On the Evolution of Time Preference

Arthur Robson*
Simon Fraser University

Balázs Szentes†
University of Chicago

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Abstract

We reexamine Rogers' (1994) analysis of the biological basis of the rate of time preference. Although his basic insight concerning the derivation of the felicity function holds up, the functional form he uses does not generate the evolutionary equilibrium behavior. Moreover, Rogers relies upon an interior solution for a particular kind of intergenerational transfer. We show such interior solutions do not generally arise. Hence Rogers most striking prediction, namely that the real interest rate should be about 2% per annum, does not follow.

Key words: Biological evolution, rate of time preference, intergenerational transfers.

1 Introduction

Why do individuals have the preferences they have? Why are they as risk-averse as they are or as impatient? Although these questions have traditionally been claimed to be inappropriate to address within economics, recent work has realized some of the potential of evolutionary biology and anthropology to provide illuminating answers.

One such path-breaking contribution is due to Rogers (1994), who examines the biological underpinnings of intertemporal preferences and of the rate of time preference, in particular. Rogers' argument can be paraphrased as follows. Consider the evolution of a sexually reproducing species in which resource availability plays a key role. Each individual uses resources to increase the probability of survival until the next period, but, as a

*Department of Economics, 8888 University Drive, Burnaby, V56 1S6, CA. Email: robson@sfu.ca.
Robson thanks the Canada Research Chairs program for support.
†Department of Economics, 1126 E. 59th St, Chicago, IL 60637. Email: szentes@uchicago.edu.

simplifying assumption, has fixed fertility at every age at which he or she is alive. Furthermore, each parent can save resources for the ultimate benefit of an offspring. Rogers supposes that savings behavior is controlled genetically, and looks for the evolutionary equilibrium level of such savings. He considers how individuals might be motivated to choose the equilibrium level of saving by means of the appropriate interdependent utility functions. These interdependent utilities are then those that would be generated by the evolutionary process.

Rogers considers that a key option is for a mother to make a "same-age transfer," that is, to save resources to benefit her daughter in the future, at a date when her daughter will be the same age as the mother is now. Since the daughter has more remaining reproductive life than her mother will at that future date, it may be advantageous for the mother to make this transfer rather than to keep the repayment for herself. This advantage is offset by the fact that the daughter is only a half-relative of her mother and by population growth. When considering the tradeoff between resources now for the mother and resources in the future for the daughter, it is only the degree of relatedness of one half and the population growth rate that matters, since the mother's biological value now and that of her daughter later are the same. Rogers assumes there is no growth in income, and that the rate of interest then equals the pure rate of time preference. With zero population growth, the overall discount factor due to time preference must then equal a half, and this is also the overall discount factor due to interest. Given a reasonable estimate of the length of a generation, he then derives a plausible estimate of annual rate of pure time preference, and of the annual real rate of interest, in the neighborhood of 2%.

Although we believe strongly in the basic biological approach adopted here, there are serious problems with Rogers' model. Altogether, these problems make much of his argument invalid as it stands. In particular, his prediction concerning the real rate of interest does not follow. Rogers' paper is frequently cited within the small but growing research field that attempts to provide a biological basis for economic phenomena. Since it is a keystone paper in this area, it is crucial to get the analysis correct. We illustrate the problems with Rogers' approach by means of an example that permits some insight into the true properties of such a model.

Perhaps Rogers' most important theoretical claim is that the utility function of an individual can be derived from "reproductive value." The overall biological success (or "inclusive fitness") of an individual must allow for the reproductive value of descendants, as well, so this utility function is interdependent. Rogers uses a notion of reproductive

value due to R.A. Fisher, which is the expected future offspring of an individual, discounted by the rate of growth of population, and conditioning on the age of the individual. However, we show that this particular notion is inappropriate here, since it presumes that all offspring are identical, which cannot be true when the mother makes resource transfers to her offspring that vary with her age. The preferences identified by Rogers do not generate evolutionarily appropriate behavior.

