An Investigation of Student Learning of Motion of Connected Particles in an Interactive Physics Environment

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[redacted]

I, Yogesswarnath SANMUKHIYA, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

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Abstract

This research explores how sixth form students learn about motion of connected particles (mcp) with *Interactive Physics* (IP). In particular, our interest focuses on students' use of intuitive knowledge and its relationship to the development of new knowledge. This research involved the construction of a microworld, consisting of a sequence of carefully crafted tasks based on IP. The aim is to foster students' insights into several ideas related to mcp, as well as offering the researcher a 'window' into the students' thinking about mcp. The theoretical approach is adapted from diSessa's knowledge-in-pieces (k-i-p), an approach which emerges as the most appropriate for illuminating students' evolving ideas.

The corpus of data is based on case studies of 10 sixth form students, aged between 17 and 19, working individually with the microworld. These students were engaged in think-aloud, task-based interviews, which were structured according to a "Predict-Interact-Reflect" strategy. In the Prediction Phase, the students made predictions about a particular system and were encouraged to explain the rationales for their predictions. In the Interaction Phase, the students were asked to observe any discrepancies, if any, between their predictions and the simulations. Finally, in the Reflection Phase the students were asked to explain these discrepancies. From these case studies, episodes were identified which then served as the basis for analysis and discussion.

The findings validate the k-i-p approach in relation to the intuitive ideas that students brought to the tasks, how students constructed new pieces of knowledge and how IP shaped these pieces. They also indicate that the generation of new knowledge, a non-linear and time-consuming process, occurred when students extracted relevant information from the crucial region of the simulation at the appropriate time, made sense of it and integrated it into their mental models.

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Chapter 1

Aims and Rationale of the Study

1.1 Introduction

One of the distinguishing features of the General Certificate of Education (GCE) Advanced Level Mathematics syllabus in British, Mauritian and many other countries' secondary schools has long been the inclusion of Newtonian mechanics. GCE A-Level Mathematics is normally taught to students of 16-19 years old who wish to pursue any maths-related course at tertiary level. It consists of pure mathematics and options from Newtonian mechanics, statistics and possibly discrete mathematics. Over the decades, though the content of Newtonian mechanics has undergone profound changes, its relevance in the GCE A-Level Mathematics syllabus cannot be undermined.

Nevertheless, it should be pointed out that mechanics also exists as a core part in physics, be it at IGCSE, GCSE and GCE Ordinary or Advanced Level. According to Truesdell (1968), it is only since the 1960s that mechanics is grouped within physics as a science of empirical origin. In the 'Age of Reason', it was not so regarded because mechanics was not discovered by physicists. It was created by a handful of 'geometers' and 'algebrists', who strove to put into mathematical form laws governing the daily physical experience. But we should not think that, content-wise, mechanics in physics differs from that in mathematics - in theory, they are almost the same. It appears that the way in which it is learnt in both subjects that makes the difference.

To date, mechanics not only provides an area of mathematics in the sixth form but also shows the important stages in real problem solving of formulating a mathematical model and associated mathematical problem, solving the mathematical problem, and interpreting and validating the model (Roper, 1994). Thus, the importance and value of mechanics lies not only in the elegance of its mathematical structure but in its success in aiding the understanding of real phenomena (Kline, 1959; Mathematical Association, 1965).

However, though mechanics is a subject with a few key ideas, these ideas take a long time for most students to learn and apply in new situations. This is because mechanics is a subject to which children come with more intuitive knowledge - from sensory experience - than to any other school subject. On-going research has demonstrated that such knowledge can become an obstacle for the adequate understanding of mechanics. Though students may be quite capable of obtaining high marks on stereotype questions, they are unable to cope with more searching questions. This is accounted for by the fact that many students have pre-instructional ideas and still hold incorrect models after conventional teaching in mechanics (Gill & Wright, 1994). I argue that such situation arises because conventional and even some innovative teaching strategies have failed to empower students to connect their intuition to formal mechanics.

A search in the literature reveals that a lot of research undertaken in mechanics has been around students' conceptions of mechanical concepts (see, for example, Clement, 1982; Gilbert et al., 1982; Graham & Berry, 1990, 1992). However, there is a handful number of articles about how students learn and understand the concepts of mechanics at sixth form level mathematics. Since 1985, especially due to the formation of the Mechanics in Action Project, there has been a series of useful books and articles for the teacher in the classroom, for example, Savage & Williams (1990). Apart from studies carried out by a few people such as Burns & Smart (1988), Graham & Berry (1996, 1997), Jagger (1994) and Roper (1985), very little research into the learning problems experienced by sixth form students has been done by mathematics educators; but much has been accomplished by physics educators in the US (McDermott et al., 1994; Sherin, 2006; White, 1983).

Interestingly, over the years several innovative strategies have been designed to improve mechanics instruction at GCE A-Level Mathematics. Yet, the popularity of mechanics still continues to decline, implying that sixth form students continue to find it difficult to learn (Kitchen et al., 1997; Rowlands, 2008). This definitely raises the issue of why most students still find mechanics difficult to learn. I believe that some light can be shed on the 'why' by tracing the evolution of student learning mechanics. And this can be achieved by capturing moments of conceptual change. It is imperative to note that in the past most research studies have concentrated on the before and after snapshots to investigate the occurrence of conceptual change. Following in the footsteps of diSessa (2002), I also find that such studies lack theoretical accountability. I argue through the following example – say a problem on fraction is set to two students, A and B.

Problem: Reduce $\frac{16}{64}$.

STUDENT	***STUDENT'S SOLUTION***	***STUDENT THINKING PROCESS***
Student A	$\frac{16}{64}=\frac{1}{4}$	16 is the common factor. So 16 ÷ 16 = 1 and 64 ÷ 16 = 4
Student B	$\frac{16}{64}=\frac{1}{4}$	6 is a common number in the numerator and denominator. So it can be cancelled.

Table 1.1: Two students' thinking processes in reducing a fraction.

The above example clearly demonstrates that though the final answer turns out to be the same for both students, the actual thinking process that is going on in both students is entirely different. As 'before/after' studies provide no detailed information about thinking processes, their results are of limited importance only. Thus, learning in such studies can be viewed as a black box – by which I mean to suggest that the thought processes involved remain hidden to the researcher.

On this basis, I argue that when it comes to mechanics, it is not appropriate to use the 'before/after' model to evaluate student learning. This is because such a model does not enable the researcher to focus on how the student's intuitive knowledge helps or hinders the construction of formal mechanics. It can only point out that there is shift in student thinking, but it cannot describe it. According to Mayer (2002), understanding how conceptual change works would make ‘important contributions both to learning theory and to educational practice’. Thus, outcomes and implications of prospective conceptual change studies largely depend on the mechanisms and methods adopted to examine conceptual change.

A third and final issue of interest to this study is the thoughtful use of computers in the teaching and learning of mathematics. There has been extensive research in finding out how children interact with computers in different areas of mathematics. It is also widely believed that it is possible to bring conceptual change in a computer-based environment (see, for example, Hennessy et al., 1995; Richards et al., 1992; Wiser & Amin, 2002). From the mechanics viewpoint there have been several studies involving computational tools to encourage and motivate students to learn. However, there is practically no research on how sixth form students make connections between their intuitive ideas and new knowledge when they are learning mechanics in computer-based settings. This study attempts to carry out such an investigation.

1.2 Issues of the Study

This section will give an account of the issues related to the study. While § 1.2.1 gives an account of teaching and learning mechanics relevant to the study, the relevance of using an appropriate model to analyse conceptual change in mechanics is elaborated in § 1.2.2. Finally, § 1.2.3 concludes with a discussion of the role that computers may play in both helping students learn mechanics and providing a window on student learning of mechanics in such settings.

1.2.1 Teaching and Learning Mechanics

Since mechanics is still an established and important part of the curriculum at GCE A-Level Mathematics (Kitchen et al., 1997), I believe that at least some part of mathematics education should be concerned with developing some 'effective' way of teaching and learning it. From this position it seems important that one should seek an understanding of the current teaching and learning practice of mechanics. It is only through such study that we, mathematical educators, may analyse the negative aspects that conventional teaching and learning of mechanics generates and from this we may begin to see ways which may rekindle an enthusiasm in mechanics learning in schools.

The PencilMathematics (see, for example, Noss & Hoyles, 1996) of solving a mechanics problem usually consists of four main steps (Stevenson, 1998, 1999). The first step is always to draw a diagram of the system being modelled. The second step is to add relevant information on the diagram - a dot to represent the object and arrows to represent all the forces acting on the particle. The third step, which is the purpose of the diagram, is to help one write down the scalar equations of motions from the forces. Finally, an appropriate mathematical method is employed to solve the equations, using any given information about initial conditions, and the resulting solution is compared with the original system to check its validity. For simplicity, Figure 1.1 shows the steps involved as a linear process, although, in practice, there may be several iterations of the process between the various elements.

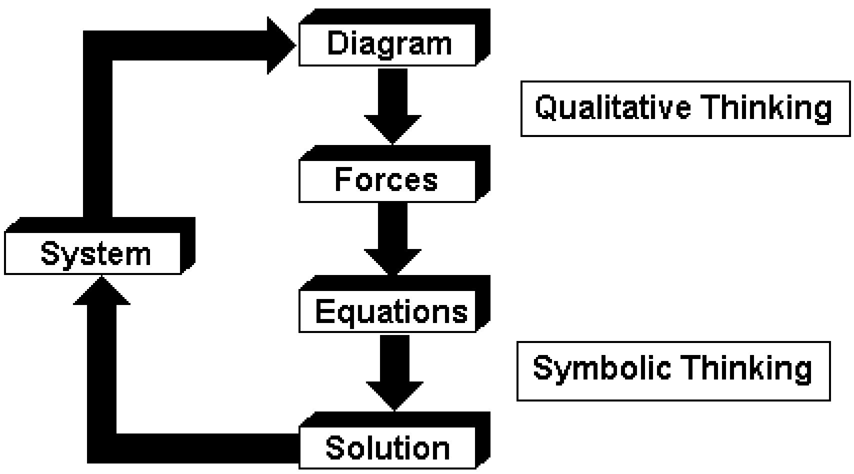


Figure 1.1 (from Stevenson, 1998): Steps involved to solve a mechanics problem.

From Figure 1.1, it would appear that the traditional setup of solving a problem in mechanics is a linear process and must involve two types of thinking: qualitative thinking and symbolic thinking.

- Qualitative thinking about a mechanical system associated with creating a diagram and analysing the forces acting on the system.
- Symbolic thinking concerned with translating the analysis into an algebraic form, so that a set of equations can be of created and solved analytically.

For a very long time the teaching of mechanics has been guided by providing examples for students to emulate. After drawing a diagram, several mathematical equations are written down from which a numerical solution is obtained by tedious calculation. This may explain why students most often believe that selection of the correct formulas is the key factor in problem solving, forcing students to make sense of mechanical principles by inference and adopt a rote-behaviour strategy. It can be seen that in conventional teaching too much emphasis is laid on symbolic thinking at the expense of qualitative thinking.

Strategies to redress this situation were created by highlighting the need for classroom discussion. Common to these 'innovative' approaches were the notion of revealing and challenging students' intuitive ideas, most often referred to as 'misconceptions', which were supposed to get 'replaced' by the correct conceptions. Yet, mechanics continues to remain a difficult subject for students to learn. It appears that mechanics, after having completely been stripped off its phenomenological behaviour, causes a lot of hardship to most students across the world – an aspect which seems not to be addressed by these innovative strategies. On this basis, as I have suggested earlier, I argue that we need to create settings where children can use their intuitive knowledge to construct new knowledge in the direction of expertise. Unless this is done in the context of mechanics, I will argue that there is no way forward.

Since mechanics is a very broad area of study, I have selected the topic 'motion of connected particles' in 1-D and 2-D for the purpose of this research. It consists of two objects connected by a string that passes over a pulley. Previous work, described in § 2.3.4, suggests that motion of connected particles can offer a rich arena for the study of intuitive knowledge in the construction of expert knowledge. An analysis of the motion of this compound system involves all three of Newton's laws. For example, applying $F = ma$ can give the acceleration of the objects. Moreover, many different concepts from both kinematics and dynamics enter into the analysis. Thus, my approach is to start at some post-basic position of understanding mechanics so that it would allow students to either progressively advance or revise relevant mechanical concepts (see Figure 1.2).

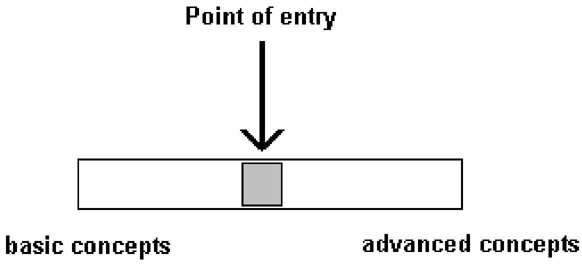


Figure 1.2: A model depicting the position of connected particles along the mechanics hierarchy.

1.2.2 Conceptual Change in Mechanics

The notion of conceptual change is as old as learning itself. It is widely accepted that without conceptual change there is no meaningful or 'effective' learning. And across the world the implicit aim of schooling has always been to bring about conceptual change in the students' minds. Therefore, the issue of how conceptual change occurs, whatever be the domain knowledge, definitely helps to raise student learning and performance in examinations. It should be noted that decades ago scholars were more apt to develop general theories of learning which did not work in practice. This explains why today it seems more appropriate to carry out research on domain-specific theories of learning.

The educational literature abounds with studies on conceptual change, especially in the field of science education. However, diSessa & Sherin (1998) have criticised many of these research studies on the following grounds:

(a) How do we define a concept and conceptual change?;

(b) Do we replace or construct on student intuitive knowledge?;
(c) Is intuitive knowledge an obstacle or vehicle for learning?; and,
(d) Do we compare data before/after snapshot or analyze data collected throughout snapshot?

The questions 'what is a concept?' and 'how do you recognise a concept when you see one?' have created a commotion among members of the research community. After having raised these pertinent questions, diSessa & Sherin (1998) create the notion of coordination class to replace the idea of concept. They persuasively argue that the term 'concept' is a highly imprecise word since it cannot be sub-divided into elements. Coordination class, on the other side, is a synergy among different elements, mainly in the form of p-prims, and is a derivation of the 'knowledge in pieces' perspective developed by diSessa (1988).

Another debate in conceptual change literature is whether an 'incorrect' concept is replaced by a 'correct' one or we construct the expert knowledge by using the 'incorrect' concept. If the notion of replacing one 'incorrect' concept by a 'correct' one was true, students would not have possessed mental models with both Aristotelian and Newtonian ideas after instruction (see, for example, Mildenhall & Williams, 2001). Students should have been able to apply the 'correct' concepts to new contexts. Since these arguments do not hold true in real-life situations, I argue that the replacement theory cannot be used in the context of mechanics.

Conceptual change, especially in the field of mechanics is directly related to the notion of intuitive knowledge. Here, we have two salient issues. First, while most people nowadays agree that students come to mechanics classes with intuitive ideas arising out of their interactions with the physical world, Rowlands et al. (2007) argue that intuitive ideas in the form of misconceptions are spontaneous rather than preformed. Second, there is the debate of whether intuitive knowledge is an obstacle or vehicle for learning. When viewed through the lens of knowledge in pieces theory, the active use of intuitive knowledge is a key strategy for supporting knowledge construction processes.

Finally, several studies appear to determine conceptual change by comparing what knowledge the students have before and after a test. In some studies we even have delayed post test. It has been argued elsewhere (diSessa, 2002) that such comparison can only suggest whether or not conceptual change has occurred. It can also indicate to what extent the conceptual change has taken place. What remains a mystery with such comparison models is how the conceptual change happens. However, the knowledge in pieces theory, as we shall see, appears to be an appropriate tool that we can adapt to analyse data collected during the snapshot.

I, therefore, argue that several innovative strategies designed to rekindle mechanics learning have failed in the sense that they have not been examined from the intuitive-expert viewpoint and have not adopted the appropriate methodology. This research will be examined in the pages which follow by taking into consideration the four criticisms discussed earlier.

1.2.3 Computers in Mechanics Instruction

It is generally recognised by most educators that computers can bring to the learning of mathematics and science new opportunities as well as new categories of problems. The advent of computers in the classroom has given rise to several key debates. The one, which is of remarkable importance to this study, is that the computer by itself is not a powerful tool for significant learning in both mathematics and science. In this respect, the idea of a microworld has been put forward for incorporating the computer as part of a setting that can positively influence students' learning (Hoyles & Noss, 1987).

From the microworld perspective, one important aspect is to consider the kind of software from which students are made to extract mathematical ideas. Noss & Hoyles (1996) argue that there is no point in employing software that: (a) provides the same type of support as that by a 'human tutor', and (b) fails to open any windows onto the processes of mathematical learning. They further emphasise that the computer-software should not be the centre of attention for it then hinders the learning process. Instead, the focus should be on what the computer-software makes possible for mathematical meaning-making, that is, to make the technology work to give students access to mathematical concepts.

It would then appear that features incorporated into the software have to be transparent (that is, can be used without cognitive effort and, therefore, do not draw all attention onto themselves) and ‘ready to hand’ (Roth et al., 1996). Students should not only know which feature to use in which circumstance, but how to use it. From this viewpoint, it is imperative to analyze the strategies devised by the students to use the features in such a way that they are able to construct new mathematical meanings. This also implies that tasks have to be designed in such a way that the available features can be fully exploited. Thus, features without appropriate tasks are useless from a pedagogical perspective. On this basis, I find that as students become more familiar with the features, more and more learning can take place. I argue that through the appropriate use of features coupled with the relevant type of tasks, students can come to construct new mathematical meanings.

The notion that mathematics must be taught only as an abstract subject has been challenged with the advent of computer-based environments. From time immemorial, the teaching of mathematics has always been dominated by the deductive method – theorems are deduced from axioms, with no reference to phenomenological aspect. This method of mathematical teaching forces students to learn mathematics in its decontextualised form. It is now argued that this distinction between the concrete and the abstract can be blurred in a computer-based setting to help students learn ‘more effectively’ (Noss & Hoyles, 1996). By supporting students to make connections between the informal and formal views of mathematics, I believe that this can help students in constructing new knowledge. This aspect is a key component for this study.

In the context of mechanics, I find that there have been two major ways in which computational tools have been employed. First, these tools have been used to probe student understanding of mechanics (Grayson, 1990). Second, they have been involved in both revealing student intuitive ideas and bringing about conceptual change (Andaloro et al., 1997; Goodchild, 1997; Gorsky & Finegold, 1992; Hennessy et al., 1995). However, it would appear that using computers to track student learning of mechanics has been largely unexplored. Not many researchers have been involved in the latter research. Burns & Smart (1988), for instance, employ the Newton Microworld not only to tease out student intuitive understanding of motion, but to make sense of how sixth form students learn mechanics in such an environment. These two principles - that of using computers for research in student understanding of mechanics and that of using computers for tracking student learning of mechanics - underlie this thesis.

In this respect, Interactive Physics^{TM}, which is the building block of this study, is one potential software that allows users to conduct motion-related experiments. It is noted that Interactive Physics^{TM} has become for the teacher an important demonstration tool for important physical concepts and the focus of whole-class discussions (Roth, 1995). It would also seem (see Roth et al., 1996) that one particular aspect of this software – 'the copresence of the phenomenal and the conceptual' – can be a step forward in bridging the gap between intuitive and expert knowledge. Though it was initially designed to be used in the science classroom, Stevenson (2000) argues that it can also be used in the instruction of mechanical concepts in the sixth form mathematics classroom. Therefore, it appears feasible to examine how students use their intuitions in the construction of new knowledge in such a technological context.

It is my hypothesis that the integration of Interactive Physics, especially its animation aspect, into the teaching and learning of mechanics at GCE A-Level Mathematics can offer an opportunity for learning experiences that are more adapted to the world of students. As such, I argue that Interactive Physics can provide a medium which combines the formal and informal aspects of mechanics which, in turn, can reduce student perception that mechanics is a difficult subject to learn and appreciate. This study intends to investigate this aspect.

1.3 Aims of the Study

There has been a growing recognition that though mechanics is located across both informal real-life situations and formal mathematics, when it comes to mechanics learning these two contexts are rarely brought into juxtaposition. As such, this study explores the issue of how both informal and formal views of mechanics may coalesce in the medium of the computational tool Interactive Physics.

I hypothesize that by concentrating on the software's features of bridging the informal and formal aspects of motion of connected particles, it is possible to develop a learning approach that may empower students to connect their intuitive knowledge to mechanical principles, thus, constructing new meanings for the said topic. I believe that this is feasible if the following conditions are in place:

(a) Students are empowered with the features available on Interactive Physics;
(b) Students are given some carefully crafted tasks;
(c) It can be observed which feature they (prefer to) use; and,
(d) It can be determined whether and how this specific feature enables them to construct new meanings about the motion of connected particles.

This research is about creating situations and ways in which Newtonian mechanics can become more accessible to sixth form students, and exploring how these computer-based tasks influence student learning of motion of connected particles. The latter aspect of this study will be viewed through the lens of an approach inspired from diSessa's 'knowledge in pieces' perspective.

Involving sixth form students in this work should not pose any difficulty as the notion of 'sixth form', which originated in the UK, is also present in Mauritius, a former British colony. To date, Mauritian students continue to sit for the GCE O-Level and A-Level examinations prepared by the University of Cambridge International Examinations (CIE). The latter is responsible for devising the syllabus, preparing and printing the examination papers, advising the Mauritius Examinations Syndicate (M.E.S.) on how to conduct the examinations, officialising the examination timetables, marking students' scripts in collaboration with the M.E.S., and issuing students' results. It is imperative to note that in Mauritius the GCE A-Level Mathematics syllabus is almost similar to that offered in the UK. On this basis, working with Mauritian (instead of British) students should not be an issue for this study as I am solely interested in their intuitions and how the latter are used in the generation of new knowledge.

The main research issue is, therefore, to investigate how sixth form students use their intuitions to develop new knowledge while learning motion of connected particles with a set of carefully crafted tasks based on the computational tool Interactive Physics. As such, there are two inter-related strands in this study: the role of intuitive knowledge in creating new knowledge and the role of Interactive Physics in this learning process.

Thus, the aims of this study are two-fold:

- How is intuitive knowledge used by students in learning motion of connected particles?

- How do the uses of Interactive Physics influence the evolution of the learning process?

1.4 Structure of the Thesis

Chapter 2 deals with the literature review which is in three parts: (a) conceptual change and learning; (b) mechanics at GCE A-Level Mathematics; and, (c) the use of computational tools in the learning of mechanics. The chapter ends with precise research questions.

Chapter 3 focuses on the research methodology developed for this research study. The chapter is divided into two main parts: theoretical framework of the research and overview of the methodology for data collection.

Chapter 4 describes the pilot study and documents its outcomes which, in turn, enhance the design of the main study. It not only describes the procedure adopted to collect data, it also discusses the analysis to be applied to the data.

Chapter 5 revolves around the findings related to student understanding of the mechanical conditions required for the system of connected particles to be in motion and at rest. Based on the participants' responses, their understanding is examined in three different situations.

Chapter 6 portrays the findings related to student understanding of acceleration in three different situations associated with the system of connected particles.

Chapter 7 synthesizes the findings of Chapters 5 and 6 so that we can start to look at the bigger picture of this thesis. The findings are reported in relation to the two research questions of this study.

Chapter 8 concludes this thesis.

Chapter 2

Review of the Literature

2.1 Introduction

The aim of this chapter is to carry out a literature review, which is presented in three parts. § 2.2 begins by considering previous work on conceptual change and learning, and then moves on to examine its implications for this study. In § 2.3, I explore the relevant issues pertaining to mechanics at GCE A-Level Mathematics and then discuss the rationale of using motion of connected particles for this study. § 2.4 looks at the possibility of using computational tools in the learning of mechanics. This is achieved by incorporating the software Interactive Physics into the microworld paradigm. The chapter ends with § 2.5 which articulates the research questions more precisely.

2.2 Conceptual Change and Learning

In this section my intention is to develop a model of conceptual change for the purpose of this study. Since conceptual change is mutually related to learning, tracing evolution of learning in mechanics can be achieved by analysing conceptual change. While § 2.2.1 and § 2.2.2 examine the notions of conceptual change and concepts respectively, § 2.2.3 reviews the literature on five theoretical approaches to analyse conceptual change. In § 2.2.4 I intend to develop a model of conceptual change that will be used in this study.

2.2.1 What does Conceptual Change mean?

Conceptual change seems to be a derivative of at least two relatively independent research traditions: the cognitive-developmental and the science education traditions (Vosniadou, 1999). While in developmental psychology it was much needed to provide an alternative to the Piagetian explanation of cognitive development, in science education it was initially associated with the instructional theory developed by Posner et al. (1982). It was also considered to be just another word for learning science (Duit, 1999).

Instead of converging towards a common meaning, it is today seen that there is a degree of disagreement among researchers on its exact definition. Ruhf (2003) questions whether conceptual change is simply about altering a particular belief or whether conceptual change occurs every time a student learns something new in the classroom. He argues that "simply altering a student's idea about some phenomenon is not what is meant by the term conceptual change" (p. 2). For Mayer (2002), conceptual change has been the driving force of science learning in terms of achieving structural insight, accommodative learning, understanding of relations, deep learning and mental model building. This is why he professes that

> Conceptual change is the mechanism underlying meaningful learning. Conceptual change occurs when a learner moves from not understanding how something works to understanding it. For decades scholars have recognized that conceptual change is at the heart of meaningful learning.
>
> (Mayer, 2002: p.101)

Biemans & Simons (1999) assert that conceptual change is about a partial or radical change of learners' existing conceptions – it is not about an integration of new information into their preconceptions without really changing these ideas. By contrast, Halldén (1999) finds that conceptual change can signify three processes: (a) abandoning an old conception and replacing it with a new one; (b) acquiring an entirely new conception; and, (c) acquiring a new way of conceptualizing the world, not in order to replace the conceptions one already entertains, but rather to enrich one's repertoire of conceptualizations of a particular phenomenon. Though these two approaches have certain differences, they have two key themes in common: use of the notion 'conception' and replacement of conception.

Approaches using these themes have been labelled as the ‘standard model of conceptual change’ by diSessa & Sherin (1998). According to them, such approaches are not appropriate, reliable and valid to pursue research with. Moreover, the standard model of conceptual change is most closely associated with a methodology that concentrates on the before and after snapshots. On this basis, diSessa (2002) claims that most research on conceptual change has lacked theoretical accountability.

> … current practice in conceptual change research is far from being able to (and rarely attempts to) match system elements and processes against the details of student reasoning and learning data. (diSessa, 2002: p. 30)

Another important issue in the conceptual change literature is the different terminologies associated with it. Terms to describe different types of conceptual change such as weak restructuring and strong restructuring (Carey, 1985); normal and radical (Chi, 1992); differentiation, class extension and reconceptualization (Dykstra, 1992); conceptual capture and conceptual exchange (Hewson, 1981); assimilation and accommodation (Posner et al., 1982); branch jumping and tree switching (Thagard, 1990); enrichment and revision (Vosniadou, 1994) emerge from literature.

Tyson et al. (1997), extending Dagher’s (1994) work of comparing the kinds of conceptual changes described by various theorists, find that there are two major categories of conceptual change: strong ones and weak ones (Figure 2.1).

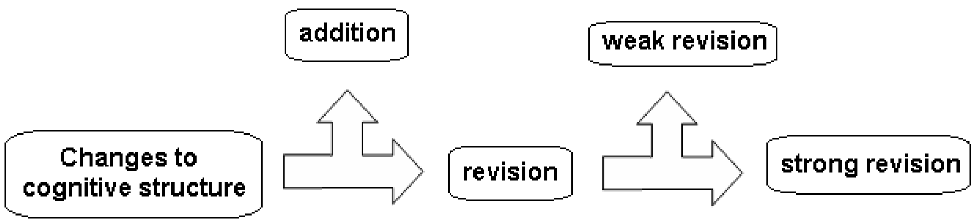


Figure 2.1 (adapted from Tyson et al., 1997):
A model of the dichotomy of kinds of conceptual change.

At the most basic level, there are changes that can occur in the conceptual structure which involve the simple addition of knowledge. This kind of conceptual change is described as knowledge accumulation that does not involve restructuring (Carey, 1985), as enrichment by the mechanism of accretion (Vosniadou, 1994), and as belief revision (Thagard, 1991). Alternatively, there are changes to the conceptual structures which involve some kind of change to the existing conceptual structures rather than simple addition. This kind of conceptual change is called revision (Vosniadou, 1994), conceptual change involving more than mere belief revision (Thagard, 1992), and knowledge accumulation that involves restructuring (Carey, 1985). The latter kind of learning is most commonly described as conceptual change and is divided intro strong revision and weak revision by most theorists.

The debate has been whether to consider weak revision as conceptual change. Tyson et al. (1997) argue that both levels of revision should be referred to as conceptual change. Similar to the position adopted by Hewson & Thorley (1989), Dagher (1994) also points out that both weak and normal conceptual changes are as worthy as radical conceptual change, and must receive equal attention from science educators.

For the purpose of my study, I will assume that

- Conceptual change is at the heart of meaningful learning (Mayer, 2002).
- Conceptual change is the interaction between intuitive knowledge and new knowledge such that it is learning generated in the direction of expertise (diSessa, 1993).

2.2.2 What are Concepts?

Though there have been numerous conceptual change studies involving the term 'concept' in the past, it remains an undisputed fact that this term does not have the same meaning for different researchers (diSessa & Sherin, 1998). The traditional view of the notion of concept assumes that humans tend to organize and categorize objects of their environments. Ferrari & Elik (2003), concurring with Rey (1998), believe that "concepts are the constituents or the smallest units of thought and that they are shared among people in a society (and sometimes, around the people)" (p. 25). They also tend to agree with diSessa & Sherin

(1998) who find that it is not straightforward to determine whether or not someone possesses a concept.

White (1994) argues that the term concept is totally different from the term conception. While the term concept is used in two ways: classification and all the knowledge a person associates with the concept's name [synonymous to addition], the term conception is to describe a more complex way of learning [synonymous to revision].

In their seminal paper, diSessa & Sherin (1998) raise this issue by first examining the nature and structure of concepts. Their debate starts with the following questions. Does the concept of dog differ from that of force? How difficult is it for the children to learn the concept of dog compared to that of force?

Some years later, diSessa (2002) raises the question: "Why do some people learn certain concepts almost effortlessly while others do not?" (p. 36). He also raises the following question: "How do we know a concept when we see one?" (p. 30). So, from this perspective, what does it mean if it is said that a student understands the concept of, say, tension in the string which is of prime importance in this study? Does it mean that a student can explain the following four relationships:

(a) the effect of the mass of the hanging object on the tension in the string;
(b) the effect of the mass of the sliding object on the tension in the string;
(c) the effect of friction between the sliding object and the surface on the tension in the string; and,
(d) the relationship between the acceleration of the objects and the tension in the string?

From this example, it can be seen that there is no sharp line between 'having' and 'not having' the concept of 'tension in the string'.

According to diSessa & Wagner (2005), if several elements and relations are involved, there always exists the possibility of a few being missing or malformed and "yet the person could exhibit generally competent performance. Indeed, there is every reason to suspect either that no humans achieve complete or perfect construction, or that no such state is specifiable.” (p. 126). On this basis, it would be much more important to look for states of partial construction rather than ‘has it’. Therefore, I agree with diSessa that the term ‘concept’, as being used currently, is misleading and not instructionally productive. Following in his footsteps, I shall assume that concepts are large and complex organized structures that coordinate the activation and use of many specific elements according to context.

2.2.3 Theoretical Approaches to (Studying) Conceptual Change

In this section I review the literature on five significant, yet very different, theoretical approaches that have been developed to analyse conceptual change in science education over the past decades. Among them are:

(a) Carey’s (1985) domain-specificity approach
(b) Chi’s (1992) ontological approach
(c) diSessa’s (1993) knowledge in pieces approach
(d) Posner et al. (1982) conceptual change model approach
(e) Vosniadou’s (1992) framework theory approach

This review will have important implications for this study and will be discussed in § 2.2.4.

(a) Carey’s (1985) Domain-Specificity Approach

Besides advocating that students’ intuitive ideas are most accurately represented as coherent, systematic and theory-like, Carey (cited in Ozdemir & Clark, 2007) supports the view that intuitive knowledge is made of two primary components: concepts and beliefs. Beliefs act as bridges among concepts. For example, “cars are animals” refer to two different concepts: cars and animals. She explains that it is relatively easy for changes in relations between the concepts to take place. However, to bring changes in the concepts is a very difficult process because intuitive theories constrain the concepts in which beliefs are formed. On this basis, she states that

> conceptual change does not occur suddenly. There is not one moment of gestalt shift. It takes time for concepts to change, sometimes centuries in the history of science, always years for the individual scientist or student or child engaged in knowledge restructuring. (Carey, 1999: p. 296)

Rejecting Piaget's view of domain-general changes, Carey (1985) argues that development involves domain-specific changes in theory. According to this view, children restructure their naïve theory structures in a specific domain. It is believed that children gradually replace their pre-existing understandings with more coherent theories when they are subjected to new experiences and instruction. This is in contrast to Piaget's notion that development is led by changes in logical capabilities. Carey's approach is consistent with the results of some studies, for instance, Voss (1989, cited in Read, 2004) finds that the differences between novices and experts in problem solving within a domain are strongly related to knowledge acquisition.

According to Carey (1991), change between concepts can be achieved through three processes: replacement, differentiation and coalescence. In replacement, an alternative concept displaces an initial concept. This does not mean that a correct belief replaces an incorrect belief (Chi & Roscoe, 2002). In this process, the two beliefs are incommensurable – implying that by accepting the new belief, the old belief is totally discarded. Differentiation is another replacement process but with a difference: the initial concept shifts into two or more new concepts, which may be incommensurable to the initial concept or to each other. These newly split concepts replace the initial concept. Coalescence is the opposite process of differentiation; it involves the coalescing of two or more original concepts into a single concept, thus, replacing the initial ones. However, Chi & Roscoe (2002) challenge Carey's notions of incommensurability and replacement. They argue that (a) the historical examples provided by Carey do not clearly specify how two concepts are incommensurable; and, (b) it is not clear whether replacement process are conceptual change processes, or whether they are the outcome of reorganization.

(b) Chi's (1992) Ontological Approach

Chi (1992) distinguishes two types of conceptual change: normal and radical. She proposes a method of defining differences between concepts on the basis of three ontological categories: matter (or things), process and mental states – each of which can be subdivided into a hierarchy of subcategories (see Chi et al., 1994). Normal conceptual change (or conceptual reorganization or perspective shift) occurs when there is restructuring of knowledge within the same ontology or hierarchy such as the migration of concepts. By contrast, radical conceptual change (or reassignment or conceptual shift) involves reassigning a given concept to a new concept to a new ontological category. As such, the initial and new concept are considered to be incommensurate. Chi & Roscoe (2002) find that their process of radical conceptual change is analogous to the two processes of conceptual change, 'branch jumping' and 'tree switching', proposed by Thagard (1990).

The driving force of this ontological approach is ontological categories are distinct, stable and constraining. It is argued (see Chi et al, 1994; Slotta et al., 1995) that some naïve conceptions are robust, for example in physics, because most students think of concepts such as electric current, heat and light within the category of matter when they belong to the ontological category of processes, or more precisely to the subcategory labelled 'constraint-based interaction'. Chi & Roscoe (2002) insist that the conceptual change process is hard because students may lack awareness of when they need to shift and may lack an appropriate category to which the concept could be shifted into. According to this view, misconceptions "are, in fact, miscategorizations of concepts" (Chi & Roscoe, 2002: p. 4) or result from novices' "commitment to substance-based conceptions" (Reiner et al., 2000: p. 1). That is, misconceptions are concepts which have been assigned to the inappropriate ontological categories – an argument which has been refuted by diSessa (1993b). On this basis, conceptual change is "merely the process of reassigning or "shifting" a miscategorised concept from one "ontological" category to another "ontological" category" (Chi & Roscoe, 2002: p. 4). This view of conceptual change suggests that it is a revolutionary process (Lautrey & Mazens, 2004). This is in contrast to research carried by Ozdemir & Clark (2007) who find that such radical changes do not happen suddenly. For them, such changes must be gradual and time-consuming "because the student must revise and restructure an entire network of beliefs and presuppositions" (Ozdemir & Clark, 2007: p. 354).

As described above, Chi claims that for students to achieve a deep understanding in science, misconceptions must be removed and replaced by the correct conceptions. Such process of conceptual change does not involve a change in the internal structure of the concept under study. It is critical to point out that this approach has serious implication for the nature and role of intuitive knowledge. First, it is seen that, akin to Carey's approach, this one also assumes that intuitive knowledge is organized into coherent theory-like structures (Chi & Slotta, 1993). Second, this view of learning science suggests that refining or developing intuitive knowledge is a futile possibility. This notion has recently been reinforced by the following comment:

> Teachers should not try to "bridge the gap" between students' misconceptions and the target instructional material, as there is no tenable pathway between distinct ontological conceptions. For example, students who understand "force" as a property of an object cannot come gradually to shift this conception until it is thought of as a process of interaction between two objects. Indeed, students' learning may actually be hindered if they are required to relate scientifically normative instruction to their existing conceptualizations.
>
> (Slotta & Chi, 2006: p. 286).

Several authors, namely Vosniadou (1994), diSessa (1993b), Duit (1999) and Gupta et al. (2010), have expressed concerns over Chi's approach. In response to Chi & Slotta (1993), diSessa (1993b) argues that there is limited explanatory power in their ontological model and the ontological discontinuity portrayed by them seems much too stark. Vosniadou (1994) claims that this approach presents only a syntactic not a semantic explanation of conceptual change. In a recent paper, Gupta et al. (2010) argue that expert and novice reasoning often and productively traverses ontological categories. They claim that learners' ontologies are better understood as dynamic and context dependent rather than as static constraints. Their finding models ontological knowledge as being flexible and ontological categories as being multiply connected. According to them, from an instructional perspective, "the dynamic view suggests building from students' everyday resources" (p. 317).

(c) diSessa's (1988) Knowledge In Pieces (k-i-p) Approach

Rather than understanding intuitive physics as highly organised theories which must be confronted, overcome and replaced, diSessa (1993) claims that it can be fragmented in 'hundreds if not thousands' of self-explanatory elements that he calls 'phenomenological primitives' or 'p-prims' for short. They are called 'phenomenological' because they are minimal abstractions of common phenomena; they are apparent to people in real-life situations. They are 'primitive' because they appear obvious and self-evident; people use them to explain what happens naturally around them. P-prims are also subconceptual in the sense that "they are much smaller and more fluid pieces of knowledge than concepts or beliefs" (diSessa, 1996: p. 715). It is imperative to note that diSessa (1996) points out that intuitive physics or 'folk physics' consists of five types of knowledge: p-prims, mental models, narratives, committed facts and nominal facts. However, he claims that p-prims are the most important for understanding intuitive physics.

> It would not be a bad approximation to say folk physics is the rather large, diverse and mildly organized collection of fairly simple phenomenological ideas, which are p-prims. On the other hand, spontaneous mental models, narratives, and nominal and committed facts exist, if relatively rarely (and dependent on the p-prim substrate).
>
> (diSessa, 1996: p. 719)

The central concept to explain the cognitive mechanism of p-prims is recognition, that is, p-prims are "being cued to an active state on the basis of perceived configurations, which are themselves previously activated knowledge structures" (diSessa, 1993: p. 112). To understand how p-prims are recognised and systematized, diSessa (1993) defines the terms cuing priority, reliability priority and structured priorities. The first term cuing priority describes the likelihood that a particular p-prim will be activated on the basis of perceived configuration of objects or events. For example, a high cuing priority implies that a p-prim is easily activated by the cognitive context. The second term reliability priority provides a measure of how likely a p-prim is to stay activated once it has been activated. For instance, a high reliability priority implies that a p-prim is in state of reinforcement rather than that of suppression. The third term structured priorities is used to combine cuing priority and reliability priority together. Based on these notions, diSessa speculates how intuitive physics can be tuned toward expertise. In this theoretical framework, conceptual change is viewed as a process where conceptual elements of the naïve state "are modified and combined in complex ways, possibly in levels and into subsystems that, together, constitute the "final" configuration of an expert concept. For reference, I call this a "complex knowledge systems" view of conceptual change" (diSessa, 2002: p. 39).

From this theoretical perspective, I find that there are two central themes. The first one is the notion that intuitive knowledge is not a highly coherent theory as claimed by Carey, Chi, Posner et al. and Vosniadou. In this respect, there are several studies (Chiou & Anderson, 2010; diSessa et al., 2004; Ozdemir & Clark, 2009; Southerland et al., 2001; Tytler, 1998) defending diSessa's viewpoint. In analysing students' explanations of a range of air pressure phenomena, Tytler (1998) concludes that the k-i-p approach was more appropriate to capture the fluidity and complexity of children's thinking. In particular, it helped to interpret "findings concerning the inconsistency in application of conceptions and the complexity of the way these were cued according to specific aspects of phenomena or contexts" (p. 923). In a similar vein, Southerland et al. (2001) argue that the k-i-p approach, unlike the conceptual frameworks, was able to provide a more explanatory construct to understand both the tentative and shifting nature of students' explanations. However, they propose that further research be carried out in answering the following question: "What are the specific characteristics of a situation that invokes use of a particular p-prim?" (p. 346). Chiou & Anderson (2010), in their study of undergraduate physics students' understanding of heat conduction, find that the emergent patterns supported diSessa's (2002) k-i-p theory and were less consistent with Vosniadou et al.'s (2008) framework theory that proposes a coherent development of human conceptions.

The second theme is the notion that intuitive knowledge is not replaced by expert knowledge; instead, it is refined in the direction of expertise. This is again in contrast to Carey's, Chi's, Posner et al.'s and Vosniadou's opinions. Observing students learn about linear functions in supportive contexts, Moschkovich (1998) finds that students' initial conceptions played a productive role in student understanding of linear function. By extending diSessa's (1993) theory to mathematics, she was able to describe the refinement of a conception in this domain. Sherin (2006), who investigated the role of intuitive knowledge in physics problem solving, makes the following two conclusions: (a) intuitive knowledge can provide a context for interpretation; and, (b) there are limits to how much it is really necessary for intuitive knowledge to be refined. He argues that in his study a new type of knowledge was developed – symbolic forms which mediated the connection between p-prims and equations. Finally, by adopting the k-i-p approach in their work, Masson & Legendre (2008) find that their historical world promoted "students conceptual change by pushing them to organize their knowledge by structuring and contextualizing the domain of validity of their intuitive conceptions" (p. 127).

(d) Posner et al. (1982) CCM Approach

There seems to be a general consensus among the conceptual-change researchers that the most influential theory of conceptual change has been that of Posner, Strike, Hewson & Gertzog (1982), referred to as the Conceptual Change Model (CCM). This model, drawing on the works of Kuhn and Piaget, distinguishes two types of conceptual change – assimilation and accommodation. Assimilation is the process through which a learner's existing concepts can be used to deal with new phenomena, analogous to Carey's (1985) weak restructuring or Hewson's (1981) conceptual capture. Accommodation involves "replacing or reorganizing the learner's central conceptions" (Posner et al., 1982: p. 212), analogous to Carey's (1985) strong restructuring or Hewson's (1981) conceptual exchange. In this case, a radical change occurs involving the abandonment of the initial conception and the acceptance of a scientific conception for successful change.

According to Posner et al. (1982), two conditions are imperative for successful conceptual change: (a) the learner must be dissatisfied with the existing conception; and, (b) the learner must find the new conception intelligible, plausible and fruitful. Hewson & Thorley (1989) describe the extent to which the new conception meets the three conditions of intelligibility, plausibility and fruitfulness as being the status of a person's conception. They state that "as more conditions are met, the conception's status is raised" (p. 542). Thus, to raise the status of a new conception, the three conditions must be fulfilled 'in a linear manner' beginning with the dissatisfaction state and proceeding through to the fruitfulness of the new conception (Tyson et al., 1997). However, in their revisionist theory of CCM, Strike & Posner (1992) add that a conceptual ecology must be interactionist in nature. On this basis, Demastes et al. (1996) explain that the linear nature of change no longer holds as originally described. More importantly, in order to initiate conceptual change Hewson (1982) argues that creating dissatisfaction with an 'irreconciable current conception' is the key to lowering its status for the new one to be incorporated.

They also describe five features of a student's conceptual ecology that influence the conceptual change process. A decade later, the theory was revised (Strike & Posner, 1992) and the authors introduced the notion that alternative conceptions may not necessarily pre-exist but "may be generated on the spot as a consequence of instruction" (p. 158). A further revision in their 1992 report included their acknowledgement of the active role played by social and motivational factors in the learning environment.

From the CCM approach, we can deduce the following observations. First, naïve knowledge, which is a subset of a person's conceptual ecology, is considered to be theory-like and coherent, and is replaced by new knowledge in the event that the former cannot explain the phenomenon under study. The role of intuitive knowledge is to determine the status of a person's existing conception as it strongly influences the person's acceptance of new conceptions (Hewson & Hewson, 1992). Second, while the original model (Posner et al., 1982) was designed to account for radical conceptual change (that is, conceptual exchange which is considered to be an abrupt process), Demastes et al. (1996) find that "there is a tension in descriptions of conceptual exchange between notions of gradual and wholesale change" (p. 408). Their findings, similar to those of Metz (1991), Nussbaum (1989) and Terry & Jones (1986), show that the process of conceptual exchange can be a gradual process. This is in contrast to the finding of Tao & Gunstone (1999) who find students giving up their alternative conceptions for the scientific conceptions within a context. However, they also point out that "students' recognition of the generality of scientific conceptions was a lengthy process during which students were exposed to a range of contexts" (p. 877). Third, according to the studies carried out by Tao & Gunstone (1999) and Venville & Treagust (1998), it is not a sufficient condition for students to become dissatisfied with an existing conception so that a scientific conception may be accommodated. Tao & Gunstone (1999) assert that students must be made to reflect on and reconstruct their conceptions. Fourth, the CCM approach accounts neither competing conceptions nor a vacillation between alternative and scientific conceptions. However, the studies of Demastes et al. (1996) and Tao & Gunstone (1999) find that these possibilities do exist. In this case, Tao & Gunstone (1999) provide evidence of how context plays a key role in helping students select which conception they need to use.

(e) Vosniadou's (1994) Framework Theory Approach

Vosniadou (1994), based on her work with Brewer (see Vosniadou & Brewer, 1992, 1994), develops a theory of conceptual change in which there are different levels of conceptual knowledge. Since "concepts are embedded into larger theoretical structures which constrain them" (Vosniadou, 1994: p. 46), she makes a distinction between specific theories and framework theories within learners' naive knowledge. While a framework theory consists of a core set of entrenched ontological and epistemological beliefs about the target domain which are built during early childhood and are continuously experienced in real-life situations, a specific theory includes specific explanations of phenomena from the target domain. The salient point in this approach is that specific theories are constrained by framework theories. From this perspective, the conceptual change process is a difficult one because it is rather difficult to modify the ontological and epistemological presuppositions of the respective framework theory. On this basis, Vosniadou (1994) assumes that conceptual change proceeds through the gradual revision of the assumptions of framework theory.

The term conceptual change, according to Vosniadou (1994), implies that conceptual development can involve enrichment of existing conceptual structures or their revision. While enrichment involves the addition of information to existing conceptual structures, revision involves changes in individual beliefs or presuppositions or the relational structure of a theory. Revision is believed to take place at the level of either the specific theory or the framework theory. However, revision occurring at the level of the framework theory is considered to be the most difficult kind of conceptual change and the one responsible for the occurrence of misconceptions or synthetic mental models. Vosniadou (1994) argues that children construct synthetic mental models when they try to reconcile new information, which is inconsistent and conflicting, with old assumptions about the domain under study. From this perspective, conceptual change is a gradual process that moves from initial mental models via synthetic mental models to scientifically correct models.

From this approach, the following implications emerge. First, intuitive knowledge involves unified coherent structures, akin to Carey's and Chi's position. Ioannides & Vosniadou (2002) claim that their findings support the coherent nature of students' knowledge. However, these findings have been challenged by the studies of diSessa et al. (2004) and Ozdemir & Clark (2009). Second, the role of intuitive knowledge in the construction of new knowledge strongly appears to be insignificant; it is clearly seen that naïve theories have to be replaced by scientific theories. Third, according to Brown & Hammer (2008), children's responses to classes of situations will be consistent, since the same framework theory would be used for a variety of instances.

2.2.4 Implications for this Study

In this section, I discuss important implications for this study in terms of the following four themes:

(a) nature of intuitive knowledge;

(b) role of intuitive knowledge;
(c) conceptual change: revolutionary or evolutionary?; and,
(d) adapting the 'Knowledge in pieces' approach.

(a) Nature of Intuitive Knowledge

As a starting point I shall take the view that students learning mechanics do already have intuitive, naïve, prior or subconscious knowledge - a type of knowledge that is formed through abstraction from exposure to real-life situations. As from this point in this thesis, I am going to refer to this type of knowledge as intuitive knowledge. To sharpen focus on the existence of intuitive knowledge, I would like to contrast two prominent but competing theoretical perspectives. On the one hand, intuitive knowledge has been described as coherent, systematic or even theory-like (Chi, 2005; Ioannides & Vosniadou, 2002) – this perspective contains the terms such as alternative frameworks (Terry & Jones, 1986), misconceptions (Hestenes, 1992), alternative conceptions (Ramadas et al., 1996), preconceptions (Arons, 1990) and student prior conceptions (Driver, 1989). On the other hand, intuitive knowledge has more aptly been described as an ecology of diverse and fragmented elements, displaying limited integration or coherence: the 'knowledge in pieces' perspective (diSessa, 1988, 1993, 2008).

The 'theory-like' perspective suggests that learners are always using several well-developed coherent naïve theories based on their everyday experiences, and that these theories suffice to make consistent predictions and explanations. From this position, conceptual change usually takes place when one concept is replaced by another one. The knowledge in pieces perspective hypothesizes that intuitive knowledge structures consist of relatively independent fragments – p-prims. From this viewpoint, the context of the situation is the determinant factor in helping the learner to spontaneously activate and connect these pieces. During the conceptual change process, the elements and interactions among the elements are revised and refined through addition, elimination and reorganization to strengthen the knowledge network.

In order to take a stand on this issue, I wish to put forward an argument of diSessa (1988). He conjectures the possibility that a misconception may be fragmented into sub-conceptual elements. To justify his argument, he successfully applies this decomposition to the 'impetus theory' of McCloskey (1983) when a ball or a coin is tossed into the air. diSessa (2008), therefore, argues that the highly controversial debate between the 'theory-like' and the 'knowledge in pieces' perspectives can be settled through two important issues: grain size and structure. On this basis, he claims that the 'knowledge in pieces' perspective is a more accurate model to analyse conceptual change. Therefore, this study firmly supports the 'knowledge in pieces' view of intuitive knowledge whereby knowledge is constituted by a multiplicity of diverse and loosely connected pieces rather than being a solid and coherent system.

This position logically results in hypothesizing that students will make up explanations (including intuitive ideas) spontaneously at the point when they are faced with a question, drawing where they can on core intuitions based on real life experience. It is also expected that in this study once students have developed their intuitive ideas, the latter may resist change. It would be interesting to see to what extent these intuitive ideas are going to be resistant to change and under what circumstances, if any, they will allow or accept changes.

(b) Role of Intuitive Knowledge

While one group of researchers has considered intuitive knowledge as an obstacle to learning (see, for example, Chi & Roscoe, 2002), the rest have found it to be a vehicle for learning (Carson, 1913; diSessa, 2002; Sherin, 2006). Chi & Roscoe (2002) find that intuitive knowledge is an obstacle to conceptual change. Very often, it is incorrect and impedes the learning of formal knowledge. They argue that some form of intuitive knowledge – preconceptions – can be readily revised or removed through instruction. Another form of intuitive knowledge – misconceptions – are highly resistant to change even when they are confronted by ingenious forms of instruction. Sherin (2006) argues that intuitive knowledge can play a variety of roles in expert problem solving, including some roles that are central and directly connected to equations. Most importantly, it can provide a context for interpretation. However, he also claims that there are limits to what we need from our intuitive knowledge. According to him, the refinement of intuitive knowledge must reach the extent where it can support and complement work with equations and other formal strategies – we should not expect the refinement of intuitive knowledge to attain the level of making perfect Newtonian predictions. On this basis, he points out that "instruction must nurture and refine intuitive physics, not confront and replace it, or simply build up a new set of frameworks" (p. 554). What has been confirmed by the work of Sherin (2006) was suggested by Carson (1913) almost one century ago. He argues that intuitive knowledge must be brought into 'full consciousness and formalised by some suitable course of training'.

It is imperative to realize the source of intuitive knowledge is none other than individual experience. So, does this imply that our lifestyle needs to be readjusted so that we acquire 'good intuitive knowledge' that does not become obstacle to learning or we should ensure not to acquire intuitive knowledge? Or does it mean that our instructional methods are not yet appropriate to fill in the gap between intuitive knowledge and expert knowledge? Or can those who believe that intuitive knowledge is an obstacle is to look for ways of hindering or slowing the formation of intuitive knowledge in children? And, finally, I am in accord with the view expressed by Ausubel (1968) that "the most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly" (p. vi). Therefore, in this study, I shall take the position of considering intuitive knowledge in mechanics as a driving force for meaningful learning. From this perspective, students' intuitive ideas are productive resources from which more systematic and integrated knowledge is constructed, and the process of conceptual change is a process of reorganization (rather than replacement), in which existing pieces of knowledge are modified in terms of the contexts in which they get activated and used.

diSessa (1993) and Smith et al. (1993) contend that confrontation is not an adequate model of learning. For the purpose of the study, I support their claim, arguing that cognitive conflict (Limon, 2001), also known as conceptual conflict, cognitive dissonance or discrepant event, is not a sufficient condition to initiate conceptual change in all learning situations. This stand is also shared by Grayson & McDermott (1996) who point out that

> merely placing students in a situation in which there is a conflict between their preconceptions and correct scientific concepts does not ensure that they will successfully resolve the conflict.
>
> (Grayson & McDermott, 1996: p. 564)

However, I argue that it can help perturb student thinking, thereby, throwing him in some kind of imbalance. As such, it can be hypothesized that through this cognitive conflict strategy the student will have to employ his intuitive knowledge to make sense of the situation in hand. In this respect, this strategy can help to investigate the intuitive knowledge activated by the student when he is presented with a learning task (Limon, 2001). At the same time it would be interesting to observe the conditions under which it may work in this study.

(c) Conceptual Change: Revolutionary or Evolutionary?

The debate of whether conceptual change is evolutionary or revolutionary is a highly controversial one. On the one hand, Chi & Roscoe (2002), for example, assert that the conceptual change process is revolutionary rather than evolutionary. In their framework, conceptual change occurs through the reassignment of a concept from one category to another. This appears to trigger a rather sudden shift: “Once a concept has been re-represented on a different ontological tree, the concept immediately inherits the attributes of that tree. This immediate inheritance can provide the aha phenomenon” (Chi, 1997: p. 230). On the other hand, the findings of several authors contest the views expressed by researchers such as Chi & Roscoe (2002). First, Demastes et al. (1996) shows that in two types of conceptual change, namely the wholesale change and the incremental change, the change was a gradual process. Second, Terry & Jones (1986) conclude that “there are good reasons to suppose that it is gradual and piecemeal process involving a gradual modification of existing ideas as the learner comes to terms with the meaning and implications of the new concept” (p. 298). Third, based on his previous works, Nussbaum (1989) supports the Toulminian viewpoint that conceptual change is an evolutionary process such that "global conceptions never shift all at once, but gradually, through changes of particular concepts" (p. 538). For the purpose of this study, I shall take the view that conceptual change is a slow gradual process. From this viewpoint, I also hypothesize that conceptual change seems to be a non-linear process.

(d) Adapting the 'Knowledge in pieces' Approach

Based on the literature review presented in Section 2.2.3, it is evident that diSessa's (1993) knowledge-in-pieces framework, through the notion of p-prims, is most appropriate for this study. Specifically, this theory's fine-grained quality suits the questions of this thesis, enabling productive examinations of processes of knowledge reorganisation and knowledge evolution. From this perspective, the researcher is empowered to zoom in on the learning process and analyse the cognitive dynamics of the transitions that occur in the processes of knowledge reorganisation and knowledge evolution. Moreover, Hammer (1996) asserts that the context sensitivity of students' discussion is easier to understand from the p-prim perspective than from the misconception perspective. If you ask students one question, they show one robust pattern of reasoning; now if you ask them a slightly different question or change the context, the same students will show a different pattern of reasoning (Brown & Hammer, 2008).

For the purpose of this study, I wish to borrow three key ideas from the knowledge in pieces approach. First, the grain size is roughly at the level of sentences containing students' expressions of intuitive ideas and principles in mechanics. From there onwards, as Schoenfeld (1992) propounds, it is not ethical to pick a subset of a student's actions and tell a story about them only; we must account for all detail, especially actions that seem anomalous. We must also try to identify any actions we cannot explain. Second, the notions of cuing priority and reliability priority can be used to probe into the student thinking process by analysing the identified grain sizes. For example, say we have three sentences A, B, C that have been uttered by a student, that is the three sentences are activated. On a closer analysis, we may see that sentence A increases the cueing priority of sentence C, but decreases that of sentence B. Third, there is no doubt that context plays a significant part in diSessa's framework. From this perspective, it would help tremendously to identify why the student says something. For example, what can account for sentence A increasing the cueing priority of sentence C, but decreasing that of sentence B in the setting? Is it the researcher intervention, the animation on the screen, the values generated on the screen, school knowledge or real-life experience?

2.3 Mechanics at GCE A-Level Mathematics

In this section I examine the relevant issues pertaining to mechanics at GCE A-Level Mathematics from the UK examining boards. While § 2.3.1 looks at the relevance and nature of mechanics in the mathematics curriculum, § 2.3.2 describes the difficulties often met by students learning mechanics. § 2.3.3 reviews the research on some key strategies developed to motivate student learning of mechanics. Finally, in § 2.3.4 I discuss the rationale of using motion of connected particles for the purpose of this study.

2.3.1 Mechanics in the Mathematics Curriculum

As a starting point I shall take the view that mechanics is still an established and important option of the curriculum at GCE A-Level Mathematics (Crighton, 1985; Kitchen et al., 1997). The fact that it is directly related to many aspects of everyday life and the physical world makes it an ideal area of application of mathematics for sixth form students (Collins, 1988; Mathematical Association, 1965). There also exists this suggestion (see, for example, Berry et al., 1989; Robinson et al, 2005) that by encouraging more and more students to read mechanics, we are directly helping to attract more students towards science and engineering courses.

It should be pointed out that mechanics is also taught as a compulsory component of physics at both GCE Ordinary Level (including GCSE and IGCSE) and GCE Advanced Level. Even though mathematics and physics appear as two distinct domains of knowledge, it is difficult to draw a line of distinction between mechanics as a topic in physics and mathematics (Lawson & Tabor, 1997). On this basis, it can be said that it is almost the same mechanics which is taught in both subjects. However, it is the way that mechanics is taught in both subjects that makes the difference. Woodrow (1965), writing over fifty years ago, explained that the difference between physics and mathematics can be suggested by the generalisation that physics is concerned with what happens and mathematics demands a knowledge of why it happens.

According to Adams (1872?), there are two methods of teaching mechanics:

(a) Mechanics may be treated as an experimental subject. From this perspective its principles are most likely established by experiment as unconnected facts.

(b) Certain mechanical principles may be assumed as axioms, and by a course of accurate reasoning, with the help of Geometry and Algebra, all other principles may be derived from them. From this viewpoint the assumptions made must always tally with the conclusions arrived at with the results obtained from observation and experiment.

These two methods have led mathematicians to debate on whether mechanics should be treated as an experimental subject or as a theoretical one. In a discussion at the Annual meeting of the Mathematical Association (1950), Brown, for instance, regards mechanics as an empirical subject and asserts that the laws of motion could be verified and checked by experiment. On this basis, he claims that teaching of mechanics should start with experiments. On the other hand, Snell, in the said meeting, argues that mechanics should be taught mathematically. That is, the approach in teaching mechanics should be ‘by appeal to intuition first’ and then subjected to experimental verification, when necessary, rather than in the reverse order. According to him, students must be asked first what they would expect, and then carry out the experiments to ascertain if they were correct rather than allowing them to perform the experiments first and then asking them why a certain thing occurred. Similar comments had earlier been made by Eggar (1911) who points out that in mechanics experiment must serve the purpose of making mathematical work more of a reality.

It is interesting, for instance, to note that historically Newton's laws of motion were deduced theoretically, from knowledge based on experience. It is clearly obvious from Newton's Principia that the principle of natural philosophy was deductively obtained. According to Hughes (1990), Newtonian mechanics is mathematical in the following two ways. "In the first place, it is laid out in the Euclidean manner: propositions are deduced from definitions and axioms in accordance with mathematical canons of proof. Second, and the reason why this mode of presentation is so successful, it investigates aspects of nature that admit a mathematical representation, and assumes them to be fundamental. Thus, the Principia deals exclusively with physical quantities (mass, force, velocity, and so on)" (p. 5).

From this perspective, it can be argued that the great objection to the experimental method when taken by itself, is the fact that it is impossible to simplify the circumstances of the experiment as to arrive at true results without making allowance for those circumstances. Therefore, for the purpose of this study, I argue that results should be obtained by general reasoning first and then verified by experiment. Furthermore, this approach, as suggested by Snell, incorporates the notion of intuitions in learning mechanics. This is in line with diSessa's approach which was discussed earlier.

2.3.2 Student Difficulties in Learning Mechanics

Despite the continuing relevance of mechanics there is however considerable evidence of its increasing unpopularity in the mathematics curriculum (Berry et al., 1989; Kitchen et al., 1997; Lee et al., 2005). This is due to the fact that mechanics has often been perceived by students to be a difficult option. With continuing changes in the GCE A-Level Mathematics, students can avoid studying mechanics by learning statistics – an option perceived as easy – and yet score a grade A. This is why Rowlands (2006) believes that mechanics will disappear from the mathematics curriculum within the next two decades. He therefore pleads for mechanics to be an integral component to secondary school mathematics.

Looking back in time, especially in the late 1970s, several reasons were advanced for the decline in the performance and interest of students in mechanics (see, for instance, Collins (1988)) – namely that students

a) find the subject dull and artificial;
b) have complex algebraic manipulation to perform;
c) lack understanding of basic concepts, for example, Newton's laws; and,
d) entertain misleading and erroneous ideas, for example, in equating direction of motion with direction of force.

According to Edsall (1992), the nature of teaching and learning of mechanics has practically not changed over time. Teachers explain Newton's laws rather like a set of axioms, and from them develop the theory, providing ample problems along the way with the conviction that this will be sufficient for their students to learn the required mechanical principles. Moreover, he argues that the conventional teaching and learning of mechanics closely matches the style of pure geometry where students are required to write down the facts, draw a force diagram, derive equations and then manipulate these equations to get the answer presented by the question. This is why Berry (1990) questions this style of teaching and learning which justifies his view that mechanics is 'just a specialist branch of pure mathematics'. For him, "mechanics is taught as a purely theoretical subject, shrouded with mysterious words such as particle, smooth, inelastic full of algebra and calculus" (p. 119).

Indeed skills in algebra and calculus are actually needed to solve a problem in mechanics. In general, the mathematical formulation of the fundamental principles of mechanics always provides a set of (differential) equations whose solutions depend on both the abilities of students and their knowledge of solving these equations. This is consistent with Binongo's (1995) remark that before students can begin to solve real-world problems they need a lot of mathematics. In the account given by Berry (1990), the author reports that it is often the application of algebra and calculus skills that makes mechanics a tortuous subject for many students. He claims that modern school curriculum puts less emphasis on skill development and more emphasis on problem solving.

In their paper, Berry & Graham (1991) highlight the weaknesses in students' understanding of mechanical principles though they can obtain the numerical answers correctly. In a similar vein, Mildenhall & Williams (2001) discover that though some students give the right prediction, their justification is not supported by a Newtonian perspective but by an inappropriate intuitive idea. They conclude that "this is a dangerous situation for teachers, for they may believe that a paradigm shift has taken place and that Newton's laws have been accommodated, whereas in fact a pseudo concept has been accepted without proper conceptual change" (p. 655). Arons (1979) also reports that some students can cope very well with the mathematics involved but they are not able to relate their knowledge to the world. He rightly observes that "successful solution of the problem is by no means equivalent to an understanding of the abstract concepts involved" (p. 651).

It seems to me that the major opportunity that is missed in the conventional teaching approach is a genuine attempt to understand the processes in the physical world and its assumptions. Expanding on this point Hennessy et al. (1995) suggest that conventional instruction in Newtonian mechanics is hardly conducive to facilitate students' understanding of the underlying principles. This is because it is very tempting to regard the physical world as something to be got out of the way as soon as possible, so as to be able to get on with the mathematics of the abstractions. This may be allowed at some stage for the future mathematical specialist, who will probably be proficient in pure mathematics, but if it is followed with all pupils it is likely to kill any interest in mechanics on the part of the less mathematically able ones (Mathematical Association, 1965).

The knowledge that students have misleading and erroneous ideas when learning mechanics does not date from yesterday. According to Siddons (in Mathematical Association, 1950), children bring more 'unconscious knowledge' to the mechanics class than any other subject. This intuitive knowledge arises because students form their own explanations of what is happening in the world around them from their own personal experiences. These explanations are often very similar, and it is this consistency among the students that is perhaps most alarming, because it means that different students are reaching the same incorrect conclusions about concepts in mechanics quite independently. A great deal of research into student intuitive understanding of mechanics has taken place in recent years (Clement, 1982; McDermott, 1984; Graham & Berry, 1990, 1992). Much of it has focused on what have become known as misconceptions, preconceptions or alternative conceptions. These have been discovered in all areas of mechanics, and consist of incorrect interpretations or explanations of physical phenomena. Not only have they been found right across the range of mechanics topics, but also in students of all abilities.

While reason (a) was solved through the introduction of mathematical modelling in the GCE A-Level Mathematics curriculum, reason (b) saw the reduction of the algebra content in mechanics. To ensure that students could easily grasp the mechanical principles, as mentioned in reason (c), the content of the mechanics syllabus was reduced. Yet, it would seem that solutions brought to reasons (a) – (c) were not sufficient to sort out the problems discussed in reason (d). I find that mechanics is often perceived by students to be a hard option because they bring to the classroom their own intuitive ideas about the physical world.

Nowadays it is widely accepted that almost all students usually start their mechanics instruction with a well established intuitive Aristotelian model, and that conventional instruction has no major influence on these students' intuitive knowledge. Hence, it can be argued that, when compared to the amount of intuitive knowledge students bring to other subjects, mechanics is difficult to learn and that this difficulty is preserved when the instructional strategy used cannot build a bridge between intuitive knowledge and expert knowledge.

2.3.3 Strategies to Rekindle the Learning of Mechanics

There have been numerous attempts to stimulate and improve student learning of mechanics at GCE A-Level Mathematics. In this section I wish to review key strategies that have been designed to promote student interest and learning in mechanics in the mathematics curriculum. Among them are practical work, video, investigations & coursework, mathematical modelling, concept & Socratic questions, development of materials and use of computers.

The Leeds Mechanics Group, which was formed in 1981 at the University of Leeds, developed a mechanics kit containing several pieces of equipment that were designed for practical work in the classroom via demonstrations by the teacher or investigations by sixth form students. The kit was accompanied by a teachers' guide which focused upon some of the common misconceptions held by students. Williams (1985) describes two pieces of equipment that were initially tested: an inclined plane for the study of friction and a loop model car track. It was believed that: (a) this use of equipment would increase the interest and motivation of students; and, (b) mechanical concepts could be demonstrated without the complications that real life situations can often bring. Trials with these pieces of equipment indicated that students found them motivating and also that they stimulated discussion between teacher and student about the concepts and the assumptions made in the models that described them. Building on the work of the Leeds group, the Practical Mechanics Project, formed in 1984 at the University of Manchester, has further developed practical work in mechanics and has aimed to answer some key questions such as 'What constitutes good classroom practice?' and 'What materials and guidance should students be given?' (Collins, 1988). An account of the Practical Mechanics Project's work and examples of materials has been given by Williams (1987) and Williams (1988).

Another means of motivating students to learn mechanics into the classroom was through the use of video. According to Berry (1990), the fact that the very nature of mechanics is the study of motion made the use of video in mechanics instruction an obvious potential. He ascertains that video was a means of bringing problems of real industrial and engineering significance that the teacher did not normally have access to in the classroom. By also capturing physical situations, video could provide examples from sport, leisure and everyday life. On this basis, the author argues that the video could:

(a) provide a valuable medium for introducing real-world applications;
(b) assist in the mathematical modelling of such problems; and,
(c) provide an insight into the physical interpretation of much of the mathematical theory.

A third way of making mechanics popular was through the use of mathematical modelling. Though the process of mathematical modelling is as old as mathematics itself, it is nearly two decades ago that it has been explicitly introduced into the GCE A-Level Mathematics curriculum. Many mathematicians (see, for example, Ford & Hall (1970); Medley (1982)) argue that mechanics represents a well-developed framework for mathematical modelling. It is widely believed that through mathematical modelling the learner would be able to build up an intuitive understanding of the situation being modelled. Broadly speaking, the process consists of a formulation phase, a solution phase and a validation phase (Figure 2.2).

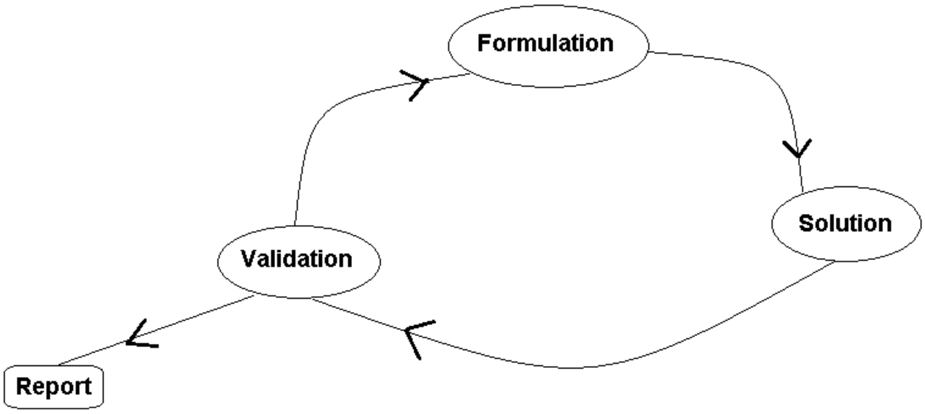


Figure 2.2 (adapted from Berry & Houston, 1995):
The three key phases of a mathematical modelling process.

Viewed through the lens of traditional mechanics teaching, students are always presented with a sequence of completed, well formulated, models for which they only need to find the one correct solution. In turn, the formulation and validation stages are almost left out. Scorer (1964) asserts that these two stages constitute the essence in mechanics.

Coursework investigations have been designed as another method of promoting greater understanding in the learning of mechanics (Edsall, 1992; Whitworth, 1988). This form of assessment involves either practical work, mathematical modelling assignments or a mixture of both. Despite the fact that they consume more classroom time and lack the rigour of external examinations, it is felt that their introduction into the examination system would provide students a means of using their mathematics to solve real-life problems. According to both authors, this measure would help students build a positive image of mechanics as modelling real-life situations, and avoid being exposed only to the stereotyped examination questions that: (a) do not allow them to discuss the arguments involved in the development of any modelling in the question; (b) remove the need to justify any assumptions they have been told or assumed they should make; (c) are almost divorced from real-life contexts; and, (d) have one and only one correct answer. Whitworth (1988) further argues that once students are placed outside an examination framework, they are not able to develop their own mathematical models. On this basis, he believes that this type of assessment should improve "students' design skills, which are increasingly being called upon in all areas of life" (p. 170). Similarly, Edsall (1992) claims that it is the coursework, rather than the written examination, that is demonstrating that they can relate the theory to the world around them. "The subject ceases to be a peculiar specialised piece of bookwork learning, and begins to explain everyday events … It also stresses to candidates that they are finding an explanation of the situation, rather than the unique solution" (p. 152).

According to Berry & Graham (1991), an important component in the learning of mechanics should be the use of concept questions. They define a concept question as a "problem designed to test student understanding of a basic concept or principle upon which the models in mechanics are based" (p. 754). Based on the intuitive approach, the concept questions may work in the following way:

(a) first, a question often exposes student intuitive knowledge;
(b) second, a group discussion, investigation or relevant practical work can help students to generate new knowledge;
(c) finally the new knowledge needs testing.

These concept questions require a qualitative approach by students preferably working in small groups; this should lead to a fairly lively discussion where each member of the group can put forward their view. Berry & Graham (1991) argue that this kind of discussion may initiate in the breaking down of incorrect intuitive ideas. During the initial discussion stage, it is important to allow the group of students time to discuss the problems themselves. Then each group may give their opinion about the problem to the whole class, thereby, giving rise to a class discussion.

The Socratic method of strategic questioning in mechanics is the asking of a concept question and a series of related parallel questions, given in response to the student's answer to the previous question but essentially having the same answer as the previous question (Rowlands et al., 1997, 1998). The Socratic method in mechanics challenges students to be consistent in their reasoning, and the much reported intransigence of student intuitive ideas may be due to the cognitive strain in forming these ideas as these students try to make sense of the questions asked.

In order to make mechanics a more appealing area of study, several groups of committed educators developed some publications that were eventually made available to the school community. First, the Spode Group (1986) developed the book ***Realistic Applications in Mechanics*** – an initiative that arose at Spode Conference Centre in December 1982. The aim of the group was to provide teachers and students a collection of 'examples that are both convincing applications of mechanics and applications that are suitable for the beginner'. Second, the Mathematics in the Everyday World (MEW) Group (1989) produced ***Exploring Mechanics***, a book with materials focusing on student understanding of key concepts and relating mechanics to real situations. Their 'What Happens If?' questions are designed to challenge student understanding of key mechanics concepts. These questions take very simple situations and ask students to describe what will happen. Many of these problems expose students to a realisation of the weaknesses of their approaches. Finally, the Mechanics in Action Project has developed a number of mechanics publications, for example ***Mechanics in Action*** by Savage & Williams (1990).

Digital technologies have also contributed in enhancing mechanics instruction (see the reviews by Stevenson (2000) and Graham & Rowlands (2000)). According to Stevenson (2000), there are four different types of computer-based environment for teaching and learning mechanics: tutoring, parameter manipulation, programming and direct manipulation environments. He finds that the first two categories adopt a model where the computer controls the process of learning with restricted student input. The final two types, he argues, are examples of open learning environments - that is, the learner has a greater control over the learning process in a computational environment in which important mathematical ideas are webbed.

One factor common to all strategies outlined above is that they trigger discussion among students and may sustain this discussion to some extent. But what are missing are the end-products and mechanisms of this discussion. For example, finding that there is discrepancy between one's prediction and one's observation will indeed trigger a discussion. Teachers may eventually end up explaining this discrepancy. But do they make use of intuitive knowledge of students to build the scientific explanation or do they replace it with the expert one? A second factor which needs close examination is what type of support is available to learners other than their teachers' and friends' feedbacks. A third factor is how we can work with friction-free situations in real-life contexts. Thus, it is not sufficient to use one or a mixture of the above strategies since we fail to see:

(a) what role intuitive knowledge plays in each strategy; and,
(b) how each strategy can blur the distinction between intuitive knowledge and the expert one.

Moreover, while I have nothing against mathematical modelling, which I find should be at the core of GCE A-Level Mathematics, it is the role that intuitive knowledge plays in mathematical modelling, especially in the formulation stage, that is of interest to me. It would appear that this part has not received much attention in the literature. There is no guarantee that discussion alone between teachers and students will suffice to bridge the gap between intuitive and expert knowledge. Needless to say, the formulation stage, which I argue can play a pivotal role in refining intuitive knowledge, has very often been left out.

Drawing on these perspectives, I find that the Predict-Observe-Explain (POE) strategy, developed by White & Gunstone (1992), is suitable to probe student understanding by encouraging students to carry out the following three tasks: first, students must predict the outcome of some event and must justify their prediction; second, they describe what they see happen; and, finally they must reconcile any conflict between prediction and observation. This well-known strategy was initially developed by Champagne, Klopfer & Anderson (1980) as

‘Demonstrate-Observe-Explain’ (DOE) to probe the thinking of first year physics students at the University of Pittsburg. Gunstone & White (1981) reworked the DOE strategy into POE. There are several research studies which have used the POE strategy in computer-based environments (Kearney, 2004; Kearney et al., 2001; Tao & Gunstone, 1999). Their findings indicate that the coupling between POE strategy and computational tool can move in the direction of building a bridge between intuitive and expert knowledge.

2.3.4 The Case Study of Motion of Connected Particles

Since mechanics provides a broad area of study and my research interest is largely influenced by the sixth form mathematics curriculum, the topic ‘motion of connected particles’ is chosen to investigate how students learn mechanics at GCE A-Level Mathematics. Motion of connected particles is a term associated with the motion of two objects connected by a coupling or a rope, which may or may not pass over a pulley. Common examples of motion of connected particles where a pulley is not involved are a vehicle towing another one, a rescuer suspended from a helicopter lifting a rescued person, and two groups of people taking part in a tug of war. Situations where motion of connected particles with a pulley can be found are on construction sites where a builder uses a pulley system to lift construction materials on upper floors of a building.

For the purpose of this study, motion of connected particles will consist of two objects connected by a string that passes over a pulley. This system is referred to as the Atwood’s machine in the physics terminology. According to the findings of several studies (see, for example, Gunstone & White (1981), McDermott (1991), Mildenhall & Williams (2001)), this device can offer a varied context for research on student intuitive understanding of mechanics. Moreover, many different concepts from both kinematics and dynamics enter into the analysis of such type of motion. For example, it can involve all three of Newton’s laws. Thus, my approach is to start at some post-basic position of understanding mechanics so that it would allow students to revise basic concepts and to progressively learn advanced concepts using those basic concepts as building blocks (see Figure 1.2).

Be it in mathematics or physics, this system of connected particles is often used as an illustration of how Newton's second law of motion can be applied to a system of two objects in which the motion of one is affected by the other. At GCE A-Level Mathematics, students are usually asked to examine one or two of the following six versions: the original (System II), the horizontal (System III), the inclined [also known as the Stevinus version] (System IV), the wedge (System VI), System I and System V (see Figure 2.3).

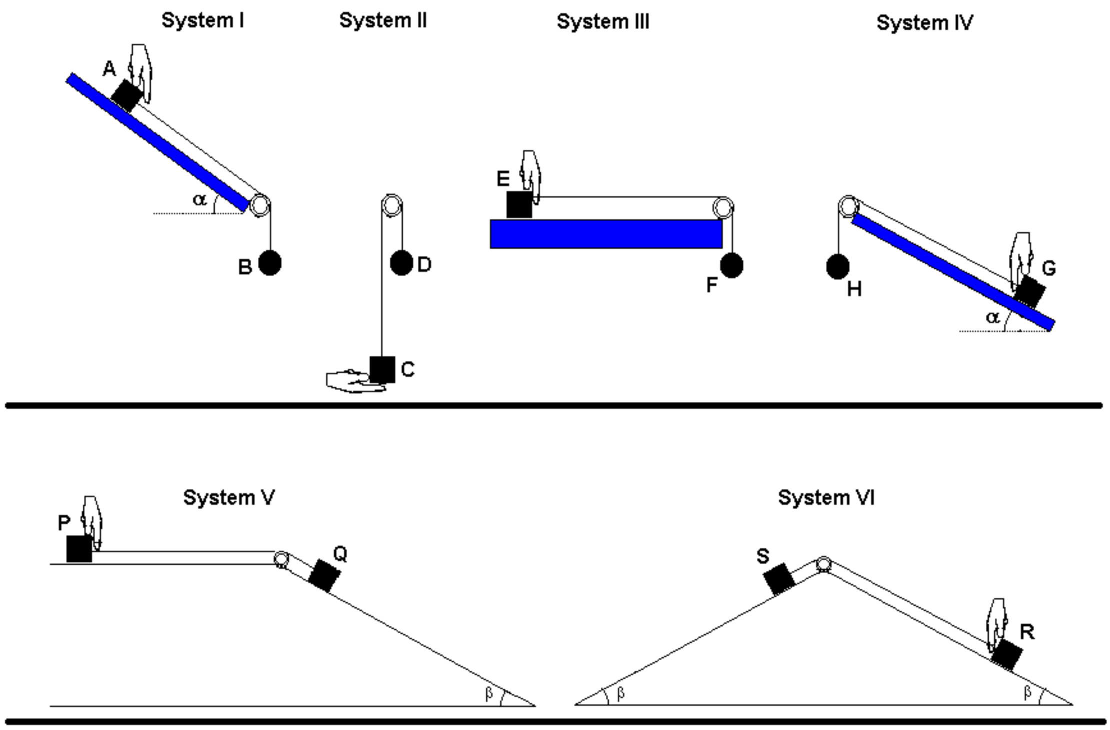


Figure 2.3: The six versions of motion of connected particles discussed at GCE A-Level Mathematics.

An analysis of A-Level examination questions from UK exam boards (AQA, CIE, Edexcel & OCR) for the period 2001 – 2008 reveals that the original, the horizontal and the inclined versions are the most popular situations for examiners (see Table 2.1). In the original version, the two objects hang vertically (System II) and the motion is in one dimension. In the horizontal version, one of the objects slides on a horizontal surface (System III) and the motion is in two dimensions. In the inclined version, one of the objects lies on an inclined plane (System IV) and the motion is in two dimensions. On this basis, this study will concentrate on these three chosen contexts.

