Sliding mode control for singular stochastic Markovian jump systems with uncertainties ⋆

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Abstract

This paper considers sliding mode control design for singular stochastic Markovian jump systems with uncertainties. A suitable integral sliding function is proposed and the resulting sliding mode dynamics is an uncertain singular stochastic Markovian jump system. A set of new sufficient conditions is developed which not only guarantees the stochastic admissibility of the sliding mode dynamics, but also determines all the parameter matrices in the integral sliding function. Then, a sliding mode control law is synthesized such that reachability of the specified sliding surface can be ensured. Finally, three examples are given to demonstrate the effectiveness of the results.

Key words: Singular systems; Stochastic systems; Markovian jump systems; Sliding mode control.

1 Introduction

Markovian jump systems (MJSs) have the advantage of better representing physical systems with random changes in both structure and parameters. Much recent attention has been paid to the investigation of these systems (Fang and Loparo, 2002; Yue and Han, 2005; Xiong and Lam, 2006). Singular systems have extensive applications in fields related to electrical circuits and power systems (Yang, Zhang and Zhou, 2006; Lewis, 1986). When singular systems experience abrupt changes in their structure, it is natural to model them as singular Markovian jump systems (SMJSs) (Boukas, 2008; Huang and Mao, 2010). In practice, these systems are often corrupted by noise, for example Brownian motion. Therefore it is of significance to study singular stochastic Markovian jump systems (SSMJSs).

Sliding mode control (SMC) has been recognized as an effective strategy for control of systems with uncertainties and nonlinearity (Hung, Gao and Hung, 1993; Ma and Boukas, 2009). The sliding mode dynamics is a reduced-order system and completely insensitive to matched uncertainties (Utkin, Guldner and Shi, 1999; Edwards and Spurgeon, 1998). Sliding mode methods can also be applied to systems in the presence of mismatched uncertainties (Yan, Spurgeon and Edwards, 2005). To obtain similar levels of robustness from a classical linear state feedback controller, high gain is required (Young, Utkin and Özgüner, 1999) which can be limiting in terms of controller saturation and practical application. A novel augmented sliding mode observer is presented for the augmented system of MJSs and is utilized to eliminate the effects of sensor faults and disturbances (Li, Gao, Shi and Zhao, 2014). Sliding mode methods are successfully applied to uncertain time-delay systems (Alwi and Edwards, 2008; Fridman, Gouaisbaut, Dambrine and Richard, 2003; Yan, Spurgeon and Edwards, 2013), interconnected systems (Yan, Spurgeon and Edwards, 2010), stochastic systems (Niu, Ho and Wang, 2007; Shi, Xia, Liu and Rees, 2006), SMJSs (Wu, Su and Shi, 2012; Wu and Daniel, 2010; Wu and Zheng, 2009). When a linear sliding function is

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used, the dimension of the resulting sliding motion will be reduced and the regular form typically used for sliding mode control design (Edwards and Spurgeon, 1998) is necessary in order to solve the corresponding existence problem. When considering singular systems, this regular form is available only if the column vector of the input matrix B is a linear representation of that of the derivative matrix E. In comparison, the integral-type sliding function introduces a compensator whose dimension is equal to the dimension of the input vector and the resulting sliding motion is of full order. In this case the regular form typically adopted for sliding mode controller design is not required and the integral-type sliding function (Wu et al., 2012; Wu and Daniel, 2010; Wu and Zheng, 2009) is suitable for any singular system. In (Wu et al., 2012; Wu and Zheng, 2009), parameter matrices G_i (G) in the sliding function need to be designed in advance. If the selection of these parameter matrices is not appropriate, additional conservatism will be introduced into the stability analysis of the resulting sliding mode dynamics. In order to decrease the conservatism, these parameter matrices need be redesigned but no constructive design approach is given. In (Wu and Daniel, 2010), although a method of how to design all the parameter matrices in the sliding function is given, a particular constraint must be satisfied so that EB_i for system matrices E and B_i must have full column rank.

This paper considers the design of a SMC for a class of uncertain SSMJSs. Key questions to be addressed are stated as follows:
Q1. How to design a suitable sliding function such that conditions developed for the stochastic admissibility of the resulting sliding mode dynamics can determine all the parameter matrices in the sliding function complementing existing design methods?
Q2. How to analyze and synthesize a SMC law so that the proposed approach can effectively reject the effect of Markovian switching on the desired dynamic performance of uncertain SSMJSs?

2 System representation and preliminaries

Consider a nonlinear SSMJS described as follows:

$$\begin{aligned} E\mathrm{d}x(t) = {} & [(A(r_t) + \Delta A(r_t,t))\,x(t) \\ & + B(r_t)(u(t) + f(x(t),r_t))]\,\mathrm{d}t \\ & + D(r_t)\,x(t)\,\mathrm{d}\varpi(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input and $\varpi(t)$ is a one-dimensional Brownian motion satisfying $\mathcal{E}\{\mathrm{d}\varpi(t)\} = 0$ and $\mathcal{E}\{\mathrm{d}\varpi^2(t)\} = \mathrm{d}t$, $\mathcal{E}\{\cdot\}$ denotes the mathematical expectation of the stochastic process or vector. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular. It is assumed that $\mathrm{rank}(E) = r \le n$. Matrices $A(r_t), B(r_t)$ and $D(r_t)$ are known and real with appropriate dimensions where $B(r_t)$ has full column rank, $\Delta A(r_t,t)$ is uncertain and satisfies

$$\Delta A(r_t,t) = M(r_t)\,F(r_t,t)\,N(r_t) \tag{2}$$

where matrices $M(r_t)$ and $N(r_t)$ are known, and the function matrix $F(r_t,t)$ is unknown and Lebesgue-measurable with

$$F^{\mathrm{T}}(r_t,t)\,F(r_t,t) \le I$$

for all $t \ge 0$; $\{r_t, t \ge 0\}$ is a continuous-time Markov process with right continuous trajectories taking values in a finite set $\mathcal{S} = \{1,2,\cdots,N\}$ with the transition rate matrix (TRM) $\Pi \triangleq \{\pi_{ij}\}$ given by

$$\mathcal{P}\{r_{t+h} = j\,|r_t = i\} = \begin{cases} \pi_{ij}h + o(h) & i \ne j \\ 1 + \pi_{ii}h + o(h) & i = j \end{cases} \tag{3}$$

where $h > 0$, $\lim\limits_{h\to 0} {o(h)}/{h} = 0$; $\pi_{ij} \ge 0$ for $j \ne i$ is the transition rate from mode i at time t to j at time $t+h$, which satisfies $\pi_{ii} = -\sum\limits_{j=1,j\ne i}^{N} \pi_{ij}$; the nonlinear term $f(x(t),r_t) \in \mathbb{R}^m$ represents the system nonlinearity satisfying

$$\|f(x(t),r_t)\| \le \vartheta_{r_t}\|x(t)\| \le \vartheta\|x(t)\|,\ r_t \in \mathcal{S} \tag{4}$$

where $\vartheta_{r_t} > 0$ is a constant and $\vartheta \triangleq \max_{i\in\mathcal{S}}(\vartheta_i)$.

For each $r_t = i \in \mathcal{S}$, corresponding matrices or vectors relating to r_t in the system (1) are denoted with the index i, for example, $A(r_t) = A_i$, $\Delta A(r_t,t) = \Delta A_i(t)$, and $f(x(t),r_t) = f_i(x)$ etc.