In analyzing our example, that is, we need a more general notion of reproductive value. This general notion allows for different types of individual, and reflects the ultimate representation of each type's descendants in the overall population. The type of an individual here is the age of his or her mother at birth. Such types must be distinguished since offspring with larger transfers are at a biological advantage and hence have a higher reproductive value than do their poorer siblings. The evolutionary equilibrium choices are supported by utility functions that are the total discounted general reproductive values of all the individuals who are affected by the choice.

In our example, furthermore, there will generally not be an interior optimum for a "same-age transfer." The example is constructed so as to produce an incentive for transfers and saving in favor of offspring. However, the optimal transfer that is positive is consumed when the daughter will be younger than her mother is now. The associated MRS does not reflect only the degree of relatedness and the population growth rate, but depends also on the underlying survival functions. Even though these survival functions are assumed to have infinite derivatives at zero, corner solutions are bound to arise for same-age transfers. The underlying reason for this is a robust one that is not particular to this example but derives from the biological assessment of the net effect of a transfer when the donor and related recipient have the same age. Since Rogers requires an interior solution to the same age transfer problem for his prediction of the long term interest rate, this prediction is not generally valid.

2 The Example

Individuals are born at age 0 and live to at most to age 3. Let $P_i(c_i)$ denote the survival probability from age i to age $i+1$, when consumption is c_i, for $i = 0, 1, 2$. In particular, it is assumed that $P_0 \equiv 1$, so that individuals surely survive until the age of one.[1] The

[1] Rogers does not consider the possibility that an intended recipient dies before receiving a transfer. The mortality rate is not then incorporated in the rate of time preference in Rogers, in contrast to Irving

functions P_1 and P_2 are assumed to be continuously differentiable, strictly increasing, strictly concave, and to have infinite derivatives at 0. This last condition favors transfers and savings.

At each age, $i = 1, 2, 3$, each individual is paired with another individual of the same age and the couple has two offspring. The transfer decisions is controlled by a single gene, with each possible choice implemented by a corresponding variety (or allele) of that gene. Sexual reproduction means that each offspring acquires either the paternal allele or the maternal allele, each with probability 1/2.[2]

Individuals have zero endowment at ages $i = 0, 1, 3$ but have one unit of endowment at age 2.[3] At age 2, the individual can transfer part of her endowment, in the total amount s, to her two newborn offspring. The transfer from each parent is shared equally between each of the two offspring, since it is not possible for a parent to distinguish between mutant and non-mutant offspring. The new born offspring will choose to save this transfer, since survival for one period is guaranteed.

The technology for intertemporal transformation is linear, with an exogenous interest factor of R. It follows that each offspring then receives $Rs/2$ at age of one, as the return from a particular parent's transfer, which they consume at once.

The question is: What value of s would arise in equilibrium? Consider a population with an allele that selects savings $\bar{s}$. A small proportion of the population is then replaced by a *mutant* allele that saves s. The question becomes: For what value of $\bar{s}$ is it true that no matter what s is, the mutant allele saving s grows no faster than the original allele that saves $\bar{s}$?

Fisher (1930, II.IV.60-62), and to the present paper.

[2] The assumption that individuals are characterized by a single allele is that individuals are "haploid," with a single locus. This is for simplicity, since, in reality, humans are diploid, having two alleles at each of a large number of position or "loci." Sexual reproduction actually entails each of two individuals randomly making a selection of one allele from each locus, and then contributing that to the new individual.

[3] Perhaps a more realistic pattern of fertility and income would reverse the situation of three year olds, giving these income but zero fertility. This would reduce savings by the new born offspring of two year olds, since the income of three year olds could serve instead. Perhaps the survival of two year olds to age three is a biological form of saving. The present assumption produces a central need for savings, which illuminates the issues that arise in Rogers model.

2.1 Transfers to Newborns

To address this question, consider the following survival probabilities. The survival probability of a mutant two-year-old who saved s is given by $P_2(1-s)$. The probability of survival of a mutant (or any) one year old who had one or three year old parents, and hence got zero transfer is simply given by $P_1(0) = P_1^1$, say.