	AQA Spec A	AQA Spec B	CIE	Edexcel	OCR
June 2001			System V		
Nov 2001			Wedge		
Jan 2001⌖	(no exams in mechanics)	Horizontal		Vertical	Horizontal
June 2001⌖	Vertical	Vertical		-	Inclined ⇨ Horizontal
Nov 2001⌖			Vertical		
Jan 2002	-			Horizontal	Vertical
June 2002	Horizontal	Horizontal	Vertical	System V	-
Nov 2002	Vertical	Vertical	-	Inclined	Vertical
Jan 2003	Horizontal	Vertical		Horizontal	Horizontal
June 2003	-	Vertical	-	-	-
Nov 2003	Vertical	Horizontal	Horizontal	Inclined	Horizontal
Jan 2004	Vertical	Inclined		Inclined	-
June 2004	Horizontal	Horizontal	-	-	Vertical
Nov 2004	Horizontal	Vertical	Horizontal	Horizontal	*missing*
Jan 2005	Vertical	Inclined		Horizontal	Inclined
June 2005	Horizontal	Horizontal	Horizontal	-	Vertical
Nov 2005			Vertical		
Jan 2005*	Horizontal	Vertical			
June 2005*	Vertical	Vertical			-
Jan 2006	Vertical	Vertical			Vertical
June 2006	Vertical	Horizontal	Horizontal		-
Nov 2006			-		
Jan 2007	-	Horizontal			Horizontal
June 2007	Vertical	-	Vertical		
Nov 2007			Inclined		
Jan 2008	Horizontal	-		Horizontal	Vertical
June 2008			Horizontal	-	
Nov 2008			Vertical		

Table 2.1: Analysis of UK A-Level examination questions between 2001 and 2008.
[⌖ denotes the start of a new syllabus]

Motion of connected particles is usually introduced via a detailed written description and force diagram of the setup. The description includes abstractions that may not have much meaning for the student: a massless, inextensible string and a massless, frictionless pulley. The connected particles is the first problem in the course that explicitly involves the tension in a string in a dynamic situation. In spite of the inherent complexity, the aims of the question asked are straightforward: find the acceleration of the masses, the tension in the string and possibly the force exerted by the string on the pulley.

The procedure for solving the problem during a conventional teaching is straightforward. Students are shown how to isolate the blocks and draw appropriate force diagrams. The teacher may state several assumptions: (1) the blocks are considered to be particles so that their weights act at a single point; (2) the string must be inextensible so as to ensure that the accelerations of the blocks are equal in magnitude; (3) the string must also be massless; and, (4) the pulley must be fixed and smooth. Without explicitly discussing the constraints imposed by the string, the teacher demonstrates how to apply Newton's second law to each body and proceeds to derive the two simultaneous equations that can be solved for the two unknown variables - tension and acceleration.

According to McDermott et al. (1994), once the equations have been introduced, there is a common tendency to avoid thinking critically about the physical situation. When asked to determine what effect certain changes in the system have on the motion, most students focus on the algebra rather than try to reason qualitatively. As a result they generally fail to recognise two important implications of the second law: (1) The presence of an acceleration indicates that an unbalanced force is acting in the same direction as the acceleration; and (2) the existence of the tension in the string limits the magnitude of this acceleration to a value smaller than in free fall.

2.3.5 Student Understanding of Motion of Connected Particles

In this section I review the literature on student understanding of motion of connected particles. This review has been categorised according to the three popular versions and will have important implications to be discussed in § 2.3.6. It should be noted that no review exists for the three least popular versions.

(a) Student Understanding of System II

Several researchers (for instance, Berry & Graham, 1991; diSessa, 2002; Gunstone, 1987; Gunstone & White, 1981; McDermott & Somers, 1992; Watts & Zylbersztajn, 1981) have reported on student understanding of the original version of connected particles (System II). Apart from diSessa (2002) who has examined the nature of student intuitive understanding, all the rest have reported how students perceive this system under different conditions.

The pioneering work of Gunstone & White (1981) reveals an unexpected set of student responses concerning the following situation: a bicycle wheel is mounted as a pulley with its axis about 2 m above the bench, and a cord, connecting a block of wood and a bucket of sand, of equal masses, is placed over the pulley. Their sample of students, reading for their first-year undergraduate courses at Monash University, are then asked various questions, including ones that required making predictions as to what would happen when certain changes would be made, and they are asked to write out the reasons for their answers.

(a) In the first experiment, after having shown the students that the pulley could rotate freely, the cord was placed over the pulley in such a way that the bucket was markedly higher than the block, and the system remained stationary. The participants were then asked to compare the weight of the bucket to that of the block. 27% of the sample stated that the block was heavier because it was nearer to the floor.

(b) In the second experiment, the students were asked to predict the behaviour of the system if a large scoop of sand were added to the bucket. Among the 54% of the participants who correctly predicted what would happen, some of them incorrectly predicted that the magnitude of the acceleration of either the bucket or the block would be equal to the acceleration due to gravity (a = g), or that either object would move with constant speed. Among the rest there was a predominant belief that the system would shift to a new equilibrium position with the bucket closer to the table and the block higher up. This notion was shown by 30% of the participants.

(c) In the third experiment, the students were required to predict the speed of the bucket at two marks. Although 90% correctly predicted that the speed would be greater at the low mark, the prediction of some students was based on their belief that the gravitational force acting on the bucket increased as the bucket lowered. Others stated that the acceleration of the bucket would be g.

(d) In the fourth experiment, the block of wood and bucket of sand, both of equal masses, were placed on the pulley in such a way that they hung freely at the same level. The block was then pulled down about 0.7 m and held. At this point the participants were asked to predict what would happen when the block was released. While 54% of the sample correctly predicted that the system would remain stationary, 35% maintained that the system would return to its original position. It was equally noted that many students saw equilibrium "as some sort of real entity contained in objects rather than as a description of a particular physical state" (p. 296).

The result obtained by Gunstone & White (1981)'s fourth experiment was confirmed by the works of both Watts & Zylbersztajn (1981) and Roper (1985). The former report that 78% of their sample students thought that two equal objects suspended by a string over a pulley would freely move to achieve equal levels. This is confirmed by the study of Roper (1985). Berry & Graham (1991) find that out of 632 sixth form students only 51% were able to correctly state that the string exerts the same force on each object, recognising that the tension in the string is constant throughout.

The work of McDermott & Somers (1992) identifies a number of specific difficulties encountered by the students they had interviewed in the area of the original version. They find that students fail to:

(a) recognise that the acceleration of the two objects in a single system does not depend on a difference in their heights;
(b) distinguish between the weight of the hanging object and the tension, and to recognise that it is the tension that acts on each object, not the weight of the other object;
(c) recognise that the tension in a massless string is the same throughout its length and that the string exerts the same force on the objects attached to each of its ends;
(d) recognise that, when there is an acceleration, the magnitude of the tension in the string must lie between the magnitudes of the two weights; and,
(e) recognise that accelerations, not forces, must be used to compare motions and that therefore mass must be taken into account.

On interviewing extensively one female freshman college student, whom he calls J, diSessa (2002) carries out an experiment similar to the second one of Gunstone & White (1981). In his version, he provides the student with a symmetrical picture of two large and equal objects attached to a rope which passes over a pulley, and asks the student to predict what happens when a small object is added to either larger object. Initially she states that the single object would move up and the combined objects would move down a certain distance before again coming to rest. When asked to justify her argument, she just changes her prediction by now claiming that the system would not come to rest but would continue to move until the combined objects hit the surface. She again could not provide any rationale for her reasoning. In order to probe J's thinking, diSessa uses the general properties of his p-prim theory to explain many details in this episode. According to him, "it uses a previously documented p-prim (generalized springiness) to explain a counter-factual prediction; it explains (momentary) sensemaking; it explains "losing track" as a general phenomenon involving p-prims (data fluidity); it explains inability to articulate" (p. 42).

(b) Student Understanding of System III

Research on student understanding of the horizontal version of connected particles (System III) has been conducted by several researchers (for instance, diSessa, 1993; Gunstone & White, 1981; Grayson & McDermott, 1996; McDermott, 1991; McDermott et al., 1994; Mestre, 2002; Mildenhall & Williams, 2001).

Using the same materials as in their four earlier experiments, Gunstone & White (1981) placed the block of wood on a smooth horizontal surface. The block and the bucket of sand, both of equal masses, were again connected by a piece of cord passing over a pulley with the bucket hanging freely. The same group of university students were asked to predict what would happen when the block was released. Out of the 289 participants, 24% predicted that the system would remain stationary because they believed that 'the weights are equal and hence cancel', or 'friction between the block and the horizontal surface will prevent motion', or a combination of these two.

Based on her work, McDermott (1991) finds that several students have many difficulties related directly to tension and acceleration - the two unknown quantities that students are asked to find. She explains four important conceptual and reasoning difficulties that students had experienced when tackling tasks based on the horizontal version. She points out that students fail to:

- (a) recognise that the magnitudes of the velocity and acceleration of the hanging and sliding blocks must be equal;
- (b) distinguish between the weight of the hanging block and the tension and to recognise that the tension, not the weight, act on the sliding block;
- (c) recognise that the tension acts on the hanging block while it is falling and thus the acceleration must be less than that of free fall; and,
- (d) recognise that the weight of the hanging block must be greater than the tension when the system is accelerating.

In another study (McDermott et al., 1994) it is also seen that students often fail to recognize that the frictional force on the sliding block does not act directly on the hanging block. McDermott and her colleagues suggest that effective conceptual questions should be part of the curriculum materials.

diSessa (1993) reports that when novices are asked to find the tension in the string, they usually assign tension to the string as if it were holding the mass at rest. They do not realise that the tension in the string is less than the static situation because there is acceleration. According to him,

> a static problem solution is here applied uncritically to a dynamic situation, which suggests that the knowledge system either does not easily recognise dynamic situations or does not know that they are essentially different from static one. (p. 163)

Mestre (2002) suggests that such belief is the outcome of the physical arrangements of the objects in the diagram – the geometry of the set-up cues the erroneous interpretation that the weight of the hanging object is simply transmitted to the attached string.

Mildenhall & Williams (2001) use both the 'two mass problem' (that is, System III) and the 'three mass problem' (Figure 2.4) to analyze students' use of intuitive and Newtonian models to predict motion.

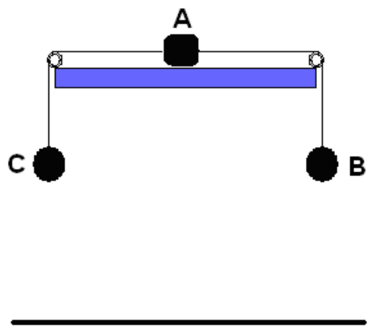


Figure 2.4: The three-mass problem of motion of connected particles.

Their findings reveal that:

(a) students describe three phases of motion: 'Static' phase (when the system does not move); 'Aristotelian' phase (when the system moves at a constant speed); and, 'Newtonian' phase (when the system accelerates);

(b) students are more at ease to work with situations involving rough surfaces than with smooth surfaces because to be completely free from friction is outside students' normal experiences and therefore of intuition.

(c) students hold three common beliefs: balance intuition, height intuition and reluctance intuition. (i) the balance intuition implies that a student thinks that the system is in equilibrium if the sums of the masses about some arbitrary point are equal; (ii) the height intuition means that the student assumes that gravity increases significantly as height decreases; and, (iii) the reluctance intuition suggests that a student believes that the magnitude of the applied force must be 'big enough' before motion can take place.

Their findings confirm that some students develop a 'hybrid mental model' in which both the Aristotelian and the Newtonian models exist. It was the problem parameters that would decide which model was to be chosen by the student. On this basis, they suggest developing instructional strategies that will prove to be effective.

> The fact that the choice of the mental model does depend upon the problem parameters might suggest a role for explicit discussion of our intuitions in the classroom; not so that they can be 'replaced', 'exchanged' or 'overcome', but so that they can be critically evaluated (p. 655).

This conclusion firmly advocates that:

(a) eliciting the intuitive knowledge of students should be at the heart of the learning process;
(b) it is a waste of time to eliminate intuitive knowledge; and,
(c) intuitive knowledge has a key role in creating new knowledge.

According to Arons (1997), after having obtained the algebraic solution for the system, students must be encouraged to bring out the mechanics by interpreting their results. They should be asked to predict what would happen to the acceleration of the system and the tension in the string if the mass of the hanging object were made very much larger (or smaller) than that of the sliding object. However, this strategy appears to be in contradiction to the observation made by McDermott et al. (1994), as mentioned earlier, that once equations are introduced, most students focus on these symbolic representations rather than try to reason qualitatively.

(c) Student Understanding of System IV

Reif (1995) discovers that several students claim that the tension in the string is simply equal to the weight of the hanging block in the inclined version, implying that these students have totally ignored the acceleration of this block.

2.3.6 Implications for this Study

From the review in the previous section, I find that understanding motion of connected particles and, therefore, its learning as well can be analysed in terms of relationships among key variables. For example, the relationship between the tension in the string and the weight of the hanging object is an important intuitive idea in the learning of motion of connected particles. For the purpose of this study, I will coin the term 'Connected Particles category' to imply such relationship. Depending on the student's intuitive knowledge, the above mentioned 'Connected Particles' category can be either a building block or a stumbling block to learning the topic.

	'Connected Particles' categories	**Original Version [System II]**	**Horizontal Version [System III]**
1	Relationship between tension & weight: (Tension = weight)	McDermott & Somers (1992)	McDermott (1991) diSessa (1993) Mestre (2002)
	Relationship between tension & weight: (Tension doesn't pull sliding object)	McDermott & Somers (1992)	McDermott (1991)
2	Tension not the same throughout string	Berry & Graham (1991) McDermott & Somers (1992)	
3	Effect of friction on motion		Gunstone & White (1981) McDermott et al. (1994)
4	Effect of mass on motion	McDermott & Somers (1992)	Gunstone & White (1981)
5	Effect of mass on height	Gunstone & White (1981) Roper (1985)	
6	Magnitude of Acceleration (a = g)	Gunstone & White (1981)	McDermott (1991)
7	Kinematical Aspects of Acceleration (both objects don't have same acc.)		McDermott (1991)
8	Kinematical Aspects of Velocity (both objects don't have same vel.)	Gunstone & White (1981)	McDermott (1991)

Table 2.2: The 8 categories act as intuitive ideas to learn motion of connected particles.

It appears (refer to Table 2.2) that eight ‘Connected Particles’ categories can be identified from the literature review. I argue that these eight categories can be used in two distinct ways: (1) They can help in designing the activities for this study; and, (2) They can be used as a starting point for analysis of data.

Since a part of this thesis will have to investigate the knowledge activated by students when they are presented with a learning task, it would be interesting to find out if students taking part in this study show similar intuitive ideas as those presented in § 2.3.5 and ‘Connected Particles’ categories just described. At the same time, it is imperative to explore whether students come up with new intuitive ideas and new ‘Connected Particles’ categories on motion of connected particles; and to examine whether they are building blocks or stumbling blocks to learning this topic.

2.4 Computer-Based Learning Environments

In this section I explore the possibility of using computational tools in the learning of mechanics at GCE A-Level Mathematics. While § 2.4.1 examines the notion of microworld for developing a computer-based environment for learning mechanics, § 2.4.2 attempts to explain the process of building new knowledge through situated abstraction and webbing. In § 2.4.3 I review the possible conditions under which animations may benefit student learning. Finally, in § 2.4.4 I justify the rationale for using the software Interactive Physics for this research study.

2.4.1 The Notion of Microworlds

A microworld can be regarded as an exploratory learning environment in which learners are actively engaged in cognitive processes for sense making (Papert, 1980). Since the early 80s, several microworlds have been designed and studied. Over the years, the term ‘microworld’ has been used in numerous ways within the mathematics and science education communities (Edwards, 1995).

Although different conceptions of microworlds exist, we can see the emergence of two central ideas. First, a microworld is a learning environment which can be both computer-based and interactive. Second, students are provided with the opportunity to actively participate in building their own meanings and to control the process of their learning through exploration of the range of possibilities and constraints that the microworld offers. As such, by interacting with the microworld, students come to understand its features - features which have been 'planted' according to a priori learning objectives. This is consonant with Hoyles' remark that

> at the core of a microworld was a knowledge domain to be investigated by interaction with the software . . . they [microworlds] aimed to facilitate the building of conceptual and strategic foundations - from simple entry points to deep ideas.
>
> (Hoyles, 1993: p. 3)

However, it should be pointed out that the microworld does not consist of the computational tool only. Hoyles & Noss (1987) define the microworld as a computer-based learning environment which consists of four aspects: the technical, the pedagogical, the pupil and the contextual. These four elements take into account the computational tools, the learner, the teacher, the setting and the analysis of the interactions which take place between them. While the technical element of the microworld consists of computational and non-computational components needed by the activities associated with the microworld, the pedagogical element of the microworld describes the aims of the microworld, the type of activity which learners engage in, and the pedagogical strategy adopted. The pupil element of the microworld refers to the knowledge and experience which children bring with them to the classroom. Finally, the contextual element describes the social setting in which the activity takes place.

Hoyles & Noss (1987) assert that the relationship between the student, computational tool and knowledge domain needs to be considered if we want to have an authentic meaning of microworlds. From this, it is vital to take into consideration students' preconceptions and intentions in designing a microworld. Hoyles et al. (1991) go on to suggest the need to anticipate students' initial conceptions of the embedded mathematics and make probes during student-computer interactions. On this basis, they explain that mathematical activities should be designed for two purposes. First, they should attempt to reveal students' meanings, strategies and intuitive mathematical frameworks. Second, they should encourage students to build ever greater understanding of the mathematical knowledge domain.

> A microworld consists of software designed to be adaptable to pupils' initial conceptions together with carefully sequenced sets of activities on and off the computer, organised in pairs, groups or whole classes each with specified learning objectives . . . There needs to be a priori analysis of the knowledge domain together with a predictive framework concerning the pupil conceptions of this domain.
>
> (Hoyles et al., 1991: p. 3)

It would appear that this remark of Hoyles et al. (1991) is in line with some researchers' (see, for example, diSessa, 1993; Clement, 1994; Hammer, 2000) claim to encourage students use their intuitive knowledge in the construction of new knowledge. This certainly suggests that the microworld must be able to integrate the informal and formal knowledge, thereby giving learners the choice of moving to and fro between the concrete and the abstract. Therefore, the microworld approach, unlike conventional pedagogies, underlies a knowledge construction metaphor of learning in which learners have the opportunity of constructing new mathematical knowledge rather than passively accepting already constructed ones.

2.4.2 Abstraction, Webbing and Situated Abstraction

The question of how learning takes place within microworlds can be approached from different angles. In this study, as stated earlier, I shall take the views of both diSessa (1988;1993) and Hoyles et al. (1991) that intuitive knowledge plays a pivotal part in the learning process and, therefore, interpret the evolution of learning under such an assumption. I shall also take the view of Mayer & Moreno (2002) that cognitive constructivism "depends on the learner's cognitive activity, not the learner's behavioural or social activity" (p. 110). From this viewpoint, hands-on activity (such as lab experiments) and social collaboration (such as working in pair) are not necessary conditions for constructivist learning to occur.

It is no secret that in mechanics intuitive knowledge is derived from daily-life experience which is context-depended. As such, context is sine qua non for building expert knowledge (see, for example, Levrini & diSessa, 2008). This is in total contrast to the conventional view of teaching and learning mathematics, which involves context-free abstraction. Empiricists view abstraction as an activity where learning may be based initially on the situated, concrete and informal, but must ultimately reach the level of context-independent, abstract and formal knowledge (Noss & Hoyles, 1996). This appears to be an ascent from the concrete to the abstract (Davydov, 1990 cited in Ozmantar & Monaghan, 2008). This view suggests that this process of abstraction is a linear one.

However, we have recently witnessed the emergence of new ways of viewing abstraction (Boero et al., 2002). van Oers (2001, cited in Ozmantar & Monaghan, 2008), for example, is against the idea of decontextualisation as the basis for abstraction. According to him, abstracting is a process of contextualising an experience and, therefore, decontextualisation implies that the learner has been removed from this process. In a similar vein, Noss & Hoyles (1996) also criticised the hierarchical and decontextualisation views of abstraction, evoking the work of Lave (1988) to support the idea that all knowledge is situated. Consequently, they argue that abstraction can be situated and, on this basis, redefine abstraction as the process of establishing new connections among objects and experiences. From this viewpoint, abstraction allows for the construction of mathematical meanings that connect with both mathematical objects and real-life objects or contexts. According to Ozmantar & Monaghan (2008), the ideas of van Oers and Noss & Hoyles are compatible with Davydov's (1990, cited in Ozmantar & Monaghan, 2008) 'method of ascent' such that "this is not an ascent from the concrete to the abstract but a dialectical relationship, a to and fro, between the concrete and abstract" (p. 109). In this study I adopt Noss & Hoyles (1996) idea of 'abstraction – mathematical meanings' which views it through the following two new theoretical constructs: the notions of webbing and situated abstraction.

Noss & Hoyles (ibid.) use the idea of webbing to describe the ways that learners choose as appropriate for the construction of new mathematical knowledge by drawing upon the resources provided by a computational environment during the activity and in reflection on it. On this basis, Pratt & Noss (2002) argue that the term webbing highlights "the central significance of tools as external resources that have been observed to be highly dependent on the particular attributes of those tools as cognized by students" (p. 456). From this view, it appears that the computer-based environment, together with the learner's intuitive and existing school knowledge, will strongly influence the kinds of actions and solutions that may be possible.

The second theoretical construct is called situated abstraction – it allows us to see the type of knowledge that emerges during the activity. According to Pratt & Noss (2002), "situated abstractions emerge during activity as internal resources that serve as relatively general devices for making sense of situations that arise within a setting" (p. 456). This view suggests that learners build abstractions within specific computational settings, rather than abstract the key ideas from the activity and learning them in a formal way. Noss & Hoyles (1996) explain that in such settings meanings are not only preserved but also extended during evolution of learning. As learners exploit the computational tools to focus their attention onto both relationships and objects, these meanings become reshaped.

2.4.3 Learning with Animated Diagrams

In this section I discuss several key issues related to dynamic (or animated) and static media. While a static diagram is a spatial representation in which information is communicated by spatial properties, an animated diagram is a sequence of static images, which when played at a relatively rapid rate, produces the effect of apparent motion. It is believed that both static and animated illustrations can be more or less isomorphic to reality (Hegarty, 2005). However, a static diagram never moves. If learners are asked to understand how a machine works from a static diagram, they will have to infer the motion of the device from the information given in the static diagram. This process of inferring motion from static diagrams is known as mental animation (Hegarty,

1992) or internal representation (Scaife & Rogers, 1996). On the other hand, comprehension of how this machine works from an animated diagram depends on learners' abilities to perceive the motion of the device rather than to infer this motion. The animation we see on the computer screen is known as external animation (Hegarty et al., 2003) or external representation (Scaife & Rogers, 1996).

It is often taken for granted that animated diagrams, which can present information about movement in a more explicit way, have enormous potential to benefit student learning as compared to static diagrams (Bodemer et al., 2004). However, previous studies have found that when both media convey the same amount of information there is no benefit of animated diagrams over static ones (see, for example: Lowe, 1999; Hegarty et al., 2003). Moreover, in a review carried out by Bétrancourt & Tversky (2000), they find that only seven of the twelve experimental studies show a rather weak effect of the animations. In this case one major factor that accounts for the slightly better performance with animations rather than with static illustrations is that there is greater information in the animated diagrams than in the static displays. For example, while the animated version shows all the micro-steps of the described process, the static version includes only one illustration of the process (Tversky et al., 2002). Two other factors – interactivity and prediction – are found to have facilitated learning in these animation studies. According to Tversky et al. (2002), these two factors must not be confused with animation and "most, if not all, of the successes of animation seem to be due to advantages in extra information conveyed or additional procedures, rather than the animation of the information per se" (p. 255).

Several authors (Hegarty, 2005; Tversky et al., 2002) find it surprising and intriguing that animations, which successfully convey change over time, are failing in the educational arena. It is argued that providing visually explicit dynamic information is not a sufficient condition for facilitating understanding of dynamic processes (Ploetzner & Lowe, 2004; Price, 2002). According to Hegarty (2005), there appears to be four reasons that can explain the ineffectiveness of animations. First, due to the transient nature of animations, the displayed information on the screen changes continuously and this means that learners can view it for a limited amount of time (Lowe, 1999). This raises the question of whether or not learners are able to keep up with the pace at which the animation is presented. Second, if during the animation there are several changes that occur simultaneously in different regions of the screen, we will be able to pay attention to one location in a screen, not to every change in other regions of the screen (Rensink (2002, cited in Hegarty, 2005)). Price (2002) also finds that the use of multiple representations can result in cognitive overload due to an unmanageable increase of available information. Third, viewing an animation is considered a passive process whereas developing a mental animation is a more active cognitive process. This is based on the self-explanation effect (Chi et al., 1994) – students learn more effectively if they are required to generate ideas or explanations. Fourth, there seems to be a mismatch between an external animation and a mental animation. It is argued that a machine, which is in motion, has many of its components moving simultaneously. However, when learners start to build mental animations, they infer the motion of components one by one, in order of a causal chain of events (Hegarty, 1992).

As mentioned earlier, animations can become more effective if the notions of interactivity and prediction-feedback are integrated with the animations. According to Bétrancourt (2005), there are two kinds of interactivity: control and interactive behaviour. While control is the capacity of learners to manipulate the pace of the animation (for example, rewind, forward, step-by-step play), interactivity encourages learners to predict hypotheses and test them by varying the parameters of the mathematical model underlying the system. In this case, the animation becomes a simulation that is used in a discovery-learning approach. With regard to the first type of interactivity – Animation Control – it is believed that by altering the pace of the external animation, the learners' comprehension processes can keep pace with the speed at which the external animation presents its information. Stopping, starting and replaying an animation can allow reinspection, focusing on specific parts and actions (Tversky et al., 2002). This is supported by the finding of Boucheix & Guignard (2005) which shows that when learners have control over the presentation of information, it can have a lasting effect on their understanding. These authors argue that by self-controlling the pace of presentation learners are more able to integrate information and becomes more active in the task. They further state that the “controlled rhythm seems especially more adequate when the processing demand of the task and the cognitive load increase” (p. 383).

Broadly speaking, there are three main types of graphical change that can occur during an external animation: (i) translation change (or extrinsic characteristics) refers to movement from one location to another; (ii) transformation change (or intrinsic characteristics) refers to change in size, shape, orientation; and, (iii) transition change (or feature presence) involves the appearance or disappearance of entities either fully or partly (Lowe, 2003; Price, 2002). In this respect the findings of Price (2002) appear to raise two salient issues for my study. First, though she detects all three types of graphical change in her data, she finds that there are 2 different kinds of transformation change – the first one involves gross motion change and the second one looks at colour change. So it is interesting to see what changes may be occurring in my study and if within one change we could see differences in processing demands. Second, after fragmenting the learning pathway into four principal levels – a visual graphical level (perceiving and identifying changes); an interpreting level (making accurate interpretations of the changes); an assimilation level; and, a conceptual understanding level – she discovers that students find it easier to identify changes than to interpret them. This is similar to Lowe’s (2003) finding that though some learners can extract some potentially useful information from the animation, they cannot retain and incorporate them into their knowledge structures. One way of interpreting both findings is that perhaps these students are not willing to interpret the changes because they do not accept it. This raises one significant issue for my study: once the changes have been identified, are students willing to accept the changes? If yes, do they accept them before or after interpreting them? Unlike Price’s study, the domain knowledge of my study is mechanics and it seems that intuitive ideas in this area appear relatively robust compared to biological ones (Chi et al., 1994). Conceptual change, I argue, is a subjective process – it is ultimately up to learners to decide whether or not they will be involved in meaningful learning. Therefore, accepting the changes is an important step in the learning process, but should it become before or after interpretation is something which we need to consider.

The studies of Lowe (1999, 2003) and of Schnotz & Grzondiel (1999) point out that if students wish to achieve a deep understanding with a static illustration, they must be able to animate the system mentally. If they are confronted with a dynamic illustration, the animation is realized externally 'instead of being realized by participants' which are then more 'passive' (Fayol, 2002; Jamet, 2002 cited in Boucheix & Guignard, 2005). However, Boucheix & Guignard (2005) hypothesize that "a static format could be more efficient than a dynamic format only if the learner possesses appropriate prior knowledge on the dynamic movement to be anticipated, in other words, if he can elaborate spontaneously an internal dynamic representation from a static representation" (p. 373). From this perspective, I concur with Hegarty (2004) that an external animation must not substitute for a mental animation, but I shall also take the view of Boucheix & Guignard (2005) that the goal of any external animation should be in helping students, who cannot mentally animate static diagrams, develop their mental animations by filling in the lack of appropriate knowledge.

2.4.4 Using Interactive Physics (Version 3.0)

The computational tool Interactive Physics can be considered as a Direct Manipulation Environment for the teaching and learning of mechanics at GCE A-Level Mathematics (Stevenson, 2000). It is a computer-based Newtonian microworld that allows users to design and run simulations of real-world physical events. Owing to its powerful and easy-to-use graphic interface, it enables the user to create a system by drawing onscreen, set its initial conditions, and observe its dynamics according to the laws of motion. More importantly, this software allows the learner to view, measure and record the properties of motion of each defined object. For example, measurable quantities such as force, displacement, velocity or acceleration can be represented by means of vectors or instruments such as strip-chart recorders and digital and analog meters. In a way, Interactive Physics can be regarded as a 'real' model of the theoretical field of Newtonian mechanics just as Cabri-géomètre is to that of Euclidean geometry (Laborde & Laborde, 1995).

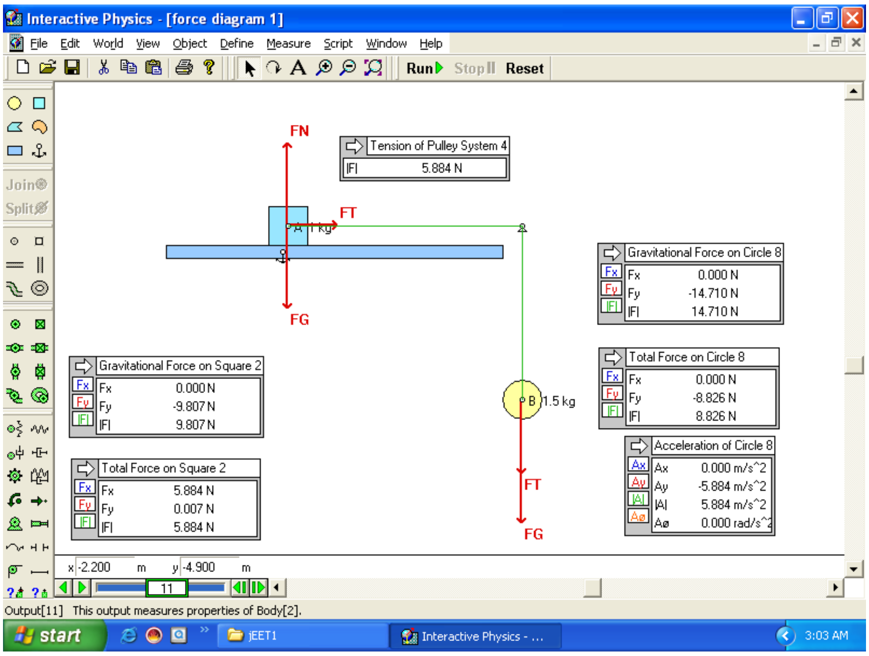


Figure 2.5: A screenshot of the software Interactive Physics showing a horizontal version of motion of connected particles, forces acting on each object and 6 digital meters.

Figure 2.5 shows two objects, one lying on a surface [the sliding object] and the other hanging in the air [the hanging object], connected by a rope which passes over a pulley located at the one end of the surface. There are six digital meters on the screen – one displays the numerical value of the tension in the string (TENSION OF PULLEY SYSTEM); one provides the acceleration of the hanging object (ACCELERATION OF CIRCLE); two others give the resultant force acting on each object (TOTAL FORCE ON CIRCLE; TOTAL FORCE ON SQUARE); and, the remaining two meters quantify the weight of each object (GRAVITATIONAL FORCE ON CIRCLE; GRAVITATIONAL FORCE ON SQUARE). An alternative way of visually displaying the resultant force on either object is through the vector arrow **FT** whose length approximates the magnitude of the said force. Interestingly, it also shows the direction in which this force is acting. While the vector arrow **FG** represents the weight acting on each object, the vector arrow **FN** depicts the normal reaction on object A by the surface. Since each of these arrows is representing a quantity that remains constant during motion, the lengths of these arrows remain unchanged. If, for instance, we put an arrow to represent the velocity of the sliding object – according to Interactive Physics, the arrow will be called **V** – we should see that as the object moves along the surface, the length of the arrow increases, implying that velocity of the sliding object increases during motion.

From the screenshot provided in Figure 2.5, we can see: (1) the external animation; (2) the six digital meters and their respective components; and, (3) the arrows representing specified quantities. Thus, we have three different types of external representations (that is, multiple representations) for the same system of connected particles. This situation, according to Kaiser et al. (1992), can result in cognitive overload because of too much information. However, we do not need to put all of these simultaneously on the screen. Students have the choice of selecting what they require, according to their learning needs. Another salient point that can be inferred from this screenshot is the type of graphical change we can see in an animation of Interactive Physics. It is clearly seen that the most important graphical change to be noticed is the translation change and we may be seeing some moments of transition change.

To create such a system (as shown in Figure 2.5) on Interactive Physics, students require no programming and need not be conversant with the algebraic equations of the system. By direct manipulation, they can create the system by selecting the appropriate drawing tools from the tool bar which is located on the left hand side of the screen. Out of the 38 drawing tools, they are required to be familiar with ten only. Once bodies and constraints have been used to develop the model, it is necessary to specify conditions such as the masses of both objects, the magnitude of the acceleration due to gravity, air resistance and the nature of contact between the sliding object and the surface. These specifications can easily be brought by either changing appropriate parameter(s) in the PROPERTIES box of the object under scrutiny or selecting the required menu (WORLD or OBJECT) at the top of the screen. In this particular example where we are interested to study how the tension in the string, the acceleration of either object and the resultant force acting on either object vary as the objects move, the six meters are easily made available through the menu MEASURE and dragged to the desired locations. The vector arrows appear on both objects by interacting with the menu DEFINE. To run and stop the simulation, either the commands located at the top of the screen or the buttons on the mouse can be used. The simulation may also be restarted by using the command RESET. In the event that the simulation happens too quickly for proper observation to occur, an alternative way is to use the forward step in the tape player control (which is located in bottom part of the screen) to move the simulation forward frame by frame instead of the command RUN. Thus, both types of interactivity – animation control and interactive behaviour – are already essential features of this microworld, and based on the review discussed in § 2.4.3, it can be hypothesized that the software Interactive Physics may have a strong impact on the learning process.

The implementation of this tool in the science curriculum has been reported in Holec et al. (2004), Pinto & Garcia (2005) and Woodrow et al. (1996). Several researchers such as Doerr (1994; 1997), Jimoyiannis & Komis (2001), Masson & Legendre (2008), Masson & Vazquez-Abad (2006), Potvin et al. (2003), Roth (1995), Roth et al. (1996) and Stevenson (2002) have made the software Interactive Physics the focus of their studies. I find that the use of Interactive Physics as employed by the mentioned researchers can broadly be classified into three groups which are of particular interest to the learning of mechanics in this study. These classifications are: (a) absence of symbolic mathematics; (b) multiple representations; and, (c) initiating and sustaining discussion.

(a) Absence of Symbolic Mathematics

Stevenson (2002) examines the role of Interactive Physics in the transitions between naïve realism and mathematical modelling of the original version of motion of connected particles. He argues that this is possible because the software does not display the equations of motion that control the behaviour of the mechanical system on screen, and this fact makes it an ideal computational tool for challenging students to build the mathematical model of the screen system, and verify that the solution of that model matches the outcomes of the tool. From this perspective, the software can help students identify key variables and hypothesise important relationships among them.

Adapting Figure 1.1 (in Chapter 1) to fit in Interactive Physics results in the Figure 2.6: SYSTEM represents the mechanics problem under investigation;

DIAGRAM, which is the first step of solving the problem, depicts the static diagram of the mechanics problem; FORCES, the second step, is about displaying all the forces acting on the particles involved in the problem – it is constructing a force diagram; had we not used Interactive Physics, the third step would have been to derive the equations from the force diagram and then use algebra/calculus/trigonometry to find their solutions. Since we now use Interactive Physics, the third step is the external animation as well as the solutions (via digital meters, for instance) to the mechanics problem under investigation.

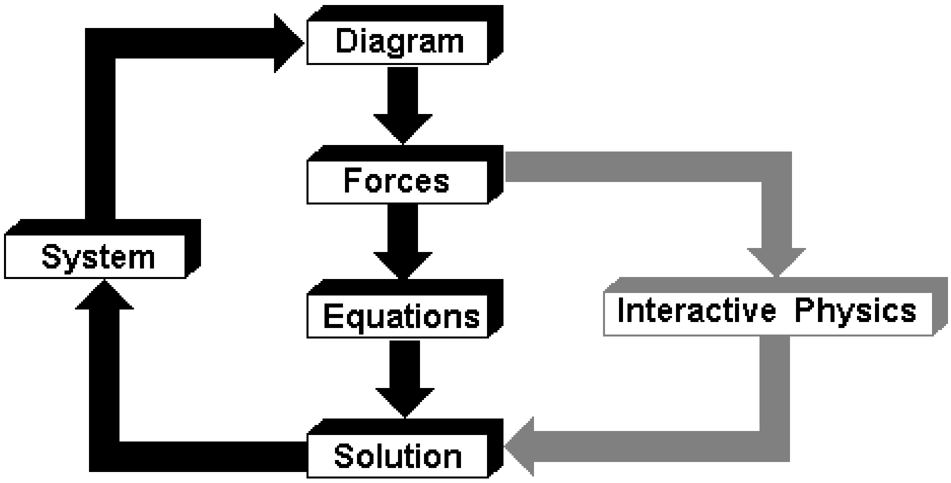


Figure 2.6 (from Stevenson, 1998):
Integrating Interactive Physics in solving a mechanics problem.

On this basis, the fact that algebraic equations governing the system under study are completely absent on the screen challenges the conventional way of learning mechanics. Instead of focusing on the equations for understanding the dynamics of the system, it is the reverse that is happening whereby the 'paper-and-pencil' technology is absent at the beginning of the learning process. This situation presents an ideal platform for probing student intuitive understanding of motion of connected particles as the software does not support students by providing algebraic cues. Thus, with the advent of Interactive Physics, students no longer need the mediation of a symbolic language to start learning mechanics. From this viewpoint, I agree with Laborde & Laborde (1995) that

> with direct manipulation this interaction process between the learner and the microworld is no longer based on the use of the symbolic language but bypasses this by allowing an immediate and physical

> access to the reified instances of abstract entities. What is important is that this direct manipulation allows the user to handle not only objects but also relations linking them.
>
> (Laborde & Laborde, 1995: p. 243)

(b) Multiple Representations

Roth et al. (1996) find that Interactive Physics supports students' sense-making activities. According to them, the computer environment can contribute substantially to the maintenance and coordination of students' physics discussions. Besides offering many possibilities for testing and exploring physical concepts, Roth et al. also claim that Interactive Physics can blur the distinction between the informal, everyday experience and the formal, school physics due to its feature to coordinate 'the phenomenal and the conceptual'. Owing to this feature, the software

> Interactive Physics™ achieves a bridge between the two domains by enabling what is impossible in the world of our everyday experience: the copresence of the phenomenal (moving object) and the conceptual (representations of velocity and force). This copresence is achieved by transforming three-dimensional real-world objects into two-dimensional drawings. At the same time, the constructs of forces and velocities are transformed into drawings (vectors) in the same two-dimensional plane of the computer display. As a result, conceptual properties and their associated microworld objects are copresent, each re-representing a different aspect of human experience (the conceptual and the phenomenal). It is this copresence which allowed students to coordinate the phenomenal and the conceptual …
>
> (p. 1002-1003)

I agree with Roth et al. (1996) that the strength of this software is its ability of making the phenomenal and the conceptual co-present. It is through this copresence that Interactive Physics has the 'potential for cognitive and discursive development and change'. As such, Roth et al. (1996) firmly emphasize that

Interactive Physics must not be considered as just another visual aid which teachers can use in their classroom or as another means of presenting students with relevant information they have to learn. On this basis, it can be argued that the multiple representation feature of Interactive Physics strongly depends on how carefully it is integrated into the learning activities. In a sense, it can be suggested that Interactive Physics becomes an alternative representational system in the learning of mechanics (Kaput et al., 2002).

(c) Initiating & Sustaining Discussion

In the account given by Roth (1995), the author explores how a teacher can use the medium of Interactive Physics to identify students' ways of seeing and talking science. From Roth's work it can be argued that the software does not only trigger discussion but can also sustain it. By using key questions, such discussion can lead to the construction of new meanings and can eventually bring about conceptual change. On this basis, it would appear that Interactive Physics can provide students with the opportunity 'to ask probing 'what if' questions' (Horwitz & Barowy, 1994).

By integrating history of science and the notion of microworlds in the context of Interactive Physics, Masson & Vazquez-Abad (2006) develop three historical microworlds (an Aristotelian microworld, a Buridanian microworld & a Newtonian microworld) to promote conceptual change in mechanics. Using diSessa's p-prim theory to analyze the conceptual dynamics that occurs when students interact with the said microworlds, Masson & Legendre (2008) find that these microworlds help promote students' conceptual change by pushing them to organize their knowledge by structuring and contextualizing the domain of validity of their intuitive conceptions. They also note that students' perception of friction and air resistance can help the students in developing expertise. From these studies by Masson and his colleagues, I find that if Interactive Physics forms part of an appropriate methodology with relevant activities, this combination can provide a medium where discussion can be initiated and can lead to the construction of new meanings.

2.5 Research Questions

The literature review carried out in this chapter clearly points out that most research carried out in the past on Newtonian mechanics has focused on the nature of student misconception but not on how students learn mechanics. At the same time, findings from research studies have laid the blame of student weak understanding of mechanics either on student intuitive knowledge or instructional strategies – very few have explored the role of intuitive knowledge in constructing new knowledge. This is actually the first strand of research for this study. Through this strand, I wish to examine two key ideas:

(a) to investigate the intuitive knowledge activated by students when they are presented with a learning task; and,

(b) to examine the role that intuitive knowledge plays during the generation of new knowledge.

The construction process is undertaken in the setting of the dynamic visualization system Interactive Physics (IP). What is of interest here and is the second strand of research for this study is the mediating role of this computational tool in the learning process. Through this strand, I wish to examine two central ideas:

(a) to explore the specific features of the software that students make use of during the learning process; and,

(b) to tease out the way(s) in which the software helps students to connect their intuitions with the new knowledge generated during the learning process.

It is anticipated that learning will occur from the tasks afforded by this IP-based environment. The term ‘affordance’, as first described by Gibson (1979), focuses on what an object or environment can possibly offer to the user. He propounds that an affordance does not arise because of the object or environment only, but because of how the user ‘exploits’ it. This implies that the same object or environment can provide different affordances for different uses (Webb, 2005). In a similar vein, Greeno (1994) explains that the term ‘affordance’ cannot be dissociated from the term ‘ability’ such that “an affordance relates attributes of something in the environment to an interactive activity by an agent who has some ability, and an ability relates attributes of an agent to an interactive activity with something in the environment that has some affordance" (p. 338).

On this basis, it can be deduced that on their own the features of Interactive Physics are powerless. It is only when they are put to 'good use' by learners that they may become powerful tools of learning. How the features of Interactive Physics will be used will potentially depend on the learners' intuitions and school knowledge, and the questions raised during the task-based interviews. So, in this thesis it appears that the affordances of Interactive Physics will be available because of the interactions between learners' intuitive ideas and school knowledge, interview questions and features of the software.

The research questions can therefore be stated as follows:

🗁 How is intuitive knowledge used by students in learning motion of connected particles?

- What intuitive ideas do students bring to motion of connected particles?
- What role, if any, do these intuitive ideas play during the generation of new knowledge?

🗁 How do the uses of Interactive Physics influence the evolution of the learning process?

- How are the affordances of Interactive Physics used in practice to develop understanding?
- How do these affordances help in the forging of new connections between intuitive knowledge and new knowledge?

Chapter 3

Research Methodology

3.1 Introduction

The aim of this chapter is to elaborate on the research methodology developed for this research study. The chapter is divided into two main parts: theoretical framework of the research and overview of the methodology for data collection. While § 3.2 explores the theoretical framework of this research, § 3.3 provides an overview of the methodology for data collection.

3.2 Theoretical Framework of the Research

The use of the adapted knowledge-in-pieces (k-i-p) approach proposed in Section 2.2.4 has direct implications for the methodology employed in this study. These implications can be grouped in the following three categories:
(a) Adopting a qualitative approach
(b) Data collection and associated methodological issues
(c) Data analysis and associated methodological issues

3.2.1 Adopting a Qualitative Approach

Since the adapted k-i-p approach is being used to analyze conceptual change in students' minds, I decided to use a qualitative approach to research the aims of the study. It is expected that a process-oriented qualitative approach will be appropriate to examine the role of intuitive knowledge that learners bring during the learning process and how Interactive Physics mediates the learners' thinking processes. The focus has been on description and interpretation rather than on measurement and prediction. To achieve these targets in this research, I involve: (i) the method of participant observation; and (ii) working with individual students.

(i) Participant Observation

According to Burgess (1984), in research involving the use of participant observation the researcher needs to accept a role within the social situation he investigates. In theory, this direct participation easily allows the researcher to enter into the social situation by reducing the resistance of the observed group and permits the researcher "to experience and observe the group's norms, values, conflicts and pressures, which (over a long period) cannot be hidden from someone playing an in-group role" (Hargreaves, 1967: p. 193). On this basis, participant observation facilitates the collection of rich detailed data on social interaction, which may be inaccessible by other methods.

It is feared that since the researcher is both involved in face-to-face relationships with those who are researched and is part of the context that is being observed, there exists a high degree of possibility that the researcher is not only influencing the research study but is being influenced by it himself or herself (Burgess, 1984). This is why participant observation studies "are often described as subjective, biased, impressionistic, idiosyncratic and lacking in the precise quantifiable measures that are the hallmark of survey research and experimentation" (Cohen & Manion, 1994: p.110). However, these criticisms can be minimized by carefully documenting interventions likely to influence the research and recognising them as another type of data for subsequent analysis.

In this study, my own role has been that of a participant-as-observer, interacting with the students as the interviewer. I observed because it was the best way I could observe these students exteriorizing their mental processes during the computer-based tasks. I participated because my aim was to gather information which has enabled me to describe what happened and why it happened. I achieved this aim by provoking students to elicit emic explanations (i.e. what was happening inside their minds). Neither was my role about taking the students through their zone of proximal development nor was my intention to be Vygotskyian more experienced other. The computer-based tasks might have mediated the construction of new knowledge, but my role was not to facilitate but to question students as they interacted with the tasks.

It should be pointed out that I have conducted this research with students in my college where I am the head teacher. Though Hargreaves (1967) asserts that there are advantages of participant observation as a research method for those carrying out studies in institutions in which they work, the methodology I have employed had certain constraints. I found that there were three issues which had to be used to minimize them:

(a) Participants in this study were volunteers. So there was not any kind of pressure on them. They were also told about the broad aims of the study. For example, there were no right or wrong answers – it was a question of what they said and why they said it.

(b) Though English is the medium of instruction and exams in Mauritius, I have mainly used CREOLE as the language of interaction during data collection. By so doing, students felt more at ease and were able to express themselves better. Translating the CREOLE to ENGLISH has not been an issue in this study because my focus was to gather the ideas that students had expressed during the computer-based tasks.

(c) I became part of the teaching staff working with these and other students before I started my research with them. This helped us enormously in getting acquainted to each other.

With these measures, I have tried to minimize power differential but I was aware that I could not eliminate the constraints completely. I had to bear that in mind when interpreting the collected data. In this respect, the finding of Southerland et al. (2001) appears to shed some light on my data analysis:

> The students viewed the interviews as a conversation with a relatively unfamiliar authority figure, and so employed the habits of conversation typical to such situations; that is, students were attempting to offer a "correct" answer to the authority and reacted to interview cues to provide that answer. As a result, students changed the nature of their explanation when they understood their answer to be insufficient or unacceptable for the interviewer.
>
> (Southerland et al., 2001: p. 342)

(ii) Individual Students

Computer simulations offer opportunities not only for collaborative learning but also for individual learning (Lee et al., 2008). The former is greatly influenced by social constructivism where knowledge is socially constructed with meaning being negotiated and agreed upon as a result of interaction and shared efforts in a group to make sense of new information. From this perspective, learning results from the reorganization of the knowledge structures of students as well as the co-operations that they carry with others (Moreno, 2009). The case of individual learning with technology is supported by cognitive constructivism where individual students are supported to build meaningful mental representations (Mayer & Moreno, 2002). According to this viewpoint, the computer-based environment plays a mediating role in scaffolding students as they construct knowledge in their minds.

However, allowing students to learn collaboratively does not guarantee effective learning. According to Harskamp & Ding (2006), collaborative learning turns into unproductive conversational learning if student interactions are not structured, and learners are less focused in their tasks if the tutor is absent. In the construction of shared knowledge, Tao & Gunstone (1999) argue that students have to personally make sense of the new understanding. This is achieved by: (1) internalizing all the group-generated views; and, (2) re-organizing and reconstructing experiences (Chou cited in Lee et al., 2008). If they cannot personally make sense of the new understanding, their knowledge is transient. On this basis, Lee et al. (2008) assert that learners must personally construct knowledge whether they operate individually or collaboratively.

Since my research is about examining the role of intuitive knowledge that students bring during the computer-based tasks and how Interactive Physics mediates the learning process, I wish to avoid the following three issues:

(a) Group dynamics: As suggested by Harskamp & Ding (2006), we need to structure student interactions in order to learn collaboratively. Or else we may end with one student doing most of the thinking and the other one the least possible. Therefore, I do not have to worry about structuring interactions which will certainly influence my research and how one peer is influencing the other.

(b) Out-of-focus discussion: By grouping students in pairs to work together, we are implicitly encouraging fruitful discussion taking place between them. As long as these discussions take place during the computer-based sessions, it will help to move towards the research aims. However, there exists no possibility at all to capture discussions occurring before or after the sessions.

(c) Individualised intuitive knowledge: No two students will have the same intuitive knowledge. The latter will certainly differ from learner to learner. It is rather difficult to study the role of intuitive knowledge in the learning process from ‘social constructivism’ perspective.

This is why I preferred to work with individual students rather than with children working in pairs.

3.2.2 Data Collection and Associated Methodological Issues

This section discusses the techniques employed to gather qualitative data relating to student intuitions and learning of motion of connected particles in an Interactive Physics environment. It is divided into four main parts:

(a) employing think-aloud protocol;
(b) using structured task-based interviews;
(c) designing tasks; and,
(d) describing tasks.

(a) Employing Think-Aloud Protocol

According to Payne (1994), think-aloud protocol remains the best technique ever developed to capture students articulating their thoughts while they engage in a task. When this method is employed, participants continually speak aloud the thoughts in their heads as they complete a given task – the aim of the researcher is to capture what they are actually thinking as they engage in the task. During the process, the researcher can engage in both 'concurrent probing' and/or 'retrospective probing' with them to elicit relevant and directed verbal data (Young, 2005). Wade (1990) argues that concurrent think-aloud protocol is more reliable than video-stimulated recall because it reduces problems associated with memory failure which may occur when one waits to collect verbal data after the task is completed. From this perspective, Young (2005) contends that "it is likely that a participant's ability to remember what they were thinking at previous point in time, even with stimulus materials, is somewhat limited" (p. 22).

It appears that three limitations have been identified with think-aloud data: issues of reactivity, participants' verbal abilities and data validity (Young, 2005). The first limitation, issues of reactivity, refers to the three main effects of asking participants to think aloud: effect of thinking and attending to a task at the same time; effect of talking aloud during the task; and, effect of drawing participants' attention to the cognitive processes. The first effect usually reduces the ability for some people to work through a task and talk at the same time. According to Young (2005), though the first effect cannot be minimised, if participants can utter some statements this may still increase our understanding of the persons under study. The second issue can be minimized by: (1) ensuring that the task being undertaken is one that is appropriate to elicit verbal data; and, (2) allowing time for participants to practise thinking aloud. The third issue refers to the influence of the researcher with regard to any verbal or non-verbal cues which may have slipped through directly or indirectly. In this case, the researcher needs to take due care with his or her actions and comments, and his or her interventions, which are likely to influence the research, have to be carefully documented and recognised as another type of data for subsequent data analysis. The second limitation is concerned with the fact that different participants will produce varying amount of think-aloud data. We need to appreciate that some participants will be more capable of producing data than others. At the same time it is to be feared that other participants are not able to produce the desired quality of think-aloud data; in this case, their participations must be reconsidered. The third limitation raises the validity issue – whether the data collected by this method reflect thinking accurately (Crutcher, 1994). This limitation can be minimized by the use of a combination of data collection methods: a post-activity interview or video-observation combined with audio-data (Young, 2005).

Since my thesis is mainly about investigating the role of intuitive knowledge in the construction of new knowledge and how the software mediates student thinking, it deals with several tasks involving cognitive processes. In this respect, I shall take the view of Payne (1994) that "the more a task involves higher level cognitive processes that take more than a few seconds to perform, and the more the task involves verbal types of information, the better" (p. 247). From this perspective, I find the think-aloud protocol mostly suits my research. Moreover, I am working with individual students because I am particularly interested in understanding the cognitive processes engaged by students as they work with the computer-based tasks. Therefore, by eliminating peer interactions, I argue that in my study the think-aloud approach enhances participants' thoughts to be highly focused, which is useful in both minimising distractions from participants' sequences of thoughts and obtaining data that are most purposeful for my research goals (Young, 2005).

(b) Using Structured Task-Based Interviews

This study relies heavily on the use of structured task-based interviews in observing and interpreting the mathematical behaviour of sixth form students learning motion of connected particles in the computer-based environment of Interactive Physics. Basically, this research instrument involves at least one student and an interviewer, interacting in relation to one or more tasks introduced proactively to the student by the interviewer. More importantly, the term task-based suggests that the student's interactions are more with the task environments than with the interviewer (Goldin, 2000).

During structured task-based interview methodology, students are asked to verbalize their reasoning and justify their practices while working through carefully crafted activities. That is, they are required to formulate their explanations in words. Provision is made for observing and recording for later analysis of what takes place during the interview through videotaping, observers' notes and the students' works. By analyzing verbal and nonverbal behaviour and interactions, the researcher hopes to make inferences about the mathematical thinking and learning trajectories of students. Thus, unlike conventional paper-and-pencil test-based methods, task-based interviews enable the researcher to focus more directly on the students' processes of tackling mathematical tasks, rather than just on the patterns of correct and incorrect answers in the results they produce. This is why this methodology offers the possibility of delving into the psychology of learning mathematics more deeply than is possible by other experimental techniques.

Another methodological reason for why I choose to work with task-based interviews is that they are flexible research instrument (Goldin, 1998). This notion of flexibility empowers the interviewer to pursue a variety of avenues of inquiry with students, depending on what happens during the interviews. Following in the footsteps of Goldin (1998) I also find that such flexibility can become the driving force for investigating the enormous differences that are known to occur in individual students' learning trajectories and meaning-making activities. Moreover, since the main goal of this methodology is to elicit and identify processes that students use spontaneously (i.e. without direct hints or coaching), flexibility becomes an important tool in the hands of the interviewer to avoid steering students in a predetermined direction in their learning trajectories.

However, we, as researchers, should aim to create task-based interviews that are not only flexible but are also reproducible. This can be achieved by designing interview scripts that sufficiently anticipate several contingencies to the students' responses. Goldin (2000) states that contingencies can take the form of heuristic questions, hints, related problems in sequence, retrospective questions, or other form of interventions by the interviewer. This explicit provision for contingencies, together with the attention to the sequence and structures of the tasks – the structured task-based interview – can be distinguished from the unstructured task-based interview where no type of contingency is provided by the interviewer to students.

If the task-based interview is structured, then we have two major implications that are inter-related:

(a) it provides a structured mathematical environment that can be controlled to some extent; and,

(b) there will be some form of intervention on the part of the interviewer.

As mentioned earlier, the task-based interview methodology can be implemented in two ways: either we choose mathematical tasks that appear relevant and then see what happens, or we explicitly describe and design tasks to elicit processes that are to some degree anticipated, and also to search for unanticipated occurrences. Following in the footsteps of several researchers, I also believe that the latter perspective will be more suited to facilitate particular interactions between mathematical structures and structures internal to students. The analysis of such structures – the possible interactions among mathematics, cognition and learning, and affect – is an important theoretical ingredient of task-based interview research. Thus, in referring to mathematical structures in the tasks presented in this study, it is not assumed that students have developed just these structures already, or that they all will interpret the tasks according to such structures. Rather, the purpose is to infer interactions between the students' internal cognitive and affective structures and the external mathematical task structures (Scaife & Rogers, 1996). The characterization of the tasks as "having" such structures means that there is reason to believe that meaningful interactions in relation to these structures are possible or likely. Whether and how they occur is then subject to empirical investigation.

It is a general feature of structured task-based interviews that interventions are a part of the task environment. Interventions are most often provided only at points in the interviews when students are no longer able to pursue the task at hand such that further progress seems unlikely. Thus, interventions can be understood to bridge the gaps in students' own, partially developed, heuristic planning competencies. According to Goldin (2000), providing meaningful interventions can increase the richness of the subsequent task solving behaviour, much further along a path toward solution than would have been possible otherwise. What is observed is always the students' behaviour in the presence of the structured interventions. What may be inferred are aspects of the subjects' internal cognitions and/or affect in the presence of the interventions. On this basis, it can be argued that the most innocent of interventions, aiming to identify the reasons behind a particular action, is likely to focus the student's thinking in a new direction, which is often the researcher's explicit aim. Following in the footsteps of Pratt (1998), my methodology also accepts that this must be the case and recognises that activity with computer-based tools is likely to increase opportunities for such interventions, and sees the exploration of the effect of such interventions as part of the study.

According to Heid et al. (1999), the interviewer's view of learning and knowing mathematics seems to have an important impact on his or her approach to interviewing. In their study they observe that teachers appear to use task-based interviews as a new mode of teaching – guiding the students arrive at an appropriate answer for a problem – instead of learning more about their students' mathematical understandings.

> They used the interview to prod students toward the correct answers, instead of, as the researchers had intended, probing students' thinking as a means of increasing their knowledge of their students.
>
> (Heid et al., 1999: p. 246)

Thus, my role as a researcher is not to ensure that students can work successfully through the activities or to make them understand motion of connected particles by any possible means. Instead my intention will be to observe how they respond to the learning environment designed for them. In the event that they cannot carry on with some part of the activities and start to appear frustrated, I will intervene to provide hints or heuristic suggestions.

(c) Designing Tasks

It is a fact that tasks play a pivotal role in any research study in mathematics education since they are responsible for data collection. For instance, Kordaki & Potari (2002) illustrate that tasks are crucial in initiating and supporting students' involvement in a computer-based environment. Yet, the irony is that the design of tasks has received scant attention in the literature. Hoyles (2001), in a commentary on a special issue of Educational Studies in Mathematics, observes that tools, tasks and 'interactions planned to take place in activities around these tools' are central to any research study. On this basis, she argues that tasks and especially their design must be overtly discussed in research studies in mathematics education. With regard to my research, the main aim of my tasks is to enable Interactive Physics perturbing what the participants think, thereby throwing them in some sort of imbalance. I am hoping that this 'imbalance' will result in cognitive processes as the participants engage in the tasks.

In this study the first salient issue is to determine the number of tasks to be worked out by participants. To a large extent, this factor is related to the degree of 'openness' of the tasks.

> Different kinds of problems will evoke different results. One gets what one deserves. The richer and more open the problems, the more they will reveal the students' understanding and abilities. A consequence is, however, that the responses may be more difficult to interpret than closed, bare problems. Therefore it is better to confine oneself to a few good problems. In the end they will reveal more than a large number that are easy to grade.
>
> (Van den Heuvel-Panhuizen, 1994: p. 369)

Since I am using structured task-based interview methodology, I suggest that tasks must not be closed, must be meaningful and accessible. Taking the view of Van den Heuvel-Panhuizen (1994) that one needs to confine oneself to a few good problems, I argue that giving students a lot of questions may bore and eventually frustrate them, thereby, causing hindrance for them to concentrate on the tasks. Moreover, the use of knowledge-in-pieces theory suggests that more contexts must be provided to students so that their learning can be investigated. On this basis, I propose that eight tasks dealing with the three contexts of motion of connected particles would suffice.

The second issue is to decide whether or not the tasks require the use of Interactive Physics. That is, do we give students the choice of using it or not when they are working through the tasks? In this matter, I shall take the view of Hmelo et al. (2000) who state that: “When we aren’t comfortable with a new practice, we often fall back on what we are comfortable with” (p. 293). This strongly suggests that students, not very conversant with manipulating Interactive Physics, can easily avoid using the software if the task does not make use of it and/or they are given the choice of avoiding it. On this basis, Interactive Physics needs to be made an integral part of the tasks – at the same time it should not be just about running simulations and collecting data. Moreover, Kaptelinin (2003) states that “mastering a tool is a process rather than a clearly defined goal” (p. 835). From this viewpoint, this learning process must start before students interact with the tasks so that they are not struggling with both the mathematical ideas embedded into the tasks and the features of Interactive Physics at the same time (Roth et al., 1996). So it is imperative that students are given ample opportunity to become aware of these features before they start using them with the tasks. This appears to be the first step towards decreasing student frustration during its use.

The third issue is about the content of the tasks which has been influenced by three factors. The first factor is the nature of recent questions set at GCE A-Level Mathematics exams – including their examiners’ reports. The second factor is the literature review reported in Chapter 2 and the third factor is the software itself. It is fundamental to take into account what can be done with Interactive Physics. For example, though examiners claim that students have difficulties in evaluating the force exerted by the string on the pulley, it is almost impossible to find this quantity in Interactive Physics. Another shortcoming in the software is that it does not allow to measure the tension in the string at different points along the string.

The fourth issue is the task sequence which can be viewed from two different, but inter-related aspects. The first one is to determine the order of the context: horizontal (System III), original (System II), inclined (System IV) – which one is first and which one is last? The second dilemma is that for the horizontal context (System III) I have six tasks which need to be sequenced. It should be noted that both the inclined context (System IV) and the original context (System II) have one task each. With regard to the second aspect, I have sequenced the six tasks according to my intuition and experience. However, I prefer that this sequence be tested during the pilot study. In connection with the first issue, one central idea that emerged during a discussion with my supervisor, Prof Richard Noss, was the order of the context: horizontal ⇒ inclined ⇒ original, instead of the conventional teaching order: original ⇒ horizontal ⇒ inclined. It is generally accepted that the vertical version is the simplest case of motion of connected particles, followed by the horizontal version and finally the inclined version. This is because the Vertical version is influenced by gravity only; the Horizontal version is influenced by gravity and the presence of a horizontal surface which can either be smooth or rough; and, the Inclined version is influenced by gravity and the presence of an inclined surface which can either be smooth or rough.

Researchers appear to have focused more on the Vertical version than the others. This version has a 'spatial symmetrical property', that is students tend to associate symmetry with this version such that according to students both objects must always be at the same level. This property is not observed in the other two versions. The conventional teaching trend is usually to start with the vertical version, proceed to the horizontal version and finally examine the inclined version. In this study I want to bring a change. I want to start with the horizontal version, then move to the inclined version and finally the vertical version. The main reason is to see what happens to student 'spatial symmetry property' intuitive idea. For example, is it influenced by knowledge built during the horizontal version and the inclined version? It is important to note that in most research studies (for instance, McDermott et al., 1994), Task 8 has often been the first question to be given to students due to its simplicity. However, in this study it will be given to students after they have completed the first seven tasks. The rationale is to observe whether or not these seven tasks have an effect on students' learning of Task 8.

The fifth and final issue is the structure of each task. One strategy that can be associated with the complex knowledge theory is the Predict-Observe-Explain strategy developed by White & Gunstone (1992). The connecting thread between the diSessa's knowledge-in-pieces theory and the Predict-Observe-Explain strategy is the relevance of intuitive knowledge in building the new knowledge in the direction of expertise. Since prediction involves the use of intuitive knowledge, this research strategy was selected as a framework for the tasks. During the computer-based tasks, students will have to interact with the software and not only observe the simulations. This is why the second phase is being called the Interaction phase. Finally, the third phase is referred to as the Reflection phase because students will have to reflect on the rationale behind the discrepancies between their intuitive knowledge and the expert knowledge generated by the simulation. Hence, for the purpose of this study we will be using the Predict-Interact-Reflect strategy in each task.

In the preliminary stage, students have got to make predictions about a particular system and explain the rationale of their predictions. They are then allowed to run the simulation and observe whether or not the behaviour of the simulated system agrees with their predictions. In the event that they get their predictions and rationale right, they will not continue with this particular task. However, if they cannot get either their predictions or the rationale of their predictions or both right, they will be expected or suggested to work with the simulation. During this process, it is significant to observe which features of the animations or simulations they consider as important for learning motion of connected particles. It is then expected that they have to make observations to extract information from the animations and finally draw inferences from their observations to learn motion of connected particles.

From this perspective, it is seen that each task involves the extraction and synthesis of perceptual information from the simulations to develop learning about motion of connected particles portrayed; information construction is precisely the type of work diSessa's theory is supposed to accomplish. The simulations present a lot of information in the form of visual, numerical and graphical representations - so students must selectively attend to the features of the simulations that can be useful for making inferences about learning motion of connected particles.

(d) Describing Tasks

The first six tasks involve the horizontal version of motion of connected particles. While the seventh task explores the inclined version, the final eighth task examines the original version. It seems like there is an imbalance of tasks in terms of their topic focus, that is six tasks focusing on the horizontal version and only 1 task on the inclined and original versions. I would like to point out that my aim is not on comparing how students perceive each version – whether students find one version easier to learn than the other in the Interactive Physics environment. Instead I wish to probe student thinking as they work through the tasks embedded in different contexts. Moreover, by providing more than eight tasks, there would have been too much data and students would have to give more time to my study.

Task 1

How may such a system behave?

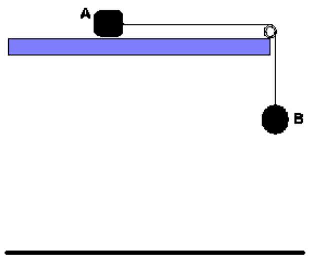


Figure 3.1: The horizontal version (System III) where a sliding object A is on the surface and the hanging object B is suspended via a string connected to object A and passing over a pulley.

The first task will begin by asking students to predict how such a system of connected particles may behave (Figure 3.1). From the finding of Gunstone & White (1981), it is expected that students will predict motion if the mass of the hanging object [mB] is greater than that of the sliding object [mA]. If mA = mB or mA > mB, it is hypothesized that students may suggest that the system will remain in equilibrium. In the event that students do not understand the question, then I shall ask them to predict the behaviour of the system when mA < mB. This is to satisfy the general perception that heavy objects displace lighter objects. By doing so, students can associate their 'perceptions' with that of the software. Having broadly discussed the effect of mass of either object on the motion of the system, I am now interested in students' responses to the following two questions:

(a) what sets the system in motion?;
(b) what is pulling the sliding object A: the hanging object B or the tension in the string?

Based on the work of McDermott (1991), I found empirically that these two questions would help me focus on the ultimate aim of this task: to probe student understanding of the relationship between the tension in the string and the weight of the hanging object B. I want to explore whether and how students use these two criteria to explain the motion of such a system. It is also important to explore the student intuitive understanding of speed and acceleration for this system. If students explain that both objects move with the same speed and same acceleration, this will suggest that they appreciate the role of the string in this system. Another issue that can be raised is whether or not the tension throughout the string is the same. In the original version, students explain that the tension cannot be the same throughout the string (Berry & Graham, 1991; McDermott & Somers, 1992). However, the choice of whether to use a smooth surface or a rough one poses a dilemma. From the work of Mildenhall & Williams (2001), it would appear that it is better to start with a rough surface as it creates a real context for students and then move to a smooth one. Contrary to their suggestion, I would prefer to start with a smooth situation, which is less complex and much simpler, and then proceed to a rough situation.

Task 2

What happens if an object C is attached to object B?

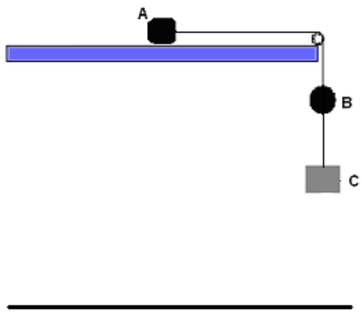


Figure 3.2: A sliding object A is on the surface and the hanging object B is suspended via a string connected to object A and passing over a pulley. Another sliding object C is attached to object B.

The second task will begin by asking students to predict the behaviour of the system when another hanging object C is attached to the existing hanging object B (Figure 3.2). Will the system remain at rest, move slower or move faster? It can be hypothesized that students will predict the system to move faster because of the additional downward force provided by object C. Students will also be asked to explain what happens to the tension in the string S_{AB} when object C is attached to object B. This task (as well as Task 6) were inspired by the work of McDermott et al. (1994) which explores student perception of the role of the string in the system shown in Figure 3.4. Within the setting of Interactive Physics, I saw the possibility of using this task to probe student understanding of the relationship between the tension in the upper string and that in the lower string. How can this relationship influence student understanding of motion of connected particles? Would investigation of this relationship shed new light on student understanding of the system? Finally, akin to Task 1, this task also explores student understanding of the relationship between the tension in the (lower) string and the weight of the hanging object C. By altering the context, I want to see what sense students make of this new situation with regard to the relationship between the tension in the string and the weight of the hanging object.

Task 3

(a) What happens if we vary the mass of the hanging block B?

(b) What happens if we vary the mass of the sliding block A?

The third task explores the behaviour of the system when the mass of either object is varied. The first part, which has its roots in the work of Gunstone & White (1981), examines the behaviour of the system when mass of the hanging object B [mB] is varied. In this case, as per the student expectation, an increase in the mass of object B will account for the increase in the tension in the string, the speed of either object and the acceleration of the system. It will also highlight the relationship between the tension in the string and the weight of the hanging object. The second part of this task examines the behaviour of the system when mass of the sliding object A [mA] is varied. This part can be expected to bring some 'shock' or cognitive conflict to the student as students will anticipate a 'no-motion' situation (Gunstone & White, 1981). In this exercise, students can see the effect of increasing the mass of object A on the tension in the string, the speed of either object and the acceleration of the system. More importantly, student understanding of this part may help to shed light on why so many students strongly believe that tension in the string must be equal to the weight of the hanging object. Why is a static problem solution applied uncritically to a dynamic situation (diSessa, 1993; Mestre, 2002)? It is expected that in the end students can start to make sense of the relationship between the tension in the string and the weight of the hanging object in order to account for the motion of the system.

Task 4

What happens if friction is now introduced in the original system?

The fourth task examines the notion of friction in the system. According to several participants in the study of Gunstone & White (1981), friction between the block and the horizontal surface will prevent motion. In their work, McDermott et al. (1994) find that several students predict that the frictional force on the sliding object acts directly on the hanging object. For Mildenhall & Williams (2001), friction provides several real-life situations where it is daily experienced by students. On this basis, it should influence their understanding of connected particles. It would be particularly interesting to probe the notion of friction on student intuitive understanding of motion of connected particles. For example, how does friction affect the motion of the system or the tension in the string? Do students expect that the tension in the string remains the same, increases or decreases when friction is introduced between the sliding object A and the surface? Does friction affects the acceleration of the hanging object B?

Task 5

What happens if acceleration due to gravity is now varied in the original system?

The fifth task is mainly an investigation about the magnitude of the acceleration of the system. Students will be asked to predict the effect of varying the acceleration due to gravity on the system. Will the system remain at rest, move faster or move slower? This task is drawn from the work of McDermott (1991) who finds that students do not agree that the magnitude of the acceleration of the sliding object is not the same as that of the hanging object during motion. It is also expected that students may come to realise the role of the string in the system of connected particles.

Task 6

What happens if an object F is attached to object A?

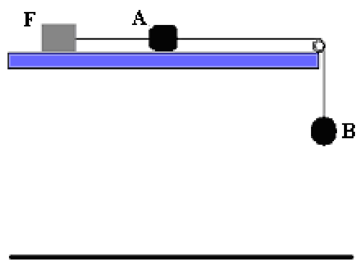


Figure 3.3: A sliding object A is on the surface and the hanging object B is suspended via a string connected to object A and passing over a pulley. Another hanging object F is attached to object A.

The sixth task will start by asking students to predict the behaviour of the system when another sliding object F is attached to the existing sliding object A (Figure 3.3). Will the system remain at rest, move slower or move faster? Students will also be asked to explain what happens to the tension in the string S_{AB} when object F is attached to object A. As briefly outlined earlier, this task (as well as Task 2) were inspired by the work of McDermott et al. (1994) which explores student perception of the role of the string in the system shown in Figure 3.4.

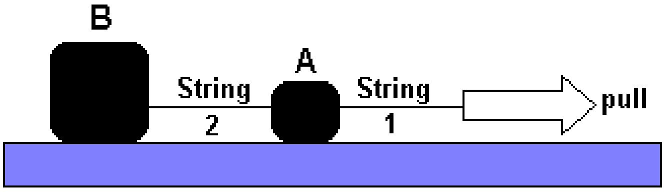


Figure 3.4: McDermott et al. (1994) use this system to elicit student difficulties related to the role of the string. Two blocks A & B, connected by a string (String 2), are pulled across a surface by another string (String 1) attached to block A. Students are told to assume that the strings are massless and the mass of block A is less than that of block B.

Within the setting of Interactive Physics, I saw the possibility of using this task to probe student understanding of the relationship between the tension in the upper string (connecting object F & A) and that in the lower string (connecting objects A & B). How can this relationship influence student understanding of motion of connected particles? Will investigation of this relationship shed new light on student understanding of the system? Another possibility with simulating this task in Interactive Physics is to gradually decrease the mass of object A and explore the magnitudes of tension in both strings. It is seen that as the mass becomes smaller, the magnitudes of tension in both strings tend to be equal. This should lead to a meaningful discussion, perhaps allowing students to explain why we can assume tension in the string to be the same throughout.

Task 7

How may such a system behave?

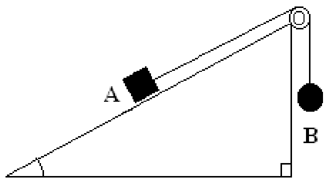


Figure 3.5: The inclined version (System IV) where a sliding object A is on the inclined surface and the hanging object B is suspended via a string connected to object A and passing over a pulley.

The seventh task considers the motion of connected particles in an inclined context (Figure 3.5: System IV). In this task, I have introduced angle of inclination into the case – object A is no longer on a flat surface, but on an inclined surface. The aim is to see how students perceive motion and other 'connected particles' issues. Will they still assert that during motion the tension in the string must be equal to the weight of hanging object (Reif, 1995)? Or will they be able to apply what was learnt in previous activities? The magnitude of acceleration of the system and introducing friction into the system can also become issues of further investigation and discussion.

Task 8

How may such a system behave?

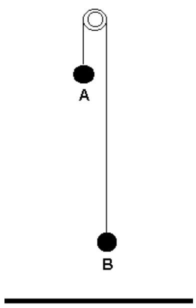


Figure 3.6: The original version (System II) where two hanging objects A and B are attached via a string passing over a fixed pulley.

The eighth task examines the original version (Figure 3.6). Though this is the simplest case of connected particles, it is the last task of this study. The first aim is to see whether (when students have reached this stage) they accept that equal masses must result in both objects being at the same level (Gunstone & White, 1981; Roper, 1985). The second aim is to see whether students can apply the relationship between the tension in the string and the weight of hanging object (that was learnt in previous tasks) in this context with unequal masses (McDermott & Somers, 1992). A third aim can be to investigate whether students think that there is more than one tension in the string (Berry & Graham, 1991). Finally, we can also explore student understanding of the magnitude of acceleration and velocity of the system (Gunstone & White, 1981). Do participants think that the object moving up will have its speed decreasing and that moving down will have its speed increasing? Such intuition, for instance, will show that for these participants the objects move freely under gravity and, therefore, they do not realise the role of the string.

3.2.3 Data Analysis and Associated Methodological Issues

This can be examined through the following two issues:
(i) creating episodes; and,
(ii) building case studies of learning trajectories of students.

(i) Creating Episodes

The first step is to use the eight 'Connected Particles' categories as a starting point to analyse the whole transcript of a student under study. The number of 'Connected Particles' categories may increase, remain the same or decrease depending on the quality of data collected.

The second step is that for each category, related part of the transcript is identified and labelled as an episode which is then used for data analysis. An episode can have the potential to act as a 'window' on the student's experiences of motion of connected particles (Noss & Hoyles, 1996).

The data analysis for each episode is carried out through the following four steps:

(a) Step 1: Identification of Mechanics Ideas;
(b) Step 2: Analysis of Force Diagrams;
(c) Step 3: Use of the software Interactive Physics; and
(d) Step 4: Researcher Intervention.

This will be discussed at length in § 4.4.

(ii) Building Case Studies of Learning Trajectories of Students

In this study I aim to track the way knowledge is constructed when 18-19 years old students interact with computer-based tasks. The first aim of this thesis is to identify what intuitive knowledge these students bring to these sessions and what new knowledge is constructed at the end. By analysis of the in-between process, I expect to explain how the new knowledge might have developed. The second aim of this thesis is to examine how Interactive Physics mediates the learning process. It is anticipated that Interactive Physics will, first of all, perturb what most of these students think, thereby inducing cognitive conflict through presenting contradictory information. In the event that students will achieve meaningful conflict, this 'imbalance' will result in cognitive processes as the former engage in the tasks.

As Cohen & Manion (1994) point out, the purpose of case studies is to empower the researcher 'to probe deeply and to analyse intensively' the target of interest. Since in this study the aims are to track student learning and to investigate the mediating role of the software in the learning process, these research aims lend themselves to a case study approach. Thus, this approach may shed light on how students learn in a particular context. Yin (1984) identifies three types of case studies: exploratory, descriptive & explanatory. In this thesis, the exploratory case study will be employed since the interventions being evaluated have no clear, single set of outcomes (Yin, (2003, cited in Baxter & Jack, 2008)). It can also offer insights in a way that conventional studies cannot (Donmoyer, 1990).

Taking the view of Stake (1978) that "the case need not be a person" (p. 7), my intention is to build case studies of learning trajectories of students, that is to examine how the affordances of Interactive Physics influence student learning of certain key ideas of motion of connected particles. It is expected that these case studies would help to carefully document the context within which learning would take place. As such, it would then be possible to explain the processes, complexities and outcomes of learning trajectories such as students accounting for the motion of the system or students experiencing the change in acceleration of either object (A or B) once the string is broken or becomes slack.

3.3 Overview of the Methodology for Data Collection

It is a fact that task-based interviews do not take place outside of a social and psychological context. In this regard, the context influences and places constraints on the interactions that occur during an interview and puts limitations on the inferences that can be drawn. This is why for task-based interviews methodology to be pursued seriously, several external factors – such as the student's willingness to participate in the study, the student's mental and physical states during the interviews, and time and place of interviews – are essential to the interview design process.

This section, which focuses on data collection for the main study, consists of three parts: participants, setting of the interview and process of collecting data.

3.3.1 Participants

In this section, I examine six issues in relation to students who took part in the main study:

1. The number of students I had worked with was initially twelve. As the data collection process progressed, this number decreased to ten. I chose all students from my college because it was easy for me to find

volunteers for this work. This methodology generated a lot of data per student since my intention was to probe each one deeply, and for that to be manageable the number of students had be kept small. What I had expected from students was the data of their interactions with the computer-based tasks. Through these interactions, I was able to discuss the central two aims of this thesis: (a) the role of intuitive knowledge in developing new knowledge, and (b) the mediating role of Interactive Physics in the learning process. It is important to note that validity in qualitative research is judged rather differently than in quantitative research. In this approach, what is of extreme importance is the notion of transparency which can be viewed through (a) the design of my research and (b) plausibility. While the design explains what I am doing, why I am doing it, with whom I am doing it, how long it is taking; the plausibility factor allows me to justify why my interpretations are plausible.

2. The participants were between 17 and 19 years old. They were sixth form students who had to sit for their GCE A-Level Cambridge examinations in the Oct/Nov 2013 session. It should be noted that these students are introduced to motion of connected particles as part of the mechanics option available for GCE A-Level Mathematics.

3. Though these students were familiar with computers they had never used Interactive Physics in the past. This is why prior to the study these students had four to five sessions on how to work with the software – probably around six hours. This was carried out in December 2012 and the beginning of April 2013. During these computer-related sessions, the students were encouraged to voice all the problems they had encountered when using this computational tool. This allowed me to take note and help them become conversant with the interface so that they were well aware of its relevant features required for this study.

4. The students were expected to have done basic mechanics (including applications of F = ma in contexts other than motion of connected particles) but they should not have done motion of connected particles.

Unfortunately this was not possible as some students already had tuition lessons on this topic. So I had to bear this point in mind during the interviews, and in the analysis and interpretation of the data collected.

5. Students had to be reassured that this was not an assessment on their performance, that the whole exercise was to explore the ways in which the software could assist student learning of mechanics.

6. If the study would have been totally separate from the curriculum, it would have been difficult to get support from the students and their parents. So I needed to highlight that participating in this study would help them learn mechanics.

3.3.2 Setting of the interviews

In this section, I consider five issues in relation to the conduct of the interviews:

1. The task-based interview was carried out by me (as a researcher, not as head of school). I had to be with them (by helping them with their maths problems) on several occasions before embarking on the main work. I am aware that there is an issue of conflict as I am both the researcher as well as the head of the school. This has already been discussed in § 3.3.1.

2. I have selected the computer lab of the institution as a venue for research because it was a familiar place for these students. The data collection took place between Friday 12th April 2013 and Monday 15th July 2013. I had to work with these students after school hours and during weekends.

3. To tackle all the eight tasks, I had to work around 8 – 10 hours per student. This duration did not include the time to learn manipulating the software. So for the eight tasks I had to plan six to eight sessions of

approximately 1.5 hours each. But given that students were different and learnt at different rates, while some of the lessons had to be shorter, others had to be longer. These sessions per student were spread over a period of three to five weeks, not over a period of two to three consecutive days. It is important to note that it was not wise to carry out all the eight tasks in one session because timeframe for conceptual restructuring is an important element (Demastes et al., 1996) – this is why we had to give students sufficient time to reflect on feedback of their previous tasks. In a way, it would have been better if we could arrange for a more longitudinal approach (for example, more than three months for each student). However, I could not ask these students to give too much of their time to my research study as they were sixth form students and were heavily under time constraint.

