Educational Quality, Communities, and Public School Choice: a Theoretical Analysis

Tarek Mostafa, Saïd Hanchane

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Abstract: In this paper, we develop a multicommunity model where public mixed finance and private schools coexist. Students are differentiated by income, ability and social capital. Schools maximize their profits under a quality constraint; the pricing function is dependent on the cost of producing education and on the position of an individual relatively to mean ability and mean social capital. Income plays an indirect role since it determines the type of schools and communities that can be afforded by a student given his ability and social capital.

Three dimensional stratification results from schools' profit maximization and individuals' utility maximization. We study majority voting over tax rates; property tax is used to finance education not only in pure public schools but also in mixed finance schools. We provide the necessary conditions for the existence of a majority voting equilibrium determined by the median voter. Finally, we analyze the consequences of introducing public school choice.

JEL Classification: I20, I21, H52, R31.

Keywords: Education market, majority voting equilibrium, peer group effects, formation of communities, school choice.

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I. INTRODUCTION.

Achieving social mixity, reducing inequalities in the access to education and in its outcomes has become a major preoccupation of educational reforms in most developed countries. Recent studies proved the existence of large disparities in the outcomes of education in most western countries. For instance in PISA 2000; Germany, the UK, the USA and France turned out to have a large students' dispersion of achievements over reading, numeracy, and basic science tests combined with low performances.[1] The recent French presidential elections triggered a vigorous debate on the quality of education in France. Particularly, the debate on the abolishment of residence requirements and the introduction of public school choice made the headlines in most French newspapers. For example, *Le Monde* titled (16 September 2006) "Residence requirements have become the instrument of social segregation", *Libération* titled (06 September 2006) "General offensive against school districts", and *Le Figaro* titled (03 January 2007) "The incoming reform of school districts".

As of September 2007 France will be experimenting new procedures in some school districts. Increased autonomy will be granted to educational institutions in order to increase educational quality, and residence requirements will be progressively abolished in order to grant parents more authority over the choice of the public school for their children. More precisely, the government announced that more students will be able to choose their public schools, and priority will be given to highly performing students.

The objective behind the abolishment of school districts is the reduction of social inequalities in the access to education. Not only highly advantaged households will be able to bypass school districts through the choice of private schools, but also ordinary students with high abilities will be able to choose their public school. It shall be said that in France students have the right to choose private schools outside their school district. Thus, sufficiently rich households have been able to bypass school districts; as a consequence segregation problems arose in poorer communities.

[1] [1] Chapter 5 in "Education, Equality and Social Cohesion" by Green, Preston and Janmatt provides interesting results on outcomes' inequalities in five groups of countries: East Asia, the Nordic countries, the Germanic region, the Mediterranean countries and the predominantly English speaking countries. Also an article of Fuchs and Wobmann (March 2004) accounted for the low performances made by these countries.

In this theoretical paper, we construct a multicommunity model in an economy where private, mixed finance and public schools coexist. We analyze the formation of communities when education is a locally provided good with decentralized finance. The interaction between students' utility maximization and schools' profit maximization leads to equilibrium on the market for schooling and that for housing and then to stratification across schools and communities. We study voting over tax rates, and we determine the necessary conditions for the existence of a majority voting equilibrium determined by the median voter. Finally, we analyze the effects of public school choice to find that only a small fraction of the population may be willing to exercise choice, we also find that some private and mixed finance schools will be losing students in favor of public ones, and that public school choice may not necessary have a negative effect on public school quality.

Yoram Barzel (1973), in a critical study of Robin Barlow (1970) introduced what is known as the "Ends against the middle phenomenon" in the provision of public goods when private alternatives coexist. The study of Barzel and that of Stiglitz (1974) constituted the starting point for a more formalized approach to school choice. Six major articles are relevant to our analysis. Fernandez and Rogerson (1996), in which education is a locally, publicly provided good and where we have a strict hierarchy of communities following educational qualities, and a complete stratification by income. Epple and Platt (1996) in which the authors construct an equilibrium in a multicommunity model where individuals have the choice between communities differentiated by housing prices and grant levels. Glomm and Ravikumar (1998), in which the authors construct a model of the provision of education when both public and private alternatives coexist while analyzing majority voting over tax rates. Epple and Romano (2000), where the authors analyze the equilibrium on the market of education in a multicommunity environment only with public schools, they study public school choice and finance decentralization. Epple and Romano (1998 and 2006) where they study stratification across the market for education with individuals differentiated by income and ability.

Relatively to Fernandez and Rogerson (1996), education can be supplied privately; as a consequence preferences are no longer single peaked, this has major effects on majority voting. In our model each community may contain a different number of public, mixed finance and private schools with different qualities. Our stratification is three dimensional allowing mixity according to income, ability and social capital, while their model has a complete stratification by income with communities having a single public school.

Relatively to Epple and Platt (1996), we construct a multicommunity model of the provision of education where students are differentiated by income, ability, and social capital. Students have to choose between public, private and mixed finance schools, the chosen school must not necessary belong to the chosen community. Housing prices, tuition, and transportation costs play a major role in the stratification of individuals across communities and schools. Students will be stratified according to their endowments in income, ability, and social capital.

In relation to the article of Glomm and Ravikumar (1998), we are studying majority voting in a multicommunity model with decentralized finance, education is a locally provided good. The level of rental in a community determines the quality of the school that can be attended. Students choosing a private school or a school outside their community have a decreasing utility in the tax rate. At very high tax rates some students will be willing to migrate. Majority equilibrium determined by the median voter would not occur unless some conditions are verified.

In comparison with Epple and Romano (2000), we allow the coexistence of private, public and mixed finance schools; thus preferences are not single peaked. Complete stratification by income does not occur. In our model we analyze the effect of introducing public school choice on the already existing equilibrium, while in their model they construct equilibrium with choice taken as a factor in its construction. Also, we find that choice may not necessary have a negative effect on educational quality. Similarly to Epple and Romano (1998) and 2006), the private sector is active, schools maximize profit under a quality constraint, and a discriminating pricing strategy is applied and is dependent on individuals' ability and social capital. However, we allow the existence of public, mixed finance and private schools in a multicommunity environment. Educational quality is three dimensional, students are stratified into communities and schools, and tax rates are chosen through majority voting.

Section II presents the model, section III presents schools' characteristics and profit maximization, section IV presents the characteristics of students and examines stratification across schools and communities, Section V analyses majority voting over tax rates, section VI introduces public school choice, and finally, section VII concluded. Mathematical details are found in the appendix.

II. THE MODEL.

In our model, we consider an economy populated with a continuum of households differentiated by income, ability and social capital; each household has one student. Social capital is defined to include the channels through which student acquisitions are affected by the social mix in their schools, these channels may include: social origins, the belonging to a professional class, the set of attitudes and mental dispositions governing different subjects like the superiority of the educational good. All these forms of social capital are represented through a single parameter, k. Individuals have identical preferences over private consumption. An individual i (with i = 1, 2, 3, 4,…,P) has an income y_i, an ability b_i and a social capital k_i. Note that i may designate a particular student or a type of students with the same combination of y, b, and k. These students live in communities, a community is designated by an index C with (C = 1, 2, 3,…, C).

Income, ability, and social capital are distributed in the population according to $f(b, y, k)$ which is positive and continuous on its support $S = (0, b_{\max}) \times (0, y_{\max}) \times (0, k_{\max})$.

Individual utility is assumed to be a function of private consumption and school quality. It is noted as $U(c, q)$; U is continuous, twice differentiable, and increasing in both arguments. We assume that $\lim_{c \to +\infty} U_c(c, q) = 0$.

School quality is determined by expenditure per pupil, peer group effect, and neighborhood effect. Quality is increasing in peer effect, and the expenditure per pupil; while it is decreasing in neighborhood effect[2]. Peer group effects are defined to be mean ability in a school; neighborhood effects are defined to be mean social capital in a school. A school is designated by an index j (with j=1, 2, 3,…, j).

Increasing quality in expenditure per pupil and in peer effect is consistent with the theoretical literature on the subject. Summers and Wolfe (1977), Henderson (1978), Sorensen and Hallinan (1986) and Kulik (1992) provided evidence that peer effects exist.

[2] Schools value diversity in term of social capital of their students.

Neighborhood effects are taken into account to shed the light on how school quality is affected by the social mix of enrolled students. Evidence from France, the USA, and other countries suggests that in order to make access to education more democratic, schools prefer a low level of mean social capital.[3]

The cost of housing in a community is given by the price of housing r^C multiplied by the number of unites of housing demanded H. r^C is the gross of tax price of housing; $r^C = (1+t)r_{net}$. Land is homogenous, and the amount of land may vary across communities. In this model we do not make a difference between renters and owners, we neglect the fact that owners may achieve a capital gain or loss from selling their house. We also neglect the existence of owners who receive the rent. All individuals pay their taxes even if they do not use public and mixed finance schools.

We consider that students face a transportation cost determined by the distance separating a student's community from his school. It is noted T_j^C. T_j^C is equal to zero for a school j geographically belonging to community C. For two schools geographically belonging to the same community, the transportation cost between these schools and another community is the same, $T_j^C = T_{j'}^C$ with j and j' two schools belonging to the same community and (C = 1, 2, 3,…, C).

Note that the number of students is larger than the number of schools and the latter is larger than the number of communities. $P > j > C$. Students can only be enrolled in one school, and they can not supplement this education elsewhere.

The proportion of students of type (b, y, k) in a particular school j is given by $\alpha_j(b, y, k)$; the number of students in school j is given by l_j. With:

$$l_j = \iiint_s \alpha_j(b, y, k) f(b, y, k) dbdydk \qquad (1)$$

And $\alpha_j(b, y, k) \in [0,1]$

[3] In France two types of scholarships are attributed to students, one is meritorious and is administered by the French ministry of education and the second is need based and is administered by the CROUS, in the latter case the scholarship is calculated using household socioeconomic data.

The proportion of students of type (b,y,k) in a particular community is given by $\alpha_C(b,y,k)$; the number of students is given by:

$$l_C = \iiint_s \alpha_C(b,y,k) f(b,y,k)\,db\,dy\,dk \tag{2}$$

And $\alpha_C(b,y,k) \in [0,1]$

The production cost of education is dependent on the number of students enrolled in the school, it is given by $Co(l_j) = V(l_j) + F = n_1 l_j + n_2 l_j^2 + F$. $V' > 0$, $V'' > 0$, F is a fixed cost, n_1 and n_2 are positive constants[4]. The absence of economies of scale in the production of education means that a large number of schools exist and will be catering to each type of students.