The unforced nominal system of the system (1) can be described as

$$E\mathrm{d}x(t) = A_i x(t)\,\mathrm{d}t + D_i x(t)\,\mathrm{d}\varpi(t) \tag{5}$$

A basic assumption and a definition are first introduced.

Assumption 1: For $i \in \mathcal{S}$, $\mathrm{rank}(E) = \mathrm{rank}([E \quad D_i])$.

Definition 1 (Xu and Lam, 2006)

(i) The continuous SSMJS (5) is said to be regular if $\det(sE - A_i)$ is not identically zero for every $i \in \mathcal{S}$.
(ii) The continuous SSMJS (5) is said to be impulse-free if $\deg(\det(sE - A_i)) = \mathrm{rank}(E)$ for every $i \in \mathcal{S}$.
(iii) The continuous SSMJS (5) is said to be stochastically stable if for any $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{S}$, there exists a scalar $\tilde{M}(x_0,r_0) > 0$ such that

$$\begin{aligned} & \lim_{t\to\infty} \mathcal{E}\left\{\int_0^t x^{\mathrm{T}}(s,x_0,r_0)\,x(s,x_0,r_0)\,\mathrm{d}s\,|x_0,r_0\right\} \\ & \le \tilde{M}(x_0,r_0) \end{aligned}$$

where $x(t, x_0, r_0)$ denotes the solution under the initial condition x_0 and r_0.
(iv) The continuous SSMJS (5) is said to be stochastically admissible if it is regular, impulse-free and stochastically stable.

Lemma 1 (Xu and Hu, 2007): For given matrices $E, X > 0, Y$, if $E^{\mathrm{T}}X + Y\Lambda^{\mathrm{T}}$ is nonsingular, then there exist matrices $S > 0, L$ such that $ES + L\Theta^{\mathrm{T}} = \left(E^{\mathrm{T}}X + Y\Lambda^{\mathrm{T}}\right)^{-1}$, where $X, S \in \mathbb{R}^{n\times n}$, $Y, L \in \mathbb{R}^{n\times(n-r)}$, and $\Lambda, \Theta \in \mathbb{R}^{n\times(n-r)}$ are any matrices with full column rank satisfying $E^{\mathrm{T}}\Lambda = 0$, $E\Theta = 0$.

Lemma 2: Let M, F, N and P be real matrices of appropriate dimensions with $P > 0$, $F^{\mathrm{T}}F \leq I$ and a scalar $\varepsilon > 0$. Then

$$MFN + N^{\mathrm{T}}F^{\mathrm{T}}M^{\mathrm{T}} \leq \varepsilon MP^{-1}M^{\mathrm{T}} + \tfrac{1}{\varepsilon}N^{\mathrm{T}}F^{\mathrm{T}}PFN$$

The proof is trivial, so it is omitted. □

Remark 1: From Lemma 2, when $F = I$, it follows that $MN + N^{\mathrm{T}}M^{\mathrm{T}} \leq \varepsilon MP^{-1}M^{\mathrm{T}} + \frac{1}{\varepsilon}N^{\mathrm{T}}PN$, and when $P = I$, it follows that $MFN + N^{\mathrm{T}}F^{\mathrm{T}}M^{\mathrm{T}} \leq \varepsilon MM^{\mathrm{T}} + \frac{1}{\varepsilon}N^{\mathrm{T}}N$.

3 SMC synthesis

In this section, a sliding surface is designed and the corresponding sliding motion is analyzed. Then sliding mode controllers are synthesized such that the closed-loop system has the desired performance.

For the system (1), consider the following integral sliding function:

$$s(t) = B_i^{\mathrm{T}}\bar{P}_iEx(t) - \int_0^t B_i^{\mathrm{T}}\bar{P}_i(A_i + B_iK_i)x(\theta)\,\mathrm{d}\theta \tag{6}$$

where $\bar{P}_i \in \mathbb{R}^{n\times n}$ and $K_i \in \mathbb{R}^{m\times n}$ are real matrices to be designed with $B_i^{\mathrm{T}}\bar{P}_iB_i$ being nonsingular. It should be noted that due to the assumption that B_i is full column rank, the nonsingularity of $B_i^{\mathrm{T}}\bar{P}_iB_i$ can be ensured if $\bar{P}_i > 0$ for $i \in \mathcal{S}$.

Considering the solution of the system (1), it is straightforward to see that the term $Ex(t)$ can be expressed by:

$$\begin{aligned} Ex(t) = {} & Ex(0) + \textstyle\int_0^t [(A_i + \Delta A_i(\theta))x(\theta) \\ & + B_i(u(\theta) + f_i(x))]\,\mathrm{d}\theta + \textstyle\int_0^t D_ix(\theta)\mathrm{d}\varpi(\theta) \end{aligned} \tag{7}$$

where the last term is the Itô stochastic integral. Thus, from (6) and (7), it follows that

$$\begin{aligned} s(t) = {} & B_i^{\mathrm{T}}\bar{P}_iEx(0) + \textstyle\int_0^t \left[B_i^{\mathrm{T}}\bar{P}_i(\Delta A_i(\theta) - B_iK_i)\right. \\ & \left.+ B_i^{\mathrm{T}}\bar{P}_iB_i(u(\theta) + f_i(x))\right]\mathrm{d}\theta + \textstyle\int_0^t B_i^{\mathrm{T}}\bar{P}_iD_ix(\theta)\mathrm{d}\varpi(\theta) \end{aligned} \tag{8}$$

This implies that the sliding function $s(t)$ in (6) is well defined for the system (1). Hence, if $B_i^{\mathrm{T}}\bar{P}_iD_i = 0$, then the sliding function is described by

$$\begin{aligned} s(t) = {} & B_i^{\mathrm{T}}\bar{P}_iEx(0) + \textstyle\int_0^t \left[B_i^{\mathrm{T}}\bar{P}_i(\Delta A_i(\theta) - B_iK_i)\right. \\ & \left.+ B_i^{\mathrm{T}}\bar{P}_iB_i(u(\theta) + f_i(x))\right]\mathrm{d}\theta \end{aligned} \tag{9}$$

From SMC theory, when the sliding motion takes place, it follows that $s(t) = 0$ and $\dot{s}(t) = 0$. From (9),

$$\begin{aligned} \dot{s}(t) = {} & B_i^{\mathrm{T}}\bar{P}_i(\Delta A_i(t) - B_iK_i)x(t) \\ & + B_i^{\mathrm{T}}\bar{P}_iB_i(u(t) + f_i(x)) \end{aligned} \tag{10}$$

Further, from $\dot{s}(t) = 0$, the equivalent control can be given by

$$u_{eq} = K_ix(t) - \left(B_i^{\mathrm{T}}\bar{P}_iB_i\right)^{-1}B_i^{\mathrm{T}}\bar{P}_i\Delta A_i(t)x(t) - f_i(x) \tag{11}$$

Thus, by substituting the equivalent control (11) into the system (1), the sliding mode dynamics can be obtained as

$$\begin{aligned} E\mathrm{d}x(t) = {} & (A_i + B_iK_i + \Delta A_i(t) \\ & -B_i\left(B_i^{\mathrm{T}}\bar{P}_iB_i\right)^{-1}B_i^{\mathrm{T}}\bar{P}_i\Delta A_i(t)\Big)x(t)\,\mathrm{d}t \\ & + D_ix(t)\,\mathrm{d}\varpi(t) \end{aligned} \tag{12}$$

The following result is ready to be presented.