What is the probability of survival of a mutant one year old who had two year old parents? Since the mutant gene is rare, essentially all mutant individuals have one mutant and one non-mutant parent. Recall that the total resources passed down by the two parents, $\bar{s}+s$, are shared equally between their two offspring. Hence the survival probability of the one year old mutant is $P_1(R\frac{\bar{s}+s}{2}) = P_1^2$, say, because the total return on savings is $(R\frac{\bar{s}+s}{2})$.

Suppose then that $\mu_i \geq 0$, for $i = 1, 2, 3$, is the proportion of $i-$year-olds in the mutant gene pool, ignoring newborns, so that $\sum_{i=1}^{3} \mu_i = 1$. These proportions need not match those in the general population. Thus $F = \left[(\mu_1 + \mu_3) P_1^1 + \mu_2 P_1^2\right]$ is the overall probability of survival of a mutant individual from age one to two.

The expected number of mutant offspring of each mutant parent is one, when mutants are rare. It follows that, if y_t is the number of one year old mutant individuals at date t, then:

$$y_t = y_{t-1} + y_{t-2}F + y_{t-3}P_2F, \tag{1}$$

This difference equation generates steady state growth in the limit. (Robson and Szentes (2007), provide more details.) That is, the mutant population converges to steady state growth with growth factor g satisfying

$$g^3 = g^2 + (g + P_2)F. \tag{2}$$

It follows that[4]

$$\mu_1 = \frac{1}{g},\ \mu_3 = \mu_2\frac{P_2}{g},\ \text{and } \mu_1 + \mu_2 + \mu_3 = 1,$$

so the steady state population proportions are given by

$$\mu_1 = \frac{1}{g};\ \mu_2 = \frac{g-1}{g+P_2},\ \text{and } \mu_3 = \frac{P_2}{g}\frac{g-1}{g+P_2}. \tag{3}$$

[4] Consider the second equation, for example. Suppose that the total population of ages 1, 2, and 3 last period is normalized to unity. It follows that $\mu_2 P_2$ is then the number of two year olds who survive to age three. Given steady state growth, this must also equal the number of three year olds from last period times the growth rate, $\mu_3 g$.

Upon substituting these values for the μ_i's into (2), we obtain the following equation implicitly determining the growth factor of the mutant population:

$$g^3 = g^2 + (1 + P_2)\, P_1^1 + (g - 1)\, P_1^2. \tag{4}$$

It follows that g as determined by this equation is maximized if and only if the right hand side is maximized for parametric choice of g. There must be a unique interior solution for this maximization problem, with first-order condition:

$$P_2'\,(1 - s)\, P_1^1 = \frac{P_1'(R\frac{\bar{s}+s}{2})\,(g - 1)\, R}{2}. \tag{5}$$

This characterizes the choice of s that yields the fastest growth rate of a small fraction of mutant alleles, where the rest of the population chooses $\bar{s}$. The right hand side of (4) is strictly concave in s. In addition, if $s = \bar{s}$, then the growth rate of the mutant alleles must match that of the original type. Hence, if the maximum of (4) occurs anywhere except at $s = \bar{s}$, the mutants can invade in the sense of initially growing faster than the original population. Conversely, if the maximum is at $s = \bar{s}$, then any nontrivial mutant with $s \neq \bar{s}$ will be strictly outdone by the original allele.

Altogether, that is, the evolutionary equilibrium value of savings and the growth factor satisfy

$$P_2'\,(1 - \bar{s})\, P_1(0) = \frac{P_1'(R\bar{s})\,(g - 1)\, R}{2}, \tag{6}$$

where $g^3 = g^2 + (1 + P_2\,(1 - \bar{s}))\, P_1(0) + (g - 1)\, P_1(R\bar{s})$.

It is clear that (6) has no simple interpretation in terms of a pure rate of time preference, since it can be written

$$\frac{2P_2'\,(1 - \bar{s})\, P_1(0)}{P_1'(R\bar{s})\,(g - 1)} = R,$$

where the left hand side is an appropriate marginal rate of substitution, MRS. However, this MRS depends in a complex way on the various survival functions, and is not simply the factor of $2g$ that is derived by Rogers.