In this model, we suppose that information is complete; individuals and schools make their decisions while having all information about educational quality, pricing, rental, tax rates and the distribution of income, ability and social capital.

III. SCHOOLS.

In our model, schools are assumed to maximize profit under a quality constraint. Funding is provided by three major sources; government subsidies, tuition paid by students and other earnings.

$$R_j = \iiint_s E_{ij}\alpha_j(b,y,k) f(b,y,k)\,db\,dy\,dk + \iiint_s p_{ij}\alpha_j(b,y,k) f(b,y,k)\,db\,dy\,dk + G_i$$

E_{ij} : Local government per student subsidies in school j.

p_{ij} : Tuition paid by student i in school j.

G_j : Other earnings.

The government local subsidies in community C for school j are given by $E_j^C = \sum E_{ij}$. The sum of government subsidies in a community C is equal to the sum of: the property tax rate in

[4] Contradicting evidence was provided by Lawrence Kenny, Ferris and West on the existence of economies of scale in the production of education. Kenny (1982) provided evidence that such economies of scale do exist. However, Ferris and West (2004) argued that large schools suffer from social problems; in other terms, external and invisible costs prevent the existence of economies of scale.

community C multiplied by the net of tax price of housing and by the number of housing units demanded $\sum E_j^C = \sum t^C r_{net} H_i$.

School quality is then given by:

$$q_j = q_j\left[\frac{R_j}{l_j}, \theta_j, \mathrm{O}_j\right] \qquad (3)$$

q_j is increasing in $\frac{R_j}{l_j}, \theta_j$, while it is decreasing in O_j.

With:

Expenditure per pupil given by $\frac{R_j}{l_j}$.

Peer group effect given by mean ability in a school:

$$\theta_j = \frac{1}{l_j}\iiint_s b_i \alpha_j(b, y, k) f(b, y, k)\, db\, dy\, dk \qquad (4)$$

Neighborhood effect given by mean social capital in a school:

$$\mathrm{O}_j = \frac{1}{l_j}\iiint_s k_i \alpha_j(b, y, k) f(b, y, k)\, db\, dy\, dk \qquad (5)$$

Optimization:

Schools maximize profit under a quality constraint; the level of quality chosen by a school and the level of local subsidies determine the price for each type of students. Schools do not select directly their students; they admit any student who is able to pay the price corresponding to his type; displaying a prohibitive price is equivalent to refusing to admit a student. The constraints include: mean ability, mean social capital, and the number of students l_j enrolled in a school since it influences expenditure per pupil. Mathematical details are found in the appendix.

The optimal level of resources is the following[5]:

$$R'^{*} = n_1 + 2n_2 l_j + \mu'(\theta_j - b_i) + \mu''(\mathrm{O}_j - k_i) \qquad (6)$$

$\mu', and \mu''$ are the Lagrangian multipliers. (7)

[5] Results (6) and (7) are obtained by forming a Lagrangian function under the constraints (1), (4), and (5), and optimizing over α_j.

$$\mu' = \frac{1}{l_j} \iiint_s \frac{\partial R'^*}{\partial \theta_j} \alpha_j(b,y,k) f(b,y,k) dbdydk$$

$$\mu'' = \frac{1}{l_j} \iiint_s \frac{\partial R'^*}{\partial \mathrm{O}_j} \alpha_j(b,y,k) f(b,y,k) dbdydk$$

μ' is positive and μ'' is negative.[6] μ' represents the per student resources change deriving from a change in θ_j; μ'' represents the per student resources change deriving from a change in O_j .For analytical convenience, they are considered to be the same for all schools.

When R'^* is replaced by its value we obtain the following equation:

For pure private schools, we have:

$p_{ij}^* = n_1 + 2n_2 l_j + \mu'(\theta_j - b_i) + \mu''(\mathrm{O}_j - k_i)$ With $E_{ij} = 0$.[7]

For mixed finance schools, we have:

$p_{ij}^* = -E_{ij} + n_1 + 2n_2 l_j + \mu'(\theta_j - b_i) + \mu''(\mathrm{O}_j - k_i)$

The level of per student subsidies E_{ij} for mixed finance schools is determined by authorities and not by optimization, schools can only choose the level of pricing to apply conditioned by its level of quality and the type of students it will enroll.

For pure public schools, we have:

$E_{ij}^* = n_1 + 2n_2 l_j + \mu'(\theta_j - b_i) + \mu''(\mathrm{O}_j - k_i)$ With $p_{ij} = 0$ for all i.

Since pure public schools are free entry free of tuition schools, the type of students enrolled in pure public schools determines his marginal cost and thus the amount of subsidies needed. Thus E is noted with an asterisk, since it is determined by the optimization.

After optimization, profit is equal to zero, $\pi_j = 0$. New entries on the market of education are possible as long as $\pi_j > 0$. When $\pi_j = 0$, no new entries are possible.

[6] See the definition of Lagrangian multipliers in Epple and Romano (1998) pp. 40.
[7] Pricing is made according to the marginal cost of admitting a student of a particular type.

In this equation, R'^{*} represents the resources needed to cover the marginal cost of admitting a student. The first term $n_1 + 2n_2 l_j$ is the part of the marginal cost resulting from the cost of producing education. It is identical for all students in the same school.

The second term, $\mu'(\theta_j - b_i)$ represents the cost of one's ability on mean ability. Students with $\theta_j < b_i$ have a negative externality cost on the school. The reverse is true for $\theta_j > b_i$.

The third term, $\mu''(\mathrm{O}_j - k_i)$ represents the cost of one's social capital on mean social capital. Students with $k_i < \mathrm{O}_j$ have a negative externality cost on the school. The reverse is true for $k_i > \mathrm{O}_j$.

Equation (6) is linear in ability and in social capital. For private and mixed finance schools, the price is decreasing in ability and increasing in social capital. For public schools, since the price is equal to zero, the ability and social capital of students would not have any effect on their disposable income but they would have an effect on the resources needed to cover the marginal cost of admitting them; E_{ij} is decreasing in ability and increasing in social capital.

This equation allows us to overcome the strict distinction between public and private schools. Different types of education finance can be considered: we can start at the level of free of tuition public schools with $p_{ij} = 0$ for all i, and go through mixed finance schools where both p and E are positive until reaching purely private schools where $E_{ij} = 0$.

Note that given the level of government subsidies; pricing will be both meritorious and need based. The second term of the equation represents the meritorious part of the pricing function; the third term of the equation represents the need based part. Another implication of equation (6) is the possibility that pricing can be negative depending on the level of government subsidies and the positioning of individual ability and social capital relatively to the means; in this case the negative price represents a scholarship.

The sequencing of decision making for individuals and schools.

The sequencing of decisions can be described through a multistage game. First, individuals choose the community in which to reside and the school to be enrolled into[8]. At this level the housing units demanded per household are determined. Second, individuals choose the property tax rate through majority voting. At this level the sum of subsidies in the community $\sum E_j^C = \sum t^C r_{net} H_i$ is determined. Thirdly, local authorities allocate their budget. At this level, it should be noted that we have three types of schools, pure publicly financed schools with $p_{ij} = 0$, mixed finance, and pure private schools with $p_{ij} \neq 0$. Local authorities attribute subsidies to pure publicly financed schools first and according to their needs, and then the rest of the funding is arbitrarily attributed to mixed finance schools. In other words, the type of students enrolled in pure public schools determines the funding needed, in this way these schools are free entry free of tuition schools, they admit all comers without any restriction. Causality in equation (6) goes from the right hand side to the left hand side; the marginal cost resulting from the type of students enrolled determines the funding allocated by local authorities. In mixed finance schools, the explanation is a little bit different; after receiving local subsidies and choosing the level of educational quality, schools determine the type of students to be admitted and the pricing that goes with it. Private and mixed finance schools do not have a restriction on the number of admitted students; they admit any student as long as he can pay the price corresponding to his type. Our model has a single period, no real time elapses between stages.

Lemma 1:

(i) Educational quality q_j is strictly increasing in the general cost of acquiring education for the same student. The general cost of acquiring education is equal to the sum of the price and the transportation cost for the same i (With i = 1, 2, 3, …, P). In this lemma, we show that in order to justify the existence of a higher cost of education a higher educational quality should be associated with it.

(ii) In each community, schools with $p_{ij} = 0$ are the lowest quality schools.

In the second part we show that, in each community, pure public schools have the lowest quality.

[8] Complete and perfect information plays an important role in the decision making of individuals and schools.

Proposition 1: The hierarchy of educational qualities in the economy follows that of educational costs (price + transportation cost). The hierarchy of educational qualities in the same community follows that of the prices. The proof for this proposition follows directly from lemma 1. Note that at equilibrium, some mixed finance schools may have higher qualities than some private ones; in this case these mixed finance schools shall have higher educational costs for all students and all communities.

IV. STUDENTS.

Individuals have two connected optimization problems; they have to choose a community and a school. They choose communities and schools simultaneously through utility maximization. It should be noted that since public, mixed finance and private schools exist in our model, individuals have a large set of choices, even outside their community. We have two cases; firstly, when public school choice is not allowed, individuals have a choice between the schools of their community and the mixed finance and private schools outside their community. Secondly, when public school choice is allowed, individuals have a choice between all schools in the economy. For sections II to VI, public school choice is not allowed; it is introduced only in section VII. The cost of transportation combined with educational pricing and rental, play an important role in determining the affordable schools.

The budget constraint is given by: $y_i = c + T_j^C + p_{ij} + r^C H_i$ (8)[9]

Maximization of U subject to constraint (8) yields the indirect utility function:

$$\begin{aligned} U^*(c,q) &= Max\left[y_i - T_j^C - p_{ij} - r^C H_i, q_j(\frac{R_j}{l_j}, \theta_j, \mathrm{O}_j) \right] \\ &= W_{ij}^C\left[(y_i, b_i, k_i), T_j^C, t^C, p_{ij}, r^C, H_i, q_j\right] \end{aligned} \tag{9}$$

Assumption 1: pining down preferences over education and housing.

a- We assume a positive (b, y, k) elasticity of the demand for educational quality. As (b, y, k) increases, the educational quality q_j demanded increases. Higher (b, y, k) prefer higher quality schools.

