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Biography

Richard Cowan is Professor of Psychology of Education at the UCL Institute of Education. He is a developmental psychologist with a particular interest in young children's number development before and during primary school. He has researched aspects of both typical and atypical mathematical cognition, including calendrical calculation in children and adults.

In addition to postgraduate teaching on UCL IOE programmes, he has been involved in the consortium (UCL Institute of Education, King's College London, and Roehampton University) that provided the higher education component of the Mathematics Specialist Teacher scheme to primary schools in allocated local authorities.

Number skills in primary school: Development and individual differences

Primary school is where most children develop the number knowledge and skills that provide the basis for subsequent educational achievement and successful management of the numeracy demands of everyday life. From the start of universal compulsory schooling, there has been dissatisfaction with the outcomes of primary education (e.g., McIntosh, 1981), dispute about the factors that have most influence on children's development, and disagreement about why some children make much slower progress (McDermott, 1993).

Successive governments have modified the curriculum, introduced attainment targets, and held primary schools publicly accountable with the results of national assessments. The data from these assessments have been used in debates about the effectiveness of primary education and the need for further policy development. Enormous amounts of public money have been spent on efforts to improve primary education since the 1990s (Burr, 2008).

Current attainments are still seen as unsatisfactory. Department for Education statistics for 2012 indicate that over 20 per cent of children failed to achieve the expected level (4) in both English and Maths at the end of Key Stage 2 (KS2). This causes concern because almost all children (97 per cent) who do not reach Level 4 at KS2 also subsequently fail to obtain passes at Grade C or higher in GCSE Mathematics and English (Burr, 2008). Level 4 is now seen as too low an attainment target: less than half (47 per cent) of children with the lowest Level 4s in primary school (Level 4c, approximately a third of those achieving Level 4), subsequently achieved five A*–C GCSEs at 16 in 2012. In contrast most children (72 per cent) with better KS2 results (> Level 4c in both English and Mathematics) did achieve five A*–C GCSEs (Department for Education, 2013b). The government is responding by introducing a new primary curriculum, changing the system of assessment, and incorporating baseline measures to provide parents with information about the amount of progress their child has made.

Another cause of public concern about primary mathematics is England's standing in international surveys such as the Trends in International Mathematics and Science Study (TIMSS). There are many reasons to be cautious about interpreting the results of these surveys (Brown, 1998; Smithers, 2013). Despite warnings by their producers that they do not provide a basis for deciding how to teach (OECD, 2009), the consistently superior performance by children from East Asian states such as Hong Kong, Japan, Singapore, South Korea, and Taiwan has led several to believe that we can learn from these places.

Research that has attempted to understand the superior mathematical skills of East Asian youngsters has thrown up many differences (Cowan and Saxton, 2010): East Asian children start learning number skills earlier; they spend more time learning number skills in and out of school; they are more likely to say they enjoy doing maths at home; they are much more likely to receive out of school tutoring; their motivation to succeed increases during primary school; their parents are more likely to value educational achievement highly and less likely to believe that maths ability is determined by nature. All of these are readily understood by Western audiences as likely to facilitate number development.

Some East Asian practices are more controversial: Japanese schools do not practise ability grouping (National Research Council, 1999). There are certainly some Western mathematics educators who oppose ability grouping (e.g., Boaler, 1997) but current practice does not respect them: experience of setting or streaming is the norm for English primary school children (Campbell, 2013; Hallam and Parsons, 2013).

East Asian children are taught in larger classes and have lower self-esteem than US children. Some might feel uncomfortable about arguing that these features are the cause of East Asian superiority. International surveys provide no more justification for believing that the differences we find easy to accept are responsible for East Asian superiority than the differences we doubt. Despite this, they have inspired efforts to imitate their curricula: Singapore maths has a good brand image.

Simply copying the cultural practices of another country in the belief that it will make our children more like them can be little more than magical thinking. It is no better than wearing the same brand of football boots as David Beckham in the belief that it will make one a better player. The problematic nature of such cultural borrowing has long been discussed in comparative education (Holmes, 1981) and developmental psychology (Hatano, 1990).

The study of children growing up in other contexts yields ideas to be discussed and hypotheses to be tested rather than 'lessons to be learned'.

What is often overlooked in discussions about national differences in educational achievement is the extraordinary variation between children in the same country (Cockcroft, 1982). This is consistently greater than the differences between countries: every country's distribution of fourth grade maths scores in TIMSS 2007 overlapped with the others (Mullis *et al*., 2009). All 36 participating countries had children who failed to achieve even the lowest benchmark and all except four had children who achieved the most advanced benchmark. Although some within-country variation is attributable to differences between schools, this only seems to play a small part.

The largest source of variation between children is within school (Goldhaber *et al*., 2010; Gutman and Feinstein, 2008). A vivid illustration of this is provided by data from an ESRC-funded project on the development and importance of proficiency in basic calculation (RES-062-23-0667). In Year 4, when most children become 9 years old, they were assessed using the Wechsler Individual Achievement Test (WIAT-II UK, Wechsler, 2005). The WIAT consists of two subtests, Numerical Operations and Mathematical Reasoning. Numerical Operations largely assesses written computation skills initially with integers and later involving fractions, decimals, and percentages. In administering the Mathematical Reasoning subtest the tester reads aloud problems accompanied by illustrations and text. They require simple arithmetical skills and the ability to interpret charts. Table 1 shows the variation in standard scores, raw scores adjusted for chronological age with a mean of 100 and a standard deviation of 15, in each class. According to the manual, standard scores above 76 are achieved by 95 per cent of UK children of that age. Standard scores above 125 are achieved by less than 5 per cent. All classes show substantial variation and this is even greater if the children who were on the schools' Special Educational Needs registers are included.

Researchers reporting the effects of interventions often use age equivalents derived from scores on standardized tests (e.g., Brooks, 2013). Although misleading if interpreted literally (Wechsler, 2005), age equivalents can provide a dramatic illustration of the differences between children. For example, Cockcroft (1982) described a 'seven year difference' at 11 in mathematical skills. Table 1 also shows the range in each class obtained by subtracting the lowest age equivalent from the highest. Given that the children in our project were to spend another two years in primary school, and that differences between children increase with age rather than decrease, the results correspond well to Cockcroft (1982).