The factor of 2 arises here from the assumption that transfers must be made blindly and therefore equally to each of the two offspring, one of whom is a fellow mutant, in expectation, but one of whom is not. This is a particular derivation of a general result known as Hamilton's rule. (See Bergstrom, 1995, for a discussion of this rule and the limits to its applicability.)

2.2 Reproductive Value

Next, we argue that the equilibrium saving behavior is a solution to a problem where the two-year old individual maximizes the *total discounted reproductive value* of those who are affected by her savings. Indeed, this total reproductive value is the utility function yielding equilibrium behavior.

In general, the reproductive values should be defined as the relative shares attained in the population, in the limit as the time into the future tends to infinity. (See Robson and Szentes, 2007.) In the present example, a heuristic approach to finding the correct reproductive values is as follows. Let $v(0_1)$ and $v(1_1)$ denote the reproductive values of individuals of ages 0 and 1, respectively, who receive no transfer; and $v(0_2)$ and $v(1_2)$ as the reproductive values of individuals of ages 0 and 1, respectively, who receive transfer $(\frac{\bar{s}+s}{2})$. In addition, let $v(2)$ and $v(3)$ denote the reproductive values of individuals of age two and three, respectively. Considering the descendants of each type one period ahead, these values must satisfy the recursive relationships

$$\begin{aligned} v(0_1) &= \frac{v(1_1)}{g}; v(0_2) = \frac{v(1_2)}{g} \\ v(1_1) &= v(0_1) + \frac{P_1^1 v(2)}{g}; v(1_2) = v(0_1) + \frac{P_1^2 v(2)}{g} \\ v(2) &= v(0_2) + \frac{P_2 v(3)}{g}; v(3) = v(0_1), \end{aligned} \tag{7}$$

Let us explain the third equality, yielding $v(1_1)$, the value of a one year old who had one or three year old parents, for example. Such a one year old produces an offspring with value $v(0_1)$. In addition, this one year old survives to the next period with probability P_1^1, when her reproductive value is $v(2)$. This expected reproductive value from the next period must finally be discounted by g because population growth deflates the evolutionary importance of any given number of individuals.

We now show that the equilibrium behavior can be derived from each two-year old individual solving the following problem

$$\max_s P_2(1-s)v(3) + \frac{P_1(R^{\frac{\bar{s}+s}{2}})v(2)}{g}. \tag{8}$$

The maximand is the relevant component of intertemporal preferences for a two year old parent. It is the expected reproductive value of the parent, as influenced by the choice of s, plus the expected discounted reproductive value of the age one recipient, also as influenced

by the choice of s.[5] Thus these reproductive values generate the utility that underpins the evolutionary equilibrium. Rogers must be given credit for this basic insight. Our disagreement stems from the appropriate notion of reproductive value to be used in such an expression. We first show that our general notion of reproductive value does generate equilibrium behavior; the next section shows that the simple notion used by Rogers does not.

A comparison of (8) and (6) implies that what is necessary and sufficient for these problems to be identical is that

$$\frac{v(3)}{v(2)} = \frac{P_1^1}{g(g-1)}. \tag{9}$$

But it is a straightforward calculation to show that the equations in (7) pin down all *relative* reproductive values and that (9) holds, in particular, as required.

2.3 Rogers/Fisher Reproductive Value

The notion of reproductive value used by Rogers is that due to R.A. Fisher. This notion, which is appropriate in models where offspring are identical, simply counts the future expected discounted total fertility of an individual at each age. In the present example, this would mean, using the notation w instead of v:

$$\begin{aligned} w(0_1) &= \frac{w(1_1)}{g} = \frac{1}{g} + \frac{P_1^1}{g^2} + \frac{P_1^1 P_2}{g^3}; w(0_2) = \frac{w(1_2)}{g} = \frac{1}{g} + \frac{P_1^2}{g^2} + \frac{P_1^2 P_2}{g^3}; \\ w(2) &= 1 + \frac{P_2}{g}; w(3) = 1. \end{aligned}$$