[9] Notation remark: small letter c is used to denote consumption, capital letter C is used to denote a community.

b- We assume a positive ability, income, and social capital elasticity of housing demand such as: $\frac{\partial H}{\partial y} > 0$, $\frac{\partial H}{\partial b} > 0$, and $\frac{\partial H}{\partial k} > 0$. Housing demand is also assumed to be increasing in (b, y, k) such as: $\frac{\partial H}{\partial (b, y, k)} > 0$. Higher (b, y, k) prefer larger houses. The case for higher (b, y, k) shall be considered because it accounts for situations where one or two of the variables b, y, or k is lower while others are higher.

Remark: The superiority of a combination (b, y, k) relatively to another one depends on how b, y, and k enter the utility function. It should be said that any (b, y, k) combination mentioned in this paper is not unique since b, y and k are substitutable in the pricing function (low ability or high social capital can be substituted by high income). Different types of students with different combinations of (b, y, k) may have the same level of utility.

Property 1: By virtue of the implicit function theorem, we obtain:

a-
$$\left.\frac{\partial (r+p+T)}{\partial q}\right|_{W=\overline{W}} = \frac{\partial r + \partial p + \partial T}{\partial q} = \frac{\partial r}{\partial q} + \frac{\partial p}{\partial q} + \frac{\partial T}{\partial q} =$$
$$-\frac{\partial W/\partial q}{\partial W/\partial r} - \frac{\partial W/\partial q}{\partial W/\partial p} - \frac{\partial W/\partial q}{\partial W/\partial T} = \frac{1}{H} + 1 + 1 = \frac{1}{H} + 2 > 0$$

Part a implies that indifference curves in the $[q,(r+p+T)]$ plane have positive slopes[10].

We are using the $[q,(r+p+T)]$ plane since community choice is not independent from school choice; the type of community chosen determines the type of schools that can be afforded, and the reverse is true.

b-
$$\partial\left(\frac{\partial (r+p+T)}{\partial q}\right) \Big/ \partial y = -\frac{H_y}{H^2} < 0$$

For students of the same ability and social capital, any indifference curve in the $[q,(r+p+T)]$ plane of a higher income student cuts any indifference curve of a lower income student from above. Thus for a given b and k, indifference curves exhibit single crossing in income. In other words, when income increases, the slope of the indifference curve decreases.

[10] This part of the assumption means that for a higher educational quality, individuals are willing to spend more on rental, education, and transportation combined.

c- $\partial\left(\frac{\partial(r+p+T)}{\partial q}\right)\Big/\partial b = -\frac{H_b}{H^2} < 0$

For students of the same income and social capital, any indifference curve in the $[q,(r+p+T)]$ plane of a higher ability student cuts any indifference curve of a lower ability student from above. Thus for a given y and k, indifference curves exhibit single crossing in ability. In other words, when ability increases, the slope of the indifference curve decreases.

d- $\partial\left(\frac{\partial(r+p+T)}{\partial q}\right)\Big/\partial k = -\frac{H_k}{H^2} < 0$

For students of the same income and ability, any indifference curve in the $[q,(r+p+T)]$ plane of a higher social capital student cuts any indifference curve of a lower social capital student from above. Thus for a given b and k, indifference curves exhibit single crossing in social capital. In other words, when social capital increases, the slope of the indifference curve decreases.

e- $\partial\left(\frac{\partial(r+p+T)}{\partial q}\right)\Big/\partial(b,y,k) = -\frac{H_{(b,y,k)}}{H^2} < 0$

Any indifference curve in the $[q,(r+p+T)]$ plane of an individual with a higher combination of (b,y,k) cuts any indifference curve of a student with a lower combination of (b,y,k) from above. Thus, indifference curves exhibit single crossing in (b,y,k). In other words, when (b,y,k) increases, the slope of the indifference curve decreases.

Lemma 2: Community and school choice: boundary indifference.

For $r^C < r^{C+1}$, and $q_j > q_{j'}$ there exists a combination $(\hat{b},\hat{y},\hat{k})$ such that:

$$W_{ij}^{C}\left\{(\hat{y}_i,\hat{b}_i,\hat{k}_i),T_j^C,t^C,p_{ij},r^C,H_i,q_j\right\} = W_{ij'}^{C+1}\left\{(\hat{y}_i,\hat{b}_i,\hat{k}_i),T_{j'}^{C+1},t^{C+1},p_{ij'},r^{C+1},H_i,q_{j'}\right\} \quad (11)$$

With $C+1 \neq C$ and $j \neq j'$. Note that $(\hat{b},\hat{y},\hat{k})$ is not unique.[11]

j and j' may or may not belong to communities C and $C+1$.

Students choose a community and a school, as long as their indirect utility is higher than that obtained in any other combination of communities and schools.

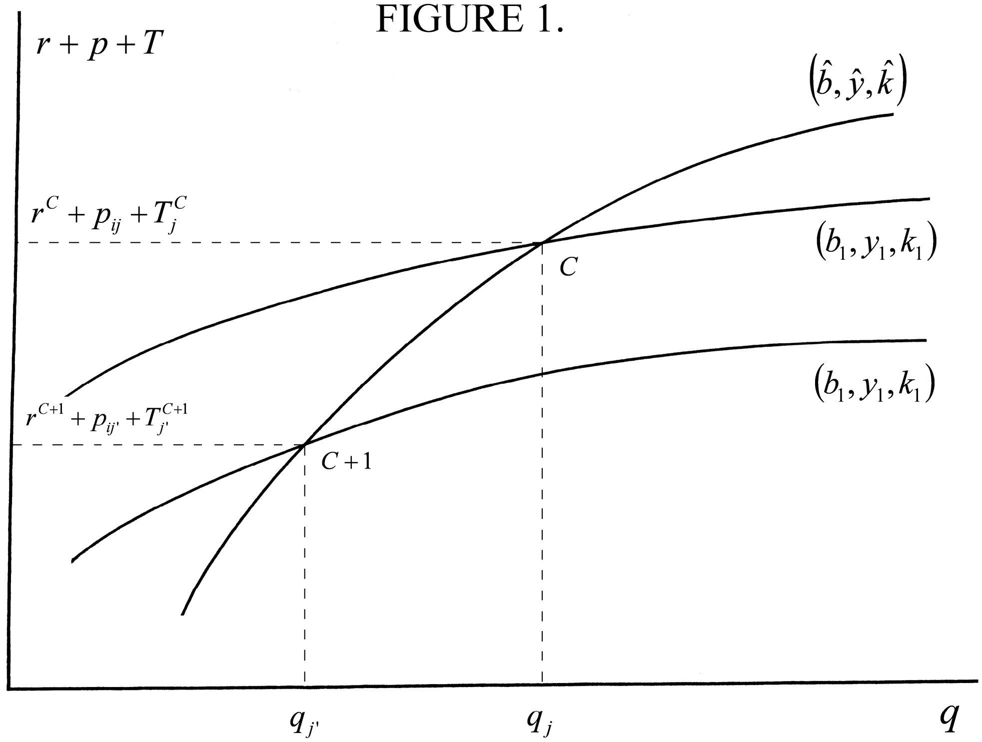
[11] $(\hat{b},\hat{y},\hat{k})$ is not unique, any other equivalent combination of b, y and k can determine the boundary indifference. See the remark on page 13.

In this Lemma we show that by choosing lower rental communities, higher (b, y, k) students would have higher numeraire consumption, higher school quality while having no less housing consumption relatively to choosing higher rental communities. Continuity of utility and of the distribution function of (b, y, k) ensures that there is a type of individuals that is indifferent between each two combinations of communities and schools.

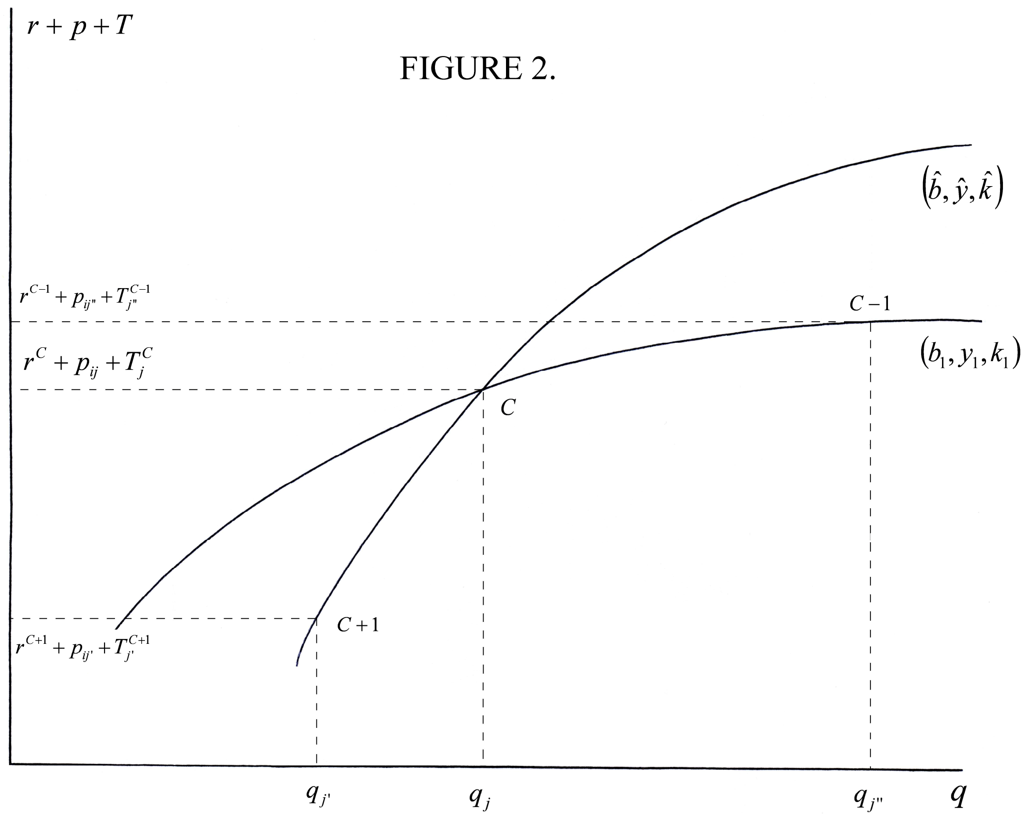
Corollary 1: $(\hat{b}, \hat{y}, \hat{k})$ is strictly monotonically decreasing in b, y, k, and (b, y, k) $\forall C$.