Theorem 1: The sliding mode dynamics (12) is stochastically admissible if there exist scalars $\varepsilon_{1i} > 0, \varepsilon_{2i} > 0$, matrices $\bar{P}_i > 0, L_i$ and $\bar{S}_i$ for $i \in \mathcal{S}$ such that

$$\begin{bmatrix} \Xi_i & * & * & * & * & * \\ \varepsilon_{2i}B_i^{\mathrm{T}} & -B_i^{\mathrm{T}}\bar{P}_iB_i & * & * & * & * \\ N_iY_i & 0 & -\varepsilon_{1i}I & * & * & * \\ N_iY_i & 0 & 0 & -\varepsilon_{2i}I & * & * \\ \Gamma^{\mathrm{T}}H^{-1}D_iY_i & 0 & 0 & 0 & -\Theta_i & * \\ \Gamma^{\mathrm{T}}H^{-1}\Psi_i^{\mathrm{T}} & 0 & 0 & 0 & 0 & -J_i \end{bmatrix} < 0 \tag{13}$$

$$\begin{bmatrix} -\bar{P}_i & * \\ M_i^{\mathrm{T}}\bar{P}_i & -\varepsilon_{2i}I \end{bmatrix} < 0 \tag{14}$$

$$B_i^{\mathrm{T}}\bar{P}_iD_i = 0 \tag{15}$$

where $\Gamma = [I_r \;\; 0]^{\mathrm{T}}$, $\Xi_i = \pi_{ii}Y_i^{\mathrm{T}}E^{\mathrm{T}} + Y_i^{\mathrm{T}}A_i^{\mathrm{T}} + L_i^{\mathrm{T}}B_i^{\mathrm{T}} + A_iY_i + B_iL_i + \varepsilon_{1i}M_iM_i^{\mathrm{T}}$, $\Theta_i = \Gamma^{\mathrm{T}}GEY_iG^{\mathrm{T}}\Gamma$, $\Psi_i = \left[\sqrt{\pi_{i1}}Y_i^{\mathrm{T}} \cdots \sqrt{\pi_{ii-1}}Y_i^{\mathrm{T}} \; \sqrt{\pi_{ii+1}}Y_i^{\mathrm{T}} \cdots \sqrt{\pi_{iN}}Y_i^{\mathrm{T}}\right]$, $J_i = \mathrm{diag}\{\Theta_1, \cdots, \Theta_{i-1}, \Theta_{i+1}, \cdots, \Theta_N\}$, $L_i = K_iY_i$ and $Y_i = \left(E\bar{P}_i + \bar{S}_i\bar{R}^{\mathrm{T}}\right)^{\mathrm{T}}$ where $\bar{R} \in \mathbb{R}^{n\times(n-r)}$ is any matrix with full column rank and satisfies $E\bar{R} = 0$. Matrices G and H are nonsingular satisfying $GEH = \mathrm{diag}\{I_r, 0\}$. Moreover, the parameter K_i in (6) is given by

$$K_i = L_iY_i^{-1} \tag{16}$$

Proof: From (13), it follows that Y_i is nonsingular. Because $Y_i = \left(E\bar{P}_i + \bar{S}_i\bar{R}^{\mathrm{T}}\right)^{\mathrm{T}}$ and $E\bar{R} = 0$,

$$Y_i^{\mathrm{T}}E^{\mathrm{T}} = EY_i = E\bar{P}_iE^{\mathrm{T}} \geq 0 \tag{17}$$

Denote

$$H^{-1}Y_iG^{\mathrm{T}} = \begin{bmatrix} Y_{i11} & Y_{i12} \\ Y_{i21} & Y_{i22} \end{bmatrix}$$

From (17), it is straightforward to see that $Y_{i12} = 0$ and Y_{i11} is symmetric, which implies that Y_{i11} and Y_{i22} are nonsingular. Therefore,

$$G^{-\mathrm{T}}Y_i^{-1}H = \begin{bmatrix} Y_{i11}^{-1} & 0 \\ -Y_{i22}^{-1}Y_{i21}Y_{i11}^{-1} & Y_{i22}^{-1} \end{bmatrix}$$

and $\Gamma^{\mathrm{T}}GEY_iG^{\mathrm{T}}\Gamma = Y_{i11}$ is nonsingular.

From Lemma 1, there exists $X_i \triangleq Y_i^{-1} = \left(E^{\mathrm{T}}P_i + S_iR^{\mathrm{T}}\right)^{\mathrm{T}}$ where $P_i > 0$, $S_i \in \mathbb{R}^{n\times(n-r)}$, and the matrix $R \in \mathbb{R}^{n\times(n-r)}$ is full column rank and satisfies $E^{\mathrm{T}}R = 0$. It is clear to see that

$$H^{-\mathrm{T}}\Gamma\left(\Gamma^{\mathrm{T}}GEY_jG^{\mathrm{T}}\Gamma\right)^{-1}\Gamma^{\mathrm{T}}H^{-1} = E^{\mathrm{T}}X_j = E^{\mathrm{T}}P_jE \tag{18}$$

By (13) and (14), it can be obtained that

$$\begin{aligned}
&(A_i + B_iK_i)^{\mathrm{T}}X_i + X_i^{\mathrm{T}}(A_i + B_iK_i) + \varepsilon_{1i}X_i^{\mathrm{T}}M_iM_i^{\mathrm{T}}X_i \\
&+\tfrac{1}{\varepsilon_{1i}}N_i^{\mathrm{T}}N_i + \varepsilon_{2i}^2X_i^{\mathrm{T}}B_i\left(B_i^{\mathrm{T}}\bar{P}_iB_i\right)^{-1}B_i^{\mathrm{T}}X_i \\
&+\tfrac{1}{\varepsilon_{2i}^2}N_i^{\mathrm{T}}F^{\mathrm{T}}(t)\,M_i^{\mathrm{T}}\bar{P}_iM_iF(t)\,N_i + D_i^{\mathrm{T}}E^{\mathrm{T}}P_iED_i \\
&+\sum_{j=1}^{N}\pi_{ij}E^{\mathrm{T}}P_jE < 0
\end{aligned} \tag{19}$$

By Lemma 2, it follows from (19) that

$$X_i^{\mathrm{T}}\Lambda_i + \Lambda_i^{\mathrm{T}}X_i + D_i^{\mathrm{T}}E^{\mathrm{T}}P_iED_i + \sum_{j=1}^{N}\pi_{ij}E^{\mathrm{T}}P_jE < 0 \tag{20}$$

where
$\Lambda_i = A_i + B_iK_i + \Delta A_i(t) - B_i\left(B_i^{\mathrm{T}}\bar{P}_iB_i\right)^{-1}B_i^{\mathrm{T}}\bar{P}_i\Delta A_i(t)$

R can be rewritten as $R = G^{\mathrm{T}}\begin{bmatrix} 0 \\ \bar{\Phi} \end{bmatrix}$ where $\bar{\Phi} \in \mathbb{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix. From Assumption 1

$$GD_iH = \begin{bmatrix} D_{i1} & D_{i2} \\ 0 & 0 \end{bmatrix}$$

Partition the matrices $G\Lambda_iH$, $G^{-\mathrm{T}}P_iG^{-1}$ and $H^{\mathrm{T}}S_i$ in a compatible way as follows

$$G\Lambda_iH = \begin{bmatrix} A_{i1}(t) & A_{i2}(t) \\ A_{i3}(t) & A_{i4}(t) \end{bmatrix}$$

$$G^{-\mathrm{T}}P_iG^{-1} = \begin{bmatrix} P_{i1} & P_{i2} \\ P_{i3} & P_{i4} \end{bmatrix} \quad \text{and} \quad H^{\mathrm{T}}S_i = \begin{bmatrix} S_{i1} \\ S_{i2} \end{bmatrix}$$

for $i \in \mathcal{S}$. It is readily concluded that $A_{i4}(t)$ is nonsingular for $i \in \mathcal{S}$. From Definition 1, the sliding mode dynamics (12) is regular and impulse-free.