Table 1: Variation in standard scores and the age equivalent ranges on WIAT Numerical Operations and Mathematical Reasoning in each class when the children were in Year 4.

Class	Numerical Operations		Mathematical Reasoning	
	Standard scores	Age equivalent ranges (years)	Standard scores	Age equivalent ranges (years)
A	74–140	4.7	82–138	5.5
B	89–133	2.7	91–130	7.7
C	78–139	4.7	76–124	6.7
D	82–149	8.7	91–128	5.7
E	85–147	6.0	79–137	8.7
F	78–149	5.0	68–138	6.7
G	81–147	4.7	91–138	8.7
H	65–142	7.0	76–130	6.3
I	81–130	6.7	70–124	4.3

Note. *N* = 212. Children on schools' registers of Special Education Needs are excluded. The number of children included in each class varies from 11 to 37.

Wechsler, 2005

In what follows I am going to consider why children might differ so much in their number development. Acknowledgement of individual differences in educational progress and ideas about what cause them go back to the beginning of education. Some ideas persist despite advances in knowledge and evidence that discredit them. Consider the headline 'Twins hold key to unravelling maths gene' and the first two sentences of an article that appeared in *The Observer* in 2005: 'Parents have long battled to persuade their children to master new spellings and learn their tables, but they may be wasting their time. A new study suggests that both maths and reading ability lies largely in the genes' (Revill, 2005).

This is just wrong on so many levels. The Twins Early Development Study (TEDS) researchers whose study this describes would not agree to this portrayal of their work. First, there is no 'maths gene': there are many genes that contribute to individual differences in mathematics. Second, there is no single 'maths ability' or 'reading ability'. There are lots of different literacy and numeracy abilities that children develop as they progress through primary school. Third, even if genetic variation explained a lot of individual variation, this does not rule out the importance of the environment. Differences in height owe much more to genes than differences in maths do, but children do not grow without food.

TEDS includes thousands of twins: some are identical and some are not. Identical twins are clones but they defy the popular myths and fears about clones (remember Dolly the sheep). In reality, identical twins do not always do the same job, marry at the same age or give their dogs the same name. In reality, if one identical twin scores very poorly on a maths test, then it is likely but by no means certain that their co-twin will do similarly (Plomin and Kovas, 2005). Genes do not determine destiny.

Many human psychological characteristics show moderate to substantial heritability but twin studies are only part of a substantial scientific journey to understanding how genes influence cognition (Fisher, 2006). Nevertheless twin studies have already produced findings that challenge long-held beliefs such as the belief that intelligence test scores are more affected by genetic influences than educational achievement (e.g., Great Britain Board of Education Consultative Committee, 1924; also known as the Hadow Report). The TEDS data indicate heritability of general cognitive ability at 7, 9, and 10 to be less than 50 per cent and to be the same or less than the heritability of educational achievements in Mathematics, English and Science (Kovas *et al*., 2007). Another assumption questioned by the TEDS data is that genetic influences just account for stability of individual differences. In the TEDS data, genetic influences account for both stability and change even within the primary years.

Another popular belief is that early experience is more important than later experience (Clarke and Clarke, 1979). Neuroscience has been wrongly recruited as support for claims that the first three years of life are particularly important for children's brain development and hence their outcomes in later life (Bruer, 1999). Now it is true that differences between children before they start school do predict later differences: this has been found in large scale correlational studies in both the UK and the US (Duncan *et al*., 2007). But there really is no basis for thinking that preschool individual differences in number skills are more stable or less reversible than differences that appear later (Howard-Jones *et al*., 2012; Knowland and Thomas, 2014). Indeed correlational studies indicate that later primary assessments are substantially better predictors of subsequent outcomes than school entry characteristics (Feinstein and Bynner, 2004).

Understanding both the continuity and the discontinuity between early and later individual differences remains an important scientific challenge. Correlational data from large cohort studies indicate associations with socio-economic factors (e.g., Feinstein, 2003). This is credited with having had a major impact on government policy but it is not clear what the interpretation should be: Jerrim and Vignoles (2013) identify problems that are inherent in large-scale studies: for example the cost of collecting information from a very large sample leads to the reliance on a small number of tests at different ages. They also suggest that the use of very different tests at different ages confounds the problem of regression to the mean. Both the 'recovery' of children who score below average initially and the 'decline' in groups who are initially above average may reflect the merely statistical phenomenon of regression to the mean, the inevitable consequence of imperfect correlation (Campbell and Kenny, 1999).

So the challenge is to understand not just why children differ at a particular time, but also what explains the relation between individual differences at different times. In what follows I am going to consider the roles played by within-child factors and characteristics of the home and school contexts in which the children develop. The within-child factors include general cognitive skills that are supposed to influence learning in many domains, not just number. I shall also consider core number skills that have been imagined to be particularly important for mathematical development in primary school. Self-beliefs are what the child thinks about their interests, their motivations, and their abilities. Socio-emotional functioning is how children manage the emotions associated with social situations such as classroom learning and peer relationships.

Within-child characteristics: General cognitive skills

In the past general cognitive skills have been studied mainly as a composite called intelligence. Indeed Colin Hindley, who was Professor of Child Development when I joined the Institute of Education, had directed a substantial longitudinal project examining stability and change in children's performance on omnibus intelligence tests: omnibus intelligence tests combine scores on separate subtests of cognitive skills to derive composite IQ scores. What this research clearly illustrated was how individual variability accompanies a substantial group correlation (Hindley and Owen, 1978). Although the combination of substantial group predictability with individual variability has been often pointed out (Rutter, 1980) it still surprises many. A recent study of teenagers found a correlation of .79 between their IQ scores assessed on two different occasions. This is consistent with over a third of the sample showing substantial change (Ramsden *et al*., 2011).