This implies that

$$\frac{w(3)}{w(2)} = \frac{g}{g+P_2} > \frac{v(3)}{v(2)} = \frac{P_1^1}{g(g-1)}$$

since, using (4),

$$g^3 - g^2 - P_1^1 g - P_1^1 P_2 = (P_1^2 - P_1^1)(g-1) > 0.$$

Thus the evolutionary equilibrium is not the solution to

$$\max_s P_2(1-s)w(3) + \frac{P_1(R^{\frac{\bar{s}+s}{2}})w(2)}{2g};$$

indeed, the solution to the problem as formulated by Rogers entails too little savings. The problem is that Fisher's notion of reproductive value is inappropriate when offspring are

[5] Although there are two offspring, each of them inherits the parent's gene with probability half. The second expression then reflects the expected number of mutant offspring.

distinguished in any important way, perhaps by their inheritance, as is true here. Thus Rogers' formulation of the utility functions that support the equilibrium level of savings is incorrect on this account.

2.4 Zero Same Age Transfer

The previous analysis assumed that the only transfer possible was from a two-year-old parent to her current newborn. Suppose now that this two-year-old parent can also put aside resources for her current one-year-old offspring. These resources must be saved for one period, and are then given to the offspring. Thus, this offspring will be the same age when she consumes the transfer as her mother is when she gives it. This combination is then a same-age transfer, as in Rogers.

We assume that there exists a competitive market for these savings by current one-year-old offspring. This market returns a fixed interest factor, R'. Since only a proportion of P_1^1 of the intended recipients survives until the age of two, and the market is competitive, $R' = R/P_1^1$.[6]

We now show that, in any evolutionary equilibrium, zero additional transfers like this will be made, given that $R \leq 2g$. Robson and Szentes (2007) show that this additional transfer is zero in equilibrium if and only the following applies for each two year old parent—

$$0 \in \arg\max_{s'} V(s') \equiv P_2 \left(1 - \bar{s} - s'\right) v\left(3\right) + \frac{P_1^1 P_2 (1 - \bar{s} + R' \frac{s'}{2}) v(3)}{g}.$$

The function V, from a biological viewpoint, is "inclusive fitness"; from an economic viewpoint, it is the utility function that underpins behavior. That is, it is the relevant part of intertemporal preferences for this two-year parent contemplating transfers to her current one year old offspring. The first term is the individual's own expected survival probability to age three multiplied by her age-three reproductive value. The second term is the survival probability of the current one year old offspring from age one to age three multiplied by the present value of her age-three reproductive value.

[6] Nothing crucial depends on this specific assumption. A similar conclusion is valid even if we assume instead that the saving is lost.

Although Rogers does not explicitly address such issues, they also arise in his model. He is simply silent as to the fate of savings made by an individual when this individual dies before repayment can be made. As noted previously, mortality does not then influence the rate of time preference, in Rogers' model.

Since $R \leq 2g$, it follows that

$$\begin{aligned} V'(0) &= -P_2'(1-\bar{s})v(3) + \frac{R'P_1^1 P_2'(1-\bar{s})\,v(3)}{2g} \\ &= \left[\frac{R}{2g} - 1\right] P_2'(1-\bar{s})\,v(3) \leq 0, \end{aligned}$$

so $s' = 0$ is an equilibrium transfer. A similar analysis shows that, if $R > 2g$, then there again cannot be an interior solution for s'. The only possibility for an interior solution is for the non-generic case where $R = 2g$.

The intuitive reason that an interior solution does not generally exist for the same age transfer is that the evolutionary criterion considers the net income position of all two-year-olds. Such transfers appear both as a debit, and as a credit, the latter multiplied by the appropriate interest and growth factor, in the single argument of the same survival function.