In this corollary, we show that any student with a better combination than $(\hat{b}, \hat{y}, \hat{k})$ can not be indifferent between the two communities C and C+1.

FIGURE 1.



Lemma 3: (i) Students with higher combinations of (b, y, k) spend more on rental, schooling, and transportation combined $(r + p + T)$ and choose higher educational qualities. (ii) Higher (b, y, k) students spend less on rental, and they have a higher housing consumption. (iii) In the third part of this Lemma we examine individuals' spending on transportation and education.



Proposition 2: Three dimensional stratification. Consider the same two students from Lemma 3. Student $(\hat{b}, \hat{y}, \hat{k})$ is indifferent between these two combinations $(C+1, j')$ and (C, j); while student (b_1, y_1, k_1) is indifferent between these two combinations (C, j) and $(C-1, j'')$. Equilibrium exists if $(b, y, k) \in C$ when $(\hat{b}, \hat{y}, \hat{k}) < (b, y, k) < (b_1, y_1, k_1)$.

By virtue of proposition 2, and corollary 1(Boundary Loci are strictly monotonically decreasing in b, y, k, and (b, y, k)), students are stratified into communities and schools according to their ability, income, and social capital. In other words, complete stratification by ability, income, or social capital would not occur. Thus, we will have a mixture of individuals with different combinations of (b, y, k) in each community.

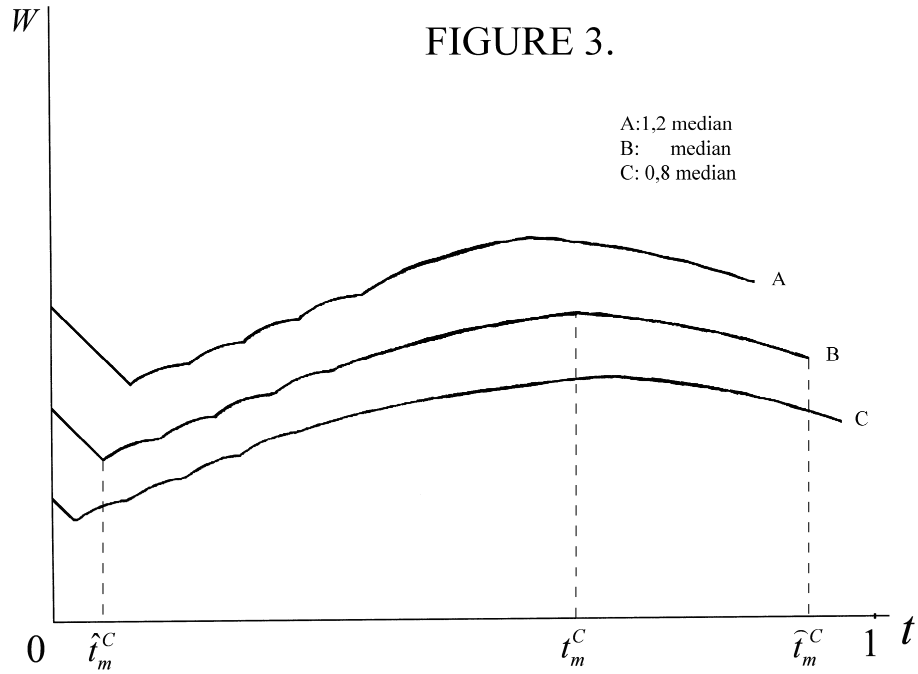
V. MAJORITY VOTING.

In this part of our model we focus on individuals' choice of tax rates. For the rest of the paper, tax rates are considered to be endogenous through majority voting. Tax rates influence educational quality through local expenditure E_j^C. Students have perfect knowledge of tax rates and school qualities; they anticipate any changes in educational qualities as a result of a change in the tax rate in their community; they assume that tax rates and educational qualities in other communities are fixed.

In our model, we considered three types of schools, pure public schools, mixed finance, and private schools. In each community, for very low tax rates the quality of public and mixed finance schools is low, students prefer private schools or even schools (private or mixed finance) outside their community. A marginal increase in tax rates will not increase public school quality that much, but it will reduce numeraire consumption and utility; private schools and schools outside the community are still preferred; at this stage students will not migrate from their community because they anticipate an increase in utility when the tax rate becomes sufficiently high. An important increase of the tax rate induces an important increase of quality in public and mixed finance schools; at some level of the tax rate, students are willing to move from their private school (or schools outside their community) to a mixed finance school inside the community, their utility increases until reaching its maximum for this school; at this point, the student will move again to the following mixed finance school, and so on. This process continues until the tax rate is high enough to encourage students to get enrolled in a public school, utility increases until reaching its highest level, from this point on, any increase in tax rates induces a decrease in utility. Once the tax rate becomes excessive such as the gross of tax price of housing is very high, students anticipate that any further increase in the tax rate will just decrease their utility more and more; high (b, y, k) students migrate from this community. Note that preferences are not single peaked, and that the "Ends against the Middle" phenomenon is present in our model. This is illustrated in Figure 3.

Remark: students enrolled in a school (private or mixed finance) outside their community or a private school inside their community have a decreasing utility in the tax rate since they do not use public or mixed finance schools inside their community. Private schools' qualities are independent of the tax rate since $E_j^C = 0$, mixed finance schools' qualities are independent of tax rates of communities to which they do not belong.

FIGURE 3.



We define $\hat{t}_i^C$ to be the tax rate at which student i is indifferent between a private school or a school (private or mixed) outside his community and a mixed finance school inside his community; note that student i lives in community C. We also define t_i^C to be the tax rate at which the utility of student i is maximized in the public school, and finally $\widehat{t}_i^C$ the tax rate at which student i is willing to migrate from community C. i=m is used to designate the student with the median combination of (b, y, k). Note that students are not willing to migrate at $\hat{t}_i^C$ because they anticipate an increase in utility when the tax rate becomes sufficiently high, conversely they are willing to migrate after $\widehat{t}_i^C$ because any further increase of the tax rate will reduce their disposable income and their utility.

Conditions for the existence of a majority voting equilibrium:

1- Students with the median combination of ability, income, and social capital $(b, y, k)_m$ prefer a positive tax rate in a public school (noted c) over a zero tax rate in a private one or a school outside his community (noted a). Note that $(b, y, k)_m$ is not unique.[12]
$W_{mc}^C\left[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c\right] > W_{ma}^C\left[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a\right]$ is true. $T_c^C = 0$, while T_a^C may be different than zero.

[12] See the remark on page 13.

2- As seen in figure 3, all students with $(b_i, y_i, k_i) > (b, y, k)_m$ have a migration tax rate $\hat{t}_i^C$ higher than t_m^C (t_m^C is the tax rate that maximizes the utility of the student with $(b, y, k)_m$ in the public school).

3- As seen in figure 3, t_i^C is decreasing in the combination (b, y, k).

4- If two students 1 and 2 with $(b_1, y_1, k_1) > (b_2, y_2, k_2)$ are enrolled in the same school, live in the same community, and have the same tax rate; student 1 has a lower utility.
$W_{2a}^C\left[(b_2, y_2, k_2), T_a^C, t^C, p_{2a}, r^C, H_2, q_a\right] > W_{1a}^C\left[(b_1, y_1, k_1), T_a^C, t^C, p_{1a}, r^C, H_1, q_a\right]$
If they are enrolled in two different schools a and b respectively with $q_a > q_b$, then student 1 has a higher utility.[13]
$W_{2b}^C\left[(b_2, y_2, k_2), T_b^C, t^C, p_{2b}, r^C, H_2, q_b\right] < W_{1a}^C\left[(b_1, y_1, k_1), T_a^C, t^C, p_{1a}, r^C, H_1, q_a\right]$

If 1 is not satisfied, then $t = 0$ is a majority voting equilibrium. If 2, 3, and 4 are not satisfied, then majority voting equilibrium may not exist. Restrictions on preferences alone do not allow the existence of a majority voting equilibrium; the following conditions represent restrictions on the parametric form of the utility function. For the rest of the paper we consider that these conditions are satisfied.

Definition: if t^C is to be a majority voting equilibrium, then there is no other tax rate that is preferred by more than 50% of the population of the community. In other terms, at least 50% and one individual choose this tax rate.

Lemma 4:

(i) $\hat{t}_i^C$ is increasing in (b, y, k).

This part applies to any indifference tax rate between a private school and a mixed finance school, between two mixed finance schools or between a mixed finance and a public school. It can be explained intuitively. Higher (b, y, k) students need higher tax rates and thus higher school qualities to be encouraged to get enrolled in a mixed finance or a public school.

(ii) We consider three schools a, b and c; a and b are two private schools belonging to community C or two schools (private or mixed finance) outside community C, c is a public school in community C.

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[13] Epple and Romano (1996) used an assumption about education being a superior or a normal good, they also used a diminishing marginal utility of numeraire consumption, see page 300. Glomm and Ravikumar (1998) used a similar assumption where $\lim_{c \to +\infty} U_c(c, q) = 0$ and a homothetic utility function.

1- If $W_{mc}^{C}\left[(b,y,k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c\right] > W_{ma}^{C}\left[(b,y,k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a\right]$ is true.[14]

Then $W_{1c}^{C}\left[(b_1,y_1,k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c\right] > W_{1b}^{C}\left[(b_1,y_1,k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b\right]$ for all $(b_1,y_1,k_1) < (b,y,k)_m$.

2- If $W_{mc}^{C}\left[(b,y,k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c\right] < W_{ma}^{C}\left[(b,y,k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a\right]$ is true.

Then $W_{1c}^{C}\left[(b_1,y_1,k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c\right] < W_{1b}^{C}\left[(b_1,y_1,k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b\right]$ for all $(b_1,y_1,k_1) > (b,y,k)_m$.

In this part of lemma 4 we prove that if the type of students with the median combination of (b,y,k) prefers "does not prefer" a positive tax rate in a public school over a zero tax rate in a private one or a school outside community C, all students with lower "higher" combinations of (b,y,k) have the same behavior.