4. It was expected that these students might not be familiar with video recording system. Some students might feel rather uncomfortable. So I started to use the video recording system as from day 1 when the students began to interact with Interactive Physics. By the time students started to work on the main study, they had become used to its presence amongst themselves.

5. It was important that students were neither exhausted nor sick when they came to these sessions. This was treated seriously.

3.3.3 Process of Collecting Data for the Main Study

For the purpose of this study, one video camera, manoeuvred by one computer technician, was used to capture as much as possible of all the selected subjects' verbalisations and researcher interventions during the learning activities. This enabled me (as a researcher) to capture and analyse important episodes including non-verbal data such as gestures, indicating degree of involvement and interest shown by the subjects, students' actions on the software (e.g. what features they used). This also captured any learning episodes that had unexpectedly arisen and were thought to be significant for the study. I took notes about the key events of the interview only after the students left the computer lab.

3.4 Concluding Remark

In this chapter, I have elaborated on the theoretical framework of this research as well as the techniques to gather data. In the following chapter, I will not only describe the data analysis process, but will also examine the lessons that were learnt from the pilot study.

Chapter 4

Lessons from the Pilot Study

4.1 Introduction

The aim of this chapter is to describe the pilot study and document its outcomes which, in turn, will enhance the design of the main study. In § 4.2, I articulate the specific aims of the pilot study. While § 4.3 describes the procedure adopted to collect data, the preliminary data analysis to be applied to the pilot data is discussed in § 4.4. In § 4.5, I examine the implications for the main study. The data analysis in relation to the main data is elaborated in § 4.6. Finally, § 4.7 provides the conclusion to this chapter.

4.2 Aims of the Pilot Study

The pilot study was designed to be an important part of this thesis in the following main ways:

(a) To test the methodology;

(b) To verify that the categories of analysis were appropriate for analysing the data collected; and,

(c) To explore whether the result generated by the data analysis was sufficient to satisfy the aims of the thesis.

Two subsidiary aims were:

(d) to investigate how the software Interactive Physics must be introduced to students so that it does not become a 'tool unready to hand' (Roth et al., 1996); and,

(e) to help me (as an interviewer) to work upon my interview techniques for enhancing the student-researcher interaction.

4.3 Data Collection for the Pilot Study

This section describes the process of collecting data for the pilot study. In § 4.3.1 I document the background of students participating in the study and in § 4.3.2 I outline the procedure of collecting the data.

4.3.1 Sample of Students

Two sixth form students, S1 and S2, volunteered to work with me and participated in this study after classes and during Saturdays. They were not chosen for their academic results, but based on their availability. They were reassured that the aim of this 'mini-project' was not to assess their mechanics understanding, but to observe how they would interact with the software and associated tasks. For this study, they were provided with the eight tasks (as described in Chapter 3). But due to technical reasons, only seven tasks were recorded.

Student S1 was approached during November 2006. He had done some elementary mechanics (that is, he had not yet done motion of connected particles). He was given the opportunity to play with the software for almost three hours in all on three occasions. Before he started to work with the set of tasks, he was allowed to review the basic features of the software for some fifteen minutes. Our interaction lasted for approximately four hours and was carried out in three sessions: on day 1, he looked at Task 1 & Task 2; on day 2, he did Task 3 & Task 4; on day 3 he completed the rest of the tasks.

After the meeting with my supervisor in June 2007, it was considered important to re-try the set of tasks on a second student. At this stage Student S2 was requested to participate and he readily accepted to work with me during August 2007. He worked with a similar set of tasks, <u>except that in this case friction was introduced right from Task 1 instead of appearing in Task 4</u> (I did not introduce friction - it was the student!). In this case, the task-based interview lasted for approximately five hours and was carried out in three sessions: on day one, he looked at Task 1 & Task 2; on day two, he did Task 3 & Task 4; on day 3 he completed the rest of the tasks.

4.3.2 Process of Collecting Data

The interviews were conducted in the computer lab of the college and the data was collected as described in § 3.3.3. My role, as an interviewer, was to encourage the student to talk about what he was thinking and doing with the task at hand. As elaborated in § 3.2.2(b), the interviews were structured, meaning that I had the general interview protocol lines in mind, but my questions depended on what the student talked about.

4.4 Data Analysis for the Pilot Study

When the interviews with the two students were finished, I listened and watched the recordings and I transcribed on paper each activity. All parts of the texts where students expressed intuitive ideas and principles in mechanics, had doubts about their intuitive ideas and principles or changed their ideas and principles were highlighted and used for data analysis.

In this study conceptual change can be regarded as the construction of new knowledge from intuitive knowledge such that the "learner moves from not understanding how something works to understanding it" (Mayer, 2002: p. 101). From this perspective, the aims of this data analysis were two-fold: (1) to seek and describe shifts in student thinking in relation to mechanics ideas; and, (2) to account for these shifts. This also included the necessity to identify plausible explanations that could account for student predictions.

From the literature review in Chapter 2 (refer to § 2.3.6), it is seen that student understanding of motion of connected particles can be viewed through the lens of how students perceive the following eight 'connected-particles' categories:

(a) Relationship between tension and weight
(b) Tension not the same throughout string

(c) Effect of friction on motion
(d) Effect of mass on motion
(e) Effect of mass on height
(f) Magnitude of acceleration
(g) Kinematical aspects of acceleration
(h) Kinematical aspects of velocity

The above eight categories were used as a starting point to analyse the transcribed data. Through an iterative process, data were coded to capture instances which could be associated with the eight ‘connected-particles’ categories. During this process it was noticed that two more categories ‘Effect of mass on tension’ and ‘Relationship between tension and friction’ emerged. In the end, the transcripts of both pilot students were analysed in terms of the ten ‘connected-particles’ categories. Related part of a transcript (which contained the categories) was identified and labelled as an episode which was then used for data analysis.

Each episode can be considered a ‘window’ (Noss & Hoyles, 1996) on a student’s experience of the ‘connected-particles’ categories. From this perspective, I shall take the view of Panorkou & Pratt (2011) that the window as a notion itself has a dual nature: first, a window can be viewed as the medium through which student could overtly and/or covertly express his or her experience of motion of connected particles; and, second, a window can be viewed as the medium through which the researcher could make sense of the student’s experience of motion of connected particles.

As from this point onwards, data for each episode were analyzed through the following four steps:

(a) Step 1: Identification of Mechanics Ideas;
(b) Step 2: Analysis of Force Diagrams;
(c) Step 3: Use of the software Interactive Physics; and,
(d) Step 4: Researcher Intervention.

(a) Step 1: Identification of Mechanics Ideas

In this step, I identified and highlighted (using different colours) students' expressions of intuitive ideas and principles in mechanics. Then I categorised these expressions into three phases: Prediction; Interaction and Reflection. This allowed me to have a clearer picture of what students predicted, what they said when interacting with the software and finally what they concluded.

(b) Step 2: Analysis of Force Diagrams

In this step, the aim was to analyse force diagrams, if any, drawn by students during the prediction phase of the activity. By asking students to draw force diagrams in my study I was not encouraging them to obtain the correct solution (Heckler, 2010). My intention was to probe into their understanding by asking them to show the forces acting on the objects and how these forces influenced the behaviour of the system. Moreover, it was used to complement and extend data, thereby supporting a triangulation approach.

(c) Use of the software Interactive Physics

From the literature review on simulations in Chapter 2 (refer to § 2.4.3), the notion of interactivity appears useful for this study. In fact, the software Interactive Physics (IP) allows a user to change parameters in a model and then observe its effects. It also allows the user to control the pace of the animation.

The literature review on the software Interactive Physics in Chapter 2 (refer to § 2.4.4) shows that its numerical and visual features are the main tools with which students can develop strategies to work through the activities.

By juxtaposing the above two literature reviews, it can be seen that student use of the software may be analysed in terms of the following three categories:

(a) Interactivity (Animation Control and Interactive Behaviour)
(b) Visual Strategy
(c) Numerical Strategy

These three categories were used as a starting point to analyse the transcribed data. Through an iterative process, data were coded to capture IP-related instances which could be associated with the three categories. During this process it was noticed that the Visual and Numerical strategies could further be sub-divided as follows:

VISUAL STRATEGY

Code	Description
	The student …
VS1	… verifies whether or not motion will take place.
VS2	… observes how the objects of the system behave.
VS3	… uses coordination of events (moving objects) with vector arrow designating a particular physical property.
VS4	… observes the behaviours of two vector arrows designating 2 physical properties (that may be inter-related) on a static or moving object.

NUMERICAL STRATEGY

Code	Description
	The student …
NS1	… compares the readings on two (or more) meters appearing simultaneously on the screen.
NS2	… compares the readings obtained from two or more simulations.
NS3	… uses the digital meter to investigate the trend of a physical property over a certain time interval (such as comparing the reading on the meter at different time intervals) or for a certain part of the simulation.
NS4	… uses the digital meter to examine the x-component and/or the y-component in order to investigate the trend of a physical property at different time intervals or for a certain part of the simulation.

Table 4.1: Descriptions of the Visual and Numerical strategies observed during pilot study.

(d) Researcher Intervention

The work of Hoyles & Sutherland (1989) points out that teacher intervention plays an important part in student learning with Logo. In this study I use their categories of intervention as an initial framework for the analysis of the pilot data. Through an iterative process, their categories were eventually adapted to suit my study as follows:

(a) Motivational;

(b) Reflection; and,

(c) Directional.

Motivational	
Code	**Description**
M1	***Reinforcement (with regard to student explanation)***: maybe you are right; you managed to prove your case
M2	***Encouragement (with regard to use of simulation proposed by student)***: try it; go ahead

Reflection	
Code	**Description**
R1	***Encourage students to reflect on their process/explanation:*** predict a value; compare two systems; clarify a doubt; reconfirm what he says [Are you sure?]; remind him of his earlier explanation; draw force diagram; enquire for more explanation; ask probing questions
R2	***Encourage students to reflect on their use of simulation:*** compare reality with simulation (do you agree with what you find on screen?); why did you do this?; observe (animation/reading) from the screen (What do we see?).

Directional	
Code	**Description**
D1	***Nudge:*** try a simulation; reset simulation; use forward/slow motion; put/drag meter; measure a value from IP; assign values to mass; make an assumption [assume object is 1 kg]; do simulation as I suggest; verify mass of object or a statement; check whether frictional force is present; add/eliminate vector arrow; help student to do something on screen; remind student of (sub-)goal to be achieved
D2	***Building:*** remind/encourage students to apply a particular piece of previously learned material or knowledge
D3	***Factual:*** supplying a particular piece of new information which is necessary to enable the pupil to continue; directing student attention to examine a specific part of simulation

Table 4.2: Categories of Researcher Intervention.

4.5 Implications for Main Study

In this section I articulate the implications for the main study in the following two themes: (a) Design of Tasks; and, (b) Introducing Interactive Physics.

4.5.1 Design of Tasks

One crucial feature of this research work is the tasks to be explored by students. This aspect can be examined through the following four groups:
(1) replacing Task 5 by a more appropriate one;
(2) the notion of friction;
(3) the sequence of tasks; and,
(4) what categories we are looking for in each task.

Task 5, which explores the behaviour of the system when the acceleration due to gravity is varied, had to be eliminated from this sequence of tasks because the interaction between this task and the students was not meaningful from the point of view of this research. This task failed to encourage both students to produce any intuitive ideas. This is why it should be replaced by another task which would be of more use to this research work. It would be more appropriate to have a task that encourages students to tease out their intuitive ideas on acceleration due to gravity. Moreover, Task 7 and Task 8, which were missing from the pilot study, must be made an integral part of the set of tasks for the main study. Since these two tasks deal with different contexts – the Vertical and Inclined versions, they will provide additional reliable and valid data.

During the pilot study, there was a difference between the way these two students came across the set of tasks. On the one hand, Student S2 introduced the notion of friction right from Task 1 and onwards. This created quite some confusion in the student’s mind since there were more than one concept to be analyzed simultaneously. On the other hand, Student S1 was introduced to friction at a later stage (Task 4 only). This allowed him to focus on one concept at a time. With regard to this aspect, it seems appropriate to adopt the strategy employed by Student S1. However, owing to my interactions with these two students, I find that the activity on friction must come after Task 1 so that there is a continuous flow of ideas. In case students wish to first work with frictional situations, as professed by Mildenhall & William (2001), they must be allowed to proceed with it and then to move to a situation with a smooth surface. Thus, by having these two activities side by side, swapping them will not disturb the sequence of tasks.

So the sequence of tasks for the main study must be as follows:

Task No.	***Description of Task***
Task 1	How may any 2-O system behave?
Task 2	What happens if friction is now introduced in the 2-O system?
Task 3	What happens if another hanging object C is attached to the existing hanging object B?
Task 4	(a) What happens if we vary the mass of the hanging block B in the 2-O system? (b) What happens if we vary the mass of the sliding block A in the 2-O system?
Task 5	What happens if another sliding object F is attached to the existing object A?
Task 6	(a) How may a 2-O system behave if the string is broken when object B is moving down? (b) Given that in one particular situation after object B strikes the floor, it starts to move up vertically. Describe the behaviour of this system.
Task 7	How may any 2-O system behave? [Inclined Version]
Task 8	How may any 2-O system behave? [Vertical Version]

Table 4.3: Sequence of tasks for the main study.

The table overleaf (Table 4.4) gives a breakdown of what 'connected-particles' category each task would be looking at. However, there is the possibility of deviating from this table in case additional categories emerge during the main study.

	Horizontal Version						***Inclined Version***	***Vertical Version***
	Task 1	**Task 2**	**Task 3**	**Task 4**	**Task 5**	**Task 6**	**Task 7**	**Task 8**
Relationship between tension & weight	✓		✓	✓	✓		✓	✓
Relationship between tension & friction		✓						
Tension not the same throughout string	✓							
Effect of friction on motion		✓						
Effect of mass on motion	✓*		✓	✓	✓		✓	✓
Effect of mass on tension			✓	✓	✓			
Magnitude of acceleration	✓					✓	✓	✓
Kinematical aspects of acceleration	✓					✓	✓	✓
Kinematical aspects of velocity	✓					✓	✓	✓

Table 4.4: Possible ‘connected-particles’ categories in each task.

✓* implies that in this instance the student will be asked to predict the behaviour of the system if (a) mA = mB; (b) mA > mB; and, (c) mA < mB. S/he will not be required to investigate all the three conditions at this stage.

4.5.2 Introducing Interactive Physics

The way Interactive Physics (IP) is introduced to students is a fundamental aspect of this study. This issue is addressed into the following five groups:
(1) Should IP be introduced to students individually or in groups?;
(2) What type of mechanics problems must be used when introducing IP to students?;
(3) Should students be allowed to familiarise with IP for some minutes prior to exploring the ‘main’ tasks?;
(4) Using FRICTION; and,
(5) Using other features relevant to the study of connected particles.

Firstly, it seems necessary to reflect on whether the software must be introduced to students individually or in groups. The benefit of introducing the software in groups is that students can probably clarify any doubts with their peers in case they feel shy to ask me.

Secondly, another issue of concern is the type of mechanics tasks that students need to work with when being introduced to the software. It should be pointed out that the aim of this introductory part is to familiarise students with key features of IP (that would help them learn connected particles) and not to test their mechanics conceptions on basic topics.

Thirdly, it was observed that while Student S1 was able to use the basic features of IP, Student S2 seemed to be at a loss at the very beginning. Had I not intervened Student S2 would not have been able to complete the tasks. This can be explained by the fact that Student S1 employed his first fifteen minutes to review the basic features of the software before embarking on the tasks. Student S2 did not find this review useful; instead, he went to tackle the main tasks directly. This has to be taken care of during the main study.

Fourthly, using FRICTION in the given activities was an issue for both students as they were not fully aware of this concept on IP. In fact, how to remove friction and introduce friction between two surfaces in contact were of relevance for the success of the activities. So it is important for students to learn how to use the meter FRICTION FORCE. During the introductory part, students should be given enough practice about this issue.

Finally, it was seen that these students were not fully conversant with some features such as TOTAL FORCE, the component of a meter and coordination of events (that is, the synergy between moving objects and the vector arrows representing specific quantities). This shortcoming definitely needs to be addressed during the main study.

4.6 Data Analysis for Main Study

In this section, I will examine two issues in relation to the data analysis for the main study. First of all, I will explain my choice for the students selected for the main study. Second, I will discuss how the pilot analysis was fine-tuned to cater for the main analysis.

In relation to the sample of students for the main study, ten students were selected: both students (S1, S2) from the pilot study and eight students (S3 – S10) from the post-pilot study, that is the 'initial' main study. The pilot ones were chosen because further analysis proved them to be a rich source of data for the thesis. From the post-pilot study, two students had to be discarded as their voices were not audible and/or they did not speak much.

Though the main analysis had all the features of the pilot analysis, it was improved in two ways. First, the moments when students remained silent were scrutinized as they appeared to provide clues (such as competing intuitive ideas) which helped to build the learning trajectories of students. Second, since the main study generated a lot of data, a selected episode was first analysed repeatedly with the aim to understand its bigger picture such as shifts in mechanics ideas, competing intuitive ideas, etc. Then the learning trajectory of the student in question was tentatively built based on all observable information available in the said episode. This learning trajectory was extended or compared repeatedly to other students' learning trajectories with the intention to gain further insights into the learning process of the student under investigation.

It is useful to add that the data analysis of the main study ultimately led to the construction of two major stories:
(1) Will the system move? Why will it move?
(2) Will the acceleration of object B (or object A) be g? When will it be g?

4.7 Concluding Remark

In the following two chapters, the results of this study will be presented in light of the data analysis described in this chapter. While Chapter 5 will examine the findings related to student understanding of the dynamics of motion, Chapter 6 will zoom in the findings related to student understanding of acceleration.

Chapter 5

Student Understanding of the Dynamics of Motion

5.1 Introduction

This chapter aims to portray the findings related to student understanding of the mechanical conditions required for the system of connected particles to be in motion and at rest. In § 5.2, based on the participants' responses, I explain how I can examine their understanding in three different situations. I describe the first situation, where the mass of the hanging object B is greater than that of the sliding object A, in § 5.3. While § 5.4 explores the second situation where the objects A and B are of equal masses, the third situation, where the mass of object A is greater than that of object B, is examined in § 5.5. Finally, § 5.6 provides the conclusion to this chapter.

5.2 Overview of the Findings

In this chapter, my central aim was to investigate whether or not students, who interacted with the Horizontal system of connected particles (Figure 5.1), realised that the weight of the hanging object B must be greater than the tension in the string during motion. In other words, the net force acting on the hanging object B or the sliding object A could not be zero when the system was in motion.

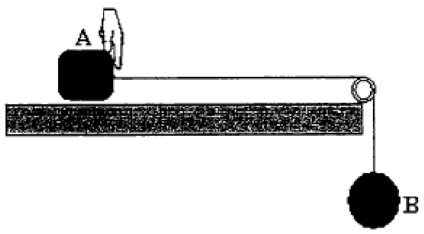


Figure 5.1: A Horizontal system of connected particles A & B being held at rest via a hand holding object A.

I also wanted to find out whether or not these students knew that the weight of the hanging object B must be equal to the tension in the string when the system was at rest. In the event that these students did not have these relevant mechanical ideas during the Prediction phase of the tasks, my intention was to see whether or not they shifted their talk from 'mass of object' to developing the appropriate register to explain the occurrence of motion.

During this research expedition, I had two subsidiary aims that I had felt were connected with my central aim. First, I wanted to explore student reasoning behind what pulled the sliding object A during motion. Was it the weight of the hanging object B, the tension in the string or something else? Second, I also intended to examine student thinking behind what created tension in the string. Was it object A, object B or something else?

When data were analysed in order to study the participants' intuitive ideas related to the conditions required for the Horizontal system of connected particles to be in motion and at rest, I found that, based on the participants' responses, this research expedition could be described via three different situations. The first situation occurred when the mass of the hanging object B [mB] was greater than that of the sliding object A [mA], that is $mB > mA$. The second one took place when we had both objects A and B of equal masses, that is $mB = mA$. Finally, the third one happened when the mass of object A was greater than that of object B, that is $mA > mB$.

For the first situation, that is $mB > mA$, I found that all the ten participants unanimously affirmed that there should be motion such that object A should move towards the pulley and object B should move down. However, none of them were able to explain the occurrence of motion via the appropriate mathematical register.

In relation to the second situation, that is $mB = mA$, I noted that the students made two different types of prediction:

(a) seven students (S1, S2, S3, S4, S5, S7, S8) predicted no motion; and,

(b) three students (S6, S9, S10) predicted some motion followed by equilibrium.

For those students who predicted no motion, I found that, after they had interacted with the software, there were some students who agreed with the motion (Students S1, S3, S4, S7) and those who did not agree with the motion (Students S2, S5). For those students who predicted some motion followed by equilibrium (Students S6, S9, S10), none of them agreed with the motion after having worked with Interactive Physics, and openly made reference to the Vertical system of connected particles (Figure 5.2). This is why it was deemed necessary to analyse these students' understanding of the Vertical system and to then determine, if ever, how this Vertical version influenced their understanding of the Horizontal version of connected particles.

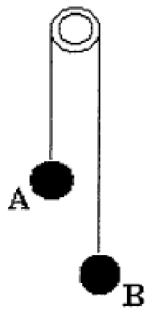


Figure 5.2: A Vertical version of connected particles A & B.

The third situation, that is mA > mB, saw all the ten students making the same prediction – all of them predicted no motion. It was observed that, after having worked through Task 1 with the software, there were some students who agreed with the motion (Students S1, S3, S6, S7, S8, S10) and those who did not agree with the motion (Students S2, S4, S5, S9). Among the later group of students, there were some who agreed with the motion and could explain it once they had worked through Task 4b (Students S2, S9).

So, in all seven episodes have been selected to portray the findings for this chapter. The table below (Table 5.1) summarises the details of each episode.

Situation	Student Prediction	Episode
mB > mA	Motion	**Episode 5.1** is about Student S9 who, after interacting with the software, could construct a 'correct' explanation of why there should be motion.
mB = mA	No motion	**Episode 5.2** deals with Student S3 who, after interacting with IP, agreed with the motion and could explain it.
		Episode 5.3 is about Student S2 who, after interacting with IP, did not agree with the motion.
	Motion followed by equilibrium	**Episode 5.4** examines Student S6 who, after interacting with IP, did not agree with the motion.
		Episode 5.5 portrays Student S6 working with the Vertical system of connected particles.
mA > mB	No motion	**Episode 5.6** deals with Student S3 who, after working with Task 1 on IP, agreed with the motion and could explain why there should be motion.
		Episode 5.7 portrays Student S2 who, after working with Task 1 on IP, did not agree with the motion. It is only when he 'developed'/integrated Task 4b at the end of his Task 1 that he agreed with the motion and was able to explain it.

Table 5.1: The details of each episode have been discussed in this table.

For each episode, it was imperative to build a detailed analysis (from the transcript) upon which the analysis presented in this chapter has been constructed. An example of such a detailed analysis for Episode 5.3 has been included in Appendix I.

5.3 First Situation: mB > mA

A thorough analysis of the participants' responses was carried out. I found that all the ten participants predicted that the Horizontal system of connected particles should be in motion whenever the mass of the hanging object B would be greater than that of the sliding object A and the hand holding object A (see Figure 5.1) would be removed – object A would move towards the pulley and object B would move down.

Nevertheless, none of the ten participants could explain the occurrence of motion by invoking the relationship between the tension in the string and the weight of the hanging object or by discussing the net force acting on either object. All of them initially thought that there should be motion simply because the mass of object B was greater than that of object A.

For this situation, in which we have only one possible scenario, I present **Episode 5.1**, where Student S9 who, after interacting with the software, could construct a 'correct' explanation of why there should be motion.

5.3.1 Unique Scenario – Motion

Brief Description of Episode 5.1

This episode began when Student S9 was asked to predict the behaviour of the system once object A would be released from rest. In this episode, I focus on the student understanding of the connected particles when the mass of the hanging object B is greater than that of the sliding object A. The transcript of this episode can be found in Appendix H.

Analysis of Episode 5.1

(a) During the initial stage of the Prediction Phase, Student S9 predicted that the system would move such that "object A will move towards the pulley and object B will move down" (lines 2 – 3) if and only if the mass of object B would be greater than that of the sliding object A. This is why when the simulation was run for the first time (between line 33 & line 34) and there was motion, he asserted that "this one [pointing to object B] is heavier" (line 35) even though he had not used any feature on Interactive Physics to verify the mass of object B. This demonstrates the activation of the intuitive idea 'a heavier load can displace a lighter one'. He also explained that the tension in the string would pull the sliding object A (lines 15 – 16; 40); however, he emphasized that the tension in the string should be equal to the weight of the hanging object B (lines 49 – 52; 171), implying that it was the weight of object B which pulled object A. From the previous statement, it can be inferred that for him the sliding object A did not contribute in creating tension in the string.

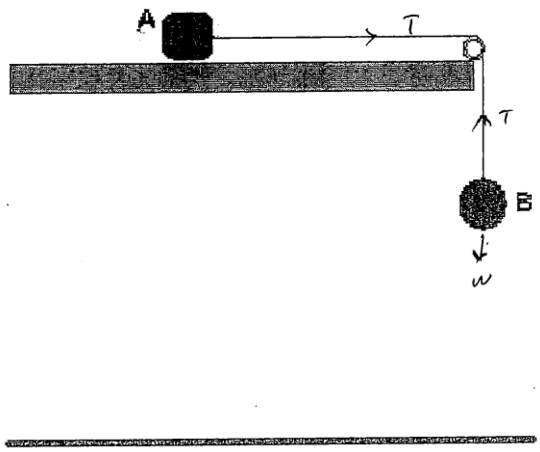


Figure 5.3: Force diagram drawn by Student S9.

On scrutinizing the student's force diagram (Figure 5.3), I find two issues of interest. First, he placed the forces acting on object B and in the string, but did not bother to label the forces acting on object A. This fact appears to confirm that for him object A did not play a key role in the behaviour of the system. Second, by analyzing the direction of the vector arrow representing the tension force in the horizontal part of the string, it seems that he had the intuitive idea 'object A was pulled by the tension in the string'. So, for him both objects were needed for the creation of a common tension in the string. However, this is in contradiction with his expressed idea that the tension in the string should only be equal to the weight of object B.

(b) During the Interaction Phase, he compared the readings of the TENSION and ACCELERATION meters (Figure 5.4) instead of the TENSION and GRAVITATIONAL FORCE meters (Figure 5.5) as I had asked him (lines 169 – 172). He was drawn to the fact that the tension in the string was equal to the acceleration of object B (lines 173 – 174; 176 – 177; 180 – 182; 184). In doing so, he did not realise that these two quantities, though of same numerical value, could not be compared as they were not of the same unit. As such, it can be hypothesized that his intuitive idea ‘the tension in the string would be equal to the weight of object B’ might have distracted his attention from the real issue.

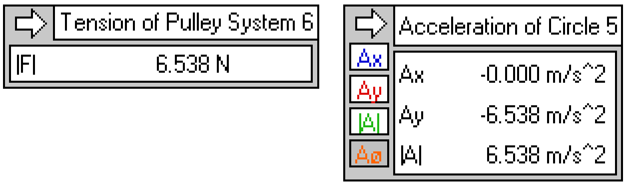


Figure 5.4: The pair of meters that were of interest to Student S9.

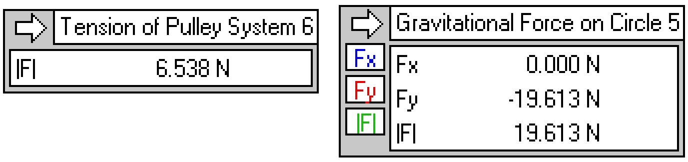


Figure 5.5: The pair of meters that should have interested Student S9.

So I had to remind him of our ‘real issue’ at this stage of the activity: to explain why on Interactive Physics the weight of object B was not equal to the tension in the string as per his prediction (lines 185 – 190).

(c) In trying to explain this discrepancy, he initially developed the idea that object A must also exert a tension in the string (lines 191; 193 – 195). However, instead of using this new fact to further construct the idea of having the same tension throughout the string, he developed the idea that there were two different tension forces in the string – the first one acting in the horizontal part of the string and the second one in the vertical part of the string (Figure 5.6). He explained that the sliding object A created a tension in the horizontal part of the string (lines 215 – 217) and the hanging object B exerted a different tension in the vertical part of the string. This is why, according to him, the weight of object B was not equal to the tension in the string.

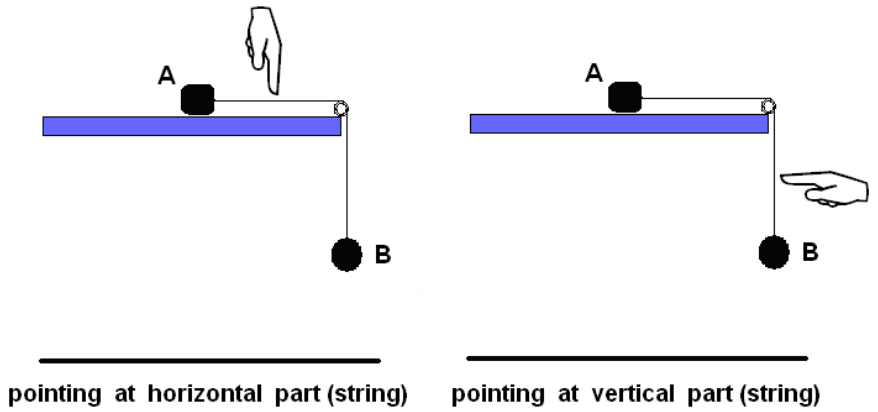


Figure 5.6: Student S9 suggested that there were 2 different tension forces in the string.

(d) When confronted with his force diagram (see diagram 5.3) where he seemed to have the intuitive idea 'there was only one tension force **T** throughout the string', he was no longer sure whether there should be one or two tension forces in the string (lines 209 – 219). As he appeared lost, I advised him to click on the string in the simulation and the whole string was selected. This strategy prompted him to realise that this system consisted of a single string (line 223) – a fact that made him realise that there had to be a single tension force in the whole string (line 225). At this point of this extract, he again suggested that the tension in the string was equal to the acceleration of object B (line 226). So I had to ask him to disregard the ACCELERATION meter and to focus on the TENSION and GRAVITATIONAL FORCE meters. Apart from stating that the tension in the string was more than half of the weight of object B (line 230), he did not make much progress along this line. This shows that he was not able to learn that motion could be explained via the relationship between tension in the string and weight of object B.

(e) As he did not know how to proceed and appeared to be a bit frustrated, I reminded him of the notion of 'net force' and advised him to find the net force acting on object B (lines 233 – 237). He put on the TOTAL FORCE meter of object B (Figure 5.7), ran the simulation for a few steps and found that the net force acting on object B was 13.076 N as object B moved down (line 239).

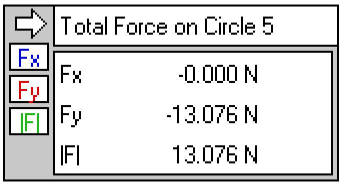


Figure 5.7: TOTAL FORCE meter showing the net force acting on object B.

When asked about the direction of this force, he stated that the net force would act downward (line 243). It can be inferred that the downward direction of motion of object B might have influenced him about the direction of the net force. This prompted me to question him about the magnitude of the net force acting on object B if the tension in the string would be equal to its weight. He stated that it should be zero, suggesting that he started to learn that net force acting on object B might be equal to the difference between tension in the string and its weight. When he was asked to explain how Interactive Physics might have calculated the value displayed (13.076 N) on the TOTAL FORCE meter, he highlighted the interplay between the weight of object B and the tension in the string (lines 253 – 265).

I: How did it [referring to Interactive Physics] get this total force value [pointing at TOTAL FORCE meter]?
S9: It equated tension and weight.
I: How did it arrive at this value?
S9: Maybe it subtracted this [pointing at TENSION meter] from this [pointing at GRAVITATIONAL FORCE meter].
I: What did it do?
S9: It took gravitational force [pointing to GRAVITATIONAL FORCE meter] minus tension [pointing to TENSION meter].

From the preceding extract (lines 253 – 258), especially from his verbal expression "Maybe it subtracted …", it appears that he gradually learnt that the net force acting on object B was equal to the difference between its weight and the tension in the string (Figure 5.8).

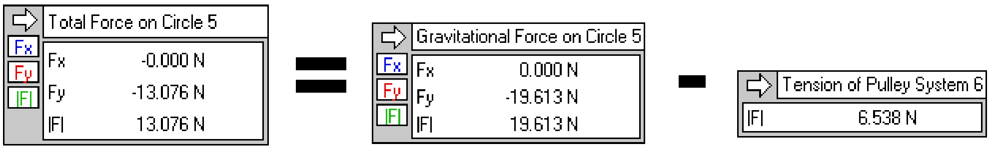


Figure 5.8: Mental calculation done by Student S9 to find the net force acting on object B via its weight and the tension in the string.

As our conversation continued, he recalled the situation where the tension in the string and the weight of object B were equal and its definite physical outcome (lines 269 – 274).

S9: If they were equal, we should have obtained zero, if tension and weight were equal.
I: It should have been zero?
S9: Yes.
I: What happens if net force is equal to zero?
S9: The objects will not move … they will remain stationary.

It is almost likely, from the above extract (lines 269 – 274), that he built the understanding that when the net force acting on object B would be equal to zero, the system of connected particles would remain at rest.

As such, it can be hypothesised that initially the student never realised that the tension in the string and the weight of the hanging object were two forces acting away from object B; he always understood that the tension in the string was solely due to the weight of the hanging object B – so the idea of finding the difference between these two quantities to explain motion must have never crossed his mind. However, in this particular case the concept of net force appeared to create a sort of bridge for the student to understand motion of connected particles in terms of the weight of object B and the tension in the string. It can also be suggested that either he developed a new mechanical idea 'At-rest ≡ equal forces acting away in opposite directions from a point' or the said idea was reactivated.

(f) It is important to note that friction was introduced in this system (as from line 376). At this point of the episode, Student S9 had already worked on Interactive Physics with the connected particles system when (1) the masses were equal and (2) the mass of object A was greater than that of object B.

When he worked with the system mA = mB, he argued that both objects would move a short distance until they would reach equilibrium (lines 9; 11 – 12; 15 – 17). It seems that for him equilibrium would be reached when the length between object A and the pulley would approximately be the same as that between object B and the pulley. When he ran the simulation, contrary to his expectation, he observed that there was motion. Though initially he did not agree with the motion (line 289), through the 'concept' of net force, he managed to successfully develop the relationship between tension in the string and weight of object B to explain motion (lines 298 – 307). However, at a later time, he did not accept that there could be motion in this system (lines 330 – 335).

When he worked with the system mA > mB, he did not agree that the system could move (lines 343 – 345; 348; 353 – 354; 358 – 359). He explained that the nature of the surface of the table could be held accountable for the motion of the system (lines 361 – 364; 374 – 375).

S9: If we had another surface, it would not have moved. In this case, there is no friction which is opposing the motion.

The above statement demonstrates that real-life experience influenced his learning process. According to him, motion was possible because of the "extra smooth, like ice" surface (line 361). This is why I immediately encouraged him to pursue with this exploration. Using the original system with mA = 1 kg and mB = 2 kg, he introduced friction in the system by making μ, the coefficient of friction, equal to 0.3.

(g) He predicted that though friction was now present in the system, the tension in the string would remain the same just as when we worked with a smooth situation.

S9: The tension 'lies' here [pointing along the vertical part of the string]. It is mostly concentrated here … It may change but I'm not sure.
I: Why is it that earlier you mentioned that friction won't affect tension?
S9: I don't think that there is any relationship between the friction here [pointing cursor around object A] and tension here [pointing along the vertical part of the string].

From the above extract (lines 385 – 390), it seems that for him the tension in the string was solely due to the weight of object B and object A had no contribution whatsoever. Though he had the idea that "object A is moving against friction, friction is opposing its motion" (lines 392 – 393), he continued to predict that the tension in the string would remain unchanged (lines 393 – 394). It is imperative to point out that just before we embarked on this system with friction, he explained that "the weight is the tension" (line 372), implying that for him the tension in the string was still due to the weight of object B.

The question that arises from this situation is why he continued to mention that the tension in the string was due to object B only, especially after having learnt that both objects moved with the same velocity (lines 95 – 116) and with the same acceleration (lines 146 – 168), and after his interaction with the concept of net force in the earlier part of the activity (refer to part (e)). It would appear that for him the relationship 'weight of B must be greater than tension to explain motion' was totally apart from his intuitive idea that 'tension was due to object B only'. That is, he could not use the relationship between weight and tension to realise the erroneous nature of his intuitive idea, and this may explain why he failed in learning that both objects contributed in creating tension in the string.

After simulating the situation on Interactive Physics, he found that the tension in the string increased when friction was present in the system. In accounting for this increase in tension, he first invoked the important role of object A (lines 414 – 415).

S9: Because object A is moving, the tension acting in this string [pointing at the horizontal part of the string] also increases.

He then suggested that the tension in the string was due to both objects A and B (lines 420 – 421) and that there were two tensions in the string (lines 423 – 424; 426), one different from the other (lines 428 – 430). This explanation is the same as previously discussed in earlier part (see Figure 5.6).

S9: … There are 2 different tensions. The value of the tension here [pointing at the horizontal part of the string] is different from the value of the tension here [pointing at the vertical part of the string].

As the discussion unfolded, it was seen that he was fully aware of the fact that when there was only one string in this system, there should be only one tension (lines 436 – 437). However, he started to profess that there should be two tensions in this string (line 439) and that Interactive Physics displayed only the one along the vertical part of the string.

S9: Perhaps there is a tension that is acting here [pointing at the horizontal part of the string] but Interactive Physics is showing only one.

The question that is worth asking here is whether or not it would have made a difference to his learning journey if he had the option of measuring the tension at any point in the string in Interactive Physics.

5.4 Second Situation: mB = mA

From the data analysis, it is seen that the participants made two different types of prediction:

(a) seven students (S1, S2, S3, S4, S5, S7, S8) predicted no motion; and,

(b) three students (S6, S9, S10) predicted some motion followed by equilibrium.

For those students who predicted no motion, I found that, after they had interacted with the software, there were two groups of students: those who agreed with the motion (Students S1, S3, S4, S7) and those who did not agree with the motion (Students S2, S5). I have chosen one student per group and have discussed their learning trajectories in the First Scenario – No Motion. While **Episode 5.2** portrays Student S3 who agreed with the motion, **Episode 5.3** deals with Student S2 who did not agree with the motion.

In relation to those students who predicted some motion followed by equilibrium (Students S6, S9, S10), they did not agree with the motion after having worked with Interactive Physics. In addition, they openly made reference to the Vertical system of connected particles (Figure 5.2). This is why it was imperative to first analyse these students' understanding of the Vertical system and to then determine, if ever, how this Vertical system influenced their understanding of the Horizontal system of connected particles. On this basis, for this Third Scenario – Some Motion followed by Equilibrium, the learning trajectory of Student S6 will be explored via two episodes: **Episode 5.4** will examine the student's interaction with the Horizontal system and **Episode 5.5** will portray the student's interaction with the Vertical system.

5.4.1 First Scenario – No Motion to Motion

Episode 5.2

Brief Description of Episode 5.2

In this episode (as from line 141), Student S3 wished to examine the behaviour of the system with equal masses on Interactive Physics. He changed the mass of object A [mA] from 1 kg to 2 kg and maintained that of object B [mB] to 2 kg. The transcript for this episode can be found in Appendix C.

Analysis of Episode 5.2

(a) At the start of Task 1, Student S3 predicted that there should be motion whenever the mass of the hanging object B [mB] would be greater than that of the sliding object A [mA] (lines 7 – 15). He also predicted that there should be no motion when the objects were of equal masses [mB = mA] (lines 27 – 32). He further stated that the weight of object B pulled object A during motion (lines 14 – 15; 19). This is in contrast to the direction of the vector arrow representing tension force in the horizontal part of the string (Figure 5.9); from the diagram, it would be deduced that it was the tension that pulled object A.

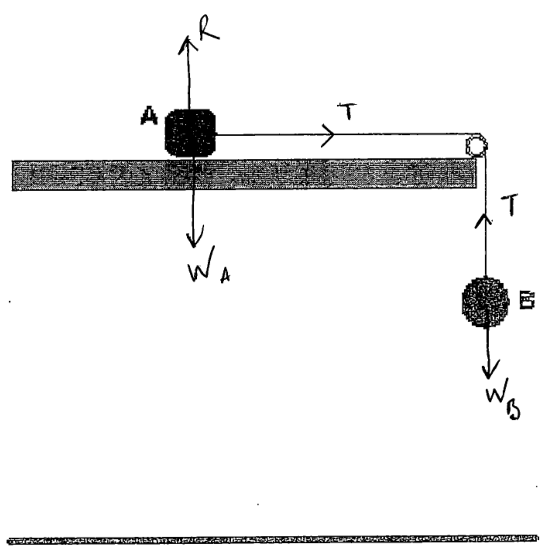


Figure 5.9: Force diagram drawn by Student S3.

When he had almost finished analysing the system mA < mB, he explained that the weight of object B and the tension in the string could not be equal (lines 139 – 140).

S3: No, they will not be equal. If object B is moving down, weight must be greater than the tension.

It appears that he knew this fact from his school mechanics (lines 55 – 56) as there is no indication from the data that he had learnt it during this activity.

(b) When he was asked whether he was still predicting a NO MOTION for the system with equal masses, he maintained his prediction (line 146). According to him, this was because both objects had the same weight (lines 148 – 149; 151). This suggests that for him the weight of each object, being equal, would balance each other and this should explain why there should be no motion. As such, it can be hypothesized that he thought of the stability of the system in terms of weights only and, therefore, had the intuitive idea ‘equal masses implied no motion’ influencing his prediction. At this point of the episode, it seems likely that the reaction force on object A, R, (see Figure 5.9) was inexistent for him.

(c) When he was allowed to run the simulation (as from line 164), he appeared surprised to discover that there was motion. When he was asked to explain why he initially predicted that there should be no motion, he maintained his justification in terms of forces.

S3: Forces acting on object A are equal to forces acting on object B. So, the objects must be in equilibrium.

The above statement (lines 170 – 171) and the one at (lines 178 – 179) suggest that he initially had the intuitive idea ‘the stability of the system was to evaluated in terms of weights only’.

When he was asked whether or not he agreed with the motion, he responded positively (line 176). On enquiry, he explained his approval of the motion in terms of forces.

S3: There is a force that will pull it [pointing at object B] down. The weight will pull it down. So it will move down. Whereas here [pointing at object A] …(pause)… there will be a reaction acting on it.
I: Can you explain it again?
S3: Weight of object B will pull object B and also pull object A.
I: But what you stated earlier in terms of their forces, will they not be the same? … Didn't you say that both objects have equal weights?
S3: Yes.
I: What do you think?
S3: But this one [pointing at object A] has a contact force.
I: So?
S3: This weight [pointing at object A] has no influence on the motion of object B.
I: Did you initially have this idea?
S3: No. I initially thought that the weight [pointing at object A] will not allow object B to move.

From the above extract (lines 178 – 193), we note two mechanical ideas developed by Student S3 in order to justify the motion. First, he assigned a key role to object B. It seems that for him the weight of object B would become "a force" that would "pull object B and also pull object A". Second, he now professed that object A did not have any effect on the motion of the system. It can be hypothesized that he initially had these intuitive ideas 'the stability of the system must be evaluated in terms of weights only' and 'object A was a burden for object B' (lines 192 – 193). However, when he realised that he could not account for the motion of the system mA = mB via his intuitive idea of comparing weights, he introduced the reaction force on object A, R, in his explanation as from line 180. He observed that there was a normal reaction acting on object A only (line 187). As such, he might have this new intuitive idea 'the weight of object A, W_A, and the normal reaction R would cancel out'.

This is why, according to him, object A had no influence on the motion of object B (lines 189 – 190). Such reflection on his part is rather odd as at this point of the episode he had already learnt that both objects moved with the same velocity and the same acceleration (lines 90 – 114). This suggests that being aware that 'both objects move with the same acceleration because they are connected' is not a sufficient condition to realise that object A would affect the motion of object B.

(d) He put on the TOTAL FORCE meters for both objects A and B, ran the simulation and found that the readings for both meters were 9.807 N (Figure 5.10).

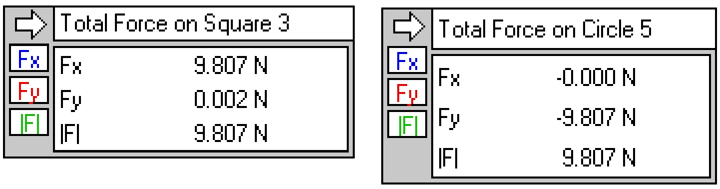


Figure 5.10: The meter TOTAL FORCE ON SQUARE 3 displayed the net force acting on object A, whereas the meter TOTAL FORCE ON CIRCLE 5 displayed the net force acting on object B.

He did not agree that "these forces be equal to g" (lines 198 – 199); he had been expecting the net force acting on object A or B to be equal to 2g (line 201). This is because for him "the total force is equal to the weight" (line 209), implying that he was being guided by his intuitive idea rather than his school knowledge on net force (lines 60 – 61). When enquired whether he also expected the net force acting on object A to be equal to its weight, he explained the dominant role played by object B in this matter (lines 210 – 224).

S3: I was expecting the total force acting on object A to be equal to the weight of this one [pointing at object B].
I: So, what would you have proven then?
S3: … This one [pointing to B] is pulling this one [pointing to A] with the same force … The force that is acting here [pointing at object B] is also

acting in this direction [pointing at the horizontal part of the string towards the pulley] and therefore pulling it [referring to object A].
I: Can you explain it again?
S3: This means that this weight [pointing at object B] is pulling this one [pointing at object A], the force that is acting on object B is pulling object A with the same magnitude.

From the above extract (lines 214 – 224), it can be seen that the student held the following intuitive ideas:

(1) it was the hanging object B that pulled the sliding object A. This is consistent with his initial idea on this issue (line 15);
(2) the tension in the string was solely due to the weight of object B;
(3) the net force acting on object B was equal to its weight;
(4) the net force acting on object B should be the same as that acting on object A; and,
(5) the weight of object A was of no importance in this system.

(e) He ran the simulation again and the readings on both TOTAL FORCE meters were the same 9.807 N (line 229). As he could not understand why the readings were not 2g as per his expectation (lines 231 – 237), I advised him to think about it. After some moment of reflection, he mentioned that he wanted to verify the value of the tension in the string. So he reset the simulation, put on the TENSION meter, ran the simulation and found that the tension in the string was equal to 9.807 N.

S3: It is the tension that is cancelling the … the weight [pointing at object B] is acting downward and the tension is acting upward, so the net force is equal to 2g minus the tension, and it will be equal to 9.81.

It can be deduced from the above statement (lines 239 – 241) that though he was fully aware of the forces weight and tension acting on object B, he did not realise that to obtain the net force in this particular case he had to subtract the tension from the weight. Interestingly, for the system mB > mA, he was fully aware of the relationship between tension in the string and weight of object B to explain motion (lines 136 – 140). So the main question is why he did not use the said relationship to explain motion for the system mB = mA. It can be hypothesized that he agreed that in the case of the system mB > mA the weight of object B would be greater than the tension in the string during motion because he expected the system to move. However, in the system mB = mA, since he initially did not expect the system to move, the relationship between weight of object B and tension in the string (from the viewpoint of motion) was of no importance to him.

It would appear that though he knew that "tension is opposing the weight" (line 60), for the system mB = mA, he never realised that the tension in the string and the weight of the hanging object were two forces acting away from object B; for him, the tension in the string was solely due to the weight of object B (lines 254 – 264) – so the idea of understanding net force as the difference between these two quantities for this system seemed to be new to him. However, in this particular case it appears that through the notion of net force he was able to build a relationship between the weight of object B and the tension in the string to make sense of the occurrence of the motion of both objects.

(f) His query "Why is tension equal to 9.807 N" (line 247) allowed me to further probe into his thinking of the origin of tension in the string. He explained that he initially held the intuitive idea 'the tension in the string was due to the weight of object B only' (lines 254 – 264). However, he then changed his idea and suggested that the tension depended on both objects (lines 264 – 270). He then reverted back to his initial idea that it depended on object B only (lines 272 – 280) and that it was because of object B that tension was created (lines 274 – 277). It can be hypothesized that the conflict between the two competing and contradictory intuitive ideas might have been resolved through the 'force of gravity' idea. For him, it appears that object B would be more influenced by 'force of gravity' because it hung in the air, whereas object A was on a surface.

At this point of the episode, he reset the simulation, added vector arrows to represent the tension in the string and ran the simulation in slow motion for a few steps. The following screenshot (Figure 5.11) appeared.

Figure 5.11: Screenshot showing the vector arrows representing tension in the string.

It is important to note that during an earlier part of this episode where he wanted "to see the forces on each object" (line 116), he found that there were no vector arrows to represent the tension in the string along the vertical part of the string (lines 121 – 122). I pointed out to him that this was a shortcoming in the software (lines 123 – 128) and I drew a correct one.

As soon as he stopped running the simulation, he wrote the following equations of motion (Figure 5.12) on a sheet of paper.

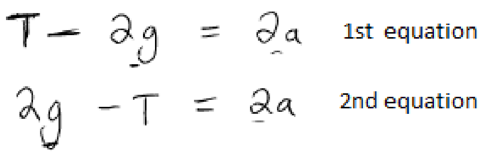


Figure 5.12: Student S3 wrote the above 2 equations of motion for the system.

He then mentioned that both objects were needed to create tension in the string (line 283). What is responsible for this change? Is it the vector arrow on object A as seen on Interactive Physics or his set of equations? It would appear that his set of equations was greatly responsible for the change because he had to consider both objects. Interestingly, these equations are written for the Vertical version of connected particles (refer to Figure 5.2). So it is observed that there has been rote learning and that he has not understood the basic principle.

(g) In order to continue exploring his ideas, he reset the simulation, added vector arrows to represent the weights of both objects A and B, and ran the simulation in slow motion for a few steps. The following screenshot (Figure 5.13) appeared.

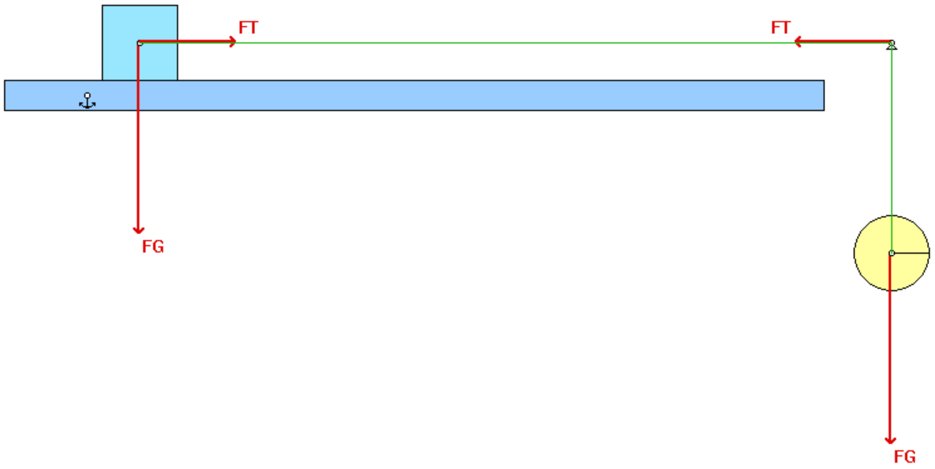


Figure 5.13: Screenshot showing the vector arrows representing tension in the string and weights of objects A & B.

He pointed out that the net force acting on object A should be the tension in the string minus the weight of object A (lines 289 – 291; 293 – 295; 297). He explained that he has learnt this relationship (refer to 1st equation in Figure 5.12) in a mechanics lesson (line 301).

S3: But now that I'm seeing it, how can this be possible since the tension is acting in this direction [pointing horizontally] and the weight is acting in another direction [pointing vertically]? […] So, one cannot minus the other.
I: Ok. So this is giving you another way of looking at it.
S3: Yes. Here [pointing at object B] both the tension and the weight are acting along the same direction [moving cursor vertically] and so we can have the weight minus the tension to have the net force.

From the above conversation (lines 303 – 310), it can be inferred that the software, through its diagrams and vector arrows, provided the student a new window through which he could make sense of his initial knowledge of the forces acting on each object simultaneously. For object A, he discovered that his

1st equation (in diagram 5.12) could not be applied in this context as the tension in the string [FT] and the weight of object A [FG] were not in the same axis (lines 303 – 306; 314). For object B, he found that his 2nd equation (in diagram 5.12) was correct as the tension in the string [FT] and the weight of object B [FG] were in the same axis (lines 308 – 310). So it appears that the to and fro movement between the visual mode (diagram and vector arrows) and the algebraic mode (equations) enabled him to make sense of his set of equations.

It should, however, be noted that Interactive Physics did not display the vector arrows representing the tension acting on object B (Figure 5.14(a)). It is imperative to add that during an earlier part of this episode (lines 118 – 131) he wanted to see the direction of the tension force in the string. As he could see no vector arrows along the vertical part of the string (Figure 5.14(a)), I discussed with him the shortcoming of the software in relation to this issue and drew an appropriate force diagram to compensate for the said shortcoming (lines 121 – 132). Yet, from his force diagram (Figure 5.9), it can be seen that he was already conversant with the vector arrow of tension moving away from the centre of object B. So it seems that the student had in mind the diagram shown in Figure 5.14(b) when he explained that “both the tension and the weight are acting along the same axis [moving cursor up and down] and so we can have the weight minus the tension to have the net force” (lines 308 – 310).

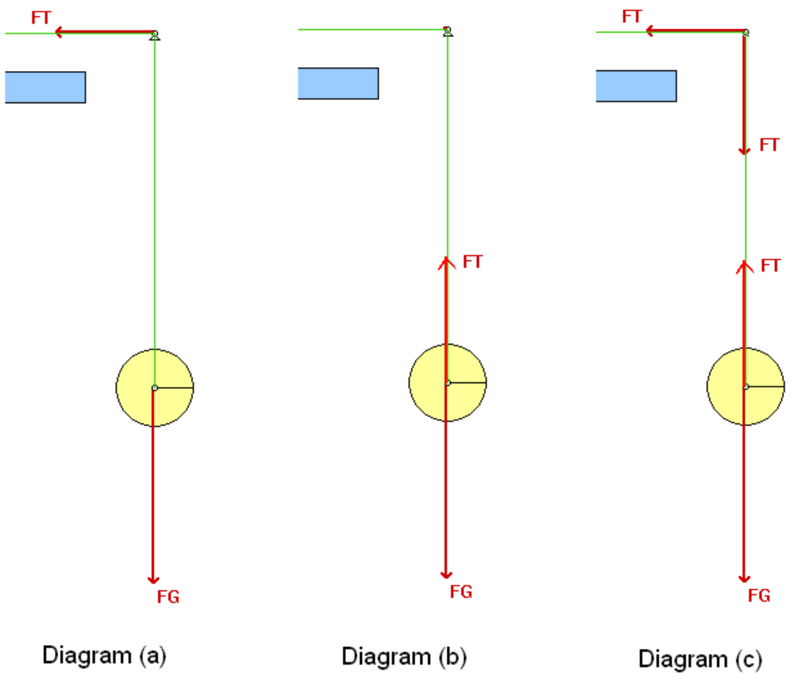


Figure 5.14: Diagram (a) displays how object B appeared on the screen of Interactive Physics. Diagram (b) displays how object B might have been pictured by the student with the direction of the tension force FT moving away from the centre of object B (refer to her force diagram – Figure 5.9). Ideally speaking, the software should have displayed Diagram (c) instead of Diagram (a).

On this basis, it can be hypothesized that had there been the said vector arrows along the vertical part of the string it might have helped the student to realise that tension in the string opposed the weight of object B and that these two vector quantities were not of equal magnitudes by virtue of their arrow length. In turn, this might have altered his learning trajectory.

(h) As he was talking about net forces, I advised him to put on the vector arrow TOTAL FORCE for each object. He removed the vector arrows GRAVITATIONAL FORCE from both objects and added the vector arrows TOTAL FORCE on both objects. He ran the simulation. I asked him to eliminate the vector arrow TENSION in the string as it had the same abbreviation [FT] as the vector arrow TOTAL FORCE on Interactive Physics and, therefore, to eliminate any sign of confusion. When he ran the simulation, the following screenshot appeared.

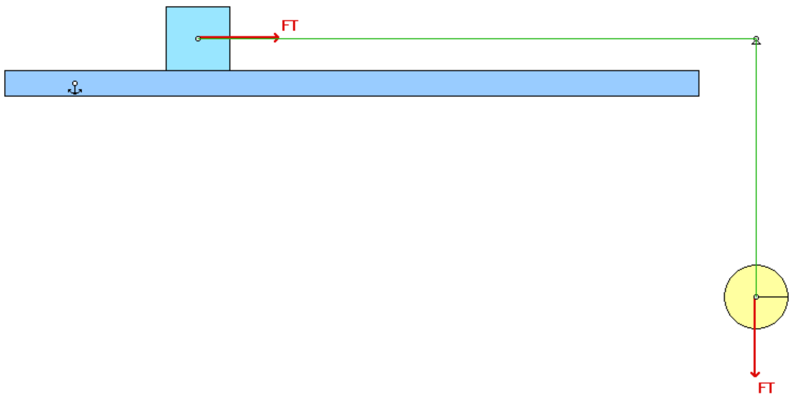


Figure 5.15: Screenshot showing the vector arrows representing the net forces acting on objects A & B.

He reiterated his initial idea ‘the net force acting on object B pulled object A’ (lines 318 – 322) and also added that the weight of object A had no effect on the net force on object B. It would appear that the vector arrows FT representing the net forces in Figure 5.15 might have contributed in cueing and reinforcing his initial idea as the length of the vector arrow FT on object B appeared to be equal to that on object A.

He reset the simulation, added the vector arrows GRAVITATIONAL FORCE on both objects and ran the simulation in slow motion for a few seconds. We had the following screenshot:

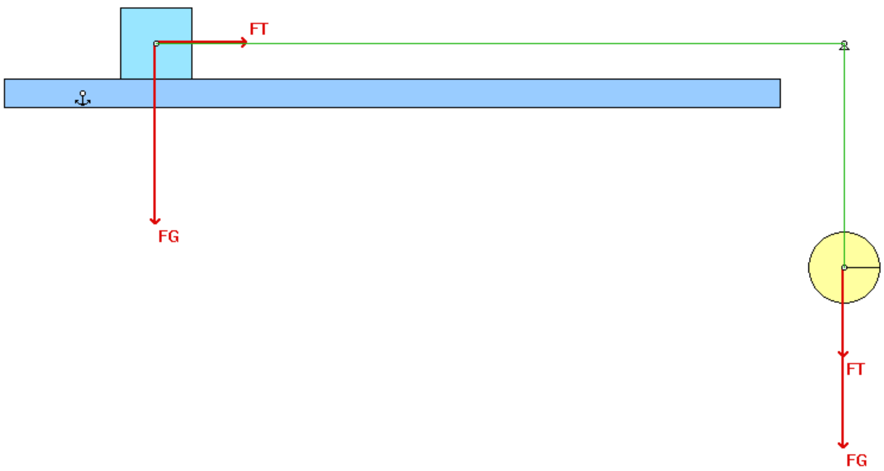


Figure 5.16: Screenshot showing the vector arrows FT representing the net forces acting on objects A & B, and the vector arrow FG representing the weights of objects A & B.

After a long time of reflection, he concluded as follows:

S3: My conclusion is that this net force [pointing at arrow on object B] is pulling object A […] the net force on object B is equal to the net force on object A.
I: So, what are you trying to clarify?
S3: That the net force on object A [pointing at arrow FT near object A] does not depend on its weight [pointing at object A].

From the above extract (lines 324 – 329), we find two mechanical issues of interest.

(a) The student now held a new intuitive idea 'it was the net force acting on object B that pulled object A'. At the start of the task, he mentioned that object B pulled object A (line 15); he then explained that it was the weight of object B that pulled object A (lines 19; 178 – 179). When he introduced the idea of net force in the task, he started to use the terms 'net force on B' and 'weight of B' interchangeably (line 209). At some point (lines 239 – 241), he learnt that the net force on object B was equal to the weight of object B minus the tension in the string.

(b) His intuitive idea 'the net force on object B is equal to the net force on object A' has been reinforced through this task. Though this intuitive idea was true in this particular case, this is not the case in general.

5.4.2 First Scenario – Persistence of No Motion

Episode 5.3

Brief Description of Episode 5.3

In this episode, which started as from line 177, Student S2 was asked to verify the masses of both objects A and B. He had just run the simulation for the first time and predicted that object A was heavier than object B (line 167). While the transcript for this episode is located in Appendix B, a (preliminary) detailed analysis can be found in Appendix I.

Analysis of Episode 5.3

(a) At the start of Task 1, Student S2 predicted that if both objects were of equal masses (mB = mA), the system would remain at rest (lines 26 – 27; 32 – 33). He accounted for this equilibrium in term of tension forces in the string (lines 33 – 36).

S2: If the table is smooth … there will be no motion. The objects won't move. Because the tension here [pointing at the vertical part of the string] will be equal to the weight of object B, and the tension here [pointing at the horizontal part of the string] will be equal to this weight [pointing at object A].

From the above statement and others (lines 50 – 60), it seems that he held the intuitive idea 'there were two tension forces in the same string, each tension being equal to the weight of the object adjacent to it' (refer to Figure 5.6). His force diagram (Figure 5.17) appears to be an exact replica of his verbal statement. His intuitive idea can account for two of his predictions: (1) that when the mass of object A would be equal to that of object B, the tension in the string would be equal to either weight (lines 73 – 74; 76 – 77; 80; 83 – 85); and, (2) that there would be no motion for a system of equal masses (lines 29 – 30; 46 – 47; 70 – 74; 85) because both tension forces would act against and cancel each other.

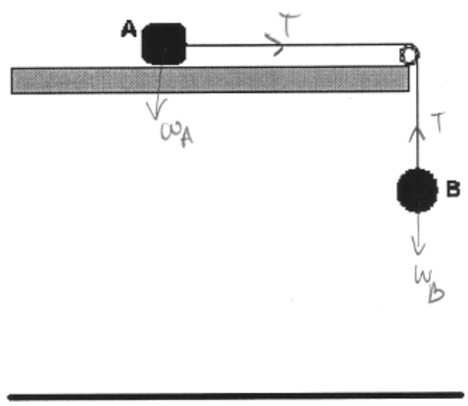


Figure 5.17: Force diagram drawn by Student S2.

As such, it may be suggested that the intuitive idea 'equal masses ≡ no motion' was activated during this activity and held a high cueing priority such that it influenced his decision to predict the no-motion situation.

(b) As he was allowed to run the simulation, he did so by using the command RUN. The simulation proceeded as follows: object A moved along the surface towards the pulley, left the surface, moved up a bit pulling object B and then started moving down (by following the arc of a simple pendulum). Finally object B was at the pulley and object A was swinging at the bottom level (Figure 5.18). I had to ask him to stop, reset and run the simulation. He again used the command RUN and the same set of events happened. He continued to watch the simulation until he was again asked to stop it. At this point I reminded him that he should analyse just the part of the simulation when object A was on the surface. When he was asked about his comments on this system, he reset the simulation, ran it using the command RUN (for the third consecutive time) and observed the same set of events.

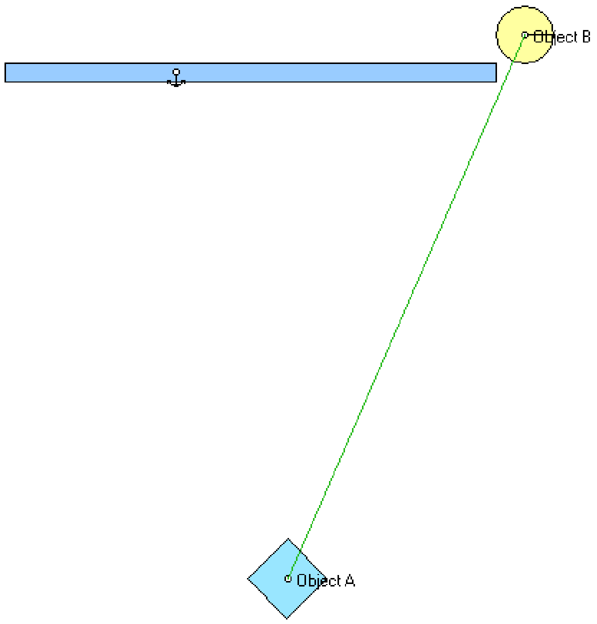


Figure 5.18: Screenshot showing object B at the pulley and object A, which was attached to the other end of the string, at the bottom level.

He then stated that object A must be heavier than object B (line 167) because "object B is near the pulley and object A is down" (line 169). This shows the activation of a new intuitive idea 'when there are two objects situated at different levels, the one lying at the bottom must be heavier than the one lying at the top'. I reminded him that according to his earlier prediction (lines 136; 140) the system must be at rest if the mass of object A was greater than that of object B. Since he could not explain this unexpected finding and remained silent for some time (line 176), I asked him to verify whether or not object A was heavier than object B in the simulation.

(c) He found that the masses of both objects were 1 kg each. At first he could not explain why the objects moved (line 186). However, after some time, he managed to develop an argument in term of forces to justify the motion.

S2: …(pause)… For motion to take place … does weight of object B not become a force that pulls object A & this sets the system in motion?
I: Say that again.
S2: The weight of object B becomes a force that pulls object A towards the pulley.
I: According to you, what is pulling object A towards the pulley?
S2: The weight of object B … This one [pointing to object A] is at a certain height compared to object B.

From the above extract (lines 192 – 199), it seems that in order to develop a rationale to justify the motion, first of all, he focused on the role of object B. For him, object B would become "a force" that would pull object A and would therefore set the system in motion (lines 192 – 193; 195 – 196). He then pointed out that object B was at a lower level than object A (lines 198 – 199) – most probably he might have implied that they were not balanced – and this difference in level might have contributed in the disequilibrium of this system.

(d) He was asked to verify his statement that "when mA = mB, tension in the string = weight of object A = weight of object B" (lines 200 – 202). After having put on the GRAVITATIONAL FORCE meters of objects A and B, and the TENSION meter, he was advised to run the simulation in slow motion and he noted the readings on the three meters (Figure 5.19)

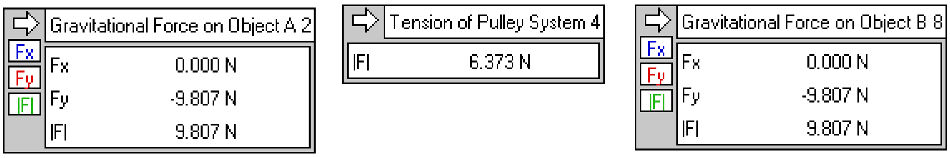


Figure 5.19: The meter GRAVITATIONAL FORCE ON OBJECT A 2 displayed the weight of object A; the meter TENSION OF PULLEY SYSTEM 4 displayed the tension in the string; and, the meter GRAVITATIONAL FORCE ON OBJECT B 8 displayed the weight of object B.

He found that the tension in the string was not equal to the weight of object B (lines 204; 206 – 207) and was not able to explain this discrepancy (line 209). He then decided to examine whether the surface was smooth or rough. When he discovered that he had been dealing with a rough situation, he explained that "the frictional force affects the tension" (line 216). Though he removed friction from the system, he found, after running the simulation again, that the tension in the string was still not equal to the weight of object B (lines 221; 223 – 224).

S2: Even now, the tension in the string is not equal to the weight of the object.

From the above statement, more specifically to the terms "even now", it seems clear that for him the tension in the string should always be equal to the weight of object B, be the system in motion or at rest. This is why he developed the following new arguments in order to defend his intuitive idea.

S2: There must be some force acting on the pulley [pointing the cursor at it].
I: Why are you saying this? … You've just said that even here the tension is less. With what are you comparing it?
S2: I'm comparing it with the weight of the hanging object B. The tension should be equal to the weight as we had assumed. But they are not equal, they are different.

From the above extract (lines 226 – 232), it can be observed that he developed the idea that "some force acting on the pulley", most probably frictional force, was responsible for the discrepancy between the tension in the string and the weight of object B. In practice, it is true that pulleys are not smooth but in the modelling world we assume it to be so – an assumption which he did not accept to work with. This new idea demonstrates the veracity of his intuitive idea 'the tension in the string should be equal to the weight of object B'. In the end, he was not able to explain this discrepancy (lines 233 – 238).

5.4.3 Second Scenario – Some Motion followed by Equilibrium

In this scenario, I have chosen Student S6 and would examine her learning trajectories in Episodes 5.4 and 5.5.

Episode 5.4

Brief Description of Episode 5.4

This episode began, as from line 154, when Student S6 showed a keen interest in exploring the simulation by varying the mass of object A [mA]. She wanted to first simulate on Interactive Physics the system mA > mB, but I advised her to first explore the system mB = mA. The transcript of this episode can be found in Appendix E1.

Analysis of Episode 5.4

(a) At the beginning of Task 1, she predicted that with the system mB = mA, "there is motion until equilibrium is reached" (line 8). She explained that at equilibrium the length between the sliding object A and the pulley should be equal to that between the pulley and the hanging object B (lines 18 – 19). On this basis, though at this point of this episode she did not overtly refer to the Vertical version of connected particles (refer to Figure 5.2), it seems that the

Vertical version was influencing her prediction for this system. It is equally important to note that before interacting with the software she already knew that (1) "when there is no motion, this means that the tension in the string is equal to the weight of the object [pointing at object B]" (lines 53 – 54); and, (2) "if it [referring to object B] is going down, this means that weight is greater than tension" (lines 119 – 120).

(b) During the Interaction Phase, after having run the simulation for the first time, she did not agree with the motion – she insisted that the system should have stopped (line 161). Yet, she tried to build an argument to explain the motion, but in vain.

S6: Because if this force [pointing at object B] is greater, it depends on this object [pointing at object B].
I: I didn't understand you.
S6: Oh God! Even I can't understand it.

From the above extract (lines 165 – 168), it can be seen that she not only failed to construct a credible explanation but also appeared confused. This confusion seems to confirm her disagreement with the motion undergone by the system on Interactive Physics. So she ran the simulation forward in slow motion until object A reached the end of the surface and then backward until it almost reached its initial position. She observed that on Interactive Physics the motion of the system mB = mA was identical to that of the system mB > mA (lines 171 – 174). When I asked her whether she thought that Interactive Physics had fooled us, she explained that this system was similar to the Vertical version of connected particles with equal masses (lines 177 – 184) and drew the diagram as shown in Figure 5.20.

S6: … Is this system not similar to the system where we have 2 objects attached to the 2 ends of a string which passes over a pivot? If they are of equal masses, they will be at the same level.
I: I didn't understand you.

S6: When we have 2 ends of a string [pointing at the vertical part of the string] and we don't have a surface, they [pointing to objects A & B] hang freely and if they are of equal masses, they will be at the same level.

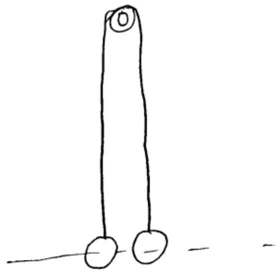


Figure 5.20: Vertical version of connected particles drawn by Student S6.

Based on these facts, it is clear that the Vertical version of connected particles had influenced her thinking in relation to this Horizontal version. In particular, she voiced out the intuitive idea that if both objects were of equal masses and were not at the same level, "they'll move up and down until they stabilize at the same level" (line 189). This suggests that in this case 'equal masses' first implied 'same height' and then 'no motion'. This is why she had earlier predicted that the length between the pulley and object B should be equal to that between the pulley and object A (lines 193 – 195). And then the system would remain at rest (line 8).

(c) As she was at a loss and did not know what to do, I asked her to verify the validity of her earlier prediction which was whether or not the weight of object B was greater than the tension in the string in the system mB > mA. She changed mA from 2 kg to 1 kg, maintained mB to 2 kg, and put on the GRAVITATIONAL FORCE meters for both objects and the TENSION meter. She then ran the simulation and found that her prediction was indeed true (lines 204 – 206). At the same time she professed that it was the net force acting on object B that pulled both objects A and B (lines 208 – 212) – a mechanical idea which she had constructed during her interaction with the system mB > mA (lines 140 – 149).

It is imperative to note that at the initial stage of the Prediction Phase she stated that it was the tension in the string that would pull object A (line 6). This is in accordance with the direction of the vector arrow representing the tension force in the horizontal part of the string (Figure 5.21).

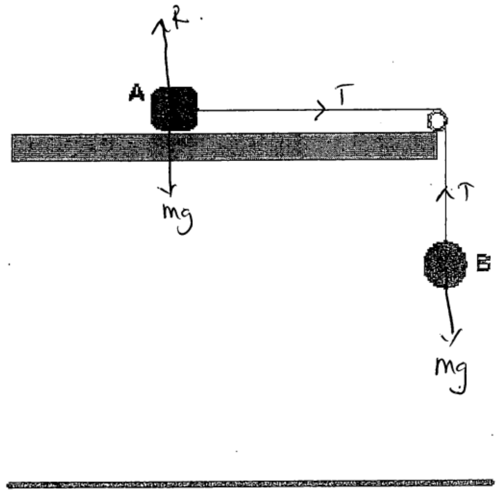


Figure 5.21: Force diagram drawn by Student S6.

However, as the interview unfolded, she mentioned that "object B is pulling object A" (line 125). She further added that 'tension in the string would pull object A' was synonymous to saying that 'object B was pulling object A' (line 127). As we proceeded with our discussion, she changed her idea.

S6: I think that tension is pulling it [referring to object A]. [laughter] Tension is on this side [pointing at the horizontal part of the string]. It will make it [referring to object A] slide in this direction [pointing towards the pulley].

I: Ok. What is pulling object A?

S6: Tension in the string … due to this hanging object [pointing at object B].

I: What is 'due to the hanging object'?

S6: The weight … the weight [pointing at object B] being greater than the tension [pointing at the vertical part of the string] … It is the resultant of the weight and tension that is pulling the object.

From the preceeding extract (lines 132 – 142), it seems that at first she held the intuitive idea 'tension in the string, which was due to object B, pulled object A'. She then appeared to learn that the tension in the string (which was lesser in magnitude) was neutralised by the weight of object B (which was greater in magnitude) such that there was a net force acting downward on object B which would pull object A (Figure 5.22). There also exists the possibility that she held the idea 'how the tension around object B, which acted upward, could pull object A towards the pulley'; in order to do so, a force was needed to act downward on object B, thereby, pulling object A towards the pulley.

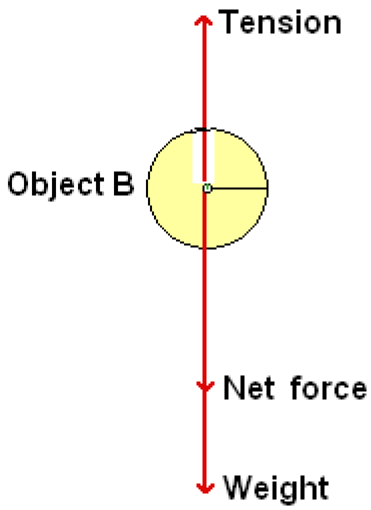


Figure 5.22: Diagram showing the direction of each force acting on object B.

When she was asked to explain why she changed her idea, she laughingly pointed out that "it changed by itself" (line 152).

In relation to which object was responsible for the creation of tension in the string, she initially hypothesised that it was due to object B only (line 33). She then quickly revised her idea to suggest that both objects were responsible for its creation (lines 35 – 36). She again reverted back to her initial idea (lines 47 – 49), but now adding that this was only true when the system would be at rest (lines 51 – 54). She explained that she had made use of an equation to reach this conclusion (lines 56 – 59). On this basis, she began to accept that both objects had created the tension in the string (lines 62 – 66). Interestingly, as mentioned in the previous paragraph, at this point of the episode, she now held the idea ‘the tension in the string was due to object B only’ (lines 137 – 140). As such, it seems that the cueing priority of her ‘equation’ idea (that she had learnt elsewhere) was decreased and that of her intuitive idea ‘force of gravity acting on object B’ was increased (lines 87 – 100).

(d) We then returned to the system with equal masses (mA = mB = 2 kg) and tried to explain why there had been motion. After having run the simulation in slow motion, she compared the system with equal masses with the previous one (mA = 1 kg; mB = 2 kg).

S6: They are the same [making an analogy to the previous system with mA = 1 kg; mB = 2 kg]. This weight [pointing at object B] is greater than tension. So, it pulls this mass [pointing at object A].

From the above statement (lines 219 – 221), it appears that she has learnt the condition required for motion to take place in the system mB = mA, especially when she had confirmed the same condition for the system mB > mA (lines 200 – 202). However, the remaining conversation showed otherwise.

I: […] So, do you agree that there is motion when the masses are equal?
S6: [nodding of head to suggest that she doesn’t agree with motion] … They will move.
I: You just refused to agree with the motion.
S6: No. I …
I: You don’t look convinced.
S6: I don’t know how to explain it.

The above extract (lines 222 – 228) shows that she did not agree with this motion. In this case, I find that she explained that weight of object B should be greater than tension in the string without conviction – she just uttered what has worked in the previous system without really being convinced. It appears certain that she was still being influenced by her intuitive idea ‘equal masses $\Rightarrow$ same level $\Rightarrow$ no motion’. On this basis, this episode suggests that knowing that ‘weight of object B should be greater than tension in the string’ is not a sufficient condition for such students to explain motion. Agreeing that motion is possible appears to be the stepping stone to allow meaningful learning to occur.

Episode 5.5

Brief Description of Episode 5.5

In this episode, Student S6 was provided with two systems (System X with mA = mB & System Y with mC = mD) of the Vertical Version on Interactive Physics – all four objects (A, B, C, D) had the same mass (see Figure 5.23). She was asked to predict and explain the behaviour of each system of the Vertical version. For her, since both systems would behave alike we concentrated solely on System X. The transcript of this episode can be found in Appendix E2.

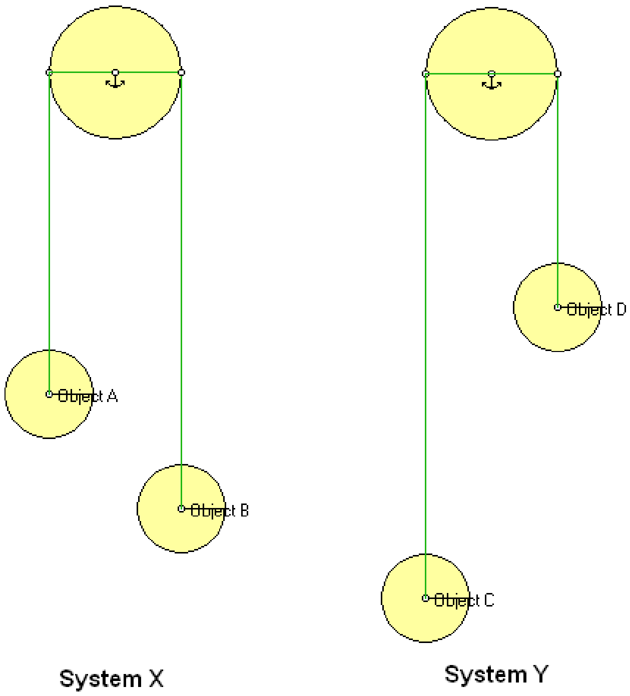


Figure 5.23: Interactive Physics simulation showing 2 systems X and Y.

Analysis of Episode 5.5

(a) Consistent with her earlier prediction in Episode 5.4 (refer to lines 7 – 19 & lines 177 – 195 in Appendix E1), at the start of this episode as well, she predicted that object A would move down and object B would move up until both objects, of equal masses, would reach equilibrium at the same height after a short time (lines 2 – 3) (see Figure 5.24).

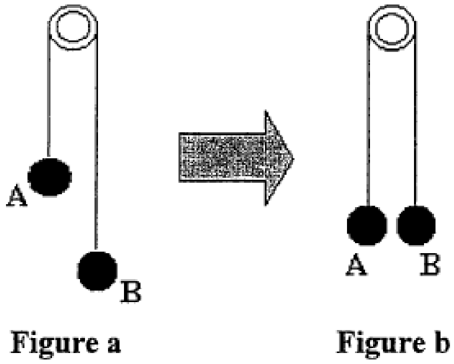


Figure 5.24: Student Thinking about the system. Figure a shows system at the start. Figure b shows student prediction after system is released from rest.

As such, for her, the objects should first move to the same height and only then they would become at rest. So it appears that she was under the influence of the intuitive idea 'equal masses $\Rightarrow$ same height $\Rightarrow$ no motion'.

(b) After having run the simulation, she appeared surprised that there was no motion.

I: What can you say?
S6: Should it not move? Is it not a system of pulley?
I: Hm? What were you expecting to see when you ran the simulation?
S6: That they should move.
I: Why were you expecting them to move?
S6: … Because they have weights to make them move.

From the above extract (lines 8 – 13), it seems that since the objects did not move as per her expectation, she developed the idea that something must be wrong with the system. This is why she questioned whether it was really "a system of pulley". According to her, the objects should have moved because "they have weights to make them move". As such, it can be hypothesized that for her the objects, when not at the same level, should momentarily produce some sort of imaginary force that would just be sufficient to bring them on the same level. However, when she was again asked to explain what had happened, she started to construct a plausible argument.

I: What is happening at the moment? […]
S6: The mass of A is equal to the mass of B and their weights are equal to tension in the string. This is why there is no motion.
I: But didn't you tell me that they would be at the same level?
S6: Yes, but they aren't moving … If they move.
I: …(pause)… But, as per your prediction, objects A & B should have been at the same height, they should have come to rest at the same height.
S6: If they are not moving, this means that tension is equal to their weights.
I: So, what should we believe?
S6: Tension is equal to weight.