Note that, although the growth rate, g, is endogenous, it must generally differ from $R/2$. For example, here it must be that $g \geq \underline{g}$ for some $\underline{g} > 1$, since all offspring survive to be one year old and have one offspring at that age, and have a positive probability of having further offspring. If the exogenous parameter $R \leq 2\underline{g}$, then $R < 2g$.

The discussion so far was based on a linear intertemporal technology. But even if this technology were strictly convex, so that corner solutions for total savings could not arise, it is clearly possible that the endogenous interest rate would be determined by the strictly positive savings of new born offspring of two year old parents. It could well then still satisfy $R \leq 2g$, and still choke off the "same age transfer." There is then no robust prediction here concerning the real rate of interest, contrary to Rogers claim.

2.5 Zero Population Growth

The Rogers approach involves assuming zero population growth, on the basis of the argument that the average growth rate for humans must have been close to zero over the two million years of our history. Our example entails a growth rate necessarily greater than zero, since all individuals survive to age one, where each couple produces two offspring, and each individual has a positive probability of producing more offspring after that. But nothing crucial depends on a positive growth rate.

In order to generate an example with zero population growth, all that is needed is to deflate all probabilities of survival by a factor of g, where g is the maximum growth factor

as found above. That is define

$$\bar{P}_i(c_i) = P_i(c_i)/g, \text{ for } i = 0, ..., 3.$$

Suppose that the interest factor R facing an individual is as before. To achieve this, the underlying technological interest factor must also be scaled down by g. That is, if this underlying factor is $\bar{R}$, then $R = \bar{R}/g$, so that $\bar{R} = Rg$.

The above analysis applies to this modified example with minor reinterpretation. That is, the same levels of transfers and savings yield an evolutionary equilibrium, but the equilibrium growth factor is now one. Rogers' analysis is still subject to the same criticisms, and there is no robust prediction on the real rate of interest, for example.

2.6 A Generalized Example

One might be concerned that our conclusions hold only because we restricted the possible transfers. Indeed, in a generalized model, the basic philosophy might be that all individuals may transfer any amount contemporaneously to any living relative, and all individuals may save any amount. Individuals cannot borrow, however. This is consistent with a stylized version of a primitive agricultural society in which savings can be made by means of reinvesting the harvest, but where borrowing is circumscribed.

In Robson and Szentes (2007) we reanalyze the present example in this light. Although the analysis is more complex, the essential points made by the above version of the example are preserved.[7] In addition to transferring resources to their new born offspring, two year old parents now make positive transfers to their one year offspring, who immediately consume it. Two year old parents also save for one period, with the return from this being transferred to the new born offspring they have at age three. These offspring reinvest this transfer and finally consume it at age one. It remains inappropriate to use the simple notion of reproductive value used by Rogers. Furthermore, all of the other myriad contemporaneous transfers and savings that might be made in principle in this generalized example may well be chosen to be zero in equilibrium. Most significantly, that

[7] One of the complications is as follows. Age two parents have a strong incentive to transfer resources to their current one year old offspring, if this is consumed immediately. However, such transfers imply a proliferation of types of such offspring. There are those with no surviving parent, those with one surviving parent, and those with two. In addition, the transfers made by one parent might be contingent on whether the other parent is still alive. This complexity seems entirely tangential to the current purposes, however.

is, there are now a number of combined transfer and savings options that amount to same age transfers, and all of these are optimally chosen to be zero.

3 Conclusion

Although we are enthusiastic about Rogers basic biological approach to deriving the rate of time preference, a number of aspects of his model do not withstand careful examination. Relatively minor issues include his neglect of mortality when considering intertemporal choice. Mortality is not then a component of the rate of time preference. There are perhaps two key aspects of his model that do not hold up. The first of these is that the reproductive value he uses is not general enough for the present circumstances. The expression he gives for the utility function is correct *only* in its basic additive form. The second issue is that there will not generally be interior solutions for "same age transfers." Although the one half degree of relatedness between mothers and daughters does play a role in determining the marginal rates of substitution between consumption now and consumption in the future, the theoretical situation here is more complex than described by Rogers. The most striking conclusion of Rogers—a particular real rate of interest—does not generally follow.

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