Essays on Microeconometrics and the Labour Market

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Declaration

I, Cathy Balfe confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Cathy Balfe

Abstract

This thesis applies and extends microeconometric methods in the analysis of economic questions related to education choice and fertility choice, and their interaction with the labour market.

Chapter 2 extends the literature on non-parametric bounds on the returns from education by allowing for non-random selection into both education and labour market participation simultaneously. Allowing for both forms of non-random selection leads to very wide bounds on the estimated returns from education. This finding highlights both the role of parametric assumptions in pinpointing the magnitude of these effects, but also the importance of rigorously validating the assumptions leading to point estimates in such a wide identified set.

Chapter 3 reviews the marginal treatment effect approach. This chapter also outlines how the MTE model can be used to estimate the selection effect, and to estimate whether an advantage exists for the treated group for either potential outcome (treated and non-treated). This chapter also rigorously discusses the comparison of ATE, OLS and IV estimates, and discusses what can be inferred from these comparisons.

Chapter 4 estimates heterogeneity in the returns to higher education in the UK. Significant heterogeneity due to observable characteristics was found, in particular with high ability individuals receiving lower returns to higher education than lower ability individuals. Since graduates have higher mean levels of ability than non-graduates this leads to negative selection; individuals who do not attend higher education stand to gain more from participation than those who do attend. This counter-intuitive finding is discussed and possible economic explanations explored.

Finally, chapter 5 found that extensions in the duration of paid maternity leave led to deteriorating female labour market conditions, with female employees receiving lower pay and experiencing higher levels of redundancy relative to males as a result of an expansion in the duration of paid maternity leave.

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Chapter 1

Introduction

This thesis applies and extends microeconometric methods in the analysis of economic questions related to education choice and fertility choice, and their interaction with the labour market.

Chapter 2 estimates the monetary returns to different levels of education, allowing for non-random selection into both the level of education and into the labour market simultaneously. Naive estimates of the return to education ignoring these sources of non-random selection could lead to severely biased estimates of the return to education. Much of the analysis in this chapter imposes less restrictive assumptions than those typically used in empirical work, which often leads to set rather than point identification of the return to education. Different assumptions about how individuals self select into education and the labour market are considered. Some assumptions considered include exogenous treatment selection (leads to point identification), monotone treatment selection (conditional on education it is assumed that there are higher mean potential earnings among the group observed to be working), monotone treatment response (potential labour market income is increasing in education for every individual), monotone instrumental variables (it is assumed that parental education changes the distribution over completed education level or labour market participation, while being monotonically related to potential earnings), with combinations of the above assumptions also being considered. This chapter builds on work by Manski (1989, 1990, 1997), Manski and Pepper (2000, 2009) (who allow for non-random selection into education level) and Blundell et al. (2007) (who allow for non-random selection into the labour market). This chapter finds that the weak assumptions imposed in the analysis do not provide a great deal of identification, with bounds of the returns to education being above zero only in cases when it is imposed by assumption. Since the data is only weakly informative about the returns to education, this finding highlights both the role of parametric assumptions in pinpointing the magnitude of these effects, but also the importance of rigorously validating the assumptions leading to point estimates in such a wide identified set.

Chapter 3 provides a detailed overview of the marginal treatment effect approach introduced into the literature by Heckman and Vytlacil (1999). The marginal treatment effect approach allows for estimation of a distribution of treatment effects over the range of unobservable characteristics that affect the probability of treatment. This chapter pays particular attention to estimation approaches and uses as a running example the use of the MTE in estimation of the returns to education, as this is the context in which it has been most frequently applied in the literature. This chapter also outlines how the MTE model can be used to estimate the selection effect, and to estimate whether an advantage exists for the treated group for either potential outcome (treated and non-treated). Furthermore, this chapter discusses how to decompose these effects into the component due to observable characteristics and the component due to unobservable characteristics. Finally, this chapter rigorously discusses the comparison of ATE, OLS and IV estimates (comparisons that are frequently made in the literature), and discusses what can be inferred from these comparisons.

Chapter 4 applies the marginal treatment effect to estimate heterogeneity in the returns to higher education in the UK. In contrast to previous work in the literature, the analysis finds little evidence of heterogeneity in the returns owing to unobservable characteristics, once an extensive set of observable characteristics were controlled for. However, significant heterogeneity owing to observable characteristics were found, in particular, with individuals with higher levels of measured ability receiving lower returns than individuals with lower levels of measured ability. Another interesting finding in this work is that there is negative selection into higher education, with those not attending higher education standing to gain more than those who choose to attend. Differential non-monetary returns was one possible explanation for this counter-intuitive finding, however, little evidence in support of this explanation was found. These results suggest that there must be either higher monetary or non-monetary costs, or barriers to entry associated with higher education for lower ability individuals.

Chapter 5 analyses the impact of an expansion in paid maternity leave on relative female-male labour market outcomes. A policy reform in the UK, which increased paid maternity leave duration by 50% from a maximum of 26 weeks to a maximum of 39 weeks from April 2007 was used to provide exogenous variation. A simple theoretical model was developed, which predicts an increasing female-male wage gap in response to an increase in maternity leave duration. A quasi-experimental difference in differences estimation approach was used to estimate the impact on the relative wage gap. Furthermore, for the the impact of the extension on discrete outcomes (employment, hiring and redundancy), an alternative estimation approach was proposed that has the advantage of providing interpretable treatment effects in the presence of substitution effects. The reform was found to significantly increase the amount of leave female employees took relative to male employees. There was an increase of about 1% in the proportion of time female employees aged 25-34 spent on leave relative to males, with almost half of this due to a fertility response and half due to an increase in maternity leave duration. The results also suggest that the expansion engendered a deterioration in relative female labour market outcomes, with empirical evidence indicating a decrease in female relative wages and an increase in relative female redundancies.

Finally, chapter 6 concludes.

Chapter 2

Bounding Returns from Education: Selection into Education and the Labour Market

2.1 Introduction

Estimates of the return to education suffer from potential bias due to non-random selection into different education levels and non-random selection into labour market participation. Both sources of bias are due to the problem of missing counterfactuals. The econometrician does not know what an individual who left school after high school would have earned if he had graduated from college, nor what this individual would earn with or without a college education if he does not participate in the labour market. This chapter provides non-parametric bound estimates on the returns from different levels of education, allowing for non-random selection into both education and the labour market.

Manski (1989, 1990, 1997), Manski and Pepper (2000, 2009) and Blundell et al. (2007) have developed a framework for estimating treatment effects under less restrictive assumptions than typically implemented in empirical work. Often, these methods set rather than point identify a region within which the treatment effect lies. These approaches have been applied to labour market earnings, with Manski and Pepper (2000) estimating the returns to distinct years of schooling, allowing for non-random selection into different education levels. Blundell et al. (2007) allow for non-random selection into the labour market when examining changes in wage distributions for different education/gender groups over time. This chapter builds on this work, by estimating bounds on the returns to education, allowing for non-random selection both into the labour market and into different education levels simultaneously. Treatment effects under various alternative assumptions that can be motivated by economic theory will be estimated, and the identifying power of the

alternative assumptions in this context will be analysed.
In this chapter the focus is on estimating conditional mean functions and average treatment effects allowing for non-random selection into both education and employment simultaneously, under various alternative assumptions about how individuals select into the labour market and different levels of education. Section 2.2 discusses related literature, section 2.3 derives bounds on the returns to education under various alternative assumptions on the selection mechanisms, section 2.4 describes the data set, section 2.5 discusses the empirical findings and section 2.6 concludes.

2.2 Literature Review

Suppose we are interested in the conditional expectation $E[y(t)|x]$, where y(t) is the outcome of an individual (e.g. hourly wages) if they received treatment $t \in T$ (e.g. years of education). The treatment they actually receive is denoted $z \in T$. $x \in X$ is a set of observable characteristics. As derived in Manski (1989, 1990) the conditional expectation under non-random selection into treatment status in the worst case (when y(z) is always observed) can be bounded as follows:

$$E[y(t)|x] = E[y(t)|x, z = t]P(z = t|x) + E[y(t)|x, z \neq t]P(z \neq t|x)$$

All identities on the right hand side of the above equation are identified with the exception of $E[y(t)|x, z \neq t]$. If y(t) is bounded between $[K_0, K_1]$ then the conditional expectation can be bounded as follows:

$$\begin{gathered} E[y|x, z = t]P(z = t|x) + K_0 P(z \neq t|x) \\ \leq E[y(t)|x] \leq \\ E[y|x, z = t]P(z = t|x) + K_1 P(z \neq t|x) \end{gathered}$$

In Manski and Pepper (2000), focus is on non-random selection into alternative education levels, and the reported empirical bounds do not deal with non-random selection into labour market participation (the estimated bounds in their paper is therefore bounding the impact of education on individuals who participate in the labour market). Assumptions including ETS, MTS, IV, MIV, MTR and MIV-MTR and MTS-MTR combinations are considered, with the MTR-MTS case being estimated empirically.
Blundell et al. (2007) focus instead on non-random selection into employment, and estimate bounds on conditional quantiles rather than the conditional mean. W denotes log wage, E denotes employment status with E=1 denoting employment and E=0 unemploy-

ment, x is again a set of observable characteristics. Blundell et al. (2007) estimate the conditional distribution of wages $F(w|x)$, and start with a similar worst case bound to Manski (1989, 1990). There is:

$$F(w|x) = F(w|x, E = 1)P(E = 1|x) + F(w|x, E = 0)(1 - P(E = 1|x))$$

All identities except $F(w|x, E = 0)$ on the right hand side of equation 3 are identified. Since $F(w|x, E = 0)$ is a conditional distribution function, it must be bounded between [0,1], so the conditional distribution function can be bounded as follows:

$$\begin{gathered} F(w|x, E = 1)P(E = 1|x) \\ \leq F(w|x) \leq \\ F(w|x, E = 1)P(E = 1|x) + (1 - P(E = 1|x)) \end{gathered}$$

Blundell et al. (2007) also consider the identifying power of various assumptions, deriving bounds on the conditional distributions imposing a stochastic dominance assumption, a median restriction, an exclusion restriction and a monotonicity assumption. The bounds on the conditional distribution functions are translated to bounds on conditional quantiles. Additionally, bounds on within group inequality and bounds on across group and time inequality based on these conditional quantiles were derived. A benefit of using quantiles rather than means is that no support assumptions are necessary. Blundell et al. (2007) also introduces specification tests for the validity of the exclusion and monotonicity assumptions. They also look at the change of education differentials over time, allowing for non-random selection into the labour market whereas Manski and Pepper (2000) estimate the impact of education on earnings allowing for non-random selection into different education levels (for the sub-population of the education who participate in the labour market). Manski and Pepper (2000) impose some structure on how individuals select into different levels of education, Blundell et al. (2007) impose some structure on how individuals select into the labour market. In this chapter, structure is imposed on both selection thresholds to try and estimate bounds on the return to different levels of education.

Structural, parametric models have estimated the returns from education allowing for endogeneous education and labour market participation e.g. (Keane and Wolpin, 1997; Adda et al., 2013). See Meghir and Rivkin (2011) for further discussion.

2.3 Allowing for both forms of selection

In this section, bounds on the conditional mean are derived, imposing different assumptions on selection into different levels of education and selection into labour market participation. In particular, the worst case scenario, exogenous treatment selection (ETS) for both the education choice and the labour market participation choice, monotone treatment response (MTR) for education, monotone treatment selection (MTS) for labour market participation and monotone instrumental variables (MIV) assumptions are considered, and identification power of each assumption is investigated. Bounds are also estimated combining MIV-MTR-MTS assumptions.

2.3.1 Worst Case Bounds

Potential wages of individuals had they received education level t; y(t), is observed only for those who actually received education level t (z=t), and for those who are in employment. No assumptions are made about the potential wages with education t (y(t)) of those individuals who do not have education level t or those who do not participate.[1]

$$
\begin{aligned}
E[y(t)|x] = & E[y|x, z = t, E = 1]P(z = t, E = 1|x) \\
& + E[y(t)|x, z \neq t, E = 1]P(z \neq t, E = 1|x) \\
& + E[y(t)|x, z = t, E = 0]P(z = t, E = 0|x) \\
& + E[y(t)|x, z \neq t, E = 0]P(z \neq t, E = 0|x)
\end{aligned}
$$

There are now more identities on the right hand side that are unidentified in comparison to the worst case bounds considered in Manski (1989, 1990), and discussed in the previous section. The unidentified identities are $E[y(t)|x, z \neq t, E = 1]$, $E[y(t)|x, z = t, E = 0]$ and $E[y(t)|x, z \neq t, E = 0]$. If y(t) is bounded between $[K_0, K_1]$, then the conditional mean can be bounded as follows:

[1]There are some assumptions imposed even in the worst case scenario; the support is assumed to lie between K_0 and K_1 and survey attrition and non-response are considered random

$$
\begin{gathered}
E[y|x, z = t, E = 1]P(z = t, E = 1|x) \\
+K_0(1 - P(z = t, E = 1|x)) \\
\leq E[y(t)|x] \leq \\
E[y|x, z = t, E = 1]P(z = t, E = 1|x) \\
+K_1(1 - P(z = t, E = 1|x))
\end{gathered}
$$

The average treatment effect $E[y(t) - y(s)|x]$ ($ATE(x)$) is bounded from below by subtracting the upper bound of $E[y(s)|x]$ from the lower bound for $E[y(t)|x]$, and bounded from above by subtracting the lower bound of $E[y(s)|x]$ from the upper bound for $E[y(t)|x]$:

$$
\begin{gathered}
\{E[y|x, z = t, E = 1]P(z = t, E = 1|x) \\
+K_0(1 - P(z = t, E = 1|x))\} - \\
\{E[y|x, z = s, E = 1]P(z = s, E = 1|x) \\
+K_1(1 - P(z = s, E = 1|x))\} \\
\leq E[y(t) - y(s)|x] \leq \\
\{E[y|x, z = t, E = 1]P(z = t, E = 1|x) \\
+K_1(1 - P(z = t, E = 1|x))\} \\
\{E[y|x, z = s, E = 1]P(z = s, E = 1|x) \\
+K_0(1 - P(z = s, E = 1|x))\}
\end{gathered}
$$

2.3.2 Exogeneous Treatment Selection (ETS)

Assumption:
$E[y(t)|x, z = t'', E = E''] = E[y(t)|x, z = t', E = E']$
$\forall x \in X$
$\forall t \times t'' \times t' \in T \times T \times T$
$\forall E' \times E'' \in (0, 1) \times (0, 1)$
This assumption implies there is no selection into either education level or labour market participation conditional on x.

This implies the conditional mean outcome of interest is point identified from:

$$E[y(t)|x] = E[y|x, z = t, E = 1]$$

And implies the ATE(x) can be point identified using:

$$E[y(t) - y(s)|x] = E[y|x, z = t, E = 1] - E[y|x, z = s, E = 1]$$

2.3.3 Monotone Treatment Selection (MTS)

Assumption:
$E[y(t)|x, z = t', E = E''] \geq E[y(t)|x, z = t', E = E']$
$\forall x \in X$
$\forall t \times t' \in T \times T$
$E'' = 1, \qquad E' = 0$
This assumption implies there is monotone treatment selection into labour market participation conditional on x and education. Individuals with the same level of education and the same observable characteristics who choose to participate in the labour market have higher mean labour market wages for any given level of education.

This implies the following bounds from observables for the conditional expectations $E[y(t)|x, z, E]$

$$\begin{aligned}
&\text{For } t' = t, E = 1 : E[y(t)|x, z = t', E = 1] = E[y|x, z = t, E = 1]\\
&\text{For } t' = t, E = 0 : K_0 \leq E[y(t)|x, z = t, E = 0] \leq E[y|x, z = t, E = 1]\\
&\text{For } t' \neq t : K_0 \leq E[y(t)|x, z = t', E] \leq K_1
\end{aligned}$$

This implies the bound:

$$\begin{gathered}
K_0(P(z \neq t|x) + P(z = t, E = 0|x)) + E[y|x, z = t, E = 1]P(z = t, E = 1|x)\\
\leq E[y(t)|x] \leq\\
K_1(1 - P(z = t|x)) + E[y|x, z = t, E = 1]P(z = t|x)
\end{gathered}$$

And the ATE(x) bound:

$$
\begin{gathered}
\{K_0(P(z \neq t|x) + P(z = t, E = 0|x)) + E[y|x, z = t, E = 1]P(z = t, E = 1|x)\} - \\
\{K_1(1 - P(z = s|x)) + E[y|x, z = s, E = 1]P(z = s|x)\} \\
\leq E[y(t) - y(s)|x] \leq \\
\{K_1(1 - P(z = t|x)) + E[y|x, z = t, E = 1]P(z = t|x)\} \\
\{K_0(P(z \neq s|x) + P(z = s, E = 0|x)) + E[y|x, z = s, E = 1]P(z = s, E = 1|x)\}
\end{gathered}
$$

2.3.4 Monotone Treatment Response (MTR)

Monotone treatment response assumes that potential labour market income is increasing in education for every individual.

$$
t_2 \geq t_1 \Rightarrow y_j(t_2) \geq y_j(t_1) \qquad \forall j
$$

Using the same notation as Manksi (1997) let $y_{0j}(t)$ and $y_{1j}(t)$ be the lower or upper bound for the potential outcome of individual j had they received treatment t. The MTR assumption implies:

$$
\begin{aligned}
y_{0j}(t) &\equiv y_j \text{ if } t \geq z_j, E_j = 1 \\
&\equiv K_0 \text{ otherwise}
\end{aligned}
$$

$$
\begin{aligned}
y_{1j}(t) &\equiv y_j \text{ if } t \leq z_j, E_j = 1 \\
&\equiv K_1 \text{ otherwise}
\end{aligned}
$$

Adjusting Proposition M1 in Manksi (1997) for the two forms of selection is straightforward.

$$z_j > t, E_j = 1, \Rightarrow K_0 \leq y_j(t) \leq y_j$$
$$z_j > t, E_j = 0, \Rightarrow K_0 \leq y_j(t) \leq K_1$$
$$z_j = t, E_j = 1, \Rightarrow y_j(t) = y_j$$
$$z_j = t, E_j = 0, \Rightarrow K_0 \leq y_j(t) \leq K_1$$
$$z_j < t, E_j = 1, \Rightarrow y_j \leq y_j(t) \leq K_1$$
$$z_j < t, E_j = 0, \Rightarrow K_0 \leq y_j(t) \leq K_1$$

Therefore,

$$y_{0j}(t) \leq y_j(t) \leq y_{1j}(t) \;\; j \in J$$

As stated in the Proposition M1, this implies $y_0(t)$ is stochastically dominated by $y(t)$, which is dominated by $y_1(t)$. Furthermore, if $D(.)$ is a function that respects stochastic dominance, then for every $t \in T$,

$$D[y_0(t)] \leq D[y(t)] \leq D[y_1(t)]$$

Therefore, since the mean respects stochastic dominance, the above can be used to derived the following bounds for the conditional expectation

$$\begin{gathered}
K_0\{P(E=0|x) + P(z > t, E=1|x)\}+ \\
E[y|x, z=t, E=1]P(z=t, E=1|x) + E[y|x, z<t, E=1]P(z<t, E=1|x) \\
\leq E[y(t)|x] \leq \\
K_1\{P(E=0|x) + P(z < t, E=1|x)\}+ \\
E[y|x, z=t, E=1]P(z=t, E=1|x) + E[y|x, z>t, E=1]P(z>t, E=1|x)
\end{gathered}$$

And following from Manksi (1997) the ATE(x) bound for $t \geq s$:

$$0 \leq E[y(t) - y(s)|x] \leq$$
$$\{K_1\{P(E=0|x) + P(z<t, E=1|x)\}+$$
$$E[y|x, z=t, E=1]P(z=t, E=1|x) + E[y|x, z>t, E=1]P(z>t, E=1|x)\}-$$
$$\{K_0\{P(E=0|x) + P(z>s, E=1|x)\}+$$
$$E[y|x, z=s, E=1]P(z=s, E=1|x) + E[y|x, z<s, E=1]P(z<s, E=1|x)\}$$

The usual computation of the lower bound of the ATE(x) found by subtracting the upper bound of $E[y(s)|x]$ from the lower bound of $E[y(t)|x]$ is always negative, which is ruled out by the MTR assumption. The upper bound estimate of the ATE(x) is always positive.

2.3.5 Monotone Instrumental Variables (MIV)

Assumption:
$E[y(t)|x, v=u''] \geq E[y(t)|x, v=u']$
$\forall x \in X$
$\forall t \in T$
$\forall u'' \geq u' \in V \times V$

In order to be more informative than the worst case bound, the instrumental variable must vary the conditional probability of obtaining education level z=t and/or choosing to participate in the labour market. However, in contrast to standard IV, under MIV the instrumental variable v is allowed to be monotonically related to potential labour market earnings.
The MIV assumption implies
$E[y(t)|x, v=u_1] \leq E[y(t)|x, v=u] \leq E[y(t)|x, v=u_2]$
$\forall u_1 \leq u \leq u_2$
Since this holds for all $u_1 \leq u$ and all $u_2 \geq u$ and combining the worst case bound and the MIV assumption:

$$sup_{(u_1 \leq u)}\{E[y|x, v=u_1, z=t, E=1]P(z=t, E=1|x, v=u_1) + K_0(1 - P(z=t, E=1|x, v=u_1))\}$$
$$\leq E[y(t)|x, v=u] \leq$$
$$inf_{(u_2 \geq u)}\{E[y|x, v=u_2, z=t, E=1]P(z=t, E=1|x, v=u_2) + K_1(1 - P(z=t, E=1|x, v=u_2))\}$$

Therefore:

$$\Sigma_{u\in V}P(v=u|x)[sup_{(u_1\le u)}\{E[y|x,v=u_1,z=t,E=1]P(z=t,E=1|x,v=u_1)+K_0(1-P(z=t,E=1|x,v=u_1))\}]$$
$$\le E[y(t)|x]\le$$
$$\Sigma_{u\in V}P(v=u|x)[inf_{(u_2\ge u)}\{E[y|x,v=u_2,z=t,E=1]P(z=t,E=1|x,v=u_2)+K_1(1-P(z=t,E=1|x,v=u_2))\}]$$

Which implies the ATE(x):

$$\{\Sigma_{u\in V}P(v=u|x)[sup_{(u_{1t}\le u)}\{E[y|x,v=u_{1t},z=t,E=1]P(z=t,E=1|x,v=u_{1t})+K_0(1-P(z=t,E=1|x,v=u_{1t}))\}]\}-$$
$$\{\Sigma_{u\in V}P(v=u|x)[inf_{(u_{2s}\ge u)}\{E[y|x,v=u_{2s},z=s,E=1]P(z=s,E=1|x,v=u_{2s})+K_1(1-P(z=s,E=1|x,v=u_{2s}))\}]\}$$
$$\le E[y(t)-y(s)|x]\le$$
$$\{\Sigma_{u\in V}P(v=u|x)[inf_{(u_{2t}\ge u)}\{E[y|x,v=u_{2t},z=t,E=1]P(z=t,E=1|x,v=u_{2t})+K_1(1-P(z=t,E=1|x,v=u_{2t}))\}]\}-$$
$$\{\Sigma_{u\in V}P(v=u|x)[sup_{(u_{1s}\le u)}\{E[y|x,v=u_{1s},z=s,E=1]P(z=s,E=1|x,v=u_{1s})+K_0(1-P(z=s,E=1|x,v=u_{1s}))\}]\}$$

2.3.6 MIV-MTR-MTS

In order to estimate the MIV-MTR-MTS bounds the MIV & MTR and the MTR & MTS bounds are first derived.

MIV & MTR

This section considers combining the MIV assumptions with the MTR assumption.
From MTR there is

$$K_0\{P(E=0|x,v=u)+P(z>t,E=1|x,v=u)\}+$$
$$E[y|x,v=u,z=t,E=1]P(z=t,E=1|x,v=u)+E[y|x,v=u,z<t,E=1]P(z<t,E=1|x,v=u)$$
$$\le E[y(t)|x,v=u]\le$$
$$K_1\{P(E=0|x,v=u)+P(z<t,E=1|x,v=u)\}+$$
$$E[y|x,v=u,z=t,E=1]P(z=t,E=1|x,v=u)+E[y|x,v=u,z>t,E=1]P(z>t,E=1|x,v=u)$$

And from MIV that $E[y(t)|x,v=u_1]\le E[y(t)|x,v=u]\le E[y(t)|x,v=u_2]$
for $u_2\ge u\ge u_1$.
Combining these assumptions:

$$sup_{(u_1\le u)}\{K_0\{P(E=0|x,v=u_1)+P(z>t,E=1|x,v=u_1)\}+$$
$$E[y|x,v=u_1,z=t,E=1]P(z=t,E=1|x,v=u_1)+E[y|x,v=u_1,z<t,E=1]P(z<t,E=1|x,v=u_1)\}$$
$$\le E[y(t)|x,v=u]\le$$
$$inf_{(u_2\ge u)}\{K_1\{P(E=0|x,v=u_2)+P(z<t,E=1|x,v=u_2)\}+$$
$$E[y|x,v=u_2,z=t,E=1]P(z=t,E=1|x,v=u_2)+E[y|x,v=u_2,z>t,E=1]P(z>t,E=1|x,v=u_2)\}$$

Therefore:

$$\Sigma_{u\in V}P(v=u|x)[sup_{(u_1\leq u)}\{K_0\{P(E=0|x,v=u_1)+P(z>t,E=1|x,v=u_1)\}+ E[y|x,v=u_1,z=t,E=1]P(z=t,E=1|x,v=u_1)+E[y|x,v=u_1,z<t,E=1]P(z<t,E=1|x,v=u_1)\}]$$
$$\leq E[y(t)|x]\leq$$
$$\Sigma_{u\in V}P(v=u|x)[inf_{(u_2\geq u)}\{K_1\{P(E=0|x,v=u_2)+P(z<t,E=1|x,v=u_2)\}+ E[y|x,v=u_2,z=t,E=1]P(z=t,E=1|x,v=u_2)+E[y|x,v=u_2,z>t,E=1]P(z>t,E=1|x,v=u_2)\}]$$

As the MTR assumption is also imposed here it implies the lower bound of $E[y(t)-y(s)|x]$ must not be smaller than zero, $E[y(t)-y(s)|x]\geq 0$ as in the MTR-only case. Unlike the MTR case, an additional complication now is that by the MTR-MIV assumption, the standard computation for the lower bound of $E[y(t)-y(s)|x,v=u]$ is not necessarily ≤ 0, so simply replacing the lower bound with 0 (as is done in the MTR and MTR-MTS cases) might result in lost information. Instead, the lower bound of $E[y(t)-y(s)|x,v=u]$ is replaced by 0 only when it is estimated to be less than zero.
Imposing these additional constraints ATE(x) is bounded:

$$\Sigma_{u\in V}P(v=u|x)*max\{0,[sup_{(u_{1t}\leq u)}\{K_0\{P(E=0|x,v=u_{1t})+P(z>t,E=1|x,v=u_{1t})\}+ E[y|x,v=u_{1t},z=t,E=1]P(z=t,E=1|x,v=u_{1t})+E[y|x,v=u_{1t},z<t,E=1]P(z<t,E=1|x,v=u_{1t})\} -\{inf_{(u_{2s}\geq u)}\{K_1\{P(E=0|x,v=u_{2s})+P(z<s,E=1|x,v=u_{2s})\}+ E[y|x,v=u_{2s},z=s,E=1]P(z=s,E=1|x,v=u_{2s})+E[y|x,v=u_{2s},z>s,E=1]P(z>s,E=1|x,v=u_{2s})\}\}]\}$$
$$\leq E[y(t)-y(s)|x]\leq$$
$$\Sigma_{u\in V}P(v=u|x)[inf_{(u_{2t}\geq u)}\{K_1\{P(E=0|x,v=u_{2t})+P(z<t,E=1|x,v=u_{2t})\}+ E[y|x,v=u_{2t},z=t,E=1]P(z=t,E=1|x,v=u_{2t})+E[y|x,v=u_{2t},z>t,E=1]P(z>t,E=1|x,v=u_{2t})\}]- \{\Sigma_{u\in V}P(v=u|x)[sup_{(u_{1s}\leq u)}\{K_0\{P(E=0|x,v=u_{1s})+P(z>s,E=1|x,v=u_{1s})\}+ E[y|x,v=u_{1s},z=s,E=1]P(z=s,E=1|x,v=u_{1s})+E[y|x,v=u_{1s},z<s,E=1]P(z<s,E=1|x,v=u_{1s})\}]\}$$

MTR & MTS

This section considers combining the MTS assumptions with the MTR assumption.
From MTS there is
$E[y(t)|x,z=t',E=1]\ \geq E[y(t)|x,z=t',E=0]$

However, $E[y(t)|x,z=t',E=e]$ is not identified for $t'\neq t\cap E\neq 1$. From MTR, $\forall j\in J, y_j(t_2)\geq y_j(t_1)$;

$$
\begin{aligned}
&\text{For } t' \leq t, E=1: && E[y(t')|x,z=t',E=1] \leq E[y(t)|x,z=t',E=1] \leq K_1\\
&\text{For } t' = t, E=1: && E[y(t)|x,z=t',E=1] = E[y(t)|x,z=t,E=1]\\
&\text{For } t' \geq t, E=1: && K_0 \leq E[y(t)|x,z=t',E=1] \leq E[y(t')|x,z=t',E=1]\\
&\text{For } t' \leq t, E=0: && K_0 \leq E[y(t)|x,z=t',E=0] \leq K_1\\
&\text{For } t' = t, E=0: && K_0 \leq E[y(t)|x,z=t',E=0] =\\
& && E[y(t)|x,z=t,E=0] \leq E[y(t)|x,z=t,E=1]\\
&\text{For } t' \geq t, E=0: && K_0 \leq E[y(t)|x,z=t',E=0] \leq\\
& && E[y(t')|x,z=t',E=0] \leq E[y(t')|x,z=t',E=1]
\end{aligned}
$$

Combining the information from the MTS and the MTR assumption, and since

$$
\begin{aligned}
E[y(t)|x] = {} & E[y|x,z=t,E=1]P(z=t,E=1|x)\\
& + E[y(t)|x,z\neq t,E=1]P(z\neq t,E=1|x)\\
& + E[y(t)|x,z=t,E=0]P(z=t,E=0|x)\\
& + E[y(t)|x,z\neq t,E=0]P(z\neq t,E=0|x)
\end{aligned}
$$

There is;

$$
\begin{gathered}
E[y|x,z=t,E=1]P(z=t,E=1|x) + \Sigma_{t'<t}\{E[y|x,z=t',E=1]P(z=t',E=1|x)\}\\
+K_0(P(E=0|x) + P(z>t,E=1|x))\\
\leq E[y(t)|x] \leq\\
E[y|x,z=t,E=1]P(z=t|x) + \Sigma_{t'>t}\{E[y|x,z=t',E=1]P(z=t'|x)\} + K_1P(z<t|x)
\end{gathered}
$$

As in the MTS & MTR bounds the lower bound is replaced by zero as the standard computation for the lower bound is always negative, which is not possible by the MTR assumption, giving the ATE(x) bounds:

$$
\begin{gathered}
0 \leq E[y(t)-y(s)|x] \leq\\
E[y|x,z=t,E=1]P(z=t|x) + \Sigma_{t'>t}\{E[y|x,z=t',E=1]P(z=t'|x)\} + K_1P(z<t|x) -\\
\{E[y|x,z=s,E=1]P(z=s,E=1|x) + \Sigma_{t'<s}\{E[y|x,z=t',E=1]P(z=t',E=1|x)\}+\\
K_0(P(E=0|x) + P(z>s,E=1|x))\}
\end{gathered}
$$

MIV-MTR-MTS

The MIV-MTR-MTS lower bound is estimated by taking the maximum of the lower MIV-MTS bound and the lower MTR-MTS bound. Similarly, the MIV-MTR-MTS upper bound is estimated by taking the minimum of the upper MIV-MTS bound and the upper MTR-MTS bound.
Therefore the MIV-MTR-MTS bound for the conditional mean function is:

$$max[\Sigma_{u \in V} P(v=u|x)[sup_{(u_1 \leq u)}\{K_0\{P(E=0|x,v=u_1)+P(z>t,E=1|x,v=u_1)\}+$$
$$E[y|x,v=u_1,z=t,E=1]P(z=t,E=1|x,v=u_1)+E[y|x,v=u_1,z<t,E=1]P(z<t,E=1|x,v=u_1)\}],$$
$$E[y|x,z=t,E=1]P(z=t,E=1|x)+\Sigma_{t'<t}\{E[y|x,z=t',E=1]P(z=t',E=1|x)\}$$
$$+K_0(P(E=0|x)+P(z>t,E=1|x))]$$
$$\leq E[y(t)|x] \leq$$
$$min[\Sigma_{u \in V} P(v=u|x)[inf_{(u_2 \geq u)}\{K_1\{P(E=0|x,v=u_2)+P(z<t,E=1|x,v=u_2)\}+$$
$$E[y|x,v=u_2,z=t,E=1]P(z=t,E=1|x,v=u_2)+E[y|x,v=u_2,z>t,E=1]P(z>t,E=1|x,v=u_2)\}],$$
$$E[y|x,z=t,E=1]P(z=t|x)+\Sigma_{t'>t}\{E[y|x,z=t',E=1]P(z=t'|x)\}+K_1 P(z<t|x)]$$

And the MIV-MTR-MTS bound for the ATE is:

$$max[\Sigma_{u \in V} P(v=u|x) * max\{0,[sup_{(u_{1t} \leq u)}\{K_0\{P(E=0|x,v=u_{1t})+P(z>t,E=1|x,v=u_{1t})\}+$$
$$E[y|x,v=u_{1t},z=t,E=1]P(z=t,E=1|x,v=u_{1t})+E[y|x,v=u_{1t},z<t,E=1]P(z<t,E=1|x,v=u_{1t})\}$$
$$-\{inf_{(u_{2s} \geq u)}\{K_1\{P(E=0|x,v=u_{2s})+P(z<s,E=1|x,v=u_{2s})\}+$$
$$E[y|x,v=u_{2s},z=s,E=1]P(z=s,E=1|x,v=u_{2s})+E[y|x,v=u_{2s},z>s,E=1]P(z>s,E=1|x,v=u_{2s})\}\}]\},$$
$$0]$$
$$\leq E[y(t)-y(s)|x] \leq$$
$$min[\Sigma_{u \in V} P(v=u|x)[inf_{(u_{2t} \geq u)}\{K_1\{P(E=0|x,v=u_{2t})+P(z<t,E=1|x,v=u_{2t})\}+$$
$$E[y|x,v=u_{2t},z=t,E=1]P(z=t,E=1|x,v=u_{2t})+E[y|x,v=u_{2t},z>t,E=1]P(z>t,E=1|x,v=u_{2t})\}]-$$
$$\{\Sigma_{u \in V} P(v=u|x)[sup_{(u_{1s} \leq u)}\{K_0\{P(E=0|x,v=u_{1s})+P(z>s,E=1|x,v=u_{1s})\}+$$
$$E[y|x,v=u_{1s},z=s,E=1]P(z=s,E=1|x,v=u_{1s})+E[y|x,v=u_{1s},z<s,E=1]P(z<s,E=1|x,v=u_{1s})\}]\},$$
$$E[y|x,z=t,E=1]P(z=t|x)+\Sigma_{t'>t}\{E[y|x,z=t',E=1]P(z=t'|x)\}+K_1 P(z<t|x)-$$
$$\{E[y|x,z=s,E=1]P(z=s,E=1|x)+\Sigma_{t'<s}\{E[y|x,z=t',E=1]P(z=t',E=1|x)\}+$$
$$K_0(P(E=0|x)+P(z>s,E=1|x))\}]$$

2.4 Data

The National Child Development Study (NCDS)[2] data is used to illustrate the identifying power of the bounds derived above, in the context of the returns to education in the UK. The NCDS is a longitudinal study that attempted to recruit all individuals born in England, Scotland and Wales in a single week in March 1958. Following the perinatal wave, data has been collected in 1965, 1969, 1974, 1981, 1991, 1999/2000, 2004, 2008 and 2013. Exam data was also collected in 1978. This analysis uses the labour market outcomes collected in 1991 to estimate the return from different levels of education. Education was measured by the "hqual33" variable, which measured highest level of education obtained by 1991 and has 7 categories: no information, no qualification, educational qualifications equivalent to NVQ1 (national vocational qualification level 1), NVQ2, NVQ3, NVQ4 and NVQ5/6. For details of how this variable was constructed see Smith (1991), and in particular see the appendix in Smith (1991) for the mapping between various qualifications and the levels of the "hqual33" variable. The only x variables (observable characteristics) controlled for are gender and region. Region is defined as region at age 16. If the individual did not participate in the 1974 round of the survey, the region recorded in the most recent round of the survey was used for that individual. There are 12 region categories; no information, North, North West, East and West Riding, North Midlands, Midlands, East, South East, South, South West, Wales and Scotland. The main outcome of interest is log hourly gross income, which is measured by dividing the reported "usual gross pay" by the usual period for usual gross pay in weeks $*$ usual hours for usual pay per week. Self employed individuals did not report their income in the same way, and so comparable hourly earnings can not be constructed for these individuals. An individual is deemed as being employed if they state that their current main economic activity is either full time or part time employee, self-employed if they state they are full-time or part-time self-employed and unemployed if they state they are unemployed or in home or family care. Individuals in full-time education, temporarily sick or disabled or permanently sick or disabled, or "other" are excluded from this analysis. The monotone instrumental variable used in this analysis is combined age that parents left education, which was reported in the 1974 wave of the study. Mid points of the response intervals were used, and those who left before age 13 were allocated 13, and those who left at age 23 or older were allocated 23. The distribution was then split into 5 categories to avoid data sparsity; less than 29, less than 30, less than 31, less than 36 and greater than 36. The thresholds were chosen to try and create a relatively even distribution over each category. (Individuals who do not have both mother's and father's level of education recorded are dropped from the MIV

[2]University of London. Institute of Education. Centre for Longitudinal Studies, National Child Development Study [computer files]. Colchester, Essex: UK Data Archive [distributor]

analysis). The analysis therefore assumes that parental education impacts selection into either/both education or the labour market. Furthermore, parental education is allowed to affect the conditional mean of potential labour market earnings but only monotonically. Of 18,555 individuals who participated in any round of the NCDS, 11,469 individuals responded to the fifth wave which is used to measure labour market outcomes in this analysis. For descriptives statistics of the variables discussed above for these individuals see Table 2.1, and for the sample distribution of observable characteristics see Table 2.2.

2.5 Empirical Analysis

All of the bounds derived above are estimated in turn and the results are illustrated in Table 2.3 - Table 2.9. Individuals for whom highest level of education is not recorded in the 1991 wave of the study are dropped. Log hourly gross income is trimmed at the 2.5th and 97.5th quantiles to remove sensitivity to outliers, and individuals reporting income at these levels are dropped from the analysis. K_0 is defined as the 2.5th quantile of the log hourly gross income distribution and K_1 as the 97.5th quantile. The majority of the analysis also drops those individuals who are self-employed, as they do not report their labour market income in the same way, as are those individuals for whom hourly gross income is not constructed (as they not report income, hours worked or period covered by most recent pay). Individuals who are temporarily/permanently sick or disabled or in full time education are dropped from all analysis. In addition, while the NCDS aims to be representative of the population who were born in a particular week in 1958, there is non-response and attrition. Without an assumption that these additional selection mechanisms are random, interpretation of the estimated bounds is difficult. The analysis in this chapter proceeds with this random selection assumption. This allows the bounds to be interpreted as the impact of education on potential formal labour market outcomes. Register data that has information on all wages, including the self-employed, could avoid some of these assumptions. Alternatively, the additional selection mechanisms could be dealt with in the same way that selection into education and selection into the labour market are treated in this chapter. For instance, Table 2.3 includes individuals who do not report their income, who fall into the income categories that are trimmed (below 2.5th quantile or above 97.5th quantile) or who are self-employed in the E=0 category under the worst case scenario. This is essentially redefining $E = 1$ as those individuals who work (not self-employed) and who report income in the 2.5th - 97.5th quantile range, and individuals with $E = 0$ as everybody else. This allocates self-employed individuals/individuals whose income is not known a maximum possible log hourly income of K_1 and a minimum possible log hourly income of K_0 in the computation of $E[y(t)|x]$.

The bounds on the returns to each level of education with no empirical data or assumptions are estimated as [K_0-K_1, K_1-K_0]. In our analysis K_0 is estimated to be 0.64 and K_1 at 2.85, resulting in bounds of [-2.21, 2.21] if no other empirical data was available. Table 2.3 - Table 2.9 show the extent to which these bounds are tightened under the various assumptions discussed above. Point identification is obtained only under ETS (exogeneous treatment selection into both education and the labour market) as shown in Table 2.5 . In this case, the point estimate of the labour market return to NVQ1 versus no qualification ranges between -.293 and 0.243 depending on the observable characteristics cell. The point estimate of NVQ2 versus NVQ1 ranges from 0.020 to 0.602, NVQ3 versus NVQ2 from 0.034 to 0.428, NVQ4 versus NVQ3 from -0.065 to 0.374, and NVQ5/6 versus NVQ4 from 0.040 to 0.310. As can be seen in the other cases, the estimated bounds are all fairly wide, and looking at Table 2.3 - 2.4 and Table 2.6 - Table 2.9, all straddle zero except in the cases where the lower bound is by assumption greater than zero.

Table 2.10 attempts to give an overview of how much identification power the alternative assumptions provide for the average treatment effects. For each assumption considered, the average bound tightness for consecutive ATEs (NVQ1 versus no qualification, NVQ2 versus NVQ1, etc.) was found by weighting the covariate specific ATE bound by the proportion of individuals with those particular characteristics in the sample. Then a straight average across ATE bounds for the different education levels was estimated to give a single average bound estimate for each assumption considered. From this table it is clear that the MTR assumption is key in tightening the bounds, with much of the tightness coming from the fact that the MTR assumption bounds the ATE above zero.

2.6 Conclusion

This chapter attempts to estimate the return to education under weak non-parametric assumptions, allowing for non-random selection into both education and the labour market. To our knowledge, dealing with both forms of selection non-parametrically has not been attempted before. Both forms of selection have been estimated in structural, parametric settings, but these approaches impose many strong assumptions.

Various restrictions were considered, and the bounds estimated under each case. In particular, the return to education was estimated under worst case bounds, with an exogenous treatment assumption, with a monotone treatment selection assumption, with a monotone treatment response assumption, with a monotone instrumental variable assumption and finally, under a combined monotone instrumental variable - monotone treatment response - monotone treatment selection assumption. In each case the estimated bounds were wide (with the obvious exception of the exogeneous treatment assumption case), and the lower bound of the ATE only above zero in the cases where it is so by assumption.

Extensions of this work could include inference on the bound estimates, development of a behavioural model that would formally provide the theoretical assumptions underpinning the imposed assumptions, and the estimation of conditional quantiles rather than the conditional mean which remove the necessity of imposing the type of support assumptions imposed in the current estimation.