Consider the Lyapunov function candidate as $V(x(t), i) = x^{\mathrm{T}}(t)\,E^{\mathrm{T}}P_iEx(t)$ for the sliding mode dynamics (12). Then the weak infinitesimal operator $\mathcal{L}$ of $V(x(t), i)$ along the solution of the system (12) is given by

$$\begin{aligned}
\mathcal{L}V(x(t), i) = x^{\mathrm{T}}(t)&\left(\Lambda_i^{\mathrm{T}}X_i + X_i^{\mathrm{T}}\Lambda_i\right. \\
&\left.+D_i^{\mathrm{T}}E^{\mathrm{T}}P_iED_i + \sum_{j=1}^{N}\pi_{ij}E^{\mathrm{T}}P_jE\right)x(t)
\end{aligned} \tag{21}$$

From (20) and (21), it is straight forward to see that there exists a scalar $\kappa > 0$, such that for each $i \in \mathcal{S}$,

$$\mathcal{L}V(x(t), i) \leq -\kappa\left\|x(t)\right\|^2$$

From the Generalized Itô formula, it yields

$$\lim_{t\to\infty}\mathcal{E}\left\{\int_0^t \left\|x(\theta)\right\|^2\mathrm{d}\theta\right\} \leq \kappa^{-1}\mathcal{E}\left\{V(x(0), r_0)\right\}$$

Then, from Definition 1, the sliding mode dynamics (12) is stochastically stable. This completes the proof. □

The objective now is to study the reachability. A SMC law will be designed to drive the trajectories of the system (1) into the designed sliding surface $s(t) = 0$ with $s(t)$ defined in (6) in finite time and maintain a sliding motion there for all subsequent time.

Theorem 2: Suppose that the sliding function is given in (6) where K_i and $\bar{P}_i$ satisfy (13)-(15). Then, the trajectories of the system (1) can be driven into the sliding surface $s(t) = 0$ in finite time and then maintain the sliding motion by employing the following SMC law:

$$\begin{aligned}
u(t) = K_ix(t) - &\left(\upsilon + \left\|\left(B_i^{\mathrm{T}}\bar{P}_iB_i\right)^{-1}B_i^{\mathrm{T}}\bar{P}_iM_i\right\|\right. \\
\cdot\left\|N_ix(t)\right\| &+ \tfrac{1}{2}\left\|\sum_{j=1}^{N}\pi_{ij}\left(B_j^{\mathrm{T}}\bar{P}_jB_j\right)^{-1}\right\|\left\|s(t)\right\| \\
&\left.+\vartheta\left\|x(t)\right\|\right)\mathrm{sign}(s(t))
\end{aligned} \tag{22}$$

where υ is a positive constant.

Proof: Choose the following Lyapunov function:

$$\bar{V}(s(t), i) = \frac{1}{2}s^{\mathrm{T}}(t)\left(B_i^{\mathrm{T}}\bar{P}_iB_i\right)^{-1}s(t)$$

From (10), the weak infinitesimal operator $\mathcal{L}$ of $\bar{V}(s(t), i)$ is given by

$$\mathcal{L}\bar{V}(s(t), i) \leq -\upsilon \|s(t)\| \leq -\tau \bar{V}^{\frac{1}{2}}(s(t), i)$$

where $\tau \triangleq \sqrt{2}\upsilon \frac{1}{\max_{i\in\mathcal{S}}\left\{\sqrt{\lambda_{\max}\left(B_i^{\mathrm{T}}\bar{P}_i B_i\right)^{-1}}\right\}} > 0$.

Then by denoting $s(t_0) \triangleq s_0$, it follows from (Kushner, 1967) that

$$\mathcal{E}\left\{\bar{V}(s(t), i)\,|s_0, t_0\right\}^{\frac{1}{2}} \leq -\frac{\tau}{2}t + \sqrt{\bar{V}(s_0, r_0)} \quad (23)$$

Since the left-hand side of (23) is nonnegative, $\mathcal{E}\left\{\bar{V}(s(t), i)\,|s_0, r_0\right\}$ reaches zero in finite time for each $i \in \mathcal{S}$, i.e., $t_r \leq 2\sqrt{\bar{V}(s_0, r_0)}\big/\tau$. This implies that $\mathcal{E}\{s(t)\} = 0$ or $\mathcal{P}\{s(t) = 0\} = 1$, for all $t \geq t_r$. This proof is completed. □

Remark 2: From SMC theory, it follows that Theorem 1 together with Theorem 2 guarantees that the corresponding closed-loop system formed by applying the control (22) to the system (1) is stochastically admissible.

4 SMC with H_∞ performance

In this section, a set of sufficient conditions will be developed under which the sliding mode dynamics of the considered system is guaranteed to be stochastically admissible with H_∞ performance.

Consider the following SSMJS in the presence of external disturbance:

$$E\mathrm{d}x(t) = [(A_i + \Delta A_i(t))\,x(t) + B_i(u(t) + f_i(x)) + F_i w(t)]\,\mathrm{d}t + D_i x(t)\,\mathrm{d}\varpi(t) \quad (24a)$$

$$z(t) = C_{1i}x(t) + C_{2i}w(t) \quad (24b)$$

where $w(t) \in \mathbb{R}^p$ is the disturbance input which belongs to $L_2^p[0, \infty)$; $z(t) \in \mathbb{R}^q$ is the controlled output; F_i, C_{1i} and C_{2i} are constant matrices with appropriate dimensions. Unless otherwise specified, the notation in (24) is as in (1).

Assume the sliding function (6) is defined for the system (24). Using a similar analysis as in Section 3, the following sliding mode dynamics can be obtained:

$$\begin{aligned}E\mathrm{d}x(t) = {} & [(A_i + B_iK_i + \Delta A_i(t) \\ & -B_i\left(B_i^{\mathrm{T}}\bar{P}_iB_i\right)^{-1}B_i^{\mathrm{T}}\bar{P}_i\Delta A_i(t)\Big)\,x(t) \\ & +\Big(I - B_i\left(B_i^{\mathrm{T}}\bar{P}_iB_i\right)^{-1}B_i^{\mathrm{T}}\bar{P}_i\Big)F_iw(t)\Big]\,\mathrm{d}t \\ & +D_ix(t)\,\mathrm{d}\varpi(t)\end{aligned} \quad (25)$$

Definition 2: Given a scalar $\gamma > 0$, the sliding mode dynamics (25) is said to be stochastically admissible with an H_∞ performance level γ if it is stochastically admissible with $w(t) = 0$, and under the zero initial condition, for nonzero $w(t) \in L_2^p[0, \infty)$,

$$\mathcal{E}\left\{\int_0^\infty z^{\mathrm{T}}(t)\,z(t)\,\mathrm{d}t\right\} < \gamma^2\int_0^\infty w^{\mathrm{T}}(t)\,w(t)\,\mathrm{d}t \quad (26)$$

The following result is ready to be presented.