What intelligence tests measure has long been a matter of controversy. Spearman deliberately avoided identifying *g*, the factor he identified as common to performance on different tests, with intelligence (Spearman, 1927). All tests measure skills that are developed (Anastasi, 1984) and Binet's warning that tests at best provide a measure of current functioning has often been disregarded by those who want them to measure a constant characteristic or provide a basis for streaming or setting. Indeed Dylan Wiliam (pers. comm.) finds even now some secondary head teachers are shocked by the implications of his demonstration of the individual unpredictability that accompanies 'good' psychometric characteristics of reliability.

Oral language

Omnibus intelligence tests often feature measures of oral language, working memory, reasoning, and processing speed. How all of these affect the development of number skills is readily imagined. Language plays a pivotal part in education generally and particularly in number development. Language skills are involved in what children have to learn as well as how they learn it.

Children's first encounters with numbers are through learning to count and master the spoken number system. These are cultural products that are inherently verbal. Although understanding number involves more than knowing the number system, it certainly does not involve less. To become competent in written arithmetic the child needs to master the Hindu-Arabic system for writing numbers. Such mastery typically depends on a grasp of the spoken system and understanding both the connections and disconnections between them.

There is a greater correspondence between spoken and written forms in Japanese, Chinese, and Korean languages: they have fewer basic number words, and do not use conjunctions such as 'and', as English-speakers are required to do following hundreds, for example saying the number '195' as 'one hundred and ninety-five' instead of 'one hundred, nine tens, five'. It has been suggested that the greater transparency of spoken number in these languages contributes to a superior development of number skills by the children who learn them (Miller *et al*., 1995).

Mathematical problem solving is commonly assessed by setting children computational problems in verbal contexts. Clearly the child has to understand the story to select the appropriate computation. Some children's difficulties in understanding the stories masks their arithmetical skills (Cooper and Dunne, 2000). This also applies to some adults: in the history of the study of calendrical calculators – people who can tell you what day a particular date will fall on – it was sometimes claimed that their skills were all the more remarkable because they did not understand arithmetic. The claim of arithmetical incompetence was based on assessments that used story problems. When assessed with pure computational problems they show arithmetical skills and the discrepancies in the two methods of assessment can be substantial (Cowan *et al*., 2003).

So oral language is involved in how children learn, what they learn, and how they are assessed. It is therefore unsurprising that differences in children's linguistic skills are related to differences in their number development (Cowan *et al*., 2005; Durand *et al*., 2005; Lewis *et al*., 1994; Plomin and Kovas, 2005). Despite this, it is common to find that older children and adults see themselves as better at one than the other: a phenomenon that is supposed to reflect internal contrasts (Marsh, 2006).

There is still much to find out about the role and importance of oral language skills. In particular, differentiating further within oral language skills may prove productive. Many have followed intelligence tests in just assessing receptive language skills, but educational progress is likely to depend on the child's expressive language skills too. Individual differences in receptive and expressive language skills cannot be assumed to be the same.

Working memory

Ideas about the role of short-term memory in educational development have evolved considerably from the inclusion of short-term memory tests in intelligence tests. Whereas short-term memory concerns the temporary storage of information, working memory implies both storage and manipulation of information (Baddeley, 2012). Many cognitive tasks involve storage and processing information, including counting a set of objects, mental and written arithmetic, and understanding speech and text. Individual differences in working memory might therefore be expected to be associated with differences in number development.

Baddeley and Hitch (1974) developed a multicomponent model with a central executive component, which works in conjunction with two subsystem components: one concerned with short-term storage of acoustic and verbal information, the phonological loop; and the other with short-term storage of visual and spatial information, the visuo-spatial sketchpad. Omnibus intelligence tests do not assess all these components. The multicomponent model has stimulated much research on individual differences in number skills, including the projects I have been involved with that have investigated number skills in primary school children with specific language impairments (funded by the Nuffield Foundation with Chris Donlan as Principal Investigator) and the development and importance of proficiency in basic calculation (funded by the ESRC with Chris Donlan as co-investigator).

In the project involving children with specific language impairments (SLI), we constructed two groups to compare with our SLI group. One group was matched on chronological age and nonverbal reasoning (Age Control, AC) and the other was matched on receptive grammar (Language Control, LC). Incidentally, consistent with the point made above about the differences between language skills, despite the match on one oral language test, the SLI group were markedly less successful on a test of past tense production that we derived from Marchman *et al*., (1999). They were also much less successful on the Children's Test of Nonword Repetition (CNRep) (Gathercole and Baddeley, 1996). CNRep is a test of phonological memory that requires children to repeat a nonword that they have just heard. It is argued that the use of unfamiliar sequences of phonemes makes this test resemble vocabulary learning much more than the conventional phonological loop tests that require children to repeat sequences of familiar number words (Baddeley, 2003).

The SLI group were less successful than the AC group on tests of every component of working memory and no better than the LC group, despite being two years older. The components of working memory showed different associations with different number tasks. Almost all zero-order correlations indicated at least moderate relationships (greater than .3) but some were considerably higher. After controlling for the associations between the components, the data patterns suggested central executive involvement in most number skills such as generating number sequences, solving story problems, and comparing multi-digit numbers. An exception was fluency in basic calculation, the rapid and correct solution of single digit addition problems (Cowan *et al*., 2005). In our sample, this skill was more associated with visuo-spatial sketchpad functioning. We did not anticipate this and have not replicated it subsequently.

Indeed there are anomalous findings and inconsistent results in research on the relation between working memory functioning and number skills (Raghubar *et al*., 2010). Some may result from the use of different measures to measure the same construct: a review of objective tests of executive functioning found little relation between them (Duckworth and Kern, 2011). It may also have something to do with the way the number skills are assessed: visually presented tests may draw on a different combination of components than purely oral presentations do. It may even be the result of purely chance factors. All of these are worth knowing but not very interesting. More interesting are the possibilities that the variation reflects changes in the importance of working memory components for learning particular skills that are age-related or concerned with how the children are taught. It may be that some ways of teaching number skills favour children with particular memory characteristics. As far as I know, this has not really been studied at all.