According to her, from the above extract (lines 17 – 28), since the objects had equal masses and therefore equal weights, the weights of both hanging objects should be equal to the tension in the string. This is why she seemed to agree that the system was at rest. It would also appear that for her had the objects moved they would have definitely stopped once they would have reached the same height. But as they did not move, she maintained that the tension in the string should be equal to the weights of the objects.

(c) When she verified the mass of each object, she found that both objects had equal masses ($mA = mB = 5$ kg). She maintained her hypothesis that since the weights were equal, tension in the string must be equal to the weight of either object (lines 34 – 39). To confirm her hypothesis, she put on the GRAVITATIONAL FORCE meter for object B and the TENSION meter. She then ran the simulation and found that the readings on both meters were equal, thereby proving her hypothesis (line 41). At this point I questioned her about why she had the intuitive idea 'these objects, of equal masses, would move to the same level when released from rest'.

I: Why did you think of same height?
S6: Equal masses implies equal weights.
I: Ok. So, how come equal weights implies same height? For you, what is the link between 'equal weights' and 'same height'?
S6: … I don't know why I said that.
I: There must be a reason […] What is it that makes you think that 'equal weights' implies 'same height'?
S6: […] I thought that it was like that.
I: What made you think like that?
S6: If they are of equal masses, they will balance each other.
I: How will they balance each other?
S6: They will be at the same height from the surface.

From the above extract (lines 45 – 56), it seems evident from the statements "I don't know why I said that" (line 49) and "I thought that it was like that" (line 52) that she could not easily express all her intuitive ideas in words. However, as the conversation continued, she showed a sort of connection between 'equal masses' and 'same height' when she explained that "they will balance each other" (line 54). Though she denied making any analogy with some sort of equipment in everyday life or science laboratory (lines 57 – 59), it would seem that for her the Vertical version of connected particles behaved like a see-saw [Figure 5.25(a)] and/or the pan balance [Figure 5.25(b)].

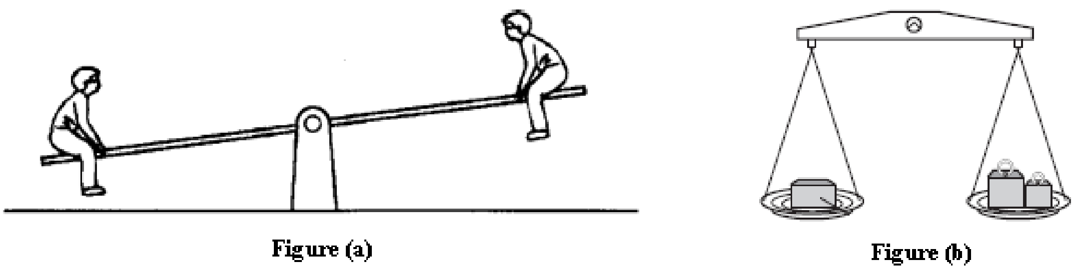


Figure 5.25: The student may have unknowingly made an analogy with these equipment or similar ones.

In either example, whenever the load on the left hand side is bigger than that on the right hand side (or vice versa), the bigger load goes down and the lighter one goes up. But when the loads on both sides are of equal magnitude, they are at the same level. So, it is most probable that analogy with these equipment or similar ones may account for her reasoning.

(d) At this stage of the episode, I reminded her that she was someone who knew that when tension would be equal to weight, there should be no motion. So my query was why, in spite of her correct mechanical idea, she held the intuitive idea ‘both objects should move to the same level and then equilibrium would be attained’. Once again she was not able to provide a plausible explanation even though she was being influenced by this intuitive idea.

(e) She was about to leave the session when she questioned whether it was “really a pulley system” (line 75) as the system did not move (line 77). This appears to show that she still held this intuitive idea. She suggested to change the mass of any object and to observe what would happen. Upon my approval, she changed mB from 5 kg to 10 kg and ran the simulation. She observed that object A moved up and object B moved down. It can be hypothesized that when she noted the uninterrupted motion of both objects until object B hit the ground and object A hit the pulley, this must have made her realise that the pulley was genuine. So she might have developed the idea that if the pulley worked with unequal masses, it should also work with equal masses. This is why I drew her attention to the fact that I was not “using some sort of fake pulley” (line 79).

S6: Yes. [laughter] It moved … This means that if they are of equal weights, whatever level they are positioned they will remain at rest. Because the forces will cancel out. If one is positioned at a lower level, it will stay there ... If both objects are at the same level, they will remain there ... I have understood it. I don’t know why I talked of same level earlier.

From the above extract (lines 80 – 85), it can be suggested that she learnt that there were no imaginary forces acting on both objects which would force them to move to the same level and would then die out to allow the system to be in equilibrium. In the end, she was still not able to understand why she had thought of “same level” (line 87), implying that not all intuitive ideas could be accounted for.

On a final note, it appears reasonable to conclude that she had applied the intuitive idea ‘equal masses ⇒ same height ⇒ no motion’ inherent in her knowledge of the Vertical version to make sense of the Horizontal version in Episode 5.4. This raises an interesting question for this research and future ones. Had the student first worked with the Vertical version and then with the Horizontal version, would she have maintained her prediction in Episode 5.4 that there would be some motion followed by equilibrium?

5.5 Third Situation: mB < mA

When the participants’ responses were analysed for this situation, it was found that all of them predicted that the system would not move at all.

For this group of students, there are two areas of interest, one being of minor importance though. First, at the initial stage of the Prediction phase, while eight of the students predicted that there would be no motion, two students (S2, S5) explained that there would be motion such that the sliding object A would move to the left and the hanging object B would move up. However, these two students quickly shifted their prediction to ‘no motion’. Second, I observed that, after having worked through Task 1 with the software, there were some students who agreed with the motion (Students S1, S3, S6, S7, S8, S10) and those who did not agree with the motion (Students S2, S4, S5, S9). Among the later group of students, there were two students who agreed with the motion & could explain it once they had worked through Task 4b (Students S2, S9).

I have chosen one student per group and have discussed their learning trajectories. While in the ‘First Scenario – No Motion’ Episode 5.6 portrays

Student S3 who agreed with the motion, in the 'Second Scenario – Persistence of No Motion' Episode 5.7 deals with Student S2 who did not agree with the motion until he had initiated Task 4b at the end of Task 1 and interacted with it. Coincidentally, Student S2 initially predicted motion such that object A would move to the left and object B would move up before changing his prediction to no motion.

5.5.1 First Scenario – No Motion to Motion

Episode 5.6

Brief Description of Episode 5.6

This episode began when Student S3 started to analyse the behaviour of the system when the mass of object A [mA] was greater than that of object B [mB] (as from line 320). It is worthwhile to note that this episode is a continuation of Episode 5.2 where he had analysed the system with equal masses. The transcript of this episode can be found in Appendix C.

Analysis of Episode 5.6

(a) At the start of the Prediction phase, Student S3 affirmed that there should be motion whenever mB > mA. On this basis, he predicted that there should be no motion when mA > mB (lines 21; 25 – 26) and when mB = mA (lines 28; 30). He also predicted that the weight of object B pulled object A during motion (lines 14 – 15; 19). When he had almost finished analysing the system mB > mA, he mentioned that "if object B is moving down, the weight of B must be greater than the tension in the string" (lines 139 – 140). It appears that he knew this fact from his school knowledge.

(b) Just before he started to explore the system mB = mA on Interactive Physics, he was asked whether he was still predicting a NO MOTION for the system mA > mB. He maintained his prediction (line 156) and provided a rationale for it.

S3: If the mass of this one [pointing at object A] is greater then it will not move … This force [pointing at object B] will not be enough to move object A.
I: Which force?
S3: This weight [pointing at object B] will not be sufficient to move object A.

From the above extract (lines 158 – 163), it seems evident that he was being influenced by the intuitive idea 'a light object cannot move a heavy one'. According to him, the system mA > mB would not move because the weight of object B "will not be sufficient to move object A". Interestingly, in the case of this student, it can be noted that he has successfully used the relationship between tension in the string and weight of object B to account for motions of the two systems namely: (1) mB > mA; and, (2) mB = mA. However, he was not able to use the said relationship to predict motion for the system mA > mB. It can be hypothesized that his intuitive idea might have decreased the cueing priority of the said relationship.

(c) He changed mA from 2 kg to 3 kg and maintained mB to 2 kg. When he ran the simulation, contrary to his prediction, he found that the system moved (lines 333 – 335). He not only accepted the motion, but managed to account for it by explaining that object A did not affect the motion of object B (line 339). This suggests a shift in his thinking in relation to object A from being a burden to object B to having no influence on the motion of object B. Interestingly, he also observed from the digital meters that the net forces acting on both objects were not equal (lines 339 – 342). He appeared not to agree with this observation (line 344). It is then that he suggested that the weight of object A might have a role to play in the system (lines 346 – 348).

(d) He reset and ran the simulation in slow motion for some steps, and then started to write on paper an equation which contained the tension in the string.

When I enquired, he observed that the tension in the string was equal to the net force acting on object A (line 350) (Figure 5.26).

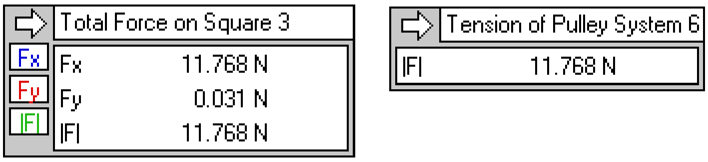


Figure 5.26: This pair of meters was of interest to Student S3.

He continued writing the following set of equations on paper.

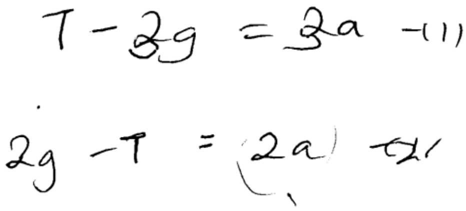


Figure 5.27: Student S3 wrote the above 2 equations of motion for the system. Equation (1) was for object A and Equation (2) was for object B.

After he finished writing the above equations (Figure 5.27), he explained that while the net forces acting on objects A and B should be different (line 352), the tension would be the same throughout the string (lines 354 – 358).

If Equation (1) is analysed, it can be seen that the net force acting on object A is 3a and if Equation (2) is considered, it is evident that the net force acting on object B is 2a. This may explain why the student accepted that "the net forces are not equal" (line 352). On this basis, it can be hypothesized that the set of equations in Figure 5.27 acted as a bridge between the readings obtained from the screen and his new idea that the net forces acting on objects A and B might not be equal.

In relation to Equation (1), it can be seen that this equation has wrongly been written – the term '3g', which represents the weight of object A, should not have been included in the equation. In fact, this set of equations is written to model the Vertical version of connected particles (refer to Figure 5.2). Interestingly, a similar issue was resolved by this student in Episode 5.2 when he had interacted with the system mB = mA. So the big question is why he repeated the same mistake in this system especially as he correctly deduced from Interactive Physics that the tension in the string was equal to the net force on object A (line 353). It can be hypothesized that there has been rote learning of this set of equations and that he has not understood the basic principle.

(e) It also appears that his intuitive idea ‘the net forces acting on both objects should be equal’ was reinforced when he interacted with the system with equal masses (lines 360 – 361). At that point in time, by sheer coincidence, the readings on the TOTAL FORCE meters of objects A and B were the same (refer to Figure 5.10). However, when he worked with the system mA > mB, he found that the net forces acting on objects A and B were not equal because the weight of object A was different compared to that of object B (line 365). He thus concluded that the net force acting on object A or B would depend on its weight (lines 371 – 377).

S3: Net force that is acting on object A depends on its mass [pointing at object A]. It is the same for object B. If I vary the mass, the net force will also vary.

The above statement (lines 375 – 377) suggests that he might have learnt that the weight of object A would influence the net force acting on it, contrary to his intuitive idea (lines 385 – 386). In relation to object B, he knew that its weight would definitely affect the net force acting on it (lines 388 – 389). However, the following statement (lines 391 – 393) appears to show that the construction of this new idea in relation to net force acting on an object had been incomplete.

S3: The weight [pointing at object A] is influencing its net force ... and it doesn’t depend on this one [pointing at object B]. The net force of each object is different.

It would seem, from the above statement (lines 391 – 393), that he learnt that the net force acting on object A was influenced by the weight of object A only; the weight of object B had no effect on the net force acting on object A. Similarly, for him the net force acting on object B was solely influenced by the weight of object B; object A had no contribution whatsoever.

(f) Our conversation focused on the net force acting on each object. For object B, he explained that the net force acting on it was equal to the "weight minus tension" (line 397). For object A, he stated that "tension minus weight is equal to net force" (line 401). At this point, I reminded him that during the previous simulation (the system with equal masses – Episode 5.2), he discovered that the net force acting on object A was not equal to tension minus weight. However, in this episode he maintained that net force acting on object A was indeed equal to tension minus weight (lines 408 – 411). Thus, it seems that both his finding in Episode 5.2 and the observation he had made earlier in this episode (line 350), in relation to the net force acting on object A being equal to the tension in the string, did not influence his learning.

So the main question is why he continued to think that the net force acting on object A should be tension minus weight despite having resolved this issue in the previous system with equal masses (refer to Episode 5.2). Interestingly, in that system, the following readings were observed (Figure 5.28):

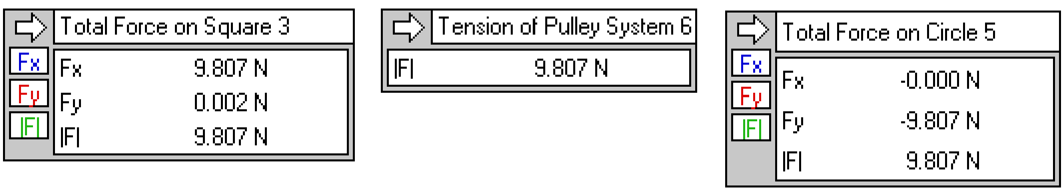


Figure 5.28: The meter TOTAL FORCE ON SQUARE 3 displayed the net force acting on object A; the meter TENSION OF PULLEY SYSTEM 6 displayed the tension in the string; and, the meter TOTAL FORCE ON CIRCLE 5 displayed the net force acting on object B.

It can be seen that during this particular simulation the net force on object B, tension in the string and net force on object A had the same magnitude (9.807 N). It would appear that these readings may have reinforced his initial idea 'the net force acting on object B must be transmitted via the tension in the string to pull object A with the same magnitude'.

(g) I advised him to put on the GRAVITATIONAL FORCE meter for object A on the screen so that he would be able to verify the veracity of his idea on the net force acting on object A (lines 414 – 415). After he had run the simulation in slow motion, I asked him to verify whether or not the net force acting on object A was equal to "tension minus weight" (line 416). As he was still comparing the net forces acting on objects A and B (lines 417 – 421), I reiterated my question (lines 427 – 431). After he found that his intuitive idea was not true (line 432), he could not explain how to calculate the net force on object A (line 434). So, on my advice, he ran the simulation in slow motion for some seconds and then noted the relationship between the net force acting on object A and the tension in the string.

S3: The tension value is equal to the net force acting on object A.
I: You were earlier talking about weight. Let's analyze and see.
S3: The net force on A is equal to the tension, not the weight.
I: Do we find the weight influencing the net force on object A?
S3: No ...(pause)… For this one [pointing at object A], net force is equal to the tension … the weight [pointing at object A] does not have any effect upon it [referring to the net force] …(silence)…

From the above extract (lines 437 – 443), it can be observed that his attention was drawn to the fact that both the TENSION and the TOTAL FORCE meters displayed the same magnitude (11.768 N), implying that "the tension value is equal to the net force acting on object A" (line 437). On this basis, he seemed to learn that the weight of object A had no influence on the net force acting on it (lines 442 – 443). It is absolutely true to say that the weight of object A had no direct influence on the net force acting on it (that is, not present in its equation), but as mentioned earlier, the weight of object A has a direct effect on the tension in the string which, in turn, affects the net force acting on object A. At this point of the episode, it appears that the student did not still build the idea of the indirect influence of the mass of object A on the net force acting on it.

(h) So I advised him to compare this system (mA = 3 kg; mB = 2 kg) with the previous system with equal masses (mA = mB = 2 kg). After changing mA from 3 kg to 2 kg and running the simulation, his attention was drawn to the fact that both the TENSION and the TOTAL FORCE meters displayed the same magnitude (9.807 N) and to the vector arrows on the screen (Figure 5.29).

S3: The net force [pointing at object A] is equal to the tension [pointing to the TENSION meter] … Weight doesn't affect the net force because it is not acting along the same axis. Net force is along the x-axis, that is moving horizontally, not up and down, that is not in a vertical line. However, here [pointing at object B] it is moving downwards and therefore weight must be taken into consideration.

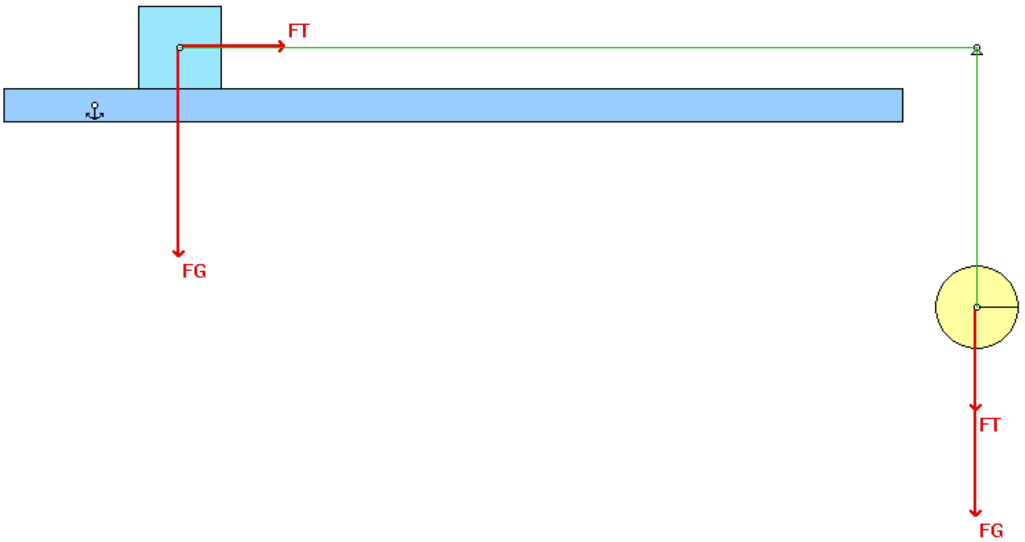


Figure 5.29: Objects A/B as appeared on the screen of Interactive Physics.

From the above extract (lines 446 – 451), it seems that there has been learning in the direction of expertise. He could now explain the rationale of why the weight of object A was not involved in writing the net force on object A, whereas that of object B appeared in the equation modelling the net force on object B. In Figure 5.29, the vector arrows FT represented the net force acting on object A and that on object B. So Student S3 realised that the net force acting on object A was along the horizontal axis just as the tension in the string and as such weight of object A (FG), which was along the vertical axis, was not needed in writing the equation of net force on object A. For object B, he observed that the weight of object B acted along the same direction as its net force, implying that its weight must be considered when evaluating its net force.

(i) When I asked him how he could confirm that the net force on object A would be equal to the tension in the string only, he suggested by changing the masses of the objects (line 454). So he changed mB to 4 kg and maintained mA to 2 kg. He noted that the net force acting object A was equal to the tension in the string (13.076 N) and that net force did not equal to tension minus weight.

S3: No … I can now understand. For this one [pointing at object B], it is weight minus tension because they are along the same axis, whereas here [pointing at object A] … Ok.

From the above statement (lines 457 – 459), it can be hypothesized that the possibility of varying mB and visualising its outcome on the software might have played a key role in the positive shift in the student thinking.

(j) At the end of the episode, he explained the occurrence of motion in this system (mA > mB) via the relationship between the tension in the string and the weight of object B (lines 463 – 464).

I: Why is there motion when mA > mB?
S3: Whenever there is a net force on B and it is independent of the weight of A.

It can be deduced that he has learnt that there was a need to analyse and resolve all the forces acting on each object in order to account for motion. At the same time, it would appear that though he held the intuitive idea 'the weight of object A would not affect the net force acting on object B directly', he did not learn that the mass of object A would definitely affect the tension in the string which, in turn, would influence the net force acting on object B.

5.5.2 Second Scenario – Persistence of No Motion (to Motion)

Episode 5.7

Brief Description of Episode 5.7

In this episode, which started as from line 332, Student S2 was asked to predict and explain what would happen to the system if the mass of the sliding object A [mA] would be greater than that of the hanging object B [mB]. It is worthwhile to note that this episode is a continuation of Episode 5.3 where he had analysed the system with equal masses. The transcript of this episode can be found in Appendix B.

Analysis of Episode 5.7

(a) During the Prediction Phase (Task 1), Student S2 predicted that if $mA > mB$, object A would move towards the left and object B would move up (lines 9 – 14; 23 – 24). However, after a short while, he changed his prediction to "there will be no motion" (lines 136; 140; 336; 338; 341; 350). At this point I wish to raise two salient issues: (1) what is the rationale behind his first prediction? (2) What can account for the shift in his prediction?

First, it appears that Student S2, during the early stage of the Prediction Phase, might have momentarily considered this system to be behaving in the same way as the Vertical version of connected particles (Figure 5.2, page 128). As elaborated in Episode 5.5, the Vertical version of connected particles behaves in similar ways as the see-saw and/or the pan balance [refer to Figure 5.25]. As such, the intuitive idea 'unequal masses $\Rightarrow$ motion' might have influenced the student thinking process during the first prediction.

Second, the shift in his prediction may be explained in terms of the order of the set of masses of connected particles. At the start of the Prediction Phase, he predicted the behaviour of the system (1) when $mA < mB$; and, then (2) when $mA > mB$ (lines 7 – 14). In both cases, he predicted motion taking place and it can be hypothesised that he might have been influenced by the intuitive idea 'unequal masses $\Rightarrow$ motion'. At a later stage, when he was asked to comment on the behaviour of the system with equal masses (line 25), he predicted, after several seconds of silence, that the system would not move (lines 26 – 27). In this case, it appears that the intuitive idea 'equal masses $\Rightarrow$ no motion' was activated.

It would seem that the second intuitive idea 'equal masses $\Rightarrow$ no motion' must have decreased the cueing priority of the first intuitive idea ''unequal masses $\Rightarrow$ motion' so that he provided a new prediction when object A was heavier than object B. If this is the case, then it appears that the second intuitive idea continued to hold a high cueing priority such that it decreased the cueing priority of the first intuitive idea, and a third intuitive idea 'a light object cannot displace a heavy one' was activated. This is why, according to him, there would be no motion because object A was heavier than object B (lines 379 – 381) and therefore object B would not be able to move object A (lines 389 – 390). How can a light object displace a heavy one? He might have thought that this was not possible in real-life situations. So the argument I wish to make is the following: Had he discussed the behaviour of a system with equal masses rather than unequal masses ($mA > mB$), his initial prediction 'object A moves to the left and object B moves up' might not have arisen.

(b) When he was asked to explain what would happen to the tension in the string if mA would be increased from 1 kg to 2 kg and mB would remain 1 kg, he predicted that there would be no change in the tension (line 355) – that is, for him the tension in the string for the system $mA = mB$ would be the same as that for the system $mA > mB$. He defended his position by emphasising the role played by object B in this matter.

I: So, tension, according to you, will be the same. Why?
S2: The string will support … (pause) …

I: The string will support what?

S2: It will support this object [pointing at object B] … (pause) … and since there is no motion, there is also no acceleration & … (pause) … tension will be equal to weight.

I: It will not be equal to what?

S2: No, it will be equal to the weight of the hanging object B, and therefore equal to g.

I: Tension will be equal to weight of the hanging object B. You're telling me that this will be true when there is motion.

S2: No. When there is no motion! The system doesn't move.

The above extract (lines 358 – 369) points to the existence of two mechanical ideas.

First, the statement "the string will support object B" (line 361) suggests that for him since object A was on the surface and object B was in the air, object B, not object A, was a 'burden' to the string (or making it become taut). So, he appeared to hold the intuitive idea 'it was solely object B that created tension in the string; object A had no role to play in the creation of tension in the string'. Therefore, owing to the fact that the mass of object B had remained unchanged, he predicted that the tension would remain the same.

It is important to note that Student S2 had other mechanical idea earlier on. At the start of the activity, when he discussed the system mA = mB, he had the idea that each object created a tension in the string adjacent to it (for example, lines 33 – 36) and, as such, there were two tension forces in the string. When he talked about the system mB > mA (as from line 248), where mA was maintained to 1 kg and mB was increased from 1 kg to 2 kg, he predicted that tension would increase if there would be friction (line 258) and if we would have a smooth situation, he first predicted that "tension will decrease" (line 272), then changed his prediction to "tension will remain the same" (line 277) and finally predicted that "tension will increase" (line 282). He was again asked to predict the tension for the smooth situation.

I: […] What are you predicting? What will happen to tension?
S2: It will remain the same, eh, tension will increase. If we double the mass, tension [pointing at the vertical part of the string] will increase.

From the above extract (lines 306 – 308), it can be seen that his initial idea "it will remain the same" re-appeared and then it shifted almost immediately to "tension will increase", implying that his initial idea was that object B did not influence the tension and his final idea was that object B affected the tension. However, a few minutes later, he again changed his prediction to "It will remain the same" (line 317).

S2: It will remain the same.
I: Why? […]
S2: … Because mass of A is still unchanged.
I: So, according to you, what factor influences the tension in the string?
S2: Object B.
I: Eh?
S2: Tension will increase. It will not remain the same.
I: What?
S2: The tension will increase.
I: Why?
S2: Because when the mass will be increased, the weight will increase.

From the above extract (lines 317 – 327), it seems evident that there is a sort of conflict going on in his mind as to whether the tension in the string was affected by object A or object B. He finally decided to ignore object A and to assume that object B was the only object that created tension in the string (line 321). It appears that this decision was due to the 'force of gravity' acting more on object B than on object A (line 327).

An important question at this point of the episode is why he did not continue to think, just as at the start of the episode, that the tension could be affected by both objects. Why did he downplay the role of object A in this matter especially as at some point of the episode (lines 113; 121 – 122) he mentioned that “the tension in the string is the same throughout”?

Second, he developed the argument that when there would be no motion, the tension in the string should be equal to the weight of object B (lines 365 – 369). Thus, it appears that he has started to realise the condition required for the system to be at rest.

(c) After having changed mA to 2 kg, he maintained mB to 1 kg and ensured that he would deal with a smooth system, he ran the simulation in slow motion and observed that there was motion. He noted the new value of the tension in the string [6.538 N] and compared it to the weight of object B [9.807 N], a value which he had recorded on a piece of paper. He compared the values of both vector quantities (line 375).

S2: Tension isn't equal to weight.
I: Why is tension not equal to weight?
S2: Because there is motion.

From the above extract (lines 375 – 377), it can be seen that he started to explain the occurrence of motion in terms of tension and weight. However, the crux of this situation was his refusal to accept that there could be motion of the system mA > mB even after having started to explain motion in terms of tension in the string and weight of object B.

I: Now you've seen that
S2: There is motion.
I: And do you believe in what you are seeing?
S2: Not all of them.
I: Why? Why is it that you don't trust everything that is being shown in the simulation?
S2: … This object [pointing at object A] is heavier & it is smooth. Therefore, this one [pointing at object B] cannot move this one [pointing at object A]. If the masses of object A & object B were equal, then object B

would have pulled object A towards the pulley. If the mass of object B would have been greater than that of object A, again there would have been motion. But now if the mass of object A is greater than that of object B, there should be no motion.

From the above extract (lines 382 – 394), he appeared to agree with the argument “weight is greater than tension to explain motion” in the case of mB > mA, but not in the case of mB < mA. As such, it would seem that if he agreed with the motion of the system, he would then find the tension/weight relationship relevant. It can also be seen that the intuitive idea ‘a light object cannot displace a heavy one’ continued to hold a high cueing priority such that it decreased the cueing priority of the tension/weight relationship and did not allow him to agree with everything that he had seen on the screen.

(d) However, after almost 28 seconds of silence, he enquired whether or not there would be motion if object A was much heavier than object B (lines 396 – 398). He suggested to run a simulation on Interactive Physics where the sliding object would be five times heavier than the hanging object, most probably to see whether or not there would be motion. As soon as I encouraged him to pursue his exploration, he reset the simulation, adjusted mA to 5 kg, and ran the simulation in slow motion. He found that there was motion, but its velocity had decreased and he noted the new value of tension [8.172 N]. He stated that the tension in the string had increased, and appeared to construct the idea ‘when the mass of either object would be increased the tension in the string would have to increase’ (lines 404 – 405). I then asked him to reflect on whether the tension in the string would increase if mA was further increased. According to him, this was possible (line 420). On this basis, I asked him what would be the tension in the string if mA was made eight times bigger. He stated that it would increase. When he was asked to suggest an approximate value for it, he mentioned 10 N. I then enquired as to whether it could also be 11 N or 12 N. To this, he replied positively (line 427). He adjusted mA to 8 kg, ran the simulation in slow motion for a few frames, stopped the simulation and finally noted the new value of tension [8.717 N].

S2: The tension increases, but by a small value. The tension is now 8.717 N. And the velocity decreases by a small value.

His expression “but by a small value” in the above statement (lines 431 – 432) indicates that he might have started to learn that the increase in tension might not be proportional to the increase in mA, as per his earlier expectation (lines 425; 427). When he suggested to increase mA to 10 kg, our conversation began to focus on the “limit on tension”.

I: According to you, the more you increase the mass the more tension will increase? There is no limit on tension?
S2: No, it must have a limit.
I: Hmm?
S2: It must have a limit.
I: Why?
S2: The string might break.

From the above extract (lines 435 – 441), he seemed to realise that the tension in the string should have a limit. However, instead of realising that no matter how big the mass of object A would be, tension would not further increase, he held the idea ‘the tension in the string must have a limit or else the string might break’. This appears to be an abstraction from real-life experience.

(e) As per our discussion, he then repeated the simulation by adjusting mA to 10 kg, 15 kg, 20 kg, 25 kg, 40 kg, 60 kg and 100 kg. He noted that for each simulation the tension in the string had increased with infinitesimal value and the motion of the system had become slower. At this point of the episode, he questioned whether this motion was possible (line 467). From this perspective, he argued that “smooth surfaces don’t exist … there should be friction … ideal situations don’t exist” (lines 472 – 476). It can be inferred that his real-life experience (in relation to friction) influenced his learning process. He then continued to repeat the simulation by adjusting mA to 150 kg, 200 kg, 300 kg, 400 kg, 800 kg and 2000 kg. At the point where mA = 2000 kg, he noted that the tension in the string has become rather close to the weight of object B (line

500) and that there was a tendency for the tension in the string to equal the weight of object B (lines 502 – 503). He also stated that the string might break. When asked if there existed the possibility of the tension in the string to be greater than the weight of object B, he mentioned that this was not feasible (line 508) and developed an argument to explain the occurrence of the at-rest situation.

S2: … velocity is decreasing and a point will be reached when tension will be equal to the weight of object B, then there will be no motion.

From the above statement (lines 512 – 513), it can be suggested that he managed to construct an argument of why there should be no motion for the system mA > mB. He was finally asked to try the simulation with mA being adjusted to 5000 kg. He noted that the tension in the string was almost equal to the weight of object B. However, he again stated that “a time will come when the string will break as the tension is equal to the weight” (lines 524 – 525). Though he was asked to justify the breaking of the string, he was not able to provide any answer. He finally concluded that the tension in the string depended on the masses of both objects (line 534).

5.6 Concluding Remark

In this chapter, the animations played a key role in the learning process. Due to their intuitive ideas, the participants initially predicted that there should be no motion for the systems mA = mB and mA > mB. So when they discovered that motion was possible via Interactive Physics, this discovery provided them with thought-provoking resources. When they ‘genuinely’ accepted the occurrence of motion, they ‘threw away’ their explanations involving masses only and started to develop new ones in terms of forces. Intuitive knowledge tended to focus on mass most probably because it is tangible and it is commonly used in daily life such that it fits easily with the students’ experience. On the other hand, force, being a product of mass and acceleration, is much more complex for people to understand. In relation to the software, it seems that it was more often the animations that gave the students the impetus to analyse and to make sense of the readings on the digital meters. On the whole, this chapter relates the story about how the use of Interactive Physics shifted students' attention from 'comparing masses of objects A & B' to 'analysing forces in the system'. In the next chapter, I will discuss student understanding of acceleration in the context of motion of connected particles.

Chapter 6

Student Understanding of Acceleration

6.1 Introduction

In this chapter, the aim is to discuss the findings related to student understanding of acceleration in three different situations associated with the system of connected particles. In § 6.2, I explain the three different situations in which the students' responses have been analysed. While § 6.3 explores the case of acceleration when the string was taut, that of acceleration when the string was broken is examined in § 6.4. In § 6.5, I discuss the case of acceleration when the string was slack. Finally, § 6.6 provides the conclusion to this chapter.

6.2 Overview of the Findings

In this chapter, my central aim was to investigate how students perceived accelerations of both objects A and B as they interacted with the Horizontal version of connected particles (Figure 5.1). I started this research expedition with two broad questions that I had come across during my literature review. First, did students believe that both objects would move with the same acceleration once the system would be released from rest? This is what I am labelling as the situation where the string was taut or there was usual motion. Second, did students believe that either one object or both objects would always move with acceleration due to gravity g?

With these two questions in mind, I set out to analyse the data related to student understanding of acceleration. During this analysis process, I found that data available could be arranged according to three different situations. The first situation arose when the string was taut in Task 1. This took place when the

Horizontal system of connected particles was in motion whenever the hand holding object A (see Figure 5.1) would be removed – object A would move towards the pulley and object B would move down such that the motion of one object would depend on the other. The second situation happened when the string was broken in Task 6a. In this situation, from t = 0 to t = 1.45 s both objects, when in motion, were connected via a string and were dependent on each other. But, as from t = 1.5 s where the string broke, both objects moved independently. The third and final situation took place when the string became slack in Task 6b. In this situation, both objects A and B, when released from rest, were in motion such that they were dependent on each other until object B hit the ground for the first time (say at point X in Figure 6.1). So, as from this time onward, both objects moved independently such that object B started to move up, reached its highest point, say point Y, and finally moved down.

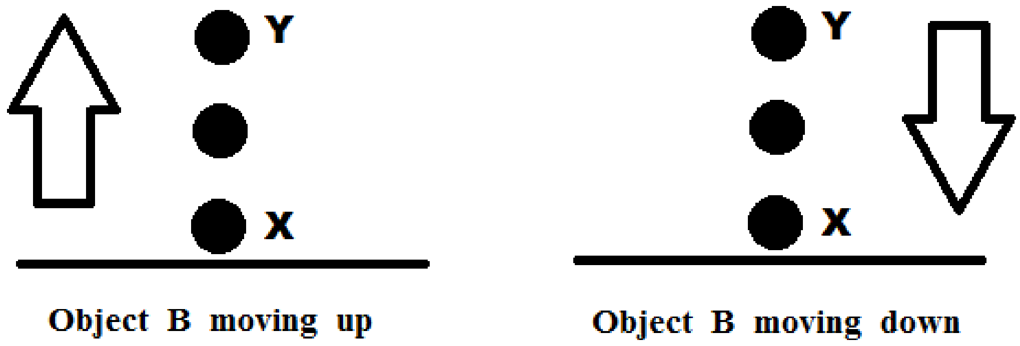


Figure 6.1: When object B hit the ground for the first time, say at point X, it started to move up until it reached its highest point, say Y. It then moved down.

For the first situation, that is when the string was taut, I found that there were two areas of interest:

(a) six out of ten students (S3, S4, S5, S8, S9, S10) predicted that objects A and B would not move with the same acceleration; and,

(b) six out of ten students (S1, S4, S7, S8, S9, S10) predicted that the acceleration of hanging object B must be equal to g.

With regard to the second situation, that is when the string was broken, I noted that the data available could be used to zoom on three different types of sub-situations (or contexts):

(a) To analyse student understanding of the acceleration of object B;
(b) To analyse student understanding of the acceleration of object A on a smooth surface; and,
(c) To analyse student understanding of the acceleration of object A on a rough surface.

It is imperative to note that Students S1 and S2 did not take part in this activity as it had not been designed during the pilot study.

In relation to the first sub-situation, I observed that there were two different types of prediction made by the students:

(a) three out of eight students (S3, S4, S9) predicted that the acceleration of object B would become g instantaneously; and,
(b) five out of eight students (S5, S6, S7, S8, S10) predicted that the acceleration of object B would gradually become g.

For the second sub-situation, it was observed that there was only one type of prediction made by six students (Students S3, S4, S5, S6, S7, S8). All of them predicted that the acceleration of object A would decrease until it would become zero. It is important to note that the data for two students (Students S9 and S10) were not taken into consideration for this part as they did not take into consideration the smoothness of the situation.

In relation to the third sub-situation, it is to be noted that Student S4 could not make any prediction. The remaining seven students predicted that the acceleration of object A would decrease until it would become zero. However, out of these seven students, only Student S6 correctly explained the negative sign of the acceleration during the deceleration process in this motion. The remaining six students only predicted that the acceleration of object A would decrease continuously until it would reach zero.

For the third situation, that is when the string was slack, I found that the participants made three different types of prediction:

(a) two students (S3, S9) predicted that as long as the string was slack, object B would move in the air with g;

(b) two students (S6, S7) predicted that upon impact at point X (Figure 6.1), object B would move up with acceleration less than 6.538 m/s^2 until it would become 0 m/s^2 at its highest point Y and then as it would move down, its acceleration would be g; and

(c) four students (S4, S5, S8, S10) predicted that upon impact at point X, object B would move up with acceleration less than 6.538 m/s^2 until it would become 0 m/s^2 at its highest point Y and then as it would move down, its acceleration would gradually increase from 0.

It is imperative to note that Students S1 and S2 did not take part in this activity as it had not been designed during the pilot study.

So, in all seven episodes have been selected to portray the findings for this chapter. The overleaf table (Table 6.1) summarises the details of each episode.

For each episode, it was imperative to build a detailed analysis (from the transcript) upon which the analysis presented in this chapter has been constructed. An example of such a detailed analysis for Episode 6.5 has been included in Appendix J.

Situation	Student Prediction	Episode
String is taut (usual motion)	Both objects will not move with the same acceleration.	**Episode 6.1** is about Student S9 who, after interacting with the software, could explain why both objects should move with the same acceleration.
	Object B will fall with acceleration due to gravity g.	**Episode 6.2** portrays Student S1 who, creates an experiment on the software to justify his prediction. Yet, his exploration taught him the opposite.
String is broken	Acceleration of object B will become g gradually.	**Episode 6.3** deals with Student S6, who predicts that the acceleration of object B will become g gradually.
	Acceleration of object A (on smooth surface) will decrease until it will become 0.	**Episode 6.4** is about Student S6, who predicts that the acceleration of object A will decrease until it will become 0.
	Acceleration of object A (on a rough surface) will decrease until it will become 0.	**Episode 6.5** is about Student S5, who is not aware of the deceleration process as object A slows down.
String is slack	Acceleration of object B will decrease to 0 as it moves up and it moves down with g.	**Episode 6.6** deals with Student S7 who predicts that upon impact the acceleration of object B would move up with acceleration less than 6.538 m/s^2 until it would become 0 m/s^2 at the highest point and then as object B would move down, its acceleration would be acceleration due to gravity g.
	Acceleration of object B will decrease to 0 as it moves up and it will increase from 0 as it moves down.	**Episode 6.7** portrays Student S8 who predicts that upon impact the acceleration of object B would move up with acceleration less than 6.538 m/s^2 until it would become 0 at the highest point and then as object B moves down, it would increase from 0.

Table 6.1: The details of each episode have been discussed in this table.

6.3 First Situation: When string was taut (usual motion)

An analysis of the participants' responses was carried out. When the string was taut, I found that there were two groups of students:

(a) six out of ten students (S3, S4, S5, S8, S9, S10) predicted that both objects would not move with the same acceleration; and,

(b) six out of ten students (S1, S4, S7, S8, S9, S10) predicted that the acceleration of object B must be equal to g.

For each group of students, I have chosen one student per group and have discussed their learning trajectories. While Episode 6.1 deals with Student S9 who predicted that both objects would not move with the same acceleration, Episode 6.2 is about Student S1 who predicted that object B would always fall with g.

6.3.1 Both objects do not move with the same acceleration.

Brief Description of Episode 6.1

In this episode, Student S9 was asked to predict whether or not both objects A & B would move with the same acceleration. The transcript of this episode can be found in Appendix H.

Analysis of Episode 6.1

(a) Student S9 predicted that the acceleration of the hanging object B would be greater than that of the sliding object A because the mass of object B was greater than that of object A (line 91). Some minutes earlier he predicted that both objects would move with different velocities, with object B moving faster than object A (line 57).

S9: It [referring to object B] will have to pull object A. So, it cannot move with the same speed. It should move quicker. It should fall quicker so that it can pull it [referring to object A].

As per his above statement (lines 59 – 61), he explained that since object B had to pull object A, the former had to move at a faster rate. It can be deduced that for him speed was associated with the amount of work done by an object. The more work an object had to do, the quicker it would have to move. He then explained that the speed of an object would depend on its mass – the larger the mass of an object, the greater the speed with which it would move (lines 64; 76 – 78; 81 – 82).

(b) When he interacted with the software to investigate whether or not both objects would move with the same velocity, he observed from both digital meters that objects A and B moved with the same velocity. He not only agreed that with this observation (line 99; 106), but he could also provide the rationale.

S9: They are in the same system. So they are directly proportional to the tension.
I: Hmm?
S9: Since both objects are linked to each other in the same system, so when one moves the other should also move.
I: Can you clarify it?
S9: The distance that this one [pointing at object B] moves, this one [pointing at object A] will also have to move the same distance, since both objects are connected in the same system via a pulley.

From the above extract (lines 108 – 116), a reorganisation of ideas seemed to occur. Through his intuitive idea of "pull" (lines 59 – 61; 102 – 104), it is evident that he already knew that both objects were connected. What he did not know was to what extent object B would affect the motion of object A. This is why he might have predicted that object B should move faster than object A (lines 57; 59 – 61; 64; 81 – 82). When he interacted with the VELOCITY meters, the readings, being of the same magnitude on both meters, might have made him realise that both objects should “move the same distance, since both objects are connected in the same system via a pulley” (lines 115 – 116). Therefore, for the time being it can be hypothesised that his intuitive idea of ‘pull’ coupled with the readings on the VELOCITY meters have enabled him to realise the key role played by the string in ensuring that both objects should move at the same pace.

(c) At this point of this episode, he stated that both objects should move with the same acceleration (line 118) – a complete departure from his initial prediction. However, when he was asked to defend his new prediction, he resorted to the initial one, that is the acceleration of the hanging object B would be greater than that of the sliding object A (lines 120; 125 – 126; 128 – 129; 144), but with more precision. He explained that object B should fall with g (lines 122 – 123; 134; 136; 138; 140; 142) “because of weight” (line 122), implying that for him the force of gravity was solely responsible for the acceleration of object B. So, it can be observed that in all he made three different types of prediction in relation to the acceleration of object B: moving at a greater acceleration than object A → moving with the same acceleration as object A → moving with g. The first prediction seemed to be based on the mass of the object – the higher the mass, the greater the acceleration (line 91). The second prediction appeared to be a direct outcome of his recent finding on Interactive Physics that both objects should move with the same velocity. I will discuss the rationale of his third prediction in the next part.

(d) During the later stage of the Interaction Phase, he ran the simulation in slow motion until object A was almost near the end of the surface. By doing so, it can be hypothesised that the student got ample time to observe via the ACCELERATION meters that both objects moved with the same acceleration during the whole motion (Figure 6.2).

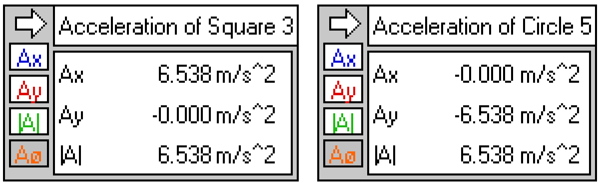


Figure 6.2: Both ACCELERATION meters displayed the same reading during the whole motion.

When he was asked to comment on this finding, he explained that object B did not move freely.

S9: I thought that its acceleration [pointing at object B] would have been 9.81 but here [pointing at ACCELERATION meter of object B] it is 6.538 … It should be that this object [pointing at object A] is opposing this one's motion [pointing at object B].
I: Can you say it again?
S9: This object [pointing at object A] is opposing its motion [pointing at object B] since it has a weight. So the accelerations must be equal … If object B was falling freely, then it would have fallen with acceleration g. But here it [pointing at object B] is attached to another object [pointing at object A] which is opposing its motion [pointing at object B].
I: Which object is opposing which object's motion?
S9: Object A [pointing at it] opposes the motion of object B [pointing at it via cursor].
I: Alright. And so you think this is the reason why object B is not falling.
S9: Freely.
I: Freely?
S9: Yes.

From the above extract (lines 151 – 168), it seems evident that the reading on the ACCELERATION meter triggered his school knowledge 'an object that would fall freely should move with g' which contributed in making the student realise the role of the string in this system, and not vice versa, that is the string did not play a key role in allowing the student to understand that both objects should move with the same acceleration. This can be observed when the student developed the ideas that object A opposed the motion of object B "since it has a weight" (line 157) and that object B did not fall freely (lines 165 – 168).

Interestingly, the above extract also provides the rationale for this third prediction. For him, object B should fall freely, implying that the string had no role to play whatsoever. From the ideas I have raised in part (b), it can be seen that the student has moved in the direction of expertise when he shifted from his first prediction to the second – the string could be seen to be playing a predominant role. However, with his third prediction, the string was no longer a key factor in the system. So the main question that arises from this discussion is the following: Why did the student not maintain his second prediction (that both objects would move with the same acceleration) instead of providing a third one? This reasoning suggests that the intuitive idea "the acceleration of B was caused by the force of gravity only" (line 122) was more powerful than the newly-developed idea 'both objects were connected by a string'.

6.3.2 Acceleration of hanging object B must be g.

Brief Description of Episode 6.2

This episode occurred when Student S1 was asked to predict the magnitude of acceleration of each object if the mass of either the hanging object B or the sliding object A would be varied. Earlier, he had explained that both objects A and B would move with the same acceleration and with the same speed because both objects were connected via the same string (line 20). The transcript for this episode can be found in Appendix A.

Analysis of Episode 6.2

(a) During the Prediction Phase, he predicted that whatever be the mass of object A or object B, the acceleration of the system would always be equal to the acceleration due to gravity g (lines 24 – 40) because the force of gravity acting on the hanging object B would always be the same (lines 42 – 43). It can be hypothesised that he first evaluated the acceleration of object B via his intuitive idea on “force of gravity” and then confirmed the acceleration of object A via his argument that both objects should move with the same acceleration.

(b) When he was asked to prove his argument, he explained that if another object (object D) was placed beside object B at the same level (see Figure 6.3) and that both objects were allowed to fall, both objects must be seen falling at the same rate (lines 64 – 65). Therefore, for him the string did not have any effect on the motion of object B such that it would fall freely under gravity. This also suggests that at this stage he considered the string to be only a tool to connect both objects and to allow both objects move with the same acceleration and speed.

(c) On Interactive Physics, after having created the object D and having positioned it beside object B (see Figure 6.3), he ran the simulation in slow motion. He observed that object D fell faster than object B.

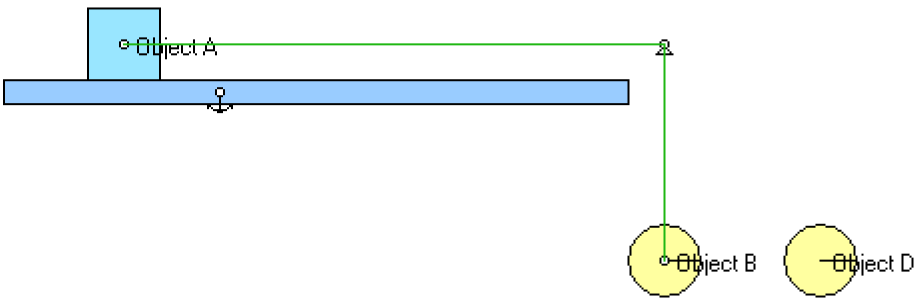


Figure 6.3: Screenshot showing experiment carried by Student S1.

He reset the simulation and investigated whether or not friction between object A and the surface was present. As a rough situation was being examined, he suggested to work with a smooth situation (line 67). So he removed friction from the simulation and then ran the simulation in slow motion. He observed that object D had again fallen faster than object B and deduced that their accelerations were not the same (line 69). It would seem that he did not agree with the simulation as he did not accept that objects B and D had fallen at different rates. For him, the simulation might be ‘wrong' for two reasons.

First, for him object A had no contribution in creating the tension in the string.

S1: The tension would have been equal to the weight of this object [pointing at object B]. Because the weight [pointing at object A] would have cancelled out the normal reaction exerted by the surface.

From the above statement (lines 9 – 11), it can be suggested that though he correctly described the forces acting on object A (Figure 6.4), the visual impact that it had on him appears to justify his thinking that the weight of the sliding object A, W_A, canceled out the normal reaction exerted by the surface on object A, R_A. So, according to him there were only two forces left - weight of object B, w_B & tension in the string, T - to describe the motion. This is why he claimed that the tension in the string would be equal to the weight of object B (lines 4 – 5; 9 – 10). So, despite his intuitive idea 'object A had no role in the system', he readily accepted that friction between object A and the surface might be held responsible for preventing object B to fall at the same rate as object D (line 67).

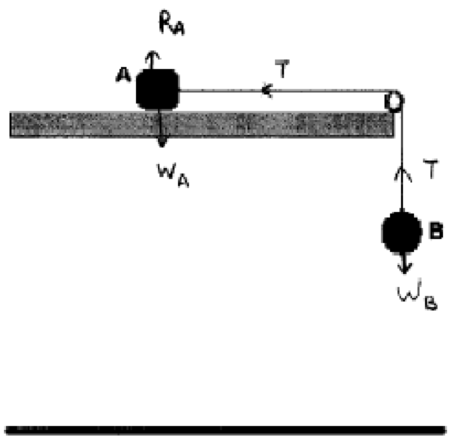


Figure 6.4: Force diagram drawn by Student S1.

Second, after removing friction from the system and running the simulation, he was astonished to discover that objects B and D continued to fall at different rates (line 69).

(d) At this point of the episode (lines 70 – 72), I professed that object D must be moving with an acceleration greater than g since as per his initial prediction object B would be moving with acceleration g. He explained that this was not the case.

S1: Oh … no, this one [referring to object D] is moving with acceleration g.
I: But didn't you tell me that this one [pointing at object B] is moving with an acceleration g?
S1: This one [pointing at object D] is moving with acceleration g and this one [pointing at object B] is moving with acceleration less than g.

From the above extract (lines 73 – 78), a change in his intuitive idea can be noted. It can be hypothesized that he was well aware of the fact that no object, which had fallen freely, could move with an acceleration greater than g. On this basis, as object D moved faster than object B, he realised that it should be object D, not object B, that had fallen with acceleration g. This in turn allowed him to think in terms of forces acting on object D (lines 81; 90).

I: Why? Why do you say that this one [pointing at object D] will move with acceleration g?
S1: Because this one is free. There is no force. There is no force on it.

From the above extract (lines 79 – 81), it can be seen that he has learnt that the acceleration of object D must be g because there was no opposing force acting on it and as such it fell freely. He further explained that the acceleration of object B must be less than g because it was not falling freely (line 99).

I: So, what do you find? What is the acceleration of this one [pointing at object B]?
S1: It is less than g because it is not falling freely.

Based on the above extract (lines 97 – 99), it can be deduced that he developed the idea that it was the string which had opposed object B from falling freely.

(e) In the end, I advised him to verify his new hypotheses. He put on the ACCELERATION meters of objects B & D and ran the simulation in slow motion. As per his hypotheses, he observed from the digital meters that the acceleration of object D was g and that of object B was less than g (line 113).

6.4 Second Situation: When the string was broken

With regard to the second situation, that is when the string was broken, I noted that the data available could be used to analyse three different types of sub-situations (or contexts):

(a) To analyse student understanding of the acceleration of object B;
(b) To analyse student understanding of the acceleration of object A on a smooth surface; and,
(c) To analyse student understanding of the acceleration of object A on a rough surface.

It is imperative to note that:

(a) Students S1 and S2 did not take part in this activity as it had not been designed during the pilot study; and
(b) The Interactive Physics version at hand was not able to represent a broken string as a dotted line. However, this issue was discussed with the participants and overcome via the concept of tension in the string.

6.4.1 First Sub-Situation: Acceleration of object B becoming g gradually

In relation to the first sub-situation, I observed that there were two different types of prediction made by the students:

(a) three out of eight students (S3, S4, S9) predicted that the acceleration of object B would become g instantaneously; and,
(b) five out of eight students (S5, S6, S7, S8, S10) predicted that the acceleration of object B would gradually become g.

For this case, I have chosen Student S6, who predicted that the acceleration of object B would gradually become g, and have discussed her learning trajectory in Episode 6.3.

Brief Description of Episode 6.3

In this episode, Student S6 was asked to predict and discuss the acceleration of the hanging object B once the string broke. She was aware that when the string would break, tension must become zero and that the acceleration of the system was 2.452 m/s^2 just before the string broke (lines 16 – 18). The transcript for this episode can be found in Appendix E3.

Analysis of Episode 6.3

(a) She predicted that the acceleration of object B would gradually increase to acceleration due to gravity g once the string would break (lines 22; 24; 26). As the discussion unfolded, she remained adamant that its acceleration would not become g instantaneously.

S6: Once the string is broken, it doesn't become g all of a sudden, it takes a short time but it will become g gradually.
I: Isn't there some sort of contradiction between 'a short time' and 'gradually'? What do you mean?
S6: It won't take that much time.
I: How much time? 1 sec, 2 sec … 3 sec, 4 sec … 5 sec?
S6 : Something like that. Around 4 seconds.

The above extract (lines 43 – 49) suggests that for her the acceleration of object B must become g after some time, not instantaneously. This would imply that she held the intuitive idea 'the change in the magnitude of acceleration of object B could not be significant all of a sudden', that is she did not agree that its acceleration could change from 2.452 m/s^2 to 9.807 m/s^2 all of a sudden. It can be hypothesized that in her experience change should take time; change could not occur instantaneously.

(b) As she was advised to verify the acceleration of object B, she put on its ACCELERATION meter and ran the simulation using the command RUN. Since the animation was fast moving and the sliding object A left the table, she reset the simulation and by using the tape player control, she moved the simulation to t = 4.05 s (object A was near the end of the table and acceleration of object B was 9.807 m/s^2). She ran the simulation backward step by step until t = 1.45 s where tension in the string was equal to 7.355 N and the acceleration of object B was 2.452 m/s^2 (see Figure 6.5). After one or two seconds she moved the simulation one step forward to t = 1.5 s where tension in the string was equal to 0 N and acceleration of object B changed to 9.807 m/s^2 (see Figure 6.6). At this specific point, she observed the sudden change in the acceleration (lines 59 – 60).

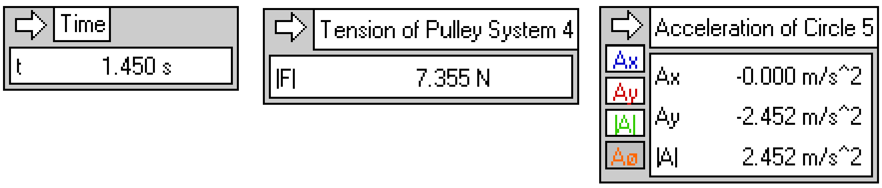


Figure 6.5: This set of meters displayed the tension in the string and the acceleration of object B just BEFORE the string was broken (t = 1.45 s).

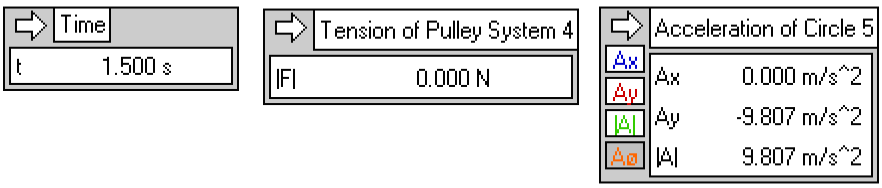


Figure 6.6: This set of meters displayed the tension in the string and the acceleration of object B just AFTER the string was broken (t = 1.5 s).

(c) When she was asked whether or not she agreed with this observation, after some seconds of silence, she provided an explanation which contained an element of uncertainty.

I: Do you accept this?

S6: … It came a bit too early.

I: It appeared to come instantaneously. What do you think?

S6: …(pause:14)… I knew that it would become 9.81, but not so quickly.

I: You don't agree with it?

S6: It's 'ok'. It should be 'ok'.

From the above extract (lines 61 – 66), it can be seen that she could not understand why the acceleration of object B became g instantaneously. However, since she pretended to accept this fact, I asked her to be more precise in her explanation. It was then that she saw the light out of the tunnel.

S6: … Oh yes! mg! Once the string breaks, there is no tension, only weight acts. This is why it changes instantaneously. Because tension becomes zero, there is only one force that acts.

A reorganisation of ideas can be noted in her above statement (lines 68 – 70) such that she could now explain the phenomenon. First, it appears that the readings in Figures 6.5 and 6.6 reminded her of the intuitive idea 'when the string would be broken, tension in the string should become zero instantaneously'. Second, from Task 1, she knew about the forces acting on object B: its weight acting downward and the tension in the string acting upward such that the net force acting on object B was its weight minus the tension. Since she realised that as soon as the string broke "there is no tension, only weight acts", she managed to construct the understanding of "why it changes instantaneously".

6.4.2 Second Sub-Situation: Acceleration of Object A on a smooth surface decreasing until reaching zero

For the second sub-situation, I observed that there was only one type of prediction by six students (S3, S4, S5, S6, S7, S8). All of them predicted that the acceleration of object A, sliding on a smooth surface, would decrease until it would become zero. I have chosen Student S6 and have discussed her learning trajectory in Episode 6.4.

Brief Description of Episode 6.4

In this episode, which began as from line 71, Student S6 was asked to predict and discuss the acceleration of the sliding object A on a smooth surface once the string would break. Earlier she had predicted that the acceleration of the hanging object B would gradually increase from 2.452 m/s^2 to 9.807 m/s^2 once the string would break (Refer to Episode 6.3). The transcript for this episode can be found in Appendix E3.

Analysis of Episode 6.4

(a) During the Prediction phase, she predicted that once the string would break, the acceleration of the sliding object A would decrease until it would become zero and object A would stop provided the table was long enough (lines 52 – 57). Her reasoning raises three interesting questions: (1) Why did she think that the acceleration of object A would decrease to zero and not become zero instantaneously, especially as she had acquired this experience in E6.3?; (2) Why did she think that the object A would eventually stop?; and, (3) Why did she think that the object A must stop once its acceleration would become zero? It is imperative to emphasize that she was reminded that she was dealing with a smooth surface (line 51). On this basis, it can be hypothesized that she must not have thought of friction to be responsible for the slowing down and stopping of object A. In turn, this implies that for her motion of object A was only possible if there was a force acting on it. That is, as long as there was a constant tension in the string, the latter was a continuing force that caused the motion. Once the string broke, there was no tension and therefore no continuous force pulling object A. According to her, when this force 'wore out', object A slowed down and eventually stopped. This appears to imply the activation of the intuitive idea 'motion implies force'.

(b) When she was asked to verify the acceleration of object A, she put on its ACCELERATION meter and then ran the simulation using RUN. As object A left the table, she reset the simulation and ran it in slow motion until t = 1.5 s. At this point the tension in the string changed to zero and the magnitude of the acceleration of object A changed to zero (see Figure 6.7). She moved the simulation one step backward, that is at t = 1.45 s, the tension in the string was again 7.355 N and the magnitude of the acceleration of object A was again 2.452 m/s^2.

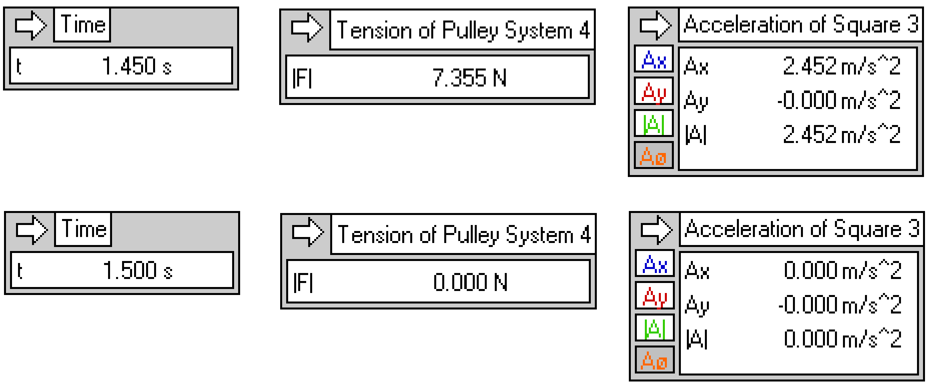


Figure 6.7: The set of meters (at t = 1.45 s) displayed the tension in the string and the acceleration of object A just BEFORE the string was broken. The set of meters (at t = 1.5 s) displayed the tension in the string and the acceleration of object A just AFTER the string was broken.

She stated that acceleration became zero, but after some ten seconds later she added that object A continued to move (line 72). It would appear that she agreed with the observation that acceleration had become zero instantaneously (most probably because she now recalled her experience in E6.3), but not with the fact that object A continued to move.

(c) She restarted to run the simulation in slow motion up to t = 1.75 s and object A continued to move. She did not agree with this motion (lines 74 – 75).

S6: Shouldn't it [pointing at object A] stop moving? … It should have stopped moving, shouldn't it?

She restarted to run the simulation in slow motion until t = 2.9 s and object A continued to move. At this point she again questioned the validity of the motion of object A (line 76), implying that for her object A should not move when its acceleration was zero. This suggests that she had the intuitive idea that motion of object A was only possible if there was a force acting on it, as discussed in part (a).

She continued to run the simulation in slow motion up to t = 3.5 s and she noted that object A moved along the table. After some twelve seconds of complete silence and inaction, she restarted to run the simulation in slow motion until object A left the table. At this point she instantly explained that object A moved with constant speed.

S6: This means that it moves with constant speed.
I: … What do you think?
S6: It does not have any acceleration; it moves with a speed with which it was being 'pulled'.

From this extract (lines 77 – 80), it can be seen that though there was no VELOCITY meter of object A on the screen, she developed the idea that when the string broke, object A must be moving with constant speed as the acceleration remained zero until object A left the table. Thus, it can be deduced that the rate at which object A moved along the surface in the simulation enabled her to construct an understanding of the speed of object A which, in turn, empowered her to make sense of its acceleration in this situation.

(d) As she was keen to verify her new hypothesis, she put on the VELOCITY meter of object A and ran the simulation using the command RUN. It was observed that the reading on the meter quickly increased from 0 to 3.657 m/s, and remained 3.657 m/s until t = 3.5 s where she stopped the simulation. As object A was about to leave the surface, she reset the simulation and ran it in slow motion up to t = 1.5 s. From the meter, it was observed that at t = 0, velocity = 0 m/s and as t increased, so did velocity; at t = 1.5 s, velocity = 3.657

m/s. At this point she was able to confirm her hypothesis about constant speed and the reason for it.

S6: Yes, it moves with the speed that it had when the string is broken.
I: According to you, why?
S6: …(pause:25)… Tension becomes zero.
I: Ok.
S6: This means … It was already moving with a certain speed.

From the above extract (lines 83 – 87), it can be seen that she not only succeeded in validating her hypothesis on constant speed but she appeared to make use of the readings that she had earlier noted from the meters: tension in the string = 0 and acceleration of object A = 0 once the string had broken (refer to Figure 6.7). It also seems likely that this situation reminded her (just as in E6.3) of her intuitive idea 'when the string would break, tension would become zero' – an idea that seemed to contribute positively in the direction of expertise. Though she did not overtly make the link, it would appear that for her because tension became zero (line 85), acceleration also became zero. She then used this mechanical idea to justify why object A moved with constant speed. Interestingly, earlier (see part (c)) she used velocity to make sense of the zero acceleration. Here she used zero tension to make sense of the zero acceleration, and in turn this allowed her to make sense of the constant speed. In the end, it is seen that she could connect this situation to her school knowledge of Newton's first law (lines 89 – 90).

6.4.3 Third Sub-Situation: Acceleration of Object A on a rough surface decreasing until reaching zero

In relation to the third sub-situation, it is to be noted that one student (Student S4) could not make any prediction. The remaining seven students predicted that the acceleration of object A would decrease until it would become zero. However, out of these seven students, only one student (S6) correctly predicted that the deceleration process would involve a negative sign. The remaining six students predicted that it would decrease continuously until it would reach zero. For this sub-situation, I present Episode 6.5 where I explore the learning trajectory of Student S5.

Brief Description of Episode 6.5

In this episode, Student S5 was asked to predict and discuss the acceleration of the sliding object A on a rough surface once the string would break. She was aware that the acceleration of the system was 1.717 m/s^2 just before the string broke. While the transcript for this episode can be found in Appendix D, a (preliminary) detailed analysis is located in Appendix J.

Analysis of Episode 6.5

(a) She predicted that the acceleration of object A would become zero as soon as the string would break (line 2). When more clarification was asked for, she added that the acceleration would decrease until it would become zero (lines 9; 11). While predicting the speed of object A, she initially stated that object A would stop once the string would break (line 14) and then quickly restated that "object A will move a bit and will then stop" (lines 17 – 19). Thus, in relation to the acceleration and the speed of object A, she quickly restated that they would become zero after some time. This tends to suggest that she still held the intuitive idea that change in acceleration (and possibly speed) could not be abrupt, but gradual (see ending note of introduction on pg 331).

(b) When she was asked to verify her hypotheses on the simulation, she put on the ACCELERATION meter for object A and ran the simulation on two occasions using the command RUN; on the screen, object A moved along the table until it became at rest and the reading on the meter changed from 1.717 m/s^2 to zero very quickly. As she did not appear to have learnt anything from these two occasions (line 22), I advised her to run the simulation in slow motion and to observe the motion just before and after the string would break.

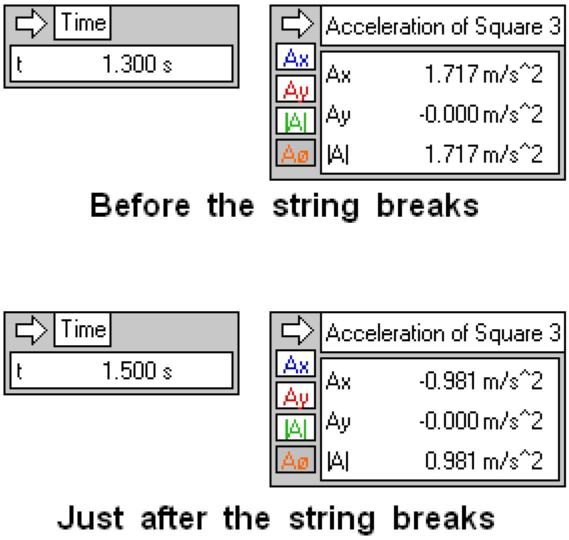


Figure 6.8: This set of meters displayed the acceleration of the sliding object A BEFORE and JUST AFTER the string was broken (between t = 1.45 s & t = 1.5 s).

After running the simulation, she mentioned that the acceleration of object A had decreased (line 26). In fact, when the ACCELERATION meter of object A is analysed (Figure 6.8), it can be seen that the reading of the modulus of acceleration, $|A|$, changed from 1.717 m/s^2 to 0.981 m/s^2, but the reading of the component of acceleration along the horizontal direction, A_X, changed from 1.717 m/s^2 to -0.981 m/s^2. Her interpretation of this situation shows that she had paid attention to the change in the reading of $|A|$ (lines 74; 153 – 156).

(c) When I enquired whether it had really decreased, she continued to run the simulation in slow motion up to t = 3.65 s. I advised her to keep running the simulation until object A stopped moving at t = 4.55 s. At this specific time, she noted that the acceleration had become zero (line 30). In practice, from the ACCELERATION meter of object A, it can be observed that the reading of the modulus of acceleration, $|A|$, was constant (0.981 m/s^2) for some seconds and then decreased to zero; in the case of the component of acceleration along the horizontal direction, A_X, it was constant (-0.981 m/s^2) for some seconds and then decreased to zero. It can be deduced that she neglected the period where its acceleration was a constant for a short time and focused only on the time when it decreased to zero. This can be confirmed by her following statement (lines 34 – 35):

S5: Once the string breaks, the acceleration of object A decreases until it becomes zero.

The question that arises is why she neglected the period during which object A experienced a constant acceleration (-0.981 m/s^2) after the string had broken. Can we attribute this 'omission' to her intuitive idea that the acceleration of object A would decrease until it would become zero (lines 9; 11)? In other words, since she had no intuitive knowledge of the 'constant acceleration' period, she did not predict this sort of behaviour and did not know which part of the animation to analyse adequately.

(d) At this point of the episode, I questioned her whether on Interactive Physics she had observed the acceleration of object A continuously decreasing until it had become zero (lines 38 – 40). This query initiated her to move the simulation backward from t = 4.55 s to t = 3.65 s in slow motion such that: (1) between t = 4.55 s and t = 4.0 s, A_X changed from 0 m/s^2 to -0.981 m/s^2 and |A| changed from 0 m/s^2 to 0.981 m/s^2; and, (2) between t = 3.95 s and t = 3.65 s, A_X remained -0.981 m/s^2 and |A| remained 0.981 m/s^2. This process appeared to make her realise the 'constant acceleration' process of object A (line 41).

S5: For some time it is a constant and then becomes zero.

It would seem that the reading of |A| between t = 3.95 s and t = 3.65 s made her reconsider her initial prediction. This may explain why, in her quest for confirming her newly-developed idea, she continued to run the simulation backward in slow motion until t = 1.5 s and ignored my question (lines 42 – 44). She must have noted via the ACCELERATION meter that the acceleration of object A remained constant throughout this motion. After some seconds, she first ran the simulation forward in slow motion up to t = 1.9 s and then moved it to t = 3.35 s. She must have observed that the acceleration of object A again remained constant throughout this motion. On this basis, she uttered the following statement (lines 46 – 47).

S5: When the string breaks, the acceleration remains constant for some time and then decreases to zero.

Since she appeared to have learnt a new fact that the acceleration should be constant for some time in this context, I asked her whether or not she agreed with this fact. She explained that when she analysed the system "in terms of velocity, it makes sense" (line 52). According to her, when the string broke, object B was no longer part of the system and therefore the velocity of object A would decrease until it would become zero. This could explain why the acceleration of object A remained constant and then became zero (lines 54 – 60). So, for her, when velocity became zero, acceleration should also become zero (lines 60 – 61). It can be hypothesized that the (dual) relationship between velocity and acceleration allowed her to make sense of the behaviour of object A when the string had broken.

(e) I advised her to put on the VELOCITY meter so that she could verify her hypothesis. She ran the simulation till t = 9.1 s using the command RUN, reset it and then ran it in slow motion up to t = 1.5 s. After a few seconds, she continued to run the simulation in slow motion up to t = 2.9 s. Using the tape player control, she moved the simulation to t = 3.55 s and then ran it backward in slow motion up to t = 2.0 s. At this point she stated that the velocity had decreased (line 67). Using the tape player control, she moved the simulation to t = 1.4 s and then ran it in slow motion up to t = 3.6 s. As she remained silent and did not make any analogy between the acceleration and velocity as expected, I asked her if she could describe the acceleration of object A just before the string would break. She explained that its acceleration was constant (1.717 m/s^2) and that velocity had increased (line 72). She noted that after the string had broken, its acceleration changed to 0.981 m/s^2, remained a constant for some time until it became zero. At this point, she further added that velocity had decreased until it became zero (lines 74 – 77). It can be hypothesized that for her it made sense that when acceleration was constant (0.981 m/s^2) velocity decreased, and that when acceleration became zero, velocity also became zero. It also seems that she did not give much emphasis to the period of the motion when the velocity had increased. Finally, the most interesting point in this situation is why she did not realize that just before the string had broken its acceleration was a constant and its velocity was increasing, whereas just after the string had broken its acceleration was again a constant, as per her explanation, and yet its velocity was decreasing.

(f) When she was asked whether the speed of an object could decrease when it would accelerate, she replied that this was only possible when the object would decelerate (line 85). Though she appeared to be familiar with the term ‘deceleration’ (lines 98; 100), there is no evidence that she had used it to make sense of this context (lines 109 – 110). However, after some time, she explained that though she “thought in terms of deceleration”, she continued to use the term ‘accelerate’ instead of the term ‘decelerate’, because I used it (lines 121 – 123). For her, the deceleration process started when the string had broken – the reading of the modulus of acceleration, $|A|$, changed from 1.717 m/s^2 to 0.981 m/s^2 (lines 136 – 146; 153 – 156) (see Figure 6.8). She held the intuitive idea that a deceleration process occurred when “there is a decrease in acceleration” (lines 150 – 152), implying that for her a decrease in the magnitude of acceleration was synonymous to deceleration. This may explain why she did not pay attention to A_X on the meter.

(g) I encouraged her to move the simulation to t = 1.45 s and to run it one step forward to t = 1.5 s, but this time by focusing on the reading of the component of acceleration along the horizontal direction, A_X (line 158). When she ran the simulation as expected, A_X changed from 1.717 m/s^2 to -0.981 m/s^2. After a few seconds, she moved the simulation one step backward (t = 1.45 s) and then one step forward (t = 1.5 s). As she seemed not to make sense of this change (line 160), I suggested to add the vector arrow for ACCELERATION on object A. Once this was implemented, I magnified the length of the vector arrow and upon her request, I also changed its colour from green to red.

(h) When she ran the simulation in slow motion, from rest (t = 0) to t = 1.45 s, the vector arrow ACCELERATION, of a fixed length, pointed towards the right.

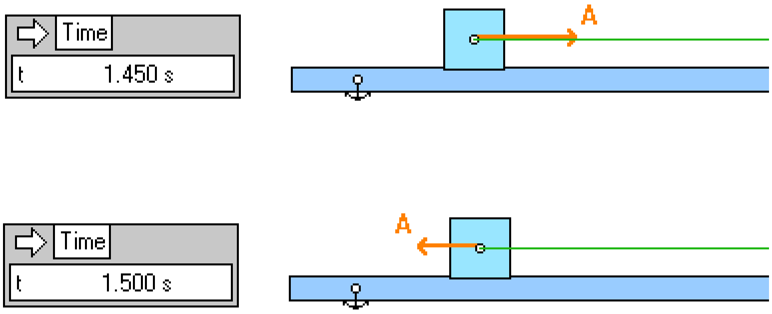


Figure 6.9: At t = 1.45 s, just before the string broke, the vector arrow A pointed toward the right. At t = 1.5 s, where the string broke, the vector arrow A, not only changed its magnitude, but also its direction (pointing toward the left).

At t = 1.5 s, the vector arrow changed direction and length – it now pointed towards the left and its length had decreased (Figure 6.9). As from t = 4.050 s, its length started to decrease and so did the readings of $|A|$ and A_X. At t = 4.15 s, the vector arrow disappeared from the screen and the reading of A_X was -0.090 m/s^2. At t = 4.550 s, $|A| = A_X = 0$ m/s^2 & v = 0 m/s. As the object had stopped moving, she moved the simulation backward (by using the tape player control) to the position where the vector arrow had changed direction (t = 1.5 s). At this point she reiterated her initial idea that when the string broke, object A decelerated (line 164). When she was asked if she had anything more to say, she continued to 'play' with the simulation at the point where the vector arrow had changed direction.

S5: The acceleration changes its direction. Its magnitude also changes.
I: What?
S5: The acceleration changes its direction.
I: What happens to the acceleration when the string breaks?
S5: It changes direction.
I: Any other thing?
S5: … It decelerates. It becomes minus.

From the above extract (lines 166 – 172), it seems evident that a change has occurred in her understanding of the deceleration of object A once the string broke. From the start, she knew about the decrease in magnitude. However, in this case she noted a change in direction of acceleration and that deceleration implied “it becomes minus”. As such, it can be hypothesized that in this situation the vector arrow **A**, unlike the ACCELERATION meter, enabled her to realise that the direction of the deceleration process should be opposite to that of the acceleration process. She could thus understand the importance of the A_X component in the ACCELERATION meter and the significance of the minus sign on A_X once the string had broken.

(i) I advised her to add the vector arrow VELOCITY on object A (line 176). In doing so, my aim was to further probe in her understanding of acceleration and deceleration processes by allowing her to investigate the relationship between the velocity and the acceleration of object A through their vector arrows appearing simultaneously on the screen. As she ran the simulation from t = 0 s, the length of the vector arrow VELOCITY increased gradually from zero, while that of the vector arrow ACCELERATION was of a fixed length – both vector arrows pointed towards the right.

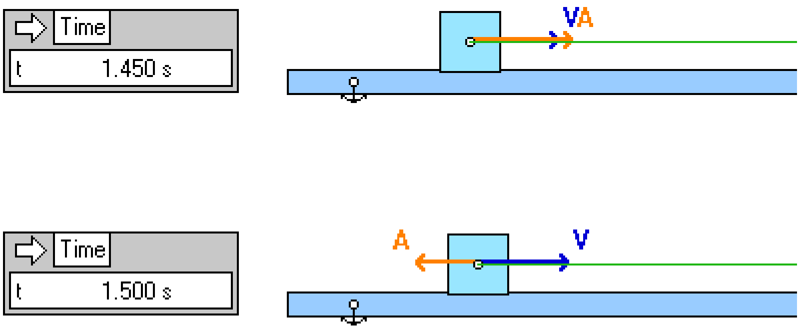


Figure 6.10: At t = 1.45 s, just before the string broke, the vector arrows V & A pointed toward the right. At t = 1.5 s, where the string broke, the vector arrow V continued to point toward the right, whereas the vector arrow A pointed toward the left.

When the simulation reached t = 1.50 s (the point just after the string broke), the vector arrow VELOCITY continued to point towards the right, but that of ACCELERATION pointed towards the left (Figure 6.10). As she continued to run the simulation from t = 1.50 s in slow motion, the length of the vector arrow VELOCITY decreased gradually until it disappeared from the screen at t = 3.8 s – at this point the readings of $|V|$ and V_X were 0.297 m/s. After the object had stopped moving, she moved the simulation to t = 1.40 s via the tape player control and then ran the simulation in slow motion up to t = 1.50 s. At this point of the episode, she seemed to have developed a new idea about the relationship between velocity and acceleration.

S5: The direction of velocity remains unchanged.
I: What have you learnt here? […]
S5: I have learnt that the direction of the acceleration changes. The direction of the velocity remains unchanged … and the acceleration, before becoming zero, is a constant when the string breaks.
I: Ok.
S5: It is a constant before becoming zero.

From the above extract (lines 178 – 184), it can be suggested that she was not aware that once the string would break, velocity and acceleration would be in opposite directions. In this instance, at the point where the string broke, while the vector arrow ACCELERATION **A** showed that acceleration changed direction, the vector arrow VELOCITY **V** showed that velocity remained in the same direction. So, the interplay between **A** and **V** allowed her to learn that at the point where the string broke, these two quantities became opposite to each other such that they could not be in the same direction during the deceleration process.

(j) At the end of this session, she explained that both features of Interactive Physics – the meters and the vector arrows – have helped learnt these ideas.

S5: When I used the meter, I did not pay attention to the acceleration along the horizontal axis [referring to Ax].
I: What happened when you looked at the meter?
S5: I looked at the modulus only [referring to |A|].
I: You looked at the modulus, at the magnitude. Alright.
S5: I did not pay attention to Ax.
I: […] But did it help you understand the system?
S5: Yes, but the vector arrows were of more help.
I: In what ways?

S5: First, it showed me that the direction of velocity remains unchanged. It always moves in the same direction. However, the direction of acceleration changed.

The above extract (lines 191 – 202) indicates how each feature has contributed in helping Student S5 to make sense of the relationship between velocity and acceleration of object A at the point where the string had broken. According to her, the digital meters were effective in conveying the magnitude of a vector quantity to her; however, they did not encourage her to think in terms of direction as well. The vector arrows were successful in conveying to her the direction of a vector quantity over a certain time interval; in particular, the vector arrow **A** allowed her to recognise the existence of A_X and to make sense of the minus sign.

6.5 Third Situation: When the string became slack

In this situation, that is when the string was slack in Task 6b, I found that the participants made three different types of prediction:

(a) two students (S3, S9) predicted that as long as the string was slack, object B would move in the air with g;

(b) two students (S6, S7) predicted that upon impact at point X (refer to Figure 6.1) object B would move up with acceleration less than 6.538 m/s^2 until it would become 0 m/s^2 at its highest point Y and then as it would move down, its acceleration would be g; and

(c) four students (S4, S5, S8, S10) predicted that upon impact at point X object B would move up with acceleration less than 6.538 m/s^2 until it would become 0 m/s^2 at its highest point Y and then as it would move down, its acceleration would gradually increase from 0.

I have chosen one student per group and have discussed their learning trajectories. Interestingly, both groups predicted that as object B would move up from point X to point Y, its acceleration would gradually decrease from 6.538

m/s^2 to zero. The difference between these two groups lies when object B would move down: while one group predicted that it would fall with g, the other suggested that its acceleration would gradually increase from zero.

While Episode 6.6 deals with Student S7 who predicted that object B would fall with g, Episode 6.7 portrays Student S8 who predicted that the acceleration of object B would gradually increase from zero as it would move down.

6.5.1 Point X → Point Y: Object B moved with acceleration g

Brief Description of Episode 6.6

In this episode, Student S7 was asked to predict and discuss the acceleration of the hanging object B between 'just before it hit the ground for the first time – the first impact' and 'just before it hit the ground for the second time – the second impact'. The transcript for this episode can be found in Appendix F.

Analysis of Episode 6.6

(a) Student S7 was aware that object B would move with an acceleration 6.538 m/s^2 just before it hit the ground for the first time (lines 18; 29; 41 – 42). He predicted that the acceleration of object B would "become negative" and be less than 6.538 m/s^2 once it would hit the ground and would move up (lines 24; 31; 44 – 45). Though he did not overtly predicted the acceleration of object B when it would hit the ground and when it would reach its highest point after the first impact, it would appear that for him it would be zero at both points. He also predicted that when object B would move down from its highest point, it would fall with g (lines 38; 47 – 48).