Theorem 3: Given a scalar $\gamma > 0$, the sliding mode dynamics (25) is stochastically admissible with an H_∞ performance level γ if there exist scalars $\varepsilon_{1i} > 0$, $\varepsilon_{2i} > 0$, matrices $\bar{P}_i > 0$, L_i and $\bar{S}_i$ for $i \in \mathcal{S}$ such that (14) and (15) hold and

$$\begin{bmatrix}\Pi_{i1} & * \\ \Pi_{i2} & \Pi_{i3}\end{bmatrix} < 0 \quad (27)$$

where

$$\Pi_{i1} = \begin{bmatrix}\Xi_i & * & * & * & * \\ F_i^{\mathrm{T}} & \Sigma_i & * & * & * \\ \varepsilon_{2i}B_i^{\mathrm{T}} & 0 & -B_i^{\mathrm{T}}\bar{P}_iB_i & * & * \\ B_i^{\mathrm{T}} & 0 & 0 & -B_i^{\mathrm{T}}\bar{P}_iB_i & * \\ C_{1i}Y_i & C_{2i} & 0 & 0 & -I\end{bmatrix}$$

$$\Pi_{i2} = \begin{bmatrix}N_iY_i & 0\ 0\ 0\ 0 \\ N_iY_i & 0\ 0\ 0\ 0 \\ \Gamma^{\mathrm{T}}H^{-1}D_iY_i & 0\ 0\ 0\ 0 \\ \Gamma^{\mathrm{T}}H^{-1}\Psi_i^{\mathrm{T}} & 0\ 0\ 0\ 0\end{bmatrix}$$

$$\Pi_{i3} = \mathrm{diag}\{-\varepsilon_{1i}I, -\varepsilon_{2i}I, -\Theta_i, -J_i\}$$

$$\Sigma_i = -\gamma^2I + F_i^{\mathrm{T}}\bar{P}_iF_i$$

and all other notation is as in Theorem 1. Moreover, the parameter K_i in (25) is given by (16).

Proof: From Theorem 1 it is straightforward to see that (25) with $w(t) = 0$ is stochastically admissible. Let $J_{zw} = \mathcal{E}\left\{\int_0^\infty\left[z^{\mathrm{T}}(\theta)\,z(\theta) - \gamma^2w^{\mathrm{T}}(\theta)\,w(\theta)\right]\mathrm{d}\theta\right\}$. Under the zero initial condition, it is straightforward to see that

$$J_{zw} \leq \mathcal{E}\left\{\left[\tilde{x}^{\mathrm{T}}(\theta)\left(\Upsilon_i + \Phi_i^{\mathrm{T}}\Phi_i\right)\tilde{x}(\theta)\right]\mathrm{d}\theta\right\}$$

where

$$\Upsilon_i = \begin{bmatrix} \begin{pmatrix} \Lambda_i^{\mathrm{T}} X_i + X_i^{\mathrm{T}} \Lambda_i + D_i^{\mathrm{T}} E^{\mathrm{T}} P_i E D_i \\ + \sum\limits_{j=1}^{N} \pi_{ij} E^{\mathrm{T}} P_j E \end{pmatrix} & * \\ F_i^{\mathrm{T}} \Omega_i^{\mathrm{T}} X_i & -\gamma^2 I \end{bmatrix},$$

$\Phi_i = [C_{1i}\ C_{2i}]$, $\Omega_i = I - B_i \left(B_i^{\mathrm{T}} \bar{P}_i B_i\right)^{-1} B_i^{\mathrm{T}} \bar{P}_i$, $\tilde{x}(t) = \begin{bmatrix} x(t) & w(t) \end{bmatrix}^{\mathrm{T}}$, the notation for X_i and Λ_i is the same as in Theorem 1.

Using the same method as in the proof of Theorem 1, from (14) and (27) it follows that $J_{zw} < 0$ for all $t > 0$. Therefore, for any nonzero $w(t) \in L_2^p[0, \infty)$, (26) holds. Hence according to Definition 2, this system is stochastically admissible with an H_∞ performance γ. This completes the proof. □

Remark 3: Theorem 3 ensures not only all the parameter matrices $\bar{P}_i$ and K_i in the sliding function (6) are determined but also the stochastic admissibility condition of the resulting sliding mode dynamics is satisfied at the same time. This shows that desired results can be obtained without repeatedly adjusting values of some parameter matrices as in (Wu et al., 2012; Wu and Zheng, 2009) and without the constraint of EB_i having full column rank as in (Wu and Daniel, 2010). This answers the first question (Q1).

Remark 4: When rank$(E) = n$, the considered system is nonsingular. In this case, without loss of generality, it is assumed that $E = I$. Then, the SSMJS (1) becomes the following uncertain stochastic MJS:

$$\begin{aligned} \mathrm{d}x(t) = {} & [(A_i + \Delta A_i(t))\, x(t) + B_i (u(t) + f_i(x))]\, \mathrm{d}t \\ & + D_i x(t)\, \mathrm{d}\varpi(t) \end{aligned} \tag{28}$$

SMC for a similar system to the stochastic MJS (28) was studied in (Niu et al., 2007; Chen, Huang and Niu, 2007), with the same sliding function adopted as in (6) with $E = I$, but the parameter matrices K_i $(i \in \mathcal{S})$ must be chosen to satisfy $A_i + B_i K_i$ being Hurwitz prior to solving matrices $\bar{P}_i$ $(i \in \mathcal{S})$. For the same reason analyzed in Remark 3, the approach proposed here for the stochastic MJS is improves the results in (Niu et al., 2007; Chen et al., 2007) in this regard.

Theorem 4: Consider the SSMJS (24), the sliding function is given in (6), where $\bar{P}_i, K_i$ $(i \in \mathcal{S})$ are solutions of (14), (15) and (27). State trajectories can be driven into the sliding surface $s(t) = 0$ in finite time and remain there by the following SMC law:

$$\begin{aligned} u(t) = {} & K_i x(t) - \Big(\upsilon + \Big\| \left(B_i^{\mathrm{T}} \bar{P}_i B_i\right)^{-1} B_i^{\mathrm{T}} \bar{P}_i M_i \Big\| \, \|N_i x(t)\| \\ & + \vartheta \|x(t)\| + \tfrac{1}{2} \Big\| \sum_{j=1}^{N} \pi_{ij} \left(B_j^{\mathrm{T}} \bar{P}_j B_j\right)^{-1} \Big\| \, \|s(t)\| \\ & + \Big\| \left(B_i^{\mathrm{T}} \bar{P}_i B_i\right)^{-1} B_i^{\mathrm{T}} \bar{P}_i F_i \Big\| \, \|w(t)\| \Big) \operatorname{sign}(s(t)) \end{aligned} \tag{29}$$

where υ is a positive constant.