Speed of processing

How quickly one performs mental operations has a plausible connection to learning and development of educational skills, from reading comprehension to mental arithmetic. An older terminology described children making less progress as slow learners. Of course most skills develop in speed of execution with practice, but the effects of practice are typically skill-specific: becoming a faster reader is not expected to make one better at mental arithmetic.

Since early psychology there have been speculations about the existence of processing speed characteristics that are more general. In the past, with the discovery of neural conduction, psychologists wondered whether differences in basic neural efficiency might explain individual differences in cognitive skills (Spearman, 1927). No measure of basic processing speed has been found that works well. Habituation is the most basic form of learning. The speed with which infants habituate excited interest for some time with findings that it could predict later child intelligence. However, as Hindley found with standardized tests of children below the age of 5, better predictions are obtained from parental education and socio-economic status (McCall and

Carriger, 1993). Also there are grounds for doubting whether differences in habituation simply reflect differences in processing speed (McCall, 1994).

Contemporary measures of processing speed in omnibus intelligence tests do not claim to measure basic neural efficiency. Instead they require children to decide whether a target symbol appears in a row (Symbol Matching, Wechsler, 1992) or identify instances where a particular pattern appears (Pair Cancellation, Woodcock *et al*., 2001). Although some numerical skills such as basic calculation proficiency require a speeded response, and slower performance of basic calculations is characteristic of primary school children making poorer progress in arithmetic (Geary and Brown, 1991; Jordan and Montani, 1997), it is unclear whether this results from differences in their general processing speed. It may instead be due to a lack of number knowledge, less developed skill in executing computational strategies, or even selection of more time consuming strategies for solving arithmetic problems. Evidence is mixed: some studies find measures of general processing speed to explain differences in arithmetic independently of working memory (Bull and Johnston, 1997), others do not (e.g., Andersson and Lyxell, 2007).

Reasoning

Reasoning is involved in any form of activity that involves combining information to draw conclusions, whether it is in developing vocabulary, understanding speech and text, or solving problems. Deductive reasoning is a special form of reasoning that is particularly important in pure mathematics and logic and other closed systems (Emmet, 1960). The difference between deductive reasoning and inductive reasoning is important. The conclusions that have been validly derived using deductive reasoning follow with logical necessity, whereas the conclusions derived from inductive reasoning do not: that '6 + 3 = 9' is not just an empirical fact, capable of revision when a disconfirming instance is found. Instead it can be deduced with logical necessity from the meanings of the symbols in the statement. Similarly logical necessity attends the transitive inference that 'Jane is taller than Anne' if 'Jane is taller than Mary' and 'Mary is taller than Anne'. It is difficult to tell when children appreciate the certainty that accompanies number facts and transitive inferences.

Interviews with children indicate that their grasp of logical necessity develops considerably during primary school (Markman, 1978; Miller *et al*., 2000; Morris and Sloutsky, 2001). Although over half of a seven-year-old group asserted that 1 + 1 = 2 is true everywhere, could not be changed, and could not be imagined to be different, they also had the same views about social conventions such as wearing shoes at school and eating peas with a fork (Miller *et al*., 2000). It may, however, be quite simple to teach even five-year-olds to appreciate logical necessity (Russell, 1982).

Reasoning problems have long been incorporated in intelligence tests and studied by developmental psychologists inspired by the work of Piaget. What performance shows remains a difficult question. Some problems, including transitive inference problems, can be correctly solved without appreciating logical necessity (Thayer and Collyer, 1978). Conversely, mistakes on reasoning problems do not always show ignorance of logical principles: in explaining why even graduate students fail to reason correctly on statistical problems, Kahneman and Tversky (1982) distinguished between errors of application and errors of comprehension. Errors of application are a failure to apply knowledge or execute procedures that one knows and accepts. Someone can fail a reasoning problem such as the THOG, but understand they made the wrong choice (Wason, 1977). Similarly someone can miscount a set of items while knowing how to count and recognizing the correct count. In contrast, errors of comprehension are made where the solver does not possess the relevant knowledge: they will not be able to understand what is wrong with their solution without further education.

Written tests with verbal reasoning problems can be tricky because they presuppose reading skills, knowledge of relevant vocabulary, and knowledge of conventions. Take for example verbal transitive inference problems such as 'Jane is taller than Mary. Mary is taller than Susan. Who is tallest?' Some children did not know it was the same Mary in both sentences (Donaldson and Withrington, 1963). They do not know the convention that all knowledge necessary to solve the problem is provided and that there would be only one Mary. This makes their failure to identify Jane as the tallest perfectly logical but wrong.

Children's arithmetic skills vary with their skill in reasoning whether this is assessed with mathematical reasoning tasks (Nunes *et al*., 2007) or non-numerical reasoning tasks (Cowan and Powell, 2014). The short form of intelligence tests typically comprises a reasoning task and a receptive vocabulary measure, consistent with Spearman's (1927) identification of both eductive and reproductive components in *g*. Therefore the considerable amount of evidence of associations between intelligence and mathematical skills (Geary *et al*., 2012) may also reflect the importance of reasoning for number development.

Relations between general cognitive skills

Associations between general cognitive skills are considerable (Cowan and Powell, 2014). While they provide justification for psychometric test developers to combine scores on them to derive composite measures they call intelligence, these associations pose theoretical problems such as whether general reasoning ability, as assessed by tests such as Raven's Progressive Matrices, is essentially different from working memory, as assessed by complex span measures (Ackerman *et al*., 2005; Engle *et al*., 1999).

The associations between general cognitive skills make it hazardous to interpret correlations between number skills and one or another general cognitive skill. All general cognitive skills tests make demands on working memory so a correlation between reasoning and number skills may reflect the importance of reasoning or working memory or both. Additionally, variation in reasoning and working memory tests may reflect differences in speed of processing (Coyle *et al*., 2011; Fry and Hale, 2000). For example, counting span is a central executive task that features in standardized tests such as the Children's Working Memory Test Battery (Pickering and Gathercole, 2001). In the counting span task children must count the spots on cards. After they finish one card they have to count another and another and so on. Subsequently they are asked to recall the numbers of spots on each card in the order that they counted them. It is not surprising that the speed with which children count the spots correlates with the number of totals they correctly recall (Hitch *et al*., 2001). This raises further questions such as how much the speed with which children count spots is a specific result of practice in counting, or a reflection of a general processing speed.