I: Alright. Why will it fall with g?
S7: Because there is weight only which is the net force.

I: What?

S7: … There is no tension which will oppose its motion.

From the above extract (lines 49 – 52), it seems that Student S7 was fully aware of the condition required for an object to fall freely under gravity. He knew that 'no tension in the string' implied that there was no force opposing object B to fall freely. So this part of the episode raises some key questions. First, why did he not realise that after the first impact object B was already under the complete influence of gravity? Second, why did he think that acceleration would decrease as object B would rise? Third, why did he not realise that the string was already slack? I will try to answer these questions in part (e).

(b) With regard to the velocity of object B, the student made several predictions. He rightly pointed out that at its initial position, its velocity would be zero (lines 57; 60 – 61; 89). As object B would move down, he predicted that its velocity would increase up to a certain height and would then become constant (lines 57; 63 – 65; 91). As such, object B would move down with a constant velocity until it would hit the ground (lines 67 – 68); at this point of impact he predicted that its velocity would become zero (lines 70; 76; 93; 100). After the first impact, object B would move up and, according to him, its velocity would increase (line 102). When object B would reach the highest point, he predicted that its velocity would become zero (lines 80 – 81; 106).

I: From here [pointing at object B as it moved up] you mentioned that the velocity is increasing.

S7: Yes.

I: So how come it becomes zero all of a sudden? […]

S7: …(pause:55)… This means that when it moves up, its velocity will decrease so that it will become zero.

In the above extract (lines 107 – 112), it can be seen that his reasoning in relation to velocity was incorrect – there was an object B moving freely under gravity; how was it possible for the velocity of object B that he had predicted to be increasing to become zero all of a sudden? It seems that he now had this new intuitive idea "when it moves up, its velocity will decrease so that it will become zero". It is worthwhile to note that he continued to ascertain that at its highest point its velocity should be zero and then reworked out the velocity with which object B would move up.

Finally, he predicted that when object B would move down, its velocity would increase and when it would hit the ground, its velocity would become zero (lines 114 – 115). He still maintained that on impact the velocity of object B would become zero.

(c) As he wished to first verify his predictions on acceleration (line 117), he put on the ACCELERATION meter of object B and ran the simulation using the command RUN. As he seemed rather loss, I advised him to move the simulation to the moment where object B was just about to hit the ground for the first time. After positioning object B to the said location, he moved the simulation 1 step forward such that object B hit the ground and the string became slack. The reading on the ACCELERATION meter changed from 6.538 m/s^2 to 9.807 m/s^2 (Figure 6.11).

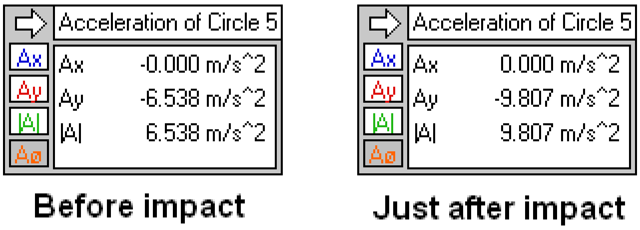


Figure 6.11: The ACCELERATION meter displayed the reading before the first impact and that just after the first impact.

After several seconds of silence, he re-started to run the simulation in slow motion and the following events took place on the screen: object B moved up, reached its peak where it appeared to remain stationary for a very short time, moved down and finally hit the ground for a second time – throughout this trajectory, the reading on the ACCELERATION meter remained 9.807 m/s^2.

(d) He noted that the acceleration of object B became g once it hit the ground (line 119). Interestingly, he argued that he had predicted this scenario (lines 120 –

122). When I reminded him of what he had earlier predicted (lines 126 – 133), he responded positively (lines 136; 138). This appears to suggest that, beside his prediction (lines 24; 31; 44 – 45), he might also have this idea of object B experiencing g once the string would hit the ground; however, the later mechanical idea was of lower cuing priority. When he was asked to explain the discrepancy between what he had predicted in relation to acceleration and what he had observed on Interactive Physics, he emphasized the relationship between acceleration and velocity (lines 139 – 145).

I: […] Why did you think that when it hits the ground it is going to be less than 6.538 m/s^2 and when it moves up it should become zero?
S7: I thought that when it hits the ground, its velocity would become zero.
I: Hmm.
S7: Acceleration is the rate of change of velocity. If velocity is zero, then acceleration must be zero.

The above extract (lines 139 – 145) demonstrates the emergence of two intuitive ideas. First, he held the intuitive idea 'when an object would hit the ground, its velocity should become zero'. Second, since he knew that acceleration was the rate of change of velocity, he assumed that in general whenever velocity would be zero, acceleration must also be zero. It can also be hypothesized that Student S7 first worked out the velocity of object B and then by his definition (lines 144 – 145) evaluated its acceleration during the trajectory.

I tried to challenge his argument that when velocity would be zero, acceleration must also be zero. After more than a minute of silence, he replied negatively (line 147). However, he could not explained why when the velocity would be zero, the acceleration would not need to be zero (line 151).

(e) He asked the question of why object B had moved up with acceleration due to gravity, g (line 153). After almost a minute of silence, he murmured that acceleration had remained constant (line 155). At this point of the episode, I enquired as to why he did not accept that object B could move up with g, but he agreed that it could move down with g. Consistent with his earlier explanations (lines 38; 47 – 48), he maintained that when it moved down, “it falls freely, it moves under gravity” (line 162). Interestingly, when he was again asked whether he agreed with the fact that object B moved up with g, he responded positively (line 167) and was able to defend his new position (lines 169 – 180).

S7: Because it is free, it moves up freely, there is no force that is acting.
I: What evidence do you have?
S7: There won’t be any tension on it [pointing at object B].
I: How do you know that there is no tension on it? …(pause)… As from when do you think that tension has no effect on it [pointing at object B]?
S7: As from when the object hits the ground and starts to move up. It is as from this point that tension doesn’t affect the system.
I: Why does it not affect the object?
S7: Because it is already zero.
I: Ok. Then?
S7: There will be only gravitational force acting on the object. This is why it moves up with 9.81.

From the above extract (lines 169 – 180), it would appear that he held the intuitive idea ‘tension continued to exist in the string even after the first impact up to when object B reached its highest point Y’. According to him, it is only when object B started to move down that there was no tension in the string (line 52). On this basis, it can be hypothesized that once object B hit the ground, he did not realise that the string became slack and the tension in the string became zero. So one question that arises from this situation is the following: How come he accepted that there was no tension in the string when object B moved down from its highest point, but that from his viewpoint there was tension in the string when object B moved up after the first impact? It is also thought-provoking to observe that he used the derivative $a = dv/dt$ when object B moved up after the first impact, but not when it moved down from its highest point. Geometrically speaking, there is some sort of symmetry between object B moving up and it moving down in this case.

Based on all these facts, it can be concluded that the slackness of the string after the first impact was of no relevance to the learning process of the student. He might have the intuitive idea 'an object could only experience g when it would fall and not when it would move up freely'. This may explain his query of why object B had moved up with g after its first impact (line 153). So it can be hypothesized that he first looked at the system (object B) in terms of acceleration and then worked out whether or not there would be tension in the string.

(f) He finally verified the velocity of object B.

S7: I compared this object [referring to object B] to one which is falling and there is weight and air resistance. At a certain time it would move with terminal velocity. That is why I thought that it would move with constant speed.

From the above statement (lines 198 – 201), it can be seen that he had the intuitive idea that object B would move with constant speed for some time. This is because he held the idea that the notion of terminal velocity could be applied to object B.

6.5.2 Point X → Point Y: Acceleration of object B increased gradually

Brief Description of Episode 6.7

This episode began when Student S8 was asked to predict and discuss the acceleration of the hanging object B between 'just before it hit the ground for the first time – the first impact' and 'just before it hit the ground for the second time – the second impact'. The transcript for this episode can be found in Appendix G.

Analysis of Episode 6.7

(a) Student S8 knew that object B would move with an acceleration 6.538 m/s^2 just before it hit the ground for the first time (line 14). She predicted that the acceleration of object B would be less than 6.538 m/s^2 once it would hit the ground and would move up (line 22). When object B would reach its highest point Y after the first impact, she predicted that its acceleration would decrease to become zero (line 38). She also predicted that when object B would move down from its highest point, its acceleration would gradually increase from zero (lines 40; 44). As such, it seems evident that she held the intuitive idea 'change in acceleration could not be abrupt, but gradual in both the ascent and descent of object B'.

(b) She put on the ACCELERATION meter of object B, ran the simulation in slow motion until object B was just about to hit the ground for the first time and found that its acceleration was 6.538 m/s^2 as per her prediction. She then ran the simulation 1 step forward such that object B just hit the surface and the string became slack. The reading on the ACCELERATION meter changed from 6.538 m/s^2 to 9.807 m/s^2 (refer to Figure 6.11). She observed for some seconds, then ran the simulation one step further, again observed for some seconds and finally ran it two more steps forward. She not only rightly observed that the acceleration of object B became g (line 49 – 50), but could also construct the reasoning behind it.

S8: … It becomes an object that is falling freely, under gravity. This explains why it has an acceleration of 9.8.

From the above statement (lines 52 – 53), it appears that the reading on the ACCELERATION meter made her realise that there was no opposing force on object B after the first impact; the only force that acted upon it was the gravitational force. She explained that initially she was under the illusion that object B was still being influenced by object A.

S8: I thought that this [pointing at object B] was connected to this one [pointing at object A].
I: They are still connected.
S8: Yes, but it [pointing at object A] has already moved … such that it is not on the table. Then it becomes a free fall.

From the above extract (lines 57 – 61), it would seem that for her the acceleration of object B would become g when object A would no longer be on the table. She appeared to have the intuitive idea 'as long as object A would be on the table, the objects would behave as a connected particles system'.

(c) She re-started to run the simulation in slow motion and I advised her to observe the simulation attentively. She continued to run the simulation in slow motion until object B just hit the ground and the string became slack; object A was still on the table. She deduced that tension "becomes zero. It becomes slack" (line 67). To this, I reminded her that it was the string that became slack, not the tension. She observed that when the string became slack, object B "acts as an object falling freely under gravity" (lines 70 – 71). She further added that she was not aware of this fact (lines 72 – 75) as she had continued to view the objects as a connected particles system and, therefore, had not taken into consideration that the string would become slack (lines 77 – 81). She ended this conversation by revealing that she could not explain why she had not realised that the string would become slack when object B would hit the ground (line 85). So it seems that the slackness of the string after the first impact was not able to decrease the level of cuing priority of her intuitive idea 'the system would behave as connected particles until object A was on the table'.

(d) She ran the simulation a few more steps such that object A was almost near the end of the table. Upon my advice, she continued to run the simulation in slow motion until object A left the table and the string continued to remain slack (Figure 6.12).

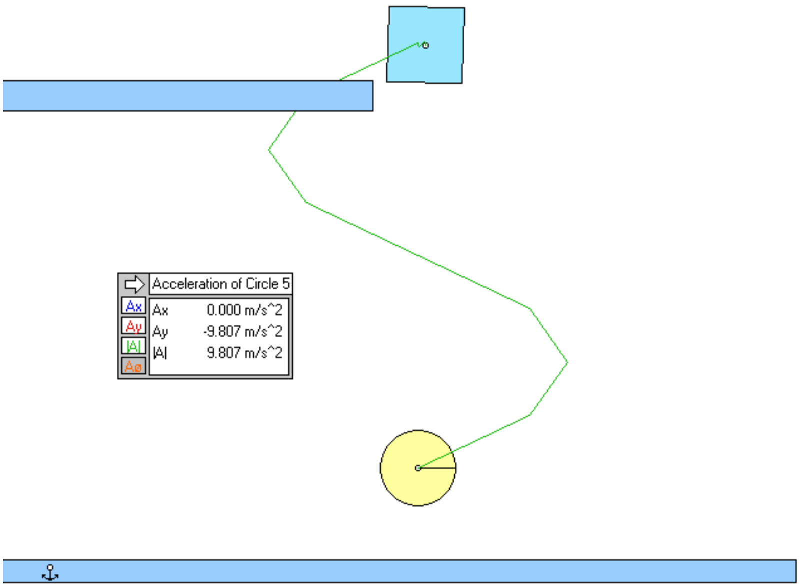


Figure 6.12: Screenshot showing object A had just left the table and the string continued to remain slack.

At this point of the episode, she observed that the acceleration of object B was still 9.807 m/s^2 (line 87). She continued to run the simulation in slow motion until the string became taut again, with objects A and B in the air (Figure 6.13).

She observed that the acceleration of object B had changed to 4.953 m/s^2 (line 89). When she was asked to explain the rationale behind this change in acceleration, she remained silent for some eight seconds. She then ran the simulation two steps backward such that the string became slack and the acceleration of object B changed to 9.807 m/s^2. Finally, when she ran the simulation one step forward, the string became taut again and the acceleration of object B became 2.049 m/s^2. She explained that there must be tension in the string to account for the change in acceleration (lines 91 – 93).

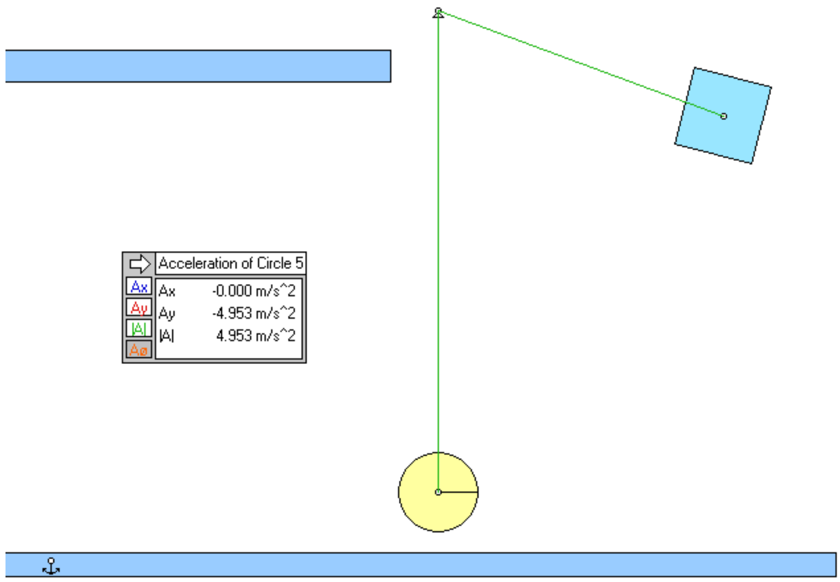


Figure 6.13: Screenshot showing that the string became taut again.

Interestingly, she was not able to explain the first change in acceleration of object B, that is from 9.807 m/s^2 to 4.953 m/s^2; however, she was successful in building an explanation for the recent (third) change in acceleration of object B, that is from 9.807 m/s^2 to 2.049 m/s^2. It may be hypothesized that during the first change in acceleration she might not have fully grasped the idea of slackness and tautness of the string. It was only after running the simulation backward and forward, and most probably observing the string changing from being slack to being taut, that she must have developed the idea that existence of tension in the string might be held responsible for the changes in acceleration.

(e) Upon my suggestion, after having reset the simulation, she put on the TENSION meter to verify her hypothesis (lines 94 – 96). She ran the simulation in slow motion until object B was just about to hit the ground. She waited for some seconds, moved the simulation one step forward such that object B just hit the ground, the string became slack, and the reading on the TENSION meter changed from 6.538 N to 0 N and that on the ACCELERATION meter changed from 6.538 m/s^2 to 9.807 m/s^2 (Figure 6.14).

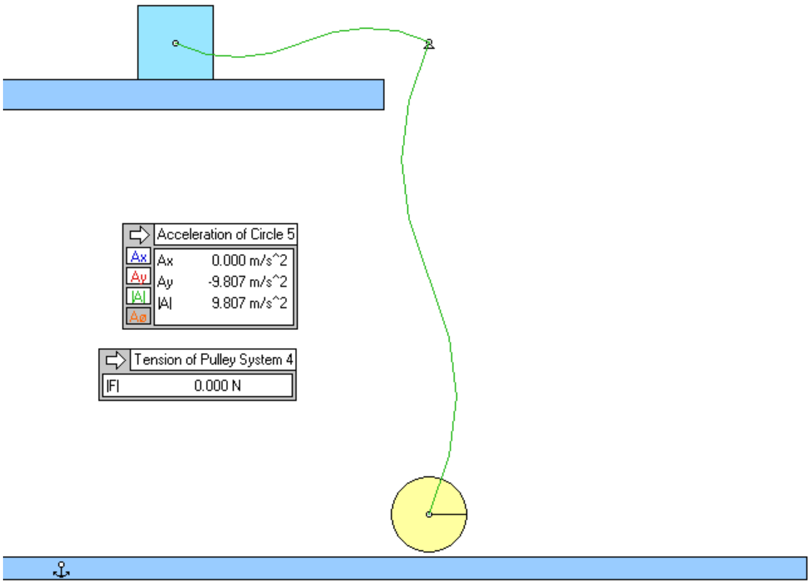


Figure 6.14: Screenshot showing that the string had just hit the ground and the string became slack. The reading on the tension meter changed from 6.538 N to 0 N.

As she re-started to run the simulation in slow motion, she noted that the tension in the string has become zero (line 97). This observation matched her new mechanical idea (line 67) that she had mentioned during the Reflection phase. She continued to run the simulation such that the tension in the string continued to remain zero until the moment when the string became taut (Figure 6.13). For some time, the readings on the TENSION and ACCELERATION meters fluctuated because the system was not stable.

As she focused on the instability of the system, I directed her attention on what happened when tension was zero.

S8: When tension is zero, the object falls freely, under gravity.

From the above statement (line 105), it can be suggested that she developed the idea that ‘when tension is zero’ is a necessary and sufficient condition for object B to fall freely, and not on the location of object A.

6.6 Concluding Remark

In this chapter, unlike Chapter 5, the animations did not play a predominant role in the learning process. Interestingly, it was more often the digital meters that stimulated the students to make sense of the slackness and tautness of the string in the animation process and their relationships with tension and acceleration. It was observed how the learning trajectories of students were influenced by some key intuitive ideas such as ‘change in magnitude of acceleration cannot be abrupt’ and ‘motion implies force’. It would also appear that the ways in which the term ‘acceleration’ is used in everyday life make it rather difficult for students to appreciate the scientific notion of acceleration. In the next chapter, based on the findings, I will look at the wider picture of this research.

Chapter 7

Discussion of Findings in relation to Aims of Thesis

7.1 Introduction

This chapter aims to discuss the findings of the previous two chapters so that we can look at the bigger picture of this thesis. In light of the findings, the definition of intuitive idea is reviewed and the study's research questions are restated in § 7.2. While § 7.3 explores the first research question in relation to intuitive ideas, § 7.4 expounds on the second research question which examines the role of the software. Finally, § 7.5 provides the concluding remark.

7.2 Setting the Scene

In the previous two chapters, the findings of this study have been presented after a thorough data analysis. It is seen that Chapter 5 relates the story of how the microworld shaped student thinking of motion of connected particles in terms of 'comparing forces', instead of 'comparing masses'. Interestingly, Chapter 6 is about the story of how the microworld contributed to make students aware of the notions of slackness and tautness of a string and their relationships to tension and acceleration.

It is clear from the data analysis that, in several cases, the students did not think in conventional ways. The word 'conventional' is used here to refer to thinking that is broadly acceptable and recognisable. We note that in this study students have constructed unconventional knowledge, but it would appear that this type of knowledge made more sense to these students.

In Chapter 2 (§ 2.2.4) I define intuitive knowledge as a type of knowledge that is formed through abstraction from exposure to real-life situations. It is with the power of this type of knowledge that students (and people in general) try to make sense of mechanics systems and motions in daily life. The data analysis reveals that in this study some school-based mechanics knowledge has become part of the learners' intuitive system which as from now on I will refer to as school-based intuitive ideas.

Again, in Chapter 2 (§ 2.2.1), through the definition of conceptual change by Mayer (2002), the term 'meaningful learning' is introduced in this thesis. It is worthwhile to note that in this study this term is used to denote learning in the direction of expertise. From the data analysis, we observe that for meaningful learning to occur, learners must genuinely agree with a new piece of correct information such that they can interpret (or make sense of) it. We find instances where the students agreed just to please the researcher. For example, in Episode 5.4 (§ 5.4.3) S6 pretended to agree with the motion of the system mA = mB.

Let us restate the research questions of this thesis:

> How is intuitive knowledge used by students in learning motion of connected particles?
>
> - What intuitive ideas do students bring to motion of connected particles?
> - What role, if any, do these intuitive ideas play during the generation of new knowledge?
>
> How do the uses of Interactive Physics influence the evolution of the learning process?
>
> - How are the affordances of Interactive Physics used in practice to develop understanding?
> - How do these affordances help in the forging of new connections between intuitive knowledge and new knowledge?

The discussion will now be carried out according to each research question. It is good to note that as from now onwards I will use the abbreviation E to imply Episode, for example E5.1 stands for Episode 5.1.

7.3 How is intuitive knowledge used by students in learning motion of connected particles?

When this first research question was formulated in Chapter 2, the aims were two-fold: (a) to identify the intuitive ideas that learners were expected to bring to the learning tasks; and, (b) to examine their role in the generation of new knowledge, be it unconventional or in the direction of expertise. Based on the findings of this study, these two aims will now be discussed.

7.3.1 What intuitive ideas do students bring to motion of connected particles?

This section focuses on what the participants predicted before developing new knowledge during their interaction with the software. As such, the big ideas have mainly come from the data collected during the Prediction Phase.

To the tasks, students brought several intuitive ideas which have been grouped in the following five categories:

(a) Explaining Motion and At-rest;
(b) Object B as a Prime Mover;
(c) Accelerations of Objects A and B;
(d) Location/Position of Object A; and,
(e) Unexplained Real-life Intuitive Ideas.

(a) Explaining Motion & At-rest

The findings point out that all the participants compared the masses of objects A and B in order to predict whether or not the connected particles system would move. It is found that all of them argued that there should be motion if and only if the mass of object B [mB] would be greater than that of object A [mA], mB > mA. It made perfect sense to all of them as they appeared to be influenced by the intuitive idea ‘a heavier load could displace a lighter one’. For the system mA > mB, all of them predicted that the system should remain at rest because they appear to be under the influence of the intuitive idea ‘a lighter object could not move a heavier object’.

For the system mA = mB, there were two types of predictions. For three students, the system should undergo some motion until equilibrium would be reached such that at rest the length between the sliding object A and the pulley should be equal to that between the pulley and the hanging object B. For the remaining seven students, the system should not move at all. So the main question that emerges from these findings is why some students, say S6 in E5.4 (§ 5.4.3) and S9 in E5.1 (§ 5.3.1), predicted that there should be some motion followed by equilibrium, whereas other students, say S3 in E5.2 (§ 5.4.1), predicted that the system should not move at all.

Those students, who predicted no motion at all, appeared to be under the influence of the idea that the weight of each object, being equal, would balance and cancel each other. This is why they had the intuitive idea ‘equal masses ⇒ no motion’. However, those, who predicted some motion followed by equilibrium, made reference to the Vertical version of connected particles such that the Vertical version influenced the students’ thinking in relation to the Horizontal version. They argued that equilibrium would be reached when the length of the string between object A and the pulley would be the same as that between object B and the pulley. This suggests that for this group of students ‘equal masses’ first implied ‘same height or length’ and then ‘no motion’. So it can be inferred that these students were under the influence of the intuitive idea ‘equal masses ⇒ same height/length ⇒ no motion’.

(b) Object B as a Prime Mover

This category is the direct outcome of two inter-related issues that were identified in the literature review in Chapter 2: (a) What pulls the sliding object A?; and, (b) Which object(s) creates tension in the string?

In relation to the first query, it is found that all the ten participants assigned a key role to object B in pulling object A. They held the intuitive idea ‘the weight of object B was responsible for pulling object A'. They, for example S2 in E5.3 (§ 5.4.2) and S3 in E5.2 (§ 5.4.1), assumed that the weight of object B became a force that pulled object A and also set the system in motion. For them, unlike object A which remained on a horizontal surface and was therefore supported, object B hanged in the air and was much more under the influence of 'force of gravity' as compared to object A. It is owing to this intuitive idea that in E5.2 S3 swayed between whether object B or both objects were needed to create tension in the string; he ultimately settled for object B only.

The findings also suggest that we have to be careful when students explain that it is the tension in the string which pulls object A. We should also not be persuaded by the correct direction of the vector arrow representing the tension force in the horizontal part of the string in the student's force diagram. If we look at the case of some students, say S9 in E5.1 (§ 5.3.1) and S6 in E5.4 (§ 5.4.3), we note that for them 'tension in the string pulls object A' was synonymous to 'object B pulls object A'; they argued that tension in the string was solely due to object B. It is also observed that during the activities two students, S3 in E5.2 (§ 5.4.1) and S6 in E5.4 (§ 5.4.3), had ultimately developed the idea that it was the net force acting on object B that pulled object A. Even from this perspective we find that these two students were convinced that object B was solely responsible for the motion of the system.

The second query 'Which object(s) creates tension in the string?' raises two interesting issues among the students: (a) whether or not tension in the string was equal to weight of object B; and, (b) whether there was one common or two tension forces in the string.

In relation to the first issue, the case of S3 in E5.2 (§ 5.4.1) is worth discussing here. His query "Why is tension equal to 9.807 N?" (line 247) demonstrates the veracity of his intuitive idea 'the tension in the string is solely due to the weight of object B' – an intuitive idea that was dominantly observed in all the ten participants. In turn, this intuitive idea did not allow students to realise that the tension in the string and the weight of object B were two forces away from the centre of object B. Though students have correctly drawn the two quantities, the tension in the string and the weight of object B, away from the centre of object B on their force diagrams, this does not mean that these students were aware that they were opposite to each other, and that the difference between them would be equal to the net force acting on object B.

In relation to the second issue, it was initially assumed that all the ten students would learn that there should be the same tension throughout the string. However, apart from S1, S3 and S6, the remaining students experienced conceptual difficulties with this mechanical idea. The cases of S2 in E5.3 (§ 5.4.2) and S9 in E5.1 (§ 5.3.1) support this observation. Interestingly, in both cases, we find that the students had the idea that the sliding object A created a tension force in the horizontal part of the string and the hanging object B exerted a different tension force in the vertical part of the string (refer to Figure 5.6). While this intuitive idea influenced S2 to assert that the tension in the string should be equal to the weight of the hanging object B, S9 developed it while trying to explain why the tension in the string was not equal to the weight of the object B. As such, it can be deduced from both cases that the intuitive idea 'having more than one tension force in the string' may be associated with the intuitive idea 'the tension in the string is solely due to the weight of object B'.

On the whole, I find that the above two inter-related issues converge to a central argument which is that in this study the students had the intuitive idea 'object A had no contribution in this system'. The example of S1 in E6.2 (§ 6.3.2) during the Prediction Phase is revealing; S1 thought that object A did not play any role in the motion of the system because its weight cancelled the normal reaction. For all the students, the only time object A played a role was when its mass was greater than that of the hanging object B such that it prevented the system from moving.

(c) Accelerations of Objects A & B

This category encompasses several intuitive ideas in relation to student understanding of accelerations of objects A and B.

Firstly, the results, as expected from § 2.3.5, show that six out of ten students predicted that both objects should move with different velocities and different accelerations. Students attributed the unequal velocities of both objects to the difference in their masses. In relation to the unequal accelerations of both objects, the main reason was because of the force of gravity. Since object B was in the air, students held the intuitive idea that force of gravity would act more on object B than on object A, which was supported by a horizontal surface. It is also found that being aware that both objects should move with the same velocity does not imply that a student would know that both objects should move with the same acceleration. Interestingly, from the findings it can be inferred that being aware that both objects move with the same velocity or acceleration is not a sufficient condition for students, say S3 in E5.2 (§ 5.4.1), to know that the motion of object A depends on object B and vice versa.

Secondly, as expected from § 2.3.5, it is found that six out of ten students predicted that object B should move with acceleration due to gravity g. The cases of S9 in E6.1 (§ 6.3.1) and S1 in E6.2 (§ 6.3.2) support this finding. For them, the force of gravity was the only factor that influenced the acceleration of object B, even though they were aware that both objects were connected. At the same time, it is observed that students had the school-based intuitive idea 'no object, which falls freely, can move with acceleration greater than g'. For them, falling freely also implied that there was no opposing force acting on the falling object. On this basis, some students, say S6 in E6.3 (§ 6.4.1) and S7 in E6.6 (§ 6.5.1), knew that when the string would break, tension in the string would become zero such that acceleration of object B would become g. It is also noted that at least one student, S7 in E6.6 (§ 6.5.1), might have the intuitive idea 'an object can only experience g when it falls and not when it moves up'.

Thirdly, it is of significant interest to note that five out of eight students, when the string was broken, and seven out of eight students, when the string became slack, reported that the change in the magnitude of acceleration of object B should be gradual, not instantaneous. Six out of eight students had the same intuitive idea about the acceleration of object A moving along a smooth surface when the string was broken. For them, it was not possible to think of an abrupt change in acceleration. The cases of S6 in E6.3 (§ 6.4.1) and E6.4 (§ 6.4.2), S5 in E6.5 (§ 6.4.3) and S8 in E6.7 (§ 6.5.2) support this finding. Since this intuitive idea was applied to both the vertical motion of object B and the horizontal motion of object A along the table, it seems to be applicable in all directions for students.

Fourthly, it is observed that six out of eight participants predicted that object A moving along a smooth horizontal surface should stop once its acceleration becomes zero. Similarly, seven out of eight students reported that when object B reached its peak, its acceleration should become zero where it stayed stationary momentarily. In this respect, the cases of S6 in E6.4 (§ 6.4.2) and S8 in E6.7 (§ 6.5.2) are worth discussing. Interestingly, in both cases, we find that the students stated that the motion of the object under study was only possible if there was a force acting on it. As such, it seems that these students held the intuitive idea 'motion implies force'.

Finally, it is student understanding of deceleration that retains considerable attention. For seven out of eight students, the deceleration process was synonymous to a decrease in the magnitude of acceleration, not a change in its direction. The case of S5 in E6.5 (§ 6.4.3) demonstrates the conceptual difficulty encountered by her just after the string had broken. According to her, object A had decelerated because she noted from the ACCELERATION meter a change from 1.717 m/s^2 to 0.981 m/s^2. In fact, she had analysed the $|A|$ component, not the A_X component, of the ACCELERATION meter. However, the real issue lies in the fact that she did not realise that her perception of deceleration from 1.717 m/s^2 to 0.981 m/s^2 should have implied that object A continued to accelerate, albeit with a lesser magnitude.

(d) Location/Position of Object A

The findings point out that for two students it was the location of object A that cued them whether or not the objects behaved as a connected particles system. For instance, in E6.7 (§ 6.5.2) S8 predicted that as long as object A was on the table, both objects had to behave as a connected particles system. It did not matter to her that the string had become slack when object B hit the ground.

Even the change in the 'straight line' to the 'curved line' of the string on the screen did not attract her attention. This suggests that the change in the physical property of the string from tautness to slackness is not a sufficient condition for some students to understand that both objects are no longer behaving as a connected particles system.

(e) Unexplained Real-life Intuitive Ideas

The results of this study point to the existence of some intuitive ideas that had been abstracted from real-life experiences and that students could not account for. This is mainly noted in the cases of S2 in E5.7 (§ 5.5.2) and S6 in E5.5 (§ 5.4.3). In the case of S2, he asserted on three occasions (lines 441; 505; 524) that the string might break when tension in the string would be equal to the weight of the hanging object. He could not justify his argument. Similarly, S6 in E5.5 (§ 5.4.3) could not explain why she had predicted that both objects, of equal masses, should have come to rest at the same level.

7.3.2 What role, if any, do these intuitive ideas play during the generation of new knowledge?

In this section I concentrate on the knowledge developed by the participants during and after their interaction with the software. So the major ideas have mainly come from the data collected during the Interaction and Reflection phases.

From the analysis in Chapters 5 & 6, I find that intuitive ideas had three key roles to play during the generation of new knowledge:

(1) They provided a context for interpretation and sense-making;
(2) They influenced the student's strategies on the software and whether or not to accept the solutions provided by the software; and,
(3) They generated unconventional knowledge in order to maintain basic intuitive ideas.

(a) A context for interpretation and sense-making

The findings show that intuitive ideas provided a window through which students could interpret existing and new mechanics situations, and could make sense of them. It did not matter to them whether the generation of new knowledge was unconventional or in the direction of expertise as long as it made sense to them.

In relation to unconventional knowledge generated, the case of all the ten participants predicting that there should be motion for the system mB > mA supports this observation. For instance, in E5.1 (§ 5.3.1) S9 applied his intuitive idea 'a heavier load can displace a lighter one' to explain the occurrence of motion of the system. As a matter of fact, it is not wrong to use this intuitive idea to start engaging with this situation, but it is not productive to use this idea to justify the motion mathematically.

As far as knowledge generated in the direction of expertise is concerned, we have two scenarios worth discussing. First, we have the case of students examining whether or not both objects would move with same velocity and/or same acceleration. In this matter, the case of S9 in E6.1 (§ 6.3.1) shows that he had the knowledge that object B should pull object A; what he did not know was how the motion of object B would affect that of object A. The readings on the VELOCITY meters enabled him to realise that B would pull A such that both objects should move at the same pace. Thus, his intuitive idea of 'pull' contributed in making him realise the key role played by the string in this situation. Second, we have the case of students using their school-based intuitive idea of net force to make sense of the forces acting on object B. In E5.2 (§ 5.4.1), S3 initially held the intuitive idea 'tension in the string was solely due to the weight of object B'. As such, he never realised that the tension in the string and the weight of object B were two forces acting away from object B. So, his intuitive idea hindered him from realising that he could use the difference between these two forces to explain motion. However, the computational environment propounded the idea of net force as a bridge between explaining motion and the relationship between tension and weight.

(b) Influencing Student’s Strategies and Solutions

Intuitive ideas had a direct impact on how students made use of Interactive Physics. The results of this study show that intuitive ideas influenced the learners in two broad ways: (a) what strategies to use in the simulation; and, (b) whether or not to accept the solutions produced by the software. In relation to the strategies adopted, it is found that there were correct ones as well as inappropriate ones. With regard to the solutions depicted by the software, it is observed that in several cases students accepted these solutions and in remaining cases students either challenged them or rejected them.

In relation to the appropriate strategies adopted, it is observed that intuitive ideas constrained their ‘owners’ to adopt a specific strategy on the simulation which ultimately led to meaningful learning. The example of S6 in E5.5 (§ 5.4.3) can shed some light on this matter. It is found that her intuitive idea ‘if we have 2 objects of equal masses, they should rest at the same level’ made her change the mass of object B so that she could deal with a system of unequal masses. By doing so, her initial intention was to verify whether it was “really a pulley system” as the system on the screen did not move. This strategy initiated a chain of events which ultimately led to meaningful learning as evidenced by her statement in lines 80 – 85.

With regard to the incorrect strategies adopted, it is found that intuitive ideas constrained their ‘owners’ to adopt a specific strategy on the simulation which did not prove to be productive. The cases of S9 in E5.1 (§ 5.3.1) and S2 in E5.3 (§ 5.4.2) provide compelling evidence for this category. Interestingly, in both cases, we find that the students held the intuitive idea ‘the tension in the string should be equal to the weight of the object B’. As such, this idea made S9 compare a wrong set of meters on two occasions. It would appear that S9 chose to work with the wrong set of meters (TENSION and ACCELERATION) as a means to deviate from the real issue of explaining the discrepancy of why the tension in the string was not equal to the weight of object B. In the case of S2, this intuitive idea made him examine whether or not the surface was smooth.

In connection with the solutions provided by Interactive Physics, it is observed that on several occasions intuitive ideas played a key role in making students accept these solutions. The example of S9 in E6.1 (§ 6.3.1) shows that his intuitive idea of 'pull', coupled with the readings from the VELOCITY meters, prompted him to almost immediately accept the idea that both objects should move with the same velocity.

Finally, those who did not agree with the solutions depicted by the software can be explained by the fact that their intuitive ideas were at the basis of these rejections. We have seen such rejections in several episodes. Let us highlight the cases of S9 in E5.1 (§ 5.3.1) and S6 in E5.4 (§ 5.4.3) who did not approve of the motion shown on the screen. Though both of them knew that the tension in the string could not be equal to the weight of object B in the system mB > mA, they did not agree with the motion of the system mB = mA. In the case of S9 only, he did not even agree with the motion of the system mA > mB.

These two ways of involving Interactive Physics may lead us to think how intuition may influence learning from it, in particular from the animation process. This will be discussed in Chapter 8.

(c) Generation of unconventional knowledge

The data analysis points out that intuitive ideas generated unconventional knowledge in order to protect and maintain initial ones. The examples of S9 in E5.1 (§ 5.3.1) and S2 in E5.3 (§ 5.4.2) can be used to support this finding. Interestingly, in both cases, we find that the students argued that the tension in the string should be equal to the weight of the object B. As such, this intuitive idea made S9 develop the idea that there must be two different tension forces in the string, one acting along the horizontal part and the other along the vertical part. It can also account for making S9 develop the idea that Interactive Physics displayed only the tension force along the vertical part of the string. In the case of S2, this intuitive idea made him construct the idea that there must be "some force acting on the pulley", most probably friction, which hindered the tension in the string to be equal to the weight of object B. This leads us to think about why some intuitive ideas are resistant to change. This will be addressed in the next chapter.

7.4 How do the uses of Interactive Physics influence the evolution of the learning process?

When this second research question was crafted in Chapter 2, the aims were two-fold: (a) to examine the features of the software which students had used during the learning process; and, (b) to explore how the software bridged the gap between students' intuitions and the new knowledge generated during the learning process. Based on the findings of this study, these two aims will now be discussed.

7.4.1 How are the affordances of Interactive Physics used in practice to develop understanding?

The data analysis suggests that the microworld provided a range of affordances that has enabled meaningful learning of motion of connected particles. These affordances were available because of the interactions between students' intuitive knowledge, interview questions and features of Interactive Physics. In this section, I focus on the key roles played by the features of Interactive Physics during their interactions with the other two components of the IP-based environment that provided the affordances. The findings reveal that animation, animation control, digital meters, vector arrows and interactive behaviour were the five main features.

Whenever the software was used, it is observed that the user was always greeted by the animation of the system under examination. From this study, we find that there were two distinct situations when the animation process could influence the learning process: first, when the student interacted with the software at the start of the Interaction Phase; and, second, when the student continued to interact with it during the Interaction Phase.

For the first situation, it was often a question of whether or not to agree with the motion depicted on the screen. From this perspective the animation became a window through which we could make the student's intuitive idea explicit. In a way, accepting the motion implied that the learner was able to make sense of the animation. This 'make sense' break-through encouraged the student to accept the readings generated by the digital meters and to, subsequently, construct plausible arguments to explain the behaviour of the system under study. As an example, the motions of the systems mA = mB and mA > mB in E5.2 (§ 5.4.1) and E5.6 (§ 5.5.1) respectively contributed in shifting S3's explanations from 'comparing masses of objects A and B' to 'comparing forces acting on object B' to explain the occurrence of motion. During this process, it is observed that the connection between 'accepting that there is motion' and 'making sense of the motion' was not a linear process.

The second situation took place when, after having interacted with the software, students predicted (idiosyncratic) hypotheses and tested them on the software. The case of S1 in E6.2 (§ 6.3.2) is such an example to support this finding. He initially hypothesised that any object, whether falling freely or falling under constraint such as attached to a string, should move with acceleration due to gravity g. After having interacted with the software for the first time and having discovered that his intuitive idea had not been correct, he developed an animation whereby, contrary to his expectation, it ultimately led him to learn that the hanging object B, when attached to the sliding object A, must move with an acceleration less than g.

Yet, it is also observed that though some animations contributed positively in the learning process, there were others who did not influence the learning trajectories. For instance, we find that the animation process failed to contribute in the learning process of these two cases: (a) in highlighting the key role played by the string in the system when both objects moved with the same acceleration and the same velocity (S9 in E5.1: § 5.3.1); and, (b) objects A and B no longer behaved as a connected particles system once the string became slack (S8 in E6.7: § 6.5.2). From this perspective, one important question that we may ask ourselves is why the animation failed to convey the message to most students that both objects moved independently once the string had become slack. Why was the change in the 'straight line' to the 'curved line' of the string on the screen not a sufficient condition for students to realise that? Through these findings, we may throw some light to understand why some animations can shape student learning more than others. This will be discussed in the next section.

The animation control of the software was another feature that was extensively used during the learning process. It is found that when the animation presented its content in a manner that was too quick to match to students' processing capacities, the animation control played a key part in allowing the students to adjust the way information was delivered to them. It is also seen that the contribution of the animation control was positive to the learning process only if the student was able to locate and identify those aspects within the animation that were relevant to the learning task. The example of S6 in E6.4 (§ 6.4.2) goes in this direction. While she agreed with the reading on the ACCELERATION meter such that it became zero when the string had broken, she did not agree with the motion of the object A. While the former was according to her expectation, the latter was contrary to her prediction. Nevertheless, it is likely that the feature of running the simulation on a number of occasions at her own pace contributed in making her notice the rate at which object A moved along the surface once the string had broken. This is why, without the use of the VELOCITY meter, she developed the idea that object A might be moving with constant speed, ultimately making sense of the zero acceleration.

The findings of this study also show that animation control proved to be of no value to the learning process when students lacked (relevant) intuitive knowledge in the topic under study. It is found that simply allowing students to use the animation control to make information more available to them was not a sufficient condition. These students required additional support within the animation so as to direct their attention where needed. The case of S5 in E6.5 (§ 6.4.3) provides sufficient evidence to support this claim. She confidently predicted that the acceleration of object A would decrease until it would become zero. When she ran the simulation on two occasions using the command RUN, she noted that acceleration changed from 1.717 m/s^2 to zero. As she did not learn anything new from this simulation, I had to make four interventions so that she could realise that the acceleration of object A was a constant between t = 1.5 s and t = 3.95 s – a conceptual idea that she had not been aware of. Among my interventions, one was about advising her to observe the motion just before and after the string would break by using slow motion, and another was about continuing to run the simulation in slow motion until object A stopped. As such, my interventions provided those cues that might partially account for the animation being finally able to shape student learning.

In most cases where students failed to abstract the relevant information from the animation process, the digital meter was another means through which students could make a clearer sense of the situation. The digital meter had the characteristics of focusing exactly at the point where and how change would be taking place. For the 'where' case, let us consider the example of S6 in E6.3 (§ 6.4.1) where she predicted that, once the string would be broken, the acceleration of object B would become g gradually as it would move down. However, with the presence of the ACCELERATION meter on the screen, she not only noted the sudden change in its acceleration at the point where the string had just broken, but she managed to construct a valid argument to justify the instantaneous change in acceleration. In relation to the 'how' case, it is found that successive use of the same meter in a number of consecutive simulations was a successful strategy for some students. To be more precise, the decimal places of the readings of these meters contributed to the learning process of the students. For instance, in E5.7 (§ 5.5.2) S2 worked with the TENSION and VELOCITY meters, which displayed readings to three decimal places. This feature allowed the student to observe that large increase in the mass of object A resulted in very small increment of the tension in the string, thereby, allowing him to develop the idea that the tension in the string had an upper limit.

In this study it is also found that digital meters were able to 'mislead' students in two ways that were linked to intuition: (a) to extract an incorrect value from a set of readings made available to them; and, (b) to refine an erroneous intuitive idea. In relation to part (a), it can be inferred that the extraction was influenced by the intuitive ideas of the students. For instance, in E6.5 (§ 6.4.3)

S5 focused on the modulus of acceleration, $|A|$, instead of the component of acceleration along the horizontal direction, A_X. With regard to part (b), it can be hypothesized that refinement of an erroneous intuitive idea in an unconventional way was possible when the students had access to multiple data that appeared to them to stem from the idea itself. The case of S3 in E5.2 (§ 5.4.1) is worth discussing here. When he worked with the system mB > mA, he brought the intuitive idea 'the weight of object B pulled object A' to the task. However, during his interaction with the system mB = mA, his thinking evolved such that he now suggested that the net force acting on object B must be transmitted via the tension in the string to pull object A. On this basis, he had the idea that the net forces acting on objects A and B, and the tension in the string should be of equal magnitude. By coincidence, both TOTAL FORCE meters had the same reading and when the TENSION meter was later added, it also had the same reading!

The fourth feature that was of importance was the vector arrow. From this study, there is no indication that when the vector arrow of a specific quantity was used standalone, it was able to contribute positively in the learning process. However, it 'became' a powerful learning tool if it was used in parallel with other representational tools such as the vector arrow of another quantity, the animation process or the digital meters. Though it had the ability to make abstract concepts (in this case forces and kinematical quantities) visible, it was not a sufficient condition to enable meaningful learning to take place. It made more sense to students when they could compare the vector arrow under study with another vector arrow of the same unit. The case of S5 in E6.5 (§ 6.4.3) shows that when the ACCELERATION vector arrow was added, it allowed the student to start engaging with the minus sign in the A_X component of the ACCELERATION meter. When the VELOCITY vector arrow was added, it allowed S5 to construct the idea that once the string had broken, the acceleration and velocity of object A could not be in the same direction during the deceleration process. This is why in the end she explained that the vector arrows were of more help to her.

The last feature that appeared to contribute to the learning process is the interactive behaviour of the software. The interactive nature of the software allowed students to change parameters and observe its effects. For this study, it was mainly to observe whether or not motion of the system was possible. The example of S2 in E5.7 (§ 5.5.2) shows that he varied the mass of the sliding object A almost fifteen times during this episode and observed its effects at a pace suitable to him. It is evident that he was not able to mentally animate the system under consideration since he claimed a 'no-motion' response – this means that his mental model was not well developed to help him understand this system. As such, the successive animations of Interactive Physics alongside the readings on the TENSION meter helped him develop his mental model. As the mass of the sliding object A was increased from 5 kg to 5000 kg, he was able to experience the effect of mass on the motion of the system.

7.4.2 How do these affordances help in the forging of new connections between intuitive knowledge and new knowledge?

In this section, I examine how the features identified in § 7.4.1 have influenced student learning of motion of connected particles. In this respect, my focus is on how these affordances supported students in bridging their intuitions to the new knowledge they had developed. In particular, I elaborate on the conditions and possibly the mechanisms under which bridging became successful.

In order for an animation to contribute in the learning process, students should be able to detect and extract task-relevant information during the animation's progress (Boucheix, 2008). From this perspective, we find that either students should look for this information or this information should be able to capture their attention, that is, it should become visible to them. It may be hypothesised that through visibility the bridging between intuitive knowledge and new knowledge could take place. It is at this period that we have either the activation of a desired idea or the development and integration of a desired idea such that meaningful learning may be expected to occur.

On this basis, we can raise the following question: When does the relevant information become visible to students? From the results, we find that there were several possibilities for it to become visible to them. However, I wish to consider some of those cases that will help us shed some light on this issue of visibility.

Firstly, I would like to use the pair of animations, E6.1 (§ 6.3.1) and E6.2 (§ 6.3.2), to zoom on the cases of S9 and S1 who held the intuitive idea ‘object B would fall with g’. Interestingly, we find that the animation in E6.1 did not contribute in student learning, whereas that in E6.2 did contribute. In the latter animation, unlike the one in E6.1, there was an additional object (Object D) which fell freely beside object B. So it appears that in E6.2 object D provided an extra visibility which lowered the cueing priority of the intuitive idea ‘object B must fall with g’ and developed a new mechanical idea ‘object B, when not falling freely, moves with acceleration less than g’.

Secondly, through the case of E6.7 (§ 6.5.2), the following question can be asked: Why was the animation not helpful at the start of the episode, but it shaped student learning at the end? It is true that the ACCELERATION meter during the early stage of the Interaction Phase enabled S8 to learn that the acceleration of object B became g upon impact. This, in turn, allowed her to construct a plausible argument to support her new finding. She realised that when the string became slack, tension became zero and object B behaved as an object falling freely. However, she failed to respond to my new query as to why the acceleration of object B had now changed to 4.953 m/s^2. At the end of the simulation, after having run it backward and forward several times and introducing the TENSION meter, she appeared very confident in applying her new idea. On this basis, I argue that the synergy between the readings on the TENSION and ACCELERATION meters, and the to-and-fro movement of the animation to make the string become taut and slack considerably increased the visibility of the relevant information ‘whenever the string is slack, tension becomes zero and object B moves with g’ such that it was extracted.

Thirdly, the ways in which the vector arrows were used by S3 in E5.2 (§ 5.4.1) and E5.6 (§ 5.5.1), and S5 in E6.5 (§ 6.4.3) can also provide us with interesting details. In E5.2, when he added the vector arrows **FT** (tension in the string), they were not of any help. However, when the vector arrows **FG** (weights of both objects) were put on, the perpendicular property between the vector arrows

FT and **FG** on object A made him questioned his equation of motion which he had written for object A. We note the same finding by S3 in E5.6. In E6.5, the vector arrow **A** (acceleration) on object A allowed the student to understand that the acceleration changed its direction when the string had broken. It also enabled S5 to make sense of the minus sign in the A_X component of the ACCELERATION meter. However, when the vector arrow **V** (velocity) on object A was added, it was the interplay between the arrows **V** and **A** that allowed her to develop the idea that once the string had broken, the acceleration and velocity of object A became opposite to each other. Interestingly, in the case of S3, the vector arrows **FT** and **FG** were static – they neither changed in direction nor in length; it was the sets of equations written by the student that caused the conceptual challenge and initiated the conceptual progress. In the case of S5, though the arrow A initiated her to make sense of the minus sign, it was the interplay between the arrows **A** and **V**, and the dynamic nature (change in both direction and length) of these arrows that allowed her to make sense of the situation at hand. So it can be seen that in the cases of both students when the visibilities of the relevant information were increased by additional support, meaningful learning took place.

Fourthly, I consider the case of S2 in E5.7 (§ 5.5.2). He was someone who did not agree with the motion of the system mA > mB and argued that tension in the string should always be equal to the weight of object B, be the system at rest or in motion. Though the TENSION meter already provided him with appropriate reading such that the information to be extracted could be considered visible, he was not convinced. So what made the difference? I argue that when he decided to vary the mass of object A and to observe its outcome, the relevant information started to make sense to him. The fifteen consecutive simulations were responsible to help him develop the relationship between the tension in the string and the weight of object B to account for motion. This was achieved when S2 discovered that the rate of increase of mass of object A was not proportional to the rate of increase of the tension in the string. He observed that large increase in the mass of object A resulted in very small increment of the tension in the string as well as made the system moved very slowly. In turn, these observations made him learn that the tension in the string would be equal to the weight of the hanging object if and only if the system was to be in equilibrium. So it can be claimed that the outcomes of the fifteen consecutive simulations greatly improved the visibility of the relevant information such that it increased the cueing priority of his new idea.

Fifthly, the learning trajectories of S6 in E5.4 and E5.5 (§ 5.4.3) can also contribute to the visibility debate. Her case is rather thought-provoking in the sense that she was well aware of the relationship between the tension in the string and the weight of object B to explain motion and at rest for the system mB > mA. However, when she analysed the system mB = mA, she did not agree with the motion though she found via the meters that the weight of object B was greater than the tension in the string. So I argue that agreeing that motion is possible appears to be the stepping stone to allow meaningful learning to occur. The visibility of the readings of the TENSION and GRAVITATIONAL FORCE meters was not enough to lower S6's erroneous intuitive idea. From this perspective, E5.5 provided S6 with enough working space to verify her intuitive idea. It is seen that when she increased the mass of object B, this set the system in motion. I argue that this motion improved the visibility of the relevant information such that it increased the cueing priority of her new idea.

The final case I wish to examine is S9 in E5.1 (§ 5.3.1). He managed to develop the correct relationship between the tension in the force and the weight of object B for the system mB > mA. Similar to S6 in E5.4 (§ 5.4.3), he also found that the relationship between tension and weight held true for the system mB = mA, but did not agree with the motion. Again he did not agree with the motion for the system mA > mB. He held the surface for being accountable for the motions of the systems mA = mB and mA > mB. Even though friction was added to the system, the system continued to move and contrary to his prediction, weight continued to be not equal to the tension. On this basis, he developed the idea that there should have been two tension forces in the string, but that Interactive Physics was showing only one. In this episode, in a way of speaking the computer-based setting was able to refute all his arguments except the last one. I argue that it would have altered his learning trajectory if the option of measuring the tension force at any point of the string would have been available.

The question that we may ask ourselves is the following: Were the relevant information not visible to these students when they did not agree with the motions or solutions provided by Interactive Physics? We find that in all the cases where the students (for example S6 in E5.4 and E5.5) were able to verify all their erroneous intuitive ideas connected to a certain ‘concept’, we observed the occurrence of meaningful learning. However, in those cases where students (for example S9 in E5.1) could not verify theirs, we observed no meaningful learning or at worst, the generation of unconventional knowledge. This leads us to think about one crucial point which is whether forging between the new ideas and the intuitive ideas depends solely on the visibility of relevant information to be extracted. This will be addressed in the next chapter.

7.5 Concluding Remark

In this chapter the findings of this study have been used to address the research questions. We saw how students learned about motion of connected particles with Interactive Physics. We isolated the intuitive ideas that students brought to the tasks; we not only examined how students constructed new ideas from their intuition, but also how the computational tool shaped these ideas. In the following chapter, we aim to broaden these findings by situating them in the literature review as well as examining them from a theoretical perspective.

Chapter 8

Conclusion

8.1 Introduction

In this closing chapter, I start by summarising the origins, stating the aims and discussing the findings of this research study. In § 8.3, I shed some light on the interventions I had to make during the learning process. In § 8.4, I explore the limitations of this study and state its implications for further research in § 8.5. While the implications for teaching are elaborated in § 8.6, an upshot finally concludes this work in § 8.7.

8.2 Origins, Aims and Findings

This study originated because of two main observations. First, when I observed the animation aspect of Interactive Physics, I was convinced that it would certainly help students learning mechanics at GCE A-Level Mathematics because it could capture their attention. Second, when I encountered in the literature the statement that intuitive 'erroneous' ideas could easily be replaced by scientifically 'correct' ideas, my experience as a learner and my interactions with students completely rejected this over-simple conclusion.

As such, this study was designed to examine how students learn motion of connected particles in an Interactive Physics environment. I had two main research questions (RQ) and two sub-questions for each RQ:

RQ1: How is intuitive knowledge used by students in learning motion of connected particles?

- What intuitive ideas do students bring to motion of connected particles?
- What role, if any, do these intuitive ideas play during the generation of new knowledge?

RQ2: How do the uses of Interactive Physics influence the evolution of the learning process?

- How are the affordances of Interactive Physics used in practice to develop understanding?
- How do these affordances help in the forging of new connections between intuitive knowledge and new knowledge?

In Chapter 2, after having expounded on five theoretical approaches in relation to conceptual change, I argue that diSessa's knowledge-in-pieces (k-i-p) approach would be the most appropriate one to analyse and discuss student learning in my work. The selected episodes in chapters 5 & 6 provide ample evidence in support of the k-i-p perspective and the findings will now be discussed according to each sub-RQ.

- **What intuitive ideas do students bring to motion of connected particles?**

In § 7.3.1, I discuss several intuitive ideas that students brought to the tasks. In this section, I wish to draw a link between them and diSessa's p-prims, to dwell on a common theme emerging across the intuitive ideas and to discuss the nature of these intuitive ideas.

I find that the intuitive ideas observed in this study correspond at least at some level to the p-prims previously described by diSessa (1993). They are comparable in the sense that the intuitive ideas seem to be minimal abstractions from real-life experience, are simple knowledge elements and are activated by the situation in question. Let us consider the intuitive idea 'change in acceleration should be gradual' which in fact corresponds quite closely to 'change takes time' p-prim, also called 'warming up' p-prim by diSessa. This p-prim involves the notion of 'building up' such that "it takes some time for any result quantity to reach its final value when a change in impetus takes place" (diSessa, 1993: p. 133). In this study, it accounts for student reasoning in relation to the idea that once the string is broken or becomes slack, the change in the magnitude of acceleration of either object B or object A should be gradual, not instantaneous. This finding is in accordance with Saldarriaga (2011) who finds that students "exhibit awareness about the need for time to pass for certain changes to occur" (p. 41). According to Lemmer (2013), this p-prim is "formed implicitly in every-day life experiences by repeated observations of moving objects" (p. 258).

We also find that the intuitive ideas observed in this study, similar to p-prims, cannot be accounted for by the students explicitly. The case of S6 in E5.5, who stated "I don't know why I said that" (line 49) and "I thought that it was like that" (line 52), demonstrates that students themselves may not be aware of the origins of their own idiosyncratic ideas and that pressing them to explain such ideas may not be informative for the researcher. As Mestre (2002) puts it, they are "taken as self-evident truths without need for explanation" (p. 14).

Interestingly, when we look at parts (a), (b) & (c) of § 7.3.1, we find a common theme emerging across them: context dependency – a crucial component in the k-i-p framework. In this study the intuitive knowledge that students brought to the learning tasks appear to depend on the context of the situation. The students' responses were determined by the contextual cues of the situation in the learning tasks. As an example, for all the ten students, motion of the system would only be possible if (and only if) the mass of object B were greater than that of object A. In this case, the value of the mass of object B as compared to that of object A was the determining factor for the occurrence of motion. As such, students held the intuitive idea that the sliding object A could play only 2 roles, depending on the context. First, if object A was heavy, that is, its mass was equal or greater than that of object B, then it was a burden for object B and would not allow the system to move. Second, if object A was light, that is, its mass was less than that of object B, it had no influence on the system and it did not contribute in creating the tension in the string. This is why for this group of learners 'the tension in the string must always be equal to the weight of the hanging object B', an intuitive idea already mentioned by diSessa (1993), Mestre (2002) and McDermott, Shaffer & Somers (1994).

The findings of this study also point out to a second possible reason for the existence of the above intuitive idea, a reason which is similar to the view of Mestre (2002). According to him, the physical arrangements of the objects invite the student to interpret that the gravitational force on object B is simply transmitted to the attached string. In this study, we find that since object B hangs in the air as compared to object A which is supported by a surface, students assumed that the ‘force of gravity’ acted solely on object B such that its acceleration had to be g. As such, it may be deduced that the ‘force of gravity’ idea decreases the cueing priority of the role of the string in this aspect.

In relation to the nature of intuitive ideas observed in this study, the findings appear to show that the knowledge structure is loosely connected. It is very likely that students’ explanations were constructed on the spot in direct response to the cues on the screen, on paper and in the interview questions. As an example, in E5.2 S3 initially predicted that the system mA = mB would not move as the weights of objects A and B were equal; however, when he saw that motion was possible, he developed an argument which involved the forces R and W acting on object A to justify the motion of the system. This is in accordance with the findings of Southerland et al. (2001) who demonstrate that students construct understandings and generate explanations based upon the intuitive ideas activated by the cues in action. Furthermore, we note in several episodes, say E5.1, E5.2 and E5.4, shifting patterns of explanation by students – in these instances, the students had conflicting views about what factors were responsible for the creation of tension in the string. In such cases, it is interesting to observe that had the students adopted the ‘theory-like’ approach, we should have seen greater consistency in their explanations. This suggests the influence of cues on student explanations, that is students construct explanations depending on what they find to be of relevance at that particular moment.

- **What role, if any, do these intuitive ideas play during the generation of new knowledge?**

In § 7.3.2, I explain that in this study intuitive ideas play 3 key roles during the learning process. In this section, I intend to discuss two findings in relation to: (1) when an intuitive idea is useful or problematic; and, (2) whether an intuitive idea can be replaced, thereby, exploring the idea of someone possessing a concept.

The results demonstrate that intuitive ideas are not always an obstacle to learning. In fact, they can become useful resources for the construction of new knowledge in various ways. We find that they can influence their ‘owners’ to view systems in specific ways, can activate other mechanics ideas and can help direct attention to the appropriate part of an animation. The results also indicate that there are instances when intuitive ideas become refined so that students can make ‘better’ sense of a particular situation. The refinement occurs when one or more mechanical ideas are developed during the learning process; these ideas enable learners to understand the condition(s) under which the intuitive idea become useful. The notion of ‘pull’ in E6.1 is an example worth recalling.

Interestingly, similar to the findings of Saldarriaga (2011), it is also found that in this study an intuitive idea can be useful or problematic, depending on how it is used by the learner. For instance, the ‘comparing masses of objects A & B’ intuitive idea is useful in the sense that it allows the student to become aware of the possible direction of motion of the system; however, it is problematic in the sense that it is not sufficient to gauge whether or not motion will occur. To refine this intuition, the student needs to analyse the forces acting on each object. Similarly, the ‘weight of object B being solely responsible for the tension in the string’ intuitive idea is useful to the learner since there will not be any tension in the string if there is no object B; it is problematic in the sense that it is not entirely responsible for the tension when the system is in motion. To refine this intuition, the student needs to become fully aware of the role of the string. The issue of refining intuition has been advocated by Sherin (2006) who explains that intuition need not be refined completely. So the question that arises from this is how learners will know up to what extent refinement is possible. One possible answer is that refinement may ‘stop’ once the relevant information to be extracted allows learners to make sense of the system under study.

Now we come to the point where we will try to answer the question of whether or not ‘erroneous’ intuitive ideas can be replaced during the generation of new knowledge. The findings show the complexity of the interaction between intuitive knowledge and formal knowledge. We know, for instance, that formal knowledge works across context. This cannot be said to be true for intuitive knowledge. In the previous section, we talked about the influence of contextual cues in shaping student explanations, thus paving the way for the argument that intuitive ideas cannot be replaced as suggested by other researchers. In fact, the limitation of intuitive knowledge illustrates just how difficult it is for someone to have a concept or not. Based on the work of diSessa (2002), I raised this question in Chapter 2: What does it mean if it is said that a student understands a concept?

This study suggests that it is not easy to determine whether or not someone 'possesses' a concept. Let us consider the relationship between the tension in the string and the weight of the hanging object B to explain motion as a concept. We find that, for the system $mB > mA$, students developed the relationship that for motion to occur the weight of object B must be greater than the tension in the string. Logically speaking, students, who had developed this concept, should have been able to apply it to explain motion occurring in the systems $mA = mB$ and $mA > mB$. However, we observe that this was not the case. In several cases, students were not only amazed at seeing the motions, but they also refused to accept these motions. This implies that students readily accepted the validity of the relationship in the case of the system $mB > mA$, but did not accept that the same relationship could hold true for the systems $mA = mB$ and $mA > mB$. It is very likely possible that since they agreed that there should be motion for the system $mB > mA$, they also accepted the relationship. But where they did not accept the motion, they rejected the relationship. In this case, it is found that possessing a concept is inter-related with agreeing that something is possible or not. Interestingly, these results appear to follow the ideas discussed in the paper by Levrini & diSessa (2008) who adopt a coordination class view of conceptual change in their work. For them, "knowing a concept is being able to use it in a broad range of circumstances." (p. 010107-4). This is why they, similar to Parnafes (2007) and Pratt & Noss (2002), find that while students begin to use the concept in some contexts, they may not able to apply it successfully in others.

- **How are the affordances of Interactive Physics used in practice to develop understanding?**

In § 7.3.3, I discuss the strengths and weaknesses of 5 features of Interactive Physics that students used during the learning process. Based on this discussion, I now present some findings in relation to: (1) strategies adopted during the learning process; (2) how intuition influences learning from Interactive Physics; and, (3) whether the visual aspect on its own can contribute positively.

The findings reveal the use of 'strategic apertures' by students in their zeal to learn about motion of connected particles. According to Hoyles & Noss (1992), strategic apertures are strategies that become available because of technology and that are not available with paper and pencil. In this study, we find six such strategies: (1) digital meter to provide a specific reading at a particular time in the animation; (2) the accuracy of the value on the digital meter as one parameter is varied; (3) testing of idiosyncratic hypotheses; (4) showing the possibility of motion via animation; (5) overlay of vector arrows on moving objects; and, (6) adjusting the pace of animation to suit learner's need. In each case, it appears that the immediacy of the feedback generated by Interactive Physics allowed the students to develop mechanical ideas so that they could ultimately develop their mental animations or mental models by filling in the lack of appropriate knowledge.

It is found that the learning process undertaken in an Interactive Physics based setting is affected by the intuitive knowledge which learners bring with them. It is observed that intuitive ideas played a central role in: (a) enabling learners to develop arguments to accept motions of the systems $mA = mB$ & $mA > mB$, and accelerations of objects A & B; and, (b) guiding learners to extract relevant information from the simulations. The latter is in agreement with the results of Hegarty & Kriz (2008) which also find that intuition may affect how well learners can extract information from animated displays.

The results also confirms two things. First, 'erroneous' intuitive knowledge can misdirect the learner's visual attention to features of the simulation that are highly salient but not relevant to the learning process. This is in line with the findings of Lowe (2003). The cases of S9 in E5.1 and S2 in E5.3 are worth recalling. Second, when learners lack intuitive knowledge, they are not able to realise what information they have to detect and extract even though they can control the pace of the animation. This is in accordance with Boucheix (2008) who suggests that cues can help students in their learning process. In this study, my interventions acted as cues to direct learners' attention towards the crucial region of depiction and to focus upon what needed to be detected and extracted.

My initial assumption was that the animation feature of the software (on its own) would contribute significantly to the learning process. The findings suggest the contrary, as reported in § 2.4.3. Be it the motion of the system mA > mB in Chapter 5 or the slackness of the string in Chapter 6 on the screen, none could immediately decrease the cueing priorities of the respective intuitive ideas. It is through other features of the software and interview questions that students managed to build meaningful learning. In a manner of speaking, meaningful learning appeared to occur when a chosen feature of a phenomenon was shown by multiple representations. As such, the findings of this study are in accordance with Hegarty & Kriz (2008) who explain that animation should be employed as part of a setting which involves other features so that students can choose what is relevant for them at the crucial point of learning.

- **How do these affordances help in the forging of new connections between intuitive knowledge and new knowledge?**

In accordance with the findings of Lowe (cited in Boucheix, 2008), this study finds that forging of new connections occurs when students identify and extract relevant information from the crucial region of the simulation at the appropriate time, and manage to integrate it into their mental model, resulting in 'good' knowledge structure. But we also discover that before students integrated new mechanical ideas, they had to make sense of (or interpret) them and agree with them.

In § 7.4.2, we ask the question of whether forging of new connections depends solely on the visibility of the relevant information to be extracted. The findings suggest that this ‘visibility’ condition is not a sufficient one. We find that the more visible the relevant information to be extracted becomes in the computer setting, the higher its cueing priority becomes and the easier its extraction occurs, thereby, deepening the forging between the new knowledge and the intuitive one. This implies that we should not rely on the ‘visibility’ condition as a means to dethrone erroneous intuitive ideas, but on the ‘degree of visibility’. I propose that the greater the ‘degree of visibility’ of the relevant information to be extracted (that is, the more visible the information becomes), the deeper the possibility of forging between the new knowledge and the intuitive one.

Yet, we did not see ‘successful’ forging of new connections in all cases of this research. This leads us back to some old questions. Why is it that some concepts are learnt more easily? Why do some people learn some concepts almost effortlessly? Why are some intuitive ideas resistant to change? I believe that in a simulation environment these three questions converge on the same idea: When and why can a simulation such as Interactive Physics shape student learning of a mechanics domain such as motion of connected particles?

We observe that during the construction process, say of whether or not there is motion, the connection between ‘accepting the mechanics idea (there is motion)’ and ‘interpreting the mechanics idea (why there is motion)’ was not a linear process. That is, each one depended on the other. So this means that the ‘degree of visibility’ of the relevant information to be extracted depends on this dual relationship of accepting-interpreting. Unless this is achieved, dissatisfaction with existing ‘erroneous’ intuitive idea will not happen such that no successful forging is possible. The findings indicate that neither cognitive conflict nor visibility alone are sufficient to allow meaningful learning to occur.

8.3 Researcher Intervention

I make no claim that the findings here are independent of my interventions although they were kept to the role as described in § 3.2.1. Using the framework of Hoyles & Sutherland (1989), which has been discussed in § 4.4, I now briefly comment on some of my interventions during the learning process. I also elaborate on how the interviews might have influenced the interviewer-expertise.

My interventions were needed when students had almost come to a dead end in their learning journeys and/or were frustrated. I intervened in three different ways with the intention to increase the richness of the task solving behaviour. First, an example of the Motivational type was to encourage learners to verify their hypotheses. The example of S1 in E6.2 shows that supporting students to test their hypotheses can have potential pay-off to the learning process. Second, encouraging students to reflect on whether or not they agreed to the readings and motions depicted by Interactive Physics can be regarded as an example of the Reflection type. The case of S6 in E6.3 suggests that we should not take students at face value when they appear to agree with the outcomes of a simulation. Third, understanding which part of the simulation to closely analyse has been a very crucial step in the learning process. In this study, directing student attention to the required part of the simulation is an example of the Directional type of intervention. The case of S5 in E6.5 demonstrates that my interventions acted as cues to direct her attention towards the crucial region of depiction and to focus upon what needed to be detected and extracted.

The interviews enriched my interviewer-expertise in two major ways: (a) refining my repertoire of questions; and, (b) making sense of student thinking process. In relation to the first one, the literature review and the pilot interviews helped me create the initial interview script for the tasks. As I interviewed students during the main study, the interview script was enhanced such that my repertoire of interview questions became refined. My ability to ask precise questions was also enhanced. From the case of S2 in E5.3, for example, I learnt how difficult it was to keep track of the student’s intuitive ideas when we had situations with both smooth and rough surfaces at the beginning of the Prediction Phase – this appeared to confuse S2 at times. In this case, I learnt not to bring in too much variables and relationships at one go during the interviews. In relation to the second one, I became more aware of potential student thinking process. This allowed me to raise relevant questions right from the beginning of tasks. For instance, I had to bring a slightly different approach to students who had already learnt motion of connected particles as compared to those who had not.

8.4 Limitations of Study

It is clear that this research has some limitations:

(1) The number of students I had interviewed was small. However, despite this, the data enabled me to focus on how students used their intuitive ideas to generate new knowledge in the computer-based environment of Interactive Physics. Based on the findings of this study, a larger study could now be explored.

(2) The length of the time the students spent using the system was relatively modest. However, in spite of the limited amount of time spent using the system, this did at least serve to shed light on the need for time in experiments of this type.

8.5 Implications for Future Research

This research has raised some interesting suggestions in relation to further research:

(1) This study finds that the notion of 'tension in the string' can create misunderstanding for many students at different stages of their learning process. We need not forget that it plays a key role in student understanding of connected particles. As such, it is recommended that more studies are designed to probe into student perception of tension in the string and its relation with other variables in the system.

(2) The results indicate that in the case of some students the geometry of the system was an issue. We find that they made use of the Vertical version of connected particles to make sense of the Horizontal version (the system under study). So, we can ask if students were made to interact, first, with the Vertical version, and then with the Horizontal version, would we get the same set of intuitive ideas? Further exploration in this area is needed to complement or nuance the findings of this study.

(3) Terminal velocity has been an idea suggested by various participants in this study in relation to object B moving down. It would be appropriate to understand the reason(s) behind such reasoning on the part of students especially when the distance of object B above ground is very short.

8.6 Implications for Teaching

Though this research had not involved any teaching aspect, the findings provide some interesting possibilities in relation to the teaching of motion of connected particles:

(1) The topic 'Motion of Connected Particles' is taught to students as an application of Newton's second law of motion. With Interactive Physics, it may be possible to discuss with students the conditions required for the system to be at rest or moving freely. This changes the approach to learning motion of connected particles. For example, how does the mass of either object affect the motion of the system?

(2) If students have to understand the relationship between the tension in the string and the weight of the hanging object B, they need to realise that the sliding object A contributes to setting the tension in the string. So, activities on the role of object A may be discussed with students before allowing them to work with the algebraic equations of the system.

(3) In this study, it is found that students did not understand the meaning of the vector arrows in force diagrams they had drawn; they are suggestive of rote learning. So activities must be designed along this direction.

(4) It is noted that students had a poor grasp of the notions of slackness and tautness of a string in relation to its tension. It is imperative that students be given sufficient opportunities through appropriate activities to understand these basic mechanics ideas.

8.7 Upshot

This study confirms that students do come to mechanics classes with intuitive ideas which can hinder or promote the learning process. Encouraging students to become aware of these ideas is useful, but designing ways to engage with these ideas is what we require the most. The refinement of intuitive ideas seems to depend largely on the quality of the interaction between students and their learning environments. In this case, the software Interactive Physics became the connecting point between intuitive ideas, interview questions and formal mechanics. It is observed that the animation process, on its own, cannot always promote meaningful learning. However, when it is used in conjunction with other tools, its participation may become productive. Finally, similar to the findings of Swanson (2015), this study reveals that the learning process can involve several micro-shifts in student thinking. We do not observe learning as a gestalt switch in which students radically change their ways of thinking.

References

Adams, W.G. (1872?) On the study and teaching of mechanics in: Lectures on education delivered before the Members of the College of Preceptors. London, C.F. Hodgson & Son.

Andaloro, G., Bellomonte, L. & Sperandeo-Mineo, R.M. (1997) 'A computer-based learning environment in the field of Newtonian mechanics', International Journal of Science Education, 19, 6, p. 661 – 680.

Arons, A.B. (1979) 'Cognitive level of college physics students', American Journal of Physics, 47, 7, p. 650 – 651.

Arons, A.B. (1997) Teaching Introductory Physics. New York, John Wiley & Sons.

Ausubel, D.P (1968) Educational Psychology: A Cognitive View. New York, Holt, Rinehart and Winston.

Baxter, P. & Jack, S. (2008) 'Qualitative Case Study Methodology: Study Design and Implementation for Novice Researchers', The Qualitative Report, 13, 4, p. 544 – 559.

Berry, J. (1990) 'Development in Mechanics Teaching', Teaching Mathematics and its Applications, 9, 3, p. 118 – 123.

Berry, J. & Graham, T. (1991) 'Using concept questions in teaching mechanics', International Journal of Mathematical Education in Science and Technology, 22, 5, p. 749 – 757.

Berry, J. & Houston, K. (1995) Mathematical Modelling. London, Edward Arnold.

Berry, J., Savage, M. & Williams, J. (1989) 'Mechanics in decline?', International Journal of Mathematical Education in Science and Technology, 20, 2, p. 289 – 296.

Betrancourt, M. (2005) The Animation and Interactivity Principles in Multimedia Learning (Chapter 18) in: Mayer, R.E. (ed) The Cambridge Handbook of Multimedia Learning. New York, Cambridge University Press.

Betrancourt, M. & Tversky, B. (2000) 'Effect of computer animation on users' performance: a review', Le Travail Humain, 63, 4, p. 311 – 329.

Biemans, H.J.A. & Simons, P.R.J. (1999) Computer-Assisted Instructional Strategies for Promoting Conceptual Change in: Schnotz, W., Vosniadou, S. & Carretero, M. (eds) New Perspectives on Conceptual Change. Amsterdam, NL, Pergamon.

Binongo, J.N.G. (1995) 'Why I Teach the Way I Do: A Statement of My Educational Philosophy', Teaching Mathematics and Its Applications, 14, 3, p. 107 – 110.

Bodemer, D., Ploetzner, R., Feuerlein, I. & Spada, H. (2004) 'The active integration of information during learning with dynamic and interactive visualisations', Learning and Instruction, 14, p. 325 – 341.

Boero, P. et al (2002) Abstraction: Theories about the emergence of knowledge structures in: Cockburn, A.D. & Nardi, E. (eds) Proceedings of the 26th annual conference of the International Group for the Psychology of Mathematics Education Volume 1. Norwich, PME.

Boucheix, J.M. (2008) Young Learners' Control of Technical Animations in: Lowe, R. & Schnotz, W. (eds) Learning with Animation. New York, Cambridge University Press.

Boucheix, J.M. & Guignard, H. (2005) 'What animated illustrations conditions can improve technical document comprehension in young students? Format, signalling and control of the presentation' European Journal of Psychology of Education, 20, 4, p. 369 – 388.

Brown, D.E. & Hammer, D. (2008) Conceptual Change in Physics in: Vosniadou, S. (ed) International Handbook of Research on Conceptual Change. New York & London, Routledge.

Bryce, T. & MacMillan, K. (2005) 'Encouraging conceptual change: the use of bridging analogies in the teaching of action-reaction forces and the 'at rest' condition in physics', International Journal of Science Education, 27, 6, p. 737 – 763.

Burgess, R.G. (1984) In the Field: An Introduction to Field Research. London, Allen & Unwin.

Burns, S. & Smart, T. (1988) Sixth-Form Girls Using Computers to Explore Newtonian Mechanics in: Hoyles, C. (ed) Girls and Computers. London, Institute of Education.

Caravita, S. (2001) 'Commentary: A re-framed conceptual change theory?', Learning & Instruction, 11, p. 421 – 429.

Caravita, S. & Hallden, O. (1994) 'Re-framing the problem of conceptual change', Learning & Instruction, 4, p. 89 – 111.

Carey, S. (1985) Conceptual Change in Childhood. Cambridge, Bradford Books, MIT Press.

Carey, S. (1986) 'Cognitive Science and Science Education', American Psychologist, 41, 10, p. 1123 – 1130.

Carey, S. (1991) Knowledge acquisition: enrichment or conceptual change? in: Carey, S. & Gelman, R. (eds) The Epigenesis of Mind: Essays in Biology and Cognition. Hillsdale, NJ, Erlbaum.

Carey, S. (1999) Sources of Conceptual Change in: Scholnick et al. (eds) Conceptual Development: Piaget's Legacy. New Jersey, Lawrence Erlbaum Associates Publishers.

Carson, G.St.L. (1913) 'Intuition', The Mathematical Gazette, 7, p. 60 – 71.

Champagne, A., Klopfer, L. & Anderson, J. (1980) 'Factors influencing the learning of classical mechanics', American Journal of Physics, 48, 12, p. 1074 – 1079.

Chi, M.T.H. (1992) Conceptual change within and across ontological categories: Examples from learning and discovery in science in: Giere, R.N. & Feigl, H. (eds) Cognitive models of science (p. 129 – 186). Minneapolis, University of Minnesota Press.

Chi, M.T.H. (1997) Creativity: Shifting Across Ontological Categories Flexibly in: Ward, T.B., Smith, S.M. & Vaid, J. (eds) Creative Thought: An investigation of conceptual structures and processes (p. 209 – 234). Washington, American Psychological Association.

Chi, M.T.H. (2005) 'Commonsense conceptions of emergent processes: Why some misconceptions are robust', Journal of the Learning Sciences, 14, p. 161 – 199.

Chi, M.T.H. & Slotta, J.D. (1993) 'The ontological coherence of intuitive physics', Cognition & Instruction, 10, 2&3, p.. 249 – 260.

Chi, M.T.H. & Roscoe, R.D. (2002) The processes and challenges of conceptual change in: Limon, M. & Mason, L. (eds) Reconsidering Conceptual Change: Issues in Theory & Practice. Dordrecht, Kluwer Academic Publishers.

Chi, M.T.H., Slotta, J.D. & Leeuw, N. de (1994) 'From things to processes: a theory of conceptual change for learning science concepts', Learning and Instruction, 4, p. 27 – 43.

Chiou, G.L. & Anderson, O.R. (2010) 'A study of undergraduate physics students' understanding of heat conduction based on mental model theory and an ontology-process analysis', Science Education, 94, p. 825 – 854.

Chiu, M.H., Chou, C.C. & Liu, C.J. (2002) 'Dynamic Processes of Conceptual Change: Analysis of Constructing Mental Models of Chemical Equilibrium', Journal of Research in Science Teaching, 39, 8, p. 688 – 712.

Clement, J. (1982) 'Students' preconceptions in introductory mechanics', American Journal of Physics, 50, 1, p. 66 – 71.

Clement, J. (1994) Use of Physical Intuition and Imagistic Simulation in Expert Problem Solving in: Tirosh, D. (ed) Implicit and Explicit Knowledge: An Educational Approach [Human Development, Volume 6]. Norwood NJ, Ablex.

Cohen, L. & Manion, L. (1994) Research Methods in Education (4th Edition). London, Routledge.

Collins, W.D. (1988) 'The Mechanics in Action Project', Bulletin of the Institute of Mathematics and its Applications, 24, 1, p. 2 – 6.

Crighton, D.G. (1985) Why Mechanics? In: Orton, A. (ed) Studies in Mechanics Learning. Leeds, The University of Leeds.

Crutcher, R.J. (1994) 'Telling What We Know: The Use of Verbal Report Methodologies in Psychological Research', Psychological Science, 5, 5, p. 241 – 242.

Dagher, Z.R. (1994) 'Does the Use of Analogies Contribute to Conceptual Change?', Science Education, 78, 6, p. 601 – 614.

Demastes, S.S., Good, R.G. & Peebles, P. (1996) 'Patterns of Conceptual Change in Evolution', Journal of Research in Science Teaching, 33, 4, p. 407 – 431.

diSessa, A.A. (1982) 'Unlearning Aristotelian Physics: A Study of Knowledge-Based Learning', Cognitive Science, 6, p. 37 – 75.

diSessa, A.A. (1983) Phenomenology and the Evolution of Intuition in: Gentner, D. & Stevens, A.L. (eds) Mental models. Hillsdale NJ, Lawrence Erlbaum.

diSessa, A.A. (1988) Knowledge in Pieces in: Forman, G. & Pufall, P.B. (eds) Constructivism in the Computer Age. Hillsdale NJ, Lawrence Erlbaum.

diSessa, A.A. (1993) 'Toward an Epistemology of Physics', Cognition & Instruction, 10, 2&3, p. 105 – 225.

diSessa, A.A. (1993b) 'Responses to commentary', Cognition & Instruction, 10, 2&3, p. 261 – 280.

diSessa, A.A. (1996) What do "just plain folk" know about physics? In: Olson, D.R. & Torrance, N. (eds) The handbook of education and human development: New models of learning, teaching and schooling. Oxford, Blackwell Publishers.

diSessa, A.A. (2002) Why "conceptual ecology" is a good idea in: Limon, M. & Mason, L. (eds) Reconsidering Conceptual Change: Issues in Theory & Practice. Dordrecht, Kluwer Academic Publishers.

diSessa, A.A. (2004) Contextuality and coordination in conceptual change in: Redish, E. & Vicentini, M. (eds) Proceedings of the International School of Physics "Enrico Fermi" Research on physics education (p. 137 – 156). Amsterdam, ISO Press/Italian Physics Society.

diSessa, A.A. (2006) A History of Conceptual Change Research: Threads and Fault Lines in: Sawyer, R.K. (ed) The Cambridge Handbook of the Learning Sciences. New York, Cambridge University Press.

diSessa, A.A. (2008) A Bird's-Eye View of the "Pieces" vs. "Coherence" Controversy (From the "Pieces" Side of the Fence) in: Vosniadou, S. (ed) International Handbook of Research on Conceptual Change. New York & London, Routledge.

diSessa, A.A & Sherin, B.L. (1998) 'What changes in conceptual change?', International Journal of Science Education, 20, 10, p. 1155 – 1191.

diSessa, A.A & Wagner, J. (2005) What coordination has to say about transfer in: Mestre, J. (ed) Transfer of learning: Research and perspectives. Greenwich, CT, Information Age Publishing.

diSessa, A.A., Gillespie, N. & Esterly, J.B. (2004) 'Coherence versus fragmentation in the development of the concept of force', Cognitive Science, 28, p. 843 – 900.

