - z_0(q_{h,b,t}) \mu_{S,t} = 0.$$

It follows that

$$\Pi_{b,t} = 1 - z_0(q_{h,b,t}) - z_1(q_{h,b,t}) - [1 - z_0(q_{h,b,t}) - z_1(q_{h,b,t})] \mu_{B,t} + [z_0(q_{h,b,t}) + z_1(q_{h,b,t})] \mu_{S,t}.$$

Similarly a fixed price seller maximizes $\Pi_{f,t}$ subject to $U_{f,t} = \bar{U}_t$. The first order condition is given by

$$z_0(q_{f,t}) - \bar{U}_t + [1 - z_0(q_{f,t})] \mu_{B,t} - z_0(q_{f,t}) \mu_{S,t} = 0,$$

which implies that

$$\Pi_{f,t} = 1 - z_0(q_{f,t}) - z_1(q_{f,t}) - [1 - z_0(q_{f,t}) - z_1(q_{f,t})] \mu_{B,t} + [z_0(q_{f,t}) + z_1(q_{f,t})] \mu_{S,t}.$$

In equilibrium sellers must earn equal profits, i.e. $\Pi_{f,t} = \Pi_{b,t}$; thus $q_{h,b,t} = q_{f,t} = q_{h,f,t} + q_{l,f,t}$. It

follows that

$$q_{h,b,t} = \lambda_t, \quad q_{h,f,t} = \lambda_t(\varphi_{f,t}^* - \eta_t)/\varphi_{f,t}^* \quad \text{and} \quad q_{l,f,t} = \lambda_t\eta_t/\varphi_{f,t}^*,$$

where $\varphi_{f,t}^*$ denotes the equilibrium fraction of fixed price sellers. Note that, $\varphi_{f,t}^*$ is indeterminate in that any value within $[\eta_t, 1]$ satisfies the equalities above; hence, there is a continuum of equilibria where any fraction $\varphi_{f,t}^* \geq \eta_t$ of sellers compete via fixed pricing while the rest compete via flexible pricing. Notice, however, in any equilibrium, the total expected demand at each firm equals to λ_t .

Now we can obtain expressions for equilibrium prices. Combining the first order condition of flexible sellers with indifference constraint $U_{h,b,t} = \bar{U}_t$ yields

$$z_0(\lambda_t) + [1 - z_0(\lambda_t)]\mu_{B,t} - z_0(\lambda_t)\mu_{S,t} = U_{h,b,t},$$

where $U_{h,b,t}$ is given by (9.14). Solving this equality for $r_{b,t}$ yields the expression for $r_{b,t}^*$ in the body of the proposition. The equilibrium fixed price $r_{f,t}^*$ is obtained likewise. The first order condition of fixed price sellers along with the indifference constraint $U_{f,t} = \bar{U}_{h,t}$ implies

$$z_0(\lambda_t) + [1 - z_0(\lambda_t)]\mu_{B,t} - z_0(\lambda_t)\mu_{S,t} = U_{f,t}$$

where $U_{f,t}$ is given by (9.15). Solving this equality for $r_{f,t}$ yields the expression for $r_{f,t}^*$ in the body of the proposition. High type buyers negotiate if $r_{b,t}^* \geq y_t^*$, which, after substituting for $r_{b,t}^*$ and y_t^* , is equivalent to $\theta \geq \bar{\theta}_t = z_1(\lambda_t) / [1 - z_0(\lambda_t)]$. Given the expressions for $r_{f,t}^*$ and $r_{b,t}^*$ one can verify that the equilibrium payoffs are as follows $\Pi_{b,t} = \Pi_{f,t} = \pi_t$ and $U_{h,b,t} = U_{f,t} \equiv u_t$, where π_t and u_t are given in the body of the proposition.

If $\theta < \bar{\theta}_t$ then $r_{b,t}^* < y_t^*$; thus no bargaining takes place as the list price $r_{b,t}^*$ is already below the bargained price y_t^* . As in the benchmark model, in this parameter region, the model collapses to a fixed-price setting.

Proof of Conjecture 1: Players transact immediately rather than waiting.