Some differentiation between general cognitive skills is possible when studies include separate measures of them. This is what we attempted to do (Cowan and Powell, 2014). Despite the considerable associations between measures of oral language, working memory, reasoning, and processing speed, we found we could discriminate between them and that the contributions they made varied with number skill. Basic calculation proficiency, the rapid and accurate solution of single digit addition problems, varied most with processing speed whereas written arithmetic varied most with reasoning and working memory, and problem solving varied most with oral language, reasoning, and working memory. The amounts of variance uniquely explained by each general cognitive skill were, however, very small: most variance was shared. Also such studies are limited by the quality of the tests they include and the variables they omit. More seriously it should be remembered that these are just correlational studies. They are silent about whether causality is involved, and if so, what causes what.

Causes or consequences: The relation between general cognitive skills and education

While some emphasize the influence of general cognitive skills on educational development (e.g., Fuchs *et al*., 2006), an older view, mental discipline, claims that education influences the development of general cognitive skills (Stanic, 1986). Belief in mental discipline used to be dominant, as depicted by Hesse in his critique of contemporary education:

> In Greek, next to the irregular verbs, the main emphasis was laid on variety of sentence structure expressed through the use of particles, in Latin they were expected to concentrate on clear and precise statements, and become familiar with countless refinements of prosody, in mathematics pride of place was given to complicated problems of arithmetic. None of these things, as his teacher was never tired of repeating, had any apparent value for his later studies, but it was only 'apparently', for in point of fact, they were very important indeed, more important than many main subjects because they developed the logical faculties and formed a basis for all clear, sober and cogent reasoning.
>
> (Hesse, 1973: 8)

The doctrine of mental discipline assumed that transfer of mental skills would be automatic, just as physical exercises to develop particular muscle groups would produce gains that automatically transferred to activities using those muscles. Early psychological research by Thorndike challenged the doctrine of mental discipline in several ways: his experiments on learning indicated that transfer was less common than expected, the limited amount of transfer that did occur could be explained without referring to general faculties, and gains in tests of thinking were not related to subjects studied as predicted by the dominant version of mental discipline theory (Thorndike, 1924a, b).

It is still common to read assertions of the value of doing mathematics at school for general thinking skills: claims such as mathematical training 'disciplines the mind, develops logical and critical reasoning, and develops analytical and problem-solving skills to a high degree' (Smith, 2004: 11) and mathematics 'nurtures both the faculties of logic and reasoning among learners' (Kaur and Vistro-Yu, 2010: 453).

While one can reject the strong version of mental discipline theory, it is still plausible that engagement in school learning has some influence on the development of general cognitive skills (Ceci, 1991): Cahan and Cohen (1989) found that children of the same chronological age differed in their performance on intelligence tests according to the amount of schooling they had received. Indeed the relations between general cognitive skills and educational development may be bidirectional. Recent accounts of development explicitly propose dynamic bidirectional relationships (e.g., Sameroff, 2010).

The years a child spends at primary school are a period of extraordinary development change in general cognitive skills. Standardized tests of reasoning memory, processing speed, and oral language indicate remarkable changes between the ages of five and ten. It is not matched by any change in later years. Little is known about what influences these changes but educational experience is plausible. Correlational studies find that reading affects growth in oral language skills and reasoning (Cain and Oakhill, 2011; Ferrer *et al*., 2007).

The claims of bidirectional causal relationships would be stronger if they were supported by evidence from intervention studies. As yet, there is no good evidence that interventions to train intelligence generally or improve working memory specifically yield any educational benefits (Brody, 1992; Melby-Lervåg and Hulme, 2013). Quite possibly this reflects the theoretical inadequacy of their approach to training. Some are based on the old mental discipline approach to education: they consider cognitive capacities to be like muscles that can be developed through exercise.

Another possibility is suggested by acknowledging that motivation and effort make important contributions to performance on any test. For example, no one is going to score well on a speed of information processing test unless they try to respond quickly and accurately. The importance of motivation and effort on cognitive tests has long been acknowledged. It is difficult to quantify the contribution of motivation to the association between cognitive tests and other measures, but an ingenious attempt has been made by Duckworth *et al*. (2011). It seems reasonable to conclude that individual differences in effort account for some of the variance that is common to general cognitive skills and for some of variance that is shared by general cognitive skills and number skills. If training programmes enhance motivation for doing particular cognitive tests then a lack of transfer may result if motivation is not generally enhanced.

In summary there is considerable correlational evidence of associations between individual differences in general cognitive skills and individual differences in number skills. Plausible accounts can be given both of how general cognitive skills affect the development of number skills and how they affect performance on tests of number skills. But plausible accounts of how mathematical education can affect the development of general cognitive skills can also be given. Whether the associations result from either of these causal relationships has yet to be established.

Within-child characteristics: Core number skills

Primary school mathematics includes some introduction to geometry, algebra, probability, and rational number, but it mainly involves developing skills in arithmetic with natural numbers. Accordingly, natural number items dominate the measures of mathematics achievement in primary school whether they assess mental arithmetic, written arithmetic, or problem solving. Although even natural number arithmetic consists of a variety of knowledge, skills, and principles (Dowker, 2005), some skills have been argued to be critical for development and individual differences in numeracy.

Approximate numerosity (Halberda and Feigenson, 2008) and exact numerosity (Butterworth, 2010) derive from considerations of non-symbolic systems that are shared with other species (Feigenson *et al*., 2004). They are claimed to support the development of number skills. Number skills proposed as core skills include basic calculation fluency and number knowledge.

The thrust behind the research has emphasized core skills as the causes of differences in mathematics achievement. Part of the justification for regarding them as core is that they are vital to the learning and performance of many mathematical skills and activities. It would follow then engaging in mathematics education is likely to involve practising and developing these skills. So the relationships between core skills and mathematics achievement may be bidirectional (Hecht *et al*., 2001).