Doerr, H.M. (1994) A model building approach to constructing student understandings of force, motion and vectors. Unpublished PhD thesis. Cornell University.

Doerr, H.M. (1997) 'Experiment, simulation and analysis: an integrated instructional approach to the concept of force', International Journal of Mathematical Education in Science and Technology, 19, 3, p. 265 – 282.

Donmoyer, R. (1990) Generalizability and the Single-Case Study in: Eisner, E.W. & Peshkin, A. (eds) Qualitative Enquiry in Education: The Continuing Debate. London, Teachers College Press.

Driver, R. (1989) 'Students' conceptions and the learning of science', International Journal of Science Education, 11, Special Issue, p. 481 – 490.

Duit, R. (1999) Conceptual Change Approaches in Science Education in : Schnotz, W., Vosniadou, S. & Carretero, M. (eds) New Perspectives on Conceptual Change (p. 263 – 282). Amsterdam, NL, Pergamon.

Duit, R. & Treagust, D.F. (2003) 'Conceptual change: a powerful framework for improving science teaching and learning', International Journal of Science Education, 25, 6, p. 671 – 688.

Duit, R., Roth, W.-M., Komorek, M. & Wilbers, J. (2001) 'Fostering conceptual change by analogies – between Scylla and Charybdis', Learning and Instruction, 11, p. 283 – 303.

Dykstra, D.I. (1992) Studying Conceptual Change: Constructing New Understandings in: Duit, R., Goldberg, F. & Niedderer (eds) Research in physics learning: Theoretical issues and empirical studies (p. 40 – 58). Proceedings of an international workshop. Kiel (Germany), IPN – Leibniz Institute for Science Education.

Dykstra, D.I., Boyle, C.F. & Monarch, I.A. (1992) 'Studying Conceptual Change in Learning Physics', Science Education, 76, 6, p. 615 – 652.

Edsall, D. (1992) 'The Role of Coursework in Sixth Form Mechanics', Teaching Mathematics and its Applications, 11, 4, p. 150 – 154.

Edwards, L. (1995) Microworlds as Representations in: diSessa, A., Hoyles, C., and Noss, R., (eds) Computers and Exploratory Learning. Berlin Heidelberg, Springer-Verlag.

Eggar, W.D. (1911) The teaching of elementary mechanics [No 7 in the teaching of mathematics in the UK]. London, HMSO.

Ferrari, M. & Elik, N. (2003) Influences on Intentional Conceptual Change in: Sinatra, G.M. & Pintrich, P.R. (eds) Intentional Conceptual Change. Mahwah, NJ, Lawrence Erlbaum Associates.

Ford, B. & Hall, G.G. (1970) 'Model Building – an Educational Philosophy for Applied Mathematics', International Journal of Mathematical Education in Science and Technology, 1, p. 77 – 83.

Gibson, J.J. (1979) The Ecological Approach to Visual Perception. Boston, MA: Houghton, Mifflin.

Gilbert, J.K. & Watts, D.M. (1983) 'Concepts, Misconceptions and Alternative Conceptions: Changing Perspectives in Science Education', Studies in Science Education, 10, p. 61 – 98.

Gilbert, J.K., Watts, D.M. & Osborne, R.J. (1982) 'Students' conceptions of ideas in mechanics', Physics Education, 17, p. 62 – 66.

Gill, S. & Wright, D. (1994) 'A hypercard based environment for the constructivist teaching of Newtonian physics', British Journal of Educational Technology, 25, 2, p. 135 – 146.

Goldin, G.A. (1998) Observing Mathematical Problem Solving Through Task-Based Interviews in: Teppo, A.R. (ed) Qualitative Research Methods in Mathematics Education (Monograph Number 9: Journal for Research in Mathematics Education). Virginia, NCTM.

Goldin, G.A. (2000) A Scientific Perspective on Structured, Task-Based Interviews in Mathematics Education Research in Kelly, A.E. & Lesh, R.A. (eds) Handbook of Research Design in Mathematics and Science Education. Mahwah, NJ, Lawrence Erlbaum.

Goodchild, S. (1997) 'Mechanics in Action for Windows', CTI Summer Newsletter. Birmingham, CTI.

Gorsky, P. & Finegold, M. (1992) 'Using Computer Simulations to Restructure Students' Conceptions of Force', Journal of Computers in Mathematics and Science Teaching, 11, p. 163 – 178.

Graham, T. & Berry, J. (1990) 'Sixth Form Students' Intuitive Understanding of Mechanics Concepts', Teaching Mathematics and its Applications, 9, 2, p. 82 – 90.

Graham, T. & Berry, J. (1992) 'Sixth-Form Students' Intuitive Understanding of Mechanics Concepts: Part 2', Teaching Mathematics and its Applications, 11, 3, p. 106 – 111.

Graham, T. & Berry, J. (1996) 'A hierarchical model of the development of student understanding of momentum', International Journal of Science Education, 18, 1, p. 75 – 89.

Graham, T. & Berry, J. (1997) 'A hierarchical model of the development of student understanding of force', International Journal of Mathematical Education in Science and Technology, 28, 6, p. 839 – 853.

Graham, T. & Rowlands, S. (2000) 'Using computer software in the teaching of mechanics', International Journal of Mathematical Education in Science and Technology, 31, 4, p. 479 – 493.

Grayson, D.J. (1990) Use of the Computer for Research on Instruction and Student Understanding in Physics. Unpublished PhD Thesis. University of Washington.

Grayson, D.J. & McDermott, L.C. (1996) 'Use of the computer for research on student thinking in physics', American Journal of Physics, 64, 5, p. 557 – 565.

Greeno, J.G. (1994) 'Gibson's Affordances', Psychological Review, 101, 2, p. 336 – 342.

Gunstone, R.F. (1987) 'Student understanding in mechanics: A large population survey', American Journal of Physics, 55, 8, p. 691 – 696.

Gunstone, R.F. & White, R.T. (1981) 'Understanding of Gravity', Science Education, 65, 3, p. 291 – 299.

Gupta, A., Hammer, D. & Redish, E.F. (2010) 'The case for dynamic models of learners' ontologies in physics', Journal of the Learning Sciences, 19, p. 285 – 321.

Halldén, O. (1999) Conceptual Change and Contextualisation in Schnotz, W., Vosniadou, S. & Carretero, M. (eds) New Perspectives on Conceptual Change (p. 53 – 65). Amsterdam, NL, Pergamon.

Hammer, D. (1996) 'Misconceptions or P-Prims: How May Alternative Perspectives of Cognitive Structure Influence Instructional Perceptions and Intentions?', The Journal of the Learning Sciences, 5, 2, p. 97 – 127.

Hammer, D. (2000) 'Student resources for learning introductory physics', American Journal of Physics, 68, 7, p. S52 – S59.

Hargreaves, D.H. (1967) Social relations in a secondary school. London, Routledge & Kegan Paul.

Harskamp, E. & Ding, N. (2006) 'Structured Collaboration versus Individual Learning in Solving Physics Problems', International Journal of Science Education, 28, 14, p. 1669 – 1688.

Heckler, A.F. (2010) 'Some Consequences of Prompting Novice Physics Students to Construct Force Diagrams', International Journal of Science Education, 32, 14, p. 1829 – 1851.

Hegarty, M. (1992) 'Mental animation: Inferring motion from static displays of mechanical systems', Journal of Experimental Psychology: Learning, Memory and Cognition, 18, 5, p. 1084 – 1102.

Hegarty, M. (2004) 'Commentary: Dynamic visualizations and learning: getting to the difficult questions', Learning and Instruction, 14, p. 343 – 351.

Hegarty, M. (2005) Multimedia Learning About Physical Systems (Chapter 28) in: Mayer, R.E. (ed) The Cambridge Handbook of Multimedia Learning. New York, Cambridge University Press.

Hegarty, M. & Kriz, S. (2008) Effects of Knowledge and Spatial Ability on Learning from Animation in: Lowe, R. & Schnotz, W. (eds) Learning with Animation. New York, Cambridge University Press.

Hegarty, M., Kriz, S. & Cate, C. (2003) 'The Roles of Mental Animations and External Animations in Understanding Mechanical Systems', Cognition and Instruction, 21, 4, p. 325 – 360.

Heid, M.K., Blume, G.W., Zbiek, R.M. & Edwards, B.S. (1999) 'Factors that influence teachers learning to do interviews to understand students' mathematical understandings', Educational Studies in Mathematics, 37, p. 223 – 249.

Hennessy, S. et al. (1995) 'Design of a computer-augmented curriculum for mechanics', International Journal of Science Education, 17, 1, p. 75 – 92.

Hestenes, D. (1992) 'Modeling games in the Newtonian World', American Journal of Physics, 60, 8, p. 732 – 748.

Hewson, P.W. (1981) 'A conceptual change to learning science', European Journal of Science Education, 3, 4, p. 383 – 396.

Hewson, P.W. (1982) 'A Case Study of Conceptual Change in Special Relativity: The Influence of Prior Knowledge in Learning', European Journal of Science Education, 4, 1, p. 61 – 78.

Hewson, P.W. & Hewson, M.G.A'B. (1992) The status of students' conceptions in: Duit, R., Goldberg, F. & Niedderer (eds) Research in physics learning: Theoretical issues and empirical studies (p. 40 – 58). Proceedings of an international workshop. Kiel (Germany), IPN – Leibniz Institute for Science Education.

Hewson, P.W. & Thorley, N.R. (1989) 'The conditions of conceptual change in the classroom', International Journal of Science Education, 11, p. 541 – 553.

Hmelo, C.E., Holton, D. & Kolodner, J.L. (2000) 'Designing to learn about complex systems', Journal of the Learning Sciences, 9, 3, p. 247 – 298.

Holec, S., Spodniakova Pfefferova, M. & Raganova, J. (2004) 'Computer Simulations in Mechanics at the Secondary School', Informatics in Education, 3, 2, p. 229 – 238.

Horwitz, P. & Barowy, B. (1994) 'Designing and using open-ended software to promote conceptual change', Journal of Science Education and Technology, 3, p. 161 – 185.

Hoyles, C. (1993) Microworlds/Schoolworlds [The Transformation of an Innovation] in: Dorfler, W., Keitel, C. & Ruthven, K. (eds) Learning from Computers [Mathematics Education & Technology]. Berlin, Springer-Verlag.

Hoyles, C. (2001) 'From describing to designing mathematical activity: the next step in developing a social approach to research in mathematics education?', Educational Studies in Mathematics, 46, p. 273 – 286.

Hoyles, C. & Noss, R. (1987) 'Synthesizing mathematical conceptions and their formalization through the construction of a Logo-based school mathematics curriculum', International Journal of Mathematical Education in Science and Technology, 18, 4, p. 581 – 595.

Hoyles, C. & Sutherland, R. (1989) LOGO mathematics in the classroom. London, Routledge.

Hoyles, C., Healy, L. & Sutherland, R. (1991) 'Patterns of discussion between pupil pairs in computer and non-computer environments', Journal of Computer Assisted Learning, 7, p. 210 – 228.

Hughes, R.I.G. (1990) Philosophical Perspectives on Newtonian Science (Chapter 1) in: Bricker, P. & Hughes, R.I.G. (eds) Philosophical Perspectives on Newtonian Science. Cambridge, The MIT Press.

Ioannides, C. & Vosniadou, S. (2002) 'The changing meanings of force', Cognitive Science Quarterly, 2, 1, p. 5 – 62.

Jagger, J.M. (1994) Students' Understanding of Acceleration in the Context of Mechanics. Unpublished PhD thesis. Leeds, University of Leeds.

Jimoyiannis, A. & Komis, V. (2001) 'Computer simulations in physics teaching and learning: a case study on students' understanding of trajectory motion', Computers and Education, 36, 2, p. 183 – 204.

Kaiser, M. et al (1992) 'Influence of animation on dynamical judgement', Journal of Experimental Psychology: Human Perception and Performance, 18, p. 669 – 690.

Kaptelinin, V. (2003) 'Learning with artefacts: integrating technologies into activities', Interacting with Computers, 15, p. 831 – 836.

Kaput, J., Hoyles, C. & Noss, R. (2002) Developing New Notations for a Learnable Mathematics in the Computational Era in: English, L. (ed) Handbook of International Research in Mathematics Education (p. 51 – 75). London, Lawrence Erlbaum.

Kearney, M. (2004) 'Classroom Use of Multimedia-Supported Predict-Observe-Explain Tasks in a Social Constructivist Learning Environment', Research in Science Education, 34, p. 427 – 453.

Kearney, M. et al. (2001) 'Student and Teacher Perceptions of the Use of Multimedia Supported Predict-Observe-Explain Tasks to Probe Understanding', Research in Science Education, 31, p. 589 – 615.

Kitchen, A., Savage, M. & Williams, J. (1997) 'The Continuing Relevance of Mechanics in A-Level Mathematics', Teaching Mathematics and its Applications, 16, 4, p. 165 – 169.

Kline, M. (1959) Mathematics and the Physical World. NY, Dover Publications.

Kordaki, M. & Potari, D. (2002) 'The effect of area measurement tools on student strategies: The role of a computer microworld', International Journal of Computers for Mathematical Learning, 7, p. 65 – 100.

Laborde, C. & Laborde, J.M. (1995) What About a Learning Environment Where Euclidean Concepts are Manipulated with a Mouse? in: diSessa, A., Hoyles, C., and Noss, R., (eds) Computers and Exploratory Learning. Berlin Heidelberg, Springer-Verlag.

Lave, J. (1988) Cognition in practice: Mind, mathematics and culture in everyday life. Cambridge, Cambridge University Press.

Lautrey, J. & Mazens, K. (2004) 'Is children's naïve knowledge consistent? A comparison of the concepts of sound and heat', Learning and Instruction, 14, p. 399 – 423.

Lawson, D.A. & Tabor, J.H. (1997) Computer-based Experiments in Mechanics in: Houston et al. (eds) Teaching and Learning Mathematical Modelling. Chichester, Albion Publishing.

Lee, S. et al. (2005) Magic of Mechanics. MSOR Connections May 2005, Vol 5, No 2.

Lee, Y.F., Guo, Y. & Ho, H.J. (2008) 'Explore Effective Use of Computer Simulations for Physics Education', Journal of Computers in Mathematics and Science Teaching, 27, 4, p. 443 – 466.

Lemmer, M. (2013) 'Nature, Cause and Effect of Students' Intuitive Conceptions Regarding Changes in Velocity', International Journal of Science Education, 35, 2, p. 239 – 261.

Levrini, O. & diSessa, A.A. (2008) 'How students learn from multiple contexts and definitions: Proper time as a coordination class', Physical Review Special Topics – Physics Education Research, 4, 010107, p. 010107-1 – 010107-18.

Limon, M. (2001) 'On the cognitive conflict as an instructional strategy for conceptual change: a critical appraisal', Learning & Instruction, 11, p. 357 – 380.

Linder, C.J. (1993) 'A Challenge to Conceptual Change', Science Education, 77, 3, p. 293 – 300.

Lowe, R.K. (1999) 'Extracting information from an animation during complex visual learning', European Journal of Psychology of Education, 14, p. 225 – 244.

Lowe, R.K. (2003) 'Animation and learning: selective processing of information in dynamic graphics', Learning & Instruction, 13, p. 157 – 176.

McCloskey, M. (1983) Naïve theories of motion in: Gentner, D. & Stevens, A.L. (eds) Mental Models. Hillsdale, NJ, Erlbaum.

McDermott, L.C. (1984) 'Research on conceptual understanding in mechanics', Physics Today, 37, 7, p. 24 – 32.

McDermott, L.C. (1991) 'Millikan Lecture 1990: What we teach and what is learned – Closing the gap', American Journal of Physics, 59, 4, p. 301 – 315.

McDermott, L.C. & Somers, M.D. (1992) Building a research base for curriculum development: an example from mechanics in: Duit, R., Goldberg, F. & Hans, N. (eds) Research in Physics Learning: Theoretical Issues and Empirical Studies. Universitat Kiel, Publications IPN.

McDermott, L.C., Shaffer, P.S. & Somers, M.D. (1994) 'Research as a guide for teaching introductory mechanics: An illustration in the context of the Atwood's machine', American Journal of Physics, 62, 1, p. 46 – 55.

Magnusson, S.J., Templin, M. & Boyle, R.A. (1997) 'Dynamic Science Assessment: A New Approach for Investigating Conceptual Change', The Journal of the Learning Sciences, 6, 1, p. 91 – 142.

Masson, S. & Legendre, M.F. (2008) 'Effects of Using Historical Microworlds on Conceptual Change: A P-prim Analysis', International Journal of Environmental & Science Education, 3, 3, p. 115 – 130.

Masson, S. & Vazquez-Abad, J. (2006) 'Integrating History of Science Education through Historical Microworlds to Promote Conceptual Change', Journal of Science Education and Technology, 15, 3, p. 257 – 268.

Mathematical Association (1950) 'The teaching of mechanics', The Mathematical Gazette, 34, p. 168 – 181.

Mathematical Association (1965) A second report on the teaching of mechanics in schools. London, G.Bell & Sons Ltd.

Mayer, R.E. (2002) Understanding conceptual change: a commentary in: Limon, M. & Mason, L. (eds) Reconsidering Conceptual Change: Issues in Theory & Practice. Dordrecht, Kluwer Academic Publishers.

Mayer, R.E. & Moreno, R. (2002) 'Aids to computer-based multimedia learning', Learning and Instruction, 12, p. 107 – 119.

Medley, D.G. (1982) 'Mechanics and Modelling', Mathematics in School, 11, 3, p. 33 – 34.

Mestre, J.P. (2002) 'Probing adults' conceptual understanding and transfer of learning via problem posing', Applied Developmental Psychology, 23, p. 9 – 50.

Metz, K.E. (1991) 'Development of explanation: Incremental and fundamental change in children's physics knowledge', Journal of Research in Science Teaching, 28, p. 785 – 797.

Mildenhall, P.T. & Williams, J.S. (2001) 'Instability in students' use of intuitive and Newtonian models to predict motion: the critical effect of the parameters involved', International Journal of Science Education, 23, 6, p. 643 – 660.

Millar, R. (1989) 'Constructive criticisms', International Journal of Science Education, 11 (Special Issue), p. 587 – 596.

Moreno, R. (2009) 'Constructing knowledge with an agent-based instructional program: A comparison of cooperative and individual meaning making', Learning and Instruction, 19, p. 433 – 444.

Mortimer, E.F. (1995) 'Conceptual change or Conceptual Profile Change?', Science & Education, 4, p. 267 – 285.

Moschkovich, J.N. (1998) 'Resources for Refining Mathematical Conceptions: Case Studies in Learning About Linear Functions', The Journal of the Learning Sciences, 7, 2, p. 209 – 237.

Noss, R. & Hoyles, C. (1996) Windows on Mathematical Meanings: Learning Cultures and Computers. Dordrecht, Kluwer Academic Publishers.

Nussbaum, J. (1989) 'Classroom conceptual change: philosophical perspectives', International Journal of Science Education, 11 (Special Issue), p. 530 – 540.

Ozdemir, G. & Clark, D.B. (2007) 'An Overview of Conceptual Change Theories', Eurasia Journal of Mathematics, Science & Technology Education, 3, 4, p. 351 – 361.

Ozdemir, G. & Clark, D.B. (2009) 'Coherence of Turkish students' understanding of force', Journal of Research in Science Teaching, 46, 5, p. 570 – 596.

Ozmantar, M.F. & Monaghan, J. (2008) Are Mathematical Abstractions Situated? in: Watson, A. & Winbourne, P. (eds) New Directions for Situated Cognition in Mathematics Education. New York, Springer-Verlag.

Panorkou, N. & Pratt, D. (2011) Designing windows for researching children's experiences of dimension in: Joubert, M., Clark-Wilson, A. & McCab, M. (eds) Proceedings of the International Conference of Technology in Mathematics Teaching. Portsmouth, ICTMT 10.

Papert, S. (1980) Mindstorms [Computers, Children and Powerful Ideas]. New York, Basic Books.

Parnafes, O. (2007) 'What Does "Fast" Mean? Understanding the Physical World Through Computational Representations', The Journal of the Learning Sciences, 16, 3, p. 415 – 450.

Payne, J.W. (1994) 'Thinking Aloud: Insights Into Information Processing', Psychological Science, 5, 5, p. 241; 245 – 248.

Pinto, R. & Garcia, P. (2005) Introducing simulation software in physics classes: risks and opportunities. [Available online].

Ploetzner, R. & Lowe, R. (2004) 'Guest Editorial: Dynamic visualizations and learning', 14, p. 235 – 240.

Posner, G.J., Strike, K.A., Hewson, P.W. & Gertzog, W.A. (1982) 'Accommodation of a Scientific Conception: Toward a Theory of Conceptual Change', Science Education, 66, 2, p. 211 – 227.

Potvin, P., Legendre, M.F. & Thouin, M. (2003) Epistemic perspective on a conceptual evolution in physics at the secondary level. Paper presented at the Sixth International Conference on Computer Based Learning in Science, Nicosia, Cyprus.

Pratt, D.C. (1998) The Construction of Meanings In and For a Stochastic Domain of Abstraction. Unpublished PhD thesis. London, Institute of Education, University of London.

Pratt, D. & Noss, R. (2002) 'The Microevolution of Mathematical Knowledge: The Case of Randomness', The Journal of the Learning Sciences, 11, 4, p. 453 – 488.

Price, S. (2002) Animated Diagrams: How Effective are Explicit Dynamics for Learners? in: Bell, P., Stevens, R. & Satwitz, T. (eds) Keeping Learning Complex: The Proceedings of the Fifth International Conference of the Learning Sciences (p. 344 – 351). Mahwah, NJ, Erlbaum.

Ramadas, J., Barve, S. & Kumar, A. (1996) 'Alternative Conceptions in Galilean Relativity: Inertial and Non-inertial Observers', International Journal of Science Education, 18, 5, p. 615 – 629.

Read, J.R. (2004) Children's Misconceptions and Conceptual Change in Science Education. Available from www.asell.org/global/docs/conceptual_change_paper.pdf

Reif, F. (1995) 'Millikan Lecture 1994: Understanding and teaching important scientific thought processes', American Journal of Physics, 63, 1, p. 17 – 32.

Reiner, M. et al (2000) 'Naïve physics reasoning: A commitment to substance-based conceptions', Cognition and Instruction, 18, 1, p. 1 – 35.

Rey, G. (1998) Concepts. Routledge Encyclopedia of Philosophy. CD-ROM.

Richards, J., Barowy, W. & Levin, D. (1992) 'Computer Simulations in the Science Classroom', Journal of Science Education and Technology, 1, 1, p. 67 – 79.

Robinson, C.L., Harrison, M.C. & Lee, S. (2005) Mechanics Report. Engineering Subject Centre Report: Responding to the Changes in the Teaching and Learning of Mechanics in Schools. Loughborough University, The Higher Education Academy.

Roper, T. (1985) Students' Understanding of Selected Mechanics Concepts in: Orton, A. (ed) Studies in Mechanics Learning. Leeds, University of Leeds.

Roper, T. (1994) Integrating Mathematics into the Wider Curriculum in: Orton, A. & Wain, G. (eds) Issues in Teaching Mathematics. London, Cassell.

Roschelle, J. (1992) 'Learning by Collaborating: Convergent Conceptual Change', The Journal of the Learning Sciences, 2, 3, p. 235 – 276.

Roschelle, J. (1998) 'Beyond romantic versus sceptic: a microanalysis of conceptual change in kinematics', International Journal of Science Education, 20, 9, p. 1025 – 1042.

Roth, W.M. (1995) 'Affordances of Computers in Teacher-Student Interactions: The Case of Interactive Physics', Journal of Research in Science Teaching, 32, 4, p. 329 – 347.

Roth, W.M., Woszczyna, C. & Smith, G. (1996) 'Affordances and Constraints of Computers in Science Education', Journal of Research in Science Teaching, 33, 9, p. 995 – 1017.

Rowlands, S. (2006) Mechanics should be integral to secondary school mathematics in: Hewitt, D. (ed) Proceedings of the British Society for Research into Learning Mathematics, 26, 2, p. 91 – 96.

Rowlands, S. (2008) 'Why mechanics should be integral to secondary school mathematics', Teaching Mathematics and Its Applications, 27, 4, p. 187 – 199.

Rowlands, S., Graham, E. & Berry, J. (1997) 'The Socratic Method of Strategic Questioning to Facilitate the Construction of the Target-Concept Within the Students' Zone of Proximal Development', Proceedings of the BSRLM Bristol Conference.

Rowlands, S., Graham, E. & Berry, J. (1998) 'Modelling, strategic questioning and the laws of student reasoning in A-Level Mechanics', Proceedings of the BSRLM London Conference.

Rowlands, S., Graham, T., Berry, J. & McWilliam, P. (2007) 'Conceptual Change Through the Lens of Newtonian Mechanics', Science & Education, 16, p. 21 – 42.

Ruhf, R.J. (2003) A General Overview of Conceptual Change Research. Unpublished document.

Saldarriaga, M. (2011) Using a water tank analogy to transform students' intuitive knowledge of dynamic systems: A qualitative study of the case of motion. Unpublished PhD thesis. University of Bergen.

Savage, M. & Williams, J. (1990) Mechanics in Action: Modelling and Practical Investigations. Cambridge, Cambridge University Press.

Scaife, M. & Rogers, Y. (1996) 'External cognition: how do graphical representations work?', International Journal of Human-Computer Studies, 45, p. 185 – 213.

Schnotz, W. & Grzondiel, H. (1999) 'Individual and co-operative learning with interactive animated pictures', European Journal of Psychology of Education, 14, p. 245 – 265.

Schoenfeld, A.H. (1992) 'On Paradigms and Methods: What Do You Do When the Ones You Know Don't Do What You Want Them to? Issues in the Analysis of Data in the Form of Videotapes', The Journal of the Learning sciences, 2, 2, p. 179 – 214.

Scorer, R.S. (1964) 'Purpose in Applied Mathematics', Mathematics Teaching, 29, p. 17 – 26, 29.

Sherin, B. (2006) 'Common Sense Clarified: The Role of Intuitive Knowledge in Physics Problem Solving', Journal of Research in Science Teaching, 43, 6, p. 535 – 555.

Sinatra, G.M. (2002) Motivational, social, and contextual aspects of conceptual change: a commentary in: Limon, M. & Mason, L. (eds) Reconsidering Conceptual Change: Issues in Theory & Practice. Dordrecht, Kluwer Academic Publishers.

Slotta, J.D. & Chi, M.T.D. (2006) 'The impact of ontology training on conceptual change: Helping students understand the challenging topics in science', Cognition and Instruction, 24, 2, p. 261 – 289.

Slotta, J.D., Chi, M.T.D. & Joram, E. (1995) 'Assessing Students' Misclassifications of Physics Concepts: An Ontological Basis for Conceptual Change', Cognition and Instruction, 13, 3, p. 373 – 400.

Smith, J.P., diSessa, A.A. & Roschelle, J. (1993) 'Misconceptions Reconceived: A Constructivist Analysis of Knowledge in Transition', The Journal of the Learning Sciences, 3, 2, p. 115 – 163.

Southerland, S.A. et al (2001) ‘Understanding Students’ Explanationa of Biological Phenomena : Conceptual Frameworks or P-Prims ?’ Science Education, 85, p. 328 – 348.

Spada, H. (1994) ‘Commentary: Conceptual change or multiple representations?’, Learning & Instruction, 4, p. 113 – 116.

Stake, R.E. (1978) ‘The Case Study Method in Social Inquiry’, Educational Researcher, 7, 2, p. 5 – 8.

Stevenson, I. (1998) Approaches to Mechanics (Modelling with Interactive Physics: INSET Course). London, Institute of Education, University of London.

Stevenson, I. (1999) Approaches to Teaching Mechanics Using New Technology. Unpublished paper.

Stevenson, I. (2000) Mechanics, Modelling and Pedagogy: Reflections on the design of contexts for new technologies in: Rowland, T. & Morgan, C. (eds) Research in Mathematics Education Volume 2. London, British Society for Research into Learning Mathematics.

Stevenson, I. (2002) ‘Microworlds and Direct Manipulation Environments: The Case of Newtonian Mechanics’, Journal of Educational Computing Research, 27, 1-2, p. 167 – 183.

Strike, K.A. & Posner, G.J. (1992) A Revisionist Theory of Conceptual Change in: Duschl, R.A. & Hamilton, R.J. (eds) Philosophy of Science, Cognitive Psychology, and Educational Theory and Practice. Albany, State University of New York Press.

Swanson, H. (2015) Leveraging Students’ Prior Knowledge in Attaining Deep Structural Understanding of Domain General Models. Unpublished PhD thesis. Berkeley, University of California.

Tao, P.K. & Gunstone, R.F. (1999) ‘The Process of Conceptual Change in Force and Motion during Computer-Supported Physics Instruction’, Journal of Research in Science Teaching, 36, 7, p. 859 – 882.

Terry, C. & Jones, G. (1986) ‘Alternative frameworks: Newton’s third law and conceptual change’, European Journal of Science Education, 8, 3, p. 291 – 298.

Thagard, P. (1990) ‘Concepts and conceptual change’, Synthese, 82, p. 255 – 274.

Thagard, P. (1991) Concepts and conceptual change in: Fetzer, J.H. (ed) Epistemology and cognition. Dordrecht, Kluwer.

Thagard, P. (1992) Conceptual revolutions. Princeton, NJ, Princeton University Press.

Treagust, D.F., Harrison, A.G. & Venville, G.J. (1996) ‘Using an analogical teaching approach to engender conceptual change’, International Journal of Science Education, 18, 2, p. 213 – 229.

Truesdell, C.A. (1968) Essays in the History of Mechanics. New York, Springer-Verlag

Tversky, B., Morrison, J.B. & Betrancourt, M. (2002) 'Animation : can it facilitate?', International Journal of Human-Computer Studies, 57, p. 247 – 262.

Tyson, L.M., Venville, G.J., Harrison, A.G. & Treagust, D.F. (1997) 'A Multidimensional Framework for Interpreting Conceptual Change Events in the Classroom', Science Education, 81, p. 387 – 404.

Tytler, R. (1998) 'Children's conceptions of air pressure: exploring the nature of conceptual change', International Journal of Science Education, 20, 8, p. 929 – 958.

Van den Heuvel-Panhuizen, M. (1994) 'Improvement of (didactical) assessment by improvement of the problems: An attempt with respect to percentage', Educational Studies in Mathematics, 27, 4, p. 341 – 372.

Venville, G.J. & Treagust, D.F. (1998) 'Exploring Conceptual Change in Genetics Using a Multidimensional Interpretive Framework', Journal of Research in Science Teaching, 35, 9, p. 1031 – 1055.

Vosniadou, S. (1992) 'Knowledge acquisition and conceptual change', Applied Psychology, 41, 4, p. 347 – 357.

Vosniadou, S. (1994) 'Capturing and modeling the process of conceptual change', Learning & Instruction, 4, p. 45 – 69.

Vosniadou, S. (1999) Conceptual Change Research: State of the Art and Future Directions in: Schnotz, W., Vosniadou, S. & Carretero, M. (eds) New Perspectives on Conceptual Change. Amsterdam, NL, Pergamon.

Vosniadou, S. (2002) On the nature of naïve physics in: Limon, M. & Mason, L. (eds) Reconsidering Conceptual Change: Issues in Theory & Practice. Dordrecht, Kluwer Academic Publishers.

Vosniadou, S. & Brewer, W.F. (1992) 'Mental models of the earth: A study of conceptual change in childhood', Cognitive Psychology, 24, 4, p. 535 – 585.

Vosniadou, S. & Brewer, W.F. (1994) 'Mental models of the day/night cycle', Cognitive Science, 18, 1, p. 123 – 183.

Vosniadou, S., Vamvakoussi, X. & Skopeliti, I. (2008) The framework theory approach to the problem of conceptual change in: Vosniadou, S. (ed) International Handbook of Research on Conceptual Change (p 3 – 34). New York, Routledge.

Vosniadou, S., Ioannides, C., Dimitrakopoulou, A. & Papademetriou, E. (2001) 'Designing learning environments to promote conceptual change in science', Learning and Instruction, 11, p. 381 – 419.

Wade, S.E. (1990) ‘Using think alouds to assess comprehension’, The Reading Teacher, 43, 7, p. 442 – 451.

Watts, D.M. & Zylbersztajn, A. (1981) ‘A survey of some children’s ideas about force’, Physics Education, 16, p. 360 – 365.

Webb, M.E. (2005) ‘Affordances of ICT in science learning: implications for an integrated pedagogy’, International Journal of Science Education, 27, 6, p. 705 – 735.

Whitworth, R. (1988) ‘Mechanics: Advanced-Level Coursework Investigations’, Teaching Mathematics and its Applications, 7, 4, p. 169 – 177.

White, B.Y. (1983) ‘Sources of Difficulty in Understanding Newtonian Dynamics’, Cognitive Science, 7, 1, p. 41 – 65.

White, R.T. (1994) ‘Commentary: Conceptual and conceptional change’, Learning & Instruction, 4, p. 117 – 121.

White, R. & Gunstone, R. (1992) Probing Understanding. London, Falmer Press.

Williams, J.S. (1985) Using equipment in teaching mechanics in: Orton, A. (ed) Studies in Mechanics Learning. Leeds, The University of Leeds.

Williams, J.S. (1987) ‘Practical Applied Mathematics: 2’, Mathematics Teaching, 118, p. 58 – 63.

Williams, J.S. (1988) ‘Practical Mechanics: Mechanics in Action’, Mathematics in School, 17, 2, p. 2 – 9.

Wiser, M. & Amin, T.G. (2002) Computer-based interactions for conceptual change in science in: Limon, M. & Mason, L. (eds) Reconsidering Conceptual Change: Issues in Theory & Practice. Dordrecht, Kluwer Academic Publishers.

Woodrow, D. (1965) ‘Life in Suburbia’, Mathematics Teaching, 33, p. 13 – 15.

Woodrow, J.E.J., Mayer-Smith, J.A. & Pedretti, E.G. (1996) ‘The Impact of Technology Enhanced Science Instruction on Pedagogical Beliefs and Practices’, Journal of Science Education and Technology, 5, 3, p. 241 – 252.

Yin, R.K. (1984) Case study research: design and methods. Sage Publications.

Young, K.A. (2005) ‘Direct from the source: the value of ‘think-aloud’ data in understanding learning’, Journal of Educational Enquiry, 6, 1, p. 19 – 33.

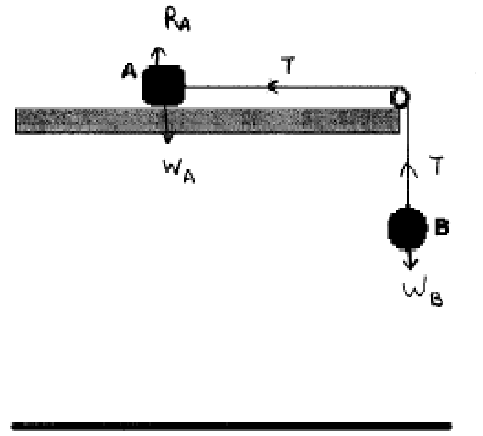
APPENDICES

Appendix A: Transcript for Student S1

I: Interviewer S1: Student

Line Transcript

[At this stage some part of Task 1 had already been completed by Student S1, including the drawing of the following force diagram.



So this transcript starts almost in the middle of Task 1.]

I: What would you have expected? If both masses had been equal, what would you have expected? … If you had not yet seen this [referring to the simulation], what would you have said about the value of the tension?

S1: The tension would be equal to the weight of this object [pointing at object B].

I: Eh? Say you had not yet seen this simulation and the mass of sliding object was 1, and that of hanging object 1, what would have been the tension in the string [pointing at the string]?

S1: The tension would have been equal to the weight of this object [pointing at object B]. Because the weight [pointing at object A] would have cancelled out the normal reaction exerted by the surface.

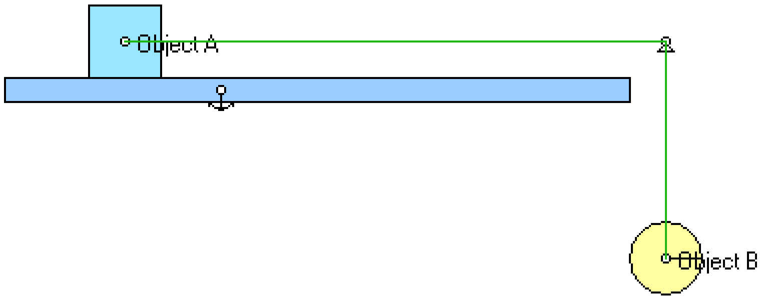
I: Would they have cancelled out?

S1: Yes

I: So, only this one [pointing at object B] is affecting the tension?

S1: Yes.

I: […] Now, let's go back to this system, the first one. Ok?



S1: Yes.

I: Let's go back to this one. Earlier you told me that both objects would have the same acceleration and same speed. Why did you think that?

S1: Because both objects are connected via the same string.

I: Alright. At this moment the mass of this object [pointing at object B] is 1 kg and that of this one [pointing at object A] is also 1 kg.

S1: Yes.

I: What may happen if I make the mass of this object [pointing at object B] become 2 kg? Do you think that both objects will continue to have the same acceleration?

S1: Yes, they will have the same acceleration.

I: And what may happen if I make the mass of this object [pointing at object A] become 4 times bigger?

S1: It will still remain the same.

I: What will remain the same? The acceleration?

S1: The acceleration will remain the same …(pause)… The downward acceleration will remain the same.

I: What do you mean by 'downward acceleration'?

S1: It means that it equals to g.

I: What is it that equals to g?

S1: This one's [pointing at object B] acceleration and this one's [pointing at object A] acceleration.

I: Their acceleration is g?

S1: Yes.

I: Eh … Why do you think that it is g?

S1: Because here [pointing at the centre of object B] force of gravity will be the same.

I: Force of gravity is?

S1: Equal.

I: Equal?

S1: Yes.

I: Ok. So this means that when this object [pointing at object B] will fall, it will fall with an acceleration g.

S1: Yes.

I: Alright. Can you prove what you are saying without taking any value?

S1: We measure the distance moved by this one [pointing at object A] in a specific time or 1 second and then we measure the distance moved by this one [pointing at object B] for the same interval. Both distances must be equal.

I: Ok. Assume we found this to be true. Then what happens?

S1: They will be the same.

I: What will be the same?

S1: Their accelerations and their velocities will be the same.

I: This is OK. I'm asking you about the magnitude of acceleration. You told me that it is equal to g.

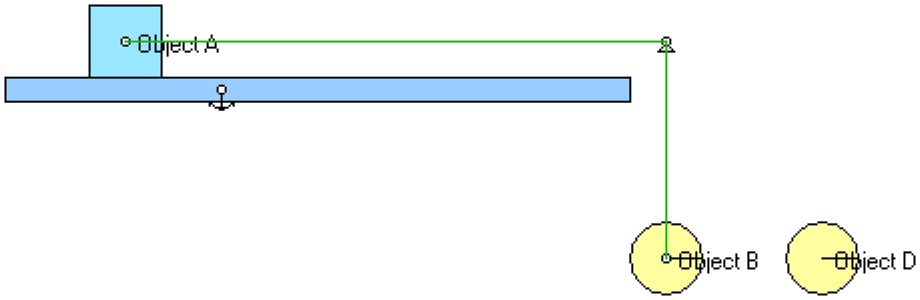
S1: Yes.

I: How can you prove that it is equal to g?

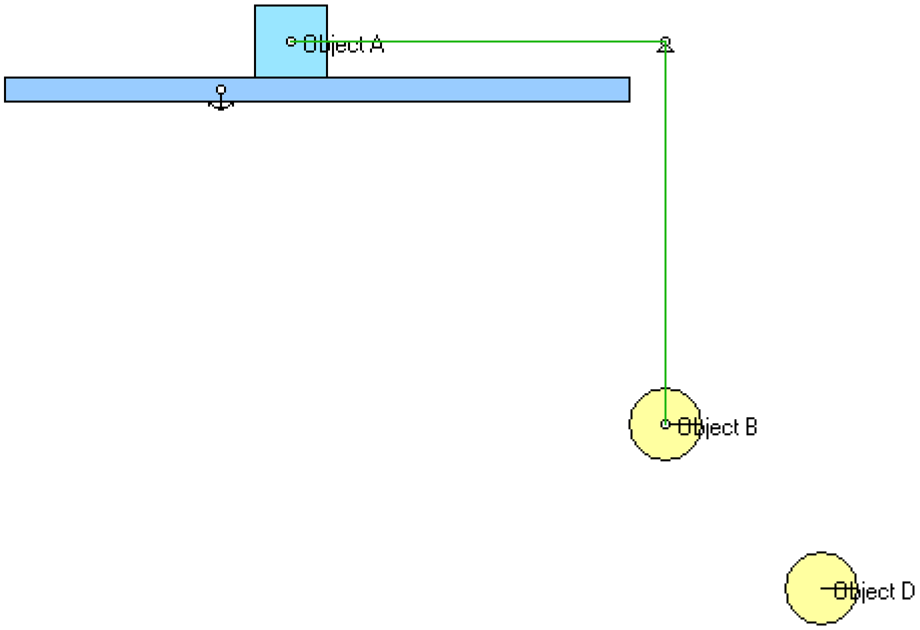
S1: We take another object and place it here [pointing at a point beside object B] and then we measure.

I: Ok. Let's try it.

[He drew another object D of equal size beside the hanging object B and then used the forward step of the tape player mode to run the simulation frame by frame.



On the screen, it was seen that the new object D fell faster compared to the hanging object B. He then reset the simulation, opened the PROPERTIES box of the table and equated the value of STATIC FRICTION to zero.]



67 **S1**: Let's make friction zero.

68 **I**: Ok. Put KINETIC FRICTION also zero.

[He changed the value of KINETIC FRICTION to zero. When he ran the simulation using the command RUN, the motion was too quick for him to study. So he reset the simulation and used the forward step of the tape player mode to run the simulation frame by frame. He noted that the object D again fell faster than the hanging object B.]

S1: They are not the same!
I: This means that this object … reset the system …

[He reset the system and object D, which had disappeared from the screen, was in its initial position.]

I: This means that this object [pointing at object D] is moving with an acceleration greater than g. Eh?
S1: Oh … no, this one [referring to object D] is moving with acceleration g.
I: But didn’t you tell me that this one [pointing at object B] is moving with an acceleration g?
S1: This one [pointing at object D] is moving with acceleration g and this one [pointing at object B] is moving with acceleration less than g.
I: Why? Why do you say that this one [pointing at object D] will move with acceleration g?
S1: Because this one is free. There is no force. There is no force on it.
I: So what can we say?
S1: This one [pointing at object B] is moving with acceleration less than g and this one [pointing at object D] is moving with acceleration 9.8 m/s^2.
I: Are you sure that this one [pointing at object D] will move with acceleration g?
S1: Yes.
I: Why do you say that the acceleration is g?
S1: Because there is no opposing force on it.
I: Some time back, you were using another term, you were saying?
S1: I think it’s free … falling freely.
I: Yes, free, falling freely.
S1: Yes.
I: I think this is what we call falling freely under gravity?
S1: Yes.
I: So, what do you find? What is the acceleration of this one [pointing at object B]?
S1: It is less than g because it is not falling freely.

100 **I**: It doesn't equal to g?

S1: No.

I: And what about the acceleration of the above object [pointing at object A]? Is it equal to g or … what do you think?

S1: It is the same as this one [pointing at object B].

105 **I**: It is the same?

S1: Yes, it is the same as this one [pointing at object B].

I: Are you sure about these?

S1: Yes.

I: Can we verify your sayings?

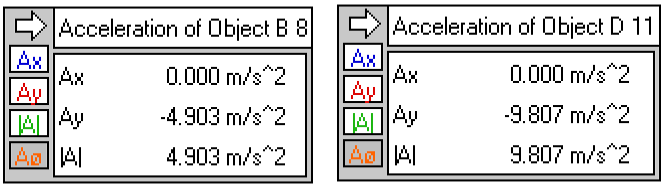
110 **S1**: The acceleration of this one [pointing at object D]?

I: We can check the accelerations of both objects. Measure them.

[He clicked on object D, then went to MEASURE, selected ACCELERATION and dragged the ACCELERATION meter to the R.H.S. of the screen beside object D. He repeated the above procedure with hanging object B and finally moved another ACCELERATION meter on the L.H.S. of hanging object B.]

112 **I**: Ok. Now use forward.

[He continuously pressed on the forward step of the tape player control and the system moved slowly. At one point, he stopped the simulation and noted the accelerations of both objects: the acceleration of object B was 4.903 m/s^2 and that of object D was 9.807 m/s^2, that is g.]



113 **S1**: They are not equal.

I: You have managed to prove your case.

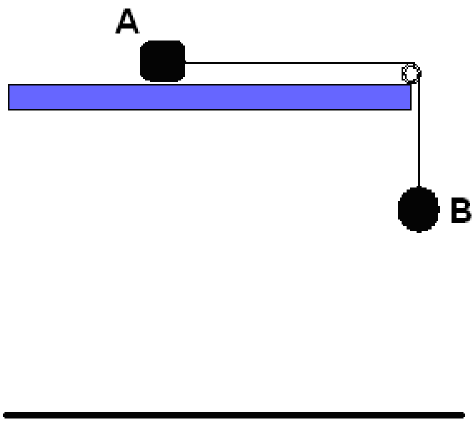
115 **S1**: Yes.

Appendix B: Transcript for Student S2

I: Interviewer S2: Student

Line Transcript

[Task 1 was introduced to Student S2.]



I: How may this system behave when it is released from rest? … According to you, how may such a system behave?

S2: Is the rope one piece or is it two pieces attached together?

I: Why are you asking this question? How does it affect the system?

S2: Because … (pause) …

I: Does it matter if it's one whole piece or two pieces attached together?

S2: No, it won't make any difference. If the mass of this object [pointing at hanging object B] is more than that of this object [pointing at sliding object A], this object [pointing at object B] will move down. But if the mass of this object [pointing at object A] is more than this one [pointing at object B] and if there is no resistance on object A, object A will move towards the left; if there is no friction along the surface (referring to the table), it is smooth, this object [pointing at object A] will tend to move towards the left.

I: So you are saying that if object B is heavier than object A, what would happen? This one [pointing at object B] will move down?

S2: This object [pointing at object A] will move towards the right & object B will move down.

I: Object B will move down. This happens when object B is heavier than object A.

S2: Yes.

I: Now what will happen if object A is heavier than object B?

S2: Object A will tend to move towards the left and object B will move up if the surface of the table is smooth.

I: […] And what happens if both objects have the same mass?

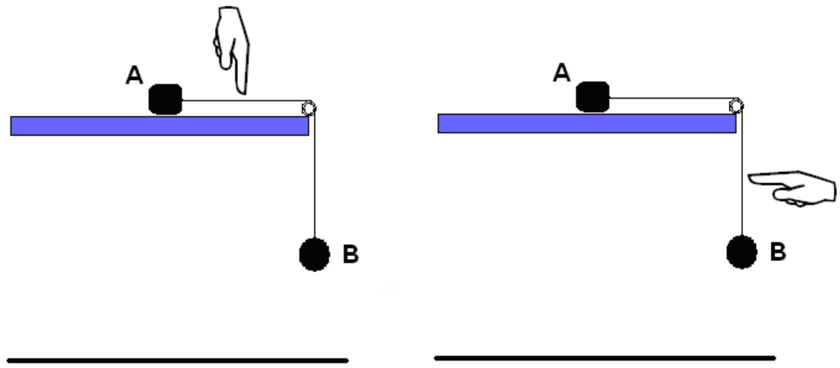
S2: … (pause) …There will be no motion in the system … Both objects won't move.

I: Are you making any assumption before stating this hypothesis?

S2: … (pause) … Assume that it is a rough surface. Then both objects will not move, if the surface of table is rough.

I: […] And what happens if the table is smooth?

S2: If the table is smooth … there will be no motion. The objects won't move. Because the tension here [pointing at the vertical part of the string] will be equal to the weight of object B, and the tension here [pointing at the horizontal part of the string] will be equal to this weight [pointing at object A].



pointing at horizontal part (string) pointing at vertical part (string)

I: The tension will be equal to what?

S2: There is a mistake.

I: Can you show me on the screen? According to you, in which direction will tension act?

S2: Tension will be here [showing that arrow is moving away from object A]; tension will be in this direction [showing that arrow is moving away from object B] & there will be a weight mg acting downwards, & this tension [pointing at the vertical part of the string] will be equal to the weight, T = mg; here also [pointing at the horizontal part of the string] there will be T = mg & then this object A will tend to … if the surface of the table is smooth, the objects will not move.

I: Why?

S2: Eh, if the mass of object B is the same as that of object A, both will have the same weight. The tension here [pointing at the horizontal part of the string] will be equal to the weight of object A.

I: So, according to you, tension in the string will be equal to weight?

S2: Yes.

I: Weight of object?

S2: Yes.

I: Ok, assume that the mass of each object is equal to 1 kg. What can we say about tension?

S2: Tension will be 10.

I: Why 10?

S2: If $g = 10\ m/s^2$, then since $W = mg = 1 \times 10 = 10$ N.

I: So what will be the tension in the string?

S2: Tension in the string will be 10 N.

I: It will be 10 N?

S2: Yes.

I: So, according to you, whether the surface is smooth or rough, if mass of object A is equal to mass of object B, the objects will not move.

S2: Say it again.

I: What I have understood is that if the surface is rough, there will be no motion. If the surface is smooth, there will be again no motion.

S2: Yes. If the surface is rough, there will be no motion. If the surface is smooth, there will again be no motion; neither object A nor object B will move […] There will be no motion, because the masses are equal & the tension in the string is equal to the weight of the object. So there will be no motion. The system will remain at rest.

I: Tension in the string will be equal to?

S2: If mass of object A is equal to mass of object B, then tension in the string is equal to the weight of object B.

I: So, when mass of A is equal to mass of B, tension in the string will be equal to what?

S2: Tension in the string will be equal to the weight of object B.

I: And because of this reason, you think there will be no motion?

S2: Yes.

I: So, are you telling me that tension in the string = weight of object B = weight of object A?

S2: Yes … and there will be no motion.

I: Ok, now let us consider the first case - where mass of object B is more than that of object A … What can be said about the speeds, velocities & accelerations of these 2 objects? What do you think?

S2: Tension in the string will be the same, i.e. tension in the string will equal to the weight of object B.

I: When mass of object B is greater than mass of object A, then, remember earlier you told me that in such a case, object B will move down & object A will move towards the right, towards the pulley.

S2: Yes.

I: Ok? So, when we have such kind of motion, what according to you, what can we say about the speeds of both objects? Will speed of object A be greater than speed of object B? Or, will speed of object A be lesser than speed of object B? At the same time, will acceleration of object A be greater than acceleration of object B or lesser than or what?

S2: Speed of object B will be equal to object A because if it is a smooth surface … speed of A = speed of B.

I: So this is possible when the surface is smooth?

S2: Yes.

I: Is this what you are saying?

S2: Yes.

I: Ok.

S2: The acceleration will be the same.

I: For what?

S2: Acceleration of object B will be the same as that of object A.

I: Why, according to you, speed of A is equal to speed of B, and acceleration, magnitude of acceleration of A is equal to that of B? How can you be sure about that?

S2: Because the tension in the string is the same throughout, the acceleration of the system must be the same.

I: Why do you think that both objects will have the same acceleration in term of magnitude? How can you be so sure about that?

S2: … (pause) … Are the masses equal?

I: No, we're referring to your first case.

S2: Both masses are equal.

I: No, mass of object B is greater than that of object A.

S2: The accelerations are the same because the tension in the string is the same throughout … (pause) … I don't think so.

I: Hmm?

S2: I don't think so, because it is one whole piece and … if the string was extensible, the string could extend, then tension wouldn't have been the same throughout.

I: So, what should we assume about the string?

S2: It is inextensible & it is one whole piece.

I: Ok … as per our discussion, acceleration of object A = acceleration of object B, when mass of object B is greater than that of object A. Right?

S2: Yes.

I: Now, what happens when this object [pointing at object A] is heavier than this object [pointing at object B]?

S2: If object A is heavier than object B?

I: What will happen to motion?

S2: There will be no motion.

I: But at the start of the activity didn't you tell me that this one [pointing at object A] will move towards the left and this one [pointing at object B] will move up?

S2: No. There will be no motion when object A is heavier than object B.

I: Ok. Now let's go back to the system where object B is heavier than object A. … You told me that the magnitude of acceleration of B will be equal to that of A. In such a case, do you think that the magnitude of acceleration of object B will be equal to, greater than or lesser than g?

S2: It'll be less than g.

I: Why?

S2: … Because in calculation, the acceleration is less than g.

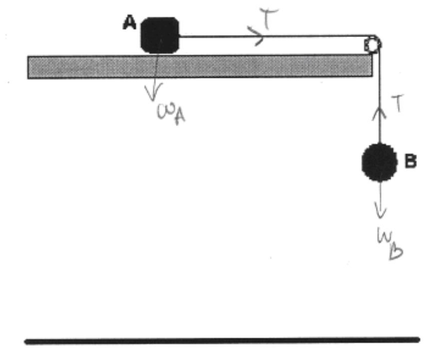
I: It is because of your calculation? … How can you account for it?

S2: … (silence) …

I: Ok. Are you sure that it is less than g?

S2: Yes. Yes.

I: Ok, we'll talk about it later. Ok? … Can we now draw a force diagram for this system? What are the forces present in this system?



S2: There are the weight of object B, the weight of object A, tension in the string.

I: Ok.

S2: If the surface is rough, there is a frictional force.

I: In which direction will it act?

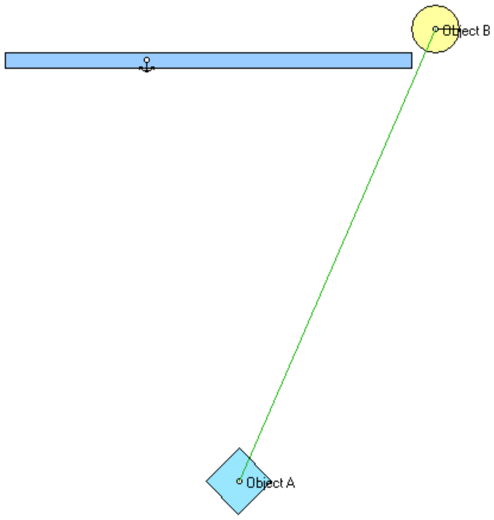
S2: If the object A will move towards the pulley, the frictional force will act in the opposite direction (referring towards the left). If object A will move away from the pulley, the frictional force will act in the direction of the pulley.

I: Ok … is there any other force in this system?

S2: No.

I: Ok. Now let's try the simulation.

[He ran the simulation using the command RUN: object A moved along the surface, left the table, went up a bit and finally started to move down in a swing. In the meantime, object B moved down and then started to move up until it was stopped by the pulley. Finally it was seen that object B was at the pulley & object A was swinging at the lowest position (see diagram overleaf). I had to ask him to stop, reset and run the simulation. He again used the command RUN and the same set of events happened. He continued to watch the simulation until he was again asked to stop it. At this point I reminded him that we were only interested with the part when object A was on the surface. When he was asked about his comments on this system, he reset the simulation, ran it using the command RUN (for the third consecutive time) and observed the same set of events.



I: So, what do you observe?
S2: The mass of object A is greater than the mass of object B.
I: Why?
S2: Because object B is near the pulley & object A is down.
I: Ok … so are you saying that object A is heavier than object B?
S2: Yes.
I: But, according to your earlier explanation, if object A is heavier than object B, there should be no motion.
S2: Yes.
I: How can you explain this?
S2: … (silence) …
I: We can verify their masses. Right now, according to your observation, object A is heavier than object B. We can verify this fact. Do you know how to do it?
S2: Yes.

[He first selected the sliding object A & opened its PROPERTIES box. He saw that its mass was 1 kg. Then he selected the hanging object B & opened its PROPERTIES box. The mass was 1 kg.]

I: What do we see?

S2: Both objects have equal masses, 1 kg each.

I: When both objects have equal masses, what do we notice?

S2: Both objects are moving. They are not stationary.

I: How can you explain this?

S2: … (silence) …

I: So it is not true to say the mass of object A is greater than that of object B. When we have verified the masses, we see that

S2: Both objects have equal masses.

I: Both objects have equal masses. When both objects have equal masses, there is motion. How can you explain this?

S2: …(pause)… For motion to take place … does weight of object B not become a force that pulls object A & this sets the system in motion?

I: Say that again.

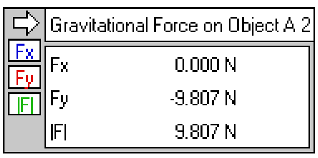
S2: The weight of object B becomes a force that pulls object A towards the pulley.

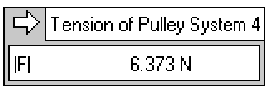
I: According to you, what is pulling object A towards the pulley?

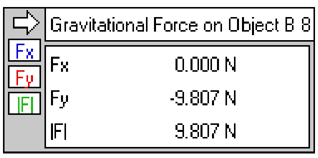
S2: The weight of object B … This one [pointing to object A] is at a certain height compared to object B.

I: Earlier you told me that when mass of object A is equal to mass of object B, tension in the string = weight of object A = weight of object B. Do you think that this is true? You can now verify this fact.

[He reset the simulation, selected the sliding object A and then seemed lost. At this point I advised him to put on the digital meter GRAVITATIONAL FORCE. He repeated the same procedure for the hanging object B. He finally selected the string and put on the digital meter TENSION. I advised him to run the system in slow motion and he noted the values on the three meters.]







I: What do we see?

S2: Tension is not equal to the weight of the hanging object B.

I: What can we say?

S2: When the masses are equal, the weights are equal but the tension is different.

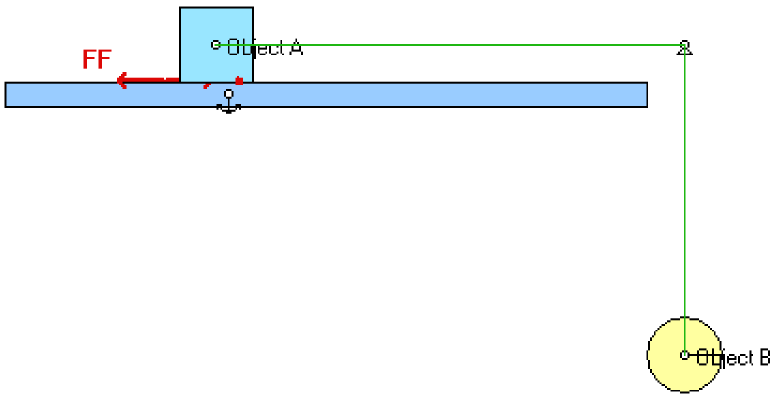
I: Why? Why is it that the tension is not equal to the weight?

S2: I have no idea.

I: What we have discovered is that when the masses are equal, the weight of object B does not equal to the tension. Right? According to you, why? … The first thing that we have seen is that when the masses are equal, there is motion.

S2: Let's verify whether the surface is smooth or rough.

[He reset the simulation, selected object A and asked for help. I went to the command DEFINE where I chose the sub-command VECTORS and clicked on FRICTIONAL FORCE. He then ran the system & a red arrow pointed in opposite direction of motion. But the object A left the surface and moved down. So he reset the simulation and ran it in slow motion. The red arrow between object A and surface could be seen in opposite direction of motion.]



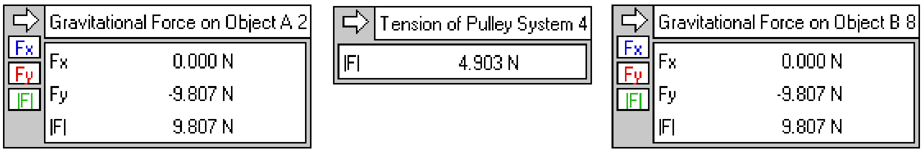
I: What can we see?

S2: Because it is a rough surface, the frictional force affects the tension.

I: Ok.

S2: Let's eliminate the frictional force and we can work with a smooth surface.

[He reset the system, selected the surface, opened its PROPERTIES box. Since both KINETIC FRICTION and STATIC FRICTION were 0.3, I advised him to make them zero. He then ran the system using the command RUN and this time no red arrow appeared implying that there was no frictional force in the system. As the motion was too quick, he could not do any proper observation. So he reset the system and this time ran it via the forward step in the tape player control until object A was a bit more than half-way along the surface.]



I: What do you find? Is there any frictional force?

S2: The tension is still not equal to the weight.

I: Huh?

S2: Even now, the tension in the string is not equal to the weight of the object.

I: … But didn't we see the same thing earlier? So, what do you mean?

S2: There must be some force acting on the pulley [pointing the cursor at it].

I: Why are you saying this? … You've just said that even here the tension is less. With what are you comparing it?

S2: I'm comparing it with the weight of the hanging object B. The tension should be equal to the weight as we had assumed. But they are not equal, they are different.

[He reset the simulation using the tape player control and then ran the system via slow motion. He verified the new values on both meters and they were still the same as before.]

S2: I have no idea … (pause) …

I: Ok. So we are not able to explain why, when the masses are equal, the tension

S2: The tension is not equal to the weight of the object.

I: We are not able to explain this … We’ll try to find an answer later. […] Ok. […]

[I opened a new screen for the previous simulation.]

I: Let’s now re-consider this system where mass of object A is 1 kg & mass of object B is 1 kg.

S2: There will be motion.

I: There is motion. Initially you told me that there would be no motion because

S2: Both masses are equal & the tension is equal to the weight of the hanging object B.

I: Is there friction in this system?

[He added the vector representing frictional force in the simulation and then ran the system. A red arrow (representing frictional force) appeared, pointing away from the pulley.]

S2: Yes, there is friction.

I: According to you, what happens if the mass of object B is doubled? What happens to the system?

S2: Do we assume that this mass [pointing at object A] remains the same?

I: Yes, it remains the same.

S2: This object [pointing at object A] will move quicker, the weight of this one [pointing at object B] will increase. If the weight is increased, the tension will change, acceleration will change & frictional force will also change.

I: Does the “change” implies an increase or a decrease?

S2: Acceleration will increase, tension will also increase.

I: … And how about speed?

S2: … Speed will also increase.

I: Are we talking about a rough surface or a smooth surface?

S2: This is a rough surface because there is friction.

I: And what happens if there is a smooth surface?

S2: If it’s a smooth surface, then we remove frictional force. Then its acceleration changes & the tension also changes.

I: Do they increase or decrease?

S2: …(pause)… If we remove the frictional force, this acceleration … (pause) … will increase.

I: What will increase?

S2: The acceleration … will increase.

I: Tension?

S2: Tension will decrease.

I: So, it seems that there are 2 possibilities. If we double the mass of the hanging object B, when there is friction, you mentioned that acceleration will increase, speed will increase & tension will increase. When we remove friction, when we’re using a smooth surface, you said that.

S2: I’ve made an error … Tension will remain the same & acceleration will increase.

I: In which case?

S2: When it is a smooth surface.

I: So, when it’s a smooth surface, acceleration will increase.

S2: Increase, tension will increase.

I: Tension will increase. And how about speed?

S2: … Speed will increase.

I: Can we try it?

S2: Yes.

[He ran this simulation using the command RUN. Since motion was too fast for observation, he reset it and used slow motion to analyse the simulation. He then tried to select object A. Since the simulation was not over, the window command TO RESET appeared. So he reset the simulation, selected object A and verified that its mass was 1 kg. He repeated this procedure for object B to ensure that its mass was also 1 kg. He then selected object A, went to the command MEASURE and added on its VELOCITY meter. Finally, he converted the rough surface into a smooth surface by selecting the surface, opening its PROPERTIES BOX and equating KINETIC/STATIC FRICTION to zero. He ran the simulation in slow motion until object A nearly reached the end of the surface. At this particular point he noted the velocity of object A.]

I: […] Earlier you mentioned that when there is friction and mass of hanging object B is doubled, tension will increase.
S2: Yes.
I: You also stated that when there is no friction and mass of object B is doubled, tension will remain the same.
S2: Yes.
I: Can we verify this?

[He highlighted the string, went to the command MEASURE and selected the meter TENSION which appeared on the top left hand-side of the screen. He dragged the meter underneath the VELOCITY meter. He ran the simulation in slow motion for some seconds.]

S2: The masses are equal. [He opened the PROPERTIES box of each object.] This one [pointing at object A] is 1 and this one [pointing at object B] is 1.
I: Have you doubled the mass of B?
S2: Not yet. I want the value of tension when we have equal masses.
I: Ok. Note the value … So the tension is 4.903 N [reading from his paper as well as from TENSION meter].
S2: When the masses are 1 kg each, tension is 4.903 N.
I: Which type of surface?
S2: Smooth.
I: Now what do you want to do?
S2: Double this mass [pointing at object B].

[He reset the simulation, selected object B, opened its PROPERTIES BOX and changed its mass from 1 kg to 2 kg.]

I: […] What are you predicting? What will happen to tension?
S2: It will remain the same, eh, tension will increase. If we double the mass, tension [pointing at the vertical part of the string] will increase.
I: But didn't you say that if it is smooth, tension will remain the same?

S2: No. I said that when this mass [referring to object B] increases, tension will change and …

I: You told me that when the surface is rough and the mass of B is doubled, tension will increase. And then you stated that when the surface is smooth and the mass of B is doubled, tension will remain the same. […] Before we try it, what do you think? Will tension remain the same or increase?

S2: It will remain the same.

I: Why? […]

S2: … Because mass of A is still unchanged.

I: So, according to you, what factor influences the tension in the string?

S2: Object B.

I: Eh?

S2: Tension will increase. It will not remain the same.

I: What?

S2: The tension will increase.

I: Why?

S2: Because when the mass will be increased, the weight will increase.

I: But earlier you told me that.

S2: It will remain the same … Can I try the simulation?

I: Yes. Try it.

[He ran the simulation in slow motion, stopped it and recorded the new value of tension in the string (6.538 N).]

S2: Tension has increased.

I: Ok ... What happens if we now do the reverse?

S2: You mean, keeping this one [pointing at object B] unchanged & doubling the mass of this one [pointing at object A].

I: Yes. What will happen?

S2: There will be no motion.

I: There won't be any motion?

S2: No, no motion.

I: Whether it's smooth surface or rough surface? There won't be any motion?

S2: No motion.

I: Why?

S2: Because the weight of this one [pointing at object A] has been increased & the weight of this one [pointing at object B] has remained the same. If we make the mass of this one [pointing at object A] become 2 kg & the mass of this one [pointing at object B] become 1 kg, the weight of this one [pointing at object B] will be g & this one [pointing at object A] will be 2g.

I: Then?

S2: There won't be any motion.

I: Why? … How is this g (referring to weight of B) related to this 2g (referring to weight of A) so that there is no motion?

S2: … (silence) …

I: […] According to you, what will happen to the tension in the string?

S2: Tension will remain 4.903 N.

I: 4.903 N? We are talking about which type of surface.

S2: Smooth.

I: So, tension, according to you, will be the same. Why?

S2: The string will support … (pause) …

I: The string will support what?

S2: It will support this object [pointing at object B] … (pause) … and since there is no motion, there is also no acceleration & … (pause) … tension will be equal to weight.

I: It will not be equal to what?

S2: No, it will be equal to the weight of the hanging object B, and therefore equal to g.

I: Tension will be equal to weight of the hanging object B. You're telling me that this will be true when there is motion.

S2: No. When there is no motion! The system doesn't move.

I: And what happens to object A of mass 2 kg?

S2: Nothing. It remains stationary.

I: It remains stationary?

S2: Yes. Let's try the simulation.

I: Ok.

[He first selected the hanging object B, opened its PROPERTIES BOX and verified that its mass was equal to 1 kg. He repeated the above procedure for the sliding object A and changed its mass to 2 kg. He ran the simulation in slow motion. He reset the simulation, selected object A, opened its PROPERTIES BOX and ensured that STATIC/KINETIC FRICTION was still zero. He again ran the simulation in slow motion until object A was a bit more than half-way along the surface. He noted the tension in the string (6.538 N) and compared it to the weight of object B (9.807 N) which he had noted on a sheet of paper.]

S2: Tension isn’t equal to weight.

I: Why is tension not equal to weight?

S2: Because there is motion.

I: There is motion?

S2: … initially I had thought that there wouldn’t be any motion because this one [pointing a object A] is heavier than this one [pointing at object B].

I: Now you’ve seen that

S2: There is motion.

I: And do you believe in what you are seeing?

S2: Not all of them.

I: Why? Why is it that you don’t trust everything that is being shown in the simulation?

S2: … This object [pointing at object A] is heavier & it is smooth. Therefore, this one [pointing at object B] cannot move this one [pointing at object A]. If the masses of object A & object B were equal, then object B would have pulled object A towards the pulley. If the mass of object B would have been greater than that of object A, again there would have been motion. But now if the mass of object A is greater than that of object B, there should be no motion.

I: So you still think that this simulation is lying to you?

S2: Not necessarily … (pause:28 s) … what if we increase the mass of the sliding object A so that it is much bigger than that of the hanging object B, will there be motion?

I: What are you thinking about?

S2: If I make this one [pointing at object A] 5 kg & this one [pointing at object B] 1 kg? Can I try it?

I: Ok. Try it.

[He selected the sliding object A, opened its PROPERTIES BOX AND adjusted its mass to 5 kg. He then verified the mass of the hanging object B & ran the simulation in slow motion. He noted the values of tension and velocity.]

S2: The tension has increased. If we increase the mass of any object, the tension increases. When its mass [pointing at object A] is 5 kg, the tension becomes 8.172 N.

I: How can you account for this? Can we find an explanation for this? Initially when you doubled the mass of the sliding block A, we saw that there is motion & then tension increased. Now when you have increased the mass of object A by five times, there is still motion & we see that.

S2: The tension also increases.

I: The tension also increases. Right? What explanation can you provide for this observation? Have you noticed any other feature?

S2: Its velocity decreases.

I: Its velocity decreases? What explanation can we provide for this? When we increase the mass of the sliding object A, we see that tension.

S2: Tension increases.

I: Tension increases … (pause) … What can we say? Ok. Do you think that the more you increase the mass of the sliding block A, the more the tension will keep on increasing?

S2: Yes.

I: Say we make the mass of the sliding object A become 8 kg. What will be the tension?

S2: It will increase.

I: What value can you suggest for the tension? … A rough estimate?

S2: Maybe 10.

I: Can it not be 11 or 12?

S2: Yes.

I: Tension in the string, can it be 11 or 12?

S2: Can we try it out?

430 **I**: Ok.

[He changed the mass of object A to 8 kg & then ran the simulation in slow motion. He noted the values of velocity and tension in the string.]

431 **S2**: The tension increases, but by a small value. The tension is now 8.717 N. And the velocity decreases by a small value.

I: So?

S2: Let's increase the mass of object A. Let's make it 10 kg.

435 **I**: According to you, the more you increase the mass the more tension will increase? There is no limit on tension?

S2: No, it must have a limit.

I: Hmm?

S2: It must have a limit.

440 **I**: Why?

441 **S2**: The string might break.

[He ran the simulation after having changed the mass of object A to 10 kg. He noted the new values of velocity and tension (8.915 N).]

442 **S2**: There is still motion but it is slow, and tension increases … Let's try 15 kg.

444 **I**: Alright.

[He changed the mass of object A to 15 kg, ran the simulation and noted the values of the velocity and tension in the string (9.194 N).]

445 **S2**: The velocity decreases & the tension increases. Say we now change the mass of object A to 20 kg.

[He changed the mass of object A to 20 kg & ran the simulation in slow motion. He noted the new values of velocity and tension (9.340 N).]

447 **I**: What do we see?

S2: Velocity further decreases.
I: How about tension?
S2: Tension increases slightly.
I: Let's equate mass of object A to be 25 kg.

[He changed the mass of object A to 25 kg & ran the simulation in slow motion. He noted the new values of velocity and tension (9.429 N).]

S2: Velocity still continues to decrease and tension increases slightly.
I: I think we better go to 40 kg instead of 30 kg … Ok? Make it become 40 kg.

[He adjusted mass of object A to 40 kg & ran the simulation. He was using slow motion, but since we could hardly see the motion, I asked him to use the command RUN. The reading on the TENSION meter was 9.567 N.]

I: What do you see?
S2: There's almost no motion! It's moving very slowly…slight increase in tension.
I: Note the values. Ok? Now we make the mass become 60 kg & let's see what happens.

[He adjusted the mass of object A to 60 kg and ran the simulation. The reading on the TENSION meter was 9.646 N.]

I: What do we see?
S2: Velocity decreases & tension increases.
I: Ok. Have you noted these values?
S2: Not yet. I'll do it now.

[He recorded the values.]

S2: Let's now make the mass become 100 kg. Can I?
I: Fine.

[He adjusted the mass of object A to 100 kg and ran the simulation. The reading on the TENSION meter was 9.710 N.]

I: Now what can we see?
S2: Motion. Increase in tension … Is this possible?
I: What?
S2: This motion?
I: Here we're talking about a smooth surface … is this possible? This is a good question. In practice, can you get a smooth surface?
S2: Smooth surfaces don't exist.
I: Maybe. Why?
S2: After all, there should be friction … (pause) …
I: Maybe it's an ideal situation.
S2: Ideal situations don't exist.
I: But what you've mentioned is quite interesting - that is, the tension will keep on increasing until the string breaks.
S2: When we increase the mass of object A by 5, the value of the tension has increased by 0.7 … (pause) …
I: Ok. Let's make it 150.

[He made the mass of object A become 150 kg, ran the system & noted the reading on the TENSION meter: 9.742 N.]

S2: Tension has again increased. We make it 200.

[He changed the mass of object A to 200 kg, ran the system & recorded the values of tension (9.758 N) and velocity.]

I: Ok. Let's make it 300.

[He changed the mass of object A to 300 kg, ran the system & recorded the values of tension (9.774 N) and velocity.]

S2: It has increased.

485 **I**: What has increased?
S2: Tension … Let's make it 400.
487 **I**: Ok.

[He changed the mass of object A to 400 kg, ran the system & recorded the values of tension (9.782 N) and velocity.]

488 **I**: Let's make it 500 & then see what happens.
S2: No. Let's make it 800.
490 **I**: Ok. Let's go for 800.

[He changed the mass of object A to 800 kg, ran the system & recorded the values of tension and velocity.]

491 **S2**: 9.794 N, that's the tension in the string
I: Let's try 1000. Or do you want to go for higher than that.
S2: Let's go for 2000 kg.
494 **I**: Ok.

[He changed the mass of object A to 2000 kg, ran the system & recorded the values of tension (9.802 N) and velocity.]

495 **I**: What is happening to tension.
S2: It has increased.
I: Yes, but by how much?
S2: By a very small value, 0.008.
I: According to you, does tension have a limit?
500 **S2**: It is becoming closer to this weight [pointing at object B], g!
I: What can you say?
S2: Tension is equal to this weight [pointing at object B] and there is a tendency to equal this weight [pointing at object B].
I: Then what happens?
505 **S2**: … The string will then break.
I: According to you, at some point, can the magnitude of tension become greater than the magnitude of weight of the hanging object B?

S2: No. Tension will not be more than weight of object B.

I: But, earlier you told me that it can be 10 or 11 … Why, as you've just mentioned, the tension in the string tends to equal the weight of the hanging object B.

S2: … velocity is decreasing and a point will be reached when tension will be equal to the weight of object B, then there will be no motion.

I: Ok … So, will there be a limit on the value of tension? In such a case?

S2: Such cases don't exist … (pause) …

I: How about mass of object A = 5000 kg & mass of object B = 1 kg ?

S2: Do we try it?

I: Ok.

[He changed the mass of object A to 5000 kg, ran the simulation using the command RUN. The system moved very, very slowly.]

I: What can you say about the value of tension?

S2: 9.805 N. It has become nearer to the value of the weight of the hanging object B.

I: Yes.

S2: A time will come when the string will break as the tension is equal to the weight.

I: Now, why do you think that the string will break? On some occasions, you've been saying that the string will break.

S2: … (silence)…

I: When will the string break?

S2: … (pause) … no idea.

I: On a conclusive note, what can you say?

S2: Tension depends on weight.

I: It depends on the weight of which object?

S2: Of both objects.

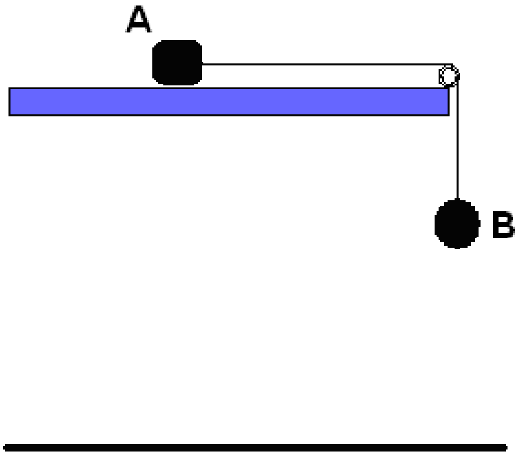
I: Ok.

Appendix C: Transcript for Student S3

I: Interviewer **S3: Student**

Line Transcript

[Task 1 was introduced to the student S3.]



I: Say I'm holding object A with my hand. How will the system behave if I remove my hand from object A? Do you think that the system will move or it will not move?

S3: It will move. Object A will be sliding along the surface and object B will move downwards.

I: Ok. Why do you say that there will be motion?

S3: There will be motion because they have different masses.

I: What do you mean by 'different masses'?

S3: They are not the same.

I: Say if this one [pointing at object A] is 1 kg, what can be the mass of this one [pointing at object B]?

S3: Eh … 3 kg.

I: Ok. So what happens?

S3: The weight of object B is greater than that of object A. On this basis, object B will pull object A.

I: Ok. So, according to you, what is pulling object A? […]
S3: B.
I: Object B?
S3: Weight of object B.
I: Ok. […] When will the system remain at rest?
S3: If mass of object A is greater than mass of object B …. And if there is friction.
I: Let’s assume that it is a smooth surface. There is no friction in the system.
S3: If mass of object A is greater than mass of object B, there is no motion.
I: What happens if both objects have equal masses?
S3: … The system will remain in equilibrium.
I: What does it mean?
S3: Neither object A nor object B will move.
I: This means that the system will remain at rest.
S3: Yes.
I: […] Let’s run the simulation and see what happens.

[He ran the simulation using the command RUN. As he allowed object A to leave the surface, I reminded him of the part of the simulation that was of relevance to us.]

I: You want to say anything with regard to the motion.
S3: I want to know the masses of the objects.
I: Eh?
S3: And the forces present in the system.
I: […] You can verify the masses.

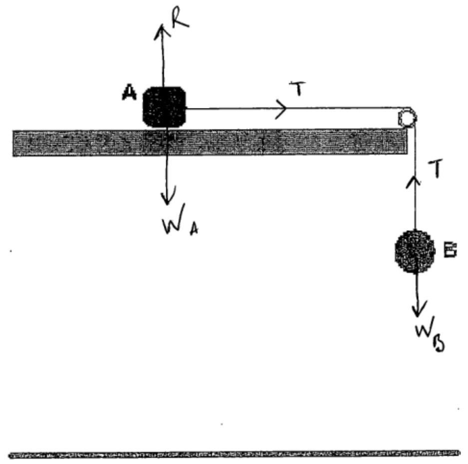
[He opened the PROPERTIES box of each object and found that the mass of object A, mA, was 1 kg and that of object B, mB, was 2 kg.]

I: Can you run the simulation in slow motion?

[He ran the simulation in slow motion for some seconds and the system moved as anticipated.]

I: I think that we can agree that there is motion when mass of object B is greater than mass of object A.
S3: Yes.
I: We can see object A moving along the surface [sliding a pen along the surface towards the pulley]. Do you think that its speed will remain constant, will increase or will decrease as it moves along the surface?
S3: Its speed increases.
I: OK. Now say, at a certain time t_1, object A is moving at 10 m/s. At the same time t_1, what will be the speed of object B? Will it be 10 m/s, or less than 10 m/s, or greater than 10 m/s?
S3: … Less.
I: Why less?
S3: … (pause) … This is because it will depend on their accelerations.
I: Ok. Can we discuss the acceleration of the hanging object B? If possible, can we assign a value to the acceleration of object B?
S3: … It will be this tension [pointing at the string above B] minus its weight [pointing at object B] divided by its mass.
I: Oh, acceleration of object B won't be g?
S3: No.
I: Why not g?
S3: Because the tension is opposing the weight. So its acceleration won't be g.
I: […] Do you think that both objects will move with same acceleration?
S3: Yes, acceleration will be the same for both objects.
I: Why do you say that they will be the same? You told me that speeds of both objects won't be equal.
S3: No, accelerations of both objects will not be equal.
I: You just told me that they should be equal.
S3: No, accelerations will not be equal.
I: Why?
S3: Because forces acting on this one [pointing at object A] are different from the forces acting on this one [pointing at object B].
I: Can you draw me a force diagram?