If $p_t \in [\beta\pi_{t+1}, 1 - \beta u_{t+1}]$ then sellers and buyers would be willing to trade in the second round instead of waiting for the next period. Indeed if $p_t \geq \beta\pi_{t+1}$ then the seller is better off transacting at p_t instead of waiting for period $t + 1$ and obtaining $\beta\pi_{t+1}$. Similarly if $p_t \leq 1 - \beta u_{t+1}$ then the buyer is better off purchasing instead of waiting for the next period and getting βu_{t+1} .

Now consider the first round. It is straightforward to show that if $\theta \geq \bar{\theta}_t$ then $r_{b,t}^* > r_{f,t}^* > y_t^*$. From a sellers' perspective the worst case scenario is transacting at y_t^* , which is the lowest price.

Similarly for a buyer the worst case scenario is purchasing at $r_{b,t}^*$. If they agree to transact under these worst case scenarios then they would agree to transact under more favorable prices.

Consider a buyer who contemplates buying at $r_{b,t}^*$. He would transact if $1 - r_{b,t}^* \geq \mu_{B,t}$, i.e. if his immediate surplus $1 - r_{b,t}^*$ exceeds his outside option $\mu_{B,t}$ associated with walking away at the end of round 1. Basic algebra reveals that this inequality is satisfied if

$$\frac{z_1(\lambda_t)(1-\theta)}{1-z_0(\lambda_t)-z_1(\lambda_t)} (1 - \mu_{B,t} - \mu_{S,t}) > 0.$$

The left hand side is positive; hence the inequality holds. Now consider a seller, whose worst case scenario is selling at y_t^* . The seller transacts if $y_t^* \geq \mu_{S,t}$, i.e. if his immediate surplus y_t^* exceeds his outside option $\mu_{S,t}$ associated with walking away at the end of round 1. Basic algebra reveals that this inequality is satisfied if

$$(1 - \theta) (1 - \mu_{B,t} - \mu_{S,t}) > 0.$$

Again, both expressions are positive; hence the seller, too, is willing to transact immediately.

Proof of Conjecture 2. Low types strictly prefer fixed price stores and high types are indifferent.

A low type's expected utility at a best offer store is given by

$$U_{l,b,t} = \frac{1-z_0(\lambda_t)}{\lambda_t} (1 - r_{b,t}^*) + \frac{\lambda_t - 1 + z_0(\lambda_t)}{\lambda_t} \mu_{B,t}$$

Substituting for $r_{b,t}^*$ it is easy to show if $\theta > \theta_t$ then $U_{l,b,t} < u_t$; confirming indeed that low types are better off staying away from best offer stores. To show that high types are indifferent between fixed and flexible stores we need to show that along the equilibrium path we have $U_{h,b,t} = U_{f,t}$. Substituting $r_{b,t}^*$ and $r_{f,t}^*$ it is a matter of basic algebra to verify that indeed this equality holds, confirming the validity of the conjecture. This completes the proof of the proposition. ■

Matching Function. Here we show how $\omega_{S,t}$ and $\omega_{B,t}$ may be derived from the fundamentals of the model if one assumes that second round meetings are governed by "urn-ball matching", where all unmatched buyers (balls) and all unmatched sellers (urns) enter into a random matching process (see Petrongolo and Pissarides (2001)). Matching frictions are due to the random nature of the process—some urns receive several balls and others none. Given the process one can pin down the probabilities $\omega_{B,t}$ and $\omega_{S,t}$ as follows. Along the equilibrium outlined in the Proposition,

at the end of the first round $s_t(1 - z_0(\lambda_t))$ sellers are matched. Players transact immediately and each transaction takes one buyer and one seller. This implies that $b_t - s_t(1 - z_0(\lambda_t))$ buyers and $s_t z_0(\lambda_t)$ sellers are not matched.¹⁸ The buyer-seller ratio in the second round is equal to

$$\lambda'_t = \frac{b_t - s_t(1 - z_0(\lambda_t))}{s_t z_0(\lambda_t)} = \frac{\lambda_t - 1 + z_0(\lambda_t)}{z_0(\lambda_t)}.$$

The second equation follows from the fact that $s_t = b_t \lambda_t$. An unmatched seller's chance of being able to transact, $\omega_{S,t}$, is equal to the probability of meeting at least a buyer, i.e.