Approximate numerosity

Approximate numerosity tasks require children to discriminate or combine numerosities in conditions that preclude success through accurate counting or judgements based on continuous dimensions such as area or contour. Evidence is mixed about the relation between preschool approximate numerosity skills and later number skills (e.g., Bonny and Lourenco, 2013; Fazio *et al*., 2014; Göbel *et al*., 2014; vanMarle *et al*., 2014) and the contribution of approximate numerosity skills to number difficulties (Mazzocco *et al*., 2011; Rousselle and Noël, 2007).

Approximate numerosity skills develop substantially throughout life (Halberda and Feigenson, 2008) and there is little evidence of the stability of individual differences in them over time. No successful intervention targeting approximate numerosity skills in children has been reported but Park and Brannon (2013) describe two studies where adults were trained to do approximate addition and subtraction without counting. The adults showed benefits in subsequent symbolic arithmetic.

If approximate numerosity skills derived from the same system, then associations between them would be expected. However, a study of adults found few associations between different measures of approximate numerosity skills (Gilmore *et al*., 2011). This may either reflect difficulties with the measures or problems with hypothesis that a single system underlies all approximate numerosity skills.

Exact numerosity

Exact numerosity skills are assessed by magnitude comparison and quantity enumeration tasks involving up to ten items. Exact numerosity efficiency, a combination of speed and accuracy, is associated with more general arithmetical skills (Reigosa-Crespo *et al*., 2012). Poor exact numerosity skills have been found in children with very poor number skills (e.g., Landerl *et al*., 2004). Two theories have been proposed to explain this: the defective number module hypothesis (Reigosa-Crespo *et al*., 2012) and the access deficit hypothesis (Rousselle and Noël, 2007). The defective number module hypothesis predicts difficulties on both symbolic and non-symbolic tasks whereas the access deficit hypothesis proposes that it is in coordinating numerals with numerosities. The balance of evidence currently supports the access deficit hypothesis (Cowan and Powell, 2014).

Interventions targeting exact numerosity skills have been reported but their effectiveness is very limited (Räsänen *et al*., 2009). There may be more to come (Butterworth and Laurillard, 2010).

Basic calculation proficiency

Basic calculation proficiency is the production of fast and accurate solutions to single digit addition and multiplication problems, such as 7 + 8 and 9 × 6 , and complementary subtraction and divisions, such as 15 −7 and 54 ÷ 9. Basic calculation fluency consistently covaries with more general mathematics achievement (e.g., Durand *et al*., 2005) and deficits in fluency are the commonest characteristic of children making poor progress in mathematics (e.g., Russell and Ginsburg, 1984).

Achieving basic calculation fluency during primary school is a universal educational aspiration. It is believed to underpin general calculation fluency, i.e. fluency in more complex mental and written calculation: for example a child who knows 6 × 7 is 42 should be able to use this to solve 60 × 70 or 4200 ÷ 600. However, as maths educators since Brownell and Chazal (1935) have pointed out, memorizing facts to achieve basic calculation fluency does not always transfer to general calculation fluency.

The development of basic calculation fluency seems to depend less on knowledge of facts, i.e. retrieving solutions from longterm memory, than is commonly imagined (Cowan *et al*., 2011). Quick solutions to basic calculation addition and subtraction problems are obtained through use of principles or reasoning with related facts (Siegler, 1987). In our study of English children in Years 3 and 4 the incidence of rapid solutions was much higher than the incidence of solutions where the child claimed to know the answer.

In reviewing interventions for primary school children struggling with mathematics, Gersten *et al*. (2009) concluded that there was moderate evidence for training that targeted the development of basic calculation fluency. They noted however that most effective interventions combined basic calculation fluency with attention to developing understanding of the number system. A more recent example of this is the substantial study by Fuchs *et al*. (2013) that targeted at-risk students in Year 2 (first grade in the US). They compared two tutoring programmes delivered in 48 lessons lasting 30 minutes each. For each lesson, 25 minutes were spent developing number knowledge. The two tutoring programmes differed in how they spent the final five minutes. One programme (non-speeded practice) emphasized thoughtful application of principles and strategies to support solution of basic addition and subtraction. The other (speeded practice) promoted quick responding and the use of efficient counting strategies. Both programmes had substantial impacts on basic and more complex calculation compared to a 'business as usual' control group. Speeded practice had more effect than non-speeded practice on calculation but the programmes did not differ in the associated benefits on number knowledge or problem solving.

Number knowledge

Number knowledge as construed by Fuchs *et al*. (2013) includes knowledge of the principles of arithmetic, the number system and number line representations. Each of these has been emphasized in previous work and each can be used to enhance calculation fluency.

Knowledge of arithmetical principles and skill in applying them supports mathematical development (Hanich *et al*., 2001; Nunes *et al*., 2007). For example, arithmetical principles such as commutativity of addition and multiplication and the inverse relations between addition and subtraction and between multiplication and division support development of calculation fluency. Differences in children's arithmetical reasoning predict later mathematical achievement with children knowing more principles making better progress (Cowan *et al*., 2011; Nunes *et al*., 2007).

Number system knowledge (Case and Okamoto, 1996) and number line estimation, an approximate number task that involves estimating the position of target numbers on a line with numerals at the endpoints (Booth and Siegler, 2006), concern understanding of the relations between numbers and their magnitudes. Children typically know some of the number word sequence and the names of numerals when they start school, but not the relative magnitudes of numbers in their counting range (Siegler and Robinson, 1982). They have some informal knowledge of fractions but do not understand them as numbers that have magnitudes.

At primary school, they master the systems for combining number words and representing numbers with numerals. This enables them to generate accurate counting and numeral sequences from numbers they have not experienced (Skwarchuk and Anglin, 2002). They learn more about the relations between numbers and become more accurate in representing their magnitudes.

Number system knowledge and number line estimation both contribute to basic calculation fluency, written arithmetic, and mathematical problem solving. In a study of Year 4 children, number system knowledge was what best differentiated children with substantially below average number skills from others (Cowan and Powell, 2014).