Note that according to condition 1, the type of students with the median combination of (b,y,k) prefer the tax rate t_m^C. It will be sufficient to prove that 50% of the population prefers t_m^C to prove the existence of a majority voting equilibrium. The type of students with the median combination of (b,y,k) splits the population into two identical fractions of 50%.

Lemma 5:

(i) Over the interval $\left[t_m^C, 1\right]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population.

(ii) Over the interval $\left[\hat{t}_m^C, t_m^C\right]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population.

An important property used in the proof of Lemma 5 is based on the fact that indirect utility is increasing in t over the interval $\left[\hat{t}_m^C, t_m^C\right]$ and decreasing over the interval $\left[t_m^C, 1\right]$.

[14] Note that since c is a public school $p_c = 0$, and for schools belonging to community C, $T = 0$.

Proposition 3:

if $W_{mc}^{C}\left[(b,y,k)_{m},T_{c}^{C},t_{m}^{C},p_{mc},r^{C},H_{m},q_{c}\right]>W_{ma}^{C}\left[(b,y,k)_{m},T_{a}^{C},0,p_{ma},r^{C},H_{m},q_{a}\right]$ is true, then t_{m}^{C} is a majority voting equilibrium.

If $W_{mc}^{C}\left[(b,y,k)_{m},T_{c}^{C},t_{m}^{C},p_{mc},r^{C},H_{m},q_{c}\right]<W_{ma}^{C}\left[(b,y,k)_{m},T_{a}^{C},0,p_{ma},r^{C},H_{m},q_{a}\right]$ is true, then all students with $(b_i,y_i,k_i)>(b,y,k)_m$ prefer a zero tax rate in a private school over a positive tax rate in a public one; then $t=0$ is a majority voting equilibrium.

Equilibrium:

At Equilibrium, all the following conditions are satisfied.

1- Students.

- Utility maximization:

$$U^{*}(c,q)=Max\left[y_i-T_j^C-p_{ij}-r^C H_i,q_j\left(\frac{R_j}{l_j},\theta_j,\mathrm{O}_j\right)\right]$$

$$=W_{ij}^{C}\left[(y_i,b_i,k_i),T_j^C,t^C,p_{ij},r^C,H_i,q_j\right]$$

- All majority voting conditions hold in equilibrium.

2- Schools.

- Profit maximization and optimal level of resources: $\max\pi_j=R_j-V(l_j)-F$.

$$R'^{*}=n_1+2n_2 l_j+\mu_j'(\theta_j-b_i)+\mu_j''(\mathrm{O}_j-k_i)$$

With μ' is positive and μ'' is negative.

- At equilibrium $\pi_j=0$; no new entries on the market are possible.
- The number of students in a school: $l_j=\iiint_s \alpha_j(b,y,k)f(b,y,k)dbdydk$.
- Mean ability in a schools: $\theta_j=\frac{1}{l_j}\iiint_s b_i\alpha_j(b,y,k)f(b,y,k)dbdydk$
- Mean social capital in a school: $\mathrm{O}_j=\frac{1}{l_j}\iiint_s k_i\alpha_j(b,y,k)f(b,y,k)dbdydk$.

3- Communities.

- The community budget is balanced: $\sum E_j^C=\sum t^C r_{net}H_i$.
- The number of students in a community: $l_C=\iiint_s \alpha_C(b,y,k)f(b,y,k)dbdydk$.

- Boundary indifference conditions are satisfied.
- The median voter is pivotal.

4- Market clearance.

- All students go to school: $\sum_{j=1}^{j} l_j = \sum_{C=1}^{C} l_C = P$
- The housing demand in a community is equal to the housing supply.

VI. INTRODUCING PUBLIC SCHOOL CHOICE.

For the rest of the paper, we suppose that all equilibrium conditions specified above are satisfied. We assume that public schools do not face a capacity constraint, they admit all new comers as seen in the sequencing of decision making on page 11. For simplicity, we are only considering two communities C and C+1. From lemma 2, and lemma 3; we know that higher (b, y, k) students choose community C, while lower (b, y, k) students choose community C+1, with $r^C < r^{C+1}$. Students with $(\hat{b}, \hat{y}, \hat{k})$ determine boundary indifference between communities. When majority voting equilibrium's conditions are satisfied, median voters are pivotal; $(b, y, k)_m^C$ is the median voter in community C, and $(b, y, k)_m^{C+1}$ is the median voter in community C+1. t_m^C and t_m^{C+1} are majority voting equilibria in each community. We know from condition 1 that median voters prefer a positive tax rate (t_m^C and t_m^{C+1}) in a public school over a zero tax rate in a private school or a school outside their community; thus at t_m^C and t_m^{C+1}, median voters choose public schools. From lemma 4 (ii), we know that all students with (b, y, k) below that of the median make the same choice, thus they choose the public school. From figure 3, we can see that at t_m^C and t_m^{C+1}, some students with a higher combination of (b, y, k) relatively to the median voter may choose the public school; thus more than 50% of the population chooses the public school. Only students with very high (b, y, k) choose mixed finance or private schools.[15]

[15] This is consistent with assumption (1-a), higher (b, y, k) students choose higher quality schools.

According to lemma 1 (ii), and proposition 1, in the same community, the hierarchy of school qualities follows that of the prices for the same student; thus public schools have the lowest quality in each community. By virtue of assumption (1-a), students with $(b, y, k) > (\hat{b}, \hat{y}, \hat{k})$ prefer higher educational qualities than students with $(b, y, k) < (\hat{b}, \hat{y}, \hat{k})$;[16] Thus all schools in community C have higher qualities than schools in community C+1. Figure 4 presents a simplifying illustration.

FIGURE 4.

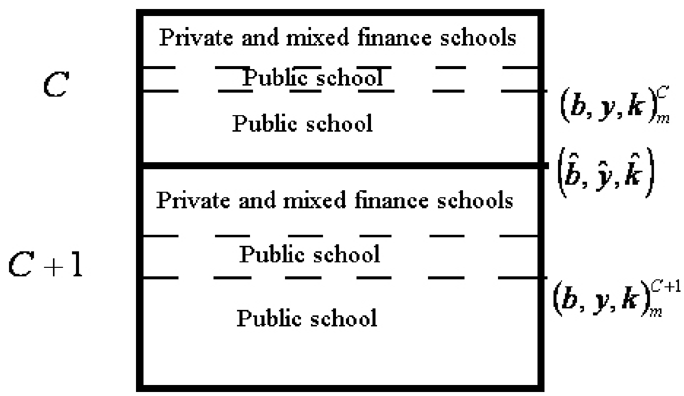

Figure 4 represents stratification across communities and schools.

When public school choice is allowed, students from C+1 will try to get enrolled in the public school of community C. Since students have already chosen their community, no one would be willing to change his residence after the introduction of public school choice. At equilibrium, those who have chosen the public school in community C+1 have their budget already balanced; the introduction of public school choice would not have any effect on them since they are unable to afford transportation to get enrolled in the public school in community C. Only those who are enrolled in private and mixed finance schools in C+1 would be able to get enrolled in the public school of community C by using the resources economized on the cost of education to finance transportation. Note that the students enrolled in private and mixed finance schools in community C+1 are the ones with high (b, y, k), thus the most advantaged in their community.

[16] The demand for educational quality is increasing in (b, y, k).

Consider a student i living in community C+1 and enrolled in a private or mixed finance school (inside or outside C+1) noted (a). If he is to exercise public school choice, he would be still living in community C+1 but will get enrolled in the public school in community C noted (c). With $q_c \geq q_a$ (this is a necessary condition, since no student would be encouraged to get enrolled in c if it has a lower quality) this is consistent with $\lim_{c \to +\infty} U_c(c,q) = 0$.

His budget constraint in (a) is the following: $c_{ia}^{C+1} = y_i - T_a^{C+1} - p_{ia} - r^{C+1} H_i$ with $T_a^{C+1} = 0$ if $a \in C+1$ and $T_a^{C+1} \neq 0$ if $a \notin C+1$.

His budget constraint in (c) is the following: $c_{ic}^{C+1} = y_i - T_c^{C+1} - r^{C+1} H_i$ with $p_{ic} = 0$ since c is a public school, and $T_c^{C+1} \neq 0$ because $c \notin C+1$.

In order for student i to exercise public school choice, he should achieve at least the same numeraire consumption while having no less educational quality. Such as:

$c_{ic}^{C+1} - c_{ia}^{C+1} = T_a^{C+1} + p_{ia} - T_c^{C+1} \geq 0$ is true; this implies: $T_a^{C+1} + p_{ia} \geq T_c^{C+1}$. In order to exercise public school choice, the transportation cost between a student's community and his new public school must not exceed the economies made by opting out of his old private or mixed finance school.

As a result, we can see that public school choice may be beneficial only to a small fraction of the population and that only high (b, y, k) students would be able to exercise their right to choose their public school. The first of these results is perhaps the most novel; it is consistent with the fact that in some countries like the UK, New Zealand, Sweden, and Ireland a large fraction of households still choose the public school of their neighborhood even if they have the choice to opt out. The final effect on school quality in both communities is ambiguous; while public school quality in community C+1 would remain unchanged, some private schools in community C+1 would be losing students, and expenditure per pupil in the public school in community C will be reduced since the number of students is growing while resources are unchanged. Since boundary loci are decreasing between communities, and thus there are no complete stratification according to one of these factors: ability, income, or social capital; it is difficult to determine the effect of public school choice on mean ability and mean social capital, it is not certain that choice may have a general negative effect on the three components of educational quality. These results can be generalized to an economy consisting of more than two communities.

VII. CONCLUSION.

In this paper, we have constructed an equilibrium on the market of education in a multicommunity environment where students are differentiated by income, ability, and social capital. We have analyzed stratification across schools and communities to find that students with higher combinations of ability, income, and social capital spend more on rental, transportation and education combined, while choosing higher educational qualities; they spend less on rental and more on the general cost of acquiring education while having higher housing consumption.

We have studied majority voting over tax rates; the presence of private, mixed finance, and public schools led to the existence of non single peaked preferences. We have provided the necessary conditions for a majority voting equilibrium to be determined by the median voter. In the final section, we have introduced public school choice to find that only a small fraction of the most advantaged students in term of income, ability, and social capital will exercise their right to choose a public school outside their community; we also found that choice may not have the dramatic negative effect on school quality feared by some critics of choice.