Remark 5: Here, the second question (Q2) will be answered, that is, how is the attraction of the sliding surface maintained when the sliding surfaces change from one to another under Markovian switching. From (29), it is straightforward to see that the designed SMC is related to the sliding surface through the matrices $\bar{P}_i$ and K_i. It should be noted that the proposed SMC law depends on the transition rates π_{ij} which reflect the effect of Markovian switching, and prescribe the desired dynamic performance of the system. Compared with this paper, in (Wu et al., 2012; Wu and Daniel, 2010) G_i $(i \in \mathcal{S})$ the designed SMC law is not related to Markovian switching and the SMC law does not depend on the transition rates π_{ij} directly.

Remark 6: It should be emphasised that it is difficult to solve equation (15) as the matrices $\bar{P}_i$ in (15) are constrained by the inequalities (13) and (14). Indeed, there is no general way to solve equation (15) with the constraints (13) and (14) (Edward, Yan and Spurgeon, 2007). In order to apply LMI techniques to find possible solutions, one possible choice is to replace the condition (15) by the following approximate constraint, which will reduce the search region for the solution P_i and facilitate the design by combining the limitations (13) and (14):

$$\left(B_i^{\mathrm{T}} \bar{P}_i D_i\right)^{\mathrm{T}} \left(B_i^{\mathrm{T}} \bar{P}_i D_i\right) < \nu I, (\text{for all } i \in \mathcal{S}) \tag{30}$$

where ν is a sufficiently small positive scalar. By using the Schur complement, (30) is equivalent to

$$\begin{bmatrix} -\nu I & D_i^{\mathrm{T}} \bar{P}_i B_i \\ B_i^{\mathrm{T}} \bar{P}_i D_i & -I \end{bmatrix} < 0 \tag{31}$$

The following minimization problems for Theorems 1 and 3 are defined:

$$\begin{cases} \min\limits_{\varepsilon_{1i}>0, \varepsilon_{2i}>0, \bar{P}_i>0, L_i, \bar{S}_i} \nu \\ \text{s.t.}(13), (14) \text{ and } (31) \end{cases}$$

$$\begin{cases} \min\limits_{\varepsilon_{1i}>0, \varepsilon_{2i}>0, \bar{P}_i>0, L_i, \bar{S}_i} \nu \\ \text{s.t.}(14), (27) \text{ and } (31) \end{cases}$$

It can be seen that if the global infimum ν approaches zero, the corresponding solutions ε_{1i}, ε_{2i}, $\bar{P}_i > 0$, $L_i, \bar{S}_i$ will satisfy (13)-(15) or (14), (15) and (27) respectively.

Remark 7: When compared with the LQR algorithm which is an automated approach to finding an appropriate linear state-feedback controller, the proposed sliding mode controllers (see for example (29)) are nonlinear and have variable structure, and thus produce a sliding motion which is totally insensitive to matched uncertainties (Edwards and Spurgeon, 1998) and also exhibits good robustness to mismatched uncertainty (Yan, Spurgeon and Edward, 2014; Yan et al., 2005). The discontinuous terms in the proposed variable structure controllers effectively reject the effects of the matched uncertainties if the magnitude of the switched element is larger than the corresponding bounds on the uncertainties. Although linear feedback control is straightforward to implement, high gain may be required to reject the effects of uncertainties if only linear state feedback is employed, particularly for systems with uncertainties where the bounds are nonlinear (Young et al., 1999).

5 Numerical examples

Example 1 Consider a SSMJS (24) with two modes and parameters as follows:

$$\text{Mode 1: } A_1 = \begin{bmatrix} 1.5 & -1 & -1.2 \\ 1.3 & 1.6 & 1.1 \\ 0.6 & 0.8 & -0.8 \end{bmatrix}, B_1 = \begin{bmatrix} 1.0 \\ 0.5 \\ 0.2 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}, D_1 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, F_1 = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 0.2 & 0.1 & 0.1 \end{bmatrix}, C_{11} = \begin{bmatrix} 0.1 & 0 & 0.2 \end{bmatrix}, C_{21} = 0;$$

$$\text{Mode 2: } A_2 = \begin{bmatrix} 0.5 & -0.6 & 0.7 \\ 1.2 & 2.4 & -0.4 \\ 0.6 & 0.2 & 1.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.8 \\ 1.0 \\ 0.3 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0.2 \\ 0 \\ 0.1 \end{bmatrix}, D_2 = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, F_2 = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}^{\mathrm{T}},$$

$$N_2 = \begin{bmatrix} 0.3 & 0.2 & 0.1 \end{bmatrix}, C_{12} = \begin{bmatrix} 0.2 & 0 & 0.3 \end{bmatrix}, C_{22} = 0;$$

The other parameters for models 1 and 2 are given as follows

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \Pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.4 & -0.4 \end{bmatrix},$$

$$f_1(x) = f_2(x) = 0.3\sin(t)\,x_1,$$

$$\gamma = 1, \quad F_1(t) = F_2(t) = 0.2\sin(t)$$

By Theorem 3 and Remark 6, it can be obtained that

$$\bar{P}_1 = \begin{bmatrix} 81.5 & 23.7 & -466.7 \\ 23.7 & 20.6 & -169.8 \\ -466.7 & -169.8 & 8830.5 \end{bmatrix},$$

$$\bar{P}_2 = \begin{bmatrix} 57.1 & 15.6 & -204.2 \\ 15.6 & 30.8 & -144.1 \\ -204.2 & -144.1 & 6328.8 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -0.6213 & -75.4896 & -18.8646 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 8.4051 & -34.5859 & 14.9472 \end{bmatrix}, \quad \nu \approx 1.2 \times 10^{-6}.$$

Then, the sliding surface can be computed by (6). Let the adjustable parameter $\upsilon = 1$, then the SMC law can be obtained by (29). For simulation purposes, the initial condition is chosen as $x(0) = [\,1\ 2\ 2.9\,]^{\mathrm{T}}$. Simulation results are shown in Figures 1 and 2.

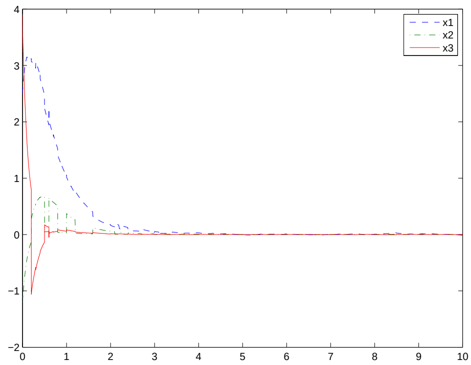

Fig. 1. The time response of the closed-loop system states

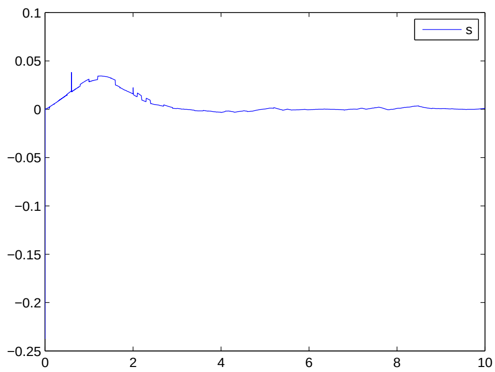

Fig. 2. The time response of the switching function $s(t)$

Example 2 Consider an electrical circuit that can be given as a SMJS (Boukas, 2006). Assuming the voltage $u(t)$ is excited by a nonlinear disturbance (where the nonlinear disturbance $f(x)$ satisfies (4)), which can be illustrated in Figure 3. It can be seen that the switch occupies three positions and switches from one position to another in a random way. This random process is the consequence of a random request which may come from the choice of an operator. For this system, it is assumed that the position of the switch follows a continuous-time Markovian process $\{r_t | t \geq 0\}$ in (3) and takes three modes in $\mathcal{S} = \{1, 2, 3\}$. For example, if r_t occupies state 2 at time t, it is depicted as $r_t = 2$. Denoting the electrical current in the circuit as $i(t)$ and using the basic electrical current laws, it is obtained that $u(t) + f(x) = u_R(t) + u_L(t) + u_C(t)$,$u_R(t) = i(t)\bar{R}$, $u_L(t)\,\mathrm{d}t = L\mathrm{d}i(t)$, $a(r_t)i(t)\,\mathrm{d}t = \mathrm{d}u_C(t)$,