Effective number system knowledge and number line estimation interventions have been conducted in the US and UK (Booth and Siegler, 2008; Fuchs *et al*., 2013; Griffin *et al*., 1994; Kucian *et al*., 2011; Laski and Siegler, 2014; Ramani *et al*., 2012; Rittle-Johnson *et al*., 2001; Whyte and Bull, 2008).

Summary

The important parts played by basic calculation proficiency and number knowledge are theoretically grounded and empirically supported by both correlational studies and interventions. As individual differences in these core skills show associations with general cognitive skills, it is likely that these core skills at least partially mediate the relationship between general cognitive skills and mathematics achievement (Cowan *et al*., 2011; Cowan and Powell, 2014).

In mathematical development, the nature of what children learn changes greatly as they progress through primary school and later through secondary school and beyond. This makes the extent of continuity surprising. Individual differences in natural number knowledge in primary school predict later differences in knowledge of fractions, which in turn predict differences in secondary school mathematics achievement and knowledge of algebra (Bailey *et al*., 2014; Siegler *et al*., 2012).

It would be remarkable if all that were needed to improve achievement in secondary mathematics and adult numeracy were some interventions in primary school. So it is likely that some of the continuity between early and later individual differences is not due to the importance of core number skills. The role of general cognitive skills has already been considered. We now turn to consider other characteristics of the child such as their self-beliefs and socio-emotional functioning and characteristics of the home and school environments in which they develop. These may also help to explain why some children respond to interventions better than others and why some show longer lasting benefits.

Within-child characteristics: Self-beliefs and socio-emotional functioning

Self-beliefs

More variation in later achievement can be explained when models include children's self-beliefs as well as earlier achievement (Gutman and Schoon, 2013). Theorists distinguish between interests, motivation, views of one's academic abilities (academic self-concepts and attributions of success and failure), expectations of success (self-efficacy), and mindset (Aunola *et al*., 2006; Fisher *et al*., 2012; Guay *et al*., 2003; Haimovitz *et al*., 2011; Luo *et al*., 2011; Morgan and Fuchs, 2007; Nunes *et al*., 2009; Ryan and Deci, 2000; Schunk, 2003; Zimmerman, 2000).

Many teachers try to encourage children to develop interests in the subjects they teach and positive beliefs about their ability to learn (Aunola *et al*., 2006). Increasing children's interests in mathematics may make it more likely that they will engage in activities that practise and develop their skills both in and out of school. Whether such engagement occurs will depend on the availability of activities and the attractiveness of alternatives, such as playing with friends or watching television.

Children may develop self-beliefs through comparing themselves to other children and through their perceptions of how their teachers think of them. School provides children with information that can shape their ideas about how well they are doing, and affect their enjoyment and confidence in relation to school-based activities in arithmetic. Their motivations may develop in various ways, for example by internalizing beliefs that doing well in school work is important for future progression, or by resolving to succeed to please their teacher and parents, or to gain recognition from their peers. Motivation and effort may be influenced by their mindset beliefs: Dweck (2008) differentiates growth mindsets, which consider abilities capable of development with effort, from fixed mindsets, which view abilities to be determined by factors outside learners' control. Fixed mindsets are associated with approaches to learning that avoid challenge, give up easily, and show fear of making mistakes.

Assessing self-beliefs is fraught with difficulty. The deliberate attempts by teachers to manipulate children's self-beliefs may result in children's reports of interests, enjoyment, and mindsets being affected by social desirability (Dweck *et al*., 1995). In recent Master's projects, 9- and 10-year-olds typically rejected fixed mindset beliefs about maths (Bohdjalian, 2013; McCabe, 2013). Their reported mindset beliefs were slightly related to their number skills. Their teachers also rated the pupils' approach to mathematics learning, specifically whether they showed fixed mindset characteristics. Achievement was more strongly related to teacher ratings of pupils' approaches to learning than to pupils' self-reported mindset beliefs. However, the teachers were of course aware of how well the pupils were doing and this may have influenced their judgements.

Changing children's self-beliefs does not by itself enhance number development or performance on number skills tests. Enhancing self-beliefs can only benefit development if it is accompanied by relevant learning activity, and can only benefit performance if it leads to additional effort or persistence. As Marsh (2006) points out, enhanced academic self-concepts will only be sustained if they are accompanied by enhanced achievement. Does this work the other way around too? Do self-beliefs moderate the extent to which interventions that target core skills lead to more general benefits? This is currently unknown.

Socio-emotional functioning

Response to constructive criticism is a characteristic that differentiates growth and fixed mindsets. It would seem also to depend on a child's socio-emotional functioning. As well as children's ability to manage the emotional challenges of working under direction of teachers, this also involves being able to work together with other children. Socio-emotional functioning can be considered as a set of social skills, such as the ability to regulate emotions. These skills undergo considerable development during primary school years. They are related to educational progress (Duckworth and Schoon, 2010; Hinshaw, 1992; Rapport *et al*., 2001).

Self-regulation is considered a core component of successful socio-emotional functioning (Eisenberg *et al*., 2010) but questions remain about the relations between self-regulation in social interaction and in cognitive tasks such as measures of executive functioning (Duckworth and Kern, 2011; Ursache *et al*., 2012).

Some recent work suggests the value of distinguishing between academic and social impulsivity (Tsukayama *et al*., 2013). Academic impulsivity is negatively related to amount of time spent doing homework and television, and ratings of attentiveness and conscientiousness. Social impulsivity is positively related to ratings of frustration and aggression. Both are negatively related to teacher-rated educational achievement with academic impulsivity being more strongly related.

Current issues

Both self-beliefs and socio-emotional functioning are measured subjectively either by the children themselves or informants. Possibly combining these assessments with objective behavioural observations could enhance the investigation of how these characteristics contribute to children's number development. Although socio-emotional functioning has been considered as a characteristic that affects school readiness, measures of it in preschoolers have not been found to predict later school progress. In discussing these findings Duncan *et al*. (2007) pointed to the need for further research on the relationships between aspects of socio-emotional functioning and educational achievement. There may well be reciprocity in these relationships.