$$\omega_{S,t} = 1 - z_0(\lambda'_t).$$

Similarly, an unmatched buyer's chance of transacting in the second round is equal to

$$\omega_{B,t} = \sum_{n=0}^{\infty} \frac{z_n(\lambda'_t)}{n+1} = \frac{1 - z_0(\lambda'_t)}{\lambda'_t}.$$

With probability $z_n(\lambda'_t)$ he encounters $n = 0, 1, 2, \dots$ other buyers there (recall that due to randomness of the process a seller may get more than one buyer), in which case he has a probability of $\frac{1}{n+1}$ obtaining the item (each buyer has an equal chance). The second equality follows from the facts that $z_{n+1}(x) = \frac{x z_n(x)}{n+1}$ and that $\sum_{n=0}^{\infty} z_n(x) = 1$. Notice that if $\lambda'_t > 1$ then $\omega_{S,t} > \omega_{B,t}$, i.e. if there are more buyers in the pool than sellers, then a seller is more likely to meet a trading partner than a buyer. The opposite is true if $\lambda'_t < 1$.

9.4 Sellers' Implementation Cost of Bargaining

In our model firms do not incur any implementation costs to sell via bargaining. However given the results on how the nature of equilibria respond to ε we can predict what would happen if sellers were to incur such a cost. Recall that if $\varepsilon = 0$ then both pricing rules are payoff equivalent and sellers are indifferent to pick either fixed pricing or flexible pricing. If, however, ε turns negative then the payoff equivalence breaks down and fixed pricing emerges as the unique outcome. From sellers' point of view the negative ε is an indirect cost. It is incurred by buyers, but nevertheless it bleeds into the sellers' profit functions and thereby induces them to switch to fixed pricing. The

¹⁸The payoff in the second round is the same for all buyers, thus we do not need to keep track of high and low types during this process. As an aside, note that along the equilibrium in Proposition 8 high and low types trade at the same rate; thus, the ratio of high types to low types remains intact among unmatched buyers. (See the analysis in the main text for a formal proof for this argument.)

implication is that if an indirect cost can disturb the payoff equivalence between fixed and flexible pricing then a direct cost will result in the same outcome, i.e. introducing a cost of implementing bargaining into the setting $\varepsilon = 0$ would cause flexible sellers to earn less, and thereby, lead to a fixed price equilibrium. Needless to say, introducing such a cost into a setting with $\varepsilon < 0$ will only reinforce the fixed price outcome.

If, however, $\varepsilon > 0$ then the outcome is less clear because along Eq-FS sellers are able to convert the positive ε into higher prices and, thereby, earn higher profits compared to a fixed price equilibrium. So, if one inserts an implementation cost into the framework with $\varepsilon > 0$ then whether or not sellers would revert back to fixed pricing depends on how this cost compares with the difference in profits. If the cost is prohibitively large then we would expect a fixed price equilibrium to emerge and if the cost is sufficiently small then Eq-FS should survive, albeit with fewer flexible stores (compared to the benchmark model with no cost).

9.5 Game with Infinite Horizon

In our model the market runs for a finite number of periods, i.e. $T < \infty$. Under this specification one can solve the model recursively by substituting the terminal payoffs $u_{T+1} = \pi_{T+1} = 0$ into the equilibrium conditions to obtain payoffs for period T , which then can be substituted to obtain payoffs for period $T - 1$, and so on. The method is straightforward, but more importantly, one does not need to worry about how market demand fluctuates over time, driven by the tuple $\{b_t^{new}, s_t^{new}, \eta_t^{new}\}_{t=2}^T$.

If $T = \infty$ then one can prove existence of equilibrium and analytically characterize a solution if the market exhibits some cyclicity, i.e. if agents face the same outlook, say, every k periods. The cyclical nature of the model would allow us to prove analogous versions of Propositions 1, 2 and 3 using induction and then, again, exploiting cyclicity we can pin down equilibrium payoffs and prices. As an example focus on the setting with $\varepsilon = 0$ and consider the simplest possible scenario where the environment is fully stationary in that outgoing agents are replaced by incoming agents one for one. With perfect replacement the number of buyers and sellers, and therefore the expected demand λ_t , remains constant at all times. Since players face the same market outlook irrespective of the calendar time, equilibrium payoffs π_t and u_t , and thereby, equilibrium prices are also time independent, which allows us to solve the model analytically. (To prove existence of the equilibrium one needs to virtually repeat the steps outlined in the proof of Proposition 1). Dropping the time

subscripts from equations (17) and (18), we have

$$\pi = 1 - \beta u - [z_0(\lambda) + z_1(\lambda)](1 - \beta u - \beta\pi) \quad \text{and} \quad u = z_0(\lambda)[1 - \beta u - \beta\pi] + \beta u.$$