where $a(r_t) = \begin{cases} \frac{1}{C_1} & if \quad r_t = 1 \\ \frac{1}{C_2} & if \quad r_t = 2 \\ \frac{1}{C_3} & if \quad r_t = 3 \end{cases}$, $\bar{R} = R + \triangle R$, R is the resistance with the uncertainty $\triangle R$.

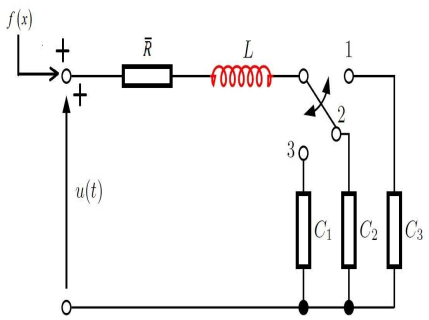

Fig. 3. Electrical circuit: singular circuit

Then the equation established above can be rewritten as follows

$$\begin{bmatrix} L & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{i}(t) \\ \dot{u}_L(t) \\ \dot{u}_C(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ a(r_t) & 0 & 0 \\ \bar{R} & 1 & 1 \end{bmatrix} \begin{bmatrix} i(t) \\ u_L(t) \\ u_C(t) \end{bmatrix} - \begin{bmatrix} 0 & 0 & u(t) + f(t) \end{bmatrix}^{\mathrm{T}}$$

If $x(t) = \begin{bmatrix} i(t) & u_L(t) + u_C(t) & u_C(t) \end{bmatrix}^{\mathrm{T}}$ is chosen, the obtained system is equivalent to the following

$$\begin{bmatrix} L & 0 & 0 \\ L & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ a(r_t) - \bar{R} & 0 & -1 \\ \bar{R} & 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 & -1 \end{bmatrix}^{\mathrm{T}} (u(t) + f(t))$$

Parameters are chosen as follows

$L = 1$, $a_1 = 1$, $a_2 = 1.2$, $a_3 = 1.5$, $R = 2$, $\triangle R = 0.01$, $f(x) = 0.5\sin(t)x_1$, $\Pi = \begin{bmatrix} -1 & 0.4 & 0.6 \\ 0.9 & -2 & 1.1 \\ 0.2 & 0.3 & -0.5 \end{bmatrix}$

By Theorem 1, it can be obtained that

$$\bar{P}_1 = \begin{bmatrix} 17.6206 & 0.5206 & -7.8191 \\ 0.5206 & 61.6055 & -2.2036 \\ -7.8191 & -2.2036 & 33.2896 \end{bmatrix},$$

$$\bar{P}_2 = \begin{bmatrix} 16.0577 & 0.5818 & -7.5931 \\ 0.5818 & 62.0916 & -2.2547 \\ -7.5931 & -2.2547 & 30.0298 \end{bmatrix},$$

$$\bar{P}_3 = \begin{bmatrix} 18.3454 & 0.4484 & -9.1911 \\ 0.4484 & 60.4777 & -1.8868 \\ -9.1911 & -1.8868 & 39.1067 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -2.6864 & -2.8694 & -0.3674 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -3.5209 & -3.8764 & -0.3897 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -2.1167 & -1.8464 & -0.5969 \end{bmatrix}$$

Then, the sliding surface can be computed by (6). Let the adjustable parameter υ be υ=1. The SMC law can be obtained by (22). Simulation results with the initial condition $x(0) = [\,0.5 \quad -0.7 \quad -1\,]^{\mathrm{T}}$, are shown in Figures 4 and 5, which demonstrate the effectiveness of the obtained results.

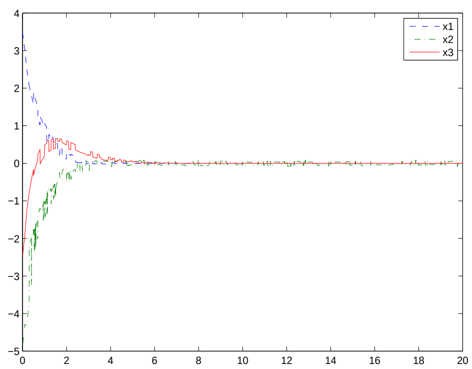

Fig. 4. The time response of the closed-loop system states

Example 3: Consider a two-mode SSMJS (24) without uncertainty:

Mode 1: $A_1 = \begin{bmatrix} 1.5 & -1 & -1.2 \\ 1.3 & 1.6 & 1.1 \\ 0.6 & 0.8 & -0.8 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 0.5 \\ 0.4 \end{bmatrix}$,

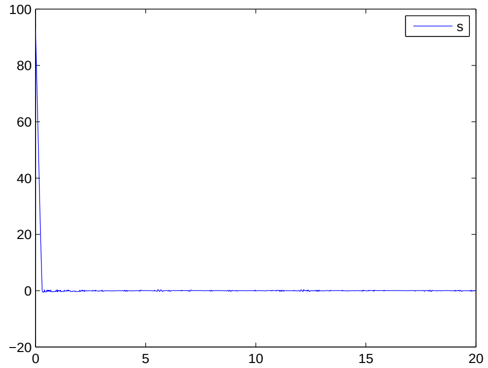

Fig. 5. The time response of the switching function $s(t)$

$$D_1 = \begin{bmatrix} 0.1 & 0.2 & 0 \\ 0.1 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, F_1 = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} 0.1 & 0 & 0.2 \end{bmatrix}, \quad C_{12} = 0;$$

$$\text{Mode 2: } A_2 = \begin{bmatrix} 0.5 & -0.6 & 0.7 \\ 1.2 & 2.4 & -0.4 \\ 0.6 & 0.2 & 1.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0.8 \\ 1 \\ 0.4 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.2 & 0.1 & 0 \\ 0.2 & 0.1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, F_2 = \begin{bmatrix} 0.1 \\ 0 \\ 0 \end{bmatrix}, C_{21} = \begin{bmatrix} 0.2 \\ 0 \\ 0.3 \end{bmatrix}^{\mathrm{T}}$$

$C_{22} = 0$.

The singular matrix E is given by

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

In order to compare the results obtained in this paper with results in (Wu and Daniel, 2010), it is assumed that $\pi_{11} = -0.5$. Then based on different values of π_{22}, the corresponding minimum values of γ will be calculated. Table 1 presents the comparison of the minimum γ, for different π_{22}, between results using Theorem 3 in this paper and Theorem 4 in (Wu and Daniel, 2010). It is clear that the results obtained in this paper improve the results given in (Xu and Lam, 2006) even when the constraint of EB_i having full column rank is satisfied.