[He drew the following force diagram for Task 1.]



I: From your force diagram, can you explain to me what you mean by 'the forces are different'?

S3: See [pointing at his force diagram] we have a 'REACTION' force on object A … and the weights of both objects are not equal, this one [pointing at object A] is 1g and this one [pointing at object B] is 2g … so different forces on each object.

I: Ok. This is why you think that due to different forces on both objects, both objects will move with different speeds and accelerations?

S3: Yes.

I: What do you want to do now?

S3: Measure the velocities of both objects.

I: Ok. Go ahead.

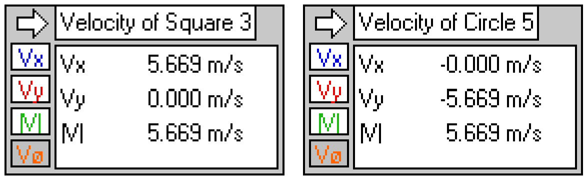
[He put on the VELOCITY meters of objects A & B.]

I: What do you predict? [laughter]

S3: The velocity of this one [pointing at object A] will be greater. [laughter]

I: Ok. Let's try it.

[He ran the simulation in slow motion for some seconds and then stopped it. He moved the cursor to and fro between the VELOCITY meters of objects A & B.]

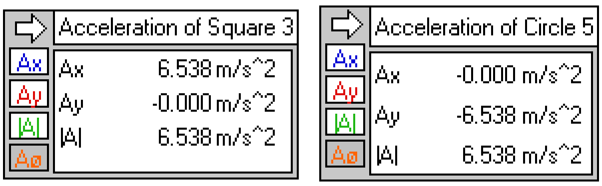


I: What happened?
S3: They have the same velocity. [laughter]

[He continued to run the simulation in slow motion and appeared to study the VELOCITY meters.]

S3: This means that this object [pointing at object A] has a horizontal velocity and this one [pointing at object B] has a vertical velocity.
I: So?
S3: The horizontal velocity of this one [pointing at object A] is equal to the vertical velocity of this one [pointing at object B].
I: […] Do you believe in this simulation?
S3: … (pause) … No.
I: Eh? You don't believe in it.
S3: No. I am not able to understand it.
I: Ok. Don't worry. We measure the accelerations now. What are we expecting?
S3: Accelerations of both objects should be different.
I: Let's try the simulation and see if the accelerations are different as per your prediction.

[He reset the simulation, put on the ACCELERATION meters for objects A & B and then ran the simulation in slow motion for a few seconds.]



S3: The accelerations are the same.

[He continued to run the simulation in slow motion.]

106 **S3**: {…} I can understand now.

I: What happened?

S3: The accelerations, at any time, will be the same.

I: Do you agree with it or not?

110 **S3**: Yes.

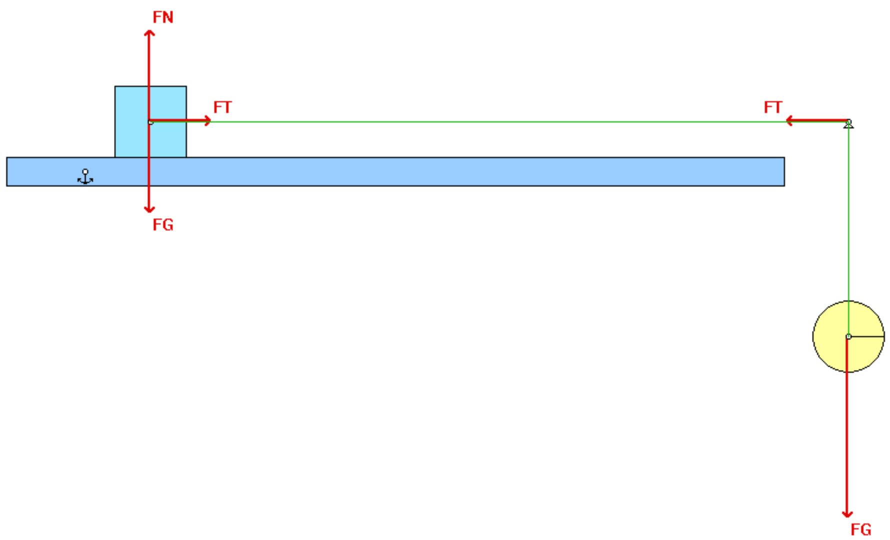
I: Why do you now accept that both objects should have equal accelerations? Initially you told me that they are not going to be equal because there are different forces acting on these objects.

S3: Because they are connected […] they move together.

115 **I**: Ok.

S3: Now I want to see the forces on each object.

[He selected the string and placed vector arrows (FT) on it to represent tension in the string via the DEFINE command. He then selected object A and added a vector arrow (FN) on it to represent CONTACT FORCE. After that he selected object B and added a vector arrow (FG) on it to represent GRAVITATIONAL FORCE. He repeated the same procedure with object A. He ran the simulation in slow motion and the following system appeared on the screen.]



[He reset the simulation and clicked on the string.]

117 **I**: What are you trying to find out?

118 **S3**: I want to see the direction of the tension along the string.

I: It is already showing you.

120 **S3**: It is only here [pointing at arrows FT in the horizontal part of the string].

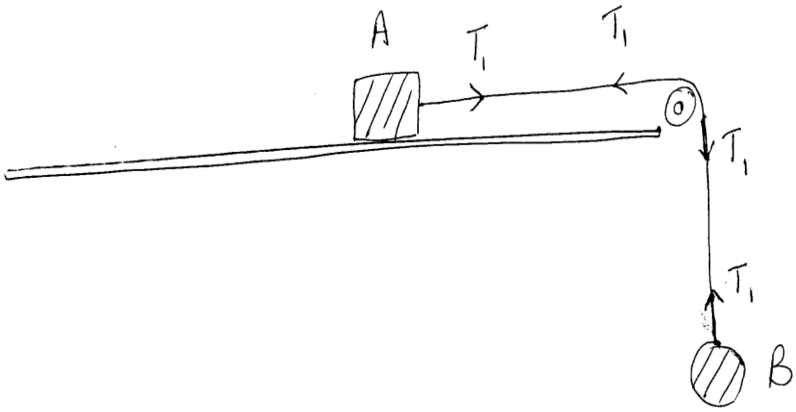
[He ran the simulation in slow motion for some seconds and then stopped it.]

121 **S3**: Why are there no arrows here [pointing at the vertical part of the string]?

I: In fact, the arrows should have been there as well. They should be on both sides of the string. But the software doesn't do it …(pause)… So
125 here [pointing at centre of object B] we should have an arrow here [pointing upwards along the string away from the centre of object B] and one arrow here [pointing downwards along the string away from the pulley]. The software doesn't do it … Ok?

S3: Yes.

130 **I**: See here. [I drew the following force diagram with the vector arrows in the string.]



132 **I**: What did you want to learn from it?

S3: {…}

I: … earlier you were talking about forces.

135 **S3**: Yes.

I: You were talking about weight and tension here [pointing at object B]. Are you thinking that weight is equal to tension here [pointing at object

B]?

S3: No, they will not be equal. If object B is moving down, weight must be greater than the tension.

I: […] What do you want to do now?

S3: To have equal masses.

I: Ok.

[He maintained the mass of object B, mB, as 2 kg, but changed the mass of object A, mA, from 1 kg to 2 kg.]

I: Earlier you told me that the system would remain at rest. Do you maintain that it will remain at rest?

S3: … Yes, it will remain at rest.

I: Why? Why do you think that it will remain at rest?

S3: Because this one [pointing at object A] is the same as this one [pointing at object B].

I: Can you explain it again?

S3: Weight of object B is the same as the weight of object A.

I: Ok.

S3: […] Let’s run the simulation.

I: No. Tell me before we run it. What happens when the mass of object A is greater than the mass of object B?

S3: It will not move.

I: Why? […]

S3: If the mass of this one [pointing at object A] is greater then it will not move … This force [pointing at object B] will not be enough to move object A.

I: Which force?

S3: This weight [pointing at object B] will not be sufficient to move object A.

I: Ok. Let’s run it.

[He ran the simulation and the system moved.]

S3: It moved!

I: […] Why did you initially predict that there would be no motion?

S3: Because of the forces acting on them.

I: Initially you suggested that there would be no motion if mA = mB and mA > mB. Why?

S3: Forces acting on object A are equal to forces acting on object B. So, the objects must be in equilibrium.

I: In both cases? Be it mA = mB and mA > mB? In both cases, will the forces be equal?

S3: If mass of A is greater, object B won't move.

I: Do you agree with this motion?

S3: … Yes.

I: Why?

S3: There is a force that will pull it [pointing at object B] down. The weight will pull it down. So it will move down. Whereas here [pointing at object A] …(pause)… there will be a reaction acting on it.

I: Can you explain it again?

S3: Weight of object B will pull object B and also pull object A.

I: But what you stated earlier in terms of their forces, will they not be the same? … Didn't you say that both objects have equal weights?

S3: Yes.

I: What do you think?

S3: But this one [pointing at object A] has a contact force.

I: So?

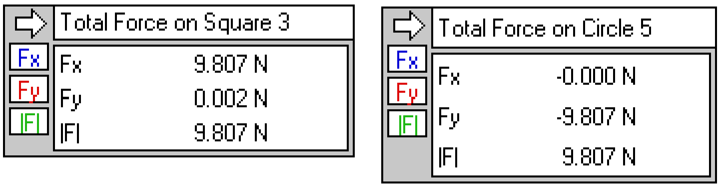
S3: This weight [pointing at object A] has no influence on the motion of object B.

I: Did you initially have this idea?

S3: No. I initially thought that the weight [pointing at object A] will not allow object B to move.

I: So how can we explain that there is motion when mA = mB? You can use Interactive Physics or any other thing to verify your reasoning.

[On a new screen, he switched on the ACCELERATION meters for both objects A & B. He then ran the simulation in slow motion for some time and reset it. He selected object A and put on its VELOCITY meter; he did the same for object B. He ran the simulation in slow motion for some seconds and then reset it. He put on the TOTAL FORCE meter for object B and ran the simulation. He reset the simulation and put on the TOTAL FORCE meter for object A. He again ran the simulation in slow motion for some time and the values on both TOTAL FORCE meters were 9.807 N.]



I: What are you trying to do?
S3: I wanted to know the forces acting on this one [pointing to object B] and on this one [pointing to object A] … how can these forces be equal to g?
I: What is your query?
S3: Should the force not be 2 times g?
I: Which force?
S3: On object A, on object B.
I: What are you looking for?
S3: Its force, its total force, that is its net force.
I: What do you mean by ‘net force’?
S3: Resultant force [moving cursor around object B].
I: What were you expecting?
S3: That the total force is equal to the weight.
I: Here [pointing at object A] as well, were you expecting total force to be equal to weight?
S3: No, not here [referring to object A].
I: So, what were you expecting here [pointing at object A]?
S3: I was expecting the total force acting on object A to be equal to the weight of this one [pointing at object B].
I: So, what would you have proven then?
S3: … This one [pointing to B] is pulling this one [pointing to A] with the same force … The force that is acting here [pointing at object B] is also acting in this direction [pointing at the horizontal part of the string towards the pulley] and therefore pulling it [referring to object A].

I: Can you explain it again?

S3: This means that this weight [pointing at object B] is pulling this one [pointing at object A], the force that is acting on object B is pulling object A with the same magnitude.

I: Oh. If we have the same magnitude at objects A and B, this would have meant that object B is pulling object A?

S3: Yes.

I: Ok.

[He ran the simulation in slow motion for a few seconds and the values on TOTAL FORCE meters were 9.807 N.]

I: The readings [pointing to both TOTAL FORCE meters] are the same. What were you expecting?

S3: Why is it not equal to 2g? It is equal to g only. Why?

I: Ok … So, if here [pointing at TOTAL FORCE meter of object A] it would have been 2g and here [pointing at TOTAL FORCE meter of object B] it would have been 2g, you would have been pleased?

S3: Yes.

I: So, your question is why TOTAL FORCE is equal to g and not 2g?

S3: Yes.

I: Ok. Think about it.

[After almost 45 seconds of complete silence, he murmured the word ‘tension’ and wished to verify its value. So, he reset the simulation, highlighted the string and put on the TENSION meter. He ran the simulation in slow motion for some seconds and the reading on the TENSION meter was 9.807 N.]

S3: It is the tension that is cancelling the … the weight [pointing at object B] is acting downward and the tension is acting upward, so the net force is equal to 2g minus the tension, and it will be equal to 9.81.

I: […] So, did you succeed in explaining why total force is not equal to 2g?

S3: Yes. Tension is cancelling, is acting here [pointing along the vertical part of the string].

I: What are you thinking?

S3: Why is tension equal to 9.807 N?

I: How do we get tension in a string?

S3: {…}

I: So what value were you expecting?

S3: Another value. Not g.

I: What other value were you expecting? … More than g? Less than g? What were you expecting?

S3: I expected the tension to depend on the string [pointing at the vertical part of the string, just above object B].

I: What do you mean by 'tension depends on the string'?

S3: I mean the characteristic of the string.

I: […] I thought that you initially said that tension is due to the weight of object B?

S3: Yes.

I: That's what I thought. I may be wrong. Do correct me if I'm wrong.

S3: Yes, that's what I said.

I: Or did you mention that it depends on both weights? I can't remember.

S3: No. It depends on this weight [pointing at object B] … on both objects.

I: What?

S3: It depends on both objects.

I: Can we discuss it again? Does tension in the string depend on both objects or on only 1 object?

S3: … It depends on both objects.

I: Eh?

S3: No. Only on one. Only on this one [pointing at object B].

I: Why are you again changing your ideas?

S3: Tension is being created in the string and it is because of this [pointing at object B] that tension is created.

I: What?

S3: This one [pointing at object B] is creating the tension and it is pulling the other [referring to object A].

I: So this means that the tension in the string is due to object B only.

280 **S3**: Yes.

I: Ok.

282 **S3**: Yes.

[He reset the simulation, added vector arrows (FT) to represent the TENSION in the string and ran the simulation in slow motion for 3 steps. The following screen shot appeared:

On a sheet of paper, he wrote the following equations of motion:

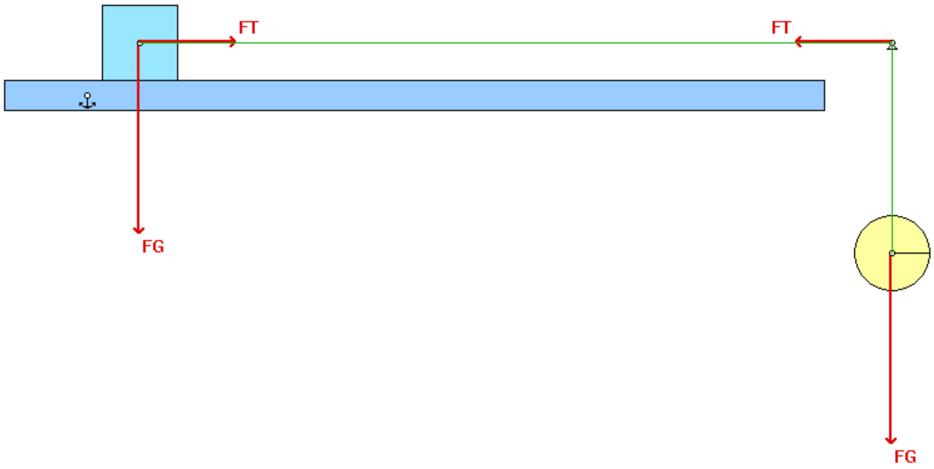
$$T - 2g = 2a$$ 1st equation

$$2g - T = 2a$$ 2nd equation

283 **S3**: Both objects are needed to create tension.

284 **I**: Why? … According to you, why?

[He resets the simulation. The TENSION arrows (FT) were already present. He selected object A and added a vector arrow (FG) for the GRAVITATIONAL FORCE on it. He then selected object B and added its GRAVITATIONAL FORCE arrow (FG). He ran the simulation in slow motion for a few steps. The following screen shot appeared:



I: What are you trying to enquire? I see that you've written these equations of motion [pointing at them].

$$2T - 2g = 0$$
$$T = g$$

S3: Through my calculations, I found that tension is equal to g.

I: [laughter] … What are you trying to investigate at the moment? […]

S3: Net force at this point [pointing at centre of object A] is the tension minus this weight [pointing at GRAVITATIONAL FORCE arrow (FG) of object A].

I: What?

S3: I take the tension here [pointing at TENSION arrow (FT) on object A] minus the weight [pointing at GRAVITATIONAL FORCE arrow of object A (FG)]. And I will call this the net force.

I: How did you find the net force here? Net force on object A is what?

S3: Tension in the string minus weight of object A.

I: And this gives you?

S3: Net force, that is the resultant force.

I: Ok. Did you learn this here?

S3: No. In our mechanics class.

I: Ok.

S3: But now that I'm seeing it, how can this be possible since the tension is acting in this direction [pointing horizontally] and the weight is acting in another direction [pointing vertically]? […] So, one cannot minus the other.

I: Ok. So this is giving you another way of looking at it.

S3: Yes. Here [pointing at object B] both the tension and the weight are acting along the same direction [moving cursor vertically] and so we can have the weight minus the tension to have the net force.

I: So, initially, was this how you were finding the net force on object A?

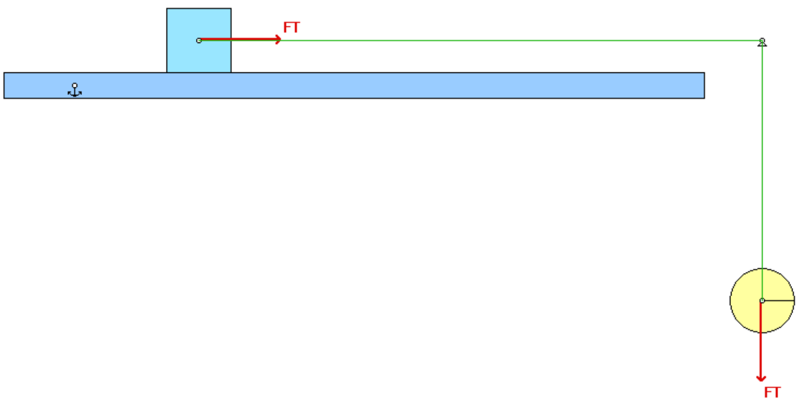
S3: Yes.

I: And what do you find here?

S3: Here [moving cursor around the centre of object A] it is not possible.

I: … Perhaps you should try putting on the vector arrow for TOTAL FORCE on each object. This may help you.

[He removed the GRAVITATIONAL FORCE arrows from both objects and added the TOTAL FORCE arrows on both objects. He ran the simulation. Since in Interactive Physics both the TOTAL FORCE arrow and the TENSION arrows use the same abbreviation FT, I asked him to eliminate the TENSION arrow in the string. He then ran the simulation. The following screen shot appeared:



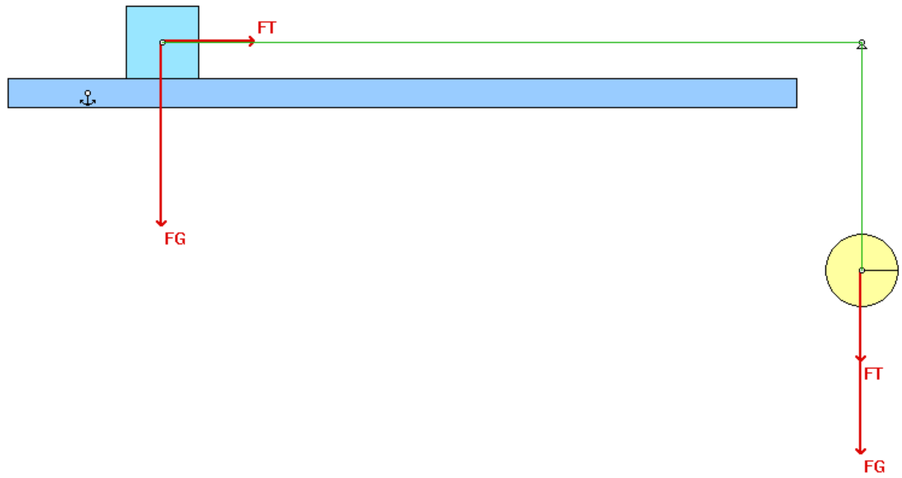
I: What do you find?

S3: The force here [pointing at TOTAL FORCE arrow on object B] is pulling this one [pointing at object A].

I: What?

S3: The force here [pointing at TOTAL FORCE arrow on object B] is acting on this one [pointing at object A]. It is the same … It is not related to the weight [pointing at object A].

[He reset the simulation, added the vector arrows for GRAVITATIONAL FORCE on both objects and ran the simulation in slow motion for a few seconds. He remained silent for a long time.]



S3: My conclusion is that this net force [pointing at arrow on object B] is pulling object A […] the net force on object B is equal to the net force on object A.

I: So, what are you trying to suggest?

S3: That the net force on object A [pointing at arrow FT near object A] does not depend on its weight [pointing at object A].

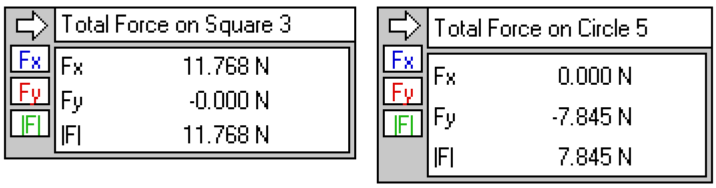
I: Ok. […] Let's change the mass of object A.

[He changed the mass of object A from 2 kg to 3 kg. mB remained 2 kg.]

I: And do we expect the system to move?

S3: No.

[He ran the simulation and the system moved. The reading on TOTAL FORCE meter for object A was 11.768 N and that for object B was 7.845 N.]

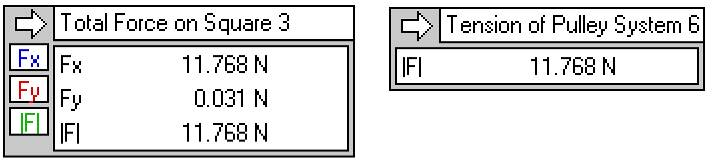


S3: Ah! It moved!

I: What happened?
S3: It moved.
I: Do you agree?
S3: … Yes.
I: Why?
S3: Because A doesn't affect motion of B […] But the forces are not equal.
I: Which forces are not equal?
S3: Net force on object A is not equal to the net force on object B.
I: What were you expecting?
S3: They should be equal since there is motion.
I: So, what can we say?
S3: The weight [pointing at object A] has a role to play in it.
I: Earlier you were saying that it doesn't play any role.
S3: No, it does [laughter].

[He reset the simulation and ran it in slow motion. He was scribbling something about tension on paper.]

I: So, what can you say? I find you working with tension.
S3: Tension is equal to the net force on object A …(silence)…



I: What are you writing?

[He wrote the following set of equations:

$$T - 3g = 3a \quad -(1)$$

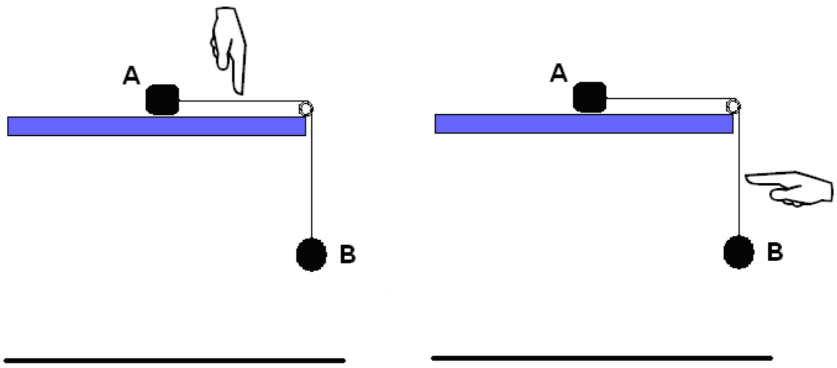
$$2g - T = 2a \quad -(2)$$

S3: The net forces are not equal.
I: Hmm?

S3: The tension will be the same for both.

I: What?

S3: The tension will be the same for both [pointing cursor at objects A & B] … because it is the same tension that is acting in both [pointing to the vertical and horizontal parts of the string].



pointing at horizontal part (string) pointing at vertical part (string)

I: What do you mean?

S3: In the previous system [referring to system mA = mB], the net forces of objects A & B were equal.

I: Ok. And now, in this system, we find that they

S3: Are not equal.

I: They are not equal.

S3: This is because its weight [pointing at object A] is different … and tension in both [pointing at the vertical and horizontal parts of the string] is the same.

I: What do you mean by ‘tension in both is the same’?

S3: Tension throughout the string is the same.

I: Did you think that it wouldn’t be the same? Can you clarify it?

S3: The net force will depend on the weight.

I: Which net force?

S3: Of object A or B.

I: Ok.

S3: Net force that is acting on object A depends on its mass [pointing at object A]. It is the same for object B. If I vary the mass, its net force will also vary.

I: Ok … Were you not expecting the net force to vary when the mass is changed?

S3: I thought that the net forces for both objects must be equal.

I: Can you be more specific?

S3: This one’s net force [pointing at object B] must be equal to this one’s net force [pointing at object A].

I: For any value of mass?

S3: I thought that the weight of this one [pointing at object A] would not influence the net force acting on it.

I: How about object B?

S3: Here [referring to object B] I knew that it would make a difference to its net force.

I: So, what can we say?

S3: The weight [pointing at object A] is influencing its net force ... and it doesn’t depend on this one [pointing at object B]. The net force of each object is different.

I: Have you been able to find how to calculate the net force on each object? For example, for this one [pointing at object B], you told me that the net force is.

S3: Weight minus tension [pointing at object B].

I: And here [pointing at object A] you were saying that initially you had thought that net force is equal to tension minus weight. Do you still maintain it?

S3: Yes … Tension minus weight is equal to net force.

I: Is this what you are inferring? … Earlier you mentioned that you had learnt about it in your mechanics class. But when you ran your last simulation, you were explaining that this could not be true – that is, net force doesn’t equal to tension minus weight.

S3: When we had equal masses?

I: Yes … So, what do you think for this system?

S3: … tension minus weight.

I: Do you get the net force on object A by removing weight from the tension?

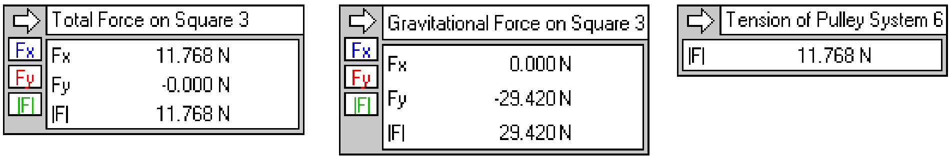
S3: Yes.

I: What is the weight of object A?

S3: 3g.

I: If you wish, you can put on the GRAVITATIONAL FORCE meter for object A on the screen.

[He put on the GRAVITATIONAL FORCE meter for object A on the screen. He ran the simulation in slow motion for a few seconds.]



I: So, does net force equal to tension minus weight?

[He worked out a calculation on paper.]

S3: No, they are not equal.

I: What?

S3: They are not equal …(pause)… Their accelerations are equal [pointing at objects A & B]. This is why their net forces must be different. If we use F = ma, we see that they are different.

I: Ok. This is by theory. But just as you (had) stated earlier that net force acting on object B is weight minus tension, in the same way how do we find the net force acting on object A? Earlier you explained that it should be tension minus weight.

S3: Yes.

I: Then you found in the previous system that it couldn't be tension minus weight. And you also found that both of them didn't lie in the same axis. Now you are telling me that it should be the tension minus weight to obtain the net force. Are you obtaining the net force in this case?

S3: No.

I: So, how do you calculate the net force on A in this case?

S3: …(silence)…

I: Try to run the simulation again … Let's analyze the TOTAL FORCE meter acting on object A. What can we observe?

[He ran the simulation in slow motion for a few seconds.]

S3: The tension value is equal to the net force acting on object A.

I: You were earlier talking about weight. Let's analyze and see.

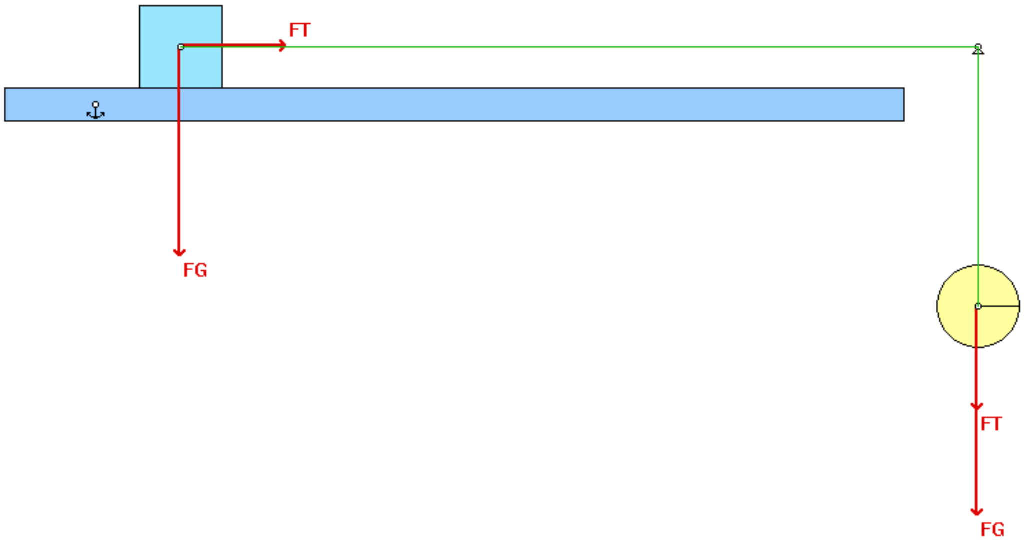
S3: The net force on A is equal to the tension, not the weight.

I: Do we find the weight influencing the net force on object A?

S3: No ...(pause)… For this one [pointing at object A], net force is equal to the tension … the weight [pointing at object A] does not have any effect upon it [referring to the net force] …(silence)…

I: If you wish, you can compare this system with your previous system [referring to the system with mA = mB = 2 kg].

[He changed mA to 2 kg and ran the simulation in slow motion for some seconds.]



S3: The net force [pointing at object A] is equal to the tension [pointing to the TENSION meter] … Weight doesn't affect the net force because it is not acting along the same axis. Net force is along the x-axis, that is moving horizontally, not up and down, that is not in a vertical line. However, here [pointing at object B] it is moving downwards and therefore weight must be taken into consideration.

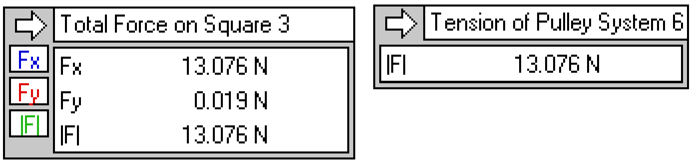
I: How can you confirm your finding that net force at this point [referring to object A] is equal to tension only? […]

S3: Change the masses.

I: Try it.

[He reset the simulation and changed the mass of object B, mB, to 4 kg.

The mass of object A, mA, continued to be 2 kg. He then ran the simulation in slow motion. He noted that the net force on A was equal to tension, which was equal to 13.076 N.]



456 **I**: So, is net force on A still tension minus weight?

S3: No … I can now understand. For this one [pointing at object B], it is weight minus tension because they are along the same axis, whereas
459 here [pointing at object A] … Ok.

[As we were about to close the session, he added the following:]

460 **S3**: Why do we use these equations [referring to T – 3g = 3a & 2g – T = 2a] that I've written to do calculations in M1?

I: […] Why is there motion when mA > mB?

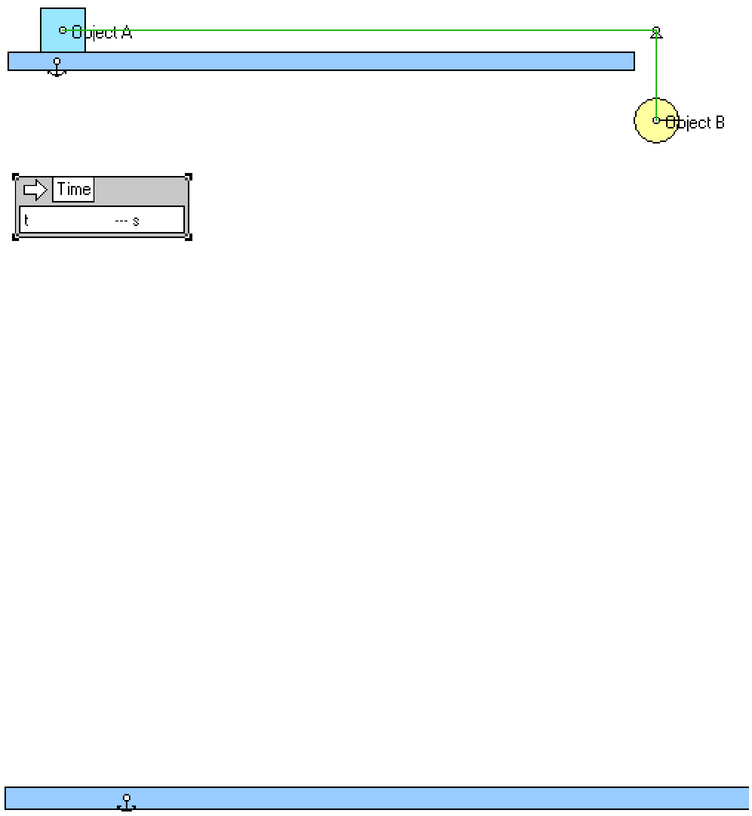
S3: Whenever there is a net force on B and it is independent of the
464 weight of A.

Appendix D: Transcript for Student S5

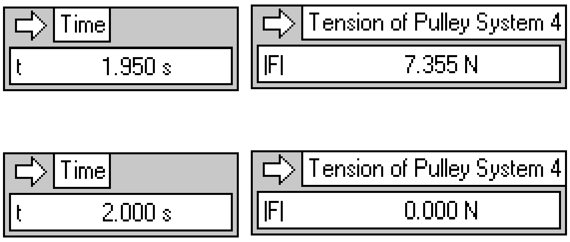
I: Interviewer S5: Student

Line Transcript

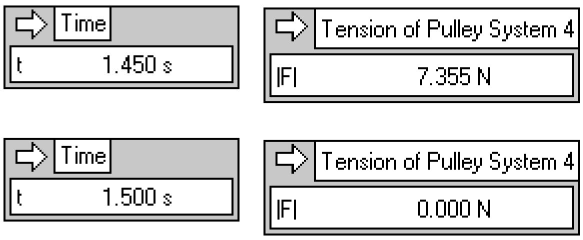
[Task 6a was discussed with Student S5 and the shortcoming of Interactive Physics with regard to this task was highlighted. When a string breaks during a simulation in Interactive Physics, it should (physically) become a dotted line. However, the software version at hand was not able to simulate this feature and therefore the broken string continued to be represented by a complete/full line.



Student S5 already knew that when the string would break, tension in the string must be equal to zero. Using this piece of knowledge, she was asked to find the time at which the string would break in the simulation provided. So she added the TENSION meter, ran the simulation using the command RUN and finally stopped it when she saw the reading on the TENSION meter becoming zero. She explained that the string broke at t = 2.750 s because tension has become zero. So I asked her to move the simulation 1 step backward. On doing so, she saw that the tension was still zero and she continued to move the simulation backward until t = 1.9 s where tension became 7.355 N. She moved the simulation 1 step forward (t = 1.95 s) and tension was still 7.355 N. She again moved the simulation 1 step forward (t = 2.0 s) and she found that tension became zero when t = 2.000 s, implying that the string broke at this instance.



I then modified the required parameter in the simulation so that the string would break at t = 1.500 s. She ran the simulation using the command RUN, reset it and then ran it via slow motion to show that the string had indeed broken at t = 1.500 s.



For the first part of this activity, she explored the accelerations of object B and object A sliding along a smooth surface. It is useful to note that during the discussion of the acceleration of object B just after the string broke, she had the intuitive idea that it was not logical for its acceleration to change from 2.452 m/s^2 to 9.807 m/s^2 all of a sudden. We then started to investigate the acceleration of object A along a rough surface. She found that the acceleration of the system was 1.717 m/s^2 just before the string broke.]

I: What may happen to the acceleration of A once the string is broken?

S5: It will become zero.

I: As soon as the string breaks?

S5: … Yes.

I: So, you think that it is similar to previous system where there was no friction ... There from 2.452 it becomes

S5: Zero.

I: Zero instantaneously. How about here?

S5: It will decrease until it will become zero.

I: From 1.717 what will happen?

S5: It will decrease until it will become zero.

I: […] How about its speed? Once the string breaks, do you think the speed will increase, will decrease or it will move with a constant speed?

S5: Once the string is broken, object A will stop.

I: You think that once the string is broken, the object will stop. Is this what you are thinking?

S5: … It won’t stop instantaneously.

I: What?

S5: When the string breaks, object A will move a bit and will then stop.

I: Ok. You want to verify the acceleration?

[She reset the simulation, put on the ACCELERATION meter for object A and then ran the simulation using RUN. She reset it and ran it using RUN again. She reset it.]

I: Did you find anything?

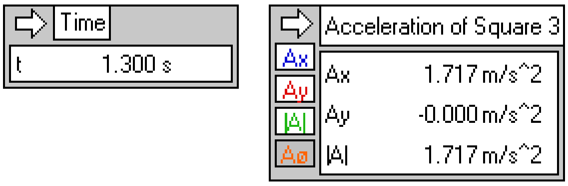
S5: …(silence)…

I: Let’s run it in slow motion between just before and after the string breaks.

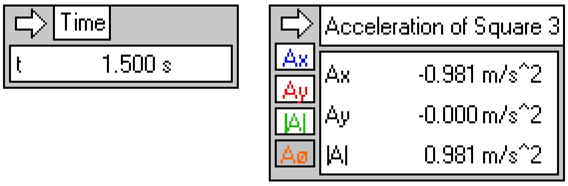
[She moved the simulation at t = 1.3 s.]

I: Now it is 1.717. What happens when the string breaks?

[She ran the simulation 4 steps forward up to t = 1.5 s.]



Before the string breaks



Just after the string breaks

S5: It decreases.
I: It decreases?

[She continued to run the simulation in slow motion up to t = 3.65 s.]

I: Continue to run it.

[She re-started to run the simulation in slow motion up to t = 4.55 s where object A stopped and Ax as well as |A| became zero.]

I: What do you find out?
S5: It becomes zero.
I: Yes, it becomes zero as you mentioned earlier. But I don't see its value decreasing from 1.717 to zero. This has not taken place. What has happened in this system?
S5: Once the string breaks, the acceleration of object A decreases until it becomes zero.
I: It decreases? Does it mean that it decreases continuously?
S5: Yes.
I: Is this what we have observed? When the string breaks, do we see on Interactive Physics the acceleration of A continuously decreasing until it becomes zero?

[She ran the simulation a few steps backward from t = 4.55 s to t = 3.65 s in slow motion. Between t = 4.55 s and t = 4.0 s, Ax changed from 0 m/s^2 to -0.981 m/s^2 and |A| changed from 0 m/s^2 to 0.981 m/s^2. Between t = 3.95 s and t = 3.65 s, Ax remained -0.981 m/s^2 and |A| remained 0.981 m/s^2.]

S5: For some time it is a constant and then becomes zero.

[She re-started to run the simulation backward in slow motion.]

I: […]. But this is not what you were saying earlier. You said that it would decrease continuously until it would become zero. This has not taken place. According to you, how is the system behaving?

[She continued to run the simulation backward in slow motion until t = 1.5 s. After some seconds. She first ran it forward in slow motion up to t = 1.9 s and then moved it to t = 3.35 s.]

I: What are you trying to understand from this?
S5: When the string breaks, the acceleration remains constant for some time and then decreases to zero.
I: Do you agree with it?
S5: Eh? …
I: What is it that you agree with and what is it that you don't agree with?
S5: If I look at it in terms of velocity, it makes sense.
I: What do you mean by 'in terms of velocity, it makes sense'?
S5: When the string breaks
I: Ok.
S5: Object B is out of the system, then velocity decreases.
I: Yes.
S5: Until it becomes zero. Acceleration remains constant.
I: Ok.
S5: And it then becomes zero. When velocity becomes zero, acceleration also becomes zero.

62 **I**: You agree with these?

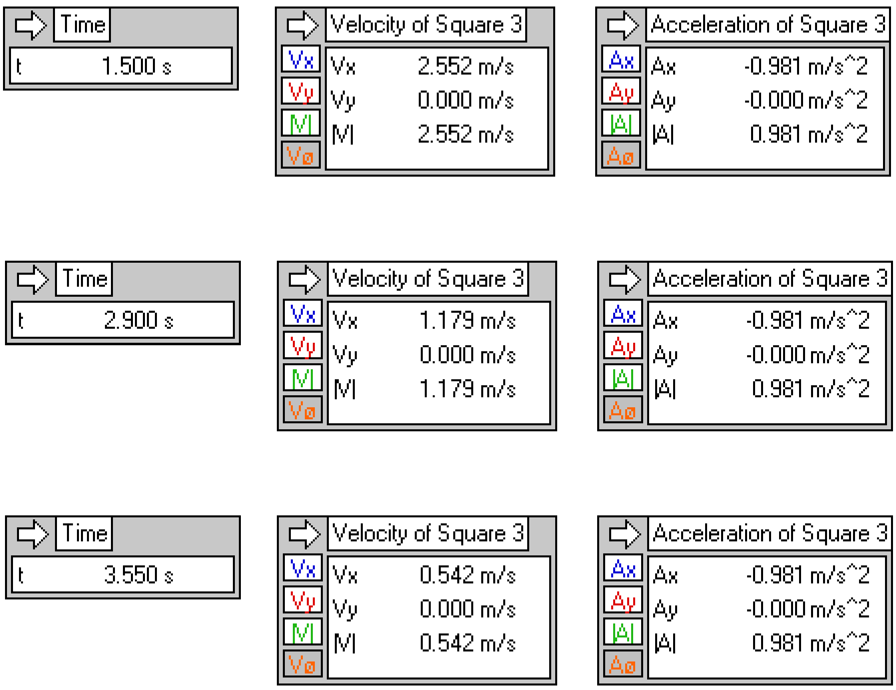
S5: Yes

64 **I**: […] Can we try it?

[I deleted the ACCELERATION meter of object B.]

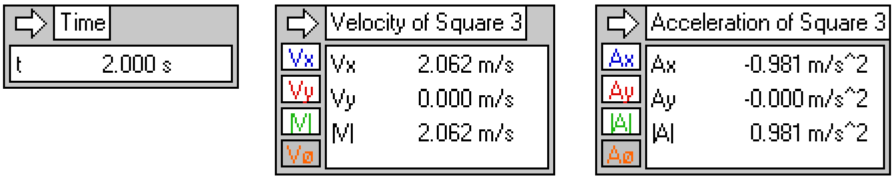
65 **I**: Put on the VELOCITY meter. […]

[She put on the VELOCITY meter of object A. She ran the simulation using RUN up to t = 9.1 s, reset it and ran it in slow motion up to t = 1.5 s. After some seconds, she first ran the simulation to t = 2.9 s and then moved it to t = 3.55 s by using the tape player control. She finally started to run the simulation backward step by step.]



66 **I**: What can you observe?

[She continued to run the simulation backward in slow motion up to t = 2.0 s.]



S5: The speed decreases.

[Using the tape player control she moved the simulation to t = 1.4 s and then ran it forward step by step up to t = 3.6 s. She remained silent.]

I: […] Can we describe the acceleration of object A just before the string breaks?

S5: […] acceleration is constant, 1.717 m/s^2, just before the string breaks.

I: Then?

S5: Velocity increases.

I: Then?

S5: When the string breaks, acceleration changes to 0.981.

I: Then?

S5: Velocity also decreases. Acceleration becomes constant, it becomes zero. Velocity also decreases until it becomes zero.

I: So there are two points of interest in this motion. The first one occurs before the string breaks where the acceleration is 1.717 and its velocity increases. The second scenario occurs after the string breaks the object, according to you, continues to accelerate but its speed is …

S5: Its speed is decreasing.

I: Is decreasing … Ok. But the question is: can the speed of an object decrease when it is accelerating?

S5: … No. Only when it decelerates ... This can happen if it decelerates.

I: […] There are two parts to this motion. The first part occurs before the string breaks and the second after the string breaks. After listening to your explanations, it appears that you have learnt from IP that before the string breaks, object A moves with an acceleration 1.717 and when it moves with this acceleration, its speed increases. Once the string is broken, the object starts to move with acceleration 0.981 but during this stage its speed decreases. So my question is when we have acceleration, does speed increase or decrease, or are both ‘cases’ possible?

S5: … Both ‘cases’ are possible.

I: Eh?

S5: Both ‘cases’ are possible.

I: And in both cases we call it acceleration?

S5: No. One will be deceleration.

I: In which case?

S5: When speed is decreasing.

I: […] Before the string breaks, do you think that the object decelerates or accelerates?

S5: It accelerates.

I: And, after the string breaks, what can we say?

S5: … It decelerates. Object A decelerates.

I: Sure?

S5: Yes.

I: Earlier you said that it was accelerating with 0.981?

S5: …(pause:12)… I didn’t pay much attention to this idea of accelerating or decelerating.

I: My understanding, after having listened to you, is that you thought that the acceleration changes from 1.717 to 0.981 and that it is still accelerating. The speed should have increased, shouldn’t it?

S5: …(pause:14)… But the string breaks when the acceleration becomes 0.981.

I: When we say an object is accelerating, does it not mean that its speed is increasing?

S5: Yes

I: … Honestly speaking, did you think that after the string breaks it continues accelerating?

S5: Yes, but I didn’t think in terms of acceleration I thought in terms of deceleration. As you were using the term accelerate, I didn’t think of the term decelerate.

I: Did you not realise from the ACCELERATION meter that it was decelerating?

S5: No.

I: Did the ACCELERATION meter not give you an indication that the

object was decelerating?
S5: Yes, it did.
I: When?
S5: When the acceleration started to decrease until it became zero.
I: How about when the string breaks?
S5: … It’s constant, the acceleration is constant.
I: Ok
S5: Then it decreases.
I: And when the string breaks?
S5: It decelerates.
I: How do you know?
S5: The acceleration changes instantaneously.
I: Ok. I agree that it changes instantaneously. But how do you know that it decelerates at that moment?
S5: Because the acceleration decreases.
I: When the acceleration changes from 1.717 to 0.981, does this mean that there is deceleration? … The magnitude of acceleration has decreased, but the speed keeps on increasing.
S5: Yes.
I: Let’s look at the ACCELERATION meter just before the string breaks.

[She moved the simulation backward from $t = 1.7$ s to $t = 0.6$ s.]

I: What is the reading on the ACCELERATION meter?
S5: 1.717
I: Ok. Let’s run it in slow motion.

[She ran it in slow motion up to $t = 1.5$ s and |A| changed from 1.717 m/s^2 to 0.981 m/s^2.

I: What can you observe?
S5: There is a decrease in acceleration.
I: Which value are you looking at in the ACCELERATION meter?
S5: This one [pointing to |A|].
I: Ok. So you are looking at modulus of A.

156 **S5**: Yes.

I: Ok. Go one step backward in the simulation.

[She ran the simulation one step backward from t = 1.5 s to t = 1.45 s.]

158 **I**: Look at the value of Ax … Now go one step forward.

[She ran the simulation one step forward from t = 1.45 s to t = 1.5 s and Ax changed from 1.717 to -0.918. After some seconds, she ran the simulation one step backward, from t = 1.5 s to t = 1.45 s, and then one step forward, from t = 1.45 s to t = 1.5 s.]

159 **I**: It seems that you haven't understood.

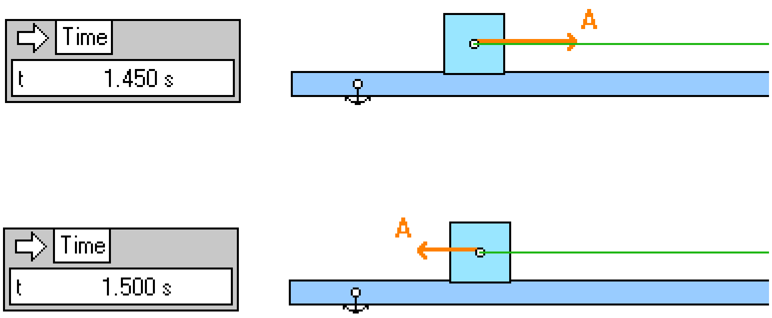
160 **S5**: No. I haven't.

I: Let's have the vector arrow for ACCELERATION on object A.

[I helped her add the vector arrow for ACCELERATION on object A and magnify its length. As per her request, I also changed the colour of the vector arrow from green to red.]

162 **I**: Let's run it in slow motion and see what happens.

[She ran the simulation in slow motion. When the simulation reached t = 1.5 s, the vector arrow changed direction: instead of pointing toward the right as it did up to t = 1.45 s, it now pointed toward the left. Its length was also reduced.



As from t = 4.05 s, the length of the vector arrow started to decrease. At the same time the values of Ax and |A| started to change. At t = 4.15 s, the vector arrow disappeared from the screen and Ax = - 0.090 m/s^2. At t = 4.55 s, |A| = Ax = 0 m/s^2; v = 0 m/s.]

163 **I**: Anything of interest?

[She moved the simulation backward up to t = 1.25 s and appeared to look for the position at which the vector arrow changed direction.]

S5: When the string breaks, it decelerates.
165 **I**: Anything more? […]

[She moved the simulation to and fro at the point where the vector arrow changed direction.]

166 **S5**: The acceleration changes its direction. Its magnitude also changes.
I: What?
S5: The acceleration changes its direction.
I: What happens to the acceleration when the string breaks?
170 **S5**: It changes direction.
I: Any other thing?
S5: … It decelerates. It becomes minus.
I: […] So when the string breaks, its direction changes and.
S5: It decreases.
175 **I**: Its magnitude changes.

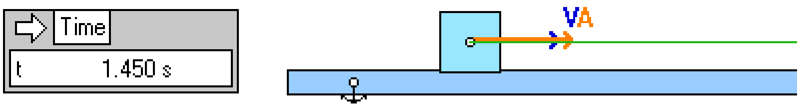
[Using the Vector Lengths feature (from DEFINE), I elongated the vector arrow for VELOCITY but not as much as that for ACCELERATION.]

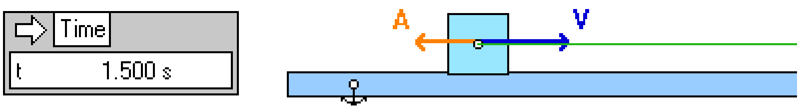
176 **I**: Let's add the vector arrow for VELOCITY on object A.

[She added the vector arrow for VELOCITY on object A.]

177 **I**: Let's run it in slow motion.

[As she ran the simulation, the length of the vector arrow for VELOCITY gradually increased. Just before string broke, that is at t = 1.45 s where v = 2.484 m/s & Ax = 1.717 m/s^2, the length of this vector arrow was a bit shorter than that for ACCELERATION. Please note that in practice, the length of the vector arrow for VELOCITY should have been longer than that for ACCELERATION. This might be seen as a shortcoming. However, this was not an issue as the main aim here was to show a change in direction.





At t = 1.5 s, when the string broke, the vector arrow for ACCELERATION changed both direction and magnitude (its length decreased as |A| changes from 1.717 to 0.918 m/s^2). As she continued to run the simulation in slow motion, the length of the vector arrow for VELOCITY started to decrease as from t = 1.5 s. At t = 3.8 s, the vector arrow for VELOCITY disappeared from the screen and the reading on the VELOCITY meter was 0.297 m/s. She ran the simulation a few steps forward and then started to run it backward until t = 1.40 s. After 2 seconds of pause, she ran it two steps forward up to t = 1.50 s. There was a change in the direction of the vector arrow for ACCELERATION, but no change in direction of the vector arrow for VELOCITY.]

S5: The direction of velocity remains unchanged.

I: What have you learnt here? […]

S5: I have learnt that the direction of the acceleration changes. The direction of the velocity remains unchanged … and the acceleration, before becoming zero, is a constant when the string breaks.

I: Ok.

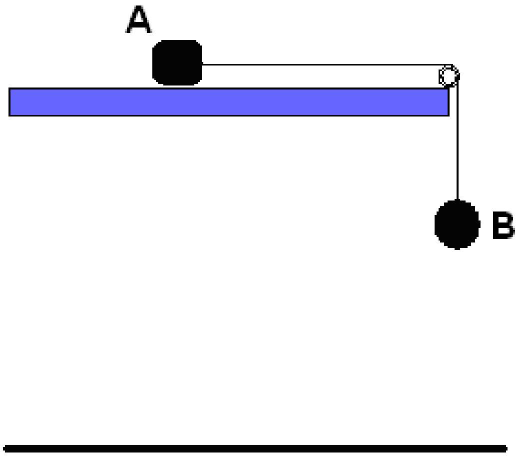
S5: It is a constant before becoming zero.
I: Any other thing?
S5: Friction influences the acceleration. [Laughter]
I: Which feature has been of more importance to help you learn these ideas: the meters or the arrows? … Or is it both?
S5: Both.
I: In what ways?
S5: When I used the meter, I did not pay attention to the acceleration along the horizontal axis [referring to Ax].
I: What happened when you looked at the meter?
S5: I looked at the modulus only [referring to |A|].
I: You looked at the modulus, at the magnitude. Alright.
S5: I did not pay attention to Ax.
I: […] But did it help you understand the system?
S5: Yes, but the vector arrows were of more help.
I: In what ways?
S5: First, it showed me that the direction of velocity remains unchanged. It always moves in the same direction. However, the direction of acceleration changed.
I: Ok.

Appendix E1: Transcript for Student S6

I: Interviewer S6: Student

Line Transcript

[Task 1 was introduced to Student S6.]



I: According to you, what may happen to such a system when it is released from rest?

S6: If mass of B is greater than mass of A, there will be motion. A will slide and B will move down.

I: What is responsible for the motion of object A [pointing at object A]?

S6: The tension in the string.

I: What happens if both masses are equal?

S6: … There is motion until equilibrium is reached.

I: What do you mean by ‘equilibrium is reached’?

S6: The 2 strings become equal.

I: What do you mean? You are allowed to use your fingers to point at anything on the screen.

S6: The strings [pointing at the horizontal and vertical parts of the string] will be equal.

I: In what respect are they equal?

S6: Length.

I: So, what do you mean?

S6: The length between the pulley and B is equal to the length between the pulley and A.

I: […] Now, what happens if the mass of A is greater than the mass of B?

S6: Nothing.

I: What do you mean by ‘nothing’?

S6: There is no motion.

I: […] Why do you think that there will be no motion when the mass of A is greater than the mass of B?

S6: It depends on the frictional force.

I: Remember we are dealing with a smooth surface. […]

S6: There will be tension here [pointing at the vertical part of the string]. This tension won’t be sufficient to pull this object [pointing at object A].

I: How is tension created in the string? […] What creates the tension in the string?

S6: This mass [pointing at object B].

I: Which mass?

S6: These 2 masses [pointing firstly at object B and hesitatingly at object A].

I: Ah. Both objects are needed to produce tension? Ok. […]

S6: Object B is responsible for the tension in the string … if we have one string the tensions on both sides [pointing at the vertical and horizontal parts of the string] are equal.

I: According to you, is it one string only here?

S6: Yes. There is only one string.

I: Yes. We have used one string to connect both objects.

S6: This implies that tension is equal on both sides.

I: Alright. So how does the tension originate in the string? Is it possible to assign a value to tension?

S6: This weight.

I: Which one?

S6: B [pointing at object B].

I: So, tension is?

S6: If the system is not moving.

I: … No. Eh … You choose the situation you wish to discuss.

S6: When there is no motion, this means that the tension in the string is equal to the weight of the object [pointing at object B].

I: Why are you saying that?

S6: I have used an equation.

I: Which equation?

S6: Tension minus weight is equal to ma. If it is not moving, a is equal to zero. Therefore, T is equal to mg.

I: Ok … Let's recap. According to you, which object is responsible for the creation of tension in the string?

S6: Both objects.

I: Earlier you were saying.

S6: Only one is responsible … Because both [pointing at objects A & B] are exerting a certain tension … Normally it is both objects that are exerting this tension … towards the pulley.

I: Alright … Now say we have a system where the mass of object B, mB, is greater than the mass of object A, mA, and there is motion … Say, object A is moving with a speed of 10 m/s at a specific time, what will be the speed of object B at that particular time? Will it be equal to 10 m/s, less than 10 m/s or greater than 10 m/s?

S6: … Equal.

I: Why equal?

S6: …(silence)…

I: Let's think about something else. Say we again have the system where the mass of object B, mB, is greater than the mass of object A, mA, and there is motion. Now, say at a specific time object A is moving with acceleration 6 m/s^2, what will be the acceleration of object B at that specific time? Will it be equal to 6 m/s^2, or less than 6 m/s^2 or greater than 6 m/s^2?

S6: Equal.

I: Why equal?

S6: … Both are connected to each other. When this one [pointing at object A] moves a certain distance, this one [pointing at object B] also moves the same distance … in the same interval as there is only 1 string.

I: Do you think that object B can move with acceleration g?

S6: …(pause)… Maybe it is … there will be acceleration due to gravity on this object [referring to object B].

I: What?

S6: Gravity will act on object B … But I have learnt that acceleration will be equal for both objects.

I: Ah. You have already learnt it.

S6: When I have worked through numerous questions, I have seen that the accelerations of both objects are equal and there is 1 tension.

I: But now that you are working with this simulation, do you find it true?

S6: No.

I: What do you find?

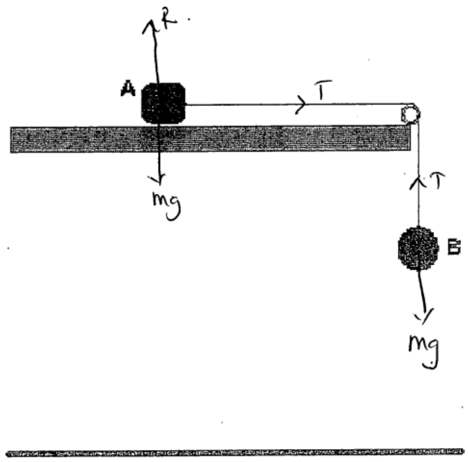
S6: This one's [pointing at object B] acceleration will be more because gravity is also accelerating it … They should be equal.

I: Eh?

S6: If this one [pointing at object B] has to move a certain distance, this one [pointing at object A] should move the same distance, because they are connected via a string.

I: Can you please draw a force diagram?

[She drew the force diagram.]



I: So do you think that the objects will move with equal speeds and equal accelerations or will they be different? How about the accelerations?

S6: Equal accelerations.

I: Why? Earlier you were not sure.

S6: Force of gravity is already taken into account.
I: So, acceleration of B won't be equal to g?
S6: No … it will be less.
I: Why not more than g?
S6: When we do the resultant, it will become less.
I: Ok […] Earlier you stated that when the system would be at rest, what would tension equal to?
S6: mg [referring to the weight of object B].
I: And if the system moves, will the tension be equal to weight?
S6: … If it [referring to object B] is going down, this means that weight is greater than tension.
I: Ok.
S6: And if it is going up, this means that tension is greater than weight.
I: Ok. Let's run the system.

[She ran the simulation using the command RUN and object A left the surface. I explained to her the part that we wished to focus on. She reset the simulation and ran it in slow motion.]

I: What can we observe?
S6: Object B is pulling object A.
I: But earlier you mentioned that tension was pulling object A.
S6: It is the same thing.
I: Eh?
S6: It is this tension [pointing at the vertical part of the string].
I: … This would imply that tension in the string is equal to the weight of object B.
S6: I think that tension is pulling it [referring to object A]. [laughter] Tension is on this side [pointing at the horizontal part of the string]. It will make it [referring to object A] slide in this direction [pointing towards the pulley].
I: Ok. What is pulling object A?
S6: Tension in the string … due to this hanging object [pointing at object B].
I: What is 'due to the hanging object'?

S6: The weight … the weight [pointing at object B] being greater than the tension [pointing at the vertical part of the string] … It is the resultant of the weight and tension that is pulling the object.

I: You're telling me that the resultant.

S6: It is the resultant force of the weight and tension that is pulling the object A.

I: So, you no longer think that it is the tension in the string which pulls the object A.

S6: Yes. If it [pointing at object B] is going down, this means mg is greater than tension. So, it is the resultant force that is pulling the object A.

I: Why have you now changed your 'idea'?

S6: It changed by itself [laughter].

I: Ok. Let's measure the mass of each object.

[She found that the mass of object B, mB, was 2 kg and that of object A, mA, was 1 kg.]

S6: Let's change this [pointing to the value of mA] and make it bigger than mB and then we see whether there is motion.

I: Let's investigate mA = mB.

S6: Ok.

[She changed the mass of object A, mA, from 1 kg to 2 kg and maintained the mass of object B, mB, as 2 kg. She ran the simulation and the system moved. She reset the simulation and verified the masses of both objects.]

I: What is your query?

S6: It did not stop.

I: Hm?

S6: It did not stop … It should have stopped.

I: It should have stopped?

S6: No.

I: Why?

S6: Because if this force [pointing at object B] is greater, it depends on this object [pointing at object B].

I: I didn't understand you.

S6: Oh God! Even I can't understand it.

[She ran the simulation in slow motion until object A reached the end of the surface and then backward until it almost reached its original position.]

S6: Both masses are equal.

I: So?

S6: But it moves in the same way.

I: … What do you mean 'it moves in the same way'?

S6: Its motion is the same as that of the system with mass of B greater than mass of A.

I: Ok … Do you think that Interactive Physics is fooling us? Do you accept what it is showing you?

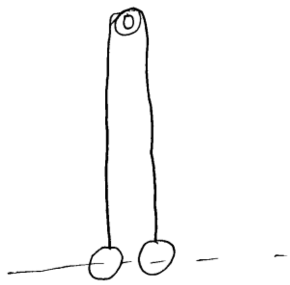
S6: … Is this system not similar to the system where we have 2 objects attached to the 2 ends of a string which passes over a pivot? If they are of equal masses, they will be at the same level.

I: I didn't understand you.

S6: When we have 2 ends of a string [pointing at the vertical part of the string] and we don't have a surface, they [pointing to objects A & B] hang freely and if they are of equal masses, they will be at the same level.

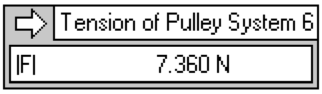
I: Can you draw the system?

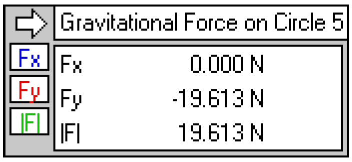
[She drew the following diagram.]



S6: If they are of equal masses, they will be at the same level.
I: Alright. So, what if they aren't at the same level, when you released them?
S6: They'll move up and down until they stabilize at the same level.
I: Ok. You were making reference to this system [referring to her drawing]?
S6: Yes. I was making reference to this one.
I: This is why earlier you were predicting that these 2 lengths [pointing at the horizontal and vertical parts of the string] should be equal.
S6: Yes.
I: What do you want to do now?
S6: …(silence)…
I: Ok. Let's return to the system with mA = 1 kg, mB = 2 kg. Is tension less than weight as you had suggested?

[She changed the mass of object A, mA, to 1 kg and maintained the mass of object B, mB, to 2 kg. She switched on the GRAVITATIONAL FORCE meters of both objects A & B, and the TENSION meter. She ran the simulation using the command RUN. She reset it and ran it in slow motion.]





I: Earlier we were talking about tension and weight of object B. Didn't you say that they were going to be equal?
S6: No! Only if it is in equilibrium!
I: Alright. So, what do we find?
S6: Since this weight [pointing at object B] is greater than this tension [pointing at the vertical part of the string], this is what is pulling this object [pointing at object A].
I: What?
S6: When we do the resultant of the weight [pointing at GRAVITATIONAL FORCE meter of object B] and the tension [pointing

to TENSION meter], weight is greater than tension. Therefore, there is a force … which pulls object B down … and this one [pointing at object A] follows it.

I: […] Ok. Let’s return to the system of equal masses and explain why there is motion. Can you now explain the motion? … What do you wish to do with Interactive Physics to explain the motion?

S6: Compare their {net forces}.

I: Ok. Go on.

[She reset the simulation, changed the mass of object A, mA, from 1 kg to 2 kg and maintained that of object B, mB, as 2 kg. She then ran the simulation using the command RUN. She reset it and used slow motion.]

I: What are you thinking?

S6: They are the same [making an analogy to the previous system with mA = 1 kg; mB = 2 kg]. This weight [pointing at object B] is greater than tension. So, it pulls this mass [pointing at object A].

I: […] So, do you agree that there is motion when the masses are equal?

S6: [nodding of head to suggest that she doesn’t agree with motion] … They will move.

I: You just refused to agree with the motion.

S6: No. I …

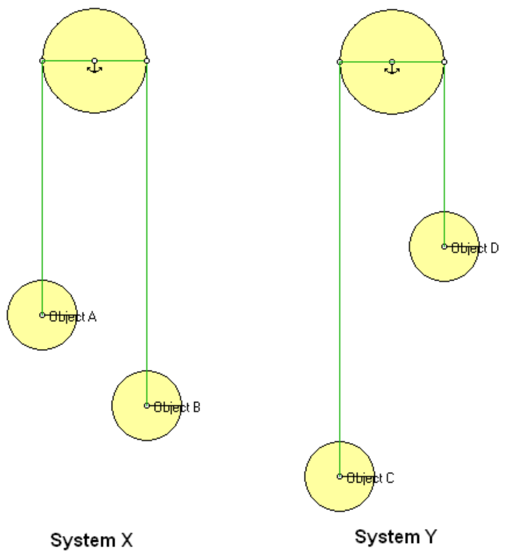
I: You don’t look convinced.

S6: I don’t know how to explain it.

Appendix E2: Transcript for Student S6

I: Interviewer S6: Student

[Task 8a was introduced to Student S6. The following diagram is a screen shot of the task as it appeared on Interactive Physics.]



[We discussed the behaviour of System X (with objects A & B) if the mass of object A would be greater than that of object B and if the mass of object A would be less than that of object B.]

I: What happens if we have equal masses?
S6: A moves down and B moves up so that they become at rest at the same height after a short time.
I: Ok … How about this system [pointing to objects C & D]? What do you think?
S6: … It will behave in the same way as System X.

[So we agreed to focus solely on System X throughout the interview.]

I: Ok. Run the system.

[She ran the simulation using the command RUN, but the system remained motionless.]

I: What can you say?
S6: Should it not move? Is it not a system of pulley?
I: Hm? What were you expecting to see when you ran the simulation?
S6: That they should move.
I: Why were you expecting them to move?
S6: … Because they have weights to make them move.
I: Ok. In which direction were you expecting them to move?
S6: If this object [pointing at object B] is heavier, it will go down and this one [pointing at object A] will go up.
I: What is happening at the moment? […]
S6: The mass of A is equal to the mass of B and their weights are equal to tension in the string. This is why there is no motion.
I: But didn't you tell me that they would be at the same level?
S6: Yes, but they aren't moving … If they move.
I: …(pause)… But, as per your prediction, objects A & B should have been at the same height, they should have come to rest at the same height.
S6: If they are not moving, this means that tension is equal to their weights.
I: So, what should we believe?
S6: Tension is equal to weight.
I: Which is heavier? I thought that object B is heavier than object A.
S6: Is it? Because it is lower?
I: Yes.
S6: No.

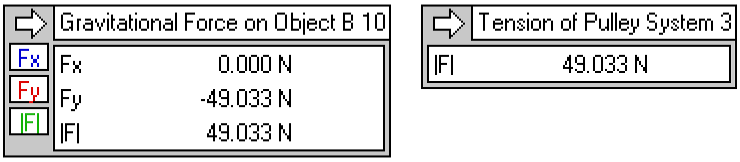
[She double-clicked on object B and its PROPERTIES box appeared on the screen. She verified the mass of object B, mB, which was 5 kg. She repeated the same procedure with object A and found that its mass, mA, was 5 kg.]

I: Ah! You are cheating.
S6: No. [laughter] They are equal. [laughter] Didn't I tell you that they have equal masses? … These 2 objects [pointing at objects C & D] also

have equal masses … This means that the tension [pointing at the vertical part of the string] is equal to the weight [pointing at object B]. For both objects [pointing at objects A & B], since they are not moving, they have equal masses and tension is equal to weight.

I: […] Ok. Let's try it.

[She switched on the GRAVITATIONAL FORCE meter for object B and TENSION meter. She ran the simulation using RUN. The readings on both meters were the same: 49.033 N.]



S6: It is correct (OR It makes sense).

I: […] Why did you think that these objects, of equal masses, will move to the same height when released from rest?

S6: Same mass.

I: Why did you think of same height?

S6: Equal masses implies equal weights.

I: Ok. So, how come equal weights implies same height? For you, what is the link between 'equal weights' and 'same height'?

S6: … I don't know why I said that.

I: There must be a reason […] What is it that makes you think that 'equal weights' implies 'same height'?

S6: […] I thought that it was like that.

I: What made you think like that?

S6: If they are of equal masses, they will balance each other.

I: How will they balance each other?

S6: They will be at the same height from the surface.

I: … Ok. I thought that you were making reference to some sort of instrument in everyday life or lab.

S6: No.

I: […] My question is what link you make between equal weights and same height … You are someone who knows that when tension is equal

to weight or when net force is equal to zero, this means that there is no motion. So how come you mentioned that there is some motion so that both objects come at rest at the same height?
S6: I thought that they should move.
I: Why did you think that they should move?
S6: Because the masses are different.
I: In what way are the masses different?
S6: I don’t know. There is a difference. But I asked myself whether they are equal.
I: Eh?
S6: I thought that they should be at the same level ... I don’t know why I said that.
I: Don’t worry.

[She was about to leave the session when she asked the following:]

S6: Is it really a pulley system?
I: Eh?
S6: It didn’t move. Let me change the mass and see what happens.
I: Ok.

[She changed the mass of object B, mB, from 5 kg to 10 kg and then ran the simulation using the command RUN. There was motion such that object A moved down and object B moved up. The system stopped moving when object B hit the pulley.]

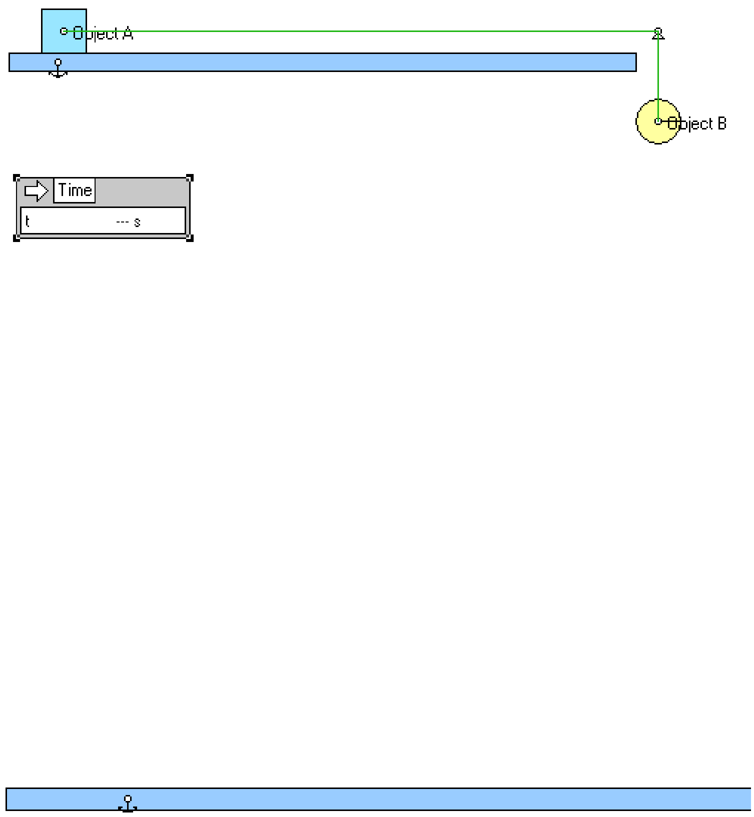
I: So you thought that I was using some sort of fake pulley?
S6: Yes. [laughter] It moved … This means that if they are of equal weights, whatever level they are positioned they will remain at rest. Because the forces will cancel out. If one is positioned at a lower level, it will stay there ... If both objects are at the same level, they will remain there ... I have understood it. I don’t know why I talked of same level earlier.
I: Think about it and tell me later.
S6: Why did I say ‘same level’?

Appendix E3: Transcript for Student S6

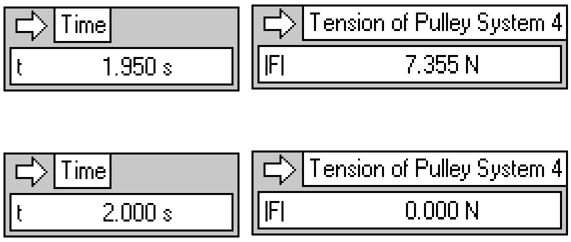
I: Interviewer S6: Student

Line Transcript

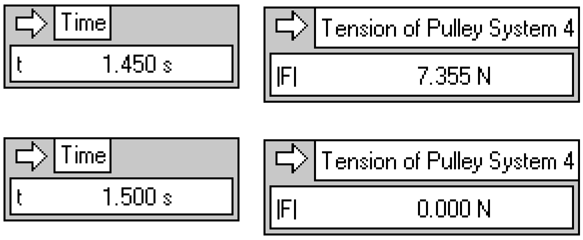
[Task 6a was discussed with Student S6 and the shortcoming of Interactive Physics with regard to this task was highlighted. When a string breaks during a simulation in Interactive Physics, it should (physically) become a dotted line. However, the software version at hand was not able to simulate this feature and therefore the broken string continued to be represented by a complete/full line.



Student S6 already knew that when the string would break, tension must be equal to zero. Using this piece of knowledge, she was asked to find the time at which the string would break in the simulation provided. So she added the TENSION meter, ran the simulation using the command RUN, reset it and finally ran it in slow motion. The reading on the TENSION meter was 7.355 N until t = 1.950 s. She found that the tension in the string became zero when t = 2.000 s, implying that the string broke at this instance.



I then modified the required parameter in the simulation so that the string would break at t = 1.500 s. She ran the simulation using the command RUN, reset it and then ran it via slow motion to show that the string had indeed broken at t = 1.500 s.



We also agreed to work with a smooth surface in the beginning and to later explore this system with a rough surface.]

I: You already know that just before the string breaks, both objects [pointing at objects A & B] must be moving with the same acceleration … Eh …Let's measure the acceleration [I put on the ACCELERATION meter of object A]. What is its value? If I run it [I started to run the simulation in slow motion], what is it?

S6: Constant.

I: What is its value [continuing to run the simulation in slow motion]?

S6: Constant.

I: 2.452 m/s^2.

S6: 2.452.

I: This is the acceleration of … which object?

S6: The sliding object.

I: How about the acceleration of B? Do you think that it is going to be equal, greater than or less than 2.452?

S6: Equal.

I: So, just before the string breaks, both objects move with an acceleration of 2.

S6: //2.452.

I: Now what will happen to the acceleration of object B once the string is broken? According to you, will it remain the same, or will it decrease, or will it increase?

S6: It will increase until it will become acceleration due to free fall.

I: What do you mean? Do you think that it will increase gradually or?

S6: It will increase gradually.

I: Or will it increase instantaneously?

S6: No, it will increase gradually.

I: Ok … Just record this value of 2.452 on your sheet of paper.

[I deleted the ACCELERATION meter of object A.]

I: […] What is the acceleration of B at this point [I moved the simulation up to t = 1.45 s and the reading on the TENSION meter was 7.355 N]?

S6: 2.452.

I: Continue to run the simulation and discuss with me the acceleration of object B as you proceed.

S6: It will increase.

I: … According to you, at what point will it become g?

S6: … It will increase until it will become 9.81.

I: So, what do you think is happening to its acceleration as B moves down gradually [I moved the simulation 2 steps forward to t = 1.55 s and the reading on the TENSION meter became 0 N.]?

S6: When the string breaks, it increases until it reaches acceleration due to free fall.

I: Alright.

S6: Once the string is broken, it doesn’t become g all of a sudden, it takes a short time but it will become g gradually.

I: Isn’t there some sort of contradiction between ‘a short time’ and ‘gradually’? What do you mean?

S6: It won’t take that much time.

I: How much time? 1 sec, 2 sec … 3 sec, 4 sec … 5 sec?

S6 : Something like that. Around 4 seconds.

I: Ok. And how about the acceleration of object A once the string is broken? It's a smooth surface.

S6: It will start to decelerate.

I: Eh? […]

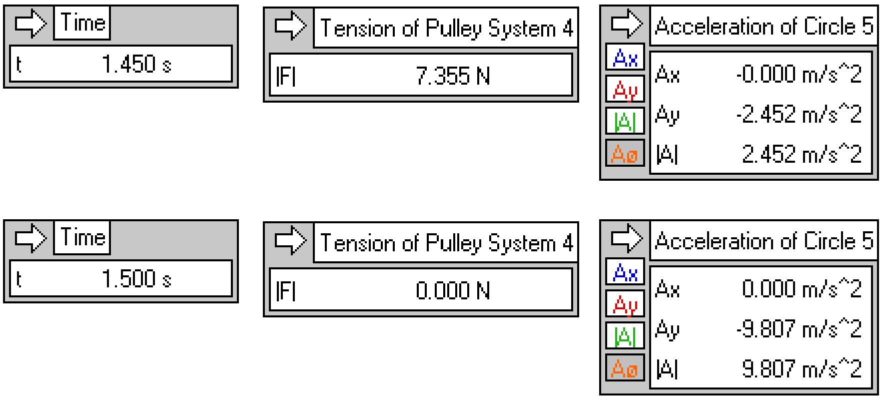
S6: It will start to decelerate until it will become zero.

I: Ok.

S6: Provided the surface is long (enough) so that it [referring to object A] does not hit the pulley and falls down.

I: Ok. Let's verify the acceleration of object B.

[She reset the simulation, put on the ACCELERATION meter of object B and ran the simulation using the command RUN. When object A left the table, she reset the simulation and by using the tape player control, she moved the simulation to t = 4.050 s, positioning object A near the end of the table. She then moved the simulation backward (until t = 1.45 s) in slow motion. At this point, tension in the string was equal to 7.355 N and acceleration was equal to 2.452 m/s^2. She moved the simulation one step forward (t = 1.5s), and tension in the string was equal to 0 N and acceleration was equal to 9.807 m/s^2.]



S6: This means that when the string breaks, acceleration becomes g instantaneously.

I: Do you accept this?

S6: … It came a bit too early.

I: It appeared to come instantaneously. What do you think?

S6: …(pause:14)… I knew that it would become 9.81, but not so quickly.

65 **I**: You don’t agree with it?

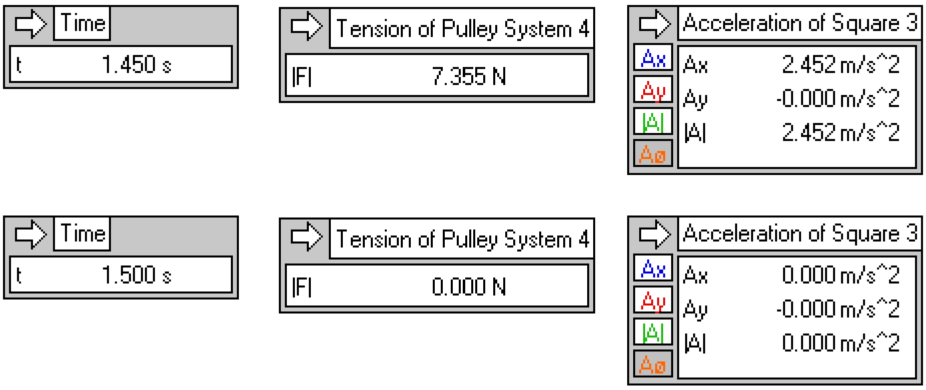
S6: It’s ‘ok’. It should be ‘ok’.

I: No. It needs not be ok. Can you explain why?

S6: … Oh yes! mg! Once the string breaks, there is no tension, only weight acts. This is why it changes instantaneously. Because tension
70 becomes zero, there is only one force that acts.

I: Ok ... Let’s look at the acceleration of A.

[She reset the simulation, put on the ACCELERATION meter of object A and then ran the simulation using the command RUN. As object A left the table, she reset the simulation and ran it in slow motion until t = 1.5 s because at this point acceleration became zero. Note at t = 1.45 s, acceleration was equal to 2.452 m/s^2.]



72 **S6**: Acceleration becomes zero …(pause)… but it continues to move!

I: Eh?

[She re-started to run the simulation in slow motion up to t = 1.75 s.]

74 **S6**: Shouldn’t it [pointing at object A] stop moving? … It should have
75 stopped moving, shouldn’t it?

[She re-started to run the simulation in slow motion until t = 2.9 s.]

76 **S6**: Shouldn’t it stop moving?

[She continued to run the simulation in slow motion up to t = 3.5 s and then remained silence for some 12 sec. She re-started to run the simulation until object A left the table.]

S6: This means that it moves with constant speed.
I: … What do you think?
S6: It does not have any acceleration; it moves with a speed with which it was being ‘pulled’.
I: … You wish to verify it?
S6: Of course!

[She reset the simulation, put on the VELOCITY meter of object A and ran the simulation using the command RUN. It was observed that the reading on the meter quickly increased from 0 m/s to 3.657 m/s until t = 3.5 s where she stopped the simulation. As object A was about to leave the surface, she reset the simulation and ran it in slow motion up to t = 1.5 s. The readings were as follows: at t = 0, $|v|$ = 0 m/s and as t increased, $|v|$ also increased. At t = 1.5 s, $|v|$ = 3.657 m/s.]