Appendix

<u>School optimization:</u>

Schools maximize profit under constraint:

$$\max \pi_j = R_j - V(l_j) - F$$

Under these constraints:

$$l_j = \iiint_s \alpha_j(b, y, k) f(b, y, k) dbdydk \qquad (1)$$

$$\theta_j = \frac{1}{l_j} \iiint_s b_i \alpha_j(b, y, k) f(b, y, k) dbdydk \qquad (4)$$

$$\mathrm{O}_j = \frac{1}{l_j} \iiint_s k_i \alpha_j(b, y, k) f(b, y, k) dbdydk \qquad (5)$$

With (i = 1, 2, 3, …, P) and (j = 1, 2, 3, …, j)

These constraints can be transformed in the following way:

1- $\theta_j \iiint_s \alpha_j(b,y,k) f(b,y,k) dbdydk - \iiint_s b_i \alpha_j(b,y,k) f(b,y,k) dbdydk = 0$

2- $\mathrm{O}_j \iiint_s \alpha_j(b,y,k) f(b,y,k) dbdydk - \iiint_s k_i \alpha_j(b,y,k) f(b,y,k) dbdydk = 0$

The Lagrangian function is of the following form:

$$\Phi = R_j - n_1 l_j - n_2 l_j^2 - F - \mu' [\theta_j \iiint_s \alpha_j(b,y,k) f(b,y,k) dbdydk - \iiint_s b_i \alpha_j(b,y,k) f(b,y,k) dbdydk]$$
$$- \mu'' [\mathrm{O}_j \iiint_s \alpha_j(b,y,k) f(b,y,k) dbdydk - \iiint_s k_i \alpha_j(b,y,k) f(b,y,k) dbdydk]$$

With $\mu', and \mu''$ the Lagrangian multipliers.

μ' is positive and μ'' is negative.

Optimization is made through the partial derivation of the Lagrangian function over $\alpha_j(b,y,k)$, the following results are obtained:

$$\frac{d\Phi}{d\alpha_j(b,y,k)} = R'^{*} - n_1 - 2n_2 l_j - \mu'(\theta_j - b_i) - \mu''(o_j - k_i) = 0$$

$$R'^{*} = n_1 + 2n_2 l_j + \mu'(\theta_j - b_i) + \mu''(o_j - k_i)$$

Proof of Lemma 1:

(i) The proof is by contradiction. Suppose that we have two schools 1 and 2 with $p_{i1} + T_1^C < p_{i2} + T_2^C$ and $q_1 \geq q_2$ (With i = 1, 2, …, P and C = 1, 2, …, C), student i lives in community C. In this case; rational students will choose school 1 since it has higher or equal quality at a lower cost for all students and all communities. No student will choose school 2; this school will be obliged to leave the market. In order for school 2 to exist it should have a quality $q_2 > q_1$. Quality is strictly increasing in the general cost of education. If two schools are of the same quality they should have the same costs of education.

For two schools 1 and 2 belonging to the same community we know that: $T_1^C = T_2^C$ (With C = 1, 2, …, C) (see page 6); thus we can write: for $q_2 > q_1$ we have $p_{i2} > p_{i1}$. Thus, in the same community, quality is increasing in the price for all students (i = 1, 2, …, P).

(ii) The second part of lemma 1 follows directly from the first part. Since we are considering schools belonging to the same community; all schools have the same transportation cost relatively to a community C (with C = 1, 2, …, C). Thus, educational quality is increasing in the price as shown above. The lowest price $p_j = 0$ of pure public schools is associated with the lowest quality. An important assumption needed to the proof of this lemma is that made in the sequencing of decisions; since pure public schools admit all comers, no student is restrained to choose a lower quality higher price school than a pure public school.

Proof of Lemma 2:

We consider a student i with (b_i, y_i, k_i), he consumes the same amount of housing in both communities C and C+1.

His budget constraint in school j and community C is given by: $y_i = c_{ij}^C + T_j^C + p_{ij} + r^C H_i$

It can be written in the following form: $c_{ij}^C = y_i - T_j^C - p_{ij} - r^C H_i$

His budget constraint in school j' and community C+1 is given by: $y_i = c_{ij'}^{C+1} + T_{j'}^{C+1} + p_{ij'} + r^{C+1} H_i$. It can be written in the following form: $c_{ij'}^{C+1} = y_i - T_{j'}^{C+1} - p_{ij'} - r^{C+1} H_i$

The difference in consumption between community C+1 and C is given by:

$c_{ij'}^{C+1} - c_{ij}^C = \left(T_j^C - T_{j'}^{C+1}\right) + \left(p_{ij} - p_{ij'}\right) + \left(r^C - r^{C+1}\right) H_i$

The sum $\left(T_j^C - T_{j'}^{C+1}\right) + \left(p_{ij} - p_{ij'}\right)$ can be positive, negative, or equal to zero. By virtue of lemma 1, if j and j' belong to the same community then $(p_{ij} - p_{ij'}) > 0$ and $\left(T_j^C - T_{j'}^{C+1}\right)$ can be positive, negative or equal to zero. Since $r^C < r^{C+1}$, we can write $r^C - r^{C+1} < 0$.

Even if the distribution of (b, y, k) has an upper bound determined by $(b_{\max}, y_{\max}, k_{\max})$ and a lower bound determined by $(0,0,0)$; it is analytically convenient for the proof of this lemma to consider $(b, y, k) \rightarrow +\infty$ and $(b, y, k) \rightarrow -\infty$.

We have two cases:

1- According to part b of assumption 1, for students with very high combinations of (b, y, k); such as $(b, y, k) \to +\infty$. We have $H_i \to +\infty$, and then $\lim_{(b,y,k)\to+\infty}\left(c_{ij'}^{C+1} - c_{ij}^{C}\right) = -\infty$.

In other terms, we can always find a (b, y, k) combination that is sufficiently high such as $\left(c_{ij'}^{C+1} - c_{ij}^{C}\right) < 0$.

According to part a of assumption 1; higher (b, y, k) students prefer higher educational qualities. As a consequence, this student chooses school j.

If this individual is to locate to C+1; he will consume the same amount of housing, while having a lower numeraire consumption. So, student (b_i, y_i, k_i) strictly prefer community C.

2- According to part b of assumption 1, for students with very low combinations of (b, y, k); such as $(b, y, k) \to -\infty$. We have $H_i \to -\infty$, and then $\lim_{(b,y,k)\to+\infty}\left(c_{ij'}^{C+1} - c_{ij}^{C}\right) = +\infty$.

In other terms, we can always find a (b, y, k) combination that is sufficiently low such as $\left(c_{ij'}^{C+1} - c_{ij}^{C}\right) > 0$.

According to part a of assumption 1, lower (b, y, k) students prefer lower educational qualities. As a consequence, this student chooses school j'.

If this student is to locate to C+1; he will consume the same amount of housing, while having a higher consumption. So, student (b_i, y_i, k_i) strictly prefer community C+1.

We have seen, that high (b, y, k) students prefer communities with lower rental and schools with higher quality, and that students with low (b, y, k) prefer communities with higher rental and schools with lower quality. The continuity of the utility function and of the distribution function of (b, y, k) implies that while going from an extreme to the other, there exist an student with $(\hat{b}, \hat{y}, \hat{k})$ that is indifferent between the two communities and the two schools. The proof that is applied to two communities and schools can be extended to multiple communities and schools; between each two consecutive communities and schools there will be a type of students indifferent between them.

Proof of corollary 1: The proof is by contradiction.

Suppose that $(\hat{b}, \hat{y}, \hat{k})$ is somewhere nondecreasing in b, y, k and (b, y, k). Then we can find an student with (b_1, y_1, k_1) indifferent between the two combinations of communities and schools, (C, j) and $(C+1, j')$ such as:

1- $b_1 > \hat{b}$, $y_1 \geq \hat{y}$, and $k_1 \geq \hat{k}$.

2- $b_1 \geq \hat{b}$, $y_1 > \hat{y}$, and $k_1 \geq \hat{k}$.

3- $b_1 \geq \hat{b}$, $y_1 \geq \hat{y}$ and, $k_1 > \hat{k}$

4- $b_1 > \hat{b}$, $y_1 > \hat{y}$, and $k_1 \geq \hat{k}$

5- $b_1 > \hat{b}$, $y_1 \geq \hat{y}$, and $k_1 > \hat{k}$

6- $b_1 \geq \hat{b}$, $y_1 > \hat{y}$, and $k_1 > \hat{k}$

7- $b_1 > \hat{b}$, $y_1 > \hat{y}$, $k_1 > \hat{k}$ or $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$

The case where $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$, shall be considered because it accounts for situations where one or two of the variables b, y, and k is lower while others are higher.