On the other hand, if it is chosen that $\gamma = 0.114$, $A_1 = \begin{bmatrix} \alpha & -1 & -1.2 \\ 1.3 & 1.6 & 1.1 \\ 0.6 & 0.8 & -0.8 \end{bmatrix}$, $\pi_{21} = \beta$, $0 \le \alpha \le 4$, $0 \le \beta \le 4$, and the other parameters are unchanged in this example, in

Table 1
Comparisons of minimum allowed γ with $\pi_{11} = -0.5$.

π_{22}	-0.2	-0.3	-0.4	-0.5	-0.6	-1
a	0.2291	0.2293	0.2295	0.2296	0.2298	0.2302
b	0.0954	0.0957	0.0960	0.0962	0.0964	0.0968

Row a: The minimum γ from Theorem 4 of (Wu and Daniel, 2010)
Row b: The minimum γ from Theorem 3 in this paper

Figure 5, " o " represents the range of feasible solutions using Theorem 3 in this paper, and " * " represents the range of the feasible solutions using Theorem 4 of (Wu and Daniel, 2010). This demonstrates that the method obtained in this paper has certain advantages in some cases.

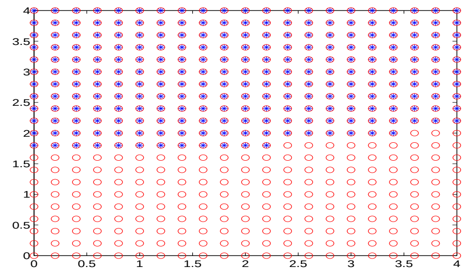

Fig. 6. Comparison of the feasible region

6 Conclusions

In this paper, SMC laws have been designed for a class of SSMJSs in the presence of uncertainties. A suitable integral-type sliding surface is designed such that the resulting sliding mode dynamics is stochastically admissible. Investigation of stochastic MJSs is considered as well using sliding mode techniques such that the corresponding closed-loop systems are stochastically stable. The conditions developed are easily testable and can be regarded as complementary to the existing results available in the literature. Moreover, the effect of Markovian switching has been overcome by the designed sliding mode controller involving the transition rates of modes. The application to several numerical examples shows the practicability of the proposed method.

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References

Alwi, H. and Edwards, C. (2008). Robust adaptive sliding control for uncertain discrete-time systems with time delay, *IEEE*

Transactions on Control Systems Technology **16(3)**: 499–510.

Boukas, E. (2006). *Stochastic Switching System: Analysis and Design*, Birkhauser, Boston.

Boukas, E. (2008). *Microscopic Staining Techniques*, Springer-Verlag, Berlin.

Chen, B., Huang, J. and Niu, Y. (2007). Reduced-order h_∞ controller design for uncertain descriptor systems, *Automatica* **33(1)**: 44–47.

Edward, C., Yan, X. and Spurgeon, S. (2007). On the solvability of the constrained lyapunov problem, *IEEE Transactions on Automatic Control* **52**: 1982–1987.

Edwards, C. and Spurgeon, S. K. (1998). *Sliding mode control: Theory and application*, Taylor & Francis Ltd, London.

Fang, Y. and Loparo, K. (2002). Stochastic stability of jump linear systems, *IEEE Transactions on Automatic Control* **47(7)**: 1204–1208.

Fridman, E., Gouaisbaut, F., Dambrine, M. and Richard, J. (2003). Sliding mode control of systems with time-varying delays via descriptor approach, *International Journal of systems Science* **34**: 553–559.

Huang, L. and Mao, X. (2010). Stability of singular stochastic systems with markovian switching, *IEEE Transactions on Automatic control* **56(2)**: 424–429.

Hung, J., Gao, W. and Hung, J. (1993). Variable structrue control: A survey, *IEEE Transactions on Industrial Electronic* **40**: 2–22.

Kushner, H. (1967). *Stochastic Stability and Control*, Academic Press, New York.

Lewis, F. (1986). A survey of linear singular systems, *Circuits Systems and Signal Processing* **5(1)**: 3–36.

Li, H., Gao, H., Shi, P. and Zhao, X. (2014). Fault-tolerant control of markovian jump stochastic systems via the augmented sliding mode observer approach, *Automatica* **50**: 1825–1834.

Ma, E. and Boukas, A. (2009). A singular system approach to robust sliding mode control for uncertain markov jump systems, *Automatica* **45**: 2707–2713.

Niu, Y., Ho, D. and Wang, X. (2007). Sliding mode control for itô stochastic systems with markovian switching, *Automatica* **43**: 1784–1790.

Shi, P., Xia, Y., Liu, G. and Rees, D. (2006). On designing of sliding-mode control for stochastic jump systems, *IEEE Transactions on Automatic Control* **51(1)**: 97–103.

Utkin, V., Guldner, J. and Shi, J. (1999). *Sliding mode control in electro-mechanical systems*, Taylor & Francis Ltd, London.

Wu, L. and Daniel, W. (2010). Sliding mode control of singular stochastic hybrid systems, *Automatica* **46**: 779–783.

Wu, L., Su, X. and Shi, P. (2012). Sliding mode control with bounded $\mathcal{L}_2$ gain of markovian jump singular time-delay systems, *Automatica* **48**: 1929–1933.

Wu, L. and Zheng, W. (2009). Passivity-based sliding mode control of uncertain singular time-delay systems, *Automatica* **45**: 2120–2127.

Xiong, J. and Lam, J. (2006). Stabilization of discrete-time markovian jump linear systems via time-delayed controllers, *Automatica* **42**: 747–753.

Xu, D. Zhang, Q. and Hu, Y. (2007). Reduced-order h_∞ controller design for uncertain descriptor systems, *Automatica* **33(1)**: 44–47.

Xu, S. and Lam, J. (2006). *Robust control filtering of singular systems*, Springer-Verlag, Berlin.

Yan, X., Spurgeon, S. and Edward, C. (2014). Memoryless static output feedback sliding mode control for nonlinear systems with delayed disturbances, *IEEE Transactions on Automatic Control* **59**: 1909–1912.

Yan, X., Spurgeon, S. and Edwards, C. (2005). Dynamic sliding mode control for a class of systems with mismatched uncertainty, *European Journal of Control* **11(1)**: 1–10.

Yan, X., Spurgeon, S. and Edwards, C. (2010). Sliding mode control for time-varying delayed systems based on a reduced-order observer, *Automatica* **46(8)**: 1354–1362.

Yan, X., Spurgeon, S. and Edwards, C. (2013). State and parameter estimation for nonlinear delay systems using sliding mode techniques, *IEEE Transactions on Automatic Control* **58(4)**: 1023–1029.

Yang, C., Zhang, Q. and Zhou, L. (2006). Stabilization of discrete-time markovian jump linear systems via time-delayed controllers, *Automatica* **42**: 747–753.

Young, K., Utkin, V. and Özgüner, U. (1999). A control engineers guide to sliding mode control, *IEEE Transactions on Control Systems Technology* **7**: 328–342.

Yue, D. and Han, Q. (2005). Delay-dependent exponential stability of stochastic systems with time-varying delay, nonlinearity, and markovian switching, *IEEE Transactions on Automatic Control* **50(2)**: 217–222.