Characteristics of the home and school contexts

Home

Correlational research in the UK and the US finds associations between familial socioeconomic status (SES) and children's educational achievement (Sirin, 2005). Analysis of UK cohort data indicates that the relationship may reflect the influences of material deprivation and parental involvement (Sacker *et al*., 2002).

Parental involvement in children's schooling is associated with achievement independently of SES (Desforges and Abouchaar, 2003). Analysis of cohort data shows that involvement by both mothers and fathers when children are aged 7 is associated with very long-term benefits (Flouri and Buchanan, 2004). Cohort studies also indicate that parental interest in education increases during primary school and that the relationship between parental involvement and children's achievement is bidirectional (Sacker *et al*., 2002).

How parents try to teach children may be more important than whether they do so. Parental involvement that supports development is constructive and focuses on understanding principles, supporting autonomy, and maintaining positive affect. In contrast unconstructive involvement focuses on performance, controls the child, and expresses frustration. Type of involvement varies with parental mindset: mothers induced to hold fixed mindset beliefs about a task were more likely to show unconstructive involvement when working with their child than those induced to hold growth mindset beliefs (Moorman and Pomerantz, 2010).

Whether parents support constructively may also be influenced by their own educational achievement and confidence in supporting their child's learning. Whether they know how to support constructively may depend on their access to knowledge about teaching through social networks (family, friends, and electronic media with sources such as Mumsnet), their confidence in approaching teachers for support, and their ability to communicate with the teacher. Whether they wish to comply with the school's suggestions of how to support their child may depend on whether they share the school's values and have confidence in their child's class teacher (Tizard *et al*., 1981).

So far, interventions to enhance parental support and involvement have not clearly yielded benefits (Gorard and See, 2013). It may be because they are ineffective in enlisting the support and cooperation of the families they target. Some parents believe that a child's number development is the responsibility of teachers: they consider requests to engage in number activities at home show the teacher is not good enough at their job. In an exploratory study of homework in New Zealand with Stuart McNaughton we found some parents enjoyed getting involved with their children's homework and provided rich diaries of family numeracy and literacy practices that involved joint activities with their children (Cowan *et al*., 1998; McNaughton, 1995). Others recorded little or no joint family practices and valued homework that the child could do without their involvement.

In relation to the socialization of early literacy, McNaughton (1995) highlighted the importance of family goals, the nature of family practices and activities, and the relations between settings. This analysis could inform research about the socialization of early numeracy, which has been less researched despite some pioneering studies (e.g., Saxe *et al*., 1987; Young-Loveridge, 1989). A key feature of McNaughton's co-construction model is the part played by the child, consistent with Bell's (1979) emphasis on reciprocal influences between parent and child.

School

Aspects of the school context that may contribute to individual differences between children include the implementation of school-level policies, the provision for children making poorer progress, and characteristics of teachers.

Although all primary schools publicly affirm the importance of parental involvement, the resources they devote to it are likely to vary with the school. The home visits by class teachers recommended by Tizard *et al*. (1981) are particularly demanding of resources. The effectiveness with which schools build relationships with parents is likely to vary with each family.

In many English primary schools the same teacher teaches all subjects for a particular class in the same year group, even when the school uses some form of ability grouping. In some schools, different teachers take responsibility for different sets. Effective support for children making poorer progress in mathematics may demand more subject knowledge (Williams, 2008), but it can be delegated to teachers with less confidence and enthusiasm (Bibby *et al*., 2007) or to teaching assistants whose interactions with pupils may be less educative (Russell *et al*., 2013).

The specification of the pedagogical knowledge that supports effective teaching is a substantial effort (Ball *et al.*, 2005). To be a good teacher involves more than having secure mathematical knowledge, however important that may be (Askew *et al*., 1997). The Learning Mathematics for Teaching project based at the University of Michigan has been engaged in developing measures, but further work is required (Hill, 2007).

There are many more aspects of teachers' expertise that could affect the development of their pupils. How successfully they manage ability grouping is an aspect about which little is known. As mentioned earlier, most English primary schools operate a form of pupil grouping such as setting for English and mathematics or streaming (Campbell, 2013; Hallam and Parsons, 2013). Whereas advocates of grouping emphasize its role in allowing children to develop at their own pace, maintaining interest and motivation, critics emphasize the inaccuracies in assignment, the demotivating effect of being in a lower group, and the reification of ability (Whitburn, 2001). Evidence of the effects of grouping is more mixed than either supporters or opponents would expect (Hallam, 2002). Quite possibly some teachers make their classroom organization, whether mixed-ability or setting, work better for all their pupils. It may also depend on the beliefs that they have about differences in ability and the ways in which they communicate these to pupils and parents. Some research indicates that the ability mindsets of staff supporting pupils making poor progress may affect their interactions (Rattan *et al*., 2012).

The value of group work depends on more than simply sitting some children around the same table and giving them the same activities. To achieve the benefits of collaborative work requires teachers to prepare children for it and monitor it effectively (Howe and Mercer, 2007; Kutnick *et al*., 2014). Advocates of collaborative group work believe it can reach more children and engage them in more effective ways of learning, but some doubt whether higher order mathematical skills are likely to be engendered (Desforges and

Cockburn, 1987). What is likely to be important is not so much the use of a particular approach: what works for some children will not work for all. Instead it is the versatility of the teacher in selecting forms of classroom arrangement to fit the learning activity that is likely to be more important (Hallam *et al*., 2004).

Conclusion

My aims in writing this were to show what an interesting period of development middle childhood is for children's psychological and educational development and to indicate how much there is to find out. The possibilities of reciprocal relationships, the range of variables that could contribute to stability and change in development, and the limitations of existing measures make progress in this field challenging. Theory-based interventions offer a way forward.

The success of an intervention does not guarantee that its effects are permanent. Some successful interventions may affect state factors, and so the effects will decay over time. Interventions may even just affect error factors, so any change is simply temporary (Campbell and Kenny, 1999). It is not enough to know what works. One also needs to know why.

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