For case 1,

According to property 1-c, the slope of the indifference curve of $(b_1, \hat{y}, \hat{k})$ is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to properties 1-b and 1-d, the slope of (b_1, y_1, k_1) is equal or lower than that of $(b_1, \hat{y}, \hat{k})$.

$$\text{Slope}\,(b_1, y_1, k_1) \leq \text{Slope}\,(b_1, \hat{y}, \hat{k}) < \text{Slope}\,(\hat{b}, \hat{y}, \hat{k})$$

For case 2:

According to property 1-b, the slope of the indifference curve of $(\hat{b}, y_1, \hat{k})$ is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to properties 1-c and 1-d, the slope of the indifference curve of (b_1, y_1, k_1) is lower or equal to that of $(\hat{b}, y_1, \hat{k})$.

$$\text{Slope}\,(b_1, y_1, k_1) \leq \text{Slope}\,(\hat{b}, y_1, \hat{k}) < \text{Slope}\,(\hat{b}, \hat{y}, \hat{k})$$

For case 3:

According to property 1-d, the slope of the indifference curve of $(\hat{b}, \hat{y}, k_1)$ is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to properties 1-b and 1-c, the slope of the indifference curve of (b_1, y_1, k_1) is lower or equal to that of $(\hat{b}, \hat{y}, k_1)$.

$$\text{Slope}\ (b_1, y_1, k_1) \leq \text{Slope}\ (\hat{b}, \hat{y}, k_1) < \text{Slope}\ (\hat{b}, \hat{y}, \hat{k})$$

For Case 4:

According to properties 1-b and 1-c, the slope of the indifference curve of $(b_1, y_1, \hat{k})$ is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to property 1-d, the slope of the indifference curve of (b_1, y_1, k_1) is equal or lower than that of $(b_1, y_1, \hat{k})$.

$$\text{Slope}\,(b_1, y_1, k_1) \leq \text{Slope}\,(b_1, y_1, \hat{k}) < \text{Slope}\,(\hat{b}, \hat{y}, \hat{k})$$

For case 5:

According to properties 1-c and 1-d, the slope of the indifference curve of $(b_1, \hat{y}, k_1)$ is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to property 1-b, the slope of the indifference curve of (b_1, y_1, k_1) is equal or lower than that of $(b_1, \hat{y}, k_1)$.

$$\text{Slope}\ (b_1, y_1, k_1) \leq \text{Slope}\,(b_1, \hat{y}, k_1) < \text{Slope}\,(\hat{b}, \hat{y}, \hat{k})$$

For case 6:

According to properties 1-b and 1-d, the slope of the indifference curve of $(\hat{b}, y_1, k_1)$ is lower than that of $(\hat{b}, \hat{y}, \hat{k})$. According to property 1-c, the slope of (b_1, y_1, k_1) is equal or lower than that of $(\hat{b}, y_1, k_1)$.

$$\text{Slope}\ (b_1, y_1, k_1) \leq \text{Slope}\ (\hat{b}, y_1, k_1) < \text{Slope}\ (\hat{b}, \hat{y}, \hat{k})$$

For case 7:

According to properties 1-b, 1-c, 1-d and 1-e. the slope of the indifference curve of (b_1, y_1, k_1) is lower than that of $(\hat{b}, \hat{y}, \hat{k})$.

$$\text{Slope}\,(b_1, y_1, k_1) < \text{Slope}\ (\hat{b}, \hat{y}, \hat{k})$$

For a student with (b_1, y_1, k_1) to be indifferent between the two combinations of communities and schools (C, j) and $(C+1, j')$; his indifference curve shall pass through the two points $[q_j, (r^C + p_{ij} + T_j^C)]$and $[q_{j'}, (r^{C+1} + p_{ij'} + T_{j'}^{C+1})]$. Thus, the slope of the indifferent curve of (b_1, y_1, k_1) must be at some point grater than that of $(\hat{b}, \hat{y}, \hat{k})$ in order to cross it at these two points. Following the 7 cases previously developed and figure 1, this is a contradiction; the slope of the indifference curve of (b_1, y_1, k_1) is always smaller than that of $(\hat{b}, \hat{y}, \hat{k})$, and there is only one intersection between the two indifference curves. The student with (b_1, y_1, k_1) can not be indifferent between these two combinations, and the boundary loci between communities and schools are strictly monotonically decreasing in b, y, k, and (b, y, k).

Proof of Lemma 3:

Consider two students with $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$; according to property (1-e), the indifference curve of student (b_1, y_1, k_1) has a lower slope than that of student $(\hat{b}, \hat{y}, \hat{k})$. There is only one intersection between their indifference curves, such that they can not be indifferent between the same two combinations of communities and schools. See figure 2.

(i) Note that $C+1 \neq C \neq C-1$ and $j'' \neq j \neq j'$.

We consider that student $(\hat{b}, \hat{y}, \hat{k})$ is indifferent between these two combinations $(C+1, j')$ and (C, j); his indifference curve passes through these two points $\left[q_{j'}, \left(r^{C+1} + p_{ij'} + T_{j'}^{C+1}\right)\right]$ and $\left[q_j, \left(r^C + p_{ij} + T_j^C\right)\right]$. According to Lemma 2 we have:

$q_j > q_{j'}$ and $r^C < r^{C+1}$.

We consider that student (b_1, y_1, k_1) is indifferent between these two combinations (C, j) and $(C-1, j'')$, his indifference curve passes through these two points $\left[q_j, \left(r^C + p_{ij} + T_j^C\right)\right]$ and $\left[q_{j''}, \left(r^{C-1} + p_{ij''} + T_{j''}^{C-1}\right)\right]$. According to Lemma 2 we have:

$q_{j''} > q_j$ and $r^{C-1} < r^C$.

We can see that there is only one intersection of indifference curves at point $\left[q_j, \left(r^C + p_{ij} + T_j^C\right)\right]$.

From the explanation mentioned above, we can write $q_{j''} > q_j > q_{j'}$. From Property (1-a), we know that indifference curves have positive slopes in the $\left[q, (r + p + T)\right]$ plane, thus $q_{j''} > q_j > q_{j'}$ implies $\left(r^{C-1} + p_{ij''} + T_{j''}^{C-1}\right) > \left(r^C + p_{ij} + T_j^C\right) > \left(r^{C+1} + p_{ij'} + T_{j'}^{C+1}\right)$.

For a student with $(\hat{b}, \hat{y}, \hat{k})$ choosing $(C+1, j')$, he would pay $\left(r^{C+1} + p_{ij'} + T_{j'}^{C+1}\right)$. For a student with (b_1, y_1, k_1) choosing (C, j), he would pay $\left(r^C + p_{ij} + T_j^C\right)$. With $(b_1, y_1, k_1) > (\hat{b}, \hat{y}, \hat{k})$, $q_j > q_{j'}$ and $\left(r^C + p_{ij} + T_j^C\right) > \left(r^{C+1} + p_{ij'} + T_{j'}^{C+1}\right)$, we deduce that students with a higher combination of (b, y, k) spend more on rental, schooling, and transportation combined, while choosing higher quality schools.

(ii) From part (i) of this Lemma we can write: $r^{C-1} < r^C < r^{C+1}$.

Higher (b, y, k) students spend less on rental.

From Assumption (1-b), we know that: $\frac{\partial H}{\partial(b,y,k)} > 0$

Thus higher (b,y,k) students have higher housing consumption.

(iii) From part (i) and (ii) of this lemma we can analyze the different possible cases of spending on transportation and education. As in part (i), the student with $(\hat{b},\hat{y},\hat{k})$ chooses $(C+1, j')$, the student with (b_1, y_1, k_1) chooses (C, j), with $(b_1, y_1, k_1) > (\hat{b},\hat{y},\hat{k})$.

We have $r^C + p_{ij} + T_j^C > r^{C+1} + p_{ij'} + T_{j'}^{C+1}$, and $r^C < r^{C+1}$.

We can write $\left(p_{ij'} + T_{j'}^{C+1}\right) - \left(p_{ij} + T_j^C\right) < r^C - r^{C+1} < 0$, this implies that $\left(p_{ij'} + T_{j'}^{C+1}\right) < \left(p_{ij} + T_j^C\right)$.

Higher (b,y,k) students spend more on education and transportation combined, in other words higher (b,y,k) students are willing to spend more on the general cost of acquiring education in order to get higher qualities. We can see that price and transportation are substitutable in acquiring higher educational qualities.

When $\left(T_{j'}^{C+1} - T_j^C\right)$ and $\left(p_{ij'} - p_{ij}\right)$ are taken separately, they can be positive, negative, or equal to zero, with $\left(T_{j'}^{C+1} - T_j^C\right) + \left(p_{ij'} - p_{ij}\right) < 0$ satisfied. For $\left(T_{j'}^{C+1} - T_j^C\right) < 0$; high (b,y,k) students choose far from school communities in comparison with lower (b,y,k) students. For $\left(T_{j'}^{C+1} - T_j^C\right) > 0$; high (b,y,k) students choose close to school communities in comparison with lower (b,y,k) students. For $\left(T_{j'}^{C+1} - T_j^C\right) = 0$; high (b,y,k) students live at the same distance from school in comparison with lower (b,y,k) students. Similarly, For $\left(p_{ij'} - p_{ij}\right) > 0$ higher (b,y,k) students spend less on education than lower (b,y,k) students; For $\left(p_{ij'} - p_{ij}\right) < 0$ higher (b,y,k) students spend more on education than lower (b,y,k) students; For $\left(p_{ij'} - p_{ij}\right) = 0$ both types of students spend the same on education.

Proof of Proposition 2:

From property 1, we know that the indifference curve of student (b,y,k) has a steeper slope than student (b_1, y_1, k_1), since $(b,y,k) < (b_1, y_1, k_1)$. From lemma 3, we know that he spends less on rental, schooling and transportation combined and chooses lower quality schools.

Similarly, the indifference curve of student (b,y,k) has a flatterer slope than student $(\hat{b},\hat{y},\hat{k})$, since $(\hat{b},\hat{y},\hat{k}) < (b,y,k)$. From lemma 3, we know that he spends more on rental, schooling and transportation combined and he chooses higher quality schools.

By virtue of Lemma 3 and figure 2, we can write.

$r^{C-1} + p_{ij''} + T_{j''}^{C-1} > r^{C} + p_{ij} + T_{j}^{C} > r^{C+1} + p_{ij'} + T_{j'}^{C+1}$ and,

$q_{j''} > q_{j} > q_{j'}$

Thus student (b, y, k) satisfying $(\hat{b}, \hat{y}, \hat{k}) < (b, y, k) < (b_1, y_1, k_1)$ must strictly prefer the combination (C, j), with $[q_j, (r^C + p_{ij} + T_j^C)]$.

Proof of lemma 4:

(i) The proof is by contradiction.

Consider two students 1 and 2 with $(b_1, y_1, k_1) > (b_2, y_2, k_2)$ living in community C, and three schools a, b, and c. a and b are two private schools belonging to community C or two schools (private or mixed finance) outside community C with $q_a > q_b$ "See the remark on page 17", c is a mixed finance school in community C .[17]

Since higher (b, y, k) students choose higher educational qualities as indicated in assumption (1-a), student (b_1, y_1, k_1) chooses school a, while student (b_2, y_2, k_2) chooses school b. In this lemma we analyze the positioning of indifference tax rates of students 1 and 2 between (a and c) and (b and c) respectively.

We suppose that $\hat{t}_1^C < \hat{t}_2^C$ is true.