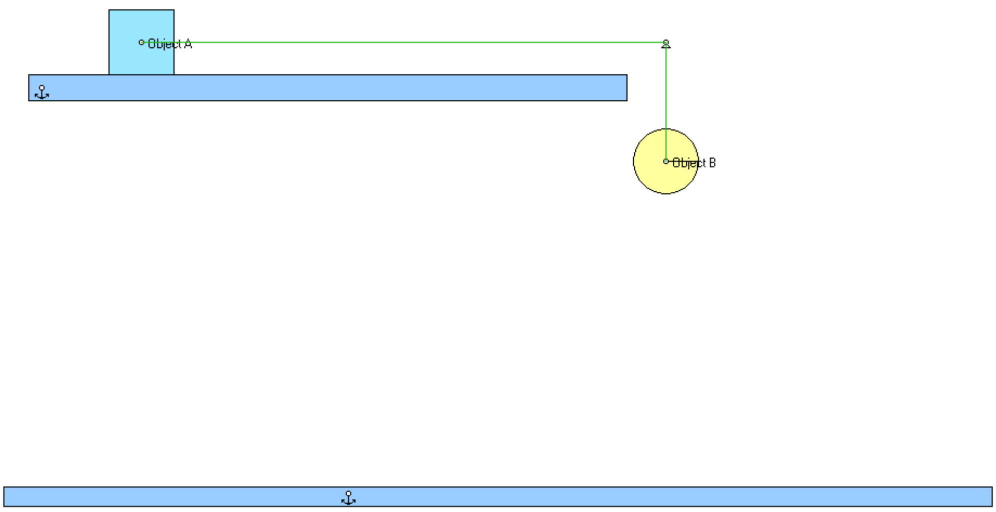
S6: Yes, it moves with the speed that it had when the string is broken.
I: According to you, why?
S6: …(pause:25)… Tension becomes zero.
I: Ok.
S6: This means … It was already moving with a certain speed.
I: Is it not a law? Why does it continue to move with the same speed?
S6: Eh! Newton’s first law. If no external force acts on the object, it continues in its state of motion.

Appendix F: Transcript for Student S7

I: Interviewer S7: Student

Line Transcript

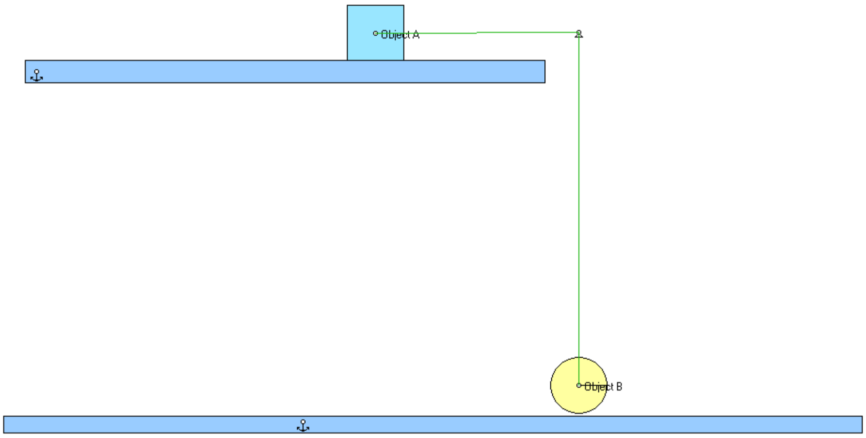
[Task 6b was introduced to Student S7. We discussed the aim of this task – to study the motion of the hanging object B between ‘just before it hit the ground for the first time’ and ‘just before it hit the ground for the second time’. He was allowed to run the simulation several times so that he became conversant with the task at hand.]



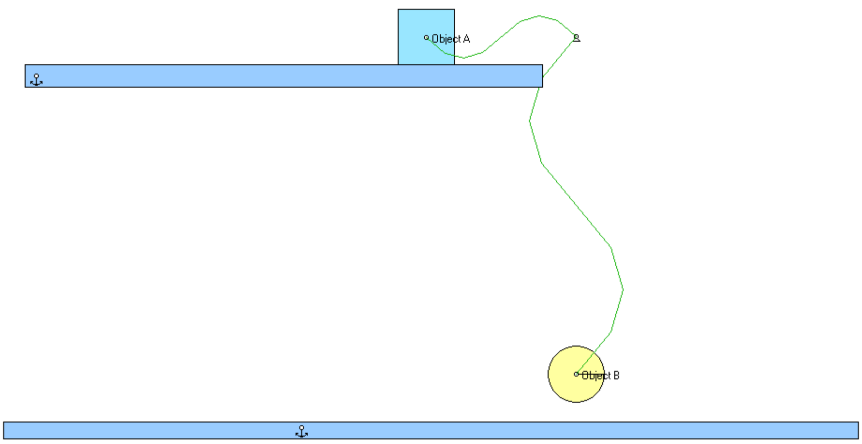
I: When the system is in motion, before object B hits the ground, what do we have in the string?
S7: Tension.
I: Alright … And how about the motion of both objects?
S7: They both move with the same velocity and the same acceleration.
I: So, if I measure the acceleration [I put on the ACCELERATION meter of object A], and I run it [I ran the simulation in slow motion for a short time], I find that acceleration is … 6.538 m/s^2.
S7: 6.538.
I: Record this value of acceleration on a sheet of paper before I delete the ACCELERATION meter.

[He recorded the acceleration and I deleted the ACCELERATION meter.]

I: As object B moves down [I ran the simulation in slow motion so as to allow object B to move down], what will be its acceleration?
S7: 6.538.
I: What will be its acceleration just before it hits the ground? [I ran the simulation in slow motion until object B was just about to hit the ground.]



S7: 6.538.
I: 6.538. Ok. Now that it has already hit the ground [I ran the simulation in slow motion, and object B hit the ground and started to move up. The string became slack], what do you think will be its acceleration? Will its acceleration continue to be 6.538, or will it be less than 6.538, or will it be more than 6.538? What do you think?



S7: …(pause:30)… Acceleration will become negative and will decrease.
I: What do you mean? Why will it become negative?
S7: Because direction of motion has changed […] It was moving down and now it is moving up.

[I moved the simulation backward in slow motion until object B was just about to hit the ground.]

I: Just before it hits the ground, its acceleration is 6.538.
S7: //6.538. Yes.

[I ran the simulation a few steps forward in slow motion, and object B hit the ground and started to move up.]

I: So, what was your prediction for its acceleration here?
S7: Less than 6.538.
I: And now what happens? Look here [I ran the simulation in slow motion for a few steps, and object B moved up, stayed at rest at its highest point, and then started to move down].
S7: Now it is positive [referring to the acceleration of object B as it started to move down].
I: And what is its magnitude? […]
S7: 9.81, eh, gravitational force.
I: Can you explain it again? Can you rewind the simulation and discuss with me as you go through it?

[He used the tape control player to move the simulation backward and forward, and positioned object B in such a way that it was at a certain height above the ground. The string became taut again.]

S7: When the object [pointing at object B] falls, it will move with acceleration 6.538.
I: Ok.

[He moved the simulation forward such that object B hit the ground and started to move up.]

S7: When it moves up, its magnitude will be negative and its acceleration will decrease.

I: Ok.

S7: When it starts to fall down, it will fall due to gravitation, that is 9.81.

I: Alright. Why will it fall with g?

S7: Because there is weight only which is the net force.

I: What?

S7: … There is no tension which will oppose its motion.

I: […] This is why it will fall with g.

S7: Yes.

I: … How about its speed? As object B moves down, what will happen to its speed? […]

S7: From rest, it will increase and will then become constant.

I: Can you explain it again? Reset the simulation and explain to me as you go through it.

[He reset the simulation using the tape player control.]

S7: Right now velocity will be zero [pointing at object B which was at its initial position].

I: Alright.

S7: It will increase [He moved object B almost halfway between the table and the ground via the tape player control] and will then become constant.

I: Ok.

S7: It will move with constant velocity [He moved object B down near the ground] until it hits the ground.

I: And what happens when it hits the ground?

S7: Its velocity will become zero.

I: Then?

S7: It starts to move up.

I: Ok. Continue the simulation in slow motion and explain to me as you

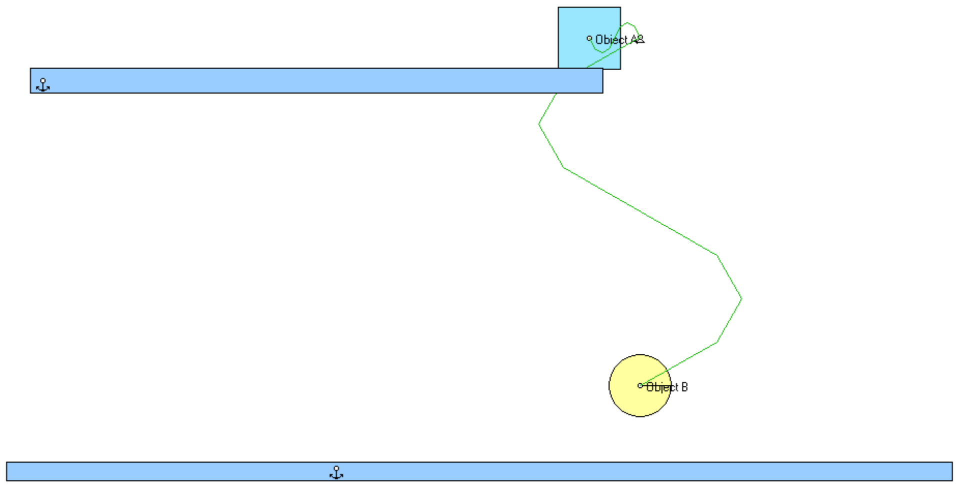
74 go through it.

[He moved the simulation in such a way that object B has just hit the ground.]

75 **I**: What happens here [object B has just hit the ground]?
S7: Here velocity will be zero.
I: Hmm.

[He moved the simulation 2 steps forward such that object B hit the ground and started to move up.]

78 **S7**: It moves up. As velocity is a vector quantity, it will become
negative. [He ran the simulation some more steps until object B appeared
80 to have reached its highest point.] … Then a point is reached where
velocity will become zero again.



82 **I**: Ok.
S7: And then when it moves down [He moved the simulation a few steps
forward so that object B could be seen moving down], it will become
85 positive.
I: And how about its magnitude? […] Let's start from the beginning [I reset the simulation.]. What happens here? Initially it [referring to the velocity of object B] will be zero.
S7: It will be zero.
90 **I**: Then?

[I ran the simulation in slow motion so that object B descended and I stopped the simulation when B reached almost half-way.]

91 **S7**: It will increase until it will become constant.
I: Then?
S7: When it hits the ground, it will become zero.
I: What do you mean?
95 **S7**: It will be constant … Its velocity won't change.
I: It will move with constant speed for the remaining height? [I ran the simulation a few steps such that object B became nearer to the ground.]
S7: Yes.
I: Then?
100 **S7**: When it hits the ground, it will become zero.

[I ran the simulation in slow motion until object B appeared to hit the ground.]

101 **I**: Ok. It has become zero. Then?

[I moved the simulation in slow motion such that object B hit the ground and started to move up, and the string had become slack.]

102 **S7**: Then it starts to increase.
I: Velocity starts to increase?
S7: Yes.
105 **I**: Then?

[I ran the simulation in slow motion until object B was at its highest point.]

106 **S7**: Its velocity becomes zero.

[I rewound the simulation to the point where object B had just hit the ground.]

I: From here [pointing at object B as it moved up] you mentioned that the velocity is increasing.

S7: Yes.

I: So how come it becomes zero all of a sudden? […]

S7: …(pause:55)… This means that when it moves up, its velocity will decrease so that it will become zero.

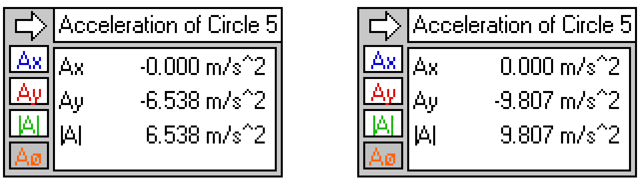
I: Hmm.

S7: Then it will increase when it comes down and will then become zero when it hits the ground.

I: Ok. Try it. What do you want to examine first: speed or acceleration?

S7: Acceleration.

[He reset the simulation, put on the ACCELERATION meter of object B and ran the simulation using the command RUN. As motion was swift, he was not able to study the ACCELERATION meter. So, he reset it and moved the simulation 'too much' forward by using the tape player control. I had to advise him to move the simulation to the moment where object B was just about to hit the ground for the first time. After positioning object B to the said location, he moved the simulation 1 step forward such that object B hit the ground and the string became slack. The reading on the ACCELERATION meter changed from 6.538 m/s^2 to 9.807 m/s^2.



Before impact　　Just after impact

After some 4-5 seconds of silence, he re-started to run the simulation in slow motion and the following events took place on the screen: object B moved up, reached a highest point where it remained stationary for a short time, moved down and finally hit the ground for a second time – throughout this trajectory, the reading on the ACCELERATION meter remained 9.807 m/s^2.]

I: What happened?

S7: Its acceleration becomes … acceleration due to gravity.

I: … It didn’t happen as you thought.

S7: No, this is what I said … When it hit the ground, it [referring to the acceleration of object B] becomes g.

[I rewound the simulation to the moment where object B was just about to hit the ground for the first time.]

I: Look here. You told me. Here it is correct [pointing at the ACCELERATION meter of object B].

S7: Yes.

I: When it hits the ground [I moved the simulation 1 step forward such that object B hit the ground and the string became slack], you told me that its acceleration would be less than 6.538 when it moves up [I ran the simulation in slow motion such that object B moved up], it would be zero when it is at its highest point [I ran the simulation in slow motion such that object B stayed stationary for a short time at its highest point], and it would fall with g [I ran the simulation in slow motion such that object B moved down]. This is what you told me.

S7: Yes … Yes.

I: This is what I understood from your earlier explanation.

S7: This is what I told you.

I: I may have misunderstood you […]

S7: No, this is what I told you.

I: […] Why did you think that when it hits the ground it is going to be less than 6.538 m/s^2 and when it moves up it should become zero?

S7: I thought that when it hits the ground, its velocity would become zero.

I: Hmm.

S7: Acceleration is the rate of change of velocity. If velocity is zero, then acceleration must be zero.

I: Ah. If velocity is zero, acceleration must be zero? […]

S7: …(pause:70)… No.

I: What are you thinking?

S7: If velocity is zero, acceleration need not be zero.

I: … Why do you think so?

S7: …(silence)…

I: What are you thinking?

S7: …(pause)… Why is it moving up with g?

I: … According to you, it should not move up with g?

S7: …(pause:55)… acceleration remains constant.

I: Eh?

S7: […]

I: Why is it that when it hits the ground, it starts to move up with g? … Why is it that here [referring to object B moving up] you don’t accept that it moves up with g but when it moves down you accept that it moves with g?

S7: Because it falls freely, it moves under gravity.

I: Ok. How about here [I moved the simulation to the moment where object B just hit the surface]? Do you have an idea why it moves up with g? … Do you agree with the magnitude of acceleration [pointing at the ACCELERATION meter] when it [pointing at object B] moves up?

S7: Yes.

I: Why?

S7: Because it is free, it moves up freely, there is no force that is acting.

I: What evidence do you have?

S7: There won’t be any tension on it [pointing at object B].

I: How do you know that there is no tension on it? …(pause)… As from when do you think that tension has no effect on it [pointing at object B]?

S7: As from when the object hits the ground and starts to move up. It is as from this point that tension doesn’t affect the system.

I: Why does it not affect the object?

S7: Because it is already zero.

I: Ok. Then?

S7: There will be only gravitational force acting on the object. This is why it moves up with 9.81.

I: And when it moves down?

S7: 9.81 … Yes … Yes.

I: Any other question?

S7: Tension is only present in strings which are straight lines. Can I
185 measure it in this one [pointing at the slack string]?

I: Measure it.

[He reset the simulation, selected the string and put on the TENSION meter. He ran the simulation using the command RUN and stopped it when object A had left the table and was in the air – the string became slack and the reading on the TENSION meter changed from 6.537 N to 0 N. Note that he did not stop the simulation as soon as the string became slack.]

187 **S7**: Yes.

I: […] What do we need to measure now?

189 **S7**: Velocity.

[He put on the VELOCITY meter of object B.]

190 **I**: So what do you predict?

S7: It will increase until it will become constant. When it hits the ground, it will become zero. When it moves up, it will decrease and it will become zero at the highest point.

194 **I**: Ok. Try it.

[He ran the simulation using the command RUN. I advised him to use slow motion from the start.]

195 **I**: What do we find?

S7: It continues to increase.

[He continued to run it until just after the second rebound of object B.]

197 **I**: […] Why did you think that it would move with the constant speed?

S7: I compared this object [referring to object B] to one which is falling and there is weight and air resistance. At a certain time it would move with terminal velocity. That is why I thought that it would move with constant speed.

I: Under what condition does an object fall with terminal velocity?

S7: When the object is falling freely, but here there is tension.

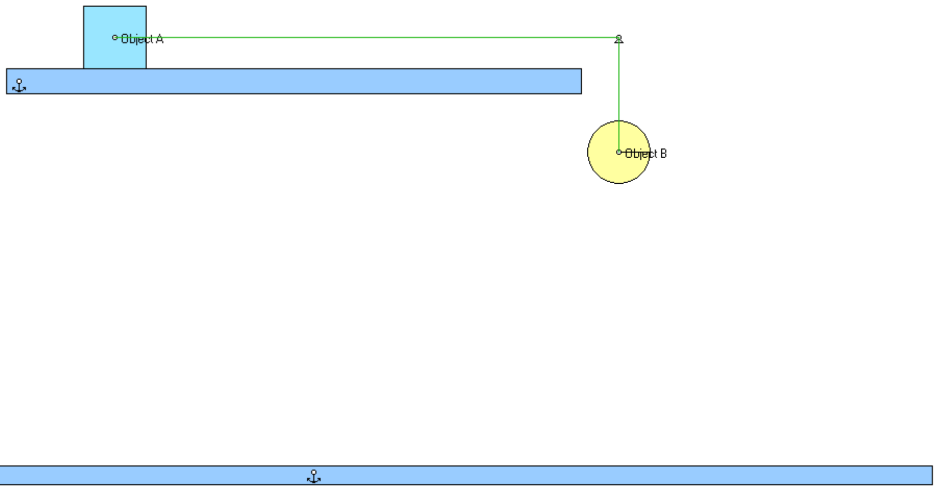
I: Alright […]

Appendix G: Transcript for Student S8

I: Interviewer **S8: Student**

Line Transcript

[Task 6b was introduced to Student S8. We discussed the aim of this task – to study the motion of the hanging object B between ‘just before it hit the ground for the first time’ and ‘just before it hit the ground for the second time’. She was allowed to run the simulation several times so that she became conversant with the task at hand.]



I: When you release the system from rest [I ran the simulation using the command RUN], there is motion … And what do we have in the string?

S8: Tension.

I: We have tension in the string. And object A must be moving with a certain acceleration [I put on the ACCELERATION meter of object A]. If I run the system [I ran the simulation in slow motion], object A is moving with an acceleration.

S8: 6.538.

I: 6.538. I think that we can agree that object A is moving with an acceleration 6.538 m/s^2 […] According to you, what will be the acceleration of object B just before it hits the surface?

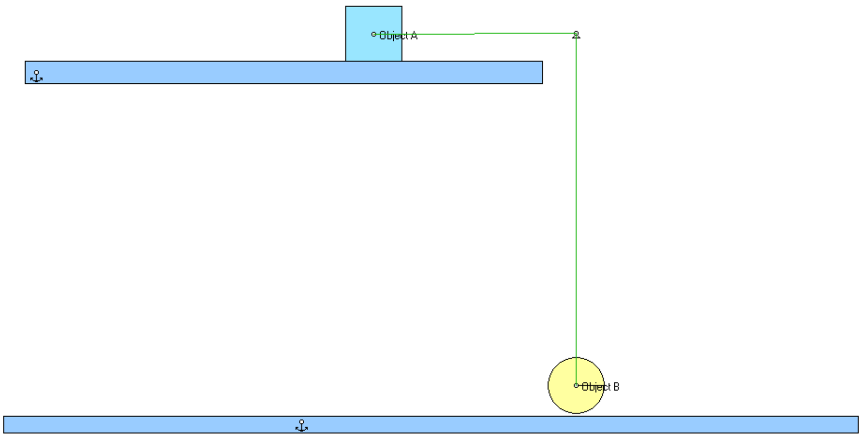
S8: Same as the acceleration of object A.

13 **I**: So you think that the acceleration of object B is …

S8: 6.538.

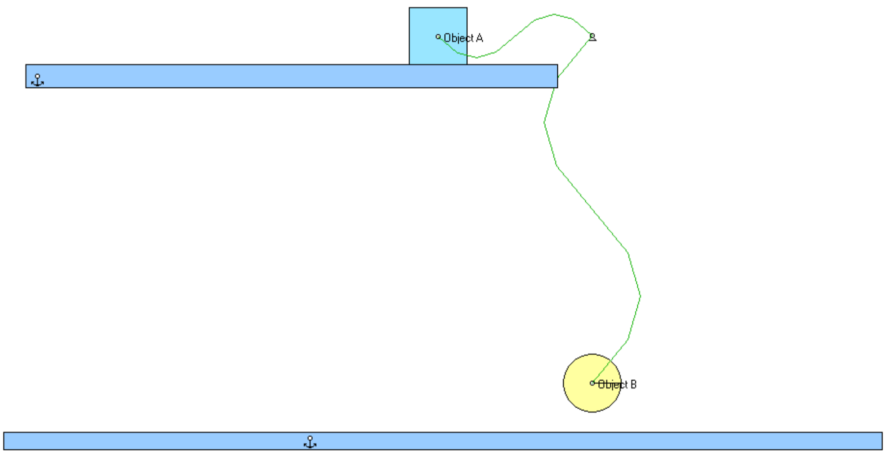
15 **I**: 6.538 just before it hits the surface … Can you record this value on a sheet of paper?

[She recorded the acceleration on a sheet of paper. I reset the simulation and deleted the ACCELERATION meter. I ran the simulation in slow motion until object B was just about to hit the ground.]



17 **I**: So, here [referring to object B which was just about to hit the ground] it is 6.538 m/s^2.

[I ran the simulation in slow motion for a few steps such that object B hit the ground and started to move up.]



I: Now that it has already hit the ground, what is going to be its acceleration? Will it continue to be 6.538 m/s^2, or will it be less than 6.538 m/s^2, or will it be more than 6.538 m/s^2?

S8: Less than 6.538.

I: Alright. As it moves up [I ran the simulation in slow motion such that object B moved up], what will be its acceleration? […] What will be its acceleration as it moves up and when it moves down?

S8: I think that when the object moves up the acceleration will be negative and when it moves down it will be positive.

I: […] Why are you saying that it will be negative?

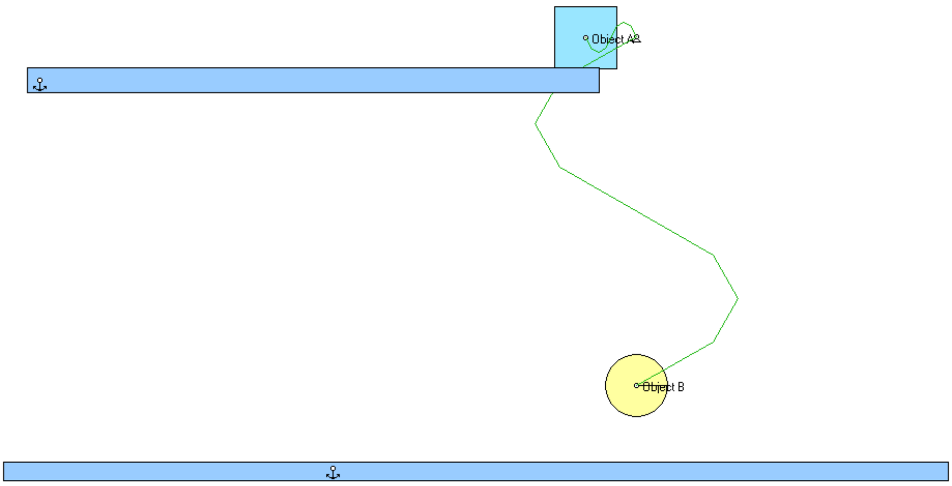
S8: Because acceleration due to gravity acts downward and here it is moving up.

I: Ok. They [referring to acceleration due to gravity and motion of object B] are in opposite directions.

S8: Yes.

I: Alright. Now how about its magnitude? […]

[I moved object B to its highest point.]



I: When it moves up, it reaches a highest point.

S8: Yes.

I: According to you, what will be its acceleration at that highest point?

S8: It will decrease to zero.

I: Ok. Then?

S8: It will increase.

I: When will it start to increase?

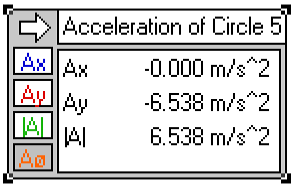
42 **S8**: When it is moving down.

I: Alright. Do you think that the increase is gradual or not?

S8: Gradual.

45 **I**: […] You want to verify it?

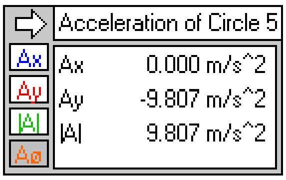
[She reset the simulation and put on the ACCELERATION meter of object B. She ran the simulation in slow motion until object B was just about to hit the surface.]



46 **S8**: Just before it hits the surface, acceleration is 6.538 [pointing at the ACCELERATION meter].

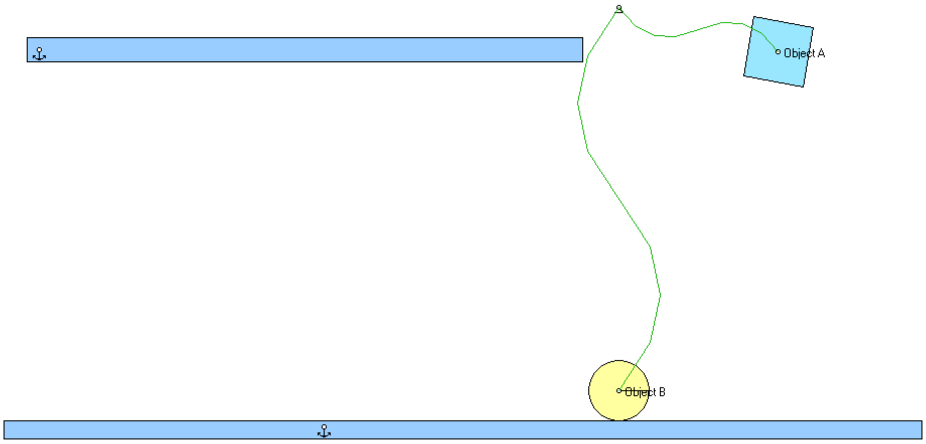
I: Alright.

[She ran the simulation one step forward such that object B just hit the surface and the string became slack. The reading on the ACCELERATION meter changed from 6.538 m/s^2 to 9.807 m/s^2. She observed for 3-4 seconds, then ran the simulation one step further, again observed for 3-4 seconds and finally ran it two more steps forward.]



49 **S8**: The acceleration has changed … now it is 9.8, acceleration due to gravity g.

[She continued to run the simulation in slow motion until object B just hit the ground for the second rebound.



She reset the simulation.]

I: What do you think?

S8: … It becomes an object that is falling freely, under gravity. This explains why it has an acceleration of 9.8.

I: Did you not think about this earlier? … What were you thinking earlier?

[She ran the simulation in slow motion and object B moved to almost 1/3 of the height.]

S8: That it [referring to acceleration of object B] would decrease.

[She continued to run the simulation in slow motion until object B moved to almost 2/3 of the height.]

S8: I thought that this [pointing at object B] was connected to this one [pointing at object A].

I: They are still connected.

S8: Yes, but it [pointing at object A] has already moved … such that it is not on the table. Then it becomes a free fall.

[She re-started to run the simulation in slow motion.]

I: Observe properly. It [referring to object A] is still on the table.

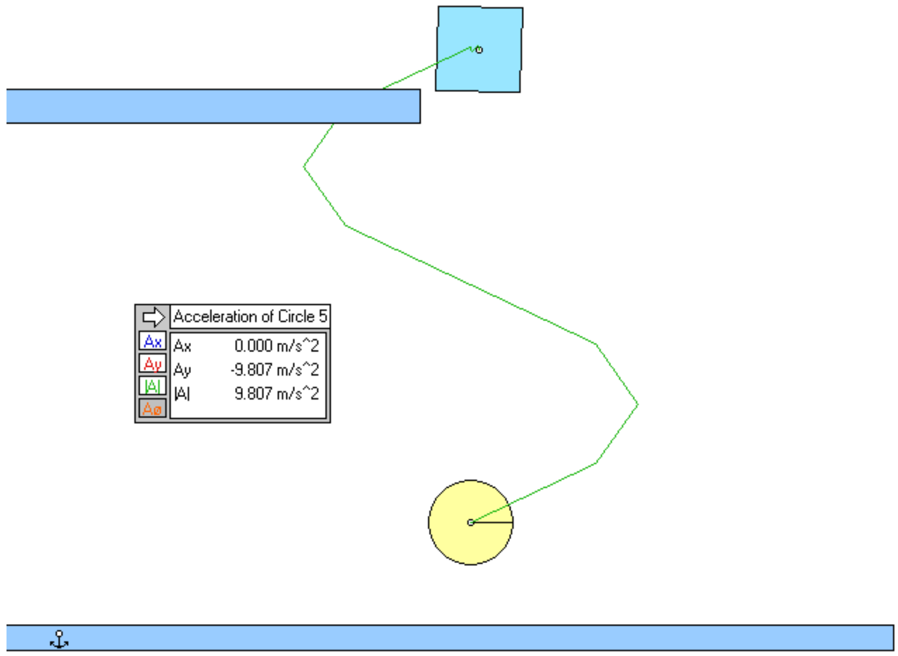
[She continued to run the simulation in slow motion until object B just hit the ground and the string became slack. Object A was still on the table.]

I: What happened?
S8: The tension decreased [pointing at the horizontal and vertical parts of the string which were slack].
I: What happens to tension?
S8: It becomes zero. It becomes slack.
I: It is the string that becomes slack, tension does not become slack. What happens when the string becomes slack?
S8: Then it [pointing at object B] acts as an object falling freely under gravity.
I: Were you not aware of this fact?
S8: No.
I: Why?
S8: I did not take it into consideration.
I: Why did you not take it into consideration?
S8: I thought it as a connected particles system and I did not realise that the tension would become slack.
I: The string would become slack.
S8: The string would become slack. I did not take this fact into consideration.
I: […] Why did you not take this fact into consideration? … What is it that ‘encouraged’ you not to pay attention to this fact and ‘encouraged’ you to focus on connected particles only?
S8: I don’t know.

[She ran the simulation a few steps and object A was almost near the end of the table.]

I: Ok […] You can continue to run the simulation.

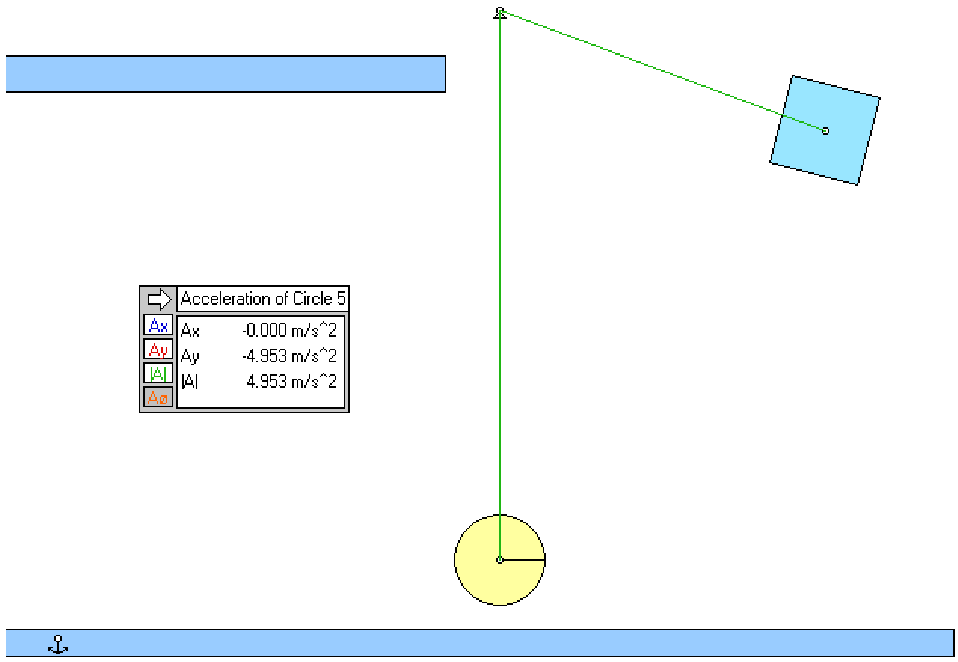
[She continued to run the simulation in slow motion and object A left the table. The string remained slack.]



87 **S8**: The acceleration is still the same.

I: And then?

[She continued to run the simulation in slow motion until the string became taut again.]



S8: Then it [pointing to ACCELERATION meter] changes.

I: Why? Why did it change?

[After some 8 seconds of silence, she ran the simulation 2 steps backward and the string became slack and the acceleration of object B became 9.807 m/s^2. Then she ran the simulation 1 step forward and the string became taut again and its acceleration became 2.049 m/s^2.]

S8: There is tension … in both objects.

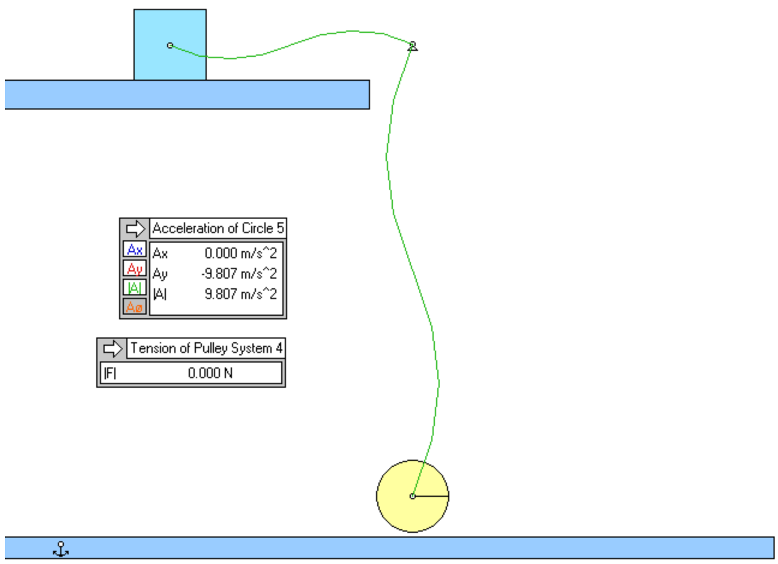
I: In both objects, there is tension?

S8: There is tension in the string.

I: Alright. Do you wish to put it [referring to the TENSION meter] on and see how it works?

S8: Yes.

[She reset the simulation, selected the string and put on the TENSION meter. She ran the simulation in slow motion until object B was just about to hit the ground and the reading on the TENSION meter was 6.538 N. She waited for around 3 seconds and ran the simulation 1 step forward such that object B just hit the ground, the string became slack and the reading on the TENSION meter became 0 N. She waited for almost 4 seconds and re-started to run the simulation in slow motion. The reading on the TENSION meter remained zero.]



S8: The tension becomes zero.

I: Ok.

[She continued to run the simulation and this is what could be seen on the screen: object B moved up, stayed stationary at its highest point for a short time, started to descend, continued to move down, hit the ground, started to move up and the string became taut. At this moment the reading on the TENSION meter was no longer zero. She started to run the simulation in slow motion for several seconds during which the string continued to remain taut. During this period, the readings on the TENSION and ACCELERATION meters fluctuated because the system was not stable.]

S8: It increases.

I: Do you find any link between acceleration and tension?

S8: Acceleration decreases [pointing at ACCELERATION meter] and tension increases [she ran the simulation 2 steps forward].

I: Ok. This observation is due to the instability of the system. What do you notice when tension is zero?

S8: When tension is zero, the object falls freely, under gravity.

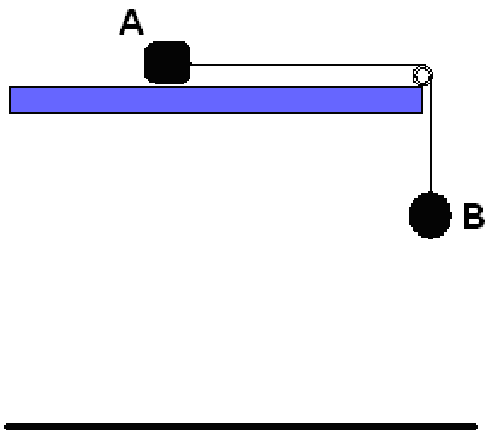
I: Ok.

Appendix H: Transcript for Student S9

I: Interviewer S9: Student

Line Transcript

[Task 1 was introduced to Student S9.]

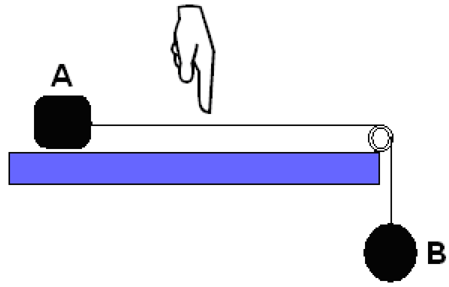


I: How may the system behave if I stop holding object A?
S9: The system will move. Object A will move towards the pulley and object B will move down.
I: Will this be always the case?
S9: No. If the mass of object A is greater than the mass of object B, the objects won't move at all.
I: What happens if the mass of object A is equal to the mass of object B?
S9: They will move until they reach equilibrium.
I: What do you mean by 'equilibrium'?
S9: They [referring to objects A & B] may move a short distance until they become stationary.
I: Why do you think that it will move a short distance and then become stationary?
S9: {…} the tension in the string [pointing at vertical part of string above object B] will pull object A down a short distance for a short

time.

I: When you're mentioning 'a short time', do you have an idea where it [referring to object A] may stop along the surface?

S9: Maybe here [pointing his finger approximately 1/3 from point of start].



I: Why here? Any idea why at this point?

S9: I don't know. {…}

I: What?

S9: According to me, it should be around here [pointing approximately 1/3 (surface) from point of start] if both objects have equal masses.

I: Ok. This will happen when both objects have equal masses.

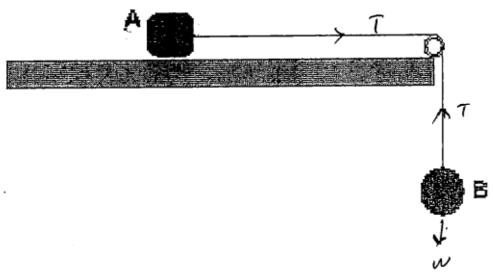
S9: Yes.

I: And what if the mass of object A is greater than the mass of object B?

S9: There will be no motion.

I: Ok. Can you please draw the force diagram?

[He drew the force diagram. During our conversation I reminded him that we were working with a smooth system.]



I: Try to run the simulation and see what happens.

[He ran the simulation using the command RUN. As he allowed object A to leave the surface and allowed the simulation to continue to run, I asked him to stop it and explained to him which part of the simulation we were interested with. I advised him to run the simulation in slow motion for some seconds and then to stop it.]

I: What can we say about the masses?
S9: This one [pointing to object B] is heavier.
I: Alright. Please reset the simulation and run it a few steps.

[He did reset the simulation and ran it in slow motion for a few seconds.]

I: Why is this object [pointing at object A] moving?
S9: It is in a system and the string is pulling it.
I: Which force is pulling the object A?
S9: Tension in the string is pulling object A.
I: […] Can you tell me what creates the tension in the string? If we want to find the value of the tension, can we assign a value to it?
S9: It is going to be nearly equal to the weight of object B.
I: Nearly equal or equal?
S9: Nearly equal, maybe a small difference.
I: Why a small difference?
S9: Friction.
I: Let's assume it's a smooth model.
S9: Then it's going to be the same.
I: So, you think that when there is motion the tension in the string is equal to
S9: weight of object B.
I: […] Now say object A is moving with 10 m/s, do you think that the speed of object B will be equal to that of object A, or will it be more than this one [pointing at object A], or will it be less than this one [pointing at object A]?

S9: It [pointing at object B] will be more.
I: Why?
S9: It [referring to object B] will have to pull object A. So, it cannot move with the same speed. It should move quicker. It should fall quicker so that it can pull it [referring to object A].
I: So, if object A is moving with speed, say 10 m/s, what can you say about the speed of object B?
S9: 20.
I: Why 20?
S9: Because it should be twice. {…}
I: Why twice? Why not thrice? Can you give me a reason?
S9: …(pause)… No.
I: Why did you say 20?
S9: It should be double. It should be twice.
I: Why twice? Why did you say 'twice'?
S9: …(pause)… Now I think that they must be equal.
I: Eh?
S9: They must be equal. I think I've made a mistake.
I: What mistake?
S9: If this one [pointing at object B] is moving with a certain speed, this one [pointing at object A] must move with the same speed because they have the same mass.
I: Why are you now changing your opinion? Why are you now saying that both objects will move with equal speeds?
S9: Their masses are not equal. So its speed [referring to object B] must be twice.
I: How do you know that their masses are not equal?
S9: The rate at which it is falling implies that it is heavier.
I: Can we verify the masses?

[He double-clicked on each object and found that the mass of object B, mB, was 2 kg and that of object A, mA, was 1 kg.]

S9: The mass of this one [pointing at object B] is greater than this one [pointing at object A].

I: Ok … How about the accelerations? If this one [pointing at object A] is moving with acceleration 1 m/s^2 at a specific time, what can we say about the acceleration of object B at that specific time?

S9: Its acceleration will be higher as its weight is greater.

I: Ok ... Can we measure the velocities of both objects? Let's measure them.

[He switched on the VELOCITY meter of object A and then that of object B. He ran the simulation using the command RUN. As the motion was swift, he could not analyze the meters properly. So he reset it and ran it in slow motion.]

I: What can we see?

S9: They are equal.

I: Hmm?

S9: The velocities are equal.

I: … This means that maybe the software is lying to us.

S9: No, maybe I am wrong.

I: Why? Why do you think that it is you who is wrong, and not the software?

S9: …(pause)… I thought that this one [pointing at object B] would move faster than this one [pointing at object A] because it [pointing at object B] has to pull it [referring to object A] … I don't know.

I: […] Do you agree with the values shown on the screen?

S9: … Yes, I agree.

I: If you agree, then why do they move with equal speeds?

S9: They are in the same system. So they are directly proportional to the tension.

I: Hmm?

S9: Since both objects are linked to each other in the same system, so when one moves the other should also move.

I: Can you clarify it?

S9: The distance that this one [pointing at object B] moves, this one [pointing at object A] will also have to move the same distance, since both objects are connected in the same system via a pulley.

I: … And how about accelerations?

S9: They should be equal.

I: Why?

S9: … According to me, it should be greater here [pointing at object B].

I: Why?

S9: Because of weight … it [pointing at object B] will fall due to gravity.

I: So?

S9: Its acceleration should be higher compared to this one [pointing at object A].

I: Can you repeat it?

S9: This one’s [pointing at object B] acceleration is greater than that of this one [pointing at object A].

I: Did you earlier mention that its acceleration [referring to object B] is equal to … g?

S9: Its weight is mg, its mass multiplied by gravity.

I: Ok. How about its acceleration?

S9: It is acceleration due to gravity, 9.81, eh, g.

I: What is going to be g?

S9: This one [pointing at object B].

I: Do you have an idea of the acceleration with which object B will fall?

S9: 9.81

I: B will fall.

S9: With acceleration due to gravity g.

I: 9.81?

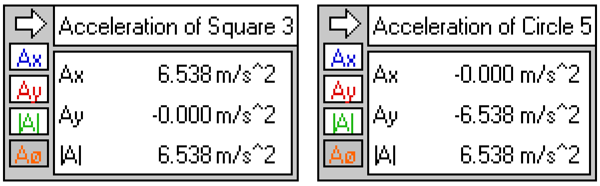
S9: Yes.

I: Alright. And how about this one [pointing to object A]?

S9: It will be less.

I: Ok. Can we measure the values?

[He first put on the ACCELERATION meter of object A and then that of object B. He ran the simulation in slow motion until object A was almost near the end of the surface. The readings on the ACCELERATION meters were as follows:]



S9: They are equal.

[He moved the simulation backward step by step until object A was near its original position on the surface.]

I: What do you find?
S9: They are equal [pointing cursor at both ACCELERATION meters].

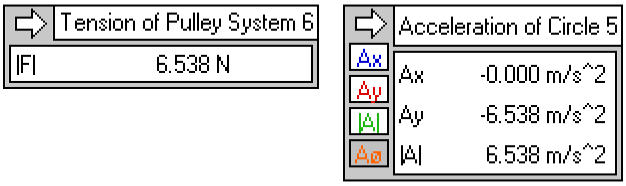
[He ran the simulation in slow motion for some steps.]

S9: They are equal [pointing cursor at both ACCELERATION meters].
I: You find that they are equal. What do you think?
S9: I thought that its acceleration [pointing at object B] would have been 9.81 but here [pointing at ACCELERATION meter of object B] it is 6.538 … It should be that this object [pointing at object A] is opposing this one’s motion [pointing at object B].
I: Can you say it again?
S9: This object [pointing at object A] is opposing its motion [pointing at object B] since it has a weight. So the accelerations must be equal … If object B was falling freely, then it would have fallen with acceleration g. But here it [pointing at object B] is attached to another object [pointing at object A] which is opposing its motion [pointing at object B].
I: Which object is opposing which object’s motion?
S9: Object A [pointing at object A] opposes the motion of object B [pointing at object B].
I: Alright. And so you think this is the reason why object B is not falling.
S9: Freely.
I: Freely?
S9: Yes.

I: Ok. Earlier you were mentioning that the tension in the string. What
170 value did you assign to it?
S9: This weight [pointing at object B].
172 **I**: Can we verify this?

[He selected the string, switched on the TENSION meter and ran the simulation in slow motion for a few steps.]

173 **S9**: Its value [pointing at TENSION meter] is the same as acceleration [pointing at ACCELERATION meter of object B].



[He continued to run the simulation in slow motion for some more steps.]

175 **I**: Hm?
S9: It [pointing at TENSION meter] is equal to the value of acceleration [pointing at ACCELERATION meter of B].
178 **I**: No, you were talking about weight. Can we measure the weight?

[He reset the simulation, switched on the GRAVITATIONAL FORCE meter for object B and ran the simulation in slow motion for a few steps.]

179 **I**: What do we find?
180 **S9**: They are equal [moving cursor between TENSION meter & ACCELERATION meter of object B] … the values of tension and acceleration are equal.
I: What?
S9: The values of tension and acceleration are equal.
185 **I**: Earlier you were saying that tension is equal to mg, the weight of the hanging object [referring to object B] and now that we have run the simulation we see that this is not the case.

S9: Yes.

I: Can we try to explain this discrepancy? Why is weight not equal to tension?

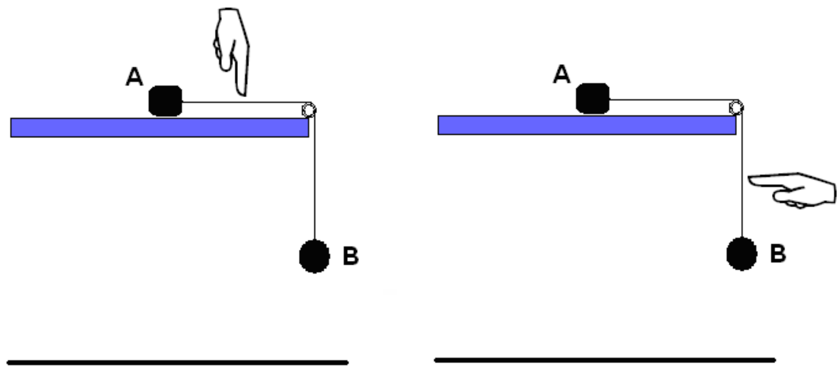
S9: … Object A is also exerting a tension.

I: I didn't understand you.

S9: This object A [pointing cursor to it] is also exerting a tension in the string [pointing cursor at the horizontal part of the string]. Maybe because of this they are not equal.

I: What are not equal?

S9: Weight [pointing to object B] is not equal to tension because there are 2 tension forces [pointing along the horizontal and vertical parts of the string].



pointing at horizontal part (string)   pointing at vertical part (string)

I: Are we talking about 1 string or 2 strings here?

S9: 1 string.

I: So, according to you is there 1 tension force or more than 1 tension force in the string?

S9: There are 2 tensions, one here [moving cursor to and fro along the horizontal part] and another one here [moving cursor to and fro along the vertical part].

I: Are they [referring to 2 tension forces] equal or unequal?

S9: They are not equal.

I: See here in your force diagram [pointing at his force diagram on the sheet of paper], you've put T here [pointing along the horizontal part of the string] and here [pointing along the vertical part of the string]. So I thought that the tension is the same throughout the string.

S9: We need to think about it. Maybe they are equal.

I: … According to you, can we have 2 tensions in the same string?

215 **S9**: … I thought that because there is another object [pointing at object A], it will exert a tension here [pointing cursor at the horizontal part of the string]. Maybe yes.

I: Hmm?

S9: Honestly I don't know.

220 **I**: […] See here [pointing at TENSION meter] when you measured the tension, you clicked on the string. Click on the string.

[He clicked on the string and the whole string was highlighted.]

222 **I**: What do we find?

S9: It is one string.

I: Which means that

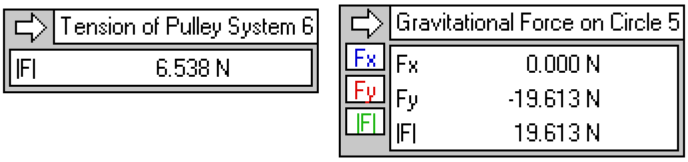
225 **S9**: There is only 1 tension.

I: There is only 1 tension throughout the string … So?

S9: Tension is the same as acceleration.

I: Let's forget about accelerations … Let's concentrate on this part

229 [pointing to TENSION and GRAVITATIONAL FORCE meters].



230 **S9**: The tension is more than half of the weight.

I: You don't have any idea about this.

S9: No.

I: Are you aware of the concept of net force? What do you understand by net force?

235 **S9**: Total force which is present on an object.

I: Can you find out the net force acting on this object [pointing at object B]?

[He switched on the TOTAL FORCE meter for object B and ran the simulation in slow motion for some steps.]

I: What do you find?

S9: The force is 13.076 N.

I: What does it mean? 13.076 N?

S9: Total force acting on object B.

I: In which direction is it acting?

S9: Downward.

I: You have a net force downward?

S9: Yes.

I: Say, as you mentioned earlier that tension is equal to weight, what would have happened? Would there have been a total force? […] Earlier you told me that tension is equal to weight.

S9: Yes.

I: Say, if tension would be equal to weight [pointing at object B], what would have been the total force?

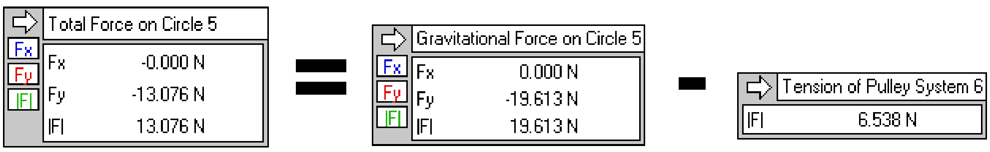
S9: Zero.

I: How did it [referring to Interactive Physics] get this total force value [pointing at TOTAL FORCE meter]?

S9: It equated tension and weight.

I: How did it arrive at this value?

S9: Maybe it subtracted this [pointing at TENSION meter] from this [pointing at GRAVITATIONAL FORCE meter].



I: What did it do?

S9: It took gravitational force [pointing to GRAVITATIONAL FORCE meter] minus tension [pointing to TENSION meter].

I: What happened then?

S9: We obtained this value [pointing at TOTAL FORCE meter]

I: Net force?

S9: Yes.

266 **I**: Then what did we observe?

S9: The difference is 13.076 N [pointing at TOTAL FORCE meter].

I: Then?

S9: If they were equal, we should have obtained zero, if tension and
270 weight were equal.

I: It should have been zero?

S9: Yes.

I: What happens if net force is equal to zero?

S9: The objects will not move … they will remain stationary.

275 **I**: Can you delete all the meters from the screen?

[He deleted all meters from the screen.]

276 **I**: Earlier you mentioned that for motion to occur, the mass of object B must be.

S9: Greater than the mass of object A.

I: And that when we have equal masses, what will happen?

280 **S9**: They will move a short distance and stop.

I: Why do you think that this will happen? […]

S9: … It clicked in my mind when you discussed this system.

283 **I**: That was your first reaction. Ok. Let’s change the mass of the object.

[He changed the mass of object B, mB, from 2 kg to 1 kg; The mass of object A, mA, remained 1 kg. He ran the simulation using the command RUN and he allowed the system to move until object A left the surface. He reset it and ran it in slow motion until object A was almost near the end of the surface.]

284 **S9**: It continues to move.

285 **I**: Eh?

[He reset the simulation, clicked on object B and its PROPERTIES box opened. He verified the mass.]

286 **I**: What happened? Are you verifying whether mass of B is still 2 kg?

S9: Yes. It continues to move.

I: … Eh?

S9: It continues to move. It should have stopped.

[He ran the simulation using RUN and stopped it when object A almost reached the end of the surface. He reset it.]

I: What do you think?

S9: … I thought that it will reach equilibrium, after some time it will stop but it continues to move just like when the masses are unequal …(pause)… I'm confused.

I: Ok. Let's try to make both masses 2 kg or 3 kg and see what happens.

[He changed the mass of object B, mB, to 2 kg. He also changed the mass of object A, mA, to 2 kg.]

I: Do you think that mass can make a difference?

S9: Perhaps.

[He ran the simulation using RUN and the system moved. He reset it.]

I: What do you think?

S9: Net force is not zero.

I: Eh?

S9: Net force is not zero.

I: Which net force?

S9: Here [pointing at object B].

I: How do you know?

S9: It [pointing at object B] continues to move. If they [referring to weight of B and tension] were equal, it should have remained at rest.

I: Can you explain it again?

S9: Net force on object B is not zero.

I: What did you think about it earlier?

S9: That it should be zero, because of equal weights and equal masses.

I: … You want to verify the total force, the net force.

S9: Ok.

[He switched on the TOTAL FORCE meter for object B and ran the simulation. Reading on the TOTAL FORCE meter was 9.807 N.]

S9: It is almost equal to gravity.
I: Eh?
S9: It [pointing to TOTAL FORCE meter] is almost equal to the value of g.
I: What is equal to the value of g?
S9: Force on B.
I: Which force?
S9: Gravity, pull due to gravity.
I: […] Ok. At the moment mA = mB = 2 kg. Let's try mA = mB = 1 kg and see what happens? … Does the total force still equal to the value of g?

[He changed the mass of object B, mB, to 1 kg, the mass of object A, mA, to 1 kg and ran the simulation. Reading on the TOTAL FORCE meter was 4.903 N.]

S9: It [pointing to TOTAL FORCE meter] has changed. They are not equal.
I: Any thought?
S9: I thought that they [referring to objects A & B] should stop but they continue to move.
I: Any idea why there is motion in a system where we have equal masses? … Do you agree that there is motion?
S9: Not really. I don't agree with it.
I: Why don't you agree with it?
S9: I think that it should stop after some time since we have equal weights.
I: And you don't agree with the simulation?
S9: Yes, I don't.
I: Ok. We'll try to find an explanation later. Let's now consider the third

case where mass of A is greater than mass of B. What do you think will happen?
S9: It should not move.
I: Let’s try it.

[He changed the mass of object A, mA, from 1 kg to 5 kg and maintained the mass of object B, mB, as 1 kg. He ran the simulation using RUN.]

S9: It is moving.
I: Eh?
S9: According to me, it should not have moved as this mass [pointing to object A] is greater.
I: The one on the surface is 5 kg and the hanging, suspended one is 1 kg … What do you think?
S9: It [pointing to object B] is pulling the object [referring to object A]. Something is wrong here … I don’t agree with this one.
I: [laughter]
S9: It should not move because this weight [pointing to object A] is bigger.
I: What?
S9: It is not possible for a small weight to pull a bigger one and it [referring to the system] continues to move.
I: So you don’t agree with it?
S9: A lighter weight is pulling a heavier weight!
I: Ok. You don’t agree with it.
S9: I’m perplexed with this. I find it strange that a heavier load is being pulled by a lighter one.
I: […] Can we try to explain it?
S9: Maybe the surface is extra smooth, like ice, it is slippery.
I: Of course the surface is smooth in this case.
S9: Maybe because of this. If we had another surface, I don’t think that it would have moved.
I: […] What is pulling object A?
S9: Object B … Force exerted by B is pulling it.

I: Object B is pulling object A? Initially didn't you state that it is the tension in the string that is pulling object A?

S9: Yes, the tension.

I: Is the weight of object B or the tension in the string that is pulling object A?

S9: The weight is the tension. It is the tension that is pulling object A.

I: […] Can we explain why there is motion?

S9: If we had another surface, it would not have moved. In this case, there is no friction which is opposing the motion.

I: Ok. Let's try it. We use the first system where mA = 1 kg and mB = 2 kg. We can introduce friction by, say, making $\mu = 0.3$.

[He changed the mass of object A, mA, to 1 kg and the mass of object B, mB, to 2 kg. He assigned μ, the coefficient of friction, the value of 0.3 to create a rough situation. He predicted and confirmed that with friction in the system the speeds of both objects would decrease.]

I: Will friction affect the tension in the string?

S9: Tension will remain the same.

I: What?

S9: Friction will not affect tension. Tension will not change.

I: Whatever be the value of μ ?

S9: We must investigate.

I: Why?

S9: The tension 'lies' here [pointing along the vertical part of the string]. It is mostly concentrated here … It may change but I'm not sure.

I: Why is it that earlier you mentioned that friction won't affect tension?

S9: I don't think that there is any relationship between the friction here [pointing cursor around object A] and tension here [pointing along the vertical part of the string].

I: Ok. And now what do you think?

S9: Perhaps friction may affect the tension in the string because object A is moving against friction, friction is opposing its motion. But I don't think so.

I: You want to try it?
S9: Yes, let's try it.

[He switched on the TENSION meter and ran the simulation in slow motion. The reading on the meter was 8.495 N (with friction, $\mu = 0.3$). After having removed friction from the system, he ran the simulation in slow motion and the reading on the TENSION meter was 6.538 N.]

S9: It is not the same.
I: Eh?
S9: It affects the tension.
I: What happened to tension when you introduce friction?
S9: It increases.
I: So, when friction is introduced.
S9: Tension increases.
I: And what do you think will happen if we increase μ to 0.6?
S9: Tension will continue to increase.

[He increased μ to 0.6 and the reading was 10.456 N.]

S9: It increases to 10.456 N.
I: What do we observe?
S9: If friction increases, tension also increases.
I: So, what can we say?
S9: They are directly proportional, tension and friction.
I: Tension and friction are directly proportional?
S9: Yes. If one increases, the other also increases.
I: How can we explain that friction affects the tension?
S9: Because object A is moving, the tension acting in this string [pointing at the horizontal part of the string] also increases.
I: But didn't you initially explain that tension in the string is due to object B only?
S9: Yes … Because of object A as well.
I: Eh?

S9: This means that tension in the string is due to object A as well as object B.

I: Can you say that again?

S9: There are 2 tensions … 2 tensions that are acting in the string … one caused by A and another one caused by B.

I: … So, what happens?

S9: There are 2 tensions that act in the string.

I: What do you mean?

S9: … There are 2 different tensions. The value of the tension here [pointing at the horizontal part of the string] is different from the value of the tension here [pointing at the vertical part of the string].

I: But we are not getting 2 values for tension on Interactive Physics.

S9: We are getting only one.

I: So, according to you, there are 2 tensions?

S9: Yes … But we have only 1 system here. {…}

I: Eh?

S9: We have only 1 string, perhaps because of this it is showing only 1 tension.

I: What do you mean?

S9: Maybe there is only 1 tension, but I’m thinking of 2 tensions.

I: Ok. This is one explanation. Any other possible explanation?

S9: Perhaps there is a tension that is acting here [pointing at the horizontal part of the string] but Interactive Physics is showing only one.

I: Ok. […]

Symbols used to transcribe data in this thesis

[ ] action taking place

[…] not relevant

{…} could not be translated

… short pause, less than 10 seconds

… (pause) … long pause, more than 10 seconds

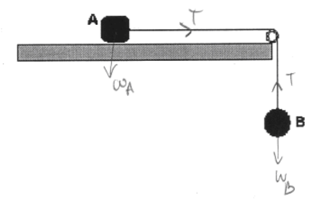
… (silence) … no talking

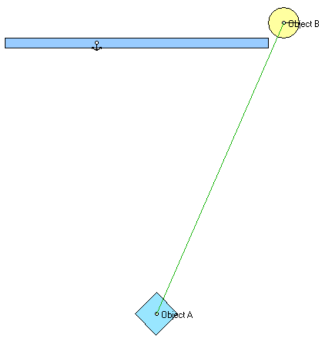
// overlapping

Appendix I: Detailed Analysis of Episode 5.3

In Chapter 5, I have discussed 7 episodes in relation to student understanding of the dynamics of motion. Here, I present the detailed analysis of Episode 5.3 upon which the analysis presented in Chapter 5 (pg 150 – 155) had been constructed. The Prediction Phase looks at what Student S2 predicts about the behaviour of the system; the Interaction Phase analyses how Interactive Physics is used during this activity; and, the Reflection Phase focuses on how S2 reacts to the IP-based setting. My commentaries appear in capital letters.

Prediction Phase	**Interaction Phase**	**Reflection Phase**
Lines 26 – 33; 70 – 74: The system would be at rest if objects A & B are of equal masses. {THIS SUGGESTS THE ACTIVATION OF THE INTUITIVE IDEA '***EQUAL MASSES IMPLIES NO MOTION***' WHICH SEEMS TO CORRESPOND TO THE P-PRIM '***DYNAMIC BALANCE***'.} Lines 33 – 36; 50 – 60: There are 2 tension forces in the string, each tension being equal to the weight of the object adjacent to it. {THIS APPEARS TO SUGGEST THE ACTIVATION OF THE P-PRIM '***CANCELING***' WHICH IN TURN ACTIVATES THE ABOVE P-PRIM '***DYNAMIC BALANCE***'.} Lines 134 – 140: The system would be at rest if object A is heavier than object B. {THIS SUGGESTS THE ACTIVATION OF THE		

INTUITIVE IDEA ‘***A LIGHTER OBJECT COULD NOT MOVE A HEAVIER ONE*** <u>OR</u> ***A HEAVIER OBJECT COULD SUPPORT A LIGHTER ONE***’ WHICH SEEMS TO CORRESPOND TO THE P-PRIM ‘***SUPPORTING***’.} He drew the following force diagram:  He explained that there were only three forces in the system: the weight of the hanging object, w_B, the tension in the string, T and the weight of the sliding object, w_A. {HIS FORCE DIAGRAM APPEARS TO BE AN EXACT REPLICA OF HIS VERBAL STATEMENT. THIS INTUITIVE IDEA OF HIS CAN ACCOUNT FOR 2 OF HIS PREDICTIONS: (1) THAT WHEN THE MASS OF OBJECT A WOULD BE EQUAL TO THAT OF OBJECT B, THE TENSION IN THE STRING WOULD BE EQUAL TO EITHER WEIGHT; AND, (2) THAT THERE WOULD BE NO MOTION FOR A SYSTEM OF EQUAL MASSES BECAUSE HE BELIEVED THAT BOTH TENSION FORCES WOULD ACT AGAINST AND CANCEL EACH OTHER.}		

Prediction Phase	Interaction Phase	Reflection Phase
	Lines 165 – 166: We find that S2 employed code VS2 (that is, he observed how objects A & B behaved during the simulation). 	Lines 166 – 171: Since object A is lower in position than object B, object A must be heavier than object B. {THIS SUGGESTS THE ACTIVATION OF THE INTUITIVE IDEA ‘***WHEN THERE ARE 2 OBJECTS SITUATED AT DIFFERENT LEVELS, THE ONE LYING AT THE BOTTOM MUST BE HEAVIER THAN THE ONE LYING AT THE TOP*** OR ***A HEAVIER LOAD COULD DISPLACE A LIGHTER ONE***’ WHICH APPEARS TO CORRESPOND TO THE P-PRIM ‘***OVERCOMING***’. THIS IS IN CONTRADICTION WITH HIS EARLIER PREDICTION THAT THE SYSTEM MUST BE AT REST IF MASS OF OBJECT A WAS GREATER THAN THAT OF OBJECT B.} Line 176: HE WAS LOST!
	Upon my advice (D1: lines 177 – 179), he verified the masses of both objects and noted that they were equal (1 kg each).	Lines 192 – 196: Weight of object B becomes a force that pulls object A and this sets the system in motion.

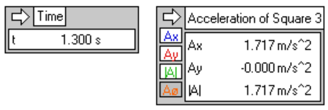
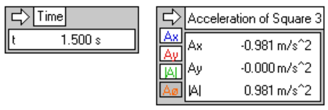
		{THIS SUGGESTS THE ACTIVATION OF THE P-PRIM '***FORCE AS MOVER***'.} Lines 198 – 199: Object B was at a lower level than object A. {THIS IMPLIES THAT BOTH OBJECTS WERE NOT BALANCED AND THIS DIFFERENCE IN LEVEL MIGHT HAVE CONTRIBUTED IN THE DISEQUILIBRIUM OF THIS SYSTEM.}
	Upon my advice (D1: lines 200 – 202), S2 started to verify his hypothesis that when mA = mB, T = wA = wB. He put on the G/FORCE meters for objects A & B, and the TENSION meter. I advised him to run the simulation in slow motion (D1). As expected, he used Code NS1, that is he compared the readings on the 3 meters appearing simultaneously. He noted that the tension in the string is not equal to the weight of object B.	He was not able to explain this discrepancy (line 209). He decided to verify the nature of the surface (line 214).
	He verified the nature of the surface and discovered that it was rough.	Line 216: The frictional force affects the tension. {THIS SUGGESTS THE ACTIVATION OF THE P-PRIM '***SPONTANEOUS RESISTANCE***'.}

Prediction Phase	Interaction Phase	Reflection Phase
	Lines 218 – 219: He removed friction from the system. We note Code NS1 in action again Lines 220 – 224: Tension is still not equal to the weight of hanging object B.	Line 226: There must be some force acting on the pulley. {THIS SUGGESTS THE ACTIVATION OF THE P-PRIM ‘***INTEFERENCE***’.}

Appendix J: Detailed Analysis of Episode 6.5

In Chapter 6, I have discussed 7 episodes in relation to student understanding of acceleration. Here, I present the detailed analysis of Episode 6.5 upon which the analysis presented in Chapter 6 (pg 205 – 213) had been constructed. The Prediction Phase looks at what Student S5 predicts about the behaviour of the system; the Interaction Phase analyses how Interactive Physics is used during this activity; and, the Reflection Phase focuses on how S5 reacts to the IP-based setting. My commentaries appear in capital letters.

Prediction Phase	**Interaction Phase**	**Reflection Phase**
Line 2: Once the string is broken, acceleration of object A will become zero. Lines 9; 11: Acceleration of object A would decrease until it would become zero. Line 14: Once the string is broken, object A will stop. Lines 17 – 19: Object A will move a bit and will then stop. {WE NOTE THAT IN BOTH CASES SHE DOES NOT ACCEPT THAT THE CHANGE (IN ACCELERATION & SPPED) CAN BE ABRUPT. HER INTUITION IS THAT THE CHANGE SHOULD BE GRADUAL. THIS SUGGESTS THE ACTIVATION OF THE P-		

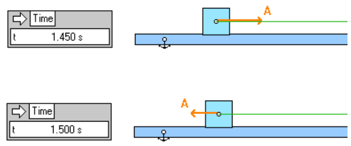
PRIM ‘***CHANGE TAKES TIME***’ <u>OR</u> ‘***WARMING UP***’.}		
	She put on the ACCELERATION meter and employed code NS3 (that is, she compared the reading on the meter at different time intervals) by using the command RUN on 2 occasions.	She did not find anything new to learn (line 22).
	{I ADVISED HER TO RUN IT IN SLOW MOTION (D1) & TO OBSERVE JUST BEFORE AND AFTER STRING WOULD BREAK (D3)}. She employed code NS3 again, but in slow motion to analyse a specific part of simulation.  Before the string breaks  Just after the string breaks	Line 26: Acceleration of object A decreases. {IN FACT, WHEN THE METER IS ANALYSED, WE SEE THAT THE READING OF THE MODULUS OF ACCELERATION, |A|, CHANGED FROM 1.717 M/S^2 TO 0.981 M/S^2, BUT THE READING OF THE COMPONENT OF ACCELERATION ALONG THE HORIZONTAL DIRECTION, A_X, CHANGED FROM 1.717

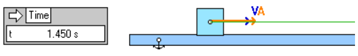
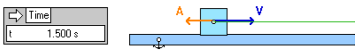
	She continued to use code NS3 and was advised to do so until object A would stop (D1).	M/S^2 TO -0.981 M/S^2. HER INTERPRETATION OF THIS SITUATION SEEMS TO SHOW THAT SHE HAD PAID ATTENTION TO THE CHANGE IN THE READING OF │A│ (LINES 74; 153 – 156).} Lines 34 – 35: Once the string breaks, the acceleration of object A decreases until it becomes zero. {I QUESTIONED HER WHETHER THIS WAS TRUE AS SHE HAD NEGLECTED THE ‘CONSTANT DECELERATION’ PART. DOES THIS IMPLY THAT SINCE SHE HAD NO INTUITIVE KNOWLEDGE OF THE ‘CONSTANT ACCELERATION’ PERIOD, SHE DID NOT PREDICT THIS SORT OF BEHAVIOUR AND DID NOT KNOW WHICH PART OF THE ANIMATION TO ANALYSE ADEQUATELY?}
	{I QUESTIONED HER WHETHER ON INTERACTIVE PHYSICS WE HAD OBSERVED THE ACCELERATION OF OBJECT A CONTINUOUSLY DECREASING UNTIL IT HAD BECOME ZERO (lines 38 – 40).} She employed code NS3 in slow motion from t = 4.55 s to t = 3.65 s.	Line 41: For some time it is a

		constant and then becomes zero. {IT WOULD SEEM THAT THE READING OF |A| BETWEEN T = 3.95 S AND T = 3.65 S MADE HER REALISE THE 'CONSTANT ACCELERATION' PROCESS OF OBJECT A.}
	She continued to use code NS3 in slow motion, ignoring my question (backward to t = 1.5 s; forward to t = 1.9s; and, forward to t = 3.35s).	Lines 46 – 47: When the string breaks, the acceleration remains constant for some time and then decreases to zero. {I ASKED HER WHETHER OR NOT SHE AGREED WITH THIS FACT. SHE EXPLAINED THAT WHEN SHE ANALYSED THE SYSTEM "IN TERMS OF VELOCITY, IT MAKES SENSE" (LINE 52). ACCORDING TO HER, WHEN THE STRING BROKE, OBJECT B WAS NO LONGER PART OF THE SYSTEM AND THEREFORE THE VELOCITY OF OBJECT A WOULD DECREASE UNTIL IT WOULD BECOME ZERO. THIS COULD EXPLAIN WHY THE ACCELERATION OF OBJECT A REMAINED CONSTANT AND THEN BECAME ZERO (LINES 54 – 60). SO, FOR HER, WHEN VELOCITY BECAME ZERO, ACCELERATION SHOULD ALSO BECOME ZERO (LINES 60 – 61). IT CAN BE HYPOTHESIZED

		THAT THE (DUAL) RELATIONSHIP BETWEEN VELOCITY AND ACCELERATION ALLOWED HER TO MAKE SENSE OF THE BEHAVIOUR OF OBJECT A WHEN THE STRING HAD BROKEN.}
	Upon my advice (D1: lines 64 – 65), she put on the VELOCITY meter to verify her hypothesis. She used code NS3 (in slow motion) again to analyse a specific part of simulation (from t = 3.55 s to t = 2.0 s). Once again she employed code NS3 (in slow motion) to analyse the simulation (from t = 1.4 s to t = 3.6 s).	Line 67: The speed decreases. Lines 70 – 77: {A SERIOUS MATTER AT THIS INSTANCE IS WHY SHE DID NOT REALISE THAT JUST BEFORE THE STRING HAD BROKEN, ITS ACCELERATION WAS A CONSTANT AND ITS VELOCITY WAS INCREASING, WHEREAS JUST AFTER THE STRING HAD BROKEN ITS ACCELERATION WAS AGAIN A CONSTANT, AS PER HER EXPLANATION AND YET ITS VELOCITY WAS DECREASING.} When she was asked whether the speed of an object could decrease when it would accelerate, she replied that this was only possible when the object would decelerate (line 85).

		{THOUGH SHE APPEARED TO BE FAMILIAR WITH THE TERM 'DECELERATION' (LINES 98; 100), THERE IS NO EVIDENCE THAT SHE HAD USED IT TO MAKE SENSE OF THIS CONTEXT (LINES 109 – 110). FOR HER, THE DECELERATION PROCESS STARTED WHEN THE STRING HAD BROKEN – THE READING OF THE MODULUS OF ACCELERATION, $\vert A \vert$, CHANGED FROM 1.717 M/S^2 TO 0.981 M/S^2 (LINES 136 – 146; 153 – 156). SHE BELIEVED THAT A DECELERATION PROCESS OCCURRED WHEN "THERE IS A DECREASE IN ACCELERATION" (LINES 150 – 152), IMPLYING THAT FOR HER A DECREASE IN THE MAGNITUDE OF ACCELERATION WAS SYNONYMOUS TO DECELERATION. THIS MAY EXPLAIN WHY SHE DID NOT PAY ATTENTION TO A_X ON THE METER.}
	Upon my advice (R2 + D1: line 158), she used code NS4 (that is, she examined the A_X component in order to investigate the acceleration of object A over a certain time interval) in slow motion. She ran the simulation one step forward from t = 1.45 s to t = 1.5 s and Ax changed from 1.717 to -0.918. After some seconds, she ran the simulation one step backward, from t = 1.5 s to t = 1.45 s, and then one step forward, from t = 1.45 s to t = 1.5 s.	

		Line 160: She appeared not to make sense of A_X changing from 1.717 m/s^2 to -0.981 m/s^2 at the point where the string broke.
	I helped her put on the vector arrow ACCELERATION on object A (D1) and she ran the simulation in slow motion (code VS3). Before the string broke, the arrow, of a fixed length, pointed towards the right. At t = 1.5 s, the arrow changed its direction and its length was reduced until it disappeared at t = 4.15 s..  At t = 4.550 s, |A| = A_X = 0 m/s^2 & v = 0 m/s. As the object had stopped moving, she moved the simulation backward (by using the tape player control) to the position where the vector arrow had changed direction (t = 1.5 s). At this point she reiterated her initial idea that when the string broke, object A decelerated (line 164). When she was asked if she had anything more to say, she continued to ‘play’ with the simulation at the point where the vector arrow had changed direction.	Lines 166 – 174: When she was asked about the behaviour of acceleration when the string had broken, she noted a change in direction of acceleration and that deceleration implied “it

		becomes minus”. {IT CAN BE HYPOTHESIZED THAT IN THIS SITUATION THE VECTOR ARROW **A**, UNLIKE THE ACCELERATION METER, ENABLED HER TO REALISE THAT THE DIRECTION OF THE DECELERATION PROCESS SHOULD BE OPPOSITE TO THAT OF THE ACCELERATION PROCESS. SHE COULD THUS UNDERSTAND THE IMPORTANCE OF THE A_X COMPONENT IN THE ACCELERATION METER AND THE SIGNIFICANCE OF THE MINUS SIGN ON A_X ONCE THE STRING HAD BROKEN.}
	Upon my advice (D1: line 176), she used code VS3, but this time she added the vector arrow VELOCITY on object A. Code VS4 was then put into practice such that S5 observed the interplay between the VELOCITY & ACCELERATION arrows as the simulation was run in slow motion.   When the simulation reached t = 1.50 s (the point where the string broke), the vector arrow VELOCITY continued to point towards the right, but that of	

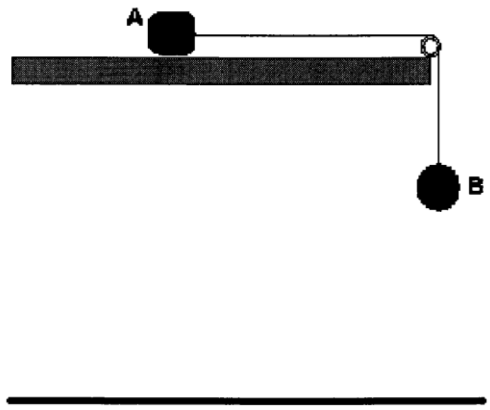
	ACCELERATION pointed towards the left. As she continued to run the simulation from t = 1.50 s in slow motion, the length of the vector arrow VELOCITY decreased gradually until it disappeared from the screen at t = 3.8 s – at this point the readings of $\lvert V \rvert$ and V_X were 0.297 m/s. After the object had stopped moving, she moved the simulation to t = 1.40 s via the tape player control and then ran the simulation in slow motion up to t = 1.50 s.	Lines 178 – 184: {IT CAN BE SUGGESTED THAT SHE WAS NOT AWARE THAT ONCE THE STRING WOULD BREAK, VELOCITY AND ACCELERATION WOULD BE IN OPPOSITE DIRECTIONS. IN THIS INSTANCE, AT THE POINT WHERE THE STRING BROKE, WHILE THE VECTOR ARROW ACCELERATION **A** SHOWED THAT ACCELERATION CHANGED DIRECTION, THE VECTOR ARROW VELOCITY **V** SHOWED THAT VELOCITY REMAINED IN THE SAME DIRECTION. SO, THE INTERPLAY BETWEEN **A** AND **V** ALLOWED HER TO CONSTRUCT THE UNDERSTANDING THAT AT THE POINT WHERE THE STRING BROKE, THESE 2 QUANTITIES BECAME OPPOSITE TO EACH OTHER SUCH THAT THEY COULD NOT BE IN THE SAME DIRECTION DURING THE DECELERATION PROCESS.}

Appendix K: Solutions to Some Mechanics Questions

In this appendix, we look at some of the basic mechanics ideas that sixth form students should develop when learning motion of connected particles in the option M1 at GCE A-Level Mathematics. Let us use Task 1 as the starting point.

Task 1

Object A is held at rest on a horizontal table. It is connected to a freely hanging object B by a piece of rope passing over a pulley at the edge of the table. The system is released from rest.

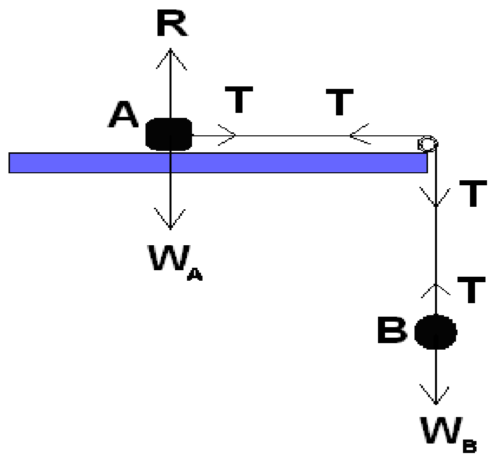


How might the system behave?

SOLUTION:

We may start by asking ourselves when motion will occur. When will this system move? We may first choose to work with a smooth surface and afterwards move to a rough situation. It is also important to discuss the assumptions made in such a system.

The force diagram for a smooth situation will be as follows:



[T represents the tension in the string; R is the normal reaction of the surface on object A; W_A is the weight of object A; and, W_B is the weight of object B.]

1) When weight of the hanging object B, W_B, is greater than the tension in the string, T, there is motion. And acceleration of either object must be less than g.

2) When $W_B = T$, the system is at rest.

3) If we alter the mass of either object (A or B) on Interactive Physics, tension in the string will change, showing that T is not a fixed quantity. For instance, if we increase the mass of either object, T will increase. This proves that T is dependent on both object, NOT on object B only.

4) It is the tension in the string, not (the weight of) object B, that acts on the sliding object A.

5) In Mechanics at GCE A-Level Mathematics, we assume that we are using a smooth, light pulley so that the tension in the string T is the same on both sides of the pulley. We also assume that the string is light.

6) During 'usual' motion (as defined in Chapter 6 pg 184), both objects move with the same speed and acceleration. In this case, we assume that the rope is inextensible. Interestingly, this system never moves with constant speed.

7) If the string becomes slack or is broken, then both objects need not move with equal speeds or equal accelerations. At the point where the string becomes slack or is broken, the change in acceleration of either object need not be gradual. For instance, once the string is broken or becomes slack, the acceleration of object B instantaneously becomes g.

TASK 8

In relation to the Vertical version, if both objects are of equal masses, there will be no motion. If they are at different levels, they will continue to remain in those positions (Figure a). They will not move to the same level (Figure b).

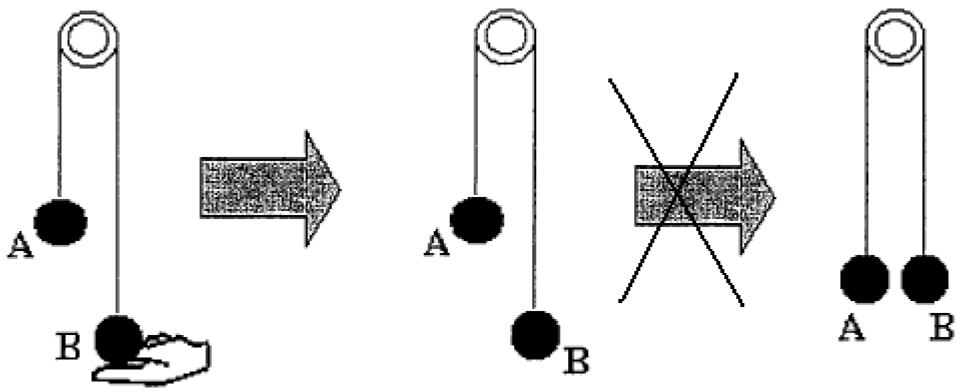


Figure a Figure b