This is a simple system with two equations and two unknowns (π and u), which can be solved easily. Once π and u are pinned down, the equilibrium prices and probabilities readily follow. This solution concept is rather straightforward, but as λ_t starts to fluctuate the system of equations grows rapidly. For instance if λ_t is high in even periods and low in odd periods then we would have a system of four equations and four unknowns ($\pi_{odd}, \pi_{even}, u_{odd}, u_{even}$) to deal with. In general if the cycles lasts k periods then one needs to solve a system of $2k$ equations and unknowns. Needless to say, as k grows large an analytic solution becomes elusive.

If the model cannot be solved analytically, then one can fix T at some large value and pick some arbitrary values for terminal payoffs u_{T+1} and π_{T+1} and solve the model via the aforementioned recursive method. The solution will be accurate for $t \ll T$ because, due to discounting, the impact of terminal payoffs vanishes if t is sufficiently far away from T . Our simulations seemed to confirm this insight. We fixed $T = 360$, $\beta = 0.95$ and ran simulations for a number of arbitrary values of $u_{T+1} \in [0, 1]$ and $\pi_{T+1} \in [0, 1]$ and saw no impact of the terminal payoffs on equilibrium objects (prices and payoffs) for $t < 350$ or so. Needless to say, the accuracy can be extended by picking a larger T or a smaller β .

9.6 Endogenous separation of hagglers and non-hagglers

Suppose that buyers are identical in terms of their bargaining skills but they differ in terms of their enjoyment for the bargaining experience, proxied by the parameter ε . Imagine that ε varies in an interval $[\underline{\varepsilon}, \bar{\varepsilon}]$, where $\underline{\varepsilon} < 0$ and $\bar{\varepsilon} \geq 0$, and that buyers are divided into N separate groups, where the fraction of group i is given by η_i , with $\sum_{i=1}^N \eta_i = 1$. Furthermore, suppose that group 1 has the lowest ε and group N has the highest, that is

$$\underline{\varepsilon} \equiv \varepsilon_1 < \varepsilon_2 < \dots < \varepsilon_N \equiv \bar{\varepsilon}.$$

The following proposition presents the main result along this variation.

Proposition 9 *The nature of the equilibria depends on the upper bound ε_N . There are two cases:*

1. *If $\varepsilon_N = 0$ then there exists a continuum of payoff-equivalent equilibria, where an indeterminate fraction $\varphi^* \geq 1 - \eta_N$ of firms trade via fixed pricing and remaining firms trade via flexible pricing. The equilibria exhibit partial segmentation in customer demographics: types 1, 2, ..., $N - 1$ shop exclusively at fixed price firms whereas type N customers shop anywhere.*

2. *If $\varepsilon_N > 0$ and if the gap $\varepsilon_N - \varepsilon_{N-1}$ is sufficiently large then there exists an equilibrium where a fraction $\varphi^* < 1 - \eta_N$ of firms trade via fixed pricing and remaining firms trade via flexible pricing. The equilibrium exhibits full segmentation of customers: types 1, 2, ..., $N - 1$ shop exclusively at fixed price firms whereas type N customers shop at flexible firms.*

The equilibrium in item 1 is largely identical to Eq-PS in the main text, which emerges when $\varepsilon = 0$. Similarly, the equilibrium in item 2 is similar to Eq-FS, which exists when $\varepsilon > 0$. The similarities indicate that the results of the main text remain robust under this variation; however there are some subtleties that we need to point out.

First, sellers designate the bargaining deals only for the most enthusiastic type (type N), which is why in equilibrium only type N customers hunt for such deals while everyone else shops at fixed price venues. To see why, note that unlike the main text, lower types in here can negotiate a deal if they are alone at a flexible store. But, if they are not alone, then they must pay the inflated list price. As hinted above, the list price is designated for the most enthusiastic type, so it is too high for everyone else. For lower types, the enjoyment they might get from negotiating a deal (proxied by their ε) is simply not enough to counter-balance the prospect of paying such a high price. Thus, lower types are better off shopping at fixed price venues.

Second, if $\varepsilon_N > 0$ then the existence of the equilibrium hinges on the condition of there being a large enough gap between ε_N and ε_{N-1} . This is to ensure that type $N - 1$ stays away from flexible stores (if type $N - 1$ stays away, then all other types will stay away). In the main text we did not need such a condition—Eq'm FS would go through as long as ε was positive. The reason is that in the main text low types were not able to bargain anyway (due to the lack of their bargaining skills), so we did not need a separate condition on their ε to keep them away from flexible stores.

Third, unlike the main text, customers in this setting are identical in their bargaining skills, yet the division of hagglers vs. non-hagglers still emerges as an endogenous phenomenon. Indeed, in equilibrium customers who enjoy bargaining the most, shop at flexible stores and haggle over the sale price whereas remaining customers shop at fixed price stores and do not haggle at all. This observation suggests that one can potentially do away with the exogenous haggler vs. non-haggler distinction in the main text, and instead start from a primitive of heterogeneous ε and still obtain qualitatively similar results.

The endogenous separation of hagglers vs. non-hagglers is indeed appealing. However, from an analytical point of view, the setup in the main text with the exogenous distinction of hagglers vs. non hagglers, has its advantages. First, the setup in the main text allows us to prove existence and uniqueness of the equilibrium. In here, unfortunately, the proof of uniqueness remains elusive. (The proposition above claims existence but it does not rule out other scenarios.)

Second, the existence of equilibrium along the current variation hinges on the condition of there being a large gap between ε_N and ε_{N-1} . If we solve this model in a dynamic setting, then there will be T similar and recursively related conditions. Analytically characterizing that many conditions would be impractical and, therefore, it will be down to numerical simulations to confirm whether an equilibrium exists for a given parameter set.

The rest of this section is devoted to the proof of the proposition. (We focus on a one shot game, where the search market operates only once, i.e. $T = 1$.)

Proof of Proposition 9. The proof consists of three steps.

Step 1. Preliminaries. Let $q_{i,m}$ denote the expected demand consisting of type i buyers at a store trading via rule m and let

$$q_m \equiv \sum_{i=1}^N q_{i,m}, \quad \text{where } m = f, b \text{ and } i = 1, 2, \dots, N$$

denote the total demand. The expected utility of a type i buyer at a flexible store is given by

$$U_{i,b} = z_0(q_b)(1 - y + \varepsilon_i) + \sum_{n=1}^{\infty} \frac{z_n(q_b)}{n+1} (1 - r_b),$$

where y is the bargained price and r_b is the flexible list price. Notice that since buyers are equally skilled in bargaining, they negotiate the same price y when dealing with a flexible seller. For the purpose of the proposition we do not need a closed form solution for y —we simply assume that buyers' bargaining power, which pins down the bargained price, is high enough for them to take advantage of flexible deals (else, the model collapses to a fixed price setting.)

Basic algebra reveals that

$$U_{i,b} = U_{N,b} - z_0(q_b)(\varepsilon_N - \varepsilon_i) \text{ for } i = 1, \dots, N$$

Since $\varepsilon_{i+1} > \varepsilon_i$ we have $U_{i+1,b} > U_{i,b}$. Now turn to sellers. Flexible sellers expect to earn

$$\Pi_b = [1 - z_0(q_b) - z_1(q_b)]r_b + z_1(q_b)y.$$

Basic algebra reveals that

$$\Pi_b = 1 - z_0(q_b) - \sum_{i=1}^N q_{i,b}U_{i,b} + z_0(q_b) \sum_{i=1}^N q_{i,b}\varepsilon_i.$$

At fixed price stores, things are the same as in the main text, i.e.

$$U_f = \frac{1 - z_0(q_f)}{q_f} (1 - r_f) \quad \text{and} \quad \Pi_f = 1 - z_0(q_f) - q_f U_f.$$

Step 2. Case: $\varepsilon_N = 0$ (item 1 in the Proposition)