For student 1; $\hat{t}_1^C$ is his indifference tax rate between a and c. we can write:

$$W_{1a}^C[(b_1, y_1, k_1), T_a^C, \hat{t}_1^C, p_{1a}, r^C, H_1, q_a] = W_{1c}^C[(b_1, y_1, k_1), T_c^C, \hat{t}_1^C, p_{1c}, r^C, H_1, q_c] \qquad \text{(a)}$$

For student 2; $\hat{t}_2^C$ is the indifference tax rate between b and c. we can write:

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, \hat{t}_2^C, p_{2b}, r^C, H_2, q_b] = W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_2^C, p_{2c}, r^C, H_2, q_c]$$

By virtue of figure 3 we know that for any $t < \hat{t}_2^C$. Student 2 prefers school b (See Figure 3).

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, t, p_{2b}, r^C, H_2, q_b] > W_{2c}^C[(b_2, y_2, k_2), T_c^C, t, p_{2c}, r^C, H_2, q_c]$$

Since we supposed that $\hat{t}_1^C < \hat{t}_2^C$, we can write:

$$W_{2b}^C[(b_2, y_2, k_2), T_b^C, \hat{t}_1^C, p_{2b}, r^C, H_2, q_b] > W_{2c}^C[(b_2, y_2, k_2), T_c^C, \hat{t}_1^C, p_{2c}, r^C, H_2, q_c] \qquad \text{(b)}$$

Since $(b_1, y_1, k_1) > (b_2, y_2, k_2)$, and by virtue condition 4 we have:

[17] Since c is in community C, then $T_c^C = 0$, while T_a^C and T_b^C may be different than zero.

$$W_{1a}^{C}\left[(b_1, y_1, k_1), T_a^C, \hat{t}_1^C, p_{1a}, r^C, H_1, q_a\right] > W_{2b}^{C}\left[(b_2, y_2, k_2), T_b^C, \hat{t}_1^C, p_{2b}, r^C, H_2, q_b\right]. \qquad \text{(c)}$$ [18]

From (a), (b) and (c), we can write:

$$W_{1c}^{C}\left[(b_1, y_1, k_1), T_c^C, \hat{t}_1^C, p_{1c}, r^C, H_1, q_c\right] > W_{2c}^{C}\left[(b_2, y_2, k_2), T_c^C, \hat{t}_1^C, p_{2c}, r^C, H_2, q_c\right]. \qquad \text{(d)}$$

We know from condition 4 that if $(b_1, y_1, k_1) > (b_2, y_2, k_2)$ is true then:

$$W_{1c}^{C}\left[(b_1, y_1, k_1), T_c^C, \hat{t}_1^C, p_{1c}, r^C, H_1, q_c\right] < W_{2c}^{C}\left[(b_2, y_2, k_2), T_c^C, \hat{t}_1^C, p_{2c}, r^C, H_2, q_c\right].$$

This is a contradiction with (d); our assumption that $\hat{t}_1^C < \hat{t}_2^C$ is false. This implies that the reverse is true: $\hat{t}_1^C > \hat{t}_2^C$. Thus the indifference tax rate is increasing in the combination (b, y, k). Note that this lemma applies to any indifference tax rate between a private school and a mixed finance one, between two mixed finance schools, and between a mixed finance and a public school. In this case, schools a and b can be private schools or mixed finance schools (inside or outside community C), school c can be a mixed finance or a public school inside community C.

(ii) Consider two students m and 1 with $(b, y, k)_m$ and (b_1, y_1, k_1) respectively; a and b are two private schools belonging to community C or two schools (private or mixed finance) outside community C "See the remark on page 17", c is a public school inside community C.

According to assumption (1-a), higher (b, y, k) students choose higher quality schools.

Part 1:

For $(b, y, k)_m > (b_1, y_1, k_1)$ and $q_a > q_b$.

By virtue of condition 4, we can write:

$$W_{mc}^{C}\left[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c\right] < W_{1c}^{C}\left[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c\right].$$ [19]

$$W_{ma}^{C}\left[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a\right] > W_{1b}^{C}\left[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b\right].$$ [20]

And if $W_{mc}^{C}\left[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c\right] > W_{ma}^{C}\left[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a\right]$ is true

We can write:

$$W_{1b}^{C}\left[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b\right] < W_{ma}^{C}\left[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a\right] <$$
$$W_{mc}^{C}\left[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c\right] < W_{1c}^{C}\left[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c\right]$$

[18] Higher (b, y, k) students achieve higher utilities when they have access to higher educational qualities and lower utilities when they are enrolled in the same school and live in the same community as lower (b, y, k) students; this is assumed to be true in condition 4.

[19] m and 1 are in the same public school in the same community, and have the same tax rate.

[20] m and 1 are in two different private schools.

Thus:

$$W_{1b}^{C}\left[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b\right] < W_{1c}^{C}\left[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c\right]$$

Part 2:

For $(b, y, k)_m < (b_1, y_1, k_1)$ and $q_a < q_b$.

By virtue of condition 4, we can write:

$$W_{mc}^{C}\left[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c\right] > W_{1c}^{C}\left[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c\right].$$

$$W_{ma}^{C}\left[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a\right] < W_{1b}^{C}\left[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b\right].$$

And if $W_{mc}^{C}\left[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c\right] < W_{ma}^{C}\left[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a\right]$ is true

We can write:

$$W_{1b}^{C}\left[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b\right] > W_{ma}^{C}\left[(b, y, k)_m, T_a^C, 0, p_{ma}, r^C, H_m, q_a\right] >$$
$$W_{mc}^{C}\left[(b, y, k)_m, T_c^C, t_m^C, p_{mc}, r^C, H_m, q_c\right] > W_{1c}^{C}\left[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c\right]$$

Thus:

$$W_{1b}^{C}\left[(b_1, y_1, k_1), T_b^C, 0, p_{1b}, r^C, H_1, q_b\right] > W_{1c}^{C}\left[(b_1, y_1, k_1), T_c^C, t_m^C, p_{1c}, r^C, H_1, q_c\right]$$

Proof of Lemma 5:

(i) For $t \in \left[t_m^C, 1\right] \Rightarrow t > t_m^C$.

Consider a student denoted 1 with, $(b_1, y_1, k_1) > (b, y, k)_m$.

From the third equilibrium condition, we know that for $(b_1, y_1, k_1) > (b, y, k)_m$ we have $t_1^C < t_m^C$.

Two situation shall be analyzed:

1- For $\hat{t}_1^C < t_1^C$.

From figure 3, we know that this student is enrolled in a public school for t_1^C. We can write: $\hat{t}_1^C < t_1^C < t_m^C < t$. Since indirect utility is decreasing in the tax rate for all students with $(b_i, y_i, k_i) > (b, y, k)_m$ over the interval $\left[t_m^C, 1\right]$; student 1 prefers the lowest tax rate over the interval which is t_m^C.

2- For $\hat{t}_1^C > t_1^C$.

From figure 3, we know that this student is enrolled in a private school for t_1^C (t_1^C is the tax rate that maximizes his utility in a public school). Since indirect utility (in a private school) is decreasing in the tax rate for all students in the economy; student 1 prefers the lowest tax rate over the interval which is t_m^C.[21]

Over the interval $\left[t_m^C, 1\right]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population.[22]

(ii) For $t \in \left[\hat{t}_m^C, t_m^C\right] \Rightarrow t < t_m^C$, consider an individual with $(b_1, y_1, k_1) < (b, y, k)_m$.

As indirect utility is increasing over this interval, individual 1 is going to choose the highest attainable tax rate over the interval, thus he chooses t_m^C.

Over the interval $\left[\hat{t}_m^C, t_m^C\right]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population.[23]

Proof of proposition 3:

We have to demonstrate that over the interval $\left[0, \hat{t}_m^C\right]$, there does not exist a tax rate different than t_m^C that is preferred by more than 50% of the population. For a student 1 with $(b_1, y_1, k_1) < (b, y, k)_m$ we have two situations to analyze corresponding to the intervals $\left[0, \hat{t}_1^C\right]$ and $\left[\hat{t}_1^C, \hat{t}_m^C\right]$.

1- Over the interval $\left[0, \hat{t}_1^C\right]$.

As indirect utility is decreasing in the tax rate over the interval $\left[0, \hat{t}_1^C\right]$, student 1 prefers a zero tax rate in a private school or a school (private or mixed finance) outside his community (noted b), over any other tax rate t in the same school. With $t \in \left[0, \hat{t}_1^C\right]$.

$$W\left[(b_1, y_1, k_1), T_b^C, 0, p_{mb}, r^C, H_m, q_b\right] > W\left[(b_1, y_1, k_1), T_b^C, t, p_{mb}, r^C, H_m, q_b\right].$$

[21] See the remark on page 17.

[22] The second and third equilibrium conditions are essential to the proof.

[23] The third equilibrium condition is essential to the proof.

In Lemma 4 we have provided the proof that if $W_{mc}^{C}\left[(b,y,k)_{m},T_{c}^{C},t_{m}^{C},p_{mc},r^{C},H_{m},q_{c}\right]>W_{ma}^{C}\left[(b,y,k)_{m},T_{a}^{C},0,p_{ma},r^{C},H_{m},q_{a}\right]$ is true,[24] all students with $(b_{i},y_{i},k_{i})<(b,y,k)_{m}$ have the same behavior, such as:

$W_{1c}^{C}\left[(b_{1},y_{1},k_{1}),T_{c}^{C},t_{m}^{C},p_{1c},r^{C},H_{1},q_{c}\right]>W_{1b}^{C}\left[(b_{1},y_{1},k_{1}),T_{b}^{C},0,p_{1b},r^{C},H_{1},q_{b}\right]$. Student 1 prefers a positive tax rate in a public school over a zero tax rate in a private school or a school (private or mixed finance) outside his community.

Thus, student 1 prefers the tax rate t_{m}^{C} over the interval $\left[0,\hat{t}_{1}^{C}\right]$.

2- Over the interval $\left[\hat{t}_{1}^{C},\hat{t}_{m}^{C}\right]$.

Indirect utility is increasing in the tax rate over this interval; t_{m}^{C} is higher than any other tax rate $t\in\left[\hat{t}_{1}^{C},\hat{t}_{m}^{C}\right]$, thus individual 1 prefers t_{m}^{C} over any other tax rate.

By Lemma 4 and proposition 2, we find that there does not exist a tax rate different than t_{m}^{C} that is preferred by more than 50% of the population. t_{m}^{C} is a majority voting equilibrium.

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[24] The median voter prefers a positive tax rate in a public school over a zero tax rate in a private school or a school (private or mixed finance) outside his community.

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