Table 2.1: Descriptive Statistics

		Male	Female
	Gender	49.12%	50.88%
Region	North	7.45%	7.04%
	North West	11.55%	12.80%
	Riding	8.93%	8.43%
	North Midlands	8.13%	7.18%
	Midlands	9.89%	9.37%
	East	8.61%	8.69%
	South East	16.68%	17.48%
	South	6.99%	6.70%
	South West	6.80%	6.70%
	Wales	5.66%	5.74%
	Scotland	9.30%	9.85%
Employment	Full-time employee	73.09%	32.31%
	Part-time employee	0.76%	28.40%
	Full-time self-employed	15.51%	3.50%
	Part-time self-employed	0.28%	3.17%
	Unemployed	5.96%	2.02%
	Full-time education	0.37%	0.82%
	Temporarily sick/disabled	0.39%	0.26%
	Permanently sick/disabled	1.69%	0.84%
	Home/family care	0.39%	27.22%
	Other	0.67%	0.60%
	Missing	0.87%	0.87%
Parental Education	Less than or equal to 29	28.65%	28.26%
	Between 29 and 30	10.60%	10.51%
	Between 30 and 31	13.53%	13.95%
	Between 31 and 36	13.08%	12.96%
	Greater than 36	4.47%	4.92%
	Missing	29.68%	29.41%
Education	No Information	2.68%	1.95%
	No Qualification	11.00%	13.42%
	NVQ1	10.77%	13.37%
	NVQ2	29.64%	36.57%
	NVQ3	17.78%	9.72%
	NVQ4	13.93%	13.57%
	NVQ5/6	13.67%	10.83%
	Missing	0.51%	0.57%
	Mean hourly gross income*	£7.46	£5.59
	K_0**	0.64	
	K_1***	2.85	
	N	11,469	

* For individuals who are part-time or full-time employed (not self-employed), for whom highest level of obtained education is known and hourly income can be constructed

** 2.5th quantile of log hourly income for individuals who are part-time or full-time employed (not self-employed), for whom highest level of obtained education is known and hourly income can be constructed

*** 97.5th quantile of log hourly income for individuals who are part-time or full-time employed (not self-employed), for whom highest level of obtained education is known and hourly income can be constructed

Table 2.2: Sample Frequencies

Sample Frequencies							
Region	**Sex**	**No Qual**	**NVQ1**	**NVQ2**	**NVQ3**	**NVQ4**	**NVQ5/6**
North	**Male**	0.40%	0.51%	1.21%	0.63%	0.56%	0.46%
North	**Female**	0.66%	0.67%	1.54%	0.21%	0.46%	0.27%
North West	**Male**	0.48%	0.62%	1.48%	0.99%	1.05%	0.66%
North West	**Female**	0.95%	0.92%	2.46%	0.55%	1.09%	0.82%
Riding	**Male**	0.53%	0.42%	1.63%	0.54%	0.49%	0.63%
Riding	**Female**	0.88%	0.68%	1.87%	0.32%	0.59%	0.38%
North Midlands	**Male**	0.47%	0.45%	1.00%	0.73%	0.58%	0.48%
North Midlands	**Female**	0.62%	0.52%	1.54%	0.29%	0.56%	0.35%
Midlands	**Male**	0.46%	0.67%	1.33%	0.76%	0.58%	0.61%
Midlands	**Female**	0.55%	0.86%	2.06%	0.42%	0.80%	0.43%
East	**Male**	0.33%	0.60%	1.03%	0.60%	0.66%	0.51%
East	**Female**	0.55%	0.81%	1.75%	0.49%	0.59%	0.51%
South East	**Male**	0.60%	0.81%	2.21%	1.47%	1.07%	1.15%
South East	**Female**	1.01%	1.46%	3.75%	1.02%	1.03%	1.39%
South	**Male**	0.22%	0.36%	0.72%	0.58%	0.48%	0.52%
South	**Female**	0.34%	0.68%	1.28%	0.43%	0.53%	0.41%
South West	**Male**	0.21%	0.28%	0.78%	0.60%	0.48%	0.51%
South West	**Female**	0.34%	0.65%	1.43%	0.34%	0.38%	0.41%
Wales	**Male**	0.52%	0.21%	0.76%	0.39%	0.34%	0.36%
Wales	**Female**	0.53%	0.33%	1.20%	0.22%	0.40%	0.32%
Scotland	**Male**	0.72%	0.07%	1.46%	1.01%	0.63%	0.73%
Scotland	**Female**	1.19%	0.02%	1.65%	1.10%	1.15%	0.56%

Table 2.3: No Assumption Bounds

		NVQ1 vs No Qualification		NVQ2 vs NVQ1		NVQ3 vs NVQ2		NVQ4 vs NVQ3		NVQ5/6 vs NVQ4	
Region	**Sex**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**
North	**Male**	-2.042	2.066	-1.848	1.884	-1.804	1.822	-1.928	1.954	-1.962	1.986
North	**Female**	-2.048	2.046	-1.902	1.845	-1.848	1.989	-2.061	2.072	-2.060	2.062
North West	**Male**	-2.058	2.071	-1.908	1.928	-1.852	1.885	-1.900	1.939	-1.956	1.957
North West	**Female**	-2.077	2.072	-1.933	1.883	-1.892	1.987	-2.032	2.064	-1.993	2.029
Riding	**Male**	-2.059	2.075	-1.830	1.861	-1.817	1.810	-2.000	2.023	-1.958	2.023
Riding	**Female**	-2.060	2.057	-1.919	1.854	-1.870	1.997	-2.054	2.087	-2.033	2.046
North Midlands	**Male**	-2.027	2.047	-1.903	1.923	-1.845	1.869	-1.947	1.945	-1.980	2.008
North Midlands	**Female**	-2.038	2.049	-1.925	1.864	-1.884	2.010	-2.061	2.081	-2.033	2.049
Midlands	**Male**	-2.038	2.031	-1.873	1.911	-1.854	1.875	-1.955	1.991	-1.983	2.009
Midlands	**Female**	-2.057	2.040	-1.893	1.846	-1.863	1.977	-2.013	2.051	-2.018	2.029
East	**Male**	-2.047	2.031	-1.870	1.946	-1.894	1.906	-1.944	1.969	-1.974	1.973
East	**Female**	-2.067	2.064	-1.927	1.877	-1.885	1.992	-2.068	2.100	-2.060	2.072
South East	**Male**	-2.073	2.099	-1.866	1.970	-1.863	1.837	-1.960	1.949	-1.974	2.000
South East	**Female**	-2.085	2.089	-1.913	1.912	-1.919	1.951	-2.085	2.104	-2.029	2.072
South	**Male**	-2.082	2.096	-1.921	1.980	-1.900	1.896	-1.958	1.967	-1.948	1.994
South	**Female**	-2.099	2.063	-1.924	1.881	-1.862	1.991	-2.030	2.056	-2.044	2.035
South West	**Male**	-2.100	2.096	-1.940	1.966	-1.855	1.876	-1.904	1.960	-1.965	1.970
South West	**Female**	-2.083	2.042	-1.886	1.853	-1.861	1.966	-2.092	2.110	-2.089	2.107
Wales	**Male**	-1.988	2.039	-1.901	1.923	-1.863	1.858	-1.957	2.000	-1.963	1.994
Wales	**Female**	-2.081	2.069	-1.929	1.883	-1.869	1.999	-2.063	2.066	-2.023	2.056
Scotland	**Male**	-2.077	2.111	-1.918	1.957	-1.765	1.770	-1.894	1.922	-1.922	1.994
Scotland	**Female**	-2.060	2.152	-2.088	1.971	-1.863	1.936	-1.867	1.965	-1.967	1.970

Table 2.4: No Assumption Bounds - Missing Income & Self Employed Dropped

		NVQ1 vs No Qualification		**NVQ2 vs NVQ1**		**NVQ3 vs NVQ2**		**NVQ4 vs NVQ3**		**NVQ5/6 vs NVQ4**	
Region	**Sex**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**
North	**Male**	-2.002	2.032	-1.760	1.806	-1.706	1.728	-1.860	1.891	-1.902	1.932
North	**Female**	-2.016	2.014	-1.841	1.773	-1.777	1.946	-2.032	2.045	-2.031	2.033
North West	**Male**	-2.006	2.024	-1.805	1.831	-1.730	1.773	-1.794	1.847	-1.870	1.870
North West	**Female**	-2.050	2.043	-1.876	1.816	-1.827	1.942	-1.995	2.034	-1.949	1.992
Riding	**Male**	-2.013	2.033	-1.713	1.754	-1.695	1.686	-1.935	1.965	-1.880	1.965
Riding	**Female**	-2.034	2.032	-1.870	1.794	-1.813	1.961	-2.028	2.066	-2.003	2.019
North Midlands	**Male**	-1.957	1.984	-1.785	1.813	-1.705	1.738	-1.846	1.844	-1.891	1.931
North Midlands	**Female**	-2.002	2.016	-1.866	1.792	-1.816	1.968	-2.030	2.055	-1.996	2.016
Midlands	**Male**	-1.973	1.963	-1.744	1.797	-1.717	1.747	-1.858	1.907	-1.897	1.933
Midlands	**Female**	-2.027	2.006	-1.829	1.773	-1.794	1.930	-1.974	2.019	-1.980	1.993
East	**Male**	-1.977	1.954	-1.723	1.832	-1.758	1.775	-1.829	1.865	-1.872	1.871
East	**Female**	-2.037	2.033	-1.867	1.806	-1.816	1.945	-2.038	2.077	-2.028	2.042
South East	**Male**	-2.017	2.054	-1.724	1.871	-1.720	1.683	-1.857	1.841	-1.876	1.914
South East	**Female**	-2.062	2.067	-1.858	1.857	-1.865	1.903	-2.062	2.085	-1.996	2.047
South	**Male**	-2.013	2.035	-1.767	1.858	-1.734	1.728	-1.824	1.838	-1.809	1.879
South	**Female**	-2.079	2.036	-1.872	1.820	-1.798	1.950	-1.996	2.027	-2.013	2.003
South West	**Male**	-2.044	2.038	-1.803	1.843	-1.675	1.707	-1.749	1.833	-1.842	1.849
South West	**Female**	-2.052	2.001	-1.805	1.765	-1.774	1.906	-2.062	2.085	-2.059	2.082
Wales	**Male**	-1.925	1.991	-1.814	1.842	-1.766	1.759	-1.886	1.940	-1.893	1.933
Wales	**Female**	-2.052	2.037	-1.865	1.809	-1.792	1.951	-2.030	2.034	-1.981	2.022
Scotland	**Male**	-2.045	2.087	-1.848	1.896	-1.658	1.664	-1.818	1.853	-1.853	1.942
Scotland	**Female**	-2.040	2.145	-2.072	1.939	-1.818	1.900	-1.822	1.933	-1.935	1.939

Table 2.5: ETS - Missing Income & Self Employed Dropped

		NVQ1 vs No Qualification	**NVQ2 vs NVQ1**	**NVQ3 vs NVQ2**	**NVQ4 vs NVQ3**	**NVQ5/6 vs NVQ4**
Region	**Sex**	**ATE**	**ATE**	**ATE**	**ATE**	**ATE**
North	**Male**	0.204	0.110	0.127	0.121	0.199
North	**Female**	0.126	0.180	0.355	0.039	0.102
North West	**Male**	0.135	0.066	0.154	0.116	0.176
North West	**Female**	0.056	0.121	0.179	0.193	0.305
Riding	**Male**	0.119	0.080	0.058	0.140	0.233
Riding	**Female**	0.143	0.129	0.204	0.251	0.159
North Midlands	**Male**	0.125	0.020	0.120	0.057	0.190
North Midlands	**Female**	0.111	0.135	0.232	0.164	0.197
Midlands	**Male**	0.009	0.159	0.111	0.240	0.099
Midlands	**Female**	0.197	0.084	0.240	0.195	0.226
East	**Male**	-0.080	0.255	0.158	0.068	0.118
East	**Female**	0.155	0.091	0.272	0.231	0.119
South East	**Male**	0.241	0.204	0.037	0.080	0.093
South East	**Female**	0.173	0.099	0.146	0.175	0.131
South	**Male**	0.173	0.170	0.045	0.097	0.162
South	**Female**	0.042	0.139	0.428	0.091	0.040
South West	**Male**	0.009	0.155	0.084	0.280	0.043
South West	**Female**	0.069	0.214	0.120	0.185	0.168
Wales	**Male**	0.243	0.026	0.034	0.226	0.109
Wales	**Female**	-0.039	0.289	0.421	-0.065	0.310
Scotland	**Male**	-0.293	0.602	0.037	0.184	0.265
Scotland	**Female**	-0.141	0.289	0.173	0.374	0.208

Table 2.6: MTS - Missing Income & Self Employed Dropped

		NVQ1 vs No Qualification		NVQ2 vs NVQ1		NVQ3 vs NVQ2		NVQ4 vs NVQ3		NVQ5/6 vs NVQ4	
Region	**Sex**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**
North	**Male**	-1.964	1.989	-1.717	1.773	-1.674	1.717	-1.848	1.889	-1.899	1.930
North	**Female**	-1.851	1.891	-1.718	1.577	-1.581	1.939	-2.026	2.030	-2.015	2.019
North West	**Male**	-1.981	2.009	-1.790	1.798	-1.697	1.753	-1.774	1.838	-1.861	1.867
North West	**Female**	-1.934	1.967	-1.800	1.660	-1.671	1.903	-1.956	2.002	-1.916	1.970
Riding	**Male**	-1.964	2.015	-1.694	1.716	-1.657	1.680	-1.929	1.962	-1.878	1.963
Riding	**Female**	-1.832	1.950	-1.789	1.585	-1.604	1.931	-1.997	2.047	-1.984	2.015
North Midlands	**Male**	-1.924	1.981	-1.781	1.797	-1.689	1.730	-1.837	1.833	-1.881	1.929
North Midlands	**Female**	-1.881	1.955	-1.805	1.586	-1.610	1.922	-1.983	2.024	-1.965	1.998
Midlands	**Male**	-1.950	1.940	-1.721	1.757	-1.677	1.742	-1.852	1.905	-1.895	1.933
Midlands	**Female**	-1.943	1.932	-1.755	1.567	-1.588	1.907	-1.950	1.995	-1.956	1.983
East	**Male**	-1.963	1.935	-1.704	1.802	-1.729	1.770	-1.824	1.863	-1.870	1.869
East	**Female**	-1.948	1.920	-1.754	1.628	-1.638	1.891	-1.983	2.040	-1.991	2.018
South East	**Male**	-1.983	2.046	-1.716	1.846	-1.694	1.675	-1.848	1.835	-1.870	1.908
South East	**Female**	-1.980	1.982	-1.773	1.679	-1.687	1.856	-2.015	2.048	-1.959	2.021
South	**Male**	-1.983	2.022	-1.754	1.854	-1.731	1.718	-1.813	1.828	-1.799	1.877
South	**Female**	-1.999	1.900	-1.735	1.660	-1.638	1.904	-1.950	1.996	-1.983	1.971
South West	**Male**	-2.029	2.028	-1.793	1.829	-1.662	1.707	-1.749	1.833	-1.842	1.838
South West	**Female**	-1.980	1.890	-1.694	1.581	-1.591	1.864	-2.021	2.035	-2.009	2.036
Wales	**Male**	-1.854	1.991	-1.814	1.809	-1.733	1.750	-1.876	1.937	-1.890	1.933
Wales	**Female**	-1.879	1.985	-1.812	1.613	-1.597	1.924	-2.003	2.013	-1.961	1.999
Scotland	**Male**	-1.990	2.087	-1.848	1.869	-1.631	1.651	-1.805	1.849	-1.848	1.938
Scotland	**Female**	-1.869	2.141	-2.068	1.780	-1.659	1.822	-1.744	1.899	-1.901	1.926

Table 2.7: MTR - Missing Income & Self Employed Dropped

		NVQ1 vs No Qualification		**NVQ2 vs NVQ1**		**NVQ3 vs NVQ2**		**NVQ4 vs NVQ3**		**NVQ5/6 vs NVQ4**	
Region	**Sex**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**
North	**Male**	0.000	1.401	0.000	1.410	0.000	1.373	0.000	1.314	0.000	1.223
North	**Female**	0.000	1.428	0.000	1.525	0.000	1.689	0.000	1.685	0.000	1.667
North West	**Male**	0.000	1.388	0.000	1.393	0.000	1.371	0.000	1.305	0.000	1.188
North West	**Female**	0.000	1.470	0.000	1.531	0.000	1.652	0.000	1.658	0.000	1.626
Riding	**Male**	0.000	1.358	0.000	1.363	0.000	1.327	0.000	1.300	0.000	1.244
Riding	**Female**	0.000	1.464	0.000	1.546	0.000	1.704	0.000	1.713	0.000	1.684
North Midlands	**Male**	0.000	1.355	0.000	1.340	0.000	1.295	0.000	1.218	0.000	1.142
North Midlands	**Female**	0.000	1.442	0.000	1.526	0.000	1.685	0.000	1.691	0.000	1.672
Midlands	**Male**	0.000	1.344	0.000	1.376	0.000	1.356	0.000	1.306	0.000	1.206
Midlands	**Female**	0.000	1.411	0.000	1.496	0.000	1.637	0.000	1.642	0.000	1.601
East	**Male**	0.000	1.405	0.000	1.428	0.000	1.343	0.000	1.241	0.000	1.102
East	**Female**	0.000	1.513	0.000	1.581	0.000	1.710	0.000	1.709	0.000	1.670
South East	**Male**	0.000	1.520	0.000	1.497	0.000	1.327	0.000	1.194	0.000	1.075
South East	**Female**	0.000	1.603	0.000	1.630	0.000	1.657	0.000	1.647	0.000	1.614
South	**Male**	0.000	1.429	0.000	1.422	0.000	1.324	0.000	1.232	0.000	1.126
South	**Female**	0.000	1.489	0.000	1.570	0.000	1.703	0.000	1.683	0.000	1.632
South West	**Male**	0.000	1.327	0.000	1.349	0.000	1.332	0.000	1.282	0.000	1.148
South West	**Female**	0.000	1.421	0.000	1.532	0.000	1.683	0.000	1.703	0.000	1.700
Wales	**Male**	0.000	1.382	0.000	1.375	0.000	1.341	0.000	1.313	0.000	1.230
Wales	**Female**	0.000	1.463	0.000	1.552	0.000	1.698	0.000	1.684	0.000	1.667
Scotland	**Male**	0.000	1.378	0.000	1.395	0.000	1.364	0.000	1.327	0.000	1.254
Scotland	**Female**	0.000	1.489	0.000	1.491	0.000	1.627	0.000	1.680	0.000	1.622

Table 2.8: MIV - Missing Income & Self Employed Dropped

		NVQ1 vs No Qualification		NVQ2 vs NVQ1		NVQ3 vs NVQ2		NVQ4 vs NVQ3		NVQ5/6 vs NVQ4	
Region	**Sex**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**
North	**Male**	-1.962	2.003	-1.740	1.740	-1.626	1.705	-1.820	1.743	-1.729	1.791
North	**Female**	-1.941	2.010	-1.848	1.725	-1.737	1.916	-1.964	1.807	-1.806	1.138
North West	**Male**	-1.957	1.920	-1.673	1.778	-1.671	1.595	-1.660	1.778	-1.822	1.689
North West	**Female**	-2.027	2.014	-1.818	1.683	-1.716	1.829	-1.858	1.956	-1.888	1.621
Riding	**Male**	-1.993	1.922	-1.531	1.709	-1.662	1.576	-1.815	1.862	-1.817	1.456
Riding	**Female**	-2.006	1.961	-1.803	1.628	-1.659	1.887	-1.922	1.969	-1.899	1.701
North Midlands	**Male**	-1.912	1.893	-1.700	1.695	-1.579	1.645	-1.787	1.768	-1.831	1.690
North Midlands	**Female**	-1.955	1.909	-1.780	1.774	-1.799	1.954	-1.963	1.993	-1.943	1.768
Midlands	**Male**	-1.931	1.954	-1.728	1.754	-1.665	1.657	-1.788	1.765	-1.767	1.747
Midlands	**Female**	-2.007	1.902	-1.758	1.754	-1.772	1.668	-1.700	1.859	-1.822	1.727
East	**Male**	-1.872	1.819	-1.630	1.736	-1.725	1.718	-1.731	1.789	-1.821	1.510
East	**Female**	-2.007	2.007	-1.791	1.703	-1.753	1.701	-1.862	1.986	-1.945	1.940
South East	**Male**	-1.967	2.045	-1.560	1.787	-1.673	1.455	-1.763	1.740	-1.757	1.637
South East	**Female**	-2.065	2.059	-1.817	1.848	-1.846	1.822	-2.010	2.023	-1.933	1.889
South	**Male**	-1.971	2.003	-1.623	1.781	-1.669	1.584	-1.773	1.750	-1.709	1.585
South	**Female**	-2.071	2.038	-1.812	1.772	-1.743	1.816	-1.903	1.763	-1.793	1.921
South West	**Male**	-1.966	1.972	-1.726	1.716	-1.547	1.622	-1.701	1.657	-1.716	1.601
South West	**Female**	-2.012	1.933	-1.687	1.696	-1.760	1.783	-2.029	1.997	-1.966	2.014
Wales	**Male**	-1.811	1.845	-1.630	1.741	-1.727	1.522	-1.692	1.858	-1.850	1.755
Wales	**Female**	-2.033	1.923	-1.661	1.754	-1.730	1.748	-1.930	1.971	-1.906	1.925
Scotland	**Male**	-2.042	2.055	-1.717	1.881	-1.588	1.370	-1.564	1.668	-1.717	1.772
Scotland	**Female**	-1.998	2.107	-2.037	1.917	-1.760	1.829	-1.758	1.744	-1.803	1.791

Table 2.9: MIV-MTR-MTS - Missing Income & Self Employed Dropped

		MIV-MTR-MTS - Missing Income \& Self Employed Dropped									
		NVQ1 vs No Qualification		NVQ2 vs NVQ1		NVQ3 vs NVQ2		NVQ4 vs NVQ3		NVQ5/6 vs NVQ4	
Region	**Sex**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**	**LB ATE**	**UB ATE**
North	**Male**	0.000	1.288	0.000	1.280	0.000	1.162	0.000	1.036	0.000	1.080
North	**Female**	0.000	1.073	0.000	1.192	0.000	0.990	0.000	0.938	0.000	0.784
North West	**Male**	0.000	1.308	0.000	1.277	0.000	1.148	0.000	1.124	0.000	0.983
North West	**Female**	0.000	1.144	0.000	1.282	0.000	1.404	0.000	1.341	0.000	1.247
Riding	**Male**	0.000	1.263	0.000	1.246	0.000	0.974	0.000	0.847	0.000	0.730
Riding	**Female**	0.000	1.120	0.000	1.283	0.000	1.358	0.000	1.434	0.000	1.363
North Midlands	**Male**	0.000	1.194	0.000	1.049	0.000	0.995	0.000	0.890	0.000	0.853
North Midlands	**Female**	0.000	1.080	0.000	1.225	0.000	1.423	0.000	1.435	0.000	1.433
Midlands	**Male**	0.000	1.274	0.000	1.269	0.000	1.092	0.000	1.008	0.000	1.006
Midlands	**Female**	0.000	1.073	0.000	1.101	0.000	1.062	0.000	1.329	0.000	1.342
East	**Male**	0.000	1.279	0.000	1.213	0.000	1.055	0.000	0.914	0.000	0.712
East	**Female**	0.000	1.105	0.000	1.286	0.000	1.287	0.000	1.494	0.000	1.511
South East	**Male**	0.000	1.449	0.000	1.336	0.000	0.804	0.000	0.699	0.000	0.714
South East	**Female**	0.000	1.231	0.000	1.342	0.000	1.460	0.000	1.450	0.000	1.424
South	**Male**	0.000	1.384	0.000	1.270	0.000	0.993	0.000	0.901	0.000	0.747
South	**Female**	0.000	1.084	0.000	1.302	0.000	1.292	0.000	1.324	0.000	1.525
South West	**Male**	0.000	1.227	0.000	1.122	0.000	1.058	0.000	0.896	0.000	0.836
South West	**Female**	0.000	0.990	0.000	1.212	0.000	1.415	0.000	1.432	0.000	1.593
Wales	**Male**	0.000	1.186	0.000	1.147	0.000	0.986	0.000	1.132	0.000	1.063
Wales	**Female**	0.000	1.145	0.000	1.287	0.000	1.511	0.000	1.524	0.000	1.468
Scotland	**Male**	0.000	1.206	0.000	1.228	0.000	1.055	0.000	1.070	0.000	1.105
Scotland	**Female**	0.000	1.200	0.000	1.206	0.000	1.339	0.000	1.339	0.000	1.475

Table 2.10: Bound Tightness

	Bound Tightness
No Data*	4.42
No Assumptions with SE and missing**	3.97
No Assumptions	3.84
ETS	0.00
MTS	3.76
MTR	1.46
MIV	3.61
MIV-MTR-MTS	1.18

* Data was used to construct K_0 and K_1, which was estimated from 2.5th quantile and 97.5th quantile of constructed log hourly gross income

** This is the only case where self employed and those with no reported income are included in the bound analysis

Chapter 3

Review and Discussion of the MTE Estimator

3.1 Introduction

The marginal treatment effect (MTE) approach to programme evaluation allows for heterogeneity in the response to treatment, and under weak assumptions identifies a distribution of heterogeneous treatment effects. This approach combines advantages of both the reduced form approach and the structural method approach, and knowledge of the distribution of marginal treatment effects allows for estimation of a range of treatment effect parameters of interest (Heckman and Vytlacil, 1999). This chapter reviews the MTE approach in a binary treatment model, paying particular attention to estimation approaches. The use of the MTE in estimation of the returns to education is used as a running example, as this is the context in which it has been most frequently applied in the literature. This chapter also contributes to the literature by outlining how the MTE model can be used to estimate the selection effect, and to estimate whether an advantage exists for the treated group in either potential outcome (treated or non-treated). Furthermore, this chapter discusses how to decompose these effects into the component due to observable characteristics and the component due to unobservable characteristics. Finally, the chapter rigorously discusses the comparison of ATE, OLS and IV estimates (comparisons that are frequently made in the literature), and discusses what can be inferred from these comparisons.

This chapter proceed as follows, section 3.2 overviews the most general MTE approach. Section 3.3 discusses alternative estimation approaches of the MTE model, and the assumptions that are often additionally imposed to make estimation tractable. This section also discusses recent developments, including the extension of the MTE model to cases where only discrete instrumental variables are available, and the use of the MTE approach to estimate the distributions of potential outcomes. Section 3.4 briefly discusses tests of constancy of the MTE model. Section 3.5 discusses treatment effect estimation. Section 3.6 and 3.7 discuss estimation of selection effects and advantage. Section 3.8 discusses the implications of OLS, ATE, and IV relative orderings and section 3.9 concludes.

3.2 Marginal Treatment Effects

Björklund and Moffitt (1987) first introduced the concept of a marginal treatment effect by identifying a treatment effect for marginal individuals in a normal selection model. Heckman and Vytlacil (1999, 2001b,c,a, 2005, 2007); Heckman et al. (2006); Carneiro et al. (2010, 2011a) develop this approach, allowing for identification of MTE under more general specifications. Furthermore, Heckman and Vytlacil (1999) show that commonly reported treatment effects (ATE, ATT, LATE) can be expressed as a weighted average of the MTE. When the full distribution of MTE cannot be estimated, often it is possible to estimate bounds of the treatment effects (Heckman and Vytlacil, 1999, 2001b). In addition, the MTE approach has facilitated the definition and estimation of a policy relevant treatment effect (PRTE), (Heckman and Vytlacil, 2001b). Rather than using commonly reported treatment effect parameters to try and answer policy relevant questions, this approach advocates directly estimating the treatment effect that would result due to implementation of the new policy. A marginal policy relevant treatment effect (MPRTE) has also been defined by Carneiro et al. (2010, 2011a), and aims to estimate the impact of a marginal increase/decrease in the currently implemented policy.

The MTE was introduced initially in the context of binary treatment models. More recent papers, Heckman et al. (2006) and Heckman and Vytlacil (2007), extend this binary treatment case to models with more than two treatments, however this chapter focuses on the binary treatment case. The MTE approach is based on a potential outcome model combined with a latent variable model of treatment choice. Under the potential outcome model, each individual is associated with two potential outcomes Y_{1i}/Y_{0i} which is the outcome that would be observed had individual i received/not received treatment. The treatment considered in this analysis is whether or not an individual has obtained a college degree. Let $C_i = 1$ represent college education and let $C_i = 0$ represent less than college education, therefore the observed outcome, Y_i, can be modelled as follows:

$$Y_i = C_i Y_{1i} + (1 - C_i) Y_{0i}$$

Let the potential outcomes be modelled:

$$Y_{1i} = \mu_1(Xa_i, Xb_i, U_{1i})$$

$$Y_{0i} = \mu_0(Xa_i, Xb_i, U_{0i})$$

Where Xa_i and Xb_i are vectors of observable characteristics determined outside of the model. Xa_i is the set of observable characteristics that will enter both the college choice model and the potential outcome model, whereas the set Xb_i enter only the potential outcome model (either vector can be null). This notation is somewhat non-standard in the literature, however since the following discussion will frequently distinguish between the two sets of control variables distinct notation is allocated to each set. U_{1i} and U_{0i} are unobserved random variables. Since this is a model with non-separable errors, continuous, discrete and ordered outcome variables can all be modelled within this framework.

Much of the following analysis rests on the assumption of a latent variable/index model for selection into treatment. Assume the following model for selection into college education:

$$\begin{aligned} &C_i^* = \mu_c(Xa_i, Z_i) - Uc_i \\ &C_i = 1[C_i^* > 0] \\ &\Rightarrow C_i = 1[\mu_c(Xa_i, Z_i) - Uc_i > 0] \end{aligned}$$

Where Z_i is a non null vector of exogeneous instruments, of which at least one element is typically required to be continuous. It is useful to interpret $\mu_c(Xa_i, Z_i)$ as a measure of the ease of attending college, since those with high values of $\mu_c(Xa_i, Z_i)$ tend to go to college. For instance, in the empirical specification used in the following chapter, distance to nearest university at age 16 is one of the instruments used, and the identification strategy rests on the intuitive assumption that the nearer you live to university the easier it is to attend college, for example, due to reduced transport costs or living costs. This identification strategy has been used before, for instance in Carneiro et al. (2011a). Uc_i is an unobserved random variable that impacts upon the treatment choice decision. It might be useful to interpret Uc_i as an unobservable measure of ability, with higher values of Uc_i indicating lower ability. Alternatively, Uc_i can be interpreted as an unobservable measure of the cost of effort, as individuals with high values of Uc_i tend not to go to college.

Given the notation defined above, the marginal treatment effect can now be defined. The marginal treatment effect at a point $(Xa = xa, Xb = xb, Uc = uc)$ is defined:

$$MTE(xa, xb, uc) = E[Y_1 - Y_0 | Xa = xa, Xb = xb, Uc = uc]$$

The marginal treatment effect can be interpreted in a number of ways (Heckman and Vytlacil, 2005). MTE(xa, xb, uc) can be interpreted as the average treatment effect for an individual who has observable characteristics $(Xa = xa, Xb = xb)$ and unobservable costs of attending college of $Uc = uc$. Alternatively, the MTE(xa, xb, uc) can be interpreted as the average treatment effect of an individual who has observable characteristics $(Xa = xa, Xb = xb)$ and who would be indifferent between attending college and not if they were randomly assigned instrument value $Z = z$, where $\mu_c(xa, z) = uc$.

In order to derive an estimator for the MTE, the following assumptions as stated in Heckman and Vytlacil (2005) are imposed:

A1: $\mu_c(Xa, Z)$ is a nondegenerate random variable conditional on Xa, Xb
A2: The random vectors (U_1, Uc) and (U_0, Uc) are independent of Z conditional on Xa, Xb
A3: The distribution of Uc is absolutely continuous with respect to Lebesgue measure
A4: Both potential outcomes Y_1 and Y_0 have finite first moments
A5: $1 > P(C = 1|Xa = xa, Xb = xb) > 0$ for all $(xa, xb) \in \Omega(Xa, Xb)$, where $\Omega(.)$ denotes support

The propensity score, or the probability that an individual with observed characteristics $(Z = z, Xa = xa, Xb = xb)$ attends college is given by:

$$\begin{aligned} P(C = 1|Xa = xa, Xb = xb, Z = z) &= P(Uc < \mu_c(Xa, Z)|Xa = xa, Xb = xb, Z = z) \\ &= F_{Uc|Xa,Xb,Z}(\mu_c(Xa, Z)) \end{aligned}$$

Also, notice the selection into college equation can be rewritten as follows:

$$C = 1[F_{Uc|Xa,Xb,Z}(\mu_c(Xa, Z)) - F_{Uc|Xa,Xb,Z}(Uc) > 0]$$

Since $F_{Uc|Xa,Xb,Z}(.)$ is a monotonic transformation.
Following the notation in Carneiro and Lee (2009), define $V = F_{Uc|Xa,Xb,Z}(Uc)$ and $P = F_{Uc|Xa,Xb,Z}(\mu_c(Xa, Z))$, where P now represents the propensity score. Therefore the selection into college equation can be rewritten as:

$$C = 1[V < P]$$

Therefore, an individual with $V < P$ attends college, and an individual with $V = P$ is just indifferent between attending college or not. The MTE distribution will now be defined over (Xa, Xb, V), rather than over (Xa, Xb, Uc). Later the fact that the distribution of V conditional on $(Xa, Xb, Z) \sim Unif[0, 1]$ will be used. To see this notice that for any $a \in [0, 1]$

$$\begin{aligned} P[F_{Uc|Xa,Xb,Z}(Uc) < a|Xa, Xb, Z] &= P[Uc < F^{-1}_{Uc|Xa,Xb,Z}(a)|Xa, Xb, Z] \\ &= F_{Uc|Xa,Xb,Z}(F^{-1}_{Uc|Xa,Xb,Z}(a)) \\ &= a \end{aligned}$$

The next step in deriving an estimator of the MTE is to write the expectation of the outcome, conditional on the observables (Xa, Xb) and conditional on the propensity score P.

$$E[Y|Xa = xa, Xb = xb, P = p] = E[Y_0|xa, xb, p] \\ + E[Y_1 - Y_0|xa, xb, p, C = 1] * Pr[C = 1|xa, xb, p]$$

Or,

$$E[Y|Xa = xa, Xb = xb, P = p] = E[\mu_0(xa, xb, uc)|xa, xb, p] \\ + E[\mu_1(xa, xb, uc) - \mu_0(xa, xb, uc)|xa, xb, p, C = 1] * Pr[C = 1|xa, xb, p]$$

Since (U_1, Uc) and (U_0, Uc) are independent of Z conditional on Xa, Xb, it is also true that (U_1, Uc) and (U_0, Uc) are independent of any function of Z conditional on Xa, Xb. P is simply a function of Z once you have already conditioned on (Xa, Xb). Additionally, plugging in the condition for selection into college the above can be rewritten as:

$$E[Y|Xa = xa, Xb = xb, P = p] = E[\mu_0(xa, xb, uc)|xa, xb] \\ + E[\mu_1(xa, xb, uc) - \mu_0(xa, xb, uc)|xa, xb, V < p] * Pr[V < p|xa, xb]$$

Notice this can also be written as:

$$E[Y|Xa = xa, Xb = xb, P = p] = E[\mu_0(xa, xb, uc)|xa, xb] \\ + \frac{\int_0^p E[\mu_1(xa, xb, uc) - \mu_0(xa, xb, uc)|xa, xb, V = p']f(V = p'|xa, xb)dp' * [\int_0^p f(V = p'|xa, xb)dp']}{\int_0^p f(V = p'|xa, xb)dp'}$$

Cancelling above and below the line and substituting for $f(V = p'|xa, xb)$:

$$E[Y|Xa = xa, Xb = xb, P = p] = E[\mu_0(xa, xb, uc)|xa, xb] \\ + \int_0^p E[\mu_1(xa, xb, uc) - \mu_0(xa, xb, uc)|xa, xb, V = p'] \int_{\mathbb{Z}} f(V = p'|xa, xb, z)f(z|xa, xb)dzdp'$$

Since V is uniform conditional on (Xa, Xb, Z) this simplifies to:

$$E[Y|Xa = xa, Xb = xb, P = p] = E[\mu_0(xa, xb, uc)|xa, xb] \\ + \int_0^p E[\mu_1(xa, xb, uc) - \mu_0(xa, xb, uc)|xa, xb, V = p']dp'$$

The derivative of this conditional expectation with respect to $P = p$ provides an expression for the MTE at the point $(xa, xb, V = p)$.

$$\begin{aligned}\frac{\partial E[Y|Xa = xa, Xb = xb, P = p]}{\partial p} &= E[\mu_1(xa, xb, uc) - \mu_0(xa, xb, uc)|xa, xb, V = p] \\ &= E[Y_1 - Y_0|Xa = xa, Xb = xb, V = p] \\ &= MTE(Xa = xa, Xb = xb, V = p)\end{aligned}$$

For every (Xa, Xb) combination, V ranges from 0 to 1, but the range over which $MTE(xa, xb, V)$ can be estimated depends on the support of P conditional on (Xa, Xb), and in this flexible approach can only be estimated at points of continuity of P. Heckman and Vytlacil (1999), Heckman and Vytlacil (2001b) and Heckman and Vytlacil (2005) show that various treatment parameters can be represented as weighted averages of the MTE(Xa,Xb,V) (e.g. the ATE(Xa,Xb), TT(Xa,Xb), LATE(Xa,Xb), TUT(Xa,Xb), PRTE(Xa,Xb), IV(Xa,Xb) and OLS(Xa,Xb)). The support of P determines which of these treatment parameters can be estimated from the MTE estimates, see Heckman and Vytlacil (1999) and Heckman and Vytlacil (2001b) for further discussion.

3.3 Estimation

Estimation of the MTE generally requires assumptions in addition to A1-A5. This is because estimation of the propensity score and the mean outcome conditional on (Xa, Xb, Z) suffers from the curse of dimensionality in finite samples. In addition, non-parametric estimation of the MTE imposes heavy requirements on the support of the propensity score conditional on (Xa, Xb). A number of different approaches have been followed in the literature to overcome these issues. Typically, the propensity score is estimated parametrically in the first step, using a logit or probit model (e.g. Carneiro et al. (2010), Carneiro et al. (2011c), Moffitt (2008)). In addition, it is often assumed that the potential outcomes are linear in parameters and have separable errors, and the independence assumption (A2) is often strengthened to full independence; (U_1, Uc) and (U_0, Uc) are independent of (Xa, Xb, Z).

Given these assumptions, a number of different estimation approaches have been followed in the literature. Three such approaches are discussed in the following. When normality is not assumed the MTE distribution can be estimated via Robinson's partially linear model or sieve estimation. Alternatively, if joint normality of the errors are assumed the MTE distribution can be estimated using the normal selection model.

Two additional estimation approaches are also discussed; that in Brinch et al. (2012) who propose an estimator for the MTE that allows for discrete instrumental variables, and that in Carneiro and Lee (2009) who estimate distributions of potential outcomes under somewhat less restrictive assumptions than those discussed above.

3.3.1 Robinson's Partially Linear Model

In the first stage of all three estimation approaches the propensity score is estimated typically using a logit or probit model. There are also the following models for the potential outcomes.[1]

$$Y_0 = (Xa\ Xb)\beta_0 + U_0$$
$$Y_1 = (Xa\ Xb)\beta_1 + U_1$$

[1] More generally, this could be written $Y_j = \mu_j(X)\beta_j + U_j$ for $j \in (0, 1)$, which is still linear in parameters but where the $\mu_j(.)$ function denotes that a flexible specification could be used, for instance with higher order terms and interactions

Therefore the expectation of Y conditional on (Xa, Xb, P) can be written as:

$$\begin{aligned} E[Y|Xa = xa, Xb = xb, P = p] = &(xa\ xb)\beta_0 + E[U_0|xa, xb, p] \\ &+ (xa\ xb)(\beta_1 - \beta_0)P(C = 1|xa, xb, p) \\ &+ E[U_1 - U_0|xa, xb, p, C = 1]P(C = 1|xa, xb, p) \end{aligned}$$

Which, due to the stronger independence assumption simplifies to:

$$\begin{aligned} E[Y|Xa = xa, Xb = xb, P = p] = &(xa\ xb)\beta_0 + E[U_0] \\ &+ (xa\ xb)(\beta_1 - \beta_0)P(V < p) + E[U_1 - U_0|V < p]P(V < p) \end{aligned}$$

Also, since $V = F_{Uc|Xa,Xb,Z}(Uc)$ was uniformly distributed on [0,1] conditional on (Xa, Xb, Z), and since U_c is now independent of (Xa, Xb, Z), this implies that $V = F_{Uc}(Uc)$, which is now unconditionally uniform. Also, denoting $E[U_1 - U_0|V < p]P(V < p)$ as K(p) the above can be rewritten as:

$$E[Y|Xa = xa, Xb = xb, P = p] = (xa\ xb)\beta_0 + E[U_0] + (xa\ xb)(\beta_1 - \beta_0)p + K(p)$$

Note that $K'(p) = E[U_1 - U_0|V = p]$
Splitting out the constant terms (since Xa,Xb contains a constant) rewrite the above as

$$E[Y|Xa = xa, Xb = xb, P = p] = \alpha_0 + (\tilde{Xa}\ \tilde{Xb})\tilde{\beta}_0 + E[U_0] + (\alpha_1 - \alpha_0)p + (\tilde{Xa}\ \tilde{Xb})(\tilde{\beta}_1 - \tilde{\beta}_0)p + K(p)$$

Where α_0, α_1 represent the constant coefficient in the potential outcome models, $\tilde{\beta}_0, \tilde{\beta}_1$ represent the remaining coefficients in the potential outcome models and where $\tilde{Xa}\ \tilde{Xb}$ represents the variables excluding the constant.

Taking the derivative with respect to p there is:

$$\begin{aligned} \frac{\partial E[Y|Xa = xa, Xb = xb, P = p]}{\partial p} &= (\alpha_1 - \alpha_0) + (\tilde{xa}\ \tilde{xb})(\tilde{\beta}_1 - \tilde{\beta}_0) + K'(p) \\ &= E[Y_1 - Y_0|Xa = xa, Xb = xb, V = p] \\ &= MTE(xa, xb, V = p) \end{aligned}$$

There is

$$\begin{aligned} E[Y|Xa = xa, Xb = xb, P] = &\alpha_0 + (\tilde{Xa}\ \tilde{Xb})\tilde{\beta}_0 + E[U_0] \\ &+ (\alpha_1 - \alpha_0)P + (\tilde{Xa}\ \tilde{Xb})(\tilde{\beta}_1 - \tilde{\beta}_0)P + K(P) \end{aligned}$$

$$\Rightarrow Y = \alpha_0 + (\tilde{Xa}\,\tilde{Xb})\tilde{\beta}_0 + E[U_0] + (\alpha_1 - \alpha_0)P + (\tilde{Xa}\,\tilde{Xb})(\tilde{\beta}_1 - \tilde{\beta}_0)P + K(P) + \epsilon$$

Where $E[\epsilon|Xa, Xb, P] = 0$

Taking expectations conditional on the propensity score:

$$E[Y|P] = \alpha_0 + E[\tilde{Xa}\,\tilde{Xb}|P]\tilde{\beta}_0 + E[U_0] + (\alpha_1 - \alpha_0)P + E[\tilde{Xa}\,\tilde{Xb}|P](\tilde{\beta}_1 - \tilde{\beta}_0)P + K(P)$$
$$\Rightarrow Y - E[Y|P] = ((\tilde{Xa}\,\tilde{Xb}) - E[\tilde{Xa}\,\tilde{Xb}|P])\tilde{\beta}_0 + ((\tilde{Xa}\,\tilde{Xb}) - E[\tilde{Xa}\,\tilde{Xb}|P])(\tilde{\beta}_1 - \tilde{\beta}_0)P + \epsilon$$

In order to estimate $\tilde{\beta}_0$ and $\tilde{\beta}_1 - \tilde{\beta}_0$, $E[W|P = \hat{p}]$ is estimated nonparametrically for $W = (Y, \tilde{Xa}, \tilde{Xb})$, (for instance using kernel estimation with cross validated bandwidth), using the fitted p̂ calculated in the first step. This is then used to estimate $W - \hat{E}[W|P = \hat{p}]$. Then $Y - \hat{E}[Y|P = \hat{p}]$ is regressed on $(\tilde{Xa}\,\tilde{Xb}) - \hat{E}[\tilde{Xa}\,\tilde{Xb}|P = \hat{p}]$ and $((\tilde{Xa}\,\tilde{Xb}) - \hat{E}[\tilde{Xa}\,\tilde{Xb}|P = \hat{p}]) * \hat{p}$, dropping a small fraction of the data for which there is low estimated density of p̂.

The K(P) is estimated in an extension of Robinson's partially linear model developed in Heckman et al. (1998). This approach is as follows:

$$Y - (\tilde{Xa}\,\tilde{Xb})\tilde{\beta}_0 - (\tilde{Xa}\,\tilde{Xb})(\tilde{\beta}_1 - \tilde{\beta}_0)P = \alpha_0 + E[U_0] + (\alpha_1 - \alpha_0)P + K(P) + \epsilon$$

Using P̂ estimated in the first step, and $\hat{\tilde{\beta}}_0, (\widehat{\tilde{\beta}_1 - \tilde{\beta}_0})$ estimated in the second step, the residual $Y - (\tilde{Xa}\,\tilde{Xb})\hat{\tilde{\beta}}_0 - (\tilde{Xa}\,\tilde{Xb})(\widehat{\tilde{\beta}_1 - \tilde{\beta}_0})\hat{P}$ is computed. The residual is estimated nonparametrically as a function of the propensity score, by analysing the relationship between this residual and P̂, using, for example, locally quadratic regression with cross validated bandwidth. Call this function L(P̂). L'(P̂) is then estimated either using analytical or numerical first order differentiation of the estimated L(P̂) function. This derivative estimates $(\alpha_1 - \alpha_0) + K'(\hat{P})$.

Recall $MTE(xa, xb, V = p) = (xa\,xb)(\beta_1 - \beta_0) + K'(p)$. Therefore, combining the above:

$$M\hat{T}E(Xa = xa, Xb = xb, V = \hat{p}) = (\tilde{xa}\,\tilde{xb})(\widehat{\tilde{\beta}_1 - \tilde{\beta}_0}) + \hat{L}'(\hat{p})$$
$$= (\tilde{xa}\,\tilde{xb})(\widehat{\tilde{\beta}_1 - \tilde{\beta}_0}) + (\alpha_1 - \widehat{\alpha_0 + K'}(\hat{p}))$$

Typically, what is reported in empirical work is MTE(V=p), which is the average marginal treatment effect for those individuals with V=p. The average MTE can be derived as follows:

$$MTE(V = \hat{p}) = E_{Xa,Xb|V}[E[Y_1 - Y_0|Xa, Xb, V = \hat{p}]]$$

$$\Rightarrow MTE(V = \hat{p}) = E[(\tilde{X}a\;\tilde{X}b)|V = \hat{p}](\tilde{\beta}_1 - \tilde{\beta}_0) + (\alpha_1 - \alpha_0 + K'(\hat{p}))$$

$$\Rightarrow MTE(V = \hat{p}) = E[(\tilde{X}a\;\tilde{X}b)](\tilde{\beta}_1 - \tilde{\beta}_0) + (\alpha_1 - \alpha_0 + K'(\hat{p}))$$

since Uc is independent of (Xa,Xb,Z), and $V = F_{Uc|Xa,Xb,Z}(Uc)$, this implies V is independent of (Xa,Xb,Z)

$$\Rightarrow M\hat{T}E(V = \hat{p}) = (\hat{\hat{\tilde{X}}}a\;\hat{\hat{\tilde{X}}}b)(\widehat{\tilde{\beta}_1 - \tilde{\beta}_0}) + (\alpha_1 - \widehat{\alpha_0 + K}'(\hat{p}))$$

Where $\hat{\hat{\tilde{X}}}a\;\hat{\hat{\tilde{X}}}b = \hat{E}[(\tilde{X}a\;\tilde{X}b)]$

3.3.2 Sieve Method:

As in the previous approach the propensity score is estimated in the first stage. As derived above:

$$E[Y|Xa = xa, Xb = xb, P = p] = (xa\;xb)\beta_0 + E[U_0] + (xa\;xb)(\beta_1 - \beta_0)p + K(p)$$

Again, splitting out the constants

$$E[Y|Xa = xa, Xb = xb, P = p] = \alpha_0 + (\tilde{X}a\;\tilde{X}b)\tilde{\beta}_0 + E[U_0] + (\alpha_1 - \alpha_0)P + (\tilde{X}a\;\tilde{X}b)(\tilde{\beta}_1 - \tilde{\beta}_0)P + K(P)$$

$$\Rightarrow Y = \alpha_0 + (\tilde{X}a\;\tilde{X}b)\tilde{\beta}_0 + E[U_0] + (\alpha_1 - \alpha_0)P + (\tilde{X}a\;\tilde{X}b)(\tilde{\beta}_1 - \tilde{\beta}_0)P + K(P) + \epsilon$$

Where $E[\epsilon|Xa, Xb, P] = 0$.

The sieve approach to estimating the MTE estimates $K(P)$ using a flexible functional form such as series/spline specifications. For example, Moffitt (2008) estimates the MTE of the returns to college education in the UK using the following $K(P)$ specifications; a linear function, a quadratic function, a cubic function, a quadratic function with a median spline break and a quadratic function with quartile spline breaks. Given a specific functional form assumption the model is then estimated by regressing Y on $(Xa\,Xb)$, $(Xa\,Xb)\hat{P}$ and the $K(\hat{P})$ function, where $\hat{P}$ is the estimated propensity score from the first step. Suppose for instance that a quadratic function is assumed for the $K(P)$ function[2]:

$$K(P) = \pi_0 + \pi_1 P + \pi_2 P^2$$

[2]The K(P) specification can be chosen by least squares cross-validation as suggested in Belloni et al. (2011)

Therefore there is the following model

$$E[Y|Xa = xa, Xb = xb, P = p] = \alpha_0 + (\tilde{Xa}\,\tilde{Xb})\tilde{\beta_0} + E[U_0] \\ + (\alpha_1 - \alpha_0)P + (\tilde{Xa}\,\tilde{Xb})(\tilde{\beta_1} - \tilde{\beta_0})P + \pi_0 + \pi_1 P + \pi_2 P^2$$

And taking the derivative with respect to p:

$$\frac{\partial E[Y|Xa = xa, Xb = xb, P = p]}{\partial p} = (\alpha_1 - \alpha_0) + (\tilde{xa}\,\tilde{xb})(\tilde{\beta_1} - \tilde{\beta_0}) + \pi_1 + 2\pi_2 p \\ = MTE(xa, xb, V = p)$$

In order to estimate the above, estimate the following by OLS:

$$Y = \theta_0 + (\tilde{Xa}\,\tilde{Xb})\beta_0 + \theta_1 P + (\tilde{Xa}\,\tilde{Xb})(\tilde{\beta_1} - \tilde{\beta_0})P + \pi_2 P^2 + \epsilon$$

Where $\theta_0 = (\alpha_0 + E[U_0] + \pi_0)$, and $\theta_1 = (\alpha_1 - \alpha_0 + \pi_1)$. The MTE is then estimated by plugging the estimated OLS coefficients from the above model into:

$$\frac{\partial E[Y|Xa = xa, Xb = xb, P = \hat{p}]}{\partial \hat{p}} = M\hat{T}E(Xa = xa, Xb = xb, V = \hat{p}) \\ = \hat{\theta_1} + (\tilde{Xa}\,\tilde{Xb})(\widehat{\tilde{\beta_1} - \tilde{\beta_0}}) + 2\hat{\pi_2}\hat{P}$$

As before, the average MTE(V=p) can be estimated:

$$MTE(V = \hat{p}) = E_{Xa,Xb|V}[E[Y_1 - Y_0|Xa, Xb, V = \hat{p}]]$$

$$\Rightarrow MTE(V = \hat{p}) = \theta_1 + E[(\tilde{Xa}\,\tilde{Xb})|V = \hat{p}](\tilde{\beta_1} - \tilde{\beta_0}) + 2\pi_2\hat{p}$$

$$\Rightarrow M\hat{T}E(V = \hat{p}) = \hat{\theta_1} + (\hat{\bar{\tilde{X}}}a\,\hat{\bar{\tilde{X}}}b)(\widehat{\tilde{\beta_1} - \tilde{\beta_0}}) + 2\hat{\pi_2}\hat{p}$$

3.3.3 Normal Selection Model

As in the previous two approaches the propensity score is estimated in the first stage. Using results from Heckman (1979), the MTE distribution can be estimated under the additional assumption of joint normality of the errors using a normal selection model. There is:

$$
\begin{aligned}
&Y_0 = (Xa\ Xb)\beta_0 + U_0 \\
&Y_1 = (Xa\ Xb)\beta_1 + U_1 \\
&C = 1[Uc < \mu_c(Xa, Z)] \\
&\begin{pmatrix} U_1 \\ U_0 \\ U_c \end{pmatrix} \Bigg| (Xa, Xb, Z) \sim N\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & \sigma_{10} & \sigma_{1c} \\ \sigma_{01} & \sigma_0^2 & \sigma_{0c} \\ \sigma_{c1} & \sigma_{c0} & 1 \end{pmatrix}\right]
\end{aligned}
$$

$$
\begin{aligned}
MTE(Xa, Xb, Uc) &= E[Y_1 - Y_0 | Xa, Xb, Uc] \\
&= E[Y_1 | Xa, Xb, Uc] - E[Y_0 | Xa, Xb, Uc] \\
&= (Xa\ Xb)(\beta_1 - \beta_0) + E[U_1 | Uc] - E[U_0 | Uc]
\end{aligned}
$$

Where, from normality

$$
\begin{aligned}
E[U_1 | Uc] &= \sigma_{1c} Uc \\
E[U_0 | Uc] &= \sigma_{0c} Uc
\end{aligned}
$$

Also,

$$
\begin{aligned}
E[Y_1 | Xa, Xb, \mu_c(Xa_i, Z_i) = a, C = 1] &= E[Y_1 | Xa, Xb, Uc < a] \\
&= (Xa\ Xb)\beta_1 + E[U_1 | Uc < a] \\
E[Y_0 | Xa, Xb, \mu_c(Xa_i, Z_i) = a, C = 0] &= E[Y_0 | Xa, Xb, Uc \geq a] \\
&= (Xa\ Xb)\beta_0 + E[U_0 | Uc \geq a]
\end{aligned}
$$

Where, from normality

$$
\begin{aligned}
E[U_1 | Uc \leq a] &= \sigma_{1c} E[Uc | Uc \leq a] \\
E[U_0 | Uc > a] &= \sigma_{0c} E[Uc | Uc > a]
\end{aligned}
$$

$$
\begin{aligned}
E[Uc|Uc \leq a] &= \int_{x-\infty}^{a} uc f_{uc}(uc|uc \leq a)duc \\
&= \int_{-\infty}^{a} uc f_{uc}(uc)/P(uc \leq a)duc \\
&= \int_{-\infty}^{a} uc\phi(uc)/\Phi(a)duc \\
&= -\phi(a)/\Phi(a) \\
\Rightarrow E[U_1|Uc \leq a] &= \sigma_{1c} * (-\phi(a)/\Phi(a))
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
E[Uc|Uc > a] &= \int_{a}^{\infty} uc f_{uc}(uc|uc > a)duc \\
&= \int_{a}^{\infty} uc f_{uc}(uc)/P(uc > a)duc \\
&= \int_{a}^{\infty} uc\phi(uc)/(1-\Phi(a))duc \\
&= \phi(a)/(1-\Phi(a)) \\
\Rightarrow E[U_0|Uc \geq a] &= \sigma_{0c}(\phi(a)/\left(1-\Phi(a)\right))
\end{aligned}
$$

Note that $P \equiv P(C = 1|Xa, Xb, Z) = \Phi(\mu_c(Xa, Z))$. And since conditioning on $\mu_c(Xa_i, Z_i) = a$ is equivalent to conditioning on $\Phi(\mu_c(Xa, Z)) = \Phi(a)$,

$$
\begin{aligned}
E[Y_1|Xa, Xb, P = \Phi(a), C = 1] &= (Xa\ Xb)\beta_1 - \sigma_{1c}(\phi(\Phi^{-1}(P))/P) \\
E[Y_0|Xa, Xb, P = \Phi(a), C = 0] &= (Xa\ Xb)\beta_0 + \sigma_{0c}(\phi(\Phi^{-1}(P))/(1-P))
\end{aligned}
$$

From a linear regression of the observed outcome for those who graduated from college on $(Xa, Xb, \phi(\Phi^{-1}(\hat{P}))/\hat{P})$, (β_1, σ_{1c}) can be estimated. From a linear regression of the observed outcome for those who did not graduate from college on $(Xa, Xb, \phi(\Phi^{-1}(\hat{P}))/(1-\hat{P}))$, (β_0, σ_{0c}) can be estimated. Therefore, the following can be estimated:

$$
M\hat{T}E(Xa, Xb, Uc) = (Xa\ Xb)(\hat{\beta}_1 - \hat{\beta}_0) + \hat{\sigma_{1c}}Uc - \hat{\sigma_{0c}}Uc
$$

or, defining over the unit interval for comparability with the previous methods,

$$
M\hat{T}E(Xa, Xb, V = \Phi(Uc)) = (Xa\ Xb)(\hat{\beta}_1 - \hat{\beta}_0) + \hat{\sigma_{1c}}\Phi^{-1}(V) - \hat{\sigma_{0c}}\Phi^{-1}(V)
$$

As before, the average $MTE(V = \hat{p})$ can be estimated:

$$MTE(V=\hat{p}) = E_{Xa,Xb|V}[E[Y_1 - Y_0|Xa, Xb, V=\hat{p}]]$$

$$\Rightarrow MTE(V=\hat{p}) = E[Xa\, Xb|V=\hat{p}](\beta_1 - \beta_0) + \sigma_{1c}\Phi^{-1}(\hat{p}) - \sigma_{0c}\Phi^{-1}(\hat{p})$$

$$\Rightarrow M\hat{T}E(V=\hat{p}) = (\hat{\hat{X}}a\, \hat{\hat{X}}b)(\hat{\beta}_1 - \hat{\beta}_0) + \hat{\sigma_{1c}}\Phi^{-1}(\hat{p}) - \hat{\sigma_{0c}}\Phi^{-1}(\hat{p})$$

3.3.4 Estimating MTE with Discrete Instrumental Variables

Brinch et al. (2012) also estimate the propensity score using a logit model in the first step and they also assume linearity in parameters and additive separability in the outcome equations. While they discuss estimation without the stronger independence assumption, their empirical illustration imposes it. The theoretical contribution of their paper is the extension of the MTE in cases where continuous instrumental variables are not available. In that case, parametric assumption about the shape of the MTE curve along with discrete instrumental variables can allow identification of the MTE curve.[3] Furthermore, this paper also shows that a separation estimation approach allows for a higher order specification of the MTE curve. In fact, with just a binary instrumental variable this approach allows a test of the external validity of a LATE estimate (i.e. whether the LATE is a consistent estimator for the ATE).

Heckman and Vytlacil (2007), Carneiro and Lee (2009) and Brinch et al. (2012) all discuss separation estimation approaches. In this approach, the participation decision is conditioned on in addition to the observable covariates and the propensity score in the conditional mean function. In Carneiro and Lee (2009) this approach is used in the estimation of distributions of potential outcomes in addition to distributions of treatment effects. In Brinch et al. (2012) it is shown that this approach allows for higher order estimation of the shape of the MTE curve when working with discrete instrumental variables.

As before the propensity score is estimated in the first stage. As before, suppose there is:

$$Y_0 = (Xa\, Xb)\beta_0 + U_0$$
$$Y_1 = (Xa\, Xb)\beta_1 + U_1$$

Therefore the expectation of Y_1 conditional on $(Xa, Xb, P, C=1)$ can be written as:

$$E[Y_1|Xa = xa, Xb = xb, P = p, C = 1] = (xa\, xb)\beta_1 + E[U_1|xa, xb, p, C = 1]$$

[3] Discrete instrumental variables are also sufficient when estimating the MTE curve using the normal selection model.

Which can be rewritten as:

$$E[Y_1|Xa = xa, Xb = xb, P = p, C = 1] = (xa\ xb)\beta_1 + E[U_1|xa, xb, V \leq p]$$

Or as:

$$\begin{aligned} E[Y_1|Xa = xa, Xb = xb, P = p, C = 1] =& (xa\ xb)\beta_1 \\ &+ \frac{\int_0^p E[U_1|xa, xb, V = p']f(V = p'|xa, xb)dp'}{\int_0^p f(V = p'|xa, xb)dp'} \\ =& (xa\ xb)\beta_1 \\ &+ \frac{\int_0^p E[U_1|xa, xb, V = p'] \int_Z f(V = p'|xa, xb, z')f(z|xa, xb)dz'dp'}{\int_0^p \int_Z f(V = p'|xa, xb, z')f(z|xa, xb)dz'dp'} \\ =& (xa\ xb)\beta_1 + \frac{\int_0^p E[U_1|xa, xb, V = p']dp'}{\int_0^p 1dp'} \\ =& (xa\ xb)\beta_1 + \frac{\int_0^p E[U_1|xa, xb, V = p']dp'}{p} \end{aligned}$$

since V is uniform conditional on (Xa, Xb, Z).
Denote

$$K_1(Xa = xa, Xb = xb, P = p) = E[U_1|xa, xb, V \leq p] = \frac{\int_0^p E[U_1|xa, xb, V = p']dp'}{p}$$

Note that

$$\begin{aligned} \frac{\partial K_1(Xa = xa, Xb = xb, P = p)}{\partial p} &= \frac{E[U_1|xa, xb, V = p]}{p} - \frac{\int_0^p E[U_1|xa, xb, V = p']dp'}{p^2} \\ \Rightarrow \frac{\partial K_1(Xa = xa, Xb = xb, P = p)}{\partial p} &= \frac{k_1(xa, xb, p)}{p} - \frac{K_1(xa, xb, p)}{p} \\ \Rightarrow k_1(xa, xb, p) &= \frac{\partial K_1(Xa = xa, Xb = xb, P = p)}{\partial p} * p + K_1(xa, xb, p) \end{aligned}$$

where $k_1(xa, xb, p) = E[U_1|xa, xb, v = p]$
And the expectation of Y_0 conditional on $(Xa, Xb, P, C = 0)$ can be written as:

$$E[Y_0|Xa = xa, Xb = xb, P = p, C = 0] = (xa\ xb)\beta_0 + E[U_0|xa, xb, p, C = 0]$$

Which can be rewritten as:

$$E[Y_0|Xa = xa, Xb = xb, P = p, C = 0] = (xa\ xb)\beta_0 + E[U_0|xa, xb, V > p]$$

Or as:

$$\begin{aligned}
E[Y_0|Xa = xa, Xb = xb, P = p, C = 0] =&(xa\ xb)\beta_0 \\
&+ \frac{\int_p^1 E[U_0|xa, xb, V = p']f(V = p'|xa, xb)dp'}{\int_p^1 f(V = p'|xa, xb)dp'} \\
=&(xa\ xb)\beta_0 \\
&+ \frac{\int_p^1 E[U_0|xa, xb, V = p'] \int_Z f(V = p'|xa, xb, z')f(z|xa, xb)dz'dp'}{\int_p^1 \int_Z f(V = p'|xa, xb, z')f(z|xa, xb)dz'dp'} \\
=&(xa\ xb)\beta_0 + \frac{\int_p^1 E[U_0|xa, xb, V = p']dp'}{\int_p^1 1dp'} \\
=&(xa\ xb)\beta_0 + \frac{\int_p^1 E[U_0|xa, xb, V = p']dp'}{1 - p}
\end{aligned}$$

Denote

$$K_0(Xa = xa, Xb = xb, P = p) = E[U_0|xa, xb, V > p] = \frac{\int_p^1 E[U_0|xa, xb, V = p']dp'}{1 - p}$$

Note that

$$\frac{\partial K_0(Xa=xa,Xb=xb,P=p)}{\partial p} = -\frac{E[U_0|xa,xb,V=p]}{1-p} + \frac{\int_p^1 E[U_0|xa,xb,V=p']dp'}{(1-p)^2}$$
$$\Rightarrow \frac{\partial K_0(Xa=xa,Xb=xb,P=p)}{\partial p} = -\frac{k_0(xa,xb,p)}{1-p} + \frac{K_0(xa,xb,p)}{1-p}$$
$$\Rightarrow k_0(xa,xb,p) = -\frac{\partial K_0(Xa=xa,Xb=xb,P=p)}{\partial p} * (1-p) + K_0(xa,xb,p)$$

where $k_0(xa,xb,p) = E[U_0|xa,xb,v=p]$
And therefore the MTE can be estimated using the separation approach from:

$$\begin{aligned} E[Y_1-Y_0|Xa=xa,Xb=xb,V=p] &= (xa\ xb)(\beta_1-\beta_0) + E[U_1-U_0|xa,xb,v=p] \\ &= (xa\ xb)(\beta_1-\beta_0) + k_1(xa,xb,p) - k_0(xa,xb,p) \\ &= (xa\ xb)(\beta_1-\beta_0) \\ &+ \frac{\partial K_1(Xa=xa,Xb=xb,P=p)}{\partial p} * p + K_1(xa,xb,p) \\ &+ \frac{\partial K_0(Xa=xa,Xb=xb,P=p)}{\partial p} * (1-p) - K_0(xa,xb,p) \end{aligned}$$

where sieve estimation methods similar to those discussed in the previous sections could be used to estimate β_1, β_0, the K_1, K_0 functions and their derivatives, k_1, k_0. Robinson's partially linear approach is no longer possible since the K_1, K_0 functions are dependent on (xa, xb) as well as the propensity score, as the stronger independence assumption has not been imposed yet.
In the case of discrete instrumental variables, the next step is to impose some functional form restriction for the K_1, K_0 functions. Suppose you want to model the MTE as a linear function. A linear specification for the K_1, K_0 functions will result in a linear specification. For instance, suppose one assumes

$$K_1(xa,xb,p) = a_{xaxb} + b_{xaxb}p$$
$$K_0(xa,xb,p) = c_{xaxb} + d_{xaxb}p$$

where the xaxb subscript denotes that these contant terms can vary across different Xa,

Xb combinations.
Therefore,

$$k_1(xa, xb, p) = a_{xaxb} + 2b_{xaxb}p$$
$$k_0(xa, xb, p) = c_{xaxb} - d_{xaxb} + 2d_{xaxb}p$$

These functions are estimated from the conditional mean expectations:

$$E[Y_1|Xa = xa, Xb = xb, P = p, C = 1] = (xa\ xb)\beta_1 + a_{xaxb} + b_{xaxb}p$$
$$\Rightarrow Y_1 = (xa\ xb)\beta_1 + a_{xaxb1} + b_{xaxb1}p$$
$$+ \sum_{xaxbj \in Xa, Xb \text{ for } j>1} 1[xaxb = xaxbj][(a_{xaxbj} - a_{xaxb1}) + (b_{xaxbj} - b_{xaxb1})p] + \epsilon_1$$
$$\text{where } E[\epsilon_1|xa, xb, p, C = 1] = 0$$
and where the set of xaxb combinations have been ordered from 1 to N
$$E[Y_0|Xa = xa, Xb = xb, P = p, C = 0] = (xa\ xb)\beta_0 + c_{xaxb} + d_{xaxb}p$$
$$\Rightarrow Y_0 = (xa\ xb)\beta_0 + c_{xaxb1} + d_{xaxb1}p$$
$$+ \sum_{xaxbj \in Xa, Xb \text{ for } j>1} 1[xaxb = xaxbj][(c_{xaxbj} - c_{xaxb1}) + (d_{xaxbj} - d_{xaxb1})p] + \epsilon_0$$
$$\text{where } E[\epsilon_0|xa, xb, p, C = 0] = 0$$

Note that a_{xaxb1}, c_{xaxb1} will not be separably identified from β_1, β_0 since there is a constant in $(xa\ xb)$. Denote

$$\tilde{\beta}_1 = \begin{pmatrix} B_{11} \\ B_{12} \\ . \\ . \\ . \\ B_{1Q} \end{pmatrix} + \begin{pmatrix} a_{xaxb1} \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}$$

and $\tilde{\beta}_0 = \begin{pmatrix} B_{01} \\ B_{02} \\ . \\ . \\ . \\ B_{0Q} \end{pmatrix} + \begin{pmatrix} c_{xaxb1} \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}$

where Xa,Xb has dimension Q, and where the constant is the first term in the (Xa, Xb)

vector.
Therefore, from above you can see that the MTE will be a linear specification, where:

$$
\begin{aligned}
E[Y_1 - Y_0|Xa = xa, Xb = xb, V = p] &= (xa\ xb)(\beta_1 - \beta_0) + a_{xaxb} - c_{xaxb} + d_{xaxb} + 2p(b_{xaxb} - d_{xaxb}) \\
&= (xa\ xb)(\tilde{\beta_1} - \tilde{\beta_0}) + d_{xaxb1} + 2p(b_{xaxb1} - d_{xaxb1}) \\
&+ \sum_{j>1} 1[xaxb = xaxbj][(a_{xaxbj} - a_{xaxb1}) - (c_{xaxbj} - c_{xaxb1}) \\
&+ (d_{xaxbj} - d_{xaxb1}) + 2p((b_{xaxbj} - b_{xaxb1}) - (d_{xaxbj} - d_{xaxb1}))]
\end{aligned}
$$

A discrete binary instrumental variable that has full support for each combination of $(xa\ xb) \in (Xa\ Xb)$ among those who go to college, and among those who do not go to college will identify the above MTE using the separation approach. Further more, this is sufficient for a test of the external validity of a LATE estimate for the ATE. If the MTE is found to be non-constant, (i.e. $b_{xaxbj} \neq d_{xaxbj} \forall j$, then the external validity of the LATE estimate is rejected.)
As pointed out in Brinch et al. (2012) if the separation approach is not taken, in order to identify a linear MTE, an instrumental variable that takes at least three distinct values is required for identification.
To see this, note that, given the linear specification assumption for K_1, K_0

$$
\begin{aligned}
E[Y|Xa = xa, Xb = xb, P = p] =& (xa\ xb)\beta_0 + E[U_0|xa, xb] + (xa\ xb)(\beta_1 - \beta_0)p \\
&+ E[U_1 - U_0|xa, xb, V \leq p]p
\end{aligned}
$$

Denoting $E[U_1 - U_0|xa, xb, V \leq p]p = K(xa, xb, p)$, note

$$
\begin{aligned}
K(xa, xb, p) &= \frac{\int_0^p (E[U_1 - U_0|xa, xb, V = p']dp')}{\int_0^p f(V = p'|xa, xb)dp'} * p \\
&= \frac{1}{p} \int_0^p E[U_1 - U_0|xa, xb, V = p']dp' * p \\
&= \int_0^p k_1(xa, xb, p)dp' - \int_0^p k_0(xa, xb, p)dp' \\
&= \int_0^p (a_x + 2b_x p)dp' - \int_0^p (c_x - d_x + 2d_x p)dp' \\
&= a_{xaxb}p + b_{xaxb}p^2 - ((c_{xaxb} - d_{xaxb})p + d_{xaxb}p^2) \\
&= a_{xaxb} - (c_{xaxb} - d_{xaxb})p + (b_{xaxb} + d_{xaxb})p^2
\end{aligned}
$$

Which, unlike the separation approach is not linear in p, and so a binary instrumental variable will not suffice for identification with this approach.

Brinch et al. (2012) also discuss the additional identification resulting from the stronger independence assumption (which they refer to as separability). The intuition here is that with this stronger assumption, the MTE curve is parallel with intercept shifts across different values of (xa, xb). Therefore, even with a binary instrumental variable a more flexible MTE curve can be estimated, since variation in the propensity score at different (Xa,Xb) points information on the shape of the MTE curve at different points of the unobservable affecting selection into treatment. The following illustrates the separation approach under the stronger independence assumption.
In contrast to before, the K_1, K_0 functions are independent of (xa, xb). There is now

$$E[U_1|Xa = xa, Xb = xb, V \leq p] = E[U_1|V \leq p] = K_1(p) = a + bp$$
$$E[U_0|Xa = xa, Xb = xb, V > p] = E[U_0|V > p] = K_0(p) = c + dp$$

Therefore,

$$E[U_1|Xa = xa, Xb = xb, V = p] = E[U_1|V = p] = k_1(p) = a + 2bp$$
$$E[U_0|Xa = xa, Xb = xb, V = p] = E[U_0|V = p] = k_0(p) = c - d + 2dp$$

These functions are estimated from the conditional mean expectations:

$$E[Y_1|Xa = xa, Xb = xb, P = p, C = 1] = (xa\ xb)\beta_1 + a + bp$$
$$\Rightarrow Y_1 = (xa\ xb)\beta_1 + a + bp + \epsilon_1$$
$$\text{where } E[\epsilon_1|xa, xb, p, C = 1] = 0$$
$$E[Y_0|Xa = xa, Xb = xb, P = p, C = 0] = (xa\ xb)\beta_0 + c + dp$$
$$\Rightarrow Y_0 = (xa\ xb)\beta_0 + c + dp + \epsilon_0$$
$$\text{where } E[\epsilon_0|xa, xb, p, C = 0] = 0$$

As before, a, c will not be separately identified from the constant coefficients in $(xa\ xb)$.

Denote $\tilde{\beta}_1 = \begin{pmatrix} B_{11} \\ B_{12} \\ . \\ . \\ . \\ B_{1Q} \end{pmatrix} + \begin{pmatrix} a \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}$

and $\tilde{\beta}_0 = \begin{pmatrix} B_{01} \\ B_{02} \\ . \\ . \\ . \\ B_{0Q} \end{pmatrix} + \begin{pmatrix} c \\ 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}$

Therefore, the MTE will be a linear specification, where:

$$
\begin{aligned}
E[Y_1 - Y_0|Xa = xa, Xb = xb, V = p] &= (xa\ xb)(\beta_1 - \beta_0) + a - c + d + 2p(b - d) \\
&= (xa\ xb)(\tilde{\beta}_1 - \tilde{\beta}_0) + d + 2p(b - d)
\end{aligned}
$$

Brinch et al. (2012) prove that without the stronger independence assumption, an instrumental variable that takes on N different values identifies an MTE of order (N-1). With the stronger independence assumption, if the X variables take on M different values, and the instrumental variable N different values, then an MTE of order (N-1)M can be identified.

3.3.5 Estimating Distributions of Potential Outcomes

Carneiro and Lee (2009) semiparametrically estimate distributions of potential outcomes using the separation approach.
In contrast to the previous approaches where most of the emphasis is on the distribution of treatment effects, this approach suggests a way of estimating distributions of potential outcomes (in addition to estimating the density of potential outcomes and quantiles of potential outcomes across different values of the unobservable affecting selection).
Interest is in estimating how $E[Y_1|Xa = xa, Xb = xb, V = p]$ and $E[Y_0|Xa = xa, Xb = xb, V = p]$ vary over V. As in the previous section, the separation approach is used in estimation.
In the empirical application, additive separability of the outcome equations are assumed, and the μ_1, μ_0 functions are modelled as quadratic functions of x, with a number of interactions terms. Furthermore, the stronger independence restriction is assumed. The

selection equation is modelled semiparametrically as a partially linear model, where some components enter additively and others are modelled non-parametrically using cubic B-splines. Cross-validation was used to determine the number of knots to use in addition to determining which interactions to include. Since the predicted propensity scores could lie outside the (0,1) interval, the propensity score estimates are trimmed above 1 and below 0.

Robinson's partially linear method was used to estimate the K_1, K_0 functions defined in the previous section as follows.

$$E[Y_1|Xa = xa, Xb = xb, C = 1, V = p] = (xa, xb)\beta_1 + E[U_1|xa, xb, V \leq p]$$
$$E[Y_1|Xa = xa, Xb = xb, C = 1, V = p] = (xa, xb)\beta_1 + E[U_1|V \leq p]$$
$$E[Y_1|Xa = xa, Xb = xb, C = 1, V = p] = (xa, xb)\beta_1 + K_1(p)$$

Note that under the model assumptions, the conditional mean outcome with treatment for individuals with some xa, xb characteristics, and a particular value of the unobservable affecting selection can be written

$$E[Y_1|Xa = xa, Xb = xb, V = p] = (xa, xb)\beta_1 + E[U_1|V = p]$$
$$E[Y_1|Xa = xa, Xb = xb, V = p] = (xa, xb)\beta_1 + k_1(p)$$

where $k_1(p) = E[U_1|V = p]$.

Furthermore, note that (as shown in the previous section):

$$E[U_1|V = p] = \frac{\partial K_1(p)}{\partial p} p + K_1(p)$$
$$\text{or } k_1(p) = \frac{\partial K_1(p)}{\partial p} p + K_1(p)$$

To see how $\beta_1, K_1(p)$ are estimated, note that

$$\begin{gathered} Y_1 = (xa, xb)\beta_1 + K_1(p) + \epsilon_1 \\ \text{where } E[\epsilon_1|Xa = xa, Xb = xb, C = 1, V = p] = 0 \\ \Rightarrow E[Y_1|p] = E[xa, xb|p]\beta_1 + K_1(p) \\ \Rightarrow Y - E[Y_1|p] = ((xa, xb) - E[xa, xb|p])\beta_1 + \epsilon_1 \end{gathered}$$

To estimate β_1, $E[W|P = \hat{p}]$ is calculated nonparametrically for $W = (Y_1, Xa, Xb)$, using the estimated propensity score calculated in the first step. This is then used to calculate $W - E[W|P = \hat{p}]$. Then $Y - E[Y|P = \hat{p}]$ is regressed on $(Xa, Xb) - E[Xa, Xb|P = \hat{p}]$, dropping a small fraction of observations for which there is low estimated density of $\hat{p}$. The residuals $Y_1 - (xa, xb)\hat{\beta}_1$ are used to estimate the $K_1(p)$ function, using:

$$Y_1 - (xa, xb)\hat{\beta}_1 = K_1(p) + \epsilon_1$$

The $K_1(p)$ function and it's derivative is estimated using local polynomial regression. Then the $\beta_1, K_1(p), p, \frac{\partial K_1(p)}{\partial p}$ estimates are plugged in to estimate:

$$\hat{E}[Y_1|Xa = xa, Xb = xb, V = \hat{p}] = (xa, xb)\hat{\beta}_1 + \frac{\partial \hat{K}_1(\hat{p})}{\partial \hat{p}}\hat{p} + \hat{K}_1(\hat{p})$$

Similarly, for $E[Y_0|Xa = xa, Xb = xb, V = p]$:

$$\begin{gathered} E[Y_0|Xa = xa, Xb = xb, V = p] = (xa, xb)\beta_0 + E[U_0|V = p] \\ E[Y_0|Xa = xa, Xb = xb, V = p] = (xa, xb)\beta_0 + k_0(p) \end{gathered}$$

where $k_0(p) = E[U_0|V = p]$.

Furthermore, note that (as shown in the previous section):

$$\begin{gathered} E[U_0|V = p] = -\frac{\partial K_0(p)}{\partial p} * (1 - p) + K_0(p) \\ \text{or } k_0(p) = -\frac{\partial K_0(p)}{\partial p} * (1 - p) + K_0(p) \end{gathered}$$

$$E[Y_0|Xa = xa, Xb = xb, C = 0, V = p] = (xa, xb)\beta_0 + E[U_0|xa, xb, V > p]$$
$$E[Y_0|Xa = xa, Xb = xb, C = 0, V = p] = (xa, xb)\beta_0 + E[U_0|V > p]$$
$$E[Y_0|Xa = xa, Xb = xb, C = 0, V = p] = (xa, xb)\beta_0 + K_0(p)$$
$$Y_0 = (xa, xb)\beta_0 + K_0(p) + \epsilon_0$$
$$\text{where } E[\epsilon_0|Xa = xa, Xb = xb, C = 0, V = p] = 0$$

Similarly to before, note that

$$Y_0 = (xa, xb)\beta_0 + K_0(p) + \epsilon_0$$
$$\text{where } E[\epsilon_0|Xa = xa, Xb = xb, C = 0, V = p] = 0$$
$$\Rightarrow E[Y_0|p] = E[xa, xb|p]\beta_0 + K_0(p)$$
$$\Rightarrow Y - E[Y_0|p] = ((xa, xb) - E[xa, xb|p])\beta_0 + \epsilon_0$$

β_0 is estimated in the same way as β_1. The $K_0(p)$ function is estimated by first forming the residuals $Y_0 - (xa, xb)\hat{\beta}_0$. Then using the following;

$$Y_0 - (xa, xb)\hat{\beta}_0 = K_0(p) + \epsilon_0$$

the $K_0(p)$ function and it's derivative is estimated using local polynomial regression. Then the $\beta_0, K_0(p), p, \frac{\partial K_0(p)}{\partial p}$ estimates are plugged in to estimate:

$$\hat{E}[Y_0|Xa = xa, Xb = xb, V = \hat{p}] = (xa, xb)\hat{\beta}_0 + \frac{\partial \hat{K}_0(\hat{p})}{\partial \hat{p}}\hat{p} + \hat{K}_0(\hat{p})$$

3.4 Testing Constancy of the MTE

Heckman et al. (2010) discuss two alternative methods of testing the constancy of the MTE estimate. The first is based on comparing LATE estimates, and the second is based on testing for linearity in the conditional mean function $E[Y|X, P]$ (if the conditional mean function is better estimated by a higher order polynomial, this is evidence against constancy of the MTE). The LATE comparison method estimates the MTE (for instance, using Robinson's partially linear method), and then compares estimates of LATEs over different intervals of the unobservable affected selection into treatment. As in the empirical illustration in Heckman et al. (2010), one might compare the LATE estimates for the interval below and above the median estimated propensity score (i.e. compare estimates of $E[Y_1 - Y_0|Xa, Xb, V \leq P_{0.5}]$ and $E[Y_1 - Y_0|Xa, Xb, V > P_{0.5}]$ where $P_{0.5}$ denotes the median propensity score - these LATEs can be calculated by taking a simple average of the MTE estimates over the relevant interval, since V is uniformly distributed when the stronger independence assumption is maintained). Similarly one might compare LATE estimates across different quartiles of the propensity score. If one rejects the null of a Wald test that the LATE estimates are the same across different intervals, then this is evidence against a constant MTE function. When the interval is split into more than two, all pairwise comparisons are made, and the null hypothesis test is based on the pair with the greatest absolute difference. Heckman et al. (2010) recommend using the Romano and Wolf (2005) stepdown procedure for multiple hypothesis testing. The second method for testing for constancy uses the spline estimation method for estimating the MTE, and tests whether the coefficients on any of the quadratic or higher order terms in P are statistically significant. In the case where the conditional mean function is only specified as a quadratic form (e.g. when it is assumed that $K(P) = \pi_0 + \pi_1 P + \pi_2 P^2$), this is simply a test for the significance of the coefficient on the quadratic term (π_2). When the $K(P)$ function is specified with higher order terms, again a stepdown procedure must be used to take into account the multiple hypothesis being tested. In the normal selection model, constancy of the MTE can be analysed by testing whether $\sigma_{1c} - \sigma_{0c}$ is significantly different from zero (Heckman et al., 2010).

Constancy of the MTE does not imply OLS is a consistent estimate of the ATE. As discussed in section 3.8.1, given a linear functional form assumption, OLS estimates

$$E[Y|X, C = 1] - E[Y|X, C = 0] = X(\beta_1 - \beta_0) + E[U_1|X, C = 1] - E[U_0|X, C = 0]$$

It can be the case that the MTE is constant:

$$\begin{aligned}
E[Y_1 - Y_0|X, V = p] &= X(\beta_1 - \beta_0) + E[U_1 - U_0|X, V = p] \\
E[Y_1 - Y_0|X, V = p] &= X(\beta_1 - \beta_0) + E[U_1 - U_0|V = p] \\
E[Y_1 - Y_0|X, V = p] &= X(\beta_1 - \beta_0) + K'(p) \\
E[Y_1 - Y_0|X, V = p] &= X(\beta_1 - \beta_0) + \pi_1
\end{aligned}$$

which implies that $E[U_1|X, V = p]$ and $E[U_0|X, V = p]$ are parallel, but not necessarily that $E[U_1|X, C = 1] = E[U_0|X, C = 0]$.

3.5 Estimating Treatment Effect Parameters

Heckman and Vytlacil (1999), Heckman and Vytlacil (2001b) and Heckman and Vytlacil (2005) show that various treatment parameters can be estimated as weighted averages of the MTE. In the following estimation of the ATE, TT and TUT are discussed.

$$
\begin{aligned}
ATE(xa,xb) &= \int_0^1 MTE(xa,xb,v)dv \\
TT(xa,xb) &= \int_0^1 MTE(xa,xb,v) f_{V|Xa,Xb}(v|Xa,Xb,C=1)dv \\
&= \int_0^1 MTE(xa,xb,v)\left(\left(\int_v^1 f(p|Xa=xa,Xb=xb)dp\right)\frac{1}{E(P|Xa=xa,Xb=xb)}\right)dv \\
TUT(xa,xb) &= \int_0^1 MTE(xa,xb,v) f_{V|Xa,Xb}(v|Xa,Xb,C=0)dv \\
&= \int_0^1 MTE(xa,xb,v)\left(\left(\int_0^v f(p|Xa=xa,Xb=xb)dp\right)\frac{1}{E((1-P)|Xa=xa,Xb=xb)}\right)dv
\end{aligned}
$$

Estimation of the weights also suffer from the curse of dimensionality, since the density of the propensity score conditional on (Xa,Xb) has to be estimated. However, Carneiro et al. (2011c) propose a simple simulation method for estimating the weights under the assumptions imposed in this section. Under the stronger independence assumption, V is assumed independent of (Xa, Xb, Z). Therefore, Carneiro et al. (2011c) suggest drawing a large number (N) from a uniform distribution for each individual observation (these draws represent draws from V for each individual), and then evaluating the $MTE(Xa, Xb, V)$ for each of the N draws for that individual. To estimate $ATE(Xa, Xb)$ for an individual, the average over the N estimated $MTE(Xa, Xb, V)$ is taken. To estimate $TT(Xa, Xb)$, the average over the N estimated $MTE(Xa, Xb, V)$ where $V < P$ is taken and to estimate $TUT(Xa, Xb)$, the average over the N estimated $MTE(Xa, Xb, V)$ where $V \geq P$ is taken. In addition, the average ATE in the population ($E[Y_1-Y_0]$) can be estimated by averaging over $ATE(Xa, Xb)$ in the sample. The average TT in the population ($\bar{TT} = E[Y_1-Y_0|C = 1]$) can be estimated by averaging over $TT(Xa, Xb)$ for those individuals in the sample who go to college, and the average TUT in the population ($T\bar{U}T = E[Y_1 - Y_0|C = 0]$) can be estimated by averaging over $TUT(Xa, Xb)$ for those individuals in the sample who do not go to college.

Two alternatives to this approach have been used in the literature. Both assume that the conditional density of the propensity score, $f(p|xa, xb)$ satisfies a single index sufficiency condition. The first approach, followed in the main analysis of Carneiro et al. (2011a) assumes that the single index is equal to the linear component of the MTE; $(xa\ xb)(\beta_1-\beta_0)$. The second approach, discussed and estimated as a misspecification test in Carneiro et al. (2011a) estimates the linear index semiparametrically using Ichimura (1993). However,

as noted in Carneiro et al. (2011a), this method is computationally intenstive, making standard error computation difficult/infeasible.
In the normal selection model you can estimate some of the parameters of interest without simulation. For instance, as shown in Heckman et al. (2003), in the normal selection model

$$ATE(Xa, Xb) = E[Y_1 - Y_0|Xa, Xb] = (Xa\,Xb)(\beta_1 - \beta_0)$$

$$TT(Xa, Xb, P) = E[Y_1 - Y_0|Xa, Xb, P, C = 1]$$

$$= E[Y_1 - Y_0|Xa, Xb, Uc < \Phi^{-1}(P)] = (Xa\,Xb)(\beta_1 - \beta_0) - (\sigma_{1c} - \sigma_{0c})(\phi(\Phi^{-1}(P))/P)$$

$$TUT(Xa, Xb, P) = E[Y_1 - Y_0|Xa, Xb, P, C = 0]$$

$$= E[Y_1 - Y_0|Xa, Xb, Uc \geq \Phi^{-1}(P)] = (Xa\,Xb)(\beta_1 - \beta_0) + (\sigma_{1c} - \sigma_{0c})(\phi(\Phi^{-1}(P))/(1 - P))$$

3.6 Selection

Maintaining the stronger assumptions discussed at the start of section 3.3 (linearity in parameters, separable errors and the stronger independence assumption), the next two sections discuss how to estimate whether those receiving treatment have a higher gain from that treatment than those not receiving it, and secondly whether the treated group have an advantage over the non-treated group in either potential outcome (treated or non-treated).
Using the running example where college is the treatment, selection into college is said to occur if the average gain from attending college is greater for those who actually attend college than for those who do not attend college. In addition, the total selection gain can be decomposed into a component due to observable characteristics and a component due to unobservable characteristics. Subtracting $TT(Xa, Xb)$ from $TUT(Xa, Xb)$ at a particular value of (Xa, Xb) gives the difference between the gain from unobservables for

individuals with that value of (Xa, Xb) who choose to go to college and the gain from unobservables for individual with that value of (Xa, Xb) who choose not to go to college.

$$TT(Xa, Xb) - TUT(Xa, Xb) = E[Y_1 - Y_0|Xa, Xb, C = 1] - E[Y_1 - Y_0|Xa, Xb, C = 0]$$

The difference between the average TT in the population and the average TUT in the population gives the total selection gain, which in the additively seperable model can be decomposed into a component owing to observable characteristics and a component owing to unobservable characteristics.

$$\begin{aligned} T\bar{T} - T\bar{U}T &= (E[(\tilde{Xa}\,\tilde{Xb})|C = 1] - E[(\tilde{Xa}\,\tilde{Xb})|C = 0])(\tilde{B_1} - \tilde{B_0}) \\ &+ (E[U_1 - U_0|C = 1] - E[U_1 - U_0|C = 0]) \end{aligned}$$

Where $(E[(\tilde{Xa}\,\tilde{Xb})|C = 1] - E[(\tilde{Xa}\,\tilde{Xb})|C = 0])(\tilde{B_1} - \tilde{B_0})$ is the selection component due to observables and $E[U_1 - U_0|C = 1] - E[U_1 - U_0|C = 0]$ is the component due to unobservables. The selection component due to observables can be estimated since $(\tilde{B_1} - \tilde{B_0})$ can be estimated using any of the estimation methods discussed previously, and by subtracting $(\hat{E}[(\tilde{Xa}\,\tilde{Xb})|C = 1] - \hat{E}[(\tilde{Xa}\,\tilde{Xb})|C = 0])(\widehat{\tilde{B_1} - \tilde{B_0}})$ from $T\hat{\bar{T}} - T\hat{\bar{U}}T$ the selection component due to unobservables can also be estimated.

3.7 Advantage

Given the model framework and assumptions, it is possible to compare whether graduates have an advantage over non-graduates in the graduate labour market, and this advantage can be decomposed into a component explained by differences in observable characteristics and a component due to differences in unobservable characteristics. It is also possible to see whether one group has an advantage over the other in the non-graduate labour market, and to test whether there is a two-skill labour market.

Graduates are said to have an advantage in the graduate labour market if $E[Y_1|C = 1] > E[Y_1|C = 0]$. Graduates are said to have an advantage in the non-graduate labour market if $E[Y_0|C = 1] > E[Y_0|C = 0]$. $E[Y_1|C = 1]$ and $E[Y_0|C = 0]$ can be estimated directly from the observed data.

The mean wages of non-graduates in the graduate labour market can be estimated using the following relationship:

$$\begin{aligned} E[Y_1|C = 0] &= E[Y_0|C = 0] + E[Y_1 - Y_0|C = 0] \\ &= E[Y_0|C = 0] + T\bar{U}T \end{aligned}$$

Similarly the mean wages of graduates in the non-graduate labour market can be estimated using the following:

$$
\begin{aligned}
E[Y_0|C=1] &= -(E[Y_1 - Y_0|C=1] - E[Y_1|C=1]) \\
&= -(\bar{TT} - E[Y_1|C=1])
\end{aligned}
$$

Furthermore, in the additively separable model, the relative advantages can be broken down into a component due to observable characteristics and a component owing to unobservable characteristics.
The graduate advantage in the graduate labour market can be broken down as follows:

$$
\begin{aligned}
E[Y_1|C=1] - E[Y_1|C=0] = {} & (E[Xa, Xb|C=1] - E[Xa, Xb|C=0])B_1 \\
& + E[U_1|C=1] - E[U_1|C=0]
\end{aligned}
$$

Once $E[Y_1|C=1] - E[Y_1|C=0]$ has been estimated, the unobservable contribution can be estimated by subtracting the observable contribution which can be estimated by plugging in $\hat{B}_1$, $E[Xa, \hat{Xb}|C=1]$ and $E[Xa, \hat{Xb}|C=0]$ estimates.[4]
Similarly, the graduate advantage in the non-graduate labour market can be broken down as follows:

$$
\begin{aligned}
E[Y_0|C=1] - E[Y_0|C=0] = {} & (E[Xa, Xb|C=1] - E[Xa, Xb|C=0])B_0 \\
& + E[U_0|C=1] - E[U_0|C=0]
\end{aligned}
$$

Once $E[Y_0|C=1] - E[Y_0|C=0]$ has been estimated, the unobservable contribution can be estimated by subtracting the observable contribution which can be estimated by plugging in $\hat{B}_0$, $\hat{E}[Xa, Xb|C=1]$ and $\hat{E}[Xa, Xb|C=0]$ estimates.
If graduates have an advantage in the graduate labour market and non-graduates in the non-graduate labour market we say there is a two-skill labour market, with individuals selecting into the sector in which they have an advantage.

3.8 ATE, OLS & IV estimates

Frequently in the literature of the return to education, IV estimates are compared with OLS estimates. It is often not clear what information this comparison provides. In this

[4]Note that in the spline/Robinson's partially linear model for estimating the MTE that although β_1 is not identified (only $\tilde{\beta_1}$, from $(\tilde{\beta_1} - \tilde{\beta_0}) + \tilde{\beta_0}$, you can still decompose the graduate advantage since $(E[Xa, Xb|C=1] - E[Xa, Xb|C=0])B_1 = (E[(\tilde{Xa}\,\tilde{Xb})|C=1] - E[(\tilde{Xa}\,\tilde{Xb})|C=0])(\tilde{B_1})$

section, the interpretation of each estimate along with the ATE parameter is discussed, and implications of the relative orderings analysed.

3.8.1 ATE

Given the assumptions of linearity and additive errors the ATE parameter is simply:

$$ATE(Xa, Xb) = E[Y_1 - Y_0|Xa, Xb] = (Xa, Xb)(B_1 - B_0) + E[U_1 - U_0|Xa, Xb]$$

3.8.2 OLS

Given the assumptions of linearity and additive errors this section discusses the interpretation of OLS estimates. The outcome Y can be written:

$$Y = (Xa, Xb)B_0 + (Xa, Xb)(B_1 - B_0)C + U_0 + (U_1 - U_0)C$$

Abadie (2003) shows that under the assumption $E[U_0 + (U_1 - U_0)C|Xa, Xb, C]$ belongs to the class of parametric functions with the specification $(Xa, Xb)\alpha_0 + (Xa, Xb)C\alpha_1$ then the OLS estimate of the following model estimates $E[Y|Xa, Xb, C]$;

$$Y = (Xa, Xb)\Pi_0 + (Xa, Xb)C\Pi_1 + \epsilon$$

where $E[\epsilon|X, C] = 0$, $\Pi_0 = B_0 + \alpha_0$ and $\Pi_1 = (B_1 - B_0) + \alpha_1$. Under functional form misspecification the above OLS estimates the best least squares approximation to that functional form specification.
Since

$$\begin{aligned} E[Y|Xa, Xb, C = 1] &= E[Y_1|Xa, Xb, C = 1] \\ E[Y|Xa, Xb, C = 0] &= E[Y_0|Xa, Xb, C = 0] \end{aligned}$$
$$\Rightarrow E[Y|Xa, Xb, C = 1] - E[Y|Xa, Xb, C = 0] = E[Y_1|Xa, Xb, C = 1] - E[Y_0|Xa, Xb, C = 0]$$

Therefore, the OLS estimate of the return to college education for a particular (Xa, Xb):

$$\begin{aligned} E[Y|Xa, Xb, C = 1] - E[Y|Xa, Xb, C = 0] &= E[Y_1|Xa, Xb, C = 1] - E[Y_0|Xa, Xb, C = 0] \\ &= (Xa, Xb)\Pi_1 \\ &= (Xa, Xb)(B_1 - B_0) + E[U_1|Xa, Xb, C = 1] - E[U_0|Xa, Xb, C = 0] \end{aligned}$$

Since $E[Y|Xa, Xb, C = 1] - E[Y|Xa, Xb, C = 0] = (Xa, Xb)\Pi_1$ is linear in (Xa,Xb), the mean OLS estimate of treatment effect in the population can be estimated from

$\bar{OLS} = (\bar{X}a, \bar{X}b)\Pi_1 = E[Y_1|\bar{X}a, \bar{X}b, C = 1] - E[Y_0|\bar{X}a, \bar{X}b, C = 0]$, where $(\bar{X}a, \bar{X}b) = E[Xa, Xb]$.[5]

3.8.3 IV

Suppose there is a binary instrument, $Z \in (Z_1, Z_2)$. If the IV model is estimated using the methodology proposed in Abadie (2003), and a number assumptions which are implied by the more restrictive version of the MTE model discussed in the previous section hold,[6] then Abadie (2003) shows that under the assumption $E[U_0 + (U_1 - U_0)C|Xa, Xb, C, C(Z_2) = 1, C(Z_1) = 0]$ belongs to a particular class of parametric functions, $E[Y|Xa, Xb, C, C(Z_2) = 1, C(Z_1) = 0]$ can be estimated, where $C(Z)$ is the treatment received had the individual been allocated instrument value Z. Furthermore, Abadie (2003) shows that

$$\begin{aligned} E[Y|Xa, Xb, C = 0, C(Z_2) = 1, C(Z_1) = 0] &= E[Y_0|Xa, Xb, Z = Z_1, C(Z_2) = 1, C(Z_1) = 0] \\ &= E[Y_0|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0] \end{aligned}$$

and

$$\begin{aligned} E[Y|Xa, Xb, C = 1, C(Z_2) = 1, C(Z_1) = 0] &= E[Y_1|Xa, Xb, Z = Z_2, C(Z_2) = 1, C(Z_1) = 0] \\ &= E[Y_1|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0] \end{aligned}$$

which implies

$$\begin{gathered} E[Y|Xa, Xb, C = 1, C(Z_2) = 1, C(Z_1) = 0] - E[Y|Xa, Xb, C = 0, C(Z_2) = 1, C(Z_1) = 0] = \\ E[Y_1 - Y_0|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0] \end{gathered}$$

Which is the treatment effect for switchers (individuals who would have gone to college if they received instrument value Z_2 but who would not have gone if they received instrument value Z_1).
Assume $E[U_0 + (U_1 - U_0)C|Xa, Xb, C, C(Z_2) = 1, C(Z_1) = 0]$ belongs to the class of parametric functions with the linear specification $(Xa, Xb)\gamma_0 + (Xa, Xb)C\gamma_1$. Therefore,

$$E[Y|Xa, Xb, C, C(Z_2) = 1, C(Z_1) = 0] = (Xa, Xb)\Psi_0 + (Xa, Xb)C\Psi_1 + \upsilon$$

where $\Psi_0 = B_0 + \gamma_0$, $\Psi_1 = (B_1 - B_0) + \gamma_1$ and $E[\upsilon|X, C, C(Z_2) = 1, C(Z_1) = 0] = 0$.
Abadie (2003) shows the following provides consistent estimates of the Ψ_0, Ψ_1 parameters:

[5] note $E[Y_1|\bar{X}a, \bar{X}b, C = 1] - E[Y_0|\bar{X}a, \bar{X}b, C = 0] \neq E[Y_1|C = 1] - E[Y_0|C = 0]$

[6] Does not impose the stronger independence assumption

$$(\hat{\Psi}_0, \hat{\Psi}_1) = \underset{(\Psi_0,\Psi_1)}{\text{argmin}} \frac{1}{n} \Sigma_{i=1}^{N} \kappa_i (y_i - (Xa, Xb)\Psi_0 + (Xa, Xb)C\Psi_1)^2$$

Where $\kappa_i = 1 - \frac{CI(Z_1)}{P(Z=Z_1|Xa,Xb)} - \frac{(1-C)I(Z_2)}{P(Z=Z_2|Xa,Xb)}$

Therefore, the treatment effect for switchers can be estimated from

$$\begin{aligned}
E[Y|Xa, Xb, C = 1, C(Z_2) = 1, C(Z_1) = 0] - E[Y|Xa, Xb, C = 0, C(Z_2) = 1, C(Z_1) = 0]\\
= E[Y_1 - Y_0|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0]\\
= (Xa, Xb)\Psi_1\\
= (Xa, Xb)(B_1 - B_0)\\
+E[U_1|Xa, Xb, C = 1, C(Z_2) = 1, C(Z_1) = 0] - E[U_0|Xa, Xb, C = 0, C(Z_2) = 1, C(Z_1) = 0]\\
= (Xa, Xb)(B_1 - B_0)\\
+E[U_1|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0] - E[U_0|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0]
\end{aligned}$$

Since $E[Y|Xa, Xb, C = 1, C(Z_2) = 1, C(Z_1) = 0] - E[Y|Xa, Xb, C = 0, C(Z_2) = 1, C(Z_1) = 0] = (Xa, Xb)\Psi_1$ is linear in (Xa,Xb), the mean IV estimate in the population for the instrumental variable Z can be estimated $\bar{IV} = E[Xa, Xb]\Psi_1 = E[Y_1 - Y_0|\bar{X}a, \bar{X}b, C(Z_2) = 1, C(Z_1) = 0]$.[7]

The above discusses how to estimate covariate specific LATE when you have a binary instrumental variable, multiple covariates and heterogeneity. More generally, with multiple instrumental variables, covariates and heterogeneous treatment effects, in order to estimate covariate specific local average treatment effects you can either estimate non-parametrically by partitioning the data for each X combination and estimating 2SLS on each subsample, or you can assume the covariate specific local average treatment effect is some deterministic function of X (Angrist and Pischke, 2009) (however this method typically involves many first stage regressions). Alternatively, Angrist and Pischke (2009) show that if you estimate 2SLS with a saturated first and second stage (dummy variable included for every possible combination of X), then the treatment estimate from the second stage estimates a weighted average of the covariate specific LATEs, where the weights are higher for values of X where the instrument has a bigger impact on treatment. However, it is not clear what the interpretation of the standard 2SLS is when you have multiple instrumental variables, covariates and heterogeneous treatment effects.

[7] note $E[Y_1 - Y_0|\bar{X}a, \bar{X}b, C(Z_2) = 1, C(Z_1) = 0] \neq E[Y_1 - Y_0|C(Z_2) = 1, C(Z_1) = 0]$

3.8.4 ATE versus OLS/IV

In empirical work estimating the returns to education it is often found that IV > OLS. Suppose for discussion purposes that the ATE lies between the IV and the OLS. In this section the implications of these orderings are discussed.

ATE versus OLS

Comparing the ATE estimate and the OLS estimate at a particular value of (Xa, Xb) estimates:

$$\begin{aligned} ATE(Xa, Xb) - OLS(Xa, Xb) &= E[U_1|Xa, Xb] - E[U_1|Xa, Xb, C = 1] - (E[U_0|Xa, Xb] - E[U_0|Xa, Xb, C = 0]) \\ &= (E[U_1|Xa, Xb, C = 0] - E[U_1|Xa, Xb, C = 1])(1 - P(C = 1|Xa, Xb)) \\ &+ (E[U_0|Xa, Xb, C = 0] - E[U_0|Xa, Xb, C = 1])P(C = 1|Xa, Xb) \end{aligned}$$

If this difference is positive, and if you assume individuals who choose to go to college have higher levels of the unobservable in the graduate market $(E[U_1|Xa, Xb, C = 0] - E[U_1|Xa, Xb, C = 1] < 0)$, this implies that individuals who choose not to go to college must have higher levels of the unobservable in the non-graduate market $(E[U_0|Xa, Xb, C = 0] - E[U_0|Xa, Xb, C = 1] > 0)$. This implies a two-skill model, where college graduates have an advantage relative to non-graduates in the graduate labour market (based on unobservables), whereas non-graduates have an advantage relative to graduate in the non-graduate labour market.
Given the model assumptions, whether or not one group have an advantage over the other in a particular market due to unobservables can be tested. Since;

$$\begin{aligned} TT(Xa, Xb) &= (Xa\ Xb)(\beta_1 - \beta_0) + E[U_1 - U_0|Xa, Xb, C = 1] \\ \text{and } OLS(Xa, Xb) &= (Xa\ Xb)(\beta_1 - \beta_0) + E[U_1|Xa, Xb, C = 1] - E[U_0|Xa, Xb, C = 0] \\ \Rightarrow TT(Xa, Xb) - OLS(Xa, Xb) &= E[U_0|Xa, Xb, C = 0] - E[U_0|Xa, Xb, C = 1] \end{aligned}$$

Similarly,

$$\begin{aligned} TUT(Xa, Xb) &= (Xa\ Xb)(\beta_1 - \beta_0) + E[U_1 - U_0|Xa, Xb, C = 0] \\ \text{and } OLS(Xa, Xb) &= (Xa\ Xb)(\beta_1 - \beta_0) + E[U_1|Xa, Xb, C = 1] - E[U_0|Xa, Xb, C = 0] \\ \Rightarrow TUT(Xa, Xb) - OLS(Xa, Xb) &= E[U_1|Xa, Xb, C = 0] - E[U_1|Xa, Xb, C = 1] \end{aligned}$$

It is possible to further interpret the comparison between conditional ATE and OLS estimates if we have some knowledge, or assume something about selection. For instance, if there is no selection into treatment, but $ATE(x) > OLS(x)$, then it must be the case that

non-graduates do better than graduates in both the graduate and non-graduate labour market due to unobservables. If there is positive selection into treatment, and the conditional ATE is higher than OLS, then it must be the case that non-graduates do better than graduates in the non-graduate labour market due to unobservables; they could do better/worse in terms of unobservables in the graduate labour market. Finally, if there is negative selection into treatment, and the conditional ATE is higher than OLS, then it must be the case that non-graduates do better than graduates in the graduate labour market due to unobservables; they could do better/worse in terms of unobservables in the non-graduate labour market.
To see the derivation for the negative selection case, notice:

$$
\begin{aligned}
ATE(Xa,Xb) - OLS(Xa,Xb) &= E[U_1|Xa,Xb,C=0] - E[U_1|Xa,Xb,C=1] \\
&+ (E[U_1|Xa,Xb,C=1] - E[U_0|Xa,Xb,C=1] \\
&- (E[U_1|Xa,Xb,C=0] - E[U_0|Xa,Xb,C=0]))P(C=1|Xa,Xb) \\
\\
&= E[U_1|Xa,Xb,C=0] - E[U_1|Xa,Xb,C=1] \\
&+ (TT(Xa,Xb) - TUT(Xa,Xb))P(C=1|Xa,Xb)
\end{aligned}
$$

Therefore, if there is negative selection $TT(Xa,Xb) - TUT(Xa,Xb) < 0$, then the result $ATE(Xa,Xb) - OLS(Xa,Xb) > 0$ must be explained by $E[U_1|Xa,Xb,C=0] - E[U_1|Xa,Xb,C=1] > 0$, i.e. non-graduates must do better in the graduate labour market than graduates.
To see the derivation for the positive selection case, notice firstly the difference between the conditional ATE and OLS could also be written as

$$
\begin{aligned}
ATE(Xa,Xb) - OLS(Xa,Xb) &= E[U_1|Xa,Xb] - E[U_1|Xa,Xb,C=1] - (E[U_0|Xa,Xb] - E[U_0|Xa,Xb,C=0]) \\
&= (E[U_1|Xa,Xb,C=0] - E[U_1|Xa,Xb,C=1])P(C=0|Xa,Xb) \\
&+ (E[U_0|Xa,Xb,C=0] - E[U_0|Xa,Xb,C=1])(1 - P(C=0|Xa,Xb)) \\
\\
&= E[U_0|Xa,Xb,C=0] - E[U_0|Xa,Xb,C=1] \\
&+ (E[U_1|Xa,Xb,C=0] - E[U_0|Xa,Xb,C=0] \\
&- (E[U_1|Xa,Xb,C=1] - E[U_0|Xa,Xb,C=1]))P(C=0|Xa,Xb) \\
\\
&= E[U_0|Xa,Xb,C=0] - E[U_0|Xa,Xb,C=1] \\
&+ (TUT(Xa,Xb) - TT(Xa,Xb))P(C=0|Xa,Xb)
\end{aligned}
$$

Therefore, if there is positive selection $TUT(Xa,Xb) - TT(Xa,Xb) < 0$, then the result $ATE(Xa,Xb) - OLS(Xa,Xb) > 0$ must be explained by $E[U_0|Xa,Xb,C=0] - E[U_0|Xa,Xb,C=1] > 0$, i.e. non-graduates must do better in the non-graduate labour market than graduates.
Finally, if there is no selection, such that $TT(Xa,Xb) = TUT(Xa,Xb)$, then from the previous two derivations it can be seen that if $ATE(Xa,Xb) - OLS(Xa,Xb) > 0$, then it must be the case that $E[U_1|Xa,Xb,C=0] - E[U_1|Xa,Xb,C=1] > 0$ and $E[U_0|Xa,Xb,C=0] - E[U_0|Xa,Xb,C=1] > 0$, i.e. non-graduates must do better in both the graduate and the non-graduate labour market than graduates.

<u>ATE versus IV</u>

Comparing the ATE estimate and the IV estimate at a particular value of (Xa,Xb):

$$
\begin{gathered}
IV(Xa,Xb) - ATE(Xa,Xb) = \\
E[U_1|Xa,Xb,C(Z_2)=1,C(Z_1)=0] - E[U_1|Xa,Xb] \\
- E[U_0|Xa,Xb,C(Z_2)=1,C(Z_1)=0] + E[U_0|Xa,Xb] \\
= E[U_1 - U_0|Xa,Xb,\mu_c(Xa,Z_1) \leq Uc < \mu_c(Xa,Z_2)](1 - P(Uc \in [\mu_c(Xa,Z_1),\mu_c(Xa,Z_2))|Xa,Xb)) \\
- E[U_1 - U_0|Xa,Xb,Uc \notin [\mu_c(Xa,Z_1),\mu_c(Xa,Z_2)])(1 - P(Uc \in [\mu_c(Xa,Z_1),\mu_c(Xa,Z_2))|Xa,Xb))
\end{gathered}
$$

If this difference is positive, it implies that the return owing to unobservable for switchers is higher than the average return for non-switchers, for the group of individuals with that level of observable characteristics. For IV to be higher than ATE, the return to switchers must be higher than either/both the return for never takers and the return for always takers. Define the region $Uc < \mu_c(Xa,Z_1) = A$ (always takers) $\mu_c(Xa,Z_1) \leq Uc < \mu_c(Xa,Z_2) = B$ (switchers), $Uc \geq \mu_c(Xa,Z_2) = C$ (never takers) for notational convenience. Therefore, note that $IV(Xa,Xb) - ATE(Xa,Xb) > 0$ implies:

$$
E[U_1 - U_0|Xa,Xb,Uc \in B] > E[U_1 - U_0|Xa,Xb,Uc \notin B]
$$

And

$$
\begin{gathered}
E[U_1 - U_0|Xa,Xb,Uc \notin B] = \\
E[U_1 - U_0|Xa,Xb,Uc \in A]P(Uc \in A|Xa,Xb,Uc \notin B) + \\
E[U_1 - U_0|Xa,Xb,Uc \in C]P(Uc \in C|Xa,Xb,Uc \notin B)
\end{gathered}
$$

Therefore,

$$E[U_1 - U_0|Xa, Xb, Uc \in B] > E[U_1 - U_0|Xa, Xb, Uc \notin B]$$

Implies

$$(E[U_1 - U_0|Xa, Xb, Uc \in B] - E[U_1 - U_0|Xa, Xb, Uc \in A]P(Uc \in A|Xa, Xb, Uc \notin B)+$$
$$(E[U_1 - U_0|Xa, Xb, Uc \in B] - E[U_1 - U_0|Xa, Xb, Uc \in C])P(Uc \in C|Xa, Xb, Uc \notin B) > 0$$

Therefore, either/both $E[U_1 - U_0|Xa, Xb, Uc \in B] - E[U_1 - U_0|Xa, Xb, Uc \in A]$, or $E[U_1 - U_0|Xa, Xb, Uc \in B] - E[U_1 - U_0|Xa, Xb, Uc \in C]$ must be greater than zero, i.e. switchers must have a higher return to participation due to unobservables than either/both always takers and never takers.

IV versus OLS

Comparing the IV estimate and the OLS estimate at a particular value of (Xa, Xb):

$$\begin{aligned} IV(Xa, Xb) - OLS(Xa, Xb) = \\ E[U_1|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0] - E[U_1|Xa, Xb, C = 1] \\ - E[U_0|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0] + E[U_0|Xa, Xb, C = 0] \\ = (E[U_1|Xa, Xb, Switcher] - E[U_1|Xa, Xb, AlwaysTaker])(1 - (\frac{A_x}{A_x + B_x})P(Z = Z_2|Xa, Xb, C = 1)) \\ - (E[U_0|Xa, Xb, Switcher] - E[U_0|Xa, Xb, NeverTaker])(1 - (\frac{A_x}{A_x + C_x})P(Z = Z_1|Xa, Xb, C = 0)) \end{aligned}$$

This derivation is not as straightforward as the previous cases, and is shown in the appendix.

A_x represents $P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb)$, (the probability of being a switcher conditional on (Xa, Xb)). B_x represents $P(U_c \leq \mu_c(Xa, Z_1)|Xa, Xb)$, (the probability of being an always taker conditional on (Xa, Xb)). C_x represents $P(U_c > \mu_c(Xa, Z_2)|Xa, Xb)$, (the probability of being a never taker conditional on (Xa, Xb)). $(1-(\frac{A_x}{A_x+B_x})P(Z = Z_2|Xa, Xb, C = 1))$ represents the proportion of college graduates who are always takers conditional on (Xa, Xb), and $(1 - (\frac{A_x}{A_x+C_x})P(Z = Z_1|Xa, Xb, C = 0))$ represents the proportion of non-college graduates who are never takers conditional on (Xa, Xb).

Suppose this difference is positive (i.e. suppose $IV(x) > OLS(x)$ as is often reported in the literature), and suppose that individuals who always choose to go to college have higher levels of the unobservable in the graduate market compared to individuals who only choose to go to college if they get a favourable instrumental variable allocation. Then the positive result implies that individuals who choose never to go to college must have higher levels of the unobservable in the non-graduate market than the switchers (individuals who only go to college if they get a favourable instrumental variable allocation). Again, this would be evidence in favour of a two skill labour market.

3.9 Conclusion

This chapter summarises the literature on estimation of the MTE model, discusses alternative estimation approaches and the assumptions necessary therein. This chapter also reviews alternative methods for testing the constancy of the MTE, and for estimating various treatment effect parameters from the estimated MTE model. This chapter contributes to the literature by discussing how the model can be used to estimate selection and advantage effects, and how to decompose these into a component due to observable/unobservable characteristics. This chapter also discusses in detail how conditional ATE, IV and OLS estimates can be interpreted in the context of the model, and what can be inferred from comparison of these treatment effect parameters.

3.A Appendix

The derivation of the OLS and IV comparison discussed in section 3.8.4 is given below: There is

$$
\begin{aligned}
E[U_1|Xa, Xb, C=1] =& E[U_1|Xa, Xb, Switcher, C=1]P(Switcher|Xa, Xb, C=1) \\
&+ E[U_1|Xa, Xb, AlwaysTaker, C=1](1 - P(Switcher|Xa, Xb, C=1)) \\
=& E[U_1|Xa, Xb, \mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2), C=1]P(Switcher|Xa, Xb, C=1) \\
&+ E[U_1|Xa, Xb, Uc \leq \mu_c(Xa, Z_1), C=1](1 - P(Switcher|Xa, Xb, C=1)) \\
=& E[U_1|Xa, Xb, \mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2), Z=Z_2]P(Switcher|Xa, Xb, C=1) \\
&+ E[U_1|Xa, Xb, Uc \leq \mu_c(Xa, Z_1), Z=(Z_1, Z_2)](1 - P(Switcher|Xa, Xb, C=1)) \\
=& E[U_1|Xa, Xb, \mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)]P(Switcher|Xa, Xb, C=1) \\
&+ E[U_1|Xa, Xb, Uc \leq \mu_c(Xa, Z_1)](1 - P(Switcher|Xa, Xb, C=1)) \\
=& E[U_1|Xa, Xb, Switcher]P(Switcher|Xa, Xb, C=1) \\
&+ E[U_1|Xa, Xb, AlwaysTaker](1 - P(Switcher|Xa, Xb, C=1))
\end{aligned}
$$

Where the fourth equality follows from the assumption that the joint distribution (U_1, U_c) is independent from Z conditional on (Xa, Xb).
Similarly,

$$
\begin{aligned}
E[U_0|Xa, Xb, C=0] =& E[U_0|Xa, Xb, Switcher, C=0]P(Switcher|Xa, Xb, C=0) \\
&+ E[U_0|Xa, Xb, NeverTaker, C=0](1 - P(Switcher|Xa, Xb, C=0)) \\
=& E[U_0|Xa, Xb, \mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2), C=0]P(Switcher|Xa, Xb, C=0) \\
&+ E[U_0|Xa, Xb, Uc > \mu_c(Xa, Z_2), C=0](1 - P(Switcher|Xa, Xb, C=0)) \\
=& E[U_0|Xa, Xb, \mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2), Z=Z_1]P(Switcher|Xa, Xb, C=0) \\
&+ E[U_0|Xa, Xb, Uc > \mu_c(Xa, Z_2), Z=(Z_1, Z_2)](1 - P(Switcher|Xa, Xb, C=0)) \\
=& E[U_0|Xa, Xb, \mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)]P(Switcher|Xa, Xb, C=0) \\
&+ E[U_0|Xa, Xb, Uc > \mu_c(Xa, Z_2)](1 - P(Switcher|Xa, Xb, C=0)) \\
=& E[U_0|Xa, Xb, Switcher]P(Switcher|Xa, Xb, C=0) \\
&+ E[U_0|Xa, Xb, NeverTaker](1 - P(Switcher|Xa, Xb, C=0))
\end{aligned}
$$

Where the fourth equality follows from the assumption that the joint distribution (U_0, U_c)

is independent from Z conditional on (Xa, Xb).
Also, note we can write

$$\begin{aligned}
P(Switcher|Xa, Xb, C = 1) =& P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, C = 1) \\
=& P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, C = 1, Z = Z_1)P(Z = Z_1|Xa, Xb, C = 1) \\
&+ P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, C = 1, Z = Z_2)P(Z = Z_2|Xa, Xb, C = 1) \\
=& P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, U_c \leq \mu_c(Xa, Z_1)P(Z = Z_1|Xa, Xb, C = 1) \\
&+ P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, U_c \leq \mu_c(Xa, Z_2)P(Z = Z_2|Xa, Xb, C = 1) \\
=& 0 \\
&+ P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, U_c \leq \mu_c(Xa, Z_2)P(Z = Z_2|Xa, Xb, C = 1) \\
=& \frac{A_x}{A_x + B_x} P(Z = Z_2|Xa, Xb, C = 1)
\end{aligned}$$

Where A_x represents $P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb)$ (the probability of being a switcher conditional on (Xa, Xb)) and B_x represents $P(U_c \leq \mu_c(Xa, Z_1)|Xa, Xb)$ (the probability of being an always taker conditional on (Xa, Xb)).
And

$$\begin{aligned}
P(Switcher|Xa, Xb, C = 0) =& P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, C = 0) \\
=& P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, C = 0, Z = Z_1)P(Z = Z_1|Xa, Xb, C = 0) \\
&+ P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, C = 0, Z = Z_2)P(Z = Z_2|Xa, Xb, C = 0) \\
=& P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, U_c > \mu_c(Xa, Z_1)P(Z = Z_1|Xa, Xb, C = 0) \\
&+ P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, U_c > \mu_c(Xa, Z_2)P(Z = Z_2|Xa, Xb, C = 0) \\
=& P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb, U_c > \mu_c(Xa, Z_1)P(Z = Z_1|Xa, Xb, C = 0) \\
&+ 0 \\
=& \frac{A_x}{A_x + C_x} P(Z = Z_1|Xa, Xb, C = 0)
\end{aligned}$$

Where as before A_x represents $P(\mu_c(Xa, Z_1) < U_c \leq \mu_c(Xa, Z_2)|Xa, Xb)$ (the probability of being a switcher conditional on (Xa, Xb)) and C_x represents $P(U_c > \mu_c(Xa, Z_2)|Xa, Xb)$ (the probability of being a never taker conditional on (Xa, Xb)).
Therefore:

$$\begin{gathered}
IV(Xa, Xb) - OLS(Xa, Xb) = \\
E[U_1|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0] - E[U_1|Xa, Xb, C = 1] \\
-E[U_0|Xa, Xb, C(Z_2) = 1, C(Z_1) = 0] + E[U_0|Xa, Xb, C = 0] \\
= (E[U_1|Xa, Xb, Switcher] - E[U_1|Xa, Xb, AlwaysTaker])(1 - P(Switcher|Xa, Xb, C = 1)) \\
-(E[U_0|Xa, Xb, Switcher] - E[U_0|Xa, Xb, NeverTaker])(1 - P(Switcher|Xa, Xb, C = 0)) \\
= (E[U_1|Xa, Xb, Switcher] - E[U_1|Xa, Xb, AlwaysTaker])(1 - (\frac{A_x}{A_x + B_x})P(Z = Z_2|Xa, Xb, C = 1)) \\
-(E[U_0|Xa, Xb, Switcher] - E[U_0|Xa, Xb, NeverTaker])(1 - (\frac{A_x}{A_x + C_x})P(Z = Z_1|Xa, Xb, C = 0))
\end{gathered}$$

Chapter 4

Heterogeneity in the Returns to Higher Education in the UK

4.1 Introduction

This chapter uses the MTE technique discussed in detail in the previous chapter to estimate heterogeneity in the returns to higher education in the United Kingdom. Recent empirical applications of the MTE approach have found a positive relationship between the return to college and values of the unobservables which make it more likely an individual attends college, both in the United States (Carneiro et al., 2011a) and in the United Kingdom (Moffitt, 2008). A positive relationship between the return to college and values of the unobservables which makes college attendance more likely may be interpreted as follows; individuals with values of the unobservables that make it more likely they attend college might be individuals with higher unobserved ability, and these high ability individuals may benefit more from participation in higher education than individuals with lower levels of unobserved ability.

In contrast to previous work, the analysis in this chapter finds little evidence of heterogeneity due to unobservables. However, there was significant heterogeneity owing to observable characteristics.

In addition to analysing heterogeneity in the returns to higher education, this chapter also looks at whether there is a two skill market, with graduates outperforming non-graduates in the graduate labour market and non-graduates outperforming graduates in the non-graduate labour market. Little evidence was found in support of this. Graduates seem to outperform non-graduates in both sections of the labour market. The differential is greatest in the non-graduate labour market.

Suggestive evidence of negative selection was found, with individuals choosing to attend college getting a lower return to higher education than individuals choosing not to at-

tend. This counter-intuitive finding is understood though considering the outcomes of graduates/non-graduates in the graduate/non-graduate labour market. Graduates do better than non-graduates in both markets, but the differential is greatest in the non-graduate labour market. Graduates are giving up more in the non-graduate labour market to earn similar amounts in the graduate labour market, relative to non-graduates.
Possible explanations for why negative selection exists are discussed. Compensating wage differentials is one avenue considered; if graduates have nicer job attributes than non-graduates in the graduate labour market this could help explain why negative selection persists. Little evidence was found to support this explanation. Alternative explanations are considered in the conclusion section.
This chapter proceeds as follows; Section 4.2 presents the model and assumptions, Section 4.3 outlines the data, Section 4.4 presents the results and Section 4.5 concludes.

4.2 Model and Assumptions

The following selection model is assumed:

$$\Rightarrow C_i = 1[\mu_c(Xa_i, Z_i) - Uc_i > 0]$$

Where C_i is a binary variable which takes a value of 1 if the individual is a college graduate and a value of 0 otherwise, Xa_i is a vector of observable characteristics, Z_i is a non null vector of exogeneous instruments, of which at least one element is typically required to be continuous and Uc_i is an unobserved random variable. Furthermore, it is assumed that this selection model can be modelled using a linear probit specification.
There are the following potential outcome models:

$$Y_{0i} = (Xa_i\ Xb_i)\beta_0 + U_{0i}$$
$$Y_{1i} = (Xa_i\ Xb_i)\beta_1 + U_{1i}$$

Where Y_{0i} is the potential non-graduate earnings of individual i, and Y_{1i} is the potential graduate earnings of individual i. As before, Xa_i is a vector of observable characteristics and Xb_i is an additional vector of observable characteristics that may appear in the potential outcome equations but not the selection equation. (U_{0i}, U_{1i}) are unobservable random variables.
The five standard MTE assumptions are imposed:
A1: $\mu_c(Xa, Z)$ is a nondegenerate random variable conditional on Xa, Xb

A2: The random vectors (U_1, Uc) and (U_0, Uc) are independent of Z conditional on Xa, Xb
A3: The distribution of Uc is absolutely continuous with respect to Lebesgue measure
A4: Both potential outcomes Y_1 and Y_0 have finite first moments
A5: $1 > P(C = 1|Xa = xa, Xb = xb) > 0$ for all $(xa, xb) \in \Omega(Xa, Xb)$, where $\Omega(.)$ denotes support

The additional assumptions typically imposed for empirical tractability (discussed in the previous chapter) are also imposed. The independence assumption (A2) is strengthened to a full independence assumption;
A2': The random vectors (U_1, Uc) and (U_0, Uc) are independent of (Xa, Xb, Z)

And, as specified in the above, linearity and separable errors are assumed for both potential outcome models.
Therefore the MTE for an individual with observable characteristics (Xa, Xb) if they had a value $Uc = uc$ for the unobservable random variable affecting the selection choice is:

$$MTE(Xa, Xb, Uc = uc) = (Xa_i\ Xb_i)(\beta_1 - \beta_0) + E[U_1 - U_0|Uc = uc]$$

There is heterogeneity in the returns to college education due to observables if $B_1 - B_0 \neq 0$. There is heterogeneity due to unobservables if $E[U_1 - U_0|Uc = uc]$ varies over the range of Uc.

4.3 Data

This analysis uses the UK National Child Development Study (NCDS)[1] to estimate the distribution of marginal treatment effects from higher education for males and females separately in the United Kingdom. The NCDS follows a group of people born in England, Scotland and Wales in a single week in March, 1958. The initial wave of the survey collected information from the mothers of babies born in that week. These children were subsequently followed in nine sweeps; in 1965 (aged 7), in 1969 (aged 11) in 1974 (aged 16), in 1981 (aged 23), in 1991 (aged 33), in 1999/2000 (aged 42), in 2004 (aged 46), in 2008 (aged 50) and most recently in 2013 (aged 55) (exam data was additionally collected in 1978 (aged 20)). The later life outcomes considered in this analysis are those reported in the 1991 wave when the respondents were aged 33.
The outcome variable of interest is gross hourly wage for employed individuals (self-

[1] University of London. Institute of Education. Centre for Longitudinal Studies, National Child Development Study [computer files]. Colchester, Essex: UK Data Archive [distributor]

employed individuals are excluded from the data as hourly wage information is not available for this group). The main explanatory variable is whether or not the individual has graduated from college (either a university/polytechnic) with a minimum of an undergraduate degree. Outcomes for males and females are estimated separately. The other variables used in this analysis are the set of Z variables (or instruments). Two sets of instrumental variables were considered in the estimation, the first set are those used in Moffitt (2008) and Blundell et al. (2005), namely, a dummy variable indicating whether the individual's teacher reported high or low parental interest in the individual's education at the age of 7, a dummy variable indicating whether at either age 11 or age 16 the individual's parents reported suffering an adverse financial shock in the previous 12 months, and the number of older siblings the individual had. The second set of instruments considered are distance from nearest college (proximity to nearest university/polytechnic are included as separate instruments) at age 16, and region at age 16.[2] The first set of instrumental variables; whether the individual's teacher reported high or low parental interest in the individual's education at the age of 7, a dummy variable indicating whether at either age 11 or age 16 the individual's parents reported suffering an adverse financial shock in the previous 12 months, and the number of older siblings the individual has, were found to be highly significant in predicting college attendance in Blundell et al. (2005). In terms of the second set of instrumental variables; proximity to college and/or local labour market conditions have been used previously in the literature to instrument for college attendance, for example in Cameron and Taber (2004), Card (1995), Cameron and Heckman (1998) and in the estimation of the MTE distribution by Carneiro et al. (2010, 2011a). The rationale behind using the proximity instrument is that distance to nearest college might proxy for the cost of attending college, which might in turn impact upon an individuals decision to attend college. Individuals living further away from college face higher commuting costs, or may be more likely to need to pay for private rented accommodation. The rationale behind using the local labour market instrument is discussed in Cameron and Taber (2004), and Carneiro et al. (2011a). As discussed in these papers, local labour market conditions have an ambigious impact on college attendance. On one hand, they proxy the opportunity cost of attending college, with individuals in better performing labour markets being less likely to attend. On the other hand, credit constraints might be less of a factor in strong local labour markets, or it might be the case that in strong labour markets the return to college is higher. Since cohort data is used, region at age 16 is included as an instrument to measure local labour market conditions when the individual was at the college-attendance decision age.

The Xa variables described in the previous section include number of siblings, ability (as measured by the Douglas general ability test score measured in 1969 when the individuals

[2]Distance variables constructed with thanks by Jon Johnson at the CLS

were aged 11), mothers' years of education and fathers' years of education. Missing variable dummies are also included for number of siblings, ability, and mothers' and fathers' education. Additional Xb variables are included in the specification with the second set of instrumental variables (distance to nearest university/polytechnic and region at age 16). The Xb variables for this specification include region at age 33, and distance to nearest pre-existing college at age 33. Cameron and Taber (2004) highlight the importance of controlling for current labour market conditions when past local labour market conditions are used as excluded instruments. This is because past local labour market conditions may be correlated with current local labour market conditions or other regional conditions which might impact upon later life outcomes. Similarly, as there is some concern that the distance to nearest university/polytechnic instrument might be proxying urban/rural status, distance to nearest pre-existing university/polytechnic at age 33 is included as an additional Xb control variable.

In total, there are 18,558 individual who answer at least one of the first three rounds of the NCDS, of whom 11,469 respond to the 1991 (age 33) round of the survey. Of these, education, region and distance variables are successfully measured for 9,158 of the 1991 respondents. Table 4.1 shows labour market participation figures in 1991 and 2008. In 1991, 75.0% of males and 62.2% of females were in employment (excluding self-employment).

Descriptives of the Xa, Xb and Z variables for these individuals are shown in Table 4.2. Hourly gross income is also displayed in these tables. In 1991, 11% of males and 19% of females report having graduated from college. Men and women who graduated from college have higher levels of mother's and father's education on average. They also have higher levels of ability as measured by the Douglas general ability test score at age 11. They have slightly fewer siblings on average. Later in life, graduates are more likely to be located in London and the South East as compared to non-graduates. Average hourly gross wages in 1991 were £10.22 for males graduates compared to £7.08 for non-graduates, and £7.66 for females graduates compared to £4.72 for non-graduates.[3]

4.4 Results

The MTE model is estimated using both the normal selection model approach and using a sieve estimation approach (both discussed in the previous chapter). Results using an OLS model are also estimated. In the sieve estimation, the model specifications estimated are; constant MTE, linear MTE, quadratic MTE, linear MTE with median break and linear MTE with quartile breaks. For the 7 different models (OLS, normal, 5 sieve specifications),

[3]2.5% of observations have been trimmed from both tails of income for entire analysis

heterogeneity due to unobservables and heterogeneity due to observable characteristics are estimated. In addition, graduate advantage in the graduate/non-graduate labour market is estimated, as is selection into higher education. Finally, average treatment effects for individuals with the mean level of observable characteristics are reported.

4.4.1 First Stage

The propensity score is estimated in the first step using a probit model. Table 4.3 tabulates the estimated model for the propensity score using the first set of instrumental variables (older siblings, financial shock and parental interest). Table 4.4 tabulates the estimated model for the propensity score using the other set of instrumental variables (distance to nearest university/polytechnic at age 16 and region at age 16), and log odds are reported. The first set of IVs was highly significant for all groups with the exception of Males in 2008. The second set of IVs were significant for females in both time periods, but not for males. Therefore, for the rest of the analysis, this chapter uses the first set of instrumental variables, as they provide significant variation in college graduation for both males and females (although not significantly for males in 2008).

4.4.2 Heterogeneity due to Unobservables

Figure 4.1 shows the shape of the MTE curves estimated under the assumption of joint normality of the unobservables (in both the outcome equations and the selection into college equation), and for the five spline specifications for Males in 1991. The MTE curves are graphed for individuals with the mean level of observed covariates (for different covariate values, given the model assumptions, the shape of the MTE curve is the same but there may be a location shift). The corresponding coefficients are shown in Table 4.5. Insignificant heterogeneity estimates in terms of unobservable characteristics were found for the normal selection model, the linear model and the linear model with a median break. While significant coefficients were estimated for the quadratic and linear quartile break models, both of these models lead to implausibly high estimates of the return to college education over some regions of the unobservables, and so it is assumed these models are incorrectly specified. Therefore, it seems that there is little evidence of heterogeneity in the returns to higher education owing to unobservable characteristics, once a rich set of observable characteristics (available in the NCDS) are controlled for. Table 4.6 reports the ATE estimated for individuals with the mean level of observed covariates. If it is assumed that there is no heterogeneity in the returns to higher education owing to unobservables, the remaining candidate models are the OLS specification and the constant MTE model. The constant MTE model does not allow for heterogeneity in the returns due to unobservables, but might lead to different estimates of the ATE from the OLS model. There could be non-constant parallel $E[U_1|X, Uc]$, $E[U_0|X, Uc]$ functions that implies homogeneity in the returns, but biased OLS estimates. However, the constant MTE model suggests implausibly high returns to higher education (twice the magnitude of the OLS estimates). Therefore, for the remainder of the discussion results from the OLS model are discussed. The OLS model suggests a return to higher education of 31.8% in aggregate for males in 1991 with the mean level of covariates.

For females the results and interpretations are very similar. There is little evidence of any heterogeneity owing to unobservables (see Table 4.15), and from the ATE estimates (Table 4.16) the sensible model choice again seems to be OLS. For females, the OLS model suggests a return to higher education of 46.7% in aggregate for individuals with the mean level of covariates.

For 2008 the results and interpretations are also very similar, and the figures and tables are not presented in this chapter for conciseness. The OLS estimate of the return to higher education for males with the mean level of the covariates in 2008 was 32.9% and for females the corresponding figure is 42.6% (when the cohort were aged 50).

4.4.3 Heterogeneity due to Observables

Table 4.8 reports the heterogeneity in the returns to higher education for males in 1991 owing to observable characteristics. The OLS results are reported in the first column, and indicate that the only significant dimension for heterogeneity is observed ability, with individuals with higher levels of ability having lower returns to education. Comparing these $B_1 - B_0$ estimates with the B_0 estimates reported in table 4.7 suggests that while there are significant gains to observed ability in the non-graduate labour market (the B_0 estimate corresponding to observed ability is 0.007), there is little relationship between ability and earnings in the graduate labour market (the $B_1 - B_0$ estimate corresponding to observed ability is -0.007, which implies $B_1 \approx 0$).

Similar results in terms of heterogeneity owing to observed ability are estimated for females in 1991. Significantly higher returns to education are estimated for individuals with lower levels of observed ability, which is explained due to the stronger relationship between ability and earnings in the non-graduate labour market relative to the graduate labour market. There are additional covariates that lead to heterogeneous returns for females however, with those with higher levels of mothers' education, those living in London at age 16 and those living in the Eastern region all receiving lower returns to education ceteris paribus.

These results hold for the sample observed in 2008, with a significant negative relationship between observed ability and returns being estimated for both males and females.

4.4.4 Graduate Advantage in the Graduate/Non-Graduate Labour Market

Male graduates in 1991 are estimated to have a small advantage over non-graduates in the graduate labour market, with the average male graduate earning approximately 4.8% more in the graduate labour market than the average male non-graduate would have earned had he graduated from higher education. The results from the OLS model are shown in column 1 of Table 4.9. However, this differential is much bigger in the non-graduate labour market. The average graduate male is estimated to earn 17.0% more in the non-graduate labour market if he had not attended higher education that the average non-graduate male earns in the non-graduate labour market. Looking at the decomposition results in column 1 Table 4.11, 67.3% of this differential is explained through observed ability. As discussed in the previous section, ability is associated with higher rewards in the non-graduate labour market than the graduate labour market. Since graduates have higher levels of observed ability than non-graduates, they earn more than non-graduates in the non-graduate labour market where ability is highly rewarded, but similar amounts in the graduate labour market.

Again, a very similar result holds for females in 1991. The average female graduate earns 1.9% more than the average female non-graduate in the graduate labour market. The differential is much greater in the non-graduate labour market, with the average female graduate earning 21.0% more than the average female non-graduate.

Similar findings were observed in 2008.

4.4.5 Selection into Higher Education

As in the previous chapter, selection into higher education is said to exist if the average gain from attending higher education is greater for those who actually attend higher education than for those who do not attend. As shown in column 1 in Table 4.10, a negative selection estimate was found for males in 1991. This surprising result implies that non-graduates would have had a higher return to attending higher education than graduates. In the decomposition analysis, differences in observed ability was found to explain 92.9% of this negative selection. This is because those with lower ability receive a higher return to attending higher education, and non-graduates have lower levels of observed ability than graduates. Intuitively, recall that those with high ability are rewarded for that ability in the non-graduate labour market, but to a lesser extent in the graduate labour market. Therefore, high ability graduates are giving up more in the non-graduate labour market to earn similar amounts in the graduate labour market, relative to non-graduates.

For females in 1991 (as shown in column 1 Table 4.21) a negative selection estimation of

19.2% was found, with half of the differential being explained through observed ability. 19.1% of the remaining differential was explained through differences in mothers years of education, and 19.3% due to growing up in London.
These significant negative selection estimates persist in the 2008 sample.

4.4.6 Compensating Differentials

The negative selection estimate found was surprising. If non-graduates stand to benefit more than graduates from participation in higher education then why don't more of them attend higher education? A rational individual chooses to invest in education if the monetary and non-monetary benefits outweigh the costs. Higher monetary benefits were estimated for non-graduates, therefore to rationalise the findings, it must be that there are higher non-monetary benefits for graduates, or higher monetary or non-monetary costs for non-graduates. Evidence of compensating differentials in the labour market could rationalise these findings if highly able graduates end up in jobs with nicer attributes than lower ability graduates. Non-graduates (with lower levels of observed ability) might anticipate working in less nice jobs if they were to attend higher education than higher ability individuals and decide not to attend higher education. This possible explanation was tested by analysing the relationship between ability and job characteristics in the graduate and non-graduate labour market. Certain job characteristics were regressed on the set of control variables in the wage equation (including observed ability), and the set of control variables interacted with graduate status. A positive coefficient on the ability-college interaction would be evidence of this type of positive relationship between ability and nice job characteristics in the graduate labour market. The set of job characteristics considered were; whether you work in the public or charity sector, whether you do night work, whether you do weekend work, whether you work fixed hours or whether you have flexibility in the hours worked, whether your job involves working with a computer and hours worked per week. In addition, whether you receive any of the following benefits was analysed; whether you have the chance to have shares in your employer's firm, whether you have private use of a company car, whether you receive other travel benefits, whether you have subsidised meals, whether you receive private medical insurance, whether you have an employer organised pension scheme, whether you receive help with child care, whether you receive discounts on goods or services, or whether you receive any other benefits.
The results are shown in Tables 4.12, 4.13 and 4.14 for males in 1991. While there is often a positive relationship between ability and what would be considered positive job characteristics (e.g. not having weekend work, having flexibility in the hours worked, having a company car, etc.) in the non-graduate labour market, it does not seem to be the case that higher ability graduates have differential levels of positive job characteristics

compared to lower ability individuals.

As can be seen in Tables 4.22, 4.23 and 4.24, similar results were found for women, although there was some evidence that higher ability graduates are more likely to receive help with childcare and receive discounts on goods and services.

4.5 Discussion and Conclusion

This chapter exploits a rich longitudinal data set to answer three related questions. Firstly, is there any evidence of heterogeneity in the returns from higher education, due to either unobservable or observable characteristics? Secondly, do one group (graduates/non-graduates) have an advantage over the other in the graduate/non-graduate labour market? Finally, if heterogeneous returns exist, are individuals selecting into higher education based on this heterogeneity?

The results presented in the previous section are somewhat surprising. The empirical evidence suggests that there is significant heterogeneity in the returns to education based on observable characteristics, in particular, with high ability individuals receiving a lower return to higher education than lower ability individuals. Since higher education participation is higher among high ability students, this creates negative selection into higher education, with individuals who attend receiving lower returns than the hypothetical returns the lower ability group would have received had they graduated. This finding is largely driven by the fact that there is a strong positive relationship between ability and earnings in the non-graduate labour market but a much smaller/negligible relationship between ability and earnings in the graduate labour market, implying that high ability graduates give up more (in the non-graduate labour market) to receive the same (in the graduate labour market). This implies a lower return to college graduation for high ability individuals relative to lower ability individuals.

There is little evidence of heterogeneity in the returns due to unobservables characteristics. This is in contrast to the previous literature in the US (Carneiro et al., 2011a). Previous work in the UK (Moffitt, 2008) found evidence of heterogeneity in the returns to higher education due to unobservables, but only when much of the heterogeneity owing to observables is assumed to be null.

A rational individual chooses to participate in higher education if the monetary and non-monetary benefits outweigh the monetary and non-monetary costs. This chapter found evidence of higher monetary benefits for non-graduates, which implies that in a rational model, graduates must have higher levels of non-monetary benefits, lower monetary costs or lower non-monetary costs in order to explain why they attend college while non-graduates who would receive higher monetary returns do not attend. Compensating differentials in the graduate labour market was ruled out, as no evidence was found that high ability graduates have nicer job attributes than lower ability graduates. An alternative explanation may be that high ability individuals have lower non-monetary costs, for instance, if they have a lower effort cost, or if there is a higher probability of course failure for low ability students. Finally, another possible explanation for this negative selection is that low ability students are facing barriers to entry. This could be indirectly though credit constraints for instance, or directly, though stringent college admission procedures.

Table 4.1: Labour Market Participation

	1991				2008			
	Male		Female		Male		Female	
Employed	90.51%		68.01%		68.23%		73.07%	
Employed (not self employed)	74.64%		61.24%		88.12%		80.66%	
	No HE	HE	No HE	HE	No HE	HE	No HE	HE
Employed	89.66%	95.71%	66.82%	77.58%	86.86%	93.26%	79.08%	87.38%
Employed (not self employed)	73.29%	82.83%	60.33%	68.52%	66.77%	74.18%	72.12%	77.09%

Table 4.2: Moffitt Descriptives

		Male 1991		Male 2008		Female 1991		Female 2008	
	College Graduate	0.11		0.16		0.19		0.23	
		HE	No HE	HE	No HE	HE	No HE	HE	No HE
Xa:	Ability	58.44	42.21	56.01	42.28	60.41	44.70	56.23	45.18
	Mother ed yrs	15.86	14.80	15.40	14.77	16.31	14.84	15.76	14.84
	Mother ed yrs m	0.02	0.02	0.03	0.02	0.01	0.02	0.03	0.01
	Father ed yrs	16.44	14.76	16.06	14.78	16.36	14.83	15.93	14.81
	Father ed yrs m	0.03	0.04	0.04	0.04	0.02	0.05	0.06	0.05
	Siblings	1.89	2.35	2.04	2.40	1.88	2.38	1.98	2.32
	North West 16	0.10	0.10	0.12	0.11	0.09	0.12	0.12	0.12
	Yorkshire and Humber 16	0.08	0.09	0.10	0.09	0.08	0.08	0.08	0.09
	East Midlands 16	0.06	0.08	0.06	0.08	0.05	0.08	0.04	0.08
	West Midlands 16	0.12	0.10	0.12	0.09	0.08	0.10	0.07	0.11
	Eastern 16	0.07	0.10	0.09	0.11	0.11	0.10	0.10	0.10
	London 16	0.15	0.09	0.12	0.09	0.20	0.09	0.17	0.08
	South East 16	0.15	0.13	0.10	0.14	0.18	0.12	0.15	0.13
	South West 16	0.09	0.08	0.09	0.08	0.07	0.09	0.08	0.09
	Wales 16	0.04	0.06	0.04	0.05	0.05	0.06	0.03	0.05
	Scotland 16	0.09	0.10	0.09	0.11	0.05	0.10	0.10	0.10
	Region m 16	0.15	0.15	0.17	0.16	0.12	0.13	0.15	0.13
Z:	Financial Shock	0.04	0.15	0.07	0.14	0.06	0.16	0.07	0.15
	Parental Interest	0.76	0.41	0.66	0.42	0.85	0.40	0.75	0.40
	Older Siblings	0.83	1.15	0.92	1.16	0.84	1.16	0.88	1.11
Y:	Hourly income	10.22	7.08	23.39	15.72	7.66	4.72	16.23	10.41
	N	2416		1761		2517		1969	

Notes
1. Individuals with non-reported income/college have been dropped from analysis. Individuals with non-reported values of the instrumental variables have also been dropped (financial shock, parental interest and older siblings in this case). In addition, individuals with income outside the 2.5th and 97.5th quantiles have been dropped from the analysis.
2: Very few individuals had missing ability once the above sample had been dropped. Therefore, these individuals were also dropped as keeping them in the sample led to perfect collinearity in the estimation.

Table 4.3: Propensity Score Estimates - Moffitt Instruments

	Male1991Moff	Female1991Moff	Male2008Moff	Female2008Moff
Financial Shock	−0.447**	−0.204	−0.190	−0.270*
	(0.154)	(0.157)	(0.131)	(0.129)
Parental Interest	0.337***	0.615***	0.119	0.456***
	(0.079)	(0.096)	(0.081)	(0.080)
Older Siblings	0.015	0.025	−0.017	−0.042
	(0.042)	(0.047)	(0.041)	(0.040)
Cons	−6.004***	−6.652***	−4.257***	−4.288***
	(0.405)	(0.456)	(0.428)	(0.395)
Ability	0.043***	0.044***	0.037***	0.026***
	(0.003)	(0.004)	(0.003)	(0.003)
Mother ed yrs	0.061*	0.121***	0.012	0.088***
	(0.026)	(0.025)	(0.030)	(0.025)
Mother ed yrs m	0.097	0.480	0.191	0.694*
	(0.281)	(0.356)	(0.272)	(0.296)
Father ed yrs	0.111***	0.050*	0.104***	0.054**
	(0.019)	(0.021)	(0.021)	(0.021)
Father ed yrs m	0.140	−0.612*	0.137	0.075
	(0.225)	(0.272)	(0.207)	(0.183)
Siblings	−0.031	−0.020	0.007	0.032
	(0.034)	(0.038)	(0.033)	(0.031)
North West 16	0.241	−0.016	0.001	−0.331
	(0.205)	(0.252)	(0.197)	(0.191)
Yorkshire and Humber 16	0.173	0.154	−0.003	−0.371
	(0.209)	(0.261)	(0.204)	(0.206)
East Midlands 16	−0.106	−0.095	−0.318	−0.646**
	(0.221)	(0.275)	(0.221)	(0.229)
West Midlands 16	0.237	0.065	0.134	−0.519*
	(0.201)	(0.260)	(0.201)	(0.205)
Eastern 16	−0.282	−0.053	−0.306	−0.493*
	(0.214)	(0.253)	(0.205)	(0.198)
London 16	0.229	0.376	−0.062	−0.026
	(0.197)	(0.240)	(0.203)	(0.193)
South East 16	0.058	0.201	−0.399*	−0.405*
	(0.192)	(0.242)	(0.199)	(0.189)
South West 16	0.102	0.002	−0.031	−0.599**
	(0.208)	(0.265)	(0.211)	(0.208)
Wales 16	−0.172	0.013	−0.340	−0.653**
	(0.241)	(0.288)	(0.247)	(0.247)
Scotland 16	0.105	−0.030	−0.201	−0.205
	(0.205)	(0.266)	(0.206)	(0.197)
Region m 16	0.166	0.181	−0.055	−0.245
	(0.187)	(0.243)	(0.183)	(0.183)
N	2416	2517	1761	1969
McFadden R-sq.	0.529	0.649	0.372	0.422
P-Value	0.000***	0.000***	0.217	0.000***

Table 4.4: Propensity Score Estimates - Distance Region Instruments

	Male1991Distreg	Female1991Distreg	Male2008Distreg	Female2008Distreg
Distance Uni 16	−0.002	−0.003	−0.001	−0.001
	(0.002)	(0.002)	(0.002)	(0.002)
Distance Poly 16	−0.000	−0.004	−0.001	−0.001
	(0.002)	(0.002)	(0.002)	(0.002)
North West 16	0.173	0.262	0.036	−0.168
	(0.165)	(0.202)	(0.164)	(0.158)
Yorkshire and Humber 16	0.207	0.261	0.021	−0.124
	(0.173)	(0.214)	(0.174)	(0.168)
East Midlands 16	0.034	0.148	−0.246	−0.460*
	(0.180)	(0.226)	(0.185)	(0.190)
West Midlands 16	0.167	0.319	0.094	−0.317
	(0.170)	(0.210)	(0.170)	(0.172)
Eastern 16	0.043	0.366	−0.181	−0.268
	(0.178)	(0.215)	(0.181)	(0.173)
London 16	0.245	0.558**	0.024	0.061
	(0.164)	(0.196)	(0.166)	(0.157)
South East 16	0.265	0.517**	−0.095	−0.213
	(0.159)	(0.198)	(0.163)	(0.158)
South West 16	0.248	0.245	0.172	−0.205
	(0.177)	(0.223)	(0.182)	(0.176)
Wales 16	0.049	0.148	−0.207	−0.455*
	(0.200)	(0.243)	(0.213)	(0.206)
Scotland 16	0.265	0.458*	−0.081	−0.013
	(0.177)	(0.215)	(0.180)	(0.169)
Cons	−6.630***	−7.294***	−4.826***	−5.209***
	(0.389)	(0.434)	(0.418)	(0.395)
Ability	0.045***	0.051***	0.039***	0.036***
	(0.003)	(0.004)	(0.003)	(0.003)
Ability m	0.300***	0.634***	0.166	0.284**
	(0.091)	(0.099)	(0.100)	(0.095)
Mother ed yrs	0.097***	0.155***	0.021	0.112***
	(0.025)	(0.024)	(0.030)	(0.024)
Mother ed yrs m	0.109	0.241	−0.061	0.382
	(0.241)	(0.257)	(0.245)	(0.233)
Father ed yrs	0.122***	0.061**	0.127***	0.069***
	(0.019)	(0.020)	(0.021)	(0.021)
Father ed yrs m	0.074	−0.288	0.145	−0.110
	(0.211)	(0.202)	(0.209)	(0.176)
Siblings	−0.048	−0.071*	−0.024	−0.003
	(0.025)	(0.029)	(0.025)	(0.024)
Siblings m	−0.282	0.112	−0.218	−0.141
	(0.199)	(0.218)	(0.198)	(0.199)
N	3073	3276	2222	2502
McFadden R-sq.	0.509	0.597	0.383	0.387
P-Value	0.529	0.000***	0.247	0.002***

Figure 4.1: MTE estimates - 1991 - Male

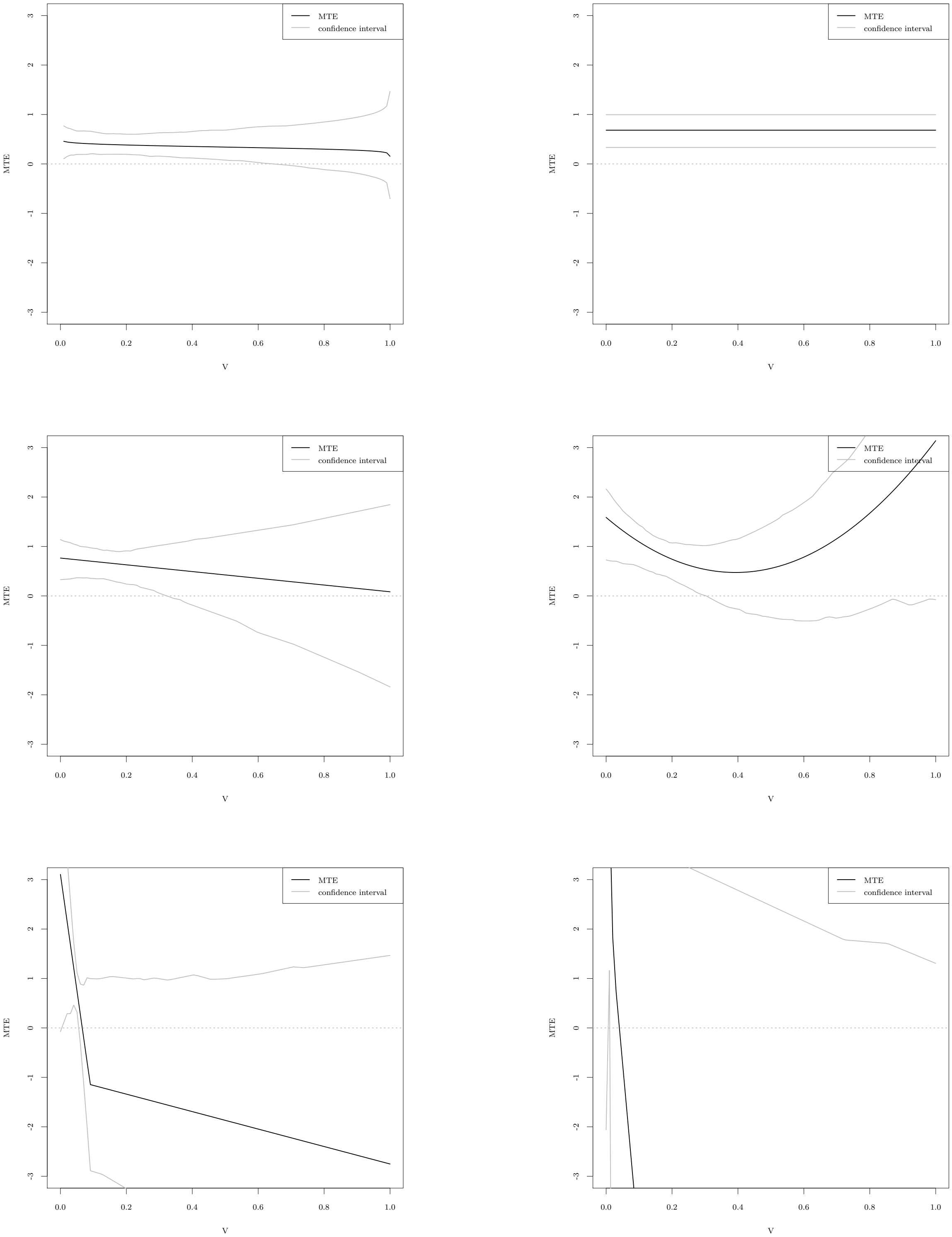


Table 4.5: Heterogeneity from Unobservables - 1991 - Male

	Normal	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
NormalCoef	−0.050					
Linear			−0.682	−5.674**	−46.906	−278.605
Quadratic				7.225**		
MedianBreak					45.139	63.944*
LowerQuartileBreak						204.634
UpperQuartileBreak						6.975*

Table 4.6: ATE comparisons - 1991 - Male

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
ATExbar	0.318***	0.341**	0.683***	0.425	1.160*	−1.683	−5.551
ATExbar − OLSxbar		0.023	0.365*	0.107	0.842	−2.000	−5.868

Table 4.7: B0 estimates - 1991 - Male

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
Cons	1.225***	1.305***	1.317***	1.353***	1.352***	1.423***	1.543***
Ability	0.007***	0.006***	0.006***	0.006***	0.004***	0.005***	0.003
Motheredyrs	0.003	0.001	−0.003	−0.004	−0.003	−0.005	−0.008
Motheredyrsm	0.006	0.003	−0.011	−0.013	−0.022	−0.017	−0.023
Fatheredyrs	0.024***	0.021***	0.022**	0.020**	0.021*	0.016*	0.012
Fatheredyrsm	−0.035	−0.035	−0.054	−0.055	−0.057	−0.055	−0.057
Siblings	−0.007	−0.007	−0.005	−0.005	−0.002	−0.003	−0.001
*NorthWest*16	−0.012	−0.016	−0.005	−0.007	−0.013	−0.015	−0.024
*YorkshireandHumber*16	−0.103**	−0.106***	−0.058	−0.060	−0.066	−0.062	−0.068
*EastMidlands*16	0.014	0.014	0.062	0.062	0.062	0.066	0.071
*WestMidlands*16	−0.037	−0.040	−0.057	−0.060	−0.066	−0.066	−0.076
*Eastern*16	0.060	0.064	0.113*	0.114**	0.118*	0.122**	0.134**
*London*16	0.170***	0.166***	0.164***	0.163***	0.154**	0.156***	0.146**
*SouthEast*16	0.102**	0.101**	0.171**	0.170***	0.163**	0.171***	0.167***
*SouthWest*16	−0.056	−0.058	−0.012	−0.013	−0.024	−0.014	−0.018
*Wales*16	−0.095**	−0.093**	−0.098*	−0.097	−0.100	−0.090	−0.083
*Scotland*16	−0.083**	−0.084**	−0.033	−0.033	−0.036	−0.036	−0.040
*Regionm*16	−0.080**	−0.082**	−0.033	−0.035	−0.043	−0.038	−0.044

Table 4.8: B1-B0 estimates - 1991 - Male

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
Cons	0.838***	0.782	2.034***	1.627	3.095**	3.408**	7.029
Ability	−0.007***	−0.006*	−0.018***	−0.013	−0.015	−0.009	−0.001
Motheredyrs	−0.016	−0.014	−0.009	−0.003	−0.021	0.005	0.014
Motheredyrsm	−0.051	−0.049	0.022	0.038	0.038	0.053	0.075
Fatheredyrs	0.002	0.005	−0.010	0.003	−0.020	0.020	0.036
Fatheredyrsm	0.143	0.142	0.298	0.311	0.315	0.320	0.331
Siblings	−0.009	−0.009	−0.007	−0.009	−0.011	−0.015	−0.024
*NorthWest*16	−0.067	−0.064	−0.246	−0.219	−0.225	−0.186	−0.151
*YorkshireandHumber*16	0.072	0.074	−0.291	−0.273	−0.244	−0.261	−0.243
*EastMidlands*16	−0.095	−0.095	−0.419	−0.428*	−0.400	−0.443	−0.469*
*WestMidlands*16	0.033	0.035	−0.001	0.028	0.028	0.060	0.090
*Eastern*16	−0.073	−0.075	−0.431	−0.456	−0.387	−0.493*	−0.546*
*London*16	−0.014	−0.010	−0.169	−0.145	−0.126	−0.120	−0.092
*SouthEast*16	−0.064	−0.064	−0.546**	−0.540**	−0.507**	−0.540**	−0.536**
*SouthWest*16	0.063	0.065	−0.304	−0.295	−0.245	−0.292	−0.287
*Wales*16	0.021	0.019	0.130	0.117	0.174	0.092	0.064
*Scotland*16	0.066	0.067	−0.329	−0.322	−0.315	−0.312	−0.302
*Regionm*16	0.111	0.112	−0.276	−0.255	−0.227	−0.239	−0.220

Table 4.9: Graduate Advantage in Graduate Labour Market - 1991 - Male

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
TotalAdvgrad	0.048***	0.038	−0.348	−0.032	−0.856	2.287	6.459
AdvgradObs	0.048***	0.045	−0.167	−0.074	−0.176	0.017	0.142
AdvgradUnobs	–	−0.007	−0.181*	0.042	−0.681	2.270	6.318
Decomposition							
Cons	–	–	–	–	–	–	–
Ability	2.85%	−2.79%	111.42%	165.57%	95.77%	−367.70%	20.95%
Motheredyrs	−27.85%	−30.57%	7.72%	9.48%	14.24%	−1.74%	4.68%
Motheredyrsm	−0.40%	−0.45%	−0.03%	−0.15%	−0.04%	0.92%	0.16%
Fatheredyrs	87.27%	92.96%	−11.14%	−51.83%	−1.81%	343.69%	55.14%
Fatheredyrsm	−1.43%	−1.54%	0.93%	2.24%	0.94%	−10.00%	−1.24%
Siblings	14.93%	15.97%	−3.17%	−8.95%	−3.58%	49.63%	8.07%
NorthWest16	−0.56%	−0.61%	0.51%	1.05%	0.46%	−4.04%	−0.42%
YorkshireandHumber16	0.61%	0.67%	−2.03%	−4.40%	−1.71%	18.45%	2.13%
EastMidlands16	3.60%	3.88%	−4.59%	−10.66%	−4.12%	47.57%	6.02%
WestMidlands16	−0.09%	−0.12%	0.46%	0.56%	0.28%	−0.50%	0.13%
Eastern16	0.57%	0.57%	−4.23%	−10.31%	−3.40%	48.44%	6.45%
London16	16.52%	17.75%	0.15%	−1.24%	−0.81%	10.90%	1.96%
SouthEast16	1.24%	1.33%	3.59%	8.04%	3.13%	−34.74%	−4.16%
SouthWest16	0.04%	0.05%	0.61%	1.35%	0.49%	−5.77%	−0.69%
Wales16	2.43%	2.61%	0.30%	0.43%	0.67%	−0.25%	0.21%
Scotland16	0.14%	0.16%	−0.90%	−2.00%	−0.83%	8.46%	0.99%
Regionm16	0.13%	0.14%	0.38%	0.81%	0.31%	−3.34%	−0.38%

Table 4.10: Graduate Advantage in Non-Graduate Labour Market - 1991 - Male

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
TotalAdvnongrad	0.170***	0.080	−0.037	−0.078	−0.360	0.970	3.302
AdvnongradObs	0.170***	0.153***	0.143***	0.136***	0.109***	0.112***	0.066
AdvnongradUnobs	–	−0.073	−0.181*	−0.213	−0.468***	0.858	3.236
Decomposition							
Cons	–	0.000	–	–	–	–	–
Ability	67.25%	68.14%	70.07%	70.62%	62.23%	72.32%	71.52%
Motheredyrs	1.73%	0.81%	−2.49%	−2.90%	−2.96%	−4.62%	−12.23%
Motheredyrsm	0.01%	0.01%	−0.03%	−0.04%	−0.09%	−0.07%	−0.15%
Fatheredyrs	22.64%	22.14%	24.56%	24.13%	32.25%	23.73%	29.86%
Fatheredyrsm	0.13%	0.15%	0.24%	0.26%	0.34%	0.32%	0.55%
Siblings	1.92%	2.01%	1.59%	1.64%	1.03%	1.41%	0.51%
NorthWest16	−0.02%	−0.04%	−0.01%	−0.02%	−0.04%	−0.04%	−0.13%
YorkshireandHumber16	0.59%	0.67%	0.40%	0.43%	0.59%	0.54%	0.99%
EastMidlands16	−0.17%	−0.20%	−0.92%	−0.98%	−1.23%	−1.26%	−2.30%
WestMidlands16	−0.28%	−0.34%	−0.52%	−0.57%	−0.79%	−0.77%	−1.49%
Eastern16	−0.79%	−0.92%	−1.75%	−1.88%	−2.42%	−2.43%	−4.50%
London16	5.14%	5.55%	5.87%	6.15%	7.26%	7.15%	11.32%
SouthEast16	0.96%	1.06%	1.90%	2.01%	2.40%	2.44%	4.04%
SouthWest16	−0.11%	−0.12%	−0.03%	−0.03%	−0.07%	−0.04%	−0.09%
Wales16	0.89%	0.96%	1.08%	1.13%	1.45%	1.27%	1.98%
Scotland16	0.20%	0.23%	0.10%	0.10%	0.14%	0.13%	0.25%
Regionm16	−0.10%	−0.11%	−0.05%	−0.05%	−0.08%	−0.07%	−0.14%

Table 4.11: Selection - 1991 - Male

	OLS	Normal	Constant	Linear	Cubic	MedianBreak	QuartileBreaks
TotalSelection	−0.121***	−0.042	−0.310***	0.046	−0.497	1.317	3.157*
SelectionObs	−0.121***	−0.108	−0.310***	−0.209	−0.284	−0.095	0.076
SelectionUnobs	–	0.066	0.000	0.255	−0.213	1.412	3.082*
Decomposition							
Cons	–	–	–	–	–	–	–
Ability	92.94%	97.57%	92.32%	104.02%	82.96%	151.13%	−23.39%
Motheredyrs	13.53%	13.83%	3.00%	1.45%	7.67%	−5.14%	19.51%
Motheredyrsm	0.18%	0.20%	−0.03%	−0.08%	−0.06%	−0.24%	0.43%
Fatheredyrs	−3.15%	−7.24%	5.35%	−2.59%	11.19%	−33.58%	77.29%
Fatheredyrsm	0.76%	0.84%	0.62%	0.96%	0.71%	2.16%	−2.81%
Siblings	−3.27%	−3.78%	−0.97%	−2.08%	−1.82%	−7.23%	14.70%
NorthWest16	0.19%	0.20%	0.27%	0.36%	0.27%	0.67%	−0.69%
YorkshireandHumber16	0.58%	0.67%	−0.91%	−1.27%	−0.83%	−2.67%	3.13%
EastMidlands16	−1.68%	−1.89%	−2.89%	−4.39%	−3.02%	−10.01%	13.32%
WestMidlands16	−0.36%	−0.43%	0.00%	−0.17%	−0.13%	−0.82%	1.55%
Eastern16	−1.33%	−1.54%	−3.08%	−4.84%	−3.03%	−11.54%	16.05%
London16	0.61%	0.49%	2.79%	3.55%	2.27%	6.47%	−6.24%
SouthEast16	0.85%	0.95%	2.81%	4.13%	2.85%	9.10%	−11.34%
SouthWest16	−0.17%	−0.19%	0.31%	0.45%	0.28%	0.99%	−1.22%
Wales16	0.27%	0.28%	0.66%	0.89%	0.97%	1.54%	−1.34%
Scotland16	0.23%	0.26%	−0.44%	−0.64%	−0.46%	−1.36%	1.65%
Regionm16	−0.19%	−0.21%	0.18%	0.25%	0.16%	0.52%	−0.60%

Table 4.12: Nicejobs A - 1991 - Male

	Pubchar	Nightwork	Weekendwork	Fixedhours	Compwork	Workhours
Cons	0.128	0.681***	0.802***	1.136***	−0.371**	40.060***
	(0.135)	(0.143)	(0.146)	(0.138)	(0.135)	(2.837)
Ability	0.000	−0.001	−0.002**	−0.004***	0.010***	0.003
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.014)
Mother ed yrs	0.007	−0.007	−0.003	0.014	0.002	0.128
	(0.009)	(0.010)	(0.010)	(0.010)	(0.009)	(0.197)
Mother ed yrs m	0.017	0.096	−0.090	−0.022	0.071	−1.309
	(0.078)	(0.083)	(0.084)	(0.080)	(0.078)	(1.649)
Father ed yrs	−0.012	0.012	0.005	0.025	−0.021	0.057
	(0.013)	(0.013)	(0.014)	(0.013)	(0.013)	(0.266)
Father ed yrs m	0.061	−0.050	−0.027	0.035	−0.104	−0.127
	(0.055)	(0.059)	(0.060)	(0.056)	(0.055)	(1.164)
Siblings	−0.010	0.017**	0.018**	0.005	−0.016**	0.322**
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.121)
North West 16	−0.050	−0.142*	−0.069	−0.111*	0.034	−0.998
	(0.053)	(0.056)	(0.057)	(0.054)	(0.052)	(1.107)
Yorkshire and Humber 16	−0.061	−0.101	−0.022	−0.065	−0.026	0.513
	(0.053)	(0.056)	(0.058)	(0.054)	(0.053)	(1.117)
East Midlands 16	−0.006	−0.001	0.011	−0.165**	0.023	0.967
	(0.054)	(0.057)	(0.059)	(0.055)	(0.054)	(1.140)
West Midlands 16	−0.047	−0.091	−0.048	−0.090	−0.032	0.139
	(0.053)	(0.056)	(0.057)	(0.054)	(0.053)	(1.111)
Eastern 16	−0.065	0.010	−0.087	−0.129*	0.136*	1.609
	(0.053)	(0.056)	(0.057)	(0.054)	(0.053)	(1.113)
London 16	−0.001	−0.125*	−0.158**	−0.139*	0.096	−0.242
	(0.054)	(0.057)	(0.058)	(0.055)	(0.054)	(1.131)
South East 16	−0.050	−0.084	−0.071	−0.131*	0.133**	0.864
	(0.050)	(0.053)	(0.055)	(0.052)	(0.050)	(1.058)
South West 16	−0.062	−0.095	−0.090	−0.062	0.016	1.934
	(0.055)	(0.058)	(0.059)	(0.056)	(0.055)	(1.152)
Wales 16	−0.002	−0.083	0.020	−0.046	−0.045	−0.013
	(0.059)	(0.063)	(0.064)	(0.061)	(0.059)	(1.246)
Scotland 16	0.032	−0.144*	−0.049	−0.023	0.009	0.151
	(0.053)	(0.056)	(0.058)	(0.054)	(0.053)	(1.117)
Region m 16	0.018	−0.040	0.000	−0.059	0.017	0.691
	(0.047)	(0.051)	(0.052)	(0.049)	(0.048)	(1.002)
Cons*C	1.075***	−1.057***	−0.577*	−0.456	1.264***	−6.293
	(0.267)	(0.284)	(0.291)	(0.275)	(0.267)	(5.642)
Ability*C	−0.008***	−0.001	−0.001	0.000	−0.009***	0.015
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.048)
Mother ed yrs*C	−0.026	0.036*	0.026	−0.015	−0.015	0.246
	(0.016)	(0.017)	(0.018)	(0.017)	(0.016)	(0.342)
Mother ed yrs m*C	−0.058	−0.119	0.192	0.391	−0.140	6.309
	(0.216)	(0.230)	(0.238)	(0.221)	(0.216)	(4.557)
Father ed yrs m*C	−0.216	0.092	−0.008	−0.268	0.195	2.371
	(0.179)	(0.190)	(0.194)	(0.183)	(0.179)	(3.771)
Siblings*C	0.014	−0.035	−0.011	−0.002	0.031	−0.551
	(0.018)	(0.019)	(0.020)	(0.019)	(0.018)	(0.381)
North West 16*C	0.126	0.303*	0.102	0.175	−0.127	1.604
	(0.143)	(0.152)	(0.157)	(0.146)	(0.143)	(3.025)
Yorkshire and Humber 16*C	0.131	0.375*	−0.045	0.133	0.063	−0.821
	(0.149)	(0.158)	(0.161)	(0.152)	(0.149)	(3.140)
East Midlands 16*C	−0.082	0.185	0.050	0.398*	−0.046	0.732
	(0.155)	(0.164)	(0.168)	(0.157)	(0.154)	(3.250)
West Midlands 16*C	0.180	0.305*	−0.074	0.292*	−0.010	−1.747
	(0.141)	(0.150)	(0.152)	(0.144)	(0.141)	(2.969)
Eastern 16*C	0.112	0.175	0.119	−0.017	−0.233	3.726
	(0.151)	(0.161)	(0.163)	(0.155)	(0.151)	(3.190)
London 16*C	0.021	0.193	0.010	0.137	−0.076	−0.330
	(0.136)	(0.145)	(0.148)	(0.140)	(0.137)	(2.880)
South East 16*C	0.043	0.262	−0.002	0.181	−0.163	1.532
	(0.136)	(0.145)	(0.147)	(0.139)	(0.136)	(2.873)
South West 16*C	0.081	0.339*	−0.020	0.049	−0.104	−0.545
	(0.147)	(0.156)	(0.159)	(0.151)	(0.147)	(3.095)
Wales 16*C	0.291	0.173	−0.135	0.075	−0.005	−4.213
	(0.175)	(0.186)	(0.192)	(0.186)	(0.175)	(3.690)
Scotland 16*C	−0.092	0.282	−0.045	0.097	−0.035	−1.242
	(0.147)	(0.155)	(0.158)	(0.149)	(0.146)	(3.085)
Region m 16*C	−0.014	0.285*	−0.074	0.051	−0.007	−1.133
	(0.133)	(0.141)	(0.143)	(0.135)	(0.133)	(2.798)
N	2532	2536	2506	2516	2540	2543

Table 4.13: Nicejobs B - 1991 - Male

	Benefits	Firmshares	Companycar	Travelben	Subsmeals
Cons	1.217*	0.030	−0.180	0.152	0.312*
	(0.494)	(0.144)	(0.138)	(0.139)	(0.149)
Ability	0.018***	0.002**	0.004***	0.003***	0.000
	(0.003)	(0.001)	(0.001)	(0.001)	(0.001)
Mother ed yrs	0.023	0.012	−0.008	−0.009	0.011
	(0.034)	(0.010)	(0.010)	(0.010)	(0.010)
Mother ed yrs m	−0.492	−0.051	−0.096	−0.038	−0.063
	(0.299)	(0.087)	(0.084)	(0.084)	(0.090)
Father ed yrs	0.012	0.016	−0.019	−0.017	0.030*
	(0.045)	(0.013)	(0.013)	(0.013)	(0.014)
Father ed yrs m	0.076	−0.006	0.003	0.124*	−0.040
	(0.214)	(0.062)	(0.060)	(0.060)	(0.064)
Siblings	−0.017	0.002	0.003	−0.010	0.004
	(0.022)	(0.006)	(0.006)	(0.006)	(0.007)
North West 16	−0.041	0.007	0.086	−0.024	0.006
	(0.199)	(0.058)	(0.056)	(0.056)	(0.060)
Yorkshire and Humber 16	−0.067	0.040	−0.022	−0.033	−0.009
	(0.202)	(0.059)	(0.056)	(0.057)	(0.061)
East Midlands 16	0.212	0.065	0.040	−0.013	0.062
	(0.202)	(0.059)	(0.056)	(0.057)	(0.061)
West Midlands 16	0.195	0.074	0.057	−0.021	0.036
	(0.200)	(0.058)	(0.056)	(0.056)	(0.060)
Eastern 16	0.394*	0.076	0.050	−0.003	0.103
	(0.199)	(0.058)	(0.056)	(0.056)	(0.060)
London 16	0.568**	0.098	0.106	0.056	0.134*
	(0.199)	(0.058)	(0.056)	(0.056)	(0.060)
South East 16	0.424*	0.112*	0.040	0.034	0.125*
	(0.189)	(0.055)	(0.053)	(0.053)	(0.057)
South West 16	−0.028	0.099	−0.004	−0.061	0.005
	(0.206)	(0.060)	(0.057)	(0.058)	(0.062)
Wales 16	−0.091	0.047	0.013	−0.010	0.026
	(0.222)	(0.065)	(0.062)	(0.062)	(0.067)
Scotland 16	0.210	0.032	0.032	0.053	0.078
	(0.200)	(0.058)	(0.056)	(0.056)	(0.060)
Region m 16	0.102	0.022	0.015	0.007	0.024
	(0.182)	(0.053)	(0.051)	(0.051)	(0.055)
Cons*C	1.294	−0.140	0.137	0.637*	0.117
	(0.968)	(0.282)	(0.270)	(0.272)	(0.291)
Ability*C	−0.007	0.004	−0.001	−0.006*	−0.001
	(0.008)	(0.002)	(0.002)	(0.002)	(0.003)
Mother ed yrs*C	−0.028	−0.026	0.023	0.001	−0.022
	(0.058)	(0.017)	(0.016)	(0.016)	(0.018)
Mother ed yrs m*C	1.109	−0.014	0.394	0.228	0.160
	(0.767)	(0.224)	(0.214)	(0.216)	(0.231)
Father ed yrs m*C	−0.021	0.263	−0.244	−0.194	−0.022
	(0.634)	(0.185)	(0.177)	(0.178)	(0.191)
Siblings*C	−0.075	0.008	−0.011	−0.002	−0.003
	(0.065)	(0.019)	(0.018)	(0.018)	(0.020)
North West 16*C	−0.473	0.029	−0.091	−0.185	−0.154
	(0.512)	(0.149)	(0.143)	(0.144)	(0.154)
Yorkshire and Humber 16*C	−0.132	−0.023	0.057	0.012	−0.164
	(0.534)	(0.156)	(0.149)	(0.150)	(0.161)
East Midlands 16*C	−0.109	0.075	−0.005	−0.049	−0.145
	(0.545)	(0.159)	(0.152)	(0.153)	(0.164)
West Midlands 16*C	−1.039*	−0.119	−0.199	−0.125	−0.168
	(0.503)	(0.146)	(0.140)	(0.141)	(0.151)
Eastern 16*C	−0.487	−0.013	0.117	0.044	−0.350*
	(0.539)	(0.157)	(0.151)	(0.151)	(0.162)
London 16*C	−0.929	0.015	−0.153	−0.179	−0.306*
	(0.487)	(0.142)	(0.136)	(0.137)	(0.147)
South East 16*C	−0.545	−0.055	−0.006	−0.108	−0.176
	(0.486)	(0.142)	(0.136)	(0.137)	(0.146)
South West 16*C	−0.236	−0.069	−0.003	0.116	−0.036
	(0.526)	(0.153)	(0.147)	(0.148)	(0.158)
Wales 16*C	−0.924	−0.153	−0.140	0.048	−0.188
	(0.630)	(0.184)	(0.176)	(0.177)	(0.190)
Scotland 16*C	−0.565	0.043	−0.130	−0.123	−0.035
	(0.521)	(0.152)	(0.145)	(0.146)	(0.157)
Region m 16*C	−0.459	0.048	−0.091	−0.020	−0.108
	(0.472)	(0.138)	(0.132)	(0.133)	(0.142)
N	2259	2259	2259	2259	2259

Table 4.14: Nicejobs C - 1991 - Male

	Medicalins	Pension	Childcare	Discounts	Otherben
Cons	−0.387**	0.730***	0.021	0.558***	−0.018
	(0.133)	(0.134)	(0.032)	(0.151)	(0.127)
Ability	0.003***	0.003***	0.000	0.001	0.002***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
Mother ed yrs	0.022*	0.001	−0.002	−0.002	−0.002
	(0.009)	(0.009)	(0.002)	(0.011)	(0.009)
Mother ed yrs m	−0.111	0.096	−0.005	−0.064	−0.160*
	(0.080)	(0.081)	(0.019)	(0.092)	(0.077)
Father ed yrs	−0.004	0.000	−0.002	0.016	−0.009
	(0.012)	(0.012)	(0.003)	(0.014)	(0.012)
Father ed yrs m	−0.050	−0.017	−0.009	0.069	0.001
	(0.057)	(0.058)	(0.014)	(0.065)	(0.055)
Siblings	−0.005	−0.015*	−0.001	0.004	0.001
	(0.006)	(0.006)	(0.001)	(0.007)	(0.006)
North West 16	−0.034	−0.115*	0.007	0.034	−0.008
	(0.053)	(0.054)	(0.013)	(0.061)	(0.051)
Yorkshire and Humber 16	0.011	−0.097	0.007	0.042	−0.006
	(0.054)	(0.055)	(0.013)	(0.062)	(0.052)
East Midlands 16	0.049	−0.066	0.014	0.041	0.018
	(0.054)	(0.055)	(0.013)	(0.062)	(0.052)
West Midlands 16	0.054	−0.078	0.001	0.081	−0.009
	(0.054)	(0.054)	(0.013)	(0.061)	(0.051)
Eastern 16	0.083	−0.036	0.013	0.078	0.030
	(0.053)	(0.054)	(0.013)	(0.061)	(0.051)
London 16	0.129*	−0.018	0.007	0.064	−0.007
	(0.053)	(0.054)	(0.013)	(0.061)	(0.051)
South East 16	0.084	−0.071	0.006	0.049	0.047
	(0.051)	(0.051)	(0.012)	(0.058)	(0.048)
South West 16	−0.030	−0.154**	0.008	0.124	−0.017
	(0.055)	(0.056)	(0.013)	(0.063)	(0.053)
Wales 16	−0.017	−0.186**	0.021	0.024	−0.008
	(0.060)	(0.060)	(0.014)	(0.068)	(0.057)
Scotland 16	−0.020	−0.063	0.007	0.067	0.024
	(0.054)	(0.054)	(0.013)	(0.061)	(0.051)
Region m 16	0.035	−0.082	0.020	0.062	−0.001
	(0.049)	(0.049)	(0.012)	(0.056)	(0.047)
Cons*C	0.607*	0.322	−0.064	−0.343	0.022
	(0.260)	(0.263)	(0.062)	(0.297)	(0.248)
Ability*C	−0.001	−0.004	0.001	0.002	−0.001
	(0.002)	(0.002)	(0.001)	(0.003)	(0.002)
Mother ed yrs*C	−0.019	−0.006	0.004	0.008	0.007
	(0.016)	(0.016)	(0.004)	(0.018)	(0.015)
Mother ed yrs m*C	0.257	0.077	−0.016	0.058	−0.035
	(0.206)	(0.209)	(0.049)	(0.235)	(0.197)
Father ed yrs m*C	−0.013	−0.135	0.007	0.188	0.128
	(0.170)	(0.173)	(0.040)	(0.194)	(0.163)
Siblings*C	−0.049**	0.006	−0.004	−0.006	−0.014
	(0.018)	(0.018)	(0.004)	(0.020)	(0.017)
North West 16*C	−0.063	0.077	0.027	−0.194	0.081
	(0.137)	(0.139)	(0.033)	(0.157)	(0.131)
Yorkshire and Humber 16*C	0.001	0.173	0.039	−0.343*	0.115
	(0.143)	(0.145)	(0.034)	(0.164)	(0.137)
East Midlands 16*C	−0.104	0.050	−0.012	−0.038	0.119
	(0.146)	(0.148)	(0.035)	(0.167)	(0.140)
West Midlands 16*C	−0.089	0.037	0.001	−0.398**	0.022
	(0.135)	(0.137)	(0.032)	(0.154)	(0.129)
Eastern 16*C	0.023	0.006	−0.015	−0.365*	0.067
	(0.145)	(0.147)	(0.034)	(0.165)	(0.138)
London 16*C	−0.125	−0.107	−0.005	−0.249	0.180
	(0.131)	(0.132)	(0.031)	(0.149)	(0.125)
South East 16*C	−0.031	0.065	0.038	−0.304*	0.031
	(0.130)	(0.132)	(0.031)	(0.149)	(0.125)
South West 16*C	−0.183	0.101	−0.003	−0.352*	0.192
	(0.141)	(0.143)	(0.034)	(0.161)	(0.135)
Wales 16*C	−0.187	0.020	−0.014	−0.369	0.058
	(0.169)	(0.171)	(0.040)	(0.193)	(0.162)
Scotland 16*C	−0.207	0.021	0.032	−0.243	0.079
	(0.140)	(0.142)	(0.033)	(0.160)	(0.134)
Region m 16*C	−0.012	0.002	0.003	−0.281	0.001
	(0.127)	(0.128)	(0.030)	(0.145)	(0.121)
N	2259	2259	2259	2259	2259

Figure 4.2: MTE estimates - 1991 - Female

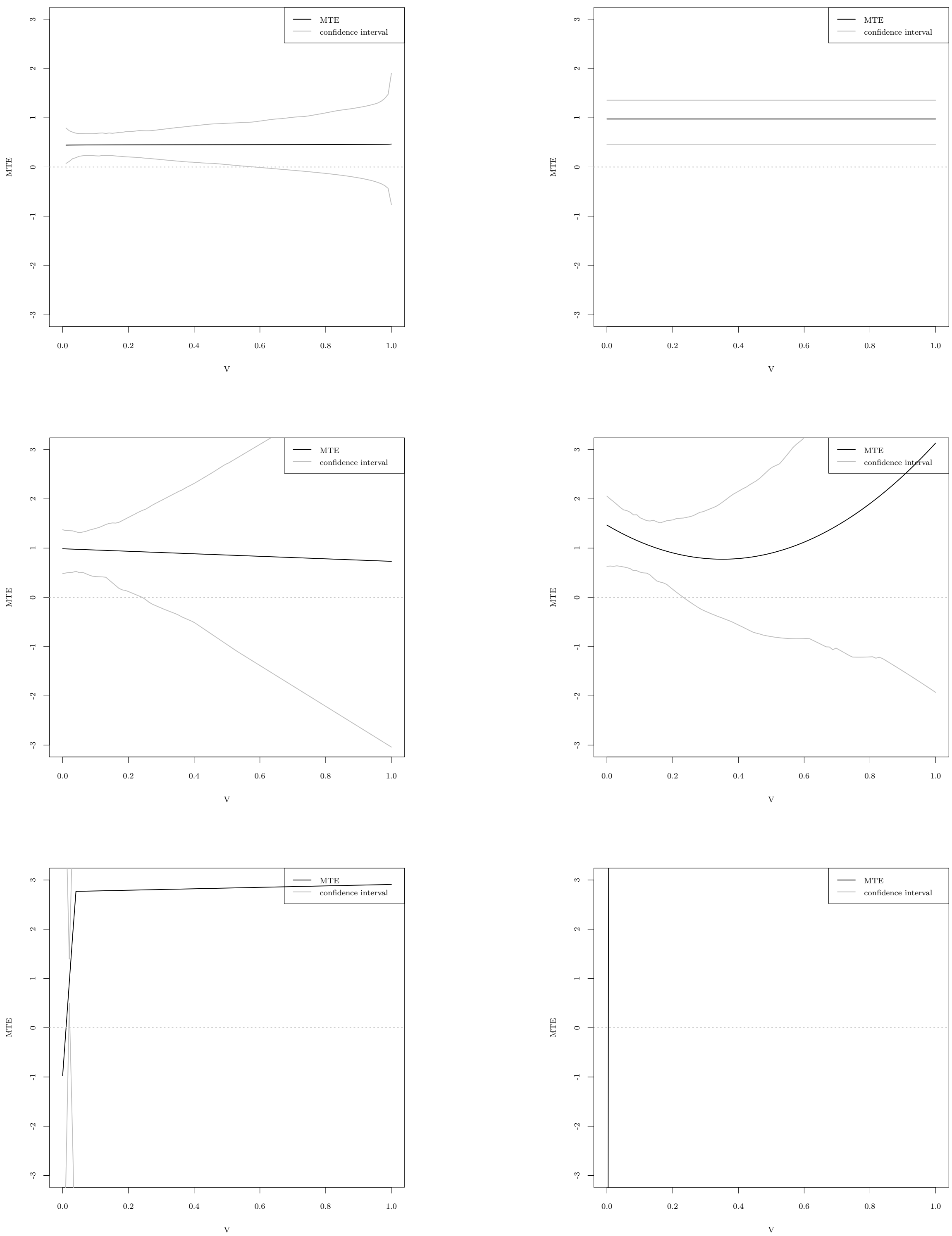


Table 4.15: Heterogeneity from Unobservables - 1991 - Female

	Normal	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
NormalCoef	0.004					
Linear			−0.255	−3.942	93.582	6437.014
Quadratic				5.609		
MedianBreak					−93.436	1.187
LowerQuartileBreak						−6463.088
UpperQuartileBreak						22.931

Table 4.16: ATE comparisons - 1991 - Female

	OLS	$Normal$	$Constant$	$Linear$	$Cubic$	$MedianBreak$	$QuartileBreaks$
$ATExbar$	0.467***	0.453*	0.974**	0.859	1.367	2.761	20.707
$ATExbar - OLSxbar$		−0.014	0.507*	0.392	0.899	2.294	20.240

Table 4.17: B0 estimates - 1991 - Female

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
Cons	0.620***	0.608***	0.861***	0.870***	0.868***	0.850***	1.037***
Ability	0.008***	0.008***	0.008***	0.008***	0.007***	0.008***	0.006***
Motheredyrs	0.018**	0.019**	0.003	0.002	0.004	0.004	−0.002
Motheredyrsm	−0.110	−0.109	−0.068	−0.069	−0.078	−0.068	−0.088
Fatheredyrs	0.015**	0.015**	0.012	0.012	0.012	0.012	0.009
Fatheredyrsm	0.006	0.005	0.042	0.043	0.052	0.040	0.065
Siblings	−0.022***	−0.022***	−0.020***	−0.020***	−0.019***	−0.020***	−0.019***
*NorthWest*16	0.057	0.057	0.012	0.012	0.011	0.011	0.011
*YorkshireandHumber*16	0.008	0.008	−0.000	−0.000	−0.002	−0.000	−0.006
*EastMidlands*16	0.027	0.026	0.039	0.039	0.037	0.038	0.037
*WestMidlands*16	0.069	0.069	0.078	0.078	0.076*	0.078	0.073
*Eastern*16	0.051	0.050	0.067	0.067	0.064	0.066	0.067
*London*16	0.312***	0.313***	0.324***	0.324***	0.319***	0.324***	0.309***
*SouthEast*16	0.064	0.064	0.036	0.035	0.031	0.036	0.028
*SouthWest*16	−0.051	−0.051	−0.025	−0.025	−0.028	−0.025	−0.027
*Wales*16	−0.010	−0.010	−0.013	−0.013	−0.017	−0.014	−0.017
*Scotland*16	0.032	0.032	0.066	0.067	0.065	0.066	0.069
*Regionm*16	0.059	0.059	0.091**	0.091*	0.088**	0.091**	0.085

Table 4.18: B1-B0 estimates - 1991 - Female

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
Cons	1.326***	1.302*	2.691**	2.505	3.231*	0.747	−18.878
Ability	−0.006**	−0.006	−0.030***	−0.028*	−0.027*	−0.029	−0.014
Motheredyrs	−0.025*	−0.025	0.010	0.015	0.003	0.009	0.044
Motheredyrsm	0.285	0.285	−0.085	−0.067	−0.044	−0.083	0.057
Fatheredyrs	−0.008	−0.008	−0.015	−0.013	−0.018	−0.015	0.003
Fatheredyrsm	0.167	0.166	−0.261	−0.283	−0.314	−0.261	−0.411
Siblings	0.021	0.021	0.020	0.019	0.017	0.019	0.011
*NorthWest*16	−0.105	−0.105	0.400	0.397	0.389	0.401	0.409
*YorkshireandHumber*16	−0.006	−0.006	−0.037	−0.034	−0.040	−0.035	−0.001
*EastMidlands*16	−0.232	−0.232	−0.402	−0.406	−0.381	−0.399	−0.412
*WestMidlands*16	0.001	0.002	−0.145	−0.144	−0.146	−0.147	−0.125
*Eastern*16	−0.301*	−0.300***	−0.514	−0.517	−0.484	−0.510	−0.526
*London*16	−0.379**	−0.378***	−0.617	−0.604*	−0.610*	−0.613	−0.514
*SouthEast*16	−0.225	−0.224**	−0.178	−0.170	−0.157	−0.174	−0.120
*SouthWest*16	−0.222	−0.221	−0.556	−0.558	−0.535	−0.556	−0.562
*Wales*16	0.015	0.015	−0.019	−0.019	0.004	−0.015	−0.004
*Scotland*16	−0.042	−0.042	−0.589	−0.593	−0.579	−0.588	−0.611
*Regionm*16	−0.192	−0.191	−0.645	−0.641*	−0.631	−0.643	−0.593

Table 4.19: Graduate Advantage in Graduate Labour Market - 1991 - Female

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
TotalAdvgrad	0.019***	0.033	−0.522*	−0.392	−0.942	−2.364	−20.148
AdvgradObs	0.019***	0.024	−0.325*	−0.285	−0.315	−0.318	−0.038
AdvgradUnobs	–	0.009	−0.197	−0.107	−0.628	−2.046	−20.110
Decomposition							
Cons	–	–	–	–	–	–	–
Ability	163.44%	143.25%	105.58%	111.06%	102.24%	106.03%	369.79%
Motheredyrs	−53.94%	−39.10%	−5.67%	−9.07%	−2.90%	−5.71%	−161.92%
Motheredyrsm	−1.25%	−0.98%	−0.06%	−0.06%	−0.05%	−0.06%	−0.11%
Fatheredyrs	54.71%	44.75%	1.43%	0.47%	2.80%	1.32%	−46.31%
Fatheredyrsm	−23.29%	−18.09%	−1.69%	−2.12%	−2.10%	−1.75%	−23.22%
Siblings	0.34%	0.56%	−0.05%	−0.12%	−0.38%	−0.12%	−11.04%
NorthWest16	5.24%	4.04%	2.56%	2.90%	2.57%	2.62%	22.64%
YorkshireandHumber16	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
EastMidlands16	21.33%	16.75%	−2.18%	−2.51%	−2.13%	−2.21%	−19.46%
WestMidlands16	−7.62%	−6.03%	−0.42%	−0.47%	−0.45%	−0.44%	−2.77%
Eastern16	−4.38%	−3.43%	0.45%	0.52%	0.44%	0.46%	3.99%
London16	−35.17%	−26.82%	8.81%	9.58%	9.02%	8.85%	53.29%
SouthEast16	−50.83%	−39.49%	2.57%	2.78%	2.35%	2.56%	14.35%
SouthWest16	22.70%	17.78%	−2.78%	−3.18%	−2.79%	−2.85%	−24.38%
Wales16	−0.18%	−0.15%	−0.08%	−0.09%	−0.03%	−0.07%	−0.43%
Scotland16	2.41%	1.92%	−6.93%	−7.95%	−7.04%	−7.08%	−62.15%
Regionm16	6.47%	5.02%	−1.55%	−1.75%	−1.57%	−1.57%	−12.28%

Table 4.20: Graduate Advantage in Non-Graduate Labour Market - 1991 - Female

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
TotalAdvnongrad	0.210***	0.227*	−0.017	−0.025	−0.175	−1.351	−20.643
AdvnongradObs	0.210***	0.213***	0.181***	0.179***	0.169***	0.185***	0.133***
AdvnongradUnobs	–	0.014	−0.197	−0.204	−0.344*	−1.536	−20.775
Decomposition							
Cons	–	0.000	–	–	–	–	–
Ability	57.35%	57.30%	67.43%	67.61%	65.29%	67.36%	66.62%
Motheredyrs	12.60%	12.76%	2.16%	1.90%	3.08%	2.76%	−2.44%
Motheredyrsm	0.07%	0.07%	0.05%	0.05%	0.06%	0.05%	0.09%
Fatheredyrs	10.56%	10.58%	9.79%	9.74%	10.26%	9.64%	9.89%
Fatheredyrsm	−0.08%	−0.06%	−0.59%	−0.60%	−0.77%	−0.54%	−1.22%
Siblings	5.13%	5.08%	5.54%	5.60%	5.68%	5.44%	7.11%
NorthWest16	−0.55%	−0.54%	−0.14%	−0.14%	−0.13%	−0.12%	−0.17%
YorkshireandHumber16	0.00%	0.00%	−0.00%	−0.00%	−0.00%	−0.00%	−0.00%
EastMidlands16	−0.25%	−0.24%	−0.42%	−0.42%	−0.42%	−0.40%	−0.54%
WestMidlands16	−0.66%	−0.66%	−0.87%	−0.88%	−0.91%	−0.86%	−1.12%
Eastern16	0.08%	0.08%	0.12%	0.12%	0.12%	0.12%	0.17%
London16	14.45%	14.34%	17.53%	17.65%	18.45%	17.13%	22.77%
SouthEast16	1.79%	1.78%	1.16%	1.16%	1.08%	1.14%	1.25%
SouthWest16	0.38%	0.37%	0.22%	0.22%	0.26%	0.21%	0.31%
Wales16	0.04%	0.04%	0.06%	0.06%	0.08%	0.06%	0.10%
Scotland16	−0.65%	−0.64%	−1.58%	−1.60%	−1.66%	−1.54%	−2.24%
Regionm16	−0.25%	−0.25%	−0.46%	−0.46%	−0.47%	−0.45%	−0.58%

Table 4.21: Selection - 1991 - Female

	OLS	*Normal*	*Constant*	*Linear*	*Cubic*	*MedianBreak*	*QuartileBreaks*
TotalSelection	−0.192***	−0.194	−0.506**	−0.367	−0.768	−1.013	0.495
SelectionObs	−0.192***	−0.189*	−0.506**	−0.464	−0.483	−0.503	−0.170
SelectionUnobs	−	−0.005	0.000	0.097	−0.284	−0.511	0.665
Decomposition							
Cons	−	−	−	−	−	−	−
Ability	47.00%	46.46%	91.96%	94.31%	89.34%	91.81%	133.52%
Motheredyrs	19.09%	19.30%	−2.87%	−4.85%	−0.81%	−2.59%	−37.63%
Motheredyrsm	0.20%	0.20%	−0.02%	−0.02%	−0.01%	−0.02%	0.04%
Fatheredyrs	6.25%	6.28%	4.42%	4.04%	5.41%	4.38%	−2.51%
Fatheredyrsm	2.19%	2.21%	−1.30%	−1.53%	−1.64%	−1.31%	−6.08%
Siblings	5.60%	5.65%	1.94%	2.08%	1.74%	1.92%	3.10%
NorthWest16	−1.11%	−1.12%	1.60%	1.73%	1.63%	1.61%	4.87%
YorkshireandHumber16	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
EastMidlands16	−2.35%	−2.38%	−1.55%	−1.70%	−1.53%	−1.55%	−4.71%
WestMidlands16	0.02%	0.02%	−0.58%	−0.63%	−0.61%	−0.59%	−1.49%
Eastern16	0.51%	0.52%	0.33%	0.36%	0.33%	0.33%	1.01%
London16	19.29%	19.53%	11.92%	12.69%	12.32%	11.90%	29.50%
SouthEast16	6.92%	6.98%	2.07%	2.15%	1.91%	2.04%	4.14%
SouthWest16	−1.80%	−1.82%	−1.71%	−1.87%	−1.72%	−1.72%	−5.13%
Wales16	0.06%	0.06%	−0.03%	−0.03%	0.01%	−0.02%	−0.02%
Scotland16	−0.95%	−0.97%	−5.02%	−5.50%	−5.16%	−5.04%	−15.47%
Regionm16	−0.91%	−0.92%	−1.16%	−1.25%	−1.18%	−1.16%	−3.16%

Table 4.22: Nicejobs A - 1991 - Female

	Pubchar	Nightwork	Weekendwork	Fixedhours	Compwork	Workhours
Cons	0.165	0.111	0.580***	0.921***	−0.212	14.994***
	(0.134)	(0.100)	(0.135)	(0.126)	(0.130)	(3.530)
Ability	0.002**	−0.001*	−0.002**	−0.004***	0.009***	0.061**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.019)
Mother ed yrs	0.012	0.007	0.005	−0.004	0.013	0.418
	(0.009)	(0.007)	(0.009)	(0.008)	(0.009)	(0.233)
Mother ed yrs m	0.012	0.057	−0.030	−0.093	0.031	−2.285
	(0.084)	(0.063)	(0.084)	(0.078)	(0.081)	(2.215)
Father ed yrs	0.003	0.011	0.019	−0.007	−0.018	−0.054
	(0.014)	(0.011)	(0.015)	(0.014)	(0.014)	(0.381)
Father ed yrs m	0.016	0.014	0.015	−0.039	−0.046	1.468
	(0.050)	(0.037)	(0.050)	(0.047)	(0.048)	(1.314)
Siblings	−0.002	0.012**	0.013*	0.016**	−0.021***	−0.049
	(0.006)	(0.004)	(0.006)	(0.005)	(0.006)	(0.154)
North West 16	0.005	0.041	−0.110*	−0.007	0.041	0.912
	(0.053)	(0.039)	(0.054)	(0.050)	(0.051)	(1.385)
Yorkshire and Humber 16	0.021	−0.045	−0.109	0.024	−0.062	−0.753
	(0.056)	(0.042)	(0.058)	(0.053)	(0.054)	(1.480)
East Midlands 16	−0.005	0.027	−0.083	−0.015	0.022	−1.406
	(0.057)	(0.043)	(0.058)	(0.054)	(0.055)	(1.501)
West Midlands 16	−0.016	0.007	−0.104	−0.028	0.055	1.857
	(0.054)	(0.040)	(0.055)	(0.051)	(0.052)	(1.420)
Eastern 16	−0.087	−0.002	−0.185***	−0.034	0.064	1.024
	(0.054)	(0.040)	(0.055)	(0.051)	(0.052)	(1.414)
London 16	0.007	0.001	−0.169**	0.007	0.179***	4.598**
	(0.055)	(0.041)	(0.056)	(0.052)	(0.053)	(1.452)
South East 16	−0.135*	0.018	−0.110*	0.001	0.052	0.301
	(0.053)	(0.039)	(0.053)	(0.050)	(0.051)	(1.385)
South West 16	−0.070	0.020	−0.106	−0.028	−0.015	−1.317
	(0.056)	(0.042)	(0.057)	(0.053)	(0.054)	(1.479)
Wales 16	0.039	0.012	−0.066	0.057	−0.031	1.260
	(0.062)	(0.046)	(0.063)	(0.059)	(0.060)	(1.627)
Scotland 16	0.056	0.006	−0.069	0.024	−0.053	1.022
	(0.054)	(0.040)	(0.055)	(0.051)	(0.052)	(1.415)
Region m 16	0.030	−0.010	−0.059	−0.005	0.024	1.721
	(0.050)	(0.037)	(0.051)	(0.047)	(0.048)	(1.324)
Cons*C	0.931**	−0.229	−0.556	0.078	0.677*	10.466
	(0.322)	(0.241)	(0.327)	(0.302)	(0.311)	(8.496)
Ability*C	−0.010**	0.004	0.002	0.002	−0.003	−0.033
	(0.003)	(0.002)	(0.003)	(0.003)	(0.003)	(0.083)
Mother ed yrs*C	−0.030	−0.011	0.003	−0.005	−0.002	0.213
	(0.017)	(0.013)	(0.017)	(0.016)	(0.016)	(0.448)
Mother ed yrs m*C	−0.210	−0.216	0.042	0.316	0.143	8.019
	(0.283)	(0.212)	(0.285)	(0.264)	(0.274)	(7.470)
Father ed yrs m*C	0.125	0.132	0.410	−0.137	−0.263	1.855
	(0.227)	(0.170)	(0.229)	(0.212)	(0.220)	(6.001)
Siblings*C	0.043	−0.011	−0.012	−0.027	−0.003	−0.307
	(0.022)	(0.017)	(0.023)	(0.021)	(0.022)	(0.590)
North West 16*C	0.196	−0.056	0.094	−0.081	−0.087	−7.969
	(0.190)	(0.142)	(0.192)	(0.177)	(0.183)	(5.010)
Yorkshire and Humber 16*C	0.290	0.090	0.154	0.194	0.116	−7.519
	(0.196)	(0.147)	(0.197)	(0.183)	(0.189)	(5.174)
East Midlands 16*C	0.148	0.286	0.171	0.172	−0.012	−1.883
	(0.212)	(0.162)	(0.217)	(0.201)	(0.206)	(5.614)
West Midlands 16*C	0.385*	−0.111	−0.043	0.241	−0.164	−8.859
	(0.193)	(0.145)	(0.195)	(0.180)	(0.187)	(5.109)
Eastern 16*C	0.288	0.050	0.163	0.142	−0.190	−5.864
	(0.189)	(0.141)	(0.191)	(0.177)	(0.182)	(4.982)
London 16*C	0.115	0.043	0.057	0.021	−0.209	−9.443*
	(0.178)	(0.133)	(0.179)	(0.166)	(0.172)	(4.690)
South East 16*C	0.335	0.014	0.134	0.046	−0.076	−5.411
	(0.179)	(0.134)	(0.180)	(0.167)	(0.173)	(4.729)
South West 16*C	0.130	−0.129	0.121	−0.002	−0.156	−3.525
	(0.196)	(0.146)	(0.197)	(0.183)	(0.189)	(5.172)
Wales 16*C	0.244	0.071	0.163	0.157	−0.062	−2.324
	(0.220)	(0.165)	(0.226)	(0.205)	(0.213)	(5.812)
Scotland 16*C	0.376	0.027	−0.138	0.041	0.036	−8.865
	(0.203)	(0.152)	(0.207)	(0.194)	(0.197)	(5.369)
Region m 16*C	0.216	−0.058	−0.037	0.025	−0.062	−10.321*
	(0.183)	(0.138)	(0.185)	(0.171)	(0.177)	(4.839)
N	2642	2640	2602	2601	2648	2651

Table 4.23: Nicejobs B - 1991 - Female

	Benefits	Firmshares	Companycar	Travelben	Subsmeals
Cons	1.053*	0.374**	−0.074	−0.355**	0.371*
	(0.414)	(0.127)	(0.079)	(0.122)	(0.155)
Ability	0.009***	0.002**	0.000	0.001	−0.002
	(0.002)	(0.001)	(0.000)	(0.001)	(0.001)
Mother ed yrs	0.004	−0.016	0.011*	0.014	0.005
	(0.027)	(0.008)	(0.005)	(0.008)	(0.010)
Mother ed yrs m	−0.295	−0.006	−0.023	−0.118	−0.027
	(0.262)	(0.080)	(0.050)	(0.077)	(0.098)
Father ed yrs	−0.052	−0.004	0.002	−0.003	−0.003
	(0.043)	(0.013)	(0.008)	(0.013)	(0.016)
Father ed yrs m	0.006	0.027	−0.010	0.046	0.105
	(0.149)	(0.046)	(0.028)	(0.044)	(0.056)
Siblings	−0.045*	−0.009	−0.006	−0.012*	0.012
	(0.019)	(0.006)	(0.004)	(0.006)	(0.007)
North West 16	0.057	−0.063	0.055	0.052	0.009
	(0.172)	(0.053)	(0.033)	(0.051)	(0.064)
Yorkshire and Humber 16	0.004	−0.069	−0.008	−0.015	0.023
	(0.184)	(0.057)	(0.035)	(0.054)	(0.069)
East Midlands 16	0.003	−0.014	0.014	0.019	0.046
	(0.187)	(0.057)	(0.036)	(0.055)	(0.070)
West Midlands 16	0.294	0.024	0.036	0.062	−0.001
	(0.176)	(0.054)	(0.034)	(0.052)	(0.065)
Eastern 16	0.263	0.047	0.035	0.007	0.021
	(0.175)	(0.054)	(0.033)	(0.052)	(0.065)
London 16	0.427*	0.014	0.016	0.086	0.085
	(0.176)	(0.054)	(0.034)	(0.052)	(0.066)
South East 16	0.217	0.028	0.015	0.020	0.067
	(0.173)	(0.053)	(0.033)	(0.051)	(0.064)
South West 16	0.165	0.012	−0.012	0.031	0.043
	(0.189)	(0.058)	(0.036)	(0.056)	(0.070)
Wales 16	0.038	−0.037	−0.001	0.086	−0.039
	(0.204)	(0.063)	(0.039)	(0.060)	(0.076)
Scotland 16	−0.142	−0.099	0.033	0.058	−0.013
	(0.177)	(0.055)	(0.034)	(0.052)	(0.066)
Region m 16	0.097	−0.008	0.020	0.083	0.003
	(0.167)	(0.051)	(0.032)	(0.049)	(0.062)
Cons*C	−0.212	−0.460	0.276	0.650*	−0.532
	(0.933)	(0.287)	(0.178)	(0.275)	(0.348)
Ability*C	0.010	−0.000	0.003	−0.003	0.001
	(0.009)	(0.003)	(0.002)	(0.003)	(0.003)
Mother ed yrs*C	0.053	0.024	−0.014	−0.021	0.026
	(0.050)	(0.015)	(0.010)	(0.015)	(0.019)
Mother ed yrs m*C	−0.351	0.242	−0.162	−0.161	−0.096
	(0.781)	(0.240)	(0.149)	(0.230)	(0.291)
Father ed yrs m*C	0.794	−0.250	0.195	0.419*	−0.015
	(0.677)	(0.208)	(0.129)	(0.199)	(0.253)
Siblings*C	0.007	0.009	−0.017	−0.005	−0.010
	(0.065)	(0.020)	(0.012)	(0.019)	(0.024)
North West 16*C	−0.258	0.059	−0.262**	0.109	0.114
	(0.526)	(0.162)	(0.101)	(0.155)	(0.196)
Yorkshire and Humber 16*C	−1.058	−0.024	−0.120	0.029	−0.076
	(0.541)	(0.166)	(0.103)	(0.159)	(0.202)
East Midlands 16*C	−0.626	−0.003	−0.149	−0.195	−0.026
	(0.579)	(0.178)	(0.111)	(0.170)	(0.216)
West Midlands 16*C	−0.640	0.028	−0.286**	−0.045	0.203
	(0.530)	(0.163)	(0.101)	(0.156)	(0.198)
Eastern 16*C	−0.765	0.010	−0.275**	0.073	0.227
	(0.536)	(0.165)	(0.102)	(0.158)	(0.200)
London 16*C	−0.989*	0.084	−0.186*	−0.154	0.042
	(0.495)	(0.152)	(0.095)	(0.146)	(0.185)
South East 16*C	−0.788	0.051	−0.219*	−0.104	0.080
	(0.498)	(0.153)	(0.095)	(0.146)	(0.186)
South West 16*C	−0.340	0.017	−0.083	0.168	0.114
	(0.556)	(0.171)	(0.106)	(0.164)	(0.207)
Wales 16*C	−0.535	0.031	−0.328**	0.191	0.224
	(0.614)	(0.189)	(0.117)	(0.181)	(0.229)
Scotland 16*C	−0.782	0.016	−0.306**	0.040	−0.118
	(0.572)	(0.176)	(0.109)	(0.168)	(0.213)
Region m 16*C	−0.802	−0.040	−0.189	−0.106	0.016
	(0.511)	(0.157)	(0.098)	(0.150)	(0.191)
N	1929	1929	1928	1927	1927

Table 4.24: Nicejobs C - 1991 - Female

	Medicalins	Pension	Childcare	Discounts	Otherben
Cons	−0.153	0.306*	−0.069	0.662***	0.025
	(0.102)	(0.156)	(0.050)	(0.155)	(0.121)
Ability	0.001*	0.005***	0.000	−0.001	0.002**
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)
Mother ed yrs	0.008	0.012	0.001	−0.018	−0.016*
	(0.007)	(0.010)	(0.003)	(0.010)	(0.008)
Mother ed yrs m	−0.044	0.041	0.051	−0.106	−0.062
	(0.065)	(0.098)	(0.031)	(0.097)	(0.076)
Father ed yrs	−0.006	0.005	−0.001	−0.019	−0.023
	(0.011)	(0.016)	(0.005)	(0.016)	(0.013)
Father ed yrs m	−0.060	−0.078	−0.008	0.011	−0.028
	(0.037)	(0.056)	(0.018)	(0.056)	(0.044)
Siblings	−0.008	−0.017*	−0.000	0.002	−0.005
	(0.005)	(0.007)	(0.002)	(0.007)	(0.005)
North West 16	0.040	−0.085	−0.001	−0.010	0.061
	(0.042)	(0.065)	(0.021)	(0.064)	(0.050)
Yorkshire and Humber 16	0.009	−0.061	0.044*	−0.023	0.105
	(0.045)	(0.069)	(0.022)	(0.069)	(0.054)
East Midlands 16	0.025	−0.102	0.005	−0.018	0.030
	(0.046)	(0.070)	(0.022)	(0.070)	(0.055)
West Midlands 16	0.075	0.001	0.020	−0.059	0.137**
	(0.043)	(0.066)	(0.021)	(0.065)	(0.051)
Eastern 16	0.087*	−0.063	0.029	−0.051	0.143**
	(0.043)	(0.066)	(0.021)	(0.065)	(0.051)
London 16	0.157***	0.029	0.011	−0.094	0.125*
	(0.043)	(0.066)	(0.021)	(0.066)	(0.051)
South East 16	0.100*	−0.066	−0.004	0.006	0.053
	(0.043)	(0.065)	(0.021)	(0.064)	(0.050)
South West 16	0.036	−0.056	0.005	−0.001	0.108
	(0.047)	(0.071)	(0.023)	(0.070)	(0.055)
Wales 16	0.061	−0.064	−0.001	−0.006	0.039
	(0.050)	(0.077)	(0.024)	(0.076)	(0.060)
Scotland 16	0.011	−0.065	0.007	−0.109	0.036
	(0.044)	(0.067)	(0.021)	(0.066)	(0.052)
Region m 16	0.049	−0.070	0.004	−0.025	0.044
	(0.041)	(0.063)	(0.020)	(0.062)	(0.049)
Cons*C	0.231	0.343	0.031	−0.498	−0.288
	(0.230)	(0.351)	(0.112)	(0.348)	(0.273)
Ability*C	0.003	−0.006	0.002*	0.007*	0.002
	(0.002)	(0.004)	(0.001)	(0.003)	(0.003)
Mother ed yrs*C	0.007	−0.008	−0.012*	0.021	0.033*
	(0.012)	(0.019)	(0.006)	(0.019)	(0.015)
Mother ed yrs m*C	0.035	−0.176	−0.117	0.361	−0.279
	(0.193)	(0.294)	(0.094)	(0.291)	(0.228)
Father ed yrs m*C	0.305	−0.305	−0.006	0.047	0.402*
	(0.167)	(0.254)	(0.081)	(0.252)	(0.198)
Siblings*C	−0.020	0.072**	0.001	−0.004	−0.020
	(0.016)	(0.025)	(0.008)	(0.024)	(0.019)
North West 16*C	−0.379**	0.133	0.108	−0.162	0.023
	(0.130)	(0.198)	(0.063)	(0.196)	(0.154)
Yorkshire and Humber 16*C	−0.399**	0.021	−0.026	−0.324	−0.140
	(0.134)	(0.204)	(0.065)	(0.202)	(0.158)
East Midlands 16*C	−0.296*	0.174	0.011	−0.098	−0.045
	(0.143)	(0.218)	(0.069)	(0.216)	(0.169)
West Midlands 16*C	−0.491***	0.139	−0.014	−0.038	−0.136
	(0.131)	(0.199)	(0.064)	(0.197)	(0.155)
Eastern 16*C	−0.548***	−0.037	0.040	−0.156	−0.090
	(0.132)	(0.202)	(0.064)	(0.200)	(0.157)
London 16*C	−0.526***	−0.062	0.012	−0.088	−0.112
	(0.122)	(0.186)	(0.059)	(0.184)	(0.145)
South East 16*C	−0.479***	0.050	0.050	−0.233	0.014
	(0.123)	(0.187)	(0.060)	(0.185)	(0.145)
South West 16*C	−0.407**	−0.049	0.138*	−0.165	−0.072
	(0.137)	(0.209)	(0.067)	(0.207)	(0.162)
Wales 16*C	−0.609***	0.329	0.106	−0.298	−0.182
	(0.151)	(0.231)	(0.074)	(0.229)	(0.179)
Scotland 16*C	−0.483***	0.150	0.021	−0.071	−0.031
	(0.141)	(0.215)	(0.069)	(0.213)	(0.167)
Region m 16*C	−0.360**	−0.027	0.023	−0.135	0.011
	(0.126)	(0.192)	(0.061)	(0.190)	(0.149)
N	1927	1928	1927	1927	1927

Chapter 5

Maternity Leave Duration and Female-Male Relative Labour Market Outcomes

5.1 Introduction

Large gender pay gaps exist in most countries, with the EU average gender pay gap standing at 16.5% in 2012. The UK gender gap stands slightly higher than the EU average, at 19.1%.[1] These gaps are persistent despite the high priority placed on closing the gender pay gap, both at an EU and a UK level, and despite female educational attainment surpassing male attainment over the last decade.[2] Motivating factors for decreasing the gender pay gap include promoting gender fairness and equality, increasing female labour market participation and decreasing litigation costs. At the same time, many countries have recently increased the generosity of maternity leave benefits, and have introduced increasingly flexible working arrangements. Numerous arguments can be made for paid parental leave/flexible working arrangements, such as increasing female labour market opportunity and participation. A key question arises as to whether the increasing flexibility of maternity leave and working arrangements (which potentially impose direct or indirect employer costs), hinders progress in closing the gender gap or impacts on other relative female-male labour market outcomes.

This chapter analyses the impact of an increase in the duration of paid maternity leave on relative female labour market outcomes. The analysis is based in the UK, which experienced a 50% increase in the duration of paid maternity leave for female employees,

[1] Eurostat

[2] In 2012 the UK average share of males aged 30-34 who had completed tertiary education was 44.0% compared to 50.2% for females. The corresponding figures for 2002 were 32.4% for males and 30.7% for females. (Eurostat)

from a maximum of 26 weeks to a maximum of 39 weeks from 1st April 2007. While similar reforms in other countries/time periods tended to increase parental leave for male and female employees simultaneously (e.g. the FMLA in the United States), this reform was unusual in changing parental rights for female employees only. Furthermore, the quasi-experimental estimation approach implemented in this chapter is unique in that it estimates the role of statistical discrimination by employers on relative female-male labour market outcomes, separately from the impact of increased human capital depreciation or higher numbers of retained job matches. The existing literature tends to estimate an impact that is an aggregate of any employer discrimination, the increased number of retained job matches and the impact of longer leave periods on human capital depreciation. A difference in differences estimation strategy is implemented, with an alternative approach being suggested for binary outcome models that respects the discrete nature of the outcome. The proposed approach builds on that of Athey and Imbens (2006) and Blundell et al. (2004b). One key benefit of the proposed approach is that it facilitates estimation of the impact of a policy change when there are possible substitution effects impacting the control group.

A simple theoretical model of the role of differential parental leave uptake by male and female employees on relative labour market outcomes is developed. While on parental leave employees do not receive pay from their firms, but employees taking longer leave periods are less profitable for the firm because the firm has equal sunk hiring and training costs for all employees. This implies a male-female wage gap for otherwise identical employees. Furthermore, in the face of expanding differentials in male-female parental leave uptake, the model predicts an increase in the male-female wage gap.

This chapter shows that the policy change increased the relative uptake of parental leave by females compared to males, particularly for those aged 25-34. The aggregate effect was decomposed into specific effects due to fertility responses and relative female-male uptake of parental leave, both of which play a significant role in the divergence. Furthermore, empirical evidence was found in support of the theoretical model, with evidence of a divergence in male-female relative wages after the expansion of paid maternity leave in the UK. This divergence was in contrast with the weak trend of converging male-female wages observed in previous years. Furthermore, evidence was found of an increase in relative female redundancies after the policy change. Although a negative impact was estimated on relative female hiring rates it was not statistically significant. The low magnitude of the hiring effect may be due to higher rates of female replacements necessitated by fertility responses and/or higher maternity leave durations, combined with the likelihood of a female employee being replaced with another female due to occupational sorting. Similarly, a negative but insignificant impact was found on relative female conditional employment rates. It is possible that a longer observation period would lead to larger

estimates on relative employment rates, as differential hiring/firing rates accumulate.
The chapter is laid out as follows; Section 5.2 provides a review of the existing literature, Section 5.3 outlines the theoretical model, Section 5.4 discusses the estimation methodology both for continuous outcomes and the proposed methodology for binary outcomes, Section 5.5 provides an overview of the legislative context in the UK, the data and empirical strategy are outlined in Section 5.6, the results are presented in Section 5.7 and finally, Section 5.8 concludes.

5.2 Previous Literature

The main economic theories of labour market discrimination are taste based models dating back to Becker (1957) on the one hand, and models of statistical discrimination introduced into the literature by Phelps (1972), Arrow (1972) and Aigner and Cain (1977) on the other. Taste based models of discrimination deal with employer taste for discrimination, employee taste for discrimination or customer taste for discrimination. More closely related to the approach followed in this chapter, statistical based discrimination presumes no prejudice. There are two main strands to the statistical discrimination model; the first which is based on different group mean productivity levels (heterogeneous productivity levels are unobservable to the employer, although they may receive a noisy signal) and the second which is based on different group precision levels relating to the signal received.
Statistical discrimination models of the male-female wage gap that assume different mean productivity levels of males and females (due to the higher levels of work absence amongst females owing to pregnancy/maternity leave) falls into the first category of different group mean productivity levels. This statistical discrimination based on different group mean productivity levels leads to labour market discrimination against women. As pointed out by Cain (1986), women who participate the most will be underpaid and women who participate the least will be overpaid relative to their productivity levels, with underpayments cancelling out overpayments. On average females are paid a rate equal to their average productivity. Therefore it is often stated that there is no group discrimination against women.[3] However, using common definitions of labour market discrimination every female experiences labour market discrimination if employers base hiring or wage decisions on this type of statistical information. This is clarified in the following.
Heckman and Siegelman (1993) define labour market discrimination as: "it occurs if persons in one groups with the same relevant productivity characteristics as persons in another group are treated unfavourably by the labor market solely as a consequence of their demo-

[3] Thurow (1975) points out that even in this case of statistical discrimination based on groups means, group discrimination may still exist when you consider employment decisions, which are a zero-one decision.

graphic status". Similarly, Altonji and Blank (1999) define labour market discrimination as "a situation in which persons who provide labor market services and who are equally productive in a physical or material sense are treated unequally in a way that is related to an observable characteristic such as race, ethnicity, or gender. By "unequal" we mean these persons receive different wages or face different demands for their services at a given wage." Either definition would lead us to conclude that women experience labour market discrimination. To clarify this point, consider a distribution of propensities to be out of the labour market on parental leave for male and female employees who have the same levels of all other productivity characteristics. Male and female employees have different distributions of the propensity to be out of the labour market, with the female distribution much more skewed to the right relative to the male distribution. However, for any given propensity to be out of the labour market, females will be paid less than a comparable male. This is because propensity to be out of the labour market is not observed by the employer, and hiring/wage decisions are based on gender means.

Regardless of how this type of employer behaviour is classified economically, this type of behaviour by employers is typically illegal in most countries, and certainly in the legal sense would tend to be classified as discriminatory.[4] Furthermore, regardless of whether this employer behaviour is considered discriminatory by economists, it is still a topic of interest for labour economists considering female labour market participation or the male-female wage gap.

With the exception of statistical discrimination models where the role of higher parental leave taking by mothers is incorporated into a lower mean productivity level, there is little in the theoretical literature about the potential role maternity leave plays in contributing to the male-female wage gap. One recent exception is a paper by Yip and Wong (2014), who introduce a search based model of labour market discrimination where female employees differ from males only in leave taking due to childbearing. In their model, a male-female wage gap occurs due to the lower productivity of a female worker.[5] They derive a number of empirical implications from their model; the first being that even in the absence of taste based discrimination females will receive lower bargaining wages than males. Furthermore, along the age specific fertility rate profile (ASFR), the male-female wage gap should be

[4]For instance, see Masselot et al. (2012) for an overview of the legislative framework in Europe. Typically, sex discrimination legislation exists in the EU member states, which prohibits differential employment treatment (recruitment/employment/payment) for male and female workers (e.g. the *Sex Discrimination Act* (1975) in the UK). Additionally, case law (for instance in the European Court of Justice), has ruled that since only a women can become pregnant, a refusal to employ her, or decision to dismiss her because of pregnancy/maternity is equivalent to sex discrimination. Furthermore the *Sex Discrimination Act* (1975) in the UK was amended by the *Employment Equality (Sex Discrimination) Regulations* (2005), to make explicit that discrimination on the grounds of pregnancy constitutes sex discrimination. See Section 5 and Appendix 5B. for a more in-depth discussion of the legislative framework in the UK.

[5]Employees continue to receive pay while on leave in this model.

largest when the ASFR is the highest. Yip and Wong (2014) do not estimate their search model, but test the empirical implications by looking at the relationship between the male-female wage gap and ASFR and find some evidence that the ASFR does indeed have a negative impact on female wages in Hong Kong. Similarly, Erosa et al. (2010) develop and calibrate a structural model of maternity leave policies and fertility, welfare and employment in the US prior to the introduction of the FMLA, and perform various counterfactual simulations to try and understand the mechanisms through which maternity leave laws affect these outcomes. The model estimates that employer costs associated with paid maternity leave (which exist even though employees do not receive pay from employers while on leave in this model) results in a lower wage for females of childbearing age.[6]

There are many empirical papers researching female labour market discrimination. Cain (1986) reviews 20 early empirical papers, which focus on estimating earnings functions. The coefficient on the group variable in the earnings functions, once other productivity characteristics have been taken into account is referred to as the wage gap rather than wage discrimination. This is in recognition of the fact that if variables measuring productivity reflect discrimination, or if productivity characteristics which are non-random across groups have been omitted, then the wage gap may be either an upwards/downwards biased estimate of wage discrimination. Altonji and Blank (1999) also review a number of empirical papers providing evidence on the extent of discrimination in the labour market, focusing largely on quasi-experimental approaches. Papers they review that deal with female labour market discrimination include audit studies (e.g. Neumark (1996) who looks at employment rates of male versus female applicants in the restaurant industry. This paper finds some evidence of discrimination, with male and female applicants more likely to be hired in high/low end restaurants respectively) and sex blind hiring (e.g Goldin and Rouse (2000), who look at the impact of blind hiring on the relative hiring rates of male and female musicians. This paper finds strong evidence of discrimination).[7] They also discuss the direct approach of Hellerstein et al. (1999) who estimate marginal productivities of various demographic groups and then compare these estimates to labour market wages directly. This paper found that the relative marginal productivities of female to male employees was higher than the corresponding relative wages, suggesting possible discrimination. However, their model does not seem to match the data very well; for instance, unskilled labour was estimated to be more productive than professional/managerial labour. In another paper, Hellerstein et al. (2002) also test directly for discrimination by

[6] One quarter of paid maternity leave was estimated to correspond to a 0.5% decrease in lifetime female wages through the bargaining channel

[7] More recent male-female audit studies include Riach and Rich (2006) who find evidence of gender sorting in the UK, with females more likely to receive call backs in female dominated occupations and vice versa for males. Petit (2007) also using an audit study, find evidence of discrimination against young women in France in high-skilled administrative jobs, and for jobs with long term contracts

comparing profitability of firms with different female/male employee compositions. Again evidence suggestive of discrimination was found, but as pointed out in the Altonji & Blank review, there may be endogeneity if the gender composition is correlated with other firm characteristics.[8]

Most closely related to this work are several papers that look at the impact of policy changes that may directly impact female labour market outcomes. Gruber (1994) analyses the impact of state and federal laws introduced in the US in the 1970s which stipulated that pregnancy must not be treated differently from comparable illness in health insurance benefits. This additional coverage resulted in higher health insurance premiums for women of childbearing age. Therefore, the cost of employing women of childbearing age by employers who provide health insurance benefits (and married men with wives of childbearing age who are covered under their employer provided health insurance) increased. State laws came into effect for 23 states between 1976 and 1977, whereas a federal law, covering all states, was passed in 1978. Using a triple differences estimation strategy (with a time-state-group dimension), Gruber (1994) estimates the impact of the introduction of this policy change first using the states that passed state laws between 1976 and 1977 as the treatment group, and states that did not pass state legislation as the control group. He also separately estimates the impact of the policy using the federal legislation, this time using the states that had already passed the legislation as the control group. The treatment group considered are married women aged 20-40 and the control group are single men aged 20-40 and everyone aged over 40. In the regression analysis, this paper found that the state (federal) policy change decreased the relative wages of the treatment group by about 4.3% (2.1%). There was no significant effects found on relative employment.

Another related study is Baum (2003), who analyses the impact of the introduction of the Family Medical Leave Act of 1993 in the US. This federal law provided access for certain employees to job-protected unpaid leave for various medical and family reasons. To qualify you must have been employed by your employer for 12 months, to have worked 1250 hours in the last 12 months and to be employed by an employer with 50 or more employees who live in a 75 mile radius of the workplace (this employee threshold did not apply to the public sector). The FMLA allows for up to 12 weeks of leave a year for reasons including caring for a new baby (birth/adopted), own health reasons and caring for an ill family member. Baum (2003) also follows a triple difference estimation strategy, exploiting the fact that some states passed maternity leave legislation before the 1993 FMLA (also uses time-state-group dimension). Two treatment groups are considered; those with children under the age of 1, and all women of childbearing age. Two control groups are also

[8]Another estimation approach reviewed is that in Altonji and Pierret (2001), who suggest a test for statistical discrimination based on observed earnings differentials over time - although this approach is applied to race rather than gender discrimination

considered; all men and alternatively single men. Sample selection is controlled for in one specification. Regardless of the specification and choice of treatment/control group the analysis found no evidence of an impact of the policy change on relative employment or wages. However, this paper does not present evidence of a significant first stage effect - a differential impact of leave taking by women relative to males as a result of the policy change. In fact, a report by The Commission on Leave (The Commission on Leave, 1995) documented similar levels of leave taking by workplaces covered/not covered by the FMLA. Furthermore, men were more likely to take FMLA leave for spousal care reasons (attributed in the report to possible care of wives before/after childbirth). Therefore, there does not seem to be clear evidence that FMLA increased leave taking, or if it did that it differentially impacted upon men and women. Waldfogel (1999) analyses the first stage effect (tests for a larger relative uptake of leave by women after the introduction of the FMLA), and finds a divergence in female-male relative leave taking only for medium sized firms, with small/large firms actually experiencing the opposite effect. Waldfogel (1999) also does not find strong evidence of any relative employment or wage effects.

A number of recent papers have analysed the impact of the introduction of the Californian Paid Family Leave policy, implemented in 2004. This policy brought in up to 6 weeks of paid leave each year. At the time the federal Family and Medical Leave Act allowed for up to 12 weeks of unpaid leave. Male and female employees were eligible for the paid leave to bond with a new child, or to care for a seriously ill child, spouse, parent or registered domestic partner. Baum and Ruhm (2013) provide evidence that the first stage effects (the impact of the policy on parental leave taking) were greater for mothers (2.4 week increase) relative to fathers (1 week increase), but otherwise focus on labour market outcomes of mothers/fathers. Using the same policy reform, a recent paper by Curtis et al. (2014) found no evidence of any impact of the policy on female earnings, using a triple difference approach with time-state-group dimensions, where young females are the treatment group and all other demographic groups the control group. The paper did find higher rates of job separations (over 3 months) for female workers. However, they do not separate redundancies from extended maternity leave in their analysis. Higher rates of recalls lead the authors to conclude that the although the policy increased the occurrence of separations, some of these women returned to their same employer after an extended leave period. Finally, evidence of an increase in new hires was found amongst young women after the policy change (which might be explained by the higher separation rates - due to occupational sorting it is more likely to hire a female to replace a female). Another recent paper, Das and Polachek (2014) using the same reform find evidence that the policy increased female labour market participation, but increased female unemployment and unemployment duration to a greater extent.

One paper that finds significant negative effects on female wages resulting from the introduction of mandatory paid maternity leave is Lai and Masters (2005). This paper analyses the introduction of the Labor Standards Law in 1984 in Taiwan. This law introduced eight weeks of paid maternity leave (paid for by employer) for employees employed for at least 6 months in covered industries (manufacturing, mining, electricity, construction, transport and mass media). The law also prohibited employers from firing women due to marriage/pregnancy. New mothers were paid two half-hour breaks a day for breastfeeding until their child was 1 year old. Finally, employers had to provide a different role for pregnant employees if their typical role had a health risk. A triple difference estimation strategy is used with time-industry-group dimensions. Women aged 20-29 are the treatment group, with one specification using men aged 20-29 as the control, and another using women aged 30-54 as the control. Negative relative wage effects are found, particularly when young men are used as the control group. A difference in difference model is used to estimate relative employment effects, with time-group dimension. Negative relative employment effects were also found.

Another interesting finding is that employers appear to reduce investment in on-the-job training of young women in response to an increase in maternity leave. Puhani and Sonderhof (2009) analyse the impact of the doubling of the German maternity leave duration from 18 months to 36 months in 1992 using a difference in difference approach (time-group dimension) where the treatment group is women aged 20-35 and the control group is women aged 40-55. A triple difference approach is also taken, with time-gender-age dimension. Across different specifications, negative effects were found for employer provided on-the-job training for women of childbearing age, with some evidence that this was partially compensated through employee arranged training.

There is also a strand of research which uses cross-country analysis to try and estimate the impact of maternity leave policies on female labour market outcomes. Ruhm (1998) studies 9 European countries during the period 1969-1993, all of which experienced large changes in maternity leave policies. Outcomes considered are employment to population ratios and hourly wages. A triple differences approach is used with time-country-gender dimension. Weeks of paid leave (regardless of replacement rate) is the key policy variable considered. 40 weeks of paid maternity leave compared to none at all was estimated to increase employment-population ratios of women by 4.2 percent and to lower hourly wages by 2.7 percent, with larger effects found for women of childbearing age. Ruhm (1998) discusses some possible econometric issues associated with this type of cross-country analysis; firstly that changes in other family policies might occur at the same time as changes in maternity leave policies within a country, confounding the results. Secondly, increases in female labour supply might increase political pressure for these types of legislative changes, leading to endogeneity. There may be intra-household substitution, changing the interpretation of the DDD estimates. Finally, he notes that some of the positive employment

effect may be due to how females are classified while on leave. Although not noted in the paper, a related issue is how wages are reported by women while on maternity leave. Often women will receive some percentage of their salary from their employers while on maternity leave, therefore longer leave periods may also lead to lower mean levels of reported wages among women, which may explain some of the downwards wage effect.
Akgunduz and Plantenga (2012) provide an update of Ruhm (1998), by analysing the impact of changes in maternity leave legislation across 16 European countries between 1970-2010. Following a very similar estimation strategy to Ruhm (1998), and using weeks of (weighted) full-replacement leave as the explanatory variable this paper also finds that longer leave entitlements increase female employment and decrease wages, particularly for high skill women. In addition to the potential issues addressed in Ruhm (1998), this paper also does not distinguish between maternity and parental leave, assuming all leave is taken up by the mother. This may also bias the results, depending on actual leave take up by females/males in the labour market.
A number of related papers look at the impact of maternity/paternity leave duration on the wages of mothers/fathers and the return to work after leave.[9,10] Another related strand of research analyses the impact of expanding parental leave policies on fertility.[11] Finally, a number of papers focus on the impact of expanding parental leave policies on child health/education outcomes.[12]

5.3 Simple Theoretical Model

In this section, a simple theoretical model is presented. Higher levels of parental leave taken by female employees leads to a predicted male-female wage gap in the model. Furthermore, greater divergence in relative female-male leave taking is predicted to increase the male-female wage gap. This prediction is tested empirically later in the chapter, facilitated by a common trends assumption. The model also predicts that a greater divergence in relative female-male leave taking is associated with higher male wages and higher male

[9] Mothers'/Fathers' wages: Ondrich and Spiess (2002) - Germany, Buligescu et al. (2008) - Germany, Schönberg and Ludsteck (2007) - Germany, Ekberg et al. (2013) - Sweden ("Daddy-Month" reform), Waldfogel (1998) - US and Britain, Joseph et al. (2013) - France, Lalive et al. (2013) - Austria, Rege and Solli (2013) - Norway (Parenity leave quota), Baum and Ruhm (2013) - US, Dahl et al. (2013) - Norway

[10] Return to work: Ondrich et al. (1996) - Germany, Pronzato (2009) - Europe, Schönberg and Ludsteck (2007) - Germany, Baker and Milligan (2008a) - Canada, Hanratty and Trzcinski (2008) - Canada, Joseph et al. (2013) - France, Lalive and Zweimüller (2009) - Austria, Lalive et al. (2013) - Austria, Rossin-Slater et al. (2013) - California, Waldfogel et al. (1999) - US, Britain and Japan, Dahl et al. (2013) - Norway

[11] Fertility: Lalive and Zweimüller (2009) - Austria, Björklund (2006) - Sweden, Dahl et al. (2013) - Norway, Cannonier (2014) - US, Raute (2014) - Germany

[12] Child outcomes: Baker and Milligan (2008b) - Canada, Berger et al. (2005) - US, Carneiro et al. (2011b) - Norway, Dustmann and Schönberg (2012) - Germany, Liu and Skans (2010) - Sweden, Rasmussen (2010) - Denmark, Dahl et al. (2013) - Norway

employment. There are ambiguous predictions for female wages and employment.

A two period model where firms can discriminate without sanction is considered. A risk neutral, profit maximizing, price taking representative firm chooses the optimal numbers of male and female hires in the first period. In this period, the firm pays hiring costs c and training costs t for each worker. It also pays male workers the market wage w_m and female workers the market wage w_f in each period. Production occurs in the second time period. All male workers are retained by the firm in the second time period. All female workers who do not take maternity leave are retained in the second period. Those that take maternity leave - a proportion $\gamma(\theta)$, do not work for the duration of their maternity leave, a fraction θ of the year. During this time they are not producing and are not being paid a wage by the employer. A higher θ is interpreted as a more generous maternity leave period. $1-\delta(\theta)$ is the proportion of productive females in the second period, where $\delta(\theta)=\gamma(\theta)*\theta$.[13] We assume $0<\delta(\theta)<1$. A firm with concave production function F, (with $F\geq 0$, $F'\geq 0$, $F''\leq 0$), where male and female workers are assumed to be perfect substitutes solves the following problem:

$$\pi=\max_{L_m,L_f}[-(c+t)(L_m+L_f)-w_mL_m-w_fL_f+\beta(-w_mL_m-w_f(1-\delta(\theta))L_f+F(L_2(L_m,L_f,\delta(\theta))))]$$

where L_2 denotes second period workers, which will be a function of the number of male and female hires in the first period, and the absence rate of females. If male and female workers are perfect substitutes, $L_2=L_m+(1-\delta(\theta))L_f$.[14,15,16]

This model could be considered a formalisation of the argument made by Thurow (1975)

[13]Note that fertility is modelled as a function of θ, this allows for the possibility that fertility rates respond to more generous maternity leave

[14]In the empirical work, it is assumed that males and female workers are perfect substitutes conditional on a set of observable characteristics including education, industry and region. Unfortunately there is no good measure of experience in the data, however analysis conditioning on employees with no children does not greatly change the estimate of the policy impact.

[15]Acemoglu et al. (2004) use female labour market mobilisation during WWII to provide one of the few quasi-experimental estimates of the degree of substitution between female and male workers. This paper found that male and female workers were imperfect substitutes. However, this research used data mainly from 1940-1960. At that time, male workers differed from female workers as they were presumably more productive in jobs where brawn was required. It is likely that as brawn has became a less important worker attribute with technological change, it is also likely that male and female workers are closer substitutes in 2006 than they were fifty year ago.

[16]Older workers do not appear in this model. Intuitively, if older male/female workers are also perfect conditional substitutes for young female workers, then the discussion of relative young female - young male labour market outcomes holds up for relative young female - young male, old female or old male labour market outcomes. If older workers are perfect complements to younger workers, and if the increase in the cost of young employment leads to decreased young employment, then there will also be decreased old employment, and lower wages. If there is perfect cost shifting among young workers, such that young employment is not impacted, there should not be any impact on old labour market outcomes. If on the other hand old and young workers are imperfect substitutes, the impact on old labour market outcomes will depend on the strength of substitution and the degree of cost shifting among young workers.

where he discusses employer's use of statistical discrimination in making employment decisions. Thurow points out that employers who invest in on-the-job training (t in the model above) are less likely to be able to recoup the investment from women.[17]
Solving the firm's optimisation problem (where L_m^D, L_f^D denotes male/female labour demand);

$$FOC[L_m^D] : w_m = \frac{\beta}{1+\beta}\frac{\partial F(L_2(L_m^D, L_f^D, \delta(\theta)))}{\partial L_2(L_m^D, L_f^D, \delta(\theta))} - \frac{c+t}{1+\beta}$$

$$FOC[L_f^D] : w_f = \frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial F(L_2(L_m^D, L_f^D, \delta(\theta)))}{\partial L_2(L_m^D, L_f^D, \delta(\theta))} - \frac{c+t}{1+\beta(1-\delta(\theta))}$$

Firms solve for the number of males and females to hire such that the above FOC are satisfied, which will be jointly determined by male and female labour market supply. Suppose male/female labour supply can be modelled as follows

$$L_m^S = S_m(w_m)$$
$$L_f^S = S_f(w_f, \theta)$$

where it is assumed that male labour supply is not influenced by θ, but that female labour supply is.[18] Female labour supply in the first period can be impacted by an increase in maternity leave allowances, θ, by increasing the payoffs to participating in the labour market. There is;

$$\frac{\partial L_m^S}{\partial w_m} = \frac{\partial S_m(w_m)}{\partial w_m} \geq 0$$
$$\frac{\partial L_m^S}{\partial \theta} = \frac{\partial S_m(w_m)}{\partial \theta} = 0$$
$$\frac{\partial L_f^S}{\partial w_f} = \frac{\partial S_f(w_f, \theta)}{\partial w_f} \geq 0$$
$$\frac{\partial L_f^S}{\partial \theta} = \frac{\partial S_m(w_f, \theta)}{\partial \theta} \geq 0$$

Equilibrium is the point $(w_m^*, w_f^*, L_m^*, L_f^*)$ that solves four equations; the firm's two optimisation equations, male labour supply = male labour demand ($L_m^* = L_m^S = L_m^D$) and

[17] Thurow (1975) also claims that although female and male workers spend similar average lengths of time with employers (due to the higher probability of job switching for males), employers still place the emphasis on lifetime labour market participation. His claim is based on the reasoning that with male employees the employer has the option to increase the wage bid to try and retain valuable workers.

[18] More generally male labour supply could also be allowed to depend on θ, which would allow for intra-household substitution between male and female labour (Ruhm, 1998). See discussion on relaxing this assumption in Section 5.3.1

female labour supply = female labour demand ($L_f^* = L_f^S = L_f^D$). Note the above could be reduced down to a problem of two equations by substituting the labour supply equilibrium conditions ($L_m^* = S_m(w_m^*)$, $L_f^* = S_f(w_f^*, \theta)$) into the first order conditions.
This model implies that in equilibrium there will be a male-female wage gap, with[19]

$$w_m^* - w_f^* = \frac{\beta\delta(\theta)}{(1+\beta)(1+\beta(1-\delta(\theta)))}\left(\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))} + (c+t)\right)$$

which is greater than zero since $F' \geq 0$, $0 < \delta(\theta) < 1$ and $0 < \beta < 1$. Therefore, in a model with training costs and hiring costs, this model predicts that when female workers are more likely to be absent from the labour market, profit maximising firms will pay female workers lower wages. The intuition behind this is that the profit maximising condition sets the marginal cost for males equal to the marginal benefit for males, and similarly for females. The marginal benefit of a female is a proportion of the marginal benefit of a male (with the proportion equal to the fraction of time spent working in the second period, 1-$\delta(\theta)$). Therefore, the marginal cost of a female will be set equal to $1 - \delta(\theta)$ times the marginal cost of a male. If wages of males and females were the same, this condition would hold in the second period (since females are only paid for the proportion of time they work) but not the first period (when males and females are paid for the full period).[20]

5.3.1 The impact of increasing maternity leave on the male-female wage gap

An increase in maternity leave increases the male-female wage gap (i.e. $d(w_m^* - w_f^*)/d\theta > 0$) if the following holds:[21]

$$\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))}{\partial \theta} \leq 0$$

Let's consider when $\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))}{\partial \theta} \leq 0$. This is the change in second period workers for a given male, female equilibrium wage in response to an increase in the length of maternity leave.

[19] See Appendix 5A for proof.

[20] More complicated models could allow for; administration costs that an employer must pay when a female worker is on maternity leave, male/female worker quits, male/female worker firing, risk adverse firms who possibly have contract commitments, for the possibility to hire short term experienced employees to cover maternity leave at a premium, and heterogeneity in fertility, leave take-up or productivity. In addition, risk of prosecution and the resulting financial costs (or reputational costs) incurred by discriminating firms could be incorporated into the model. With more time periods return behaviour of mothers could be modelled, allowing for depreciation of human capital.

[21] See Appendix 5A for proof.

$$
\begin{aligned}
\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \theta} =& \frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)} \frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \\
& \frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \delta(\theta)} \frac{\partial \delta(\theta)}{\partial \theta} \\
=& (1-\delta(\theta))\frac{\partial S_f(w_f^*,\theta)}{\partial \theta} - S_f(w_f^*,\theta)\frac{\partial \delta(\theta)}{\partial \theta}
\end{aligned}
$$

Since $L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)) = S_m(w_m^*) + (1-\delta(\theta))S_f(w_f^*,\theta)$.

This will be negative if there is either no female labour supply response to an increase in maternity leave duration (θ) at the equilibrium wages w_m^*, w_f^*, or if the increase is not too large. The first term on the right hand side of the above expression is the increase in second period workers due to an increasing maternity leave duration drawing more female workers into the labour market. The second term is the decrease in second period workers due to female workers who have children taking longer maternity leave periods (and possibly female workers having more children and hence taking up maternity leave more frequently). Therefore if the decrease due to increased leave outweighs the increase due to increased supply, the above condition will hold and it can be concluded that the male-female wage gap must increase in response to an increase in maternity leave length in this model.[22]

If we had allowed male labour market supply to react to changing θ (for instance, it is possible that households substitute female labour supply for male labour supply in response to such a policy change), then the condition required for $\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \theta} \leq 0$ to hold is

$$
\begin{aligned}
\frac{\partial L_2(S_m(w_m^*,\theta), S_f(w_f^*,\theta), \delta(\theta))}{\partial \theta} =& \frac{\partial L_2(S_m(w_m^*,\theta), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)} \frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \\
& \frac{\partial L_2(S_m(w_m^*,\theta), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*,\theta)} \frac{\partial S_m(w_m^*,\theta)}{\partial \theta} + \\
& \frac{\partial L_2(S_m(w_m^*,\theta), S_f(w_f^*,\theta), \delta(\theta))}{\partial \delta(\theta)} \frac{\partial \delta(\theta)}{\partial \theta} \\
=& (1-\delta(\theta))\frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \frac{\partial S_m(w_m^*,\theta)}{\partial \theta} - S_f(w_f^*,\theta)\frac{\partial \delta(\theta)}{\partial \theta}
\end{aligned}
$$

As before, this will be negative if the female labour supply response is not too large. If

[22] Even if there is a large female supply response, there will still be a positive impact on the male-female wage gap as long as the female labour supply response does not cause second period labour to increase so much so that the downwards impact on the male-female wage gap from the falling marginal productivity outweighs the upwards impact on the male-female wage gap due to the decreasing productivity of female workers.

there is intra-household labour supply substitution then the condition becomes weaker, since now the *sum* of the decrease in second period labour from female labour workers who are taking more frequent/longer maternity leave periods and the decrease due to male labour being substituted for by female labour is required to outweigh the increase in second period labour due to increased female labour supply.
The following assumption is imposed to facilitate the interpretation of the model implications for the remainder of this section.

Assumption: $\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial\theta} \leq 0$

The intuition for the mechanisms behind the impact of increasing maternity leave on the male-female wage gap is as follows. An increase in maternity length is assumed to decrease period two labour, which increases marginal productivity and hence applies upwards pressure on both male and female wages. However, male wages get a bigger upwards boost from this mechanism as they are more likely to be producing in the second period. Secondly, increasing maternity leave length decreases female productivity, reducing their wages. Furthermore, since male and female labour are substitutes, employers substitute away from female labour towards male labour, providing upwards pressure on male wages, but exacerbating the downwards pressure on female wages.[23]

Proposition 1: Increases in maternity leave increase the male-female wage gap

Proposition 2: Increases in maternity leave increase male wages and employment[24]

Proposition 3: Increases in maternity leave have an ambiguous effect on female wages and employment[25]

In the empirical section a difference in differences estimation approach is implemented,

[23]Note that in a non-discriminatory model, an increase in maternity leave should have the same impact on male and female wages, regardless of any labour supply responses. Therefore, if any impact is found on relative male-female wages in response to an increase in maternity leave duration this could be interpreted as evidence of discriminating employers. That in itself would not necessarily imply support for the above model - which predicts an *increasing* male-female wage gap in response to an increase in maternity leave duration (unless there are very large increases in female labour supply).

[24]Unless there are large negative male supply responses due to intra-household substitution of labour. Proof in Appendix 5A

[25]Unless maternity leave increases were found to increase female wages, in which case the employment effect is unambiguously positive. Proof in Appendix 5A

where identification is through a common trends assumption. As discussed in the methodology section, this allows for estimation of the treatment effect on relative female-male labour outcomes (rather than gender specific treatment effects). Therefore, the empirical section focusses on the key testable model implication given the common trends assumption: increases in maternity leave duration lead to increases in the male-female wage gap (proposition 1). The empirical section will also estimate the treatment effect on relative female-male employment, redundancies and hiring rates.

5.4 Methodology

The impact of the expansion in maternity leave duration in the UK on relative female labour market outcomes is estimated using a difference in differences approach. A time-gender dimension is used. A standard difference in differences estimation approach is used for the continuous outcome variable (hourly wages). The standard difference in differences treatment effect estimator provides an estimate of the relative impact of the treatment when there are substitution effects on the control group.

Let $Y_i^P(T)$ denote the potential outcome of an individual i at time T with policy P, and let $Y_i(T)$ denote the observed outcome. There are two time periods - before and after the policy change (T=0, T=1 respectively), two policy environments - pre and post policy change (P=0, P=1 respectively), and two groups, females and males (F=1, F=0 respectively). The policy change occurs between time period T=0 and time period T=1. Let $W_{iT} = 1$ if an individual i is observed in the data at time T, and 0 otherwise. X is a set of observable characteristics. It is shown in Appendix 5C, that given the following assumptions, the standard difference in differences estimator provides an estimate of the relative impact of the treatment.

Assumptions:

1. Conditional common time trend assumption
2. No composition effects conditionally
3. No heterogeneity in mean conditional treatment effects for males or females.
4. No heterogeneity in mean conditional time trends for males or females.
5. Linear index restrictions are imposed on conditional mean non-treated male and female outcomes in period T=0.
6. Covariates enter the same way into male and female conditional mean non-treated outcomes in period T=0.

The difference in differences model can then be estimated with the following regression;

$$Y_i(T) = a_1 + a_2F_i + a_3X_i + (b_1 + c_1)T_i + b_2F_iT_i + \varepsilon_{it}$$

Where

$$b_2 = E[Y^1(1) - Y^0(1)|F = 1, W_1 = 1, X] - E[Y^1(1) - Y^0(1)|F = 0, W_1 = 1, X]$$

is the estimate of the impact of the policy change on females relative to males.

5.4.1 Binary Outcome Variables

There have been a number of approaches recently suggested in the literature for estimating binary outcomes models within the difference in differences framework. The problem with using the standard approach discussed in the above, is that the assumptions can lead to predicted counterfactual probabilities outside the zero-one interval which can bias the results (discussed for instance in Athey and Imbens (2006)).

In this section the two main alternatives to point identification of binary outcome difference in differences models are discussed (Athey and Imbens (2006) and Blundell et al. (2004b)), and an alternative approach which builds upon this previous literature is suggested. This alternative approach is based on the assumption that the odds ratio of the treatment group and the odds ratio of the control group have the same growth rate in the absence of a policy change. In contrast with the other methods, this approach facilitates interpretation when one allows for the control group to be impacted by substitution effects. Similarly to Athey and Imbens (2006) but in contrast with Blundell et al. (2004b), trend assumptions are non-parametrically specified and have an intuitive interpretation, however, unlike Athey and Imbens (2006) the trend assumptions do not have a switch point. Similarly to Blundell et al. (2004b) but (generally) in contrast with Athey and Imbens (2006), the suggested approach retains the intuitive condition that when macro conditions are such that non-treated outcomes for the control group are constant across two time periods, then the predicted non-treated outcomes for the treatment group are also constant across the same two time periods.

The no composition effect assumption is assumed throughout this discussion.

Athey and Imbens (2006)

Athey and Imbens (2006) assume the following[26]

(1) The control group is not impacted by the policy change

[26] Athey and Imbens (2006) also discuss bound estimation of the policy impact under alternative assumptions.

$$E[Y^1(1) - Y^0(1)|F = 0, X] = 0$$

(2) If the probability of success (Y=1) decreases for males (the non-treated group) then the rate of decrease in the probability of success for females (the treated group) had they not been treated would have been the same as the rate of decrease in the probability of success for males

$$\frac{E[Y^0(1)|F = 1, X]}{E[Y^0(0)|F = 1, X]} = \frac{E[Y^0(1)|F = 0, X]}{E[Y^0(0)|F = 0, X]} \text{ if } E[Y^0(1)|F = 0, X] \leq E[Y^0(0)|F = 0, X]$$

On the other hand, if the probability of success increases for males, then the method assumes the rate of decrease in the probability of failure for females had they not been treated would have been the same as the rate of decrease in the probability of failure for males

$$\frac{1 - E[Y^0(1)|F = 1, X]}{1 - E[Y^0(0)|F = 1, X]} = \frac{1 - E[Y^0(1)|F = 0, X]}{1 - E[Y^0(0)|F = 0, X]} \text{ if } E[Y^0(1)|F = 0, X] > E[Y^0(0)|F = 0, X]$$

Therefore, if the probability of success for males decreases, the counterfactual for females can be written as

$$E[Y^0(1)|F = 1, X] = \frac{E[Y|F = 0, T = 1, X]}{E[Y|F = 0, T = 0, X]} E[Y|F = 1, T = 0, X]$$

And the impact of the policy can be estimated from

$$E[Y^1(1)|F = 1, X] - E[Y^0(1)|F = 1, X] =$$
$$E[Y|F = 1, T = 1, X] - \frac{E[Y|F = 0, T = 1, X]}{E[Y|F = 0, T = 0, X]} E[Y|F = 1, T = 0, X]$$

And if the probability of success for males increases, the counterfactual for females can be written as

$$E[Y^0(1)|F = 1, X] = 1 - \left(\frac{1 - E[Y|F = 0, T = 1, X]}{1 - E[Y|F = 0, T = 0, X]}\right) (1 - E[Y|F = 1, T = 0, X])$$

And the impact of the policy can be estimated from

$$E[Y^1(1)|F = 1, X] - E[Y^0(1)|F = 1, X] =$$
$$E[Y|F = 1, T = 1, X] - 1 + \left(\frac{1 - E[Y|F = 0, T = 1, X]}{1 - E[Y|F = 0, T = 0, X]}\right) (1 - E[Y|F = 1, T = 0, X])$$

In general, the approach in Athey and Imbens (2006) does not preserve the condition that when macro conditions are such that non-treated outcomes for the control group are constant across two time periods, then the predicted non-treated outcomes for the treatment group are also constant across the same two time periods.

In fact, their approach assumes convergence of the non-treated outcomes over time (one exception of this rule is when in the initial period (T=0) mean non-treated outcomes for males and females are the same, in which case the approach assumes that the non-treated mean outcomes of males and females will always be equal. Therefore, if the non-treated outcomes for males are constant across two time periods it must also be the case for females).

To understand the intuition behind this convergence point, note that imposing the same rate of decrease of non-treated outcomes implies a higher absolute decrease for whichever group had the highest starting value.[27]

Blundell et al. (2004b)

Blundell et al. (2004b) impose a common trends assumption on the inverse probability function for non-treated outcomes, in addition to the assumption that the control group are not impacted by the policy change. They assume:

(1) The control group are not impacted by the policy change

$$E[Y^1(1) - Y^0(1)|F = 0, X] = 0$$

(2) Common trends on the inverse probability function

$$E[Y^0(1)|F = 1, X] = f(g(F = 1, T = 1, X))$$

$$E[Y^0(0)|F = 1, X] = f(g(F = 1, T = 0, X))$$

[27] To further consider this point, consider male and female employment rates. Suppose male employment rates are observed to decrease. Then this approach assumes the rate of decrease of female employment rates would have been the same as that of males if there had been no policy change. Since the same rate of decrease implies that the group with the highest starting level of employment would have the largest absolute decrease in employment rates, this implies convergence of male-female employment rates. Similarly, if male employment rates were observed to increase, then the method predicts the rate of decrease of female *unemployment* rates in the absence of a policy change would have been the same as the rate of decrease in *unemployment* rates observed for males. Since the same rate of decrease implies that the group with the highest starting level of unemployment would have the largest absolute decrease in unemployment rates, this implies convergence of male-female unemployment rates (and therefore also convergence of employment rates).

$$E[Y^0(1)|F=0,X] = f(g(F=0,T=1,X))$$

$$E[Y^0(0)|F=0,X] = f(g(F=0,T=0,X))$$

where

$$g(F=1,T=1,X) - g(F=1,T=0,X) = g(F=0,T=1,X) - g(F=0,T=0,X)$$

If additional assumptions are imposed (that are somewhat analogous to the heterogeneity assumptions imposed in the linear case); specifically, linear index restrictions on how the covariates enter into the $g(.)$ function, covariates enter into the $g(.)$ index function in the same way for males and females and if there is no interaction between covariates and time in the index function, then

$$g(F,T,X) = \alpha_0 + \alpha_1 F + \alpha_2 T + \beta X$$

The choice of the $f(.)$ function is left up to the researcher. Typical specifications include the logistic function or the cumulative normal distribution function. The α and β parameters are estimated using observed data on non-treated outcomes for males in time periods T=0 and T=1, and on non-treated females in time period T=0.
Therefore the non-treated mean conditional outcomes can be written;

$$E[Y^0(T)|F,X] = f(g(F,T,X)) = f(\alpha_0 + \alpha_1 F + \alpha_2 T + \beta X)$$

and the counterfactual of the no-treatment case for females in time period T=1 can be estimated from

$$E[Y^0(1)|F=1,X] = f(g(F=1,T=1,X)) = f(\alpha_0 + \alpha_1 + \alpha_2 + \beta X)$$

Therefore the impact of the policy on females can be estimated from

$$E[Y^1(1)|F=1,X] - E[Y^0(1)|F=1,X] = E[Y|F=1,T=1,X] - f(\alpha_0 + \alpha_1 + \alpha_2 + \beta X)$$

Suppose it is also assumed that the conditional mean treated outcome for females in time period T=1 (treated outcomes) can be modelled using the same $f(.)$ function as used in the non-treated outcome,

$$E[Y^1(1)|F=1,X] = f(h(F=1,T=1,X))$$

if there are also linear index restrictions on how the covariates enter into the $h(.)$ function and if covariates enter into the $h(.)$ index function in the same way as they enter into the $g(.)$ function, then the four observed conditional mean outcomes can be modelled as:

$$E[Y|F,T,X] = f(\alpha_0 + \alpha_1 F + \alpha_2 T + \alpha_3 FT + \beta X)$$

Where now the α and β parameters are estimated using observed data on non-treated outcomes for males in time periods T=0 and T=1, on non-treated females in time period T=0 *and* on treated females in time period T=1.
The impact of the policy on females can then be estimated from

$$E[Y^1(1)|F=1,X] - E[Y^0(1)|F=1,X] = f(\alpha_0+\alpha_1+\alpha_2+\alpha_3+\beta X) - f(\alpha_0+\alpha_1+\alpha_2+\beta X)$$

Alternative Assumptions for Binary Outcome DD Model

Alternative identifying assumptions for the policy impact are considered in this section. The main assumption is that the odds ratio of the treatment group and the odds ratio of the control group have the same growth rate in the absence of a policy change, or equivalently, that non-treated relative female-male odds ratios are constant over time. This approach allows for some interpretation of the policy effect when substitution effects for males are not assumed away. Similarly to Athey and Imbens (2006), assumptions are non-parametrically specified. Similarly to Blundell et al. (2004b) when macro conditions are such that the non-treated outcomes for one group are constant across two time periods, then this assumption implies non-treated outcomes for the other group are also constant. There are close similarities between this approach and that suggested by Blundell et al. (2004b), which is discussed in more detail in the following.
Assume to begin with
(1) The control group are not impacted by the policy change

$$E[Y^1(1) - Y^0(1)|F=0,X] = 0$$

(2) The conditional relative odds ratio is constant over time in the absence of the policy. This can be expressed as

$$\frac{E[Y^0(T)|F,X]}{1-E[Y^0(T)|F,X]} \equiv l(F,T,X) = l_1(F,X)l_2(T,X)$$

or

$$\frac{\frac{E[Y^0(1)|F=1,X]}{1-E[Y^0(1)|F=1,X]}}{\frac{E[Y^0(1)|F=0,X]}{1-E[Y^0(1)|F=0,X]}} = \frac{\frac{E[Y^0(0)|F=1,X]}{1-E[Y^0(0)|F=1,X]}}{\frac{E[Y^0(0)|F=0,X]}{1-E[Y^0(0)|F=0,X]}}$$

Therefore the counterfactual for females can be estimated from

$$\frac{E[Y^0(1)|F=1,X]}{1-E[Y^0(1)|F=1,X]} = \frac{\frac{E[Y|F=1,T=0,X]}{1-E[Y|F=1,T=0,X]}}{\frac{E[Y|F=0,T=0,X]}{1-E[Y|F=0,T=0,X]}} * \frac{E[Y|F=0,T=1,X]}{1-E[Y|F=0,T=1,X]}$$

which implies

$$E[Y^0(1)|F=1,X] = \frac{\frac{\frac{E[Y|F=1,T=0,X]}{1-E[Y|F=1,T=0,X]}}{\frac{E[Y|F=0,T=0,X]}{1-E[Y|F=0,T=0,X]}} * \frac{E[Y|F=0,T=1,X]}{1-E[Y|F=0,T=1,X]}}{1+\frac{\frac{E[Y|F=1,T=0,X]}{1-E[Y|F=1,T=0,X]}}{\frac{E[Y|F=0,T=0,X]}{1-E[Y|F=0,T=0,X]}} * \frac{E[Y|F=0,T=1,X]}{1-E[Y|F=0,T=1,X]}}$$

And the impact of the policy can be estimated from

$$\begin{aligned} E[Y^1(1)|F=1,X] - E[Y^0(1)|F=1,X] =& E[Y|F=1,T=1,X] \\ &- \frac{\frac{\frac{E[Y|F=1,T=0,X]}{1-E[Y|F=1,T=0,X]}}{\frac{E[Y|F=0,T=0,X]}{1-E[Y|F=0,T=0,X]}} * \frac{E[Y|F=0,T=1,X]}{1-E[Y|F=0,T=1,X]}}{1+\frac{\frac{E[Y|F=1,T=0,X]}{1-E[Y|F=1,T=0,X]}}{\frac{E[Y|F=0,T=0,X]}{1-E[Y|F=0,T=0,X]}} * \frac{E[Y|F=0,T=1,X]}{1-E[Y|F=0,T=1,X]}} \end{aligned}$$

With iid assumptions (either on joint/separate observations over individuals) and application of CLT and delta theorems, confidence intervals on the above policy impact can be estimated.

However, it is possible to show that under the imposed assumptions, the conditional expectations (conditioning on group and time) can be written as a logistic function with group and time additive effects which suggests an alternative estimation approach.

Proof (ignoring covariates for now):

In the current setting with two groups and two time periods $\frac{E[Y^0(T)|F,X]}{1-E[Y^0(T)|F,X]} = l_1(F)l_2(T)$ can be written without additional assumptions as

$$l_1(F)l_2(T) = (\beta_{11} + \beta_{12}F)(\beta_{21} + \beta_{22}T)$$

Which can be written as

$$\begin{aligned} l_1(F)l_2(T) &= \frac{1}{\beta_{11}}(1 + \frac{\beta_{12}}{\beta_{11}}F)(\beta_{21} + \beta_{22}T) \\ &= \frac{1}{\beta_{11}\beta_{21}}(1 + \frac{\beta_{12}}{\beta_{11}}F)(1 + \frac{\beta_{22}}{\beta_{21}}T) \end{aligned}$$

And since F,T are binary the above can be written as

$$
\begin{aligned}
l_1(F)l_2(T) &= \frac{1}{\beta_{11}\beta_{21}}(1+\frac{\beta_{12}}{\beta_{11}})^F(1+\frac{\beta_{22}}{\beta_{21}})^T \\
\Rightarrow ln(l_1(F)l_2(T)) &= ln(\frac{1}{\beta_{11}\beta_{21}}) + F*ln((1+\frac{\beta_{12}}{\beta_{11}})) + T*ln(1+\frac{\beta_{22}}{\beta_{21}}) \\
\Rightarrow e^{ln(l_1(F)l_2(T))} &= e^{ln(\frac{1}{\beta_{11}\beta_{21}})+F*ln((1+\frac{\beta_{12}}{\beta_{11}}))+T*ln(1+\frac{\beta_{22}}{\beta_{21}})} \\
\Rightarrow l_1(F)l_2(T) &= e^{\alpha_0+\alpha_1 F+\alpha_2 T}
\end{aligned}
$$

where

$$
\begin{aligned}
\alpha_0 &= ln(\frac{1}{\beta_{11}\beta_{21}}) \\
\alpha_1 &= ln(1+\frac{\beta_{12}}{\beta_{11}}) \\
\alpha_2 &= ln(1+\frac{\beta_{22}}{\beta_{21}})
\end{aligned}
$$

Finally, if

$$
\begin{aligned}
\frac{E[Y^0(T)|F]}{1-E[Y^0(T)|F]} &= l_1(F)l_2(T) \\
\Rightarrow E[Y^0(T)|F] &= \frac{l_1(F)l_2(T)}{1+l_1(F)l_2(T)} \\
\Rightarrow E[Y^0(T)|F] &= \frac{e^{\alpha_0+\alpha_1 F+\alpha_2 T}}{1+e^{\alpha_0+\alpha_1 F+\alpha_2 T}}
\end{aligned}
$$

Also, going in the other direction, if the conditional expectation takes a logistic form with group and time additive effects then the odds ratio is multiplicatively separable in group and time effects.
Proof; if

$$E[Y^0(T)|F] = \frac{e^{\alpha_0+\alpha_1 F+\alpha_2 T}}{1+e^{\alpha_0+\alpha_1 F+\alpha_2 T}}$$

$$
\begin{aligned}
\Rightarrow \frac{E[Y^0(T)|F]}{1-E[Y^0(T)|F]} &= e^{\alpha_0+\alpha_1 F+\alpha_2 T} \\
&= e^{\alpha_0+\alpha_1 F}e^{\alpha_2 T} \qquad = l_1(F)l_2(T)
\end{aligned}
$$

Where $l_1(F) = e^{\alpha_0+\alpha_1 F}$ and $l_2(T) = e^{\alpha_2 T}$. This explains the close relationship between this approach and that in Blundell et al. (2004b) - if the f(.) function in the Blundell et al. (2004b) approach is assumed to be the logistic function, then in a model with no covariates the above shows that the rate of change of non-treated odds ratios must be the same for both groups. Therefore, this approach provides interpretable restrictions that lead to the estimation approach proposed in Blundell et al. (2004b).

The α coefficients are estimated using observed data on non-treated outcomes for males in time periods T=0 and T=1 and on non-treated outcomes for females in time periods T=0.

The impact of the policy can then be estimated from

$$E[Y^1(1)|F=1] - E[Y^0(1)|F=1] = E[Y|F=1,T=1] - \frac{e^{\alpha_0+\alpha_1+\alpha_2}}{1+e^{\alpha_0+\alpha_1+\alpha_2}}$$

If observations are iid then the standard error of the policy impact can be estimated using a combination of ML, a CLT for $E[Y|F=1,T=1]$, the delta method, and the formula for the variance of the sum of independent variables.

Note if it is also assumed that the conditional mean outcome for females in time period T=1 (treated outcomes) can be modelled as a logistic function then the four conditional mean outcomes can be modelled as

$$E[Y|F,T] = \frac{e^{\alpha_0+\alpha_1 F+\alpha_2 T+\alpha_3 FT}}{1+e^{\alpha_0+\alpha_1 F+\alpha_2 T+\alpha_3 FT}}$$

where now the coefficients are estimated using observed data on non-treated outcomes for males in time periods T=0 and T=1, on non-treated outcomes for females in time periods T=0 and on treated outcomes for females in time period T=1.

And the impact of the policy can be estimated from

$$E[Y^1(1)|F=1] - E[Y^0(1)|F=1] = \frac{e^{\alpha_0+\alpha_1+\alpha_2+\alpha_3}}{1+e^{\alpha_0+\alpha_1+\alpha_2+\alpha_3}} - \frac{e^{\alpha_0+\alpha_1+\alpha_2}}{1+e^{\alpha_0+\alpha_1+\alpha_2}}$$

If the observations are assumed iid, then ML estimation is efficient, and application of the delta method allows estimation of the standard error of the estimate of the policy impact. If observations are not assumed to be iid, then pseudo maximum likelihood estimation can be implemented, and subsequent application of the delta method allows estimation of the standard error of the estimate of the policy impact.[28]

[28] A pseudo or quasi MLE estimator is defined as an estimator that maximises a log-likelihood function that is misspecified (Cameron and Trivedi, 2005). For the class of linear exponential family densities (of which the Bernoulli density belongs), the pseudo MLE estimator is consistent as long as the conditional mean is correctly specified (Gourieroux et al., 1984). Suppose there is a dependence between some observations. Therefore, the log-likelihood function based on an LEF density which assumes independence between observations is misspecified, but will nonetheless provide a consistent estimator for the parameter values of the conditional mean function, so long as the conditional mean function is correctly specified.

With discrete covariates all the above follows through with the following specification:

$$l_1(F,X)l_2(T,X) = \sum_{x\in X} 1[X=x][(\beta_{11x}+\beta_{12x}F)(\beta_{21x}+\beta_{22x}T)]$$

However, for empirically tractability assumptions analogous to those discussed previously are imposed; it is assumed that the covariates enter multiplicatively into $l_1(F,X)$ and $l_2(T,X)$. In other words, it is assumed that

$$\frac{E[Y^0(T)|F,X]}{1-E[Y^0(T)|F,X]} \equiv l(F,T,X) = l_1(F)l_2(T)l_3(X)$$

and furthermore, the function $l_3(X)$ is assumed log-linear.[29] Therefore, note that the conditional means of non-treated outcomes can be modelled:

$$E[Y^0(T)|F,X] = \frac{e^{\alpha_0+\alpha_1 F+\alpha_2 T+\beta X}}{1+e^{\alpha_0+\alpha_1 F+\alpha_2 T+\beta X}}$$

and the impact of the policy can be estimated from

$$E[Y^1(1)|F=1,X] - E[Y^0(1)|F=1,X] = E[Y|F=1,T=1,X] - \frac{e^{\alpha_0+\alpha_1+\alpha_2+\beta X}}{1+e^{\alpha_0+\alpha_1+\alpha_2+\beta X}}$$

If it is also assumed that covariates enter multiplicatively into the odds ratio for treated outcomes for females in period T=1 in the same way as they enter into the non-treated odds ratios, then

$$\frac{E[Y^1(1)|F=1,X]}{1-E[Y^1(1)|F=1,X]} = l_4 l_3(X)$$

Therefore:

$$\frac{E[Y^1(1)|F=1,X]}{1-E[Y^1(1)|F=1,X]} = e^{\alpha_3\prime+\beta X}$$

Where $\alpha_3\prime = ln(l_4)$ and

$$E[Y^1(1)|F=1,X] = \frac{e^{\alpha_3\prime+\beta X}}{1+e^{\alpha_3\prime+\beta X}}$$

And the four observed conditional mean outcomes can be modelled as:

$$E[Y|F,T,X] = \frac{e^{\alpha_0+\alpha_1 F+\alpha_2 T+\alpha_3 FT+\beta X}}{1+e^{\alpha_0+\alpha_1 F+\alpha_2 T+\alpha_3 FT+\beta X}}$$

where $\alpha_3 = \alpha_3\prime - \alpha_0 - \alpha_1 - \alpha_2$.

[29] $l_3(X) = e^{\beta X}$

And the impact of the policy can be estimated from

$$E[Y^1(1)|F=1,X] - E[Y^0(1)|F=1,X] = \frac{e^{\alpha_0+\alpha_1+\alpha_2+\alpha_3+\beta X}}{1+e^{\alpha_0+\alpha_1+\alpha_2+\alpha_3+\beta X}} - \frac{e^{\alpha_0+\alpha_1+\alpha_2+\beta X}}{1+e^{\alpha_0+\alpha_1+\alpha_2+\beta X}}$$

What if there are substitution effects on the control group?

In this case, and maintaining the other assumptions, the impact of the policy on the relative odds ratio can be estimated. There is by assumption,

$$\frac{\frac{E[Y^0(1)|F=1,X]}{1-E[Y^0(1)|F=1,X]}}{\frac{E[Y^0(1)|F=0,X]}{1-E[Y^0(1)|F=0,X]}} = \frac{\frac{E[Y^0(0)|F=1,X]}{1-E[Y^0(0)|F=1,X]}}{\frac{E[Y^0(0)|F=0,X]}{1-E[Y^0(0)|F=0,X]}}$$

Also, as shown in the above (with the multiplicative and log linearity assumptions on how X enters the non-treated odds ratios)

$$\frac{E[Y^0(T)|F,X]}{1-E[Y^0(T)|F,X]} = e^{\alpha_0+\alpha_1 F+\alpha_2 T+\beta X}$$

Therefore,

$$\begin{aligned}\frac{\frac{E[Y^0(1)|F=1,X]}{1-E[Y^0(1)|F=1,X]}}{\frac{E[Y^0(1)|F=0,X]}{1-E[Y^0(1)|F=0,X]}} &= \frac{e^{\alpha_0+\alpha_1+\alpha_2+\beta X}}{e^{\alpha_0+\alpha_2+\beta X}} \\ &= e^{\alpha_1}\end{aligned}$$

Therefore, the impact of the policy on the relative odds ratio in time period T=1 can be estimated from[30]

$$\begin{aligned}&\frac{\frac{E[Y^1(1)|F=1,X]}{1-E[Y^1(1)|F=1,X]}}{\frac{E[Y^1(1)|F=0,X]}{1-E[Y^1(1)|F=0,X]}} - \frac{\frac{E[Y^0(1)|F=1,X]}{1-E[Y^0(1)|F=1,X]}}{\frac{E[Y^0(1)|F=0,X]}{1-E[Y^0(1)|F=0,X]}} \\ &= \\ &\frac{\frac{E[Y^1(1)|F=1,X]}{1-E[Y^1(1)|F=1,X]}}{\frac{E[Y^1(1)|F=0,X]}{1-E[Y^1(1)|F=0,X]}} - e^{\alpha_1}\end{aligned}$$

If this is zero then the policy has no impact. If it is negative then it means that the policy change decreased the odds ratio of females relative to males. If it is positive then it means

[30] Where $\alpha_0, \alpha_1, \beta$ are estimated from the following (note that $E[Y^0(T)|F,X]$ is observed now only for (F=1,T=0) and (F=0,T=0))

$$E[Y^0(T)|F,T=0,X] = \frac{e^{\alpha_0+\alpha_1 F+\beta X}}{1+e^{\alpha_0+\alpha_1 F+\beta X}}$$

that the policy change increased the odds ratio of females relative to males.
If it is also assumed that covariates enter multiplicatively into the odds ratio for treated outcomes for males and females in period T=1 in the same way as they enter into the non-treated odds ratios, so

$$\frac{E[Y^1(1)|F,X]}{1-E[Y^1(1)|F,X]} = l_4(F)l_3(X)$$

then:

$$\frac{E[Y^1(1)|F,X]}{1-E[Y^1(1)|F,X]} = e^{\alpha_2\prime\prime+\alpha_3\prime\prime F+\beta X}$$

Where $\alpha_2\prime\prime = ln(l_4(F=0))$, $\alpha_3\prime\prime = ln(l_4(F=1)) - ln(l_4(F=0))$ and

$$E[Y^1(1)|F,X] = \frac{e^{\alpha_2\prime\prime+\alpha_3\prime\prime F+\beta X}}{1+e^{\alpha_2\prime\prime+\alpha_3\prime\prime F+\beta X}}$$

And the four observed conditional mean outcomes can be modelled as:

$$E[Y|F,T,X] = \frac{e^{\alpha_0+\alpha_1 F+\alpha_2 T+\alpha_3 FT+\beta X}}{1+e^{\alpha_0+\alpha_1 F+\alpha_2 T+\alpha_3 FT+\beta X}}$$

where $\alpha_2 = \alpha_2\prime\prime - \alpha_0$
and $\alpha_3 = \alpha_3\prime\prime - \alpha_1$

and the impact of the policy on the relative odds ratio in time period T=1 can be estimated from

$$\frac{\frac{E[Y^1(1)|F=1,X]}{1-E[Y^1(1)|F=1,X]}}{\frac{E[Y^1(1)|F=0,X]}{1-E[Y^1(1)|F=0,X]}} - \frac{\frac{E[Y^0(1)|F=1,X]}{1-E[Y^0(1)|F=1,X]}}{\frac{E[Y^0(1)|F=0,X]}{1-E[Y^0(1)|F=0,X]}} =$$
$$\frac{e^{\alpha_0+\alpha_1+\alpha_2+\alpha_3+\beta X}}{e^{\alpha_0+\alpha_2+\beta X}} - \frac{e^{\alpha_0+\alpha_1+\beta X}}{e^{\alpha_0+\beta X}} =$$
$$e^{\alpha_1+\alpha_3} - e^{\alpha_1} =$$
$$e^{\alpha_1}(e^{\alpha_3} - 1)$$

As before, if the observations are assumed iid, then ML estimation is efficient, and application of the delta method allows estimation of the standard error of the impact of the policy. If observations are not assumed to be iid, pseudo maximum likelihood estimation can be used, and subsequent application of the delta method allows estimation of the standard error of the impact of the policy.

Beyond Relative Effects

In the standard difference in differences model (for continuous outcomes), a positive relative treatment effect in the presence of substitution effects can be found when the treatment and control groups are both positively impacted by the policy change, both negatively impacted by the policy change or when the treatment group is positively effected and the control group negatively impacted. Without further assumptions nothing more can be inferred about the direction or magnitude of the treatment effect for either group. However, if the theory suggests that the treatment group are positively impacted by the policy change, then the impact of the policy effect can be bounded from below for both the treatment and the control group. The treatment group has a lower bound simply of zero, and the control group has a lower bound of $-b_2$, where b_2 is the estimate from the interaction term in the standard difference in differences model; $b_2 = E[Y_1 - Y_0|G = T] - E[Y_1 - Y_0|G = C]$ where G=T for the treatment group and G=C for the control group.

If the theory suggests that the control group are negatively impacted by the policy change, then the impact of the policy effect can be bounded from above for both the treatment and the control groups. The control group has an upper bound of zero, and the treatment group has an upper bound of b_2.

If both assumptions hold (the treatment group are positively impacted and the control group negatively impacted), then the treatment effect for the treated group lies in the interval [0, b_2], and the treatment effect for the control group lies in the interval [$-b_2$,0].[31]

The same intuition holds in the binary difference in differences model. An increase in the relative odds ratio due to a policy change in the presence of substitution effects could be found when there are positive treatment effects for both the treatment and control group, negative treatment effects for both the treatment and control group, or when there are positive treatment effects for the treatment group and negative treatment effects for the control group. Note that an increase in the relative odds ratio corresponds to the case where $\alpha_3 > 0$. Under the same assumptions (the treatment group (females) are positively impacted and the control group (males) are negatively impacted), then the treatment effect for females lies in the interval [0, $\frac{e^{\alpha_0+\alpha_1+\alpha_2+\alpha_3+\beta X}}{1+e^{\alpha_0+\alpha_1+\alpha_2+\alpha_3+\beta X}} - \frac{e^{\alpha_0+\alpha_1+\alpha_2+\beta X}}{1+e^{\alpha_0+\alpha_1+\alpha_2+\beta X}}$] and the treatment effect for males lies in the interval [$\frac{e^{\alpha_0+\alpha_2+\beta X}}{1+e^{\alpha_0+\alpha_2+\beta X}} - \frac{e^{\alpha_0+\alpha_2+\alpha_3+\beta X}}{1+e^{\alpha_0+\alpha_2+\alpha_3+\beta X}}$, 0].[32]

[31] If a negative relative treatment effect was estimated ($b_2 < 0$), and it is assumed that the treatment group are negatively impacted and the control group positively impacted, then the treatment effect for the treatment group lies in the interval [b_2,0], and the treatment effect for the control group lies in the interval [0,$-b_2$].

[32] If there is an estimated decrease in the relative odds ratio, and the impact on the treated group (females) is assumed negative and the impact on the control group (males) assumed positive, then the treatment effect for females lies in the interval [$\frac{e^{\alpha_0+\alpha_1+\alpha_2+\alpha_3+\beta X}}{1+e^{\alpha_0+\alpha_1+\alpha_2+\alpha_3+\beta X}} - \frac{e^{\alpha_0+\alpha_1+\alpha_2+\beta X}}{1+e^{\alpha_0+\alpha_1+\alpha_2+\beta X}}$,0] and the treatment effect for males lies in the interval [0, $\frac{e^{\alpha_0+\alpha_2+\beta X}}{1+e^{\alpha_0+\alpha_2+\beta X}} - \frac{e^{\alpha_0+\alpha_2+\alpha_3+\beta X}}{1+e^{\alpha_0+\alpha_2+\alpha_3+\beta X}}$].

5.5 Legislative Environment

Some form of legislated maternity benefits have existed in the United Kingdom since the introduction of the *National Insurance Act* (1911). There have been many changes to the legislation and provisions since then, which are reviewed in detail in Appendix 5B. This section focuses on changes to maternity benefits between 1996 and 2007. The period from 1996 - 2006 is used as a placebo period in the empirical analysis, with the policy change under analysis being implemented in 2007.

There were two major policy changes impacting maternity benefits in the time period over which the placebo analysis is carried out (1996-2006). Prior to this period, all female employees had the right to 14 weeks of maternity leave (which was not necessarily paid).[33] To receive statutory maternity pay (SMP) (paid for up to 18 weeks) from your employer you typically had to be continuously employed for 26 weeks before the expected week of childbirth. To receive state maternity allowance (also paid for up to 18 weeks) you typically had to have made at least 26 national insurance contributions in the previous year. The first 6 weeks of statutory maternity pay were paid at 90% of average weekly earnings, with the remaining 12 weeks being paid at a flat rate set by the government each year (which was £54.55 in 1996, which was the equivalent of £89.35 in 2013). The state maternity allowance was paid at the flat SMP rate.[34] 92% of SMP was reclaimable by the employer through a rebate from the government. However small employers (defined as those whose annual contributing payments did not exceed £20,000) received a rebate of 104% of the SMP paid.[35] If an employee had been continuously employed for 2 years they typically qualified for an additional leave period, which ended 29 weeks after birth.

The first major policy change in the placebo period occurred in 1999, and applied to women whose expected week of childbirth began on or after 30th April 2000. The amount of maternity leave all employees were entitled to was increased from 14 to 18 weeks. The tenure qualifying condition for the additional leave period was decreased from 2 years of continuous employment to 1 year of continuous employment. Unpaid parental leave of up to 13 weeks was also introduced, with a maximum of 4 weeks in any one year. Additionally, in 1999 an exemption was introduced for small employees which stated an employee being dismissed for any reason connected with her pregnancy or maternity leave was not considered to have been unfairly dismissed if her employer had fewer than five employees. Finally, the earnings qualifying rule for the state maternity allowance was changed, and an earnings related aspect was introduced which impacted low earners.[36]

[33] 2 weeks of which were compulsory immediately after birth

[34] There were complicated earnings qualifying rules to qualify for SMP, the higher rate of SMP and the state maternity allowance - see Appendix 5B for more details

[35] This percentage was changed slightly over time, and the annual contribution level was increased to £45,000 in 2004 - see Appendix 5B for more details

[36] See Appendix 5B for more details

The second major policy change in the policy period occurred in 2002, and applied to women whose expected week of childbirth began on or after 6th April 2003. The amount of maternity leave all employees were entitled to was increased from 18 to 26 weeks. The additional leave period was changed from being up until the 29th week after birth to being the 26 week period continuing on from the first 26 week leave period. The tenure qualifying condition for this leave period was changed from 1 year of continuous employment to 26 weeks of continuous employment. In 2002, paid paternity leave of 2 weeks was also introduced, which had a tenure qualifying condition of 26 weeks of continuous employment.[37] Since the policy changes in 1999 and 2002 changed parental leave rights for both male and female employees simultaneously, the predicted direction of initial relative labour market effects would depend on the anticipated impact of the policy changes on relative male-female leave taking.

The key policy change of interest occurred in 2006. This legislative change was passed into law on the 1st October 2006, with women who qualified for SMP/the state maternity allowance, and whose expected week of childbirth fell on or after the 1st April 2007, being eligible for an additional 13 weeks of paid maternity leave/maternity allowance. This gave a maximum statutory paid maternity leave duration of 39 weeks compared to 26 weeks previously. In addition from the 1st April 2007, the tenure required to qualify for the additional maternity leave period was abolished, implying that all employed women were entitled to a job-protected leave period of up to 52 weeks. At this juncture, the small employer exemption introduced in 1999 was also removed, which meant that all employees had the right to return to the same or similar job regardless of the size of her employer's firm. As with the other changes, this affected employees whose expected week of childbirth began on or after the 1st April 2007.

The intention to increase paid maternity leave duration was published in the Labour Party's 2005 election manifesto on 13th April 2005 (The Labour Party, 2005), where they stated their intention to increase paid maternity leave from 26 to 39 weeks. The election took place on 5th May 2005, resulting in a Labour majority. In the analysis it is assumed that employers react to the actual legislative change that took place on 1st October 2006 rather than proposed legislative change. If this assumption is invalid, and in fact employers pre-empt the legislative change then it is possible that the estimation approach results in a downwards biased estimate of the impact of the policy change.

[37] The rate of statutory paternity pay was equivalent to that of female employee on the state maternity allowance, and had the same earnings qualifying rule

5.6 Data and Empirical Specification

The analysis in this chapter is based on data from the UK Labour Force Survey (LFS)[38]. The analysis uses a quasi-experimental difference in differences approach, comparing the change in female versus male outcomes during a period in which an expansion in maternity leave legislation occurred.

Most of the analysis uses LFS data from quarters 2 and 3 (April - September) in 2006 and 2007. The legislative change occurred in October 2006, but only started affecting women whose expected week of childbirth began on or after the 1st April 2007. Therefore, employers looking to avoid the additional costs associated with women taking longer periods of maternity leave did not have to react immediately.[39] Therefore, the six month period running from the 1st April 2007 until the 30th September 2007 is taken as the after period, and the corresponding period in 2006 as the before period. Using only this 6 month window is beneficial as it avoids the potentially confounding effect of higher numbers of retained job matches (amongst women who wanted to take more than 6 months of maternity leave), and also the effect of greater human capital depreciation among women coming back into the labour market after taking longer periods of maternity leave.

The analysis focuses on relative male-female outcomes aged between 25-34. This age category is chosen as the majority of births (over 50%) are to mothers in this age range in 2007 (see Figure 5.1). The probability of giving birth each year for women in this age category was over 10% in 2007 (compared to 3.5% for females aged under 20, 7.9% for females aged 20-24, 4.8% for females aged 35-39 and 0.9% for females aged over 40 - see Figure 2). Estimates for those aged 16-24, 35-44 and over 45 are also presented as a comparison.

The key outcomes considered are hourly wages, employment conditional on participation, redundancy and hiring (new starts and job changers combined). Hourly wages of employed individuals are considered (excluding self-employed). The hourly wage outcome is measured using the hourpay variable in the LFS dataset for the most part, which is constructed using gross reported last earnings, the period of time that payment covered and paid hours of work (including paid overtime). From 1999 individuals in the LFS were asked whether their gross reported last earnings was the same as that received each similar period. For those that reported no, this analysis replaces the hourpay variable with the

[38]Office for National Statistics. Social and Vital Statistics Division and Northern Ireland Statistics and Research Agency. Central Survey Unit, Quarterly Labour Force Survey, [multiple computer files]. Colchester, Essex: UK Data Archive [distributor]

[39]Estimates using the entire October 2006-September 2007 period as the after period (and the corresponding period 12 months earlier for the before period) may provide an estimate of an average of the total effect and the effect during the phase-in period. The size of the effect on hourly wages was found to be approximately $\frac{2}{3}$ of the magnitude of the effect estimated when using the period April 2007-September 2007 was used as the after period (and the corresponding period 12 months earlier for the before period), and statistically significant at the same level.

hourly wage corresponding to the gross reported typical earnings. Since there may have been a fertility response to the legislative change, and while on maternity leave employees often receive some proportion of their typical earnings, this approach avoids picking up this confounding effect in the comparison of male and female earnings. All earnings are adjusted for inflation using the ONS annual RPI figures.

Individuals are counted as in employment in accordance with the LFS definitions of employment. Therefore, employees, self-employed, those in government employment or training programmes and unpaid family workers are treated as being employed. Those in the labour force (seeking and available for work) are treated as unemployed.

Experience of redundancy in the previous three months is analysed. This time period is analysed because employees are only asked if and why they left a paid job if they started a new job in the previous 3 months (whereas unemployed individuals are asked if they have become unemployed in the previous 8 years). Individuals are treated as having experienced redundancy if they stated that the reason they left their last job was due to being dismissed, or they were made redundant or took voluntary redundancy (voluntary redundancy was unfortunately not asked as a separate category).

Experience of starting a new job (either from unemployment or job change) in the previous three months is also analysed. Unemployed individuals are assumed not to have started a job within the previous three months.

The impact of the policy change on continuous variables is analysed using the following difference in differences equation

$$Y_i(T) = a_1 + a_2 F_i + a_3 X_i + (b_1 + c_1)T_i + b_2 F_i T_i + \varepsilon_{it}$$

where b_2 is the estimate of the differential impact of the policy change on females compared to males (as discussed in the methodology section). There is an overlapping panel structure in the LFS, whereby participants recruited into the LFS are surveyed over 5 quarters. Therefore, some individuals may appear in the data set twice. It is not possible to identify these individuals in the standard LFS data files. This introduces potential serial correlation. To account for this, cluster-robust standard errors are reported, with clustering on region-industry level. Under the assumption that individuals observed twice remain in the same region-industry group, this method will account for serial correlation. This approach also allows for group errors at the region-industry level.

The impact of the policy change for binary outcome variables is analysed using a logit model

$$E[Y|F,T,X] = \frac{e^{\alpha_0+\alpha_1 F+\alpha_2 T+\alpha_3 FT+\beta X}}{1+e^{\alpha_0+\alpha_1 F+\alpha_2 T+\alpha_3 FT+\beta X}}$$

where the impact of the policy on the relative odds ratio is estimated from $e^{\alpha_1}(e^{\alpha_3} - 1)$ (as discussed in the methodology section). To allow for repeated observations and possible error correlation, pseudo maximum likelihood was used to estimate the above model. Results for discrete variables assuming a linear probability model with the above specification are also estimated, with robust standard errors reported.
The set of control variables common to all analysis are; age finished continuous full time education, government office region and an indicator for whether the individual was born in the UK. For the wage analysis, industry-region fixed effects are included, as well as an indicator for part time/full time status and an indicator for public or private sector. The redundancy analysis controls for the industry in which you were made redundant from (or if you were not made redundant, your current or last industry). Furthermore, 1% of wages are trimmed above and below to avoid undue influence of outliers. The analysis does not include individuals who report they left full time education at an age younger than 12 or older than 30.[40] See Table 5.1 for summary statistics of the outcome and control variables.

5.7 Results

5.7.1 First Stage

The first set of results look at whether the policy change impacted the amount of leave taken by females relative to males of the same age. A difference in differences model is used to analyse this, with the after period being April - September 2008 and the before period being April - September 2006. It is assumed that the rate of change in the female and male odds ratio of being on leave between 2006 and 2008 would have been the same in the absence of a policy change. Note that the after period is different from that used in the main outcome analysis. The justification for this is because the aim is to compare a group of women who are affected by the policy change to a group of women not affected. The LFS has information on whether the youngest child in your family unit is under 1, but the age is not more specifically determinable. Therefore, whereas all women from April 2008 with a child under the age of 1 qualified for the more generous leave allowance, this was not the case for all women with a child under the age of 1 in the previous year. Some of these women would have had their child after April 2007 and qualified, and some before.
In the first instance, the impact of the policy change on being on leave from paid work is estimated. Individuals who had a paid job in the reference week are considered. Individuals who had a paid job in the reference week, which they were away from temporarily, were

[40] This only results in 25 people aged 25-34 being dropped from the wage analysis, and does not affect the results.

said to be on leave.[41] Column 2 of Table 5.2 shows a significant increase in the relative odds ratio of a female employee being on leave compared to a male employee over the period 2006-2008. Table 5.3 shows the corresponding results from an LPM model, which estimates a 1.3% increase in the fraction of 25-34 year old females on leave relative to males over the period 2006-2008.

An alternative approach is also followed, where the impact of the policy change on parental leave taken by females relative to males is decomposed into an increase due to a fertility response and an increase due to changing lengths of maternity/paternity leave taken. The average length of time out of the labour market for a female employee due to maternity leave is given by:

$$E[ML] = \lambda_F \gamma_{FL}$$

where γ_{FL} denotes the fertility rate of a female employee and λ_F represents the average leave taken by a female worker who gives birth that year. The average length of time out of the labour market for a male employee due to paternity leave is given by:

$$E[PL] = \lambda_M \gamma_{ML}$$

where γ_{ML} denotes the probability a male employee has a child, and λ_M represents the average leave taken by a male worker who has a child that year. Therefore note that the gap in the average length of time out of the labour market for a female employee compared to a male employee due to parental leave is given by:

$$E[ML - PL] = \lambda_F \gamma_{FL} - \lambda_M \gamma_{ML}$$

Therefore,

$$\begin{aligned} \frac{dE[ML-FL]}{d\theta} &= \frac{\partial E[ML-PL]}{\partial \lambda_F}\frac{d\lambda_F}{d\theta} + \frac{\partial E[ML-PL]}{\partial \gamma_{FL}}\frac{d\gamma_{FL}}{d\theta} \\ &+ \frac{\partial E[ML-PL]}{\partial \lambda_M}\frac{d\lambda_M}{d\theta} + \frac{\partial E[ML-PL]}{\partial \gamma_{ML}}\frac{d\gamma_{ML}}{d\theta} \end{aligned}$$

$$\rightarrow \frac{dE[ML-PL]}{d\theta} = \gamma_{FL}\frac{d\lambda_F}{d\theta} + \lambda_F\frac{d\gamma_{FL}}{d\theta} - \gamma_{ML}\frac{d\lambda_M}{d\theta} - \lambda_M\frac{d\gamma_{ML}}{d\theta}$$

The results from this analysis are shown in Table 5.4.[42] γ_{FL} (γ_{ML}) is estimated from the

[41] They are many leave reasons besides parental leave, however, if patterns of uptake of non-parental leave did not change for females relative to males across the comparison period then comparing aggregate leave before and after should give an estimate of the relative impact of the policy change on parental leave uptake of females relative to males.

[42] Since the policy reform brought about a discrete change in the duration of maternity leave, this decomposition analysis implicitly assumes a linear response of fertility/parental leave to changes in the duration of maternity leave

proportion of females (males) who have a paid job (which they may have been away from) and who have a child under the age of 1 in 2006. λ_F (λ_M) is estimated from the mean number of females (males) with a child under the age of 1 and who have a paid job in 2006 from which they were on leave from. $\frac{d\gamma_{FL}}{d\theta}$ ($\frac{d\gamma_{ML}}{d\theta}$) is estimated from a single difference linear probability model of females (males), estimating the increase in the fertility rate of females who have a paid job (paternity rate of males) from 2006 to 2008, controlling for age finished full time continuous education, region of residence and indicator for whether UK born. $\frac{d\lambda_F}{d\theta}$ ($\frac{d\lambda_M}{d\theta}$) is estimated from a single difference linear probability model of females (males), estimating the increase in the proportion of female (male) individuals on leave from a paid job amongst those with a child under the age of 1 between 2006 and 2008, also controlling for age finished full time continuous education, region of residence and indicator for whether UK born. Since the sample of individuals used to estimate fertility rates and leave rates in 2006 are also used in the regression analysis to estimate the changes in fertility rates and leave rates, the female/male set of parameters are estimated using a GMM model (a separate model is used for males/females, and the male and female samples are assumed to be independent). Results are reported only for the age categories 25-34 and 35-44. This is because the GMM estimation did not converge for the age category 16-24 due to the low probability of men in this age category having children. Similarly, the procedure did not converge for the age category 45-64 due to the low probability of women in this age category having children.

As shown in Table 5.4, the maternity-paternity leave gap was estimated to increase by 1.00% for the 25-34 age group and by 0.43% for the 35-44 age group. Approximately half of the divergence in leave taking is explained by the increase in the number of females having children and half explained by the increase in the duration of maternity leave taken.[43]

5.7.2 Impact on Wages

The key testable model prediction is that an increase in maternity leave uptake will lead to an increase in the male-female wage gap. In this section whether or not there is any empirical evidence to support this is discussed. Table 5.5 shows the results from the difference in differences analysis, which compares female and male wage growth over the

[43]Note that in the model male/female wages are assumed not to impact upon fertility rates or length of maternity leave. A more complicated model could have allowed for this dependence. For instance, the probability a female worker has a child could be modelled as $\gamma(\theta, w_m^*, w_f^*)$ instead of $\gamma(\theta)$. Similarly, the average length of leave taken could have been modelled as a function of equilibrium wages as well as legislated leave. This would not change the models implications hugely, so long as the partial derivative of second period labour with respect to male (and female) wages remains positive. What happens to the derivation is that $\frac{\partial L_2(S_m(w_m^*,\theta),S_f(w_f^*,\theta),\delta(\theta,w_m^*,w_f^*)))}{\partial w_m^*}$ becomes $\frac{\partial L_2(S_m(w_m^*,\theta),S_f(w_f^*,\theta),\delta(\theta,w_m^*,w_f^*)))}{\partial S_m(w_m^*,\theta)}\frac{\partial S_m(w_m^*,\theta)}{\partial w_m^*} + \frac{\partial L_2(S_m(w_m^*,\theta),S_f(w_f^*,\theta),\delta(\theta,w_m^*,w_f^*)))}{\partial \delta(\theta,w_m^*,w_f^*)}\frac{\partial \delta(\theta,w_m^*,w_f^*)}{\partial w_m^*}$ instead of $\frac{\partial L_2(S_m(w_m^*,\theta),S_f(w_f^*,\theta),\delta(\theta)))}{\partial S_m(w_m^*,\theta)}\frac{\partial S_m(w_m^*,\theta)}{\partial w_m^*}$. The conclusions in the model section remain unchanged if $\frac{\partial L_2}{\partial w_m^*}$ remains positive.

period 2006-2007. The empirical results suggest there was a large and significant negative impact on the wages of females relative to males in the age group 25-34. This is in line with the model implications. The model estimated that the male-female wage gap for those aged 25-34 increased by £0.29.[44] There was a smaller negative impact estimated for the relative wages of females compared to males for the 16-24 age group (this age group also experienced a large divergence in male-female leave uptake after the policy change), however the impact was not estimated to be significant.

In Figure 5.3 the results from a placebo analysis are presented for the age group 25-34. This placebo analysis estimates the same difference in differences model using previous comparison years (so the 2006 figure represents the 2005-2006 comparison, 2005 represents the 2004-2005 comparison, etc.). There are a couple of observations to make. Firstly, the estimated impact for the relevant year (2007 versus 2006) is the largest negative impact across the 11 estimates. Secondly, across the 11 year period there is only one other year for which a statistically significant estimate is found; 2005. However, this significant estimate went in the opposite direction to the estimated policy effect. Therefore, to the extent that the significant value estimate for 2005 suggests the common trends assumption may be invalid, it in fact suggests that the male-female wage gap is converging. In light of this, the diverging estimate found for 2007 is even stronger evidence in favour of a negative impact of the policy on relative female wages.

The large convergence in the male-female wage gap estimated between 2004-2005 might be due to a number of factors. The then Prime Minister Tony Blair established a Women and Work Commission which focused on narrowing the gender pay gap in July 2004.[45] One proposal that received some media coverage at the time was the possible implementation of equal pay reviews, which may have impacted relative male-female wages.[46] There were also a number of high profile equal pay cases, for instance the North Cumbria Acute NHS Trust v Unison Trade Union case[47] and the Home Office v Bailey case[48]. Additionally, the Equal Opportunities Commission (subsumed by the Equality and Human Rights Commission in 2007) launched a pregnancy discrimination campaign in January 2005, which they estimated was heard or seen by 50% of the population at least twice (Equal Opportunities Commission, 2005). Finally, the *Sex Discrimination Act* (1975) was amended in 2005 by the *Employment Equality (Sex Discrimination) Regulations* (2005), which explicitly stated that differential treatment due to pregnancy or maternity leave amounted to sex discrimination (although this had previously been established by case law).

More evidence supporting the common trends assumption is shown in Figure 5.4a - Figure

[44] This corresponds to 2.4% of average male wages, or 2.7% of average female wages
[45] BBC News (2004)
[46] The Sunday Times (2005)
[47] The Guardian (2005)
[48] The Times (2005)

5.4f. Figures 5.4a, 5.4c and 5.4e show the time trend of male and female conditional wages in the age group 25-34 over the period 1996-2006, allowing for a cubic, quadratic or linear time trend respectively.[49] Mean male and female wages seem to follow parallel trends over the period, however further analysis of the predicted mean male-female wage gap over this time period (Figures 5.4b, 5.4d and 5.4f) suggests that the gap may have been converging during this period. Again, as with the placebo analysis, evidence of the diverging impact of the policy change on the male-female wage gap is made stronger in view of this converging trend.

Table 5.6 shows the difference in differences estimation for the sample without any dependent children. Although the estimate is no longer statistically significant (due to a halving of the sample size), the magnitude of the estimated relative treatment effect is of the same order (the male-female wage gap for those without children aged 25-34 was estimated to increase by £0.23).

There are a number of channels through which selection on unobservables could affect the results. Heterogeneous fertility responses, participation choices and experience of redundancy and hiring could imply that the group of workers in 2007 are not comparable to those observed in 2006. Intuitively, it might be assumed that positive fertility responses are most likely amongst those with weak labour market attachment and lower wages. Similarly, redundancies might be more common among the group of females with the lowest match surplus, which might also be expected to be the lowest wage workers. By the same argument, newly hired female workers may have to generate a higher surplus in order to be hired (higher productivity workers). Working in the opposite direction, the largest labour market participation effects might be expected among the group of low potential wage earners. Therefore, it is not clear a priori in which direction the selection effect would work. However, there is little evidence of any positive participation effects in response to the policy change. Estimates of the impact of the policy change on relative participation effects are shown in Table 5.7, under the assumption that the rate of change in the odds ratio for male/female labour market participation would have been the same if there had been no policy change.[50] A small negative estimate was found, implying that the odds of female participation actually decreased relative to males. This suggests that negative selection effects may be ruled out. To the extent that there are remaining selection effects through the fertility/redundancy/hiring mechanisms, they are expected to have a positive impact on the difference in differences estimate. And so the negative difference in differences estimate still stands as evidence of deteriorating female labour market outcomes as a result of the policy.

[49] This is estimated by assuming covariates affect male and female wages in the same way over time, but allowing for separate male/female time trends otherwise.

[50] Results from an LPM model are presented in Table 5.8.

Further evidence in favour of this argument comes from comparing predicted wages of the sample of employed individuals before and after the policy change. To the extent that there is any selection on unobservables you might reasonably expect the selection on observables to work in the same direction. A t-test comparing the difference in mean predicted female wages of the sample of working females after the policy change and the sample of working females before the policy change (excluding the impact of the time trend/estimated policy effect) suggests that the sample of women working after the policy change were positively selected relative to the sample working before the policy change. The predicted mean hourly wage for females in the pre-period was £10.48 compared to £10.53 in the post-period. The £0.05 gap has a p-value of 0.08. This is evidence in support of the above discussion.[51]

5.7.3 Impact on Employment

Although there are no model predictions relating to relative male/female employment rates, a common trends assumption can still allow for estimation of the impact of the policy change on relative male/female employment outcomes. Table 5.9 shows that although negative, the impact on the relative odds ratio of female-male employment was not found to be statistically significant. The placebo analysis in Figure 5.5 also suggests that 2007 was nothing out of the ordinary in the evolution of male-female relative odds ratio of employment (in the age category 25-34). The trend analysis in Figure 5.6 also corroborates this; the analysis of the male-female relative odds ratio of conditional employment suggests that the relative odds ratio was decreasing over time, and so the small (insignificant) decrease in relative conditional employment rates in 2007 seem to be in line with other years. The corresponding analysis using the LPM is shown in Table 5.10 and in Figures 5.7 and 5.8.

As noted in Curtis et al. (2014), a change in policy that impacts the labour market should be observed more quickly on labour market flows (redundancies and hires) than on aggregate levels. While it may take time to adjust to a new aggregate equilibrium, short term effects may be more quickly observable in flow data. Therefore, how relative female-male redundancy and hiring rates changed between 2006 and 2007 are analysed.

Table 5.11 suggests that the odds ratio of female redundancy increased significantly relative to males over the period in question for the age group 25-34. The placebo analysis shown in Figure 5.9 shows that the positive impact on the female-male relative odds ratio was the largest absolute impact estimated over the 11 year period. Furthermore, the trend analysis in Figure 5.10 suggests that the discrete model version of the common trends assumption

[51]A similar analysis for males suggest the sample of males observed working after the policy change were in fact negatively selected relative to the sample working before the policy change.

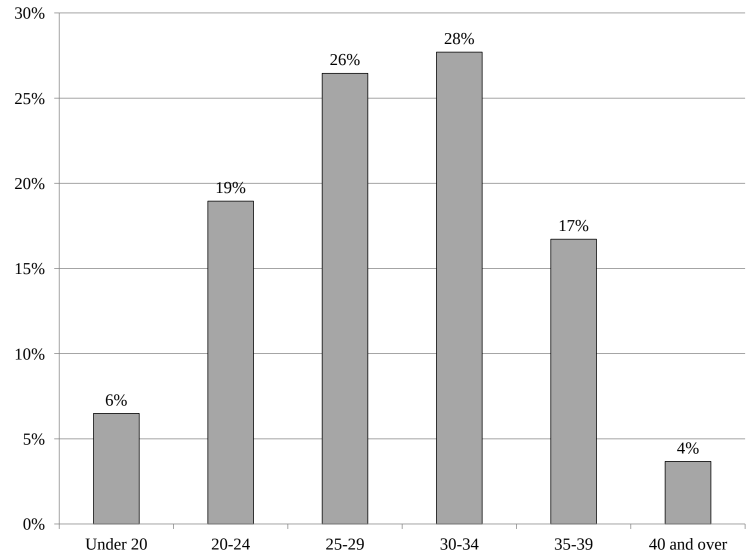
is not rejected (constant relative odds ratio). The same analysis using the LPM model is presented in Table 5.12, with the placebo analysis in Figure 5.11 and the trend analysis in Figure 5.12. These results corroborate the finding that female redundancies increased relative to males as a result of the longer paid maternity leave duration.
Finally, Table 5.13 suggests that the policy change had little impact on the odds ratio of female hiring relative to males. The estimate is slightly negative, but is not statistically significant. This outcome variable measures both hiring from unemployment and job switches. The placebo analysis in Figure 5.13 also suggests that 2007 was nothing out of the ordinary in the evolution of male-female relative odds ratio of hiring (in the age category 25-34). The trend analysis in Figure 5.14 also corroborates this. One explanation for this finding in the context of the other results, which suggest female labour market outcomes deteriorated, is that the employees hired to cover the higher number of female employees taking longer or more frequent maternity leave tended to female workers (due to occupational sorting), thus hiding any potential negative impact of the policy change on relative female-male hiring rates. Table 5.14 and Figure 5.15 and Figure 5.16 show the corresponding analysis using an LPM model.

5.8 Conclusion

A simple theoretical model was proposed, which suggests that differential levels of parental leave taken by male and female employees could contribute to the persistent male-female wage gap. A key model implication is that an increase in maternity leave uptake is associated with an increasing male-female wage gap.
A legislative change in the length of paid maternity leave/state maternity allowance that created an exogenous expansion in the uptake of maternity leave was used in a quasi-experimental estimation approach. The policy change increased the statutory maternity pay period and the state maternity allowance period from a maximum of 26 to 39 weeks. Empirical evidence of a significant first stage effect was found, with the policy change engendering a divergence in the amount of parental leave taken by female compared to male employees. A decomposition analysis provides evidence that most of this divergence was explained by female behaviour - both through increasing fertility and an increase in maternity leave duration.
Empirical evidence suggests that the extension in paid maternity leave led to a decrease in relative female wages for the age group 25-34. Empirical evidence was also found of an impact on relative female redundancy rates, with females relatively more likely than males to experience redundancy after the maternity leave expansion.
There was little evidence of an impact on aggregate employment levels, perhaps because of the longer time it would take for the impact of the legislative change to show up

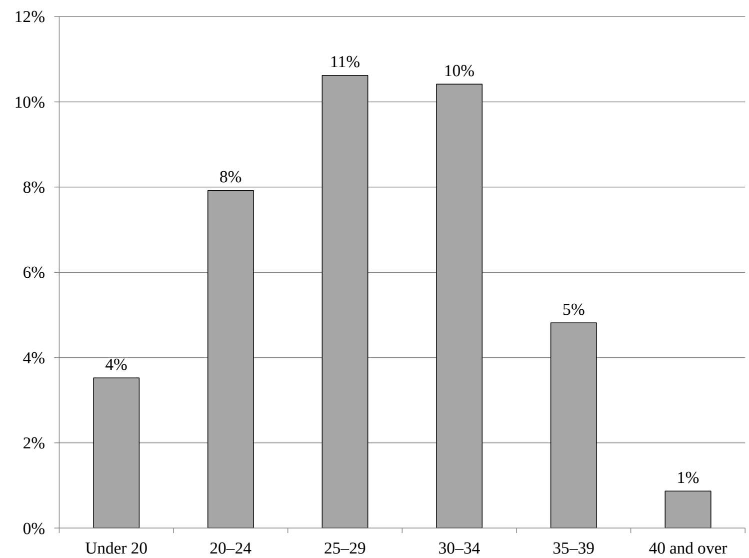
in aggregate employment figures (Curtis et al., 2014). Furthermore, the policy change did not appear to have a large impact on relative female hiring rates. This might be explained by the higher number of hires needed to replace females going on more frequent, longer maternity leave, and that due to occupational sorting females employees tend to be replaced by other females.

Figure 5.1: Births by age of mother - 2007



Source: ONS Birth Summary Tables, England and Wales 2013

Figure 5.2: Fertility by age of mother - 2007



Source: ONS Vital Statistics: Population and Health Reference Tables February 2014

Table 5.1: Descriptives - aged 25-34

	2006		2007		2008	
	Male	Female	Male	Female	Male	Female
Age	29.75	29.78	29.79	29.71	29.67	29.65
Age left full time education	18.61	18.61	18.64	18.72	18.75	18.81
Region						
North East	4.17%	4.49%	4.47%	4.17%	4.50%	4.15%
North West	8.44%	8.79%	9.19%	8.70%	9.15%	8.99%
Merseyside	1.89%	1.87%	1.64%	2.04%	1.98%	2.35%
Yorkshire and Humberside	8.97%	9.11%	8.93%	9.01%	9.00%	8.91%
East Midlands	7.09%	7.32%	6.83%	7.13%	7.87%	7.66%
West Midlands	8.64%	8.78%	7.46%	8.04%	8.11%	8.58%
Eastern	9.31%	8.31%	9.90%	9.48%	9.17%	8.72%
London	12.86%	12.88%	12.91%	13.02%	12.88%	12.90%
South East	13.00%	13.03%	12.87%	13.67%	12.77%	12.94%
South West	7.71%	8.17%	8.02%	7.77%	7.83%	7.47%
Wales	4.68%	4.79%	4.36%	5.03%	4.41%	4.88%
Scotland	8.53%	8.02%	8.19%	7.75%	7.72%	7.77%
Northern Ireland	4.72%	4.44%	5.23%	4.20%	4.62%	4.68%
UK born	84.28%	84.63%	82.25%	82.64%	81.53%	81.19%
Child under 1	8.68%	10.43%	9.00%	10.97%	9.93%	11.51%
On leave	6.14%	14.50%	6.23%	14.90%	5.69%	15.25%
On leave - child under 1	5.68%	62.29%	7.81%	61.74%	6.88%	68.02%
Participation	93.57%	76.51%	93.91%	76.08%	93.48%	76.87%
Employed	95.04%	95.28%	95.74%	95.42%	95.06%	95.01%
Hourlywage	11.27	10.17	11.92	10.64	12.08	10.71
Paid hours	41.10	32.61	41.15	32.74	40.88	32.88
Unpaid hours	1.63	1.57	1.67	1.57	1.69	1.61
Redundancy	0.84%	0.44%	0.60%	0.57%	0.91%	0.43%
Redundancy incl temp	1.46%	1.12%	1.29%	1.11%	1.51%	0.95%
Hired	6.47%	6.67%	7.14%	7.05%	6.01%	5.80%
Permanency	95.39%	94.57%	95.70%	94.13%	95.98%	95.05%
Public sector	15.21%	32.66%	14.53%	32.33%	15.63%	32.07%
Self employed	12.76%	5.80%	12.41%	5.88%	12.51%	5.88%
Small firm	24.72%	22.22%	25.76%	21.92%	25.95%	21.76%
Small firm *leq* 10	14.03%	11.90%	14.63%	12.10%	14.66%	11.43%
Part time	4.97%	35.65%	4.79%	34.01%	5.42%	33.95%
Current Industry						
Agriculture and fishing	1.73%	0.41%	1.61%	0.39%	1.70%	0.70%
Energy and water	1.33%	0.72%	1.75%	0.93%	1.58%	0.80%
Manufacturing	17.93%	7.73%	17.11%	8.27%	16.51%	8.04%
Construction	13.28%	2.11%	13.85%	1.58%	13.25%	1.69%
Distribution, hotels and restaurants	16.81%	17.61%	16.63%	17.05%	17.26%	17.34%
Transport and communication	8.79%	4.21%	8.46%	4.18%	8.20%	3.83%
Banking, finance and insurance	19.92%	18.25%	20.59%	19.50%	19.62%	19.54%
Public admin, educ. health & other	20.22%	48.95%	20.00%	48.09%	21.89%	48.07%
N	11,019	12,982	10,806	12,715	9,357	10,836

Table 5.2: Leave 2006-2008 - Logit Model

VARIABLES	(1) 16-24	(2) 25-34	(3) 35-44	(4) 45-64
female	0.398***	0.942***	0.666***	0.340***
	(0.086)	(0.051)	(0.040)	(0.032)
yearb	-0.069	-0.079	-0.022	-0.046
	(0.097)	(0.063)	(0.047)	(0.034)
femaleyearb	0.106	0.146*	0.026	-0.011
	(0.128)	(0.077)	(0.060)	(0.047)
Constant	-2.613***	-2.998***	-3.493***	-2.958***
	(0.337)	(0.167)	(0.143)	(0.111)
Observations	15,970	35,347	49,060	74,205
Impact on R.O.R.	0.167	0.404*	0.050	-0.015
	(0.203)	(0.214)	(0.119)	(0.066)

Robust standard errors in parentheses

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

(1) Other controls include age finished education, GOR region and dummy for whether born in UK

(2) Could be away for work for many reasons - parental leave, holiday, sick/injured, training course, started new/changing job, etc.

(3) yearb = 0 if observation year is 2006, yearb = 1 if observation year is 2008

(4) Estimation is by quasi-maximum likelihood.

(5) The impact on R.O.R. measures the change in the odds ratio of a female being on leave relative to a male

Table 5.3: Leave 2006-2008 - LPM Model

	(1)	(2)	(3)	(4)
VARIABLES	16-24	25-34	35-44	45-64
female	0.025***	0.083***	0.064***	0.034***
	(0.005)	(0.004)	(0.004)	(0.003)
yearb	-0.004	-0.004	-0.002	-0.004
	(0.005)	(0.004)	(0.003)	(0.003)
femaleyearb	0.006	0.013*	0.002	-0.002
	(0.008)	(0.006)	(0.006)	(0.005)
Constant	0.075***	0.040***	-0.020	0.025**
	(0.023)	(0.015)	(0.013)	(0.011)
Observations	15,970	35,347	49,060	74,205
R-squared	0.005	0.023	0.013	0.004

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

(1) Other controls include age finished education, GOR region and dummy for whether born in UK

(2) Could be away for work for many reasons - parental leave, holiday, sick/injured, training course, started new/changing job, etc.

(3) yearb = 0 if observation year is 2006, yearb = 1 if observation year is 2008

Table 5.4: Decomposition of change in parental leave of females relative to males 2006-2008

	(1) 25-34	(2) 35-44
Fertility in 2006	0.088***	0.031***
γ_{FL}	(0.003)	(0.002)
Paternity in 2006	0.090***	0.051***
γ_{ML}	(0.003)	(0.002)
Maternity leave in 2006	0.623***	0.624***
λ_F	(0.017)	(0.024)
Paternity leave in 2006	0.057***	0.098***
λ_M	(0.008)	(0.011)
Change in fertility 2006-2008	0.010**	0.003
$\frac{d\gamma_{FL}}{d\theta}$ (LPM)	(0.004)	(0.002)
Change in paternity 2006-2008	0.012***	0.004
$\frac{d\gamma_{ML}}{d\theta}$ (LPM)	(0.004)	(0.003)
Change in maternity leave 2006-2008	0.066***	0.053
$\frac{d\lambda_F}{d\theta}$ (LPM)	(0.024)	(0.034)
Change in paternity leave 2006-2008	0.013	-0.020
$\frac{d\lambda_M}{d\theta}$ (LPM)	(0.012)	(0.015)
Due to Increase in fertility	0.006**	0.002
$\lambda_F \frac{d\gamma_{FL}}{d\theta}$	(0.003)	(0.001)
Due to Increase in maternity leave	0.006***	0.002
$\gamma_{FL} \frac{d\lambda_F}{d\theta}$	(0.002)	(0.001)
Aggregate Female Response	0.012***	0.004**
	(0.003)	(0.002)
Due to Increase in paternity (-ve effect)	0.001**	0.000
$\lambda_M \frac{d\gamma_{ML}}{d\theta}$	(0.000)	(0.000)
Due to Increase in paternity leave (-ve effect)	0.001	-0.001
$\gamma_{ML} \frac{d\lambda_M}{d\theta}$	(0.001)	(0.001)
Aggregate Male Response (-ve effect)	0.002*	-0.001
	(0.001)	(0.001)
Estimated increase in maternity-paternity leave gap	0.010***	0.004**
	(0.003)	(0.002)

(1) Estimated increase in maternity-paternity leave gap may differ from estimated increase in total leave gap

(2) Independence between the male and female samples is assumed

(3) Robust standard errors reported

(4) GMM does not converge for age 16-25 men or for age 44-65 women

Table 5.5: Hourly wages 2006-2007

VARIABLES	(1) 16-24	(2) 25-34	(3) 35-44	(4) 45-64
female	-0.103	-0.790***	-1.886***	-2.012***
	(0.101)	(0.127)	(0.157)	(0.149)
year	0.067	0.132	-0.192	-0.167
	(0.126)	(0.107)	(0.132)	(0.115)
femaleyear	-0.091	-0.288**	0.243	0.026
	(0.148)	(0.140)	(0.151)	(0.130)
Constant	1.391**	-0.861*	-5.266***	-6.136***
	(0.614)	(0.451)	(0.704)	(0.592)
Observations	4,702	11,213	14,675	20,948
R-squared	0.254	0.285	0.318	0.332
Female Bound 25-34		[-0.288**, 0.000]		
Male Bound 25-34		[0.000, 0.288**]		

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(1) Other controls include age finished education, region-industry fixed effects, a dummy for whether born in the UK, a dummy for whether working part time and a dummy for whether working in the public or private sector

(2) year = 0 if observation year is 2006, year = 1 if observation year is 2007

(3) Cluster robust standard errors with clustering at the region-industry level are reported

Figure 5.3: Placebo analysis - Hourly wages age 25-34

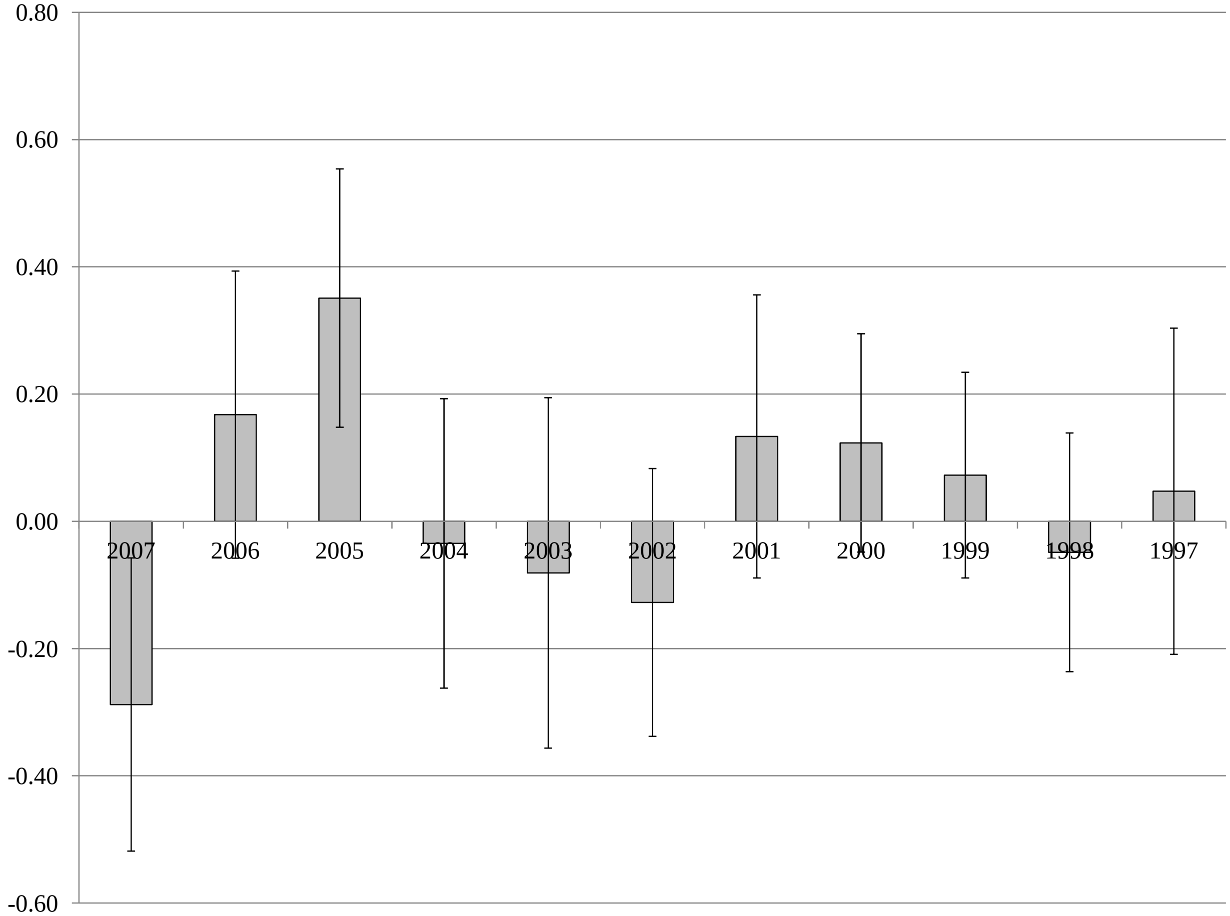