Consider fixed price firms. Along our conjecture (to be verified later) types 1 through N shop at at fixed price firms, i.e. $q_{i,f} > 0$ for all $i = 1, \dots, N$. Recall that all buyers earn the same expected payoff at fixed price firms, that is $U_{i,f} = U_f$ for all i . The fixed price firm solves

$$\max_{q_f \in \mathbb{R}_+} 1 - z_0(q_f) - q_f U_f \quad \text{s.t.} \quad U_f = \bar{U},$$

where \bar{U} is the market utility offered by the fixed price firm. The FOC is given by $\bar{U} = z_0(q_f)$; thus the firm earns

$$\Pi_f = 1 - z_0(q_f) - z_1(q_f).$$

Per our conjecture, flexible firms attract type N only, i.e. $q_{N,b} > 0$ and $q_{i,b} = 0$ for $i = 1, \dots, N - 1$. Substituting these into the expression for Π_b yields

$$\Pi_b = 1 - z_0(q_{N,b}) - q_{N,b}U_{N,b}.$$

The problem of a flexible seller is given by

$$\max_{q_{N,b} \in \mathbb{R}_+} 1 - z_0(q_{N,b}) - q_{N,b}U_{N,b} \quad \text{s.t.} \quad U_{N,b} = \bar{U}_N,$$

where \bar{U}_N is the market utility promised to type N customers. The FOC is given by $\bar{U}_N = z_0(q_{N,b})$; thus, the firm earns

$$\Pi_b = 1 - z_0(q_{N,b}) - z_1(q_{N,b}).$$

In equilibrium, sellers must earn equal profits. The equality $\Pi_f = \Pi_b$ yields $q_{N,b} = q_f$. Let φ denote the fraction of fixed price firms and recall that η_i denotes the fraction of type i buyers. The feasibility condition, analogous to the one in the main text requires

$$\varphi q_{i,f} + (1 - \varphi)q_{i,b} = \eta_i \lambda \quad \text{for } i = 1, \dots, N$$

Noting that $q_{i,b} = 0$ for $i = 1..N - 1$, we have

$$q_{i,f} = \frac{\eta_i}{\varphi} \lambda, \quad \text{for } i = 1, \dots, N - 1.$$

We know $q_f = q_{N,b}$. In addition, recall that $q_f = \sum_{i=1}^N q_{i,f}$. Thus

$$\frac{\lambda}{\varphi} \sum_{n=1}^{N-1} \eta_n + q_{N,f} = q_{N,b} \Rightarrow \frac{\lambda}{\varphi} (1 - \eta_N) + q_{N,f} = q_{N,b}.$$

For type N , we have

$$\varphi q_{N,f} + (1 - \varphi)q_{N,b} = \eta_N \lambda.$$

Substituting for $q_{N,b}$ from above

$$q_{N,f} = \frac{\lambda}{\varphi} [\varphi - (1 - \eta_N)].$$

It follows that $q_{N,b} = q_f = \lambda$. Note that, the equilibrium value of φ , denoted by φ^* , is indeterminate and can take any value within $[1 - \eta_N, 1]$, hence, there is a continuum of equilibria where an indeterminate fraction $\varphi^* \geq 1 - \eta_N$ of sellers compete via fixed pricing while the rest compete via flexible pricing. Notice, however, in any equilibrium, the total expected demand at each firm equals to λ . Along this equilibria buyers and sellers earn

$$U_f = U_{N,b} = z_0(\lambda) \quad \text{and} \quad \Pi_f = \Pi_b = 1 - z_0(\lambda) - z_1(\lambda).$$

The final task is to verify the conjectures made above. First, the fact that $U_f = U_{N,b} = z_0(\lambda)$ implies that type N customers are indifferent between fixed and flexible stores; thus, as we conjectured, they can shop at both types of stores. The second conjecture pertains types $1, \dots, N - 1$ staying away from flexible stores. At fixed price stores they earn $U_f = z_0(\lambda)$. If they were to visit flexible stores, they would earn

$$U_{i,b} = U_{N,b} - z_0(\lambda) (\varepsilon_N - \varepsilon_i),$$

which is less than U_f because $\varepsilon_N > \varepsilon_i$ and $U_{N,b} = U_f$; hence they are justified to stay away. This completes the proof of item 1 in the proposition.

Step 3. Case: $\varepsilon_N > 0$ (item 2 in the Proposition)

Consider fixed price firms. Along our conjecture (to be verified later) types 1 through $N - 1$ shop at fixed price firms, i.e. $q_{i,f} > 0$ for all $i = 1, \dots, N - 1$. Type N , on the other hand, stays away, i.e. $q_{N,f} = 0$; thus $q_f = \sum_{i=1}^{N-1} q_{i,f}$. Recall that all buyers earn the same expected payoff at fixed price firms, that is $U_{i,f} = U_f$ for all $i = 1, \dots, N - 1$. A fixed price firm solves

$$\max_{q_f \in \mathbb{R}_+} 1 - z_0(q_f) - q_f U_f \quad \text{s.t.} \quad U_f = \bar{U},$$

where \bar{U} is the market utility of all types but type N . The FOC is given by $\bar{U} = z_0(q_f)$; thus the firm earns

$$\Pi_f = 1 - z_0(q_f) - z_1(q_f).$$

Per our conjecture, flexible firms attract type N only, i.e. $q_{N,b} > 0$ whereas $q_{i,b} = 0$ for $i = 1, \dots, N-1$. Substituting these equalities into the expression for Π_b (and noting that $\varepsilon_N > 0$), a flexible firm's expected profit becomes

$$\Pi_b = 1 - z_0(q_{N,b}) - q_{N,b}U_{N,b} + z_0(q_{N,b})q_{N,b}\varepsilon_N.$$

The flexible seller solves

$$\max_{q_{N,b} \in \mathbb{R}_+} 1 - z_0(q_{N,b}) - q_{N,b}U_{N,b} + z_0(q_{N,b})q_{N,b}\varepsilon_N \quad \text{s.t. } U_{N,b} = \bar{U}_N,$$

where \bar{U}_N is the market utility of type N customers. The FOC is given by

$$\bar{U}_N = z_0(q_{N,b}) + [z_0(q_{N,b}) - z_1(q_{N,b})]\varepsilon_N$$

thus, the firm earns

$$\Pi_b = 1 - z_0(q_{N,b}) - z_1(q_{N,b}) + q_{N,b}z_1(q_{N,b})\varepsilon_N.$$

Let φ denote the fraction of fixed price firms and recall that η_i denotes the fraction of type i buyers.

Feasibility requires

$$\varphi q_{i,f} + (1 - \varphi)q_{i,b} = \eta_i \lambda \quad \text{for } i = 1, \dots, N.$$

Noting that $q_{i,b} = 0$ for $i = 1..N - 1$, we have

$$q_{i,f} = \frac{\eta_i}{\varphi} \lambda, \quad \text{for } i = 1, \dots, N - 1 \quad \Rightarrow \quad q_f = \sum_{i=1}^{N-1} q_{i,f} = \frac{1 - \eta_N}{\varphi} \lambda$$

In addition, since $q_{N,f} = 0$ we have

$$q_{N,b} = \frac{\eta_N}{1 - \varphi} \lambda.$$

In equilibrium sellers must earn equal profits, i.e. $\Delta(\varphi) \equiv \Pi_b - \Pi_f = 0$. Note that Π_b rises in $q_{N,b}$, which itself rises in φ and that Π_f rises in q_f , which in turn falls in φ . It follows that Δ rises in φ .

In addition, note that $\Delta(1 - \eta_N) > 0$ since $\varepsilon_N > 0$ and that $\Delta(0) < 0$. The Intermediate Value Theorem guarantees existence of a unique $\varphi^* < 1 - \eta_N$ such that $\Delta(\varphi^*) = 0$.

Since $\varphi^* < 1 - \eta_N$ we have $q_{N,b} < \lambda < q_f$. Solving $U_{N,b} = z_0(q_{N,b}) + [z_0(q_{N,b}) - z_1(q_{N,b})] \varepsilon_N$ and $U_f = z_0(q_f)$ for r_f and r_b yields equilibrium list prices:

$$r_f^* = 1 - \frac{z_1(q_f)}{1 - z_0(q_f)} \quad \text{and} \quad r_b^* = 1 - \frac{z_1(q_{N,b}) y - q_{N,b} z_1(q_{N,b}) \varepsilon_N}{1 - z_0(q_{N,b}) - z_1(q_{N,b})}$$

Observe that, as in the main text, the flexible list price r_b^* increases in ε_N , which indicates that flexible sellers pass type N 's enthusiasm about getting a deal on to their prices (which explains why lower types may want to stay away from a flexible store).

To complete the proof, we need to verify the conjectures made earlier. First, we need to show that, at the margin, type N buyers stay away from fixed price stores. At flexible firms they earn

$$U_{N,b} = z_0(q_{N,b}) + [z_0(q_{N,b}) - z_1(q_{N,b})] \varepsilon_N.$$

At fixed price firms they earn $U_f = z_0(q_f)$. The equal profit condition gives us

$$\varepsilon_N = \frac{z_0(q_{N,b}) + z_1(q_{N,b}) - z_0(q_f) - z_1(q_f)}{q_{N,b} z_1(q_{N,b})}.$$

Our conjecture would hold if $U_{N,b} > U_f$, i.e. if

$$(1 - q_{N,b}) \varepsilon_N > \frac{z_0(q_f) - z_0(q_{N,b})}{z_0(q_{N,b})}.$$

Recall that $q_f > q_{N,b}$. Thus $z_0(q_f) < z_0(q_{N,b})$, implying that the right hand side is negative. If $1 - q_{N,b} > 0$ then we are done; so suppose that $1 - q_{N,b} < 0$. The inequality holds if

$$\varepsilon_N < \frac{z_0(q_f) - z_0(q_{N,b})}{z_0(q_{N,b})(1 - q_{N,b})}.$$

Substituting for ε from above, we need

$$\frac{z_0(q_{N,b})}{z_0(q_f)} > q_{N,b}^2 + 1 - q_{N,b} + q_f - q_f q_{N,b}$$

We know $q_{N,b} < q_f$; thus a sufficient condition is

$$\frac{z_0(q_{N,b})}{z_0(q_f)} > 1 - q_{N,b} + q_f \Leftrightarrow e^{q_f - q_{N,b}} > 1 - (q_f - q_{N,b}).$$

Let $x \equiv q_f - q_{N,b} > 0$. The question is whether $e^x + x - 1 > 0$. The expression on the left is positive for all $x > 0$; hence the inequality holds. This verifies the conjecture that type N buyers strictly prefer to shop at flexible stores.

The second conjecture pertains remaining types, i.e. we need to show that types $1, \dots, N - 1$, would not want to shop at flexible stores. For that we need $U_f > U_{i,b}$. Recall that

$$U_{i,b} = U_{N,b} - z_0(q_{N,b})(\varepsilon_N - \varepsilon_i).$$

Thus, we need

$$U_f > U_{N,b} - z_0(q_{N,b})(\varepsilon_N - \varepsilon_i).$$

We already know that $U_{N,b} > U_f$. Thus, for the above inequality to go through, the gap $\varepsilon_N - \varepsilon_i$ must be sufficiently large. Since ε_i rises in i , the gap is the smallest for type $N - 1$ customers, so pick $i = N - 1$. In what follows we will show that if ε_{N-1} is smaller than a threshold, which itself is smaller than ε_N , then the inequality holds. To start, recall that

$$U_{N,b} = z_0(q_{N,b}) + [z_0(q_{N,b}) - z_1(q_{N,b})]\varepsilon_N \text{ and } U_f = z_0(q_f)$$

and keep in mind that $q_f > q_{N,b}$, so $z_0(q_f) < z_0(q_{N,b})$. The inequality holds if

$$z_1(q_{N,b})\varepsilon_N - z_0(q_{N,b})\varepsilon_{N-1} > z_0(q_{N,b}) - z_0(q_f)$$

Note that: (i) if $\varepsilon_{N-1} = \varepsilon_N$ then the inequality is the other way around (from above) and (ii) if $\varepsilon_{N-1} = 0$ then the inequality holds (this step can be proved by substituting for ε_N and going through the same steps as above). Thus, by the Intermediate Value Theorem, there exists some $\bar{\varepsilon}_{N-1} \in (0, \varepsilon_N)$, such that if $\varepsilon_{N-1} < \bar{\varepsilon}_{N-1}$ then the inequality holds. This verifies the second conjecture and completes the proof. ■

References

Petrongolo, B. and Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, 39(2), 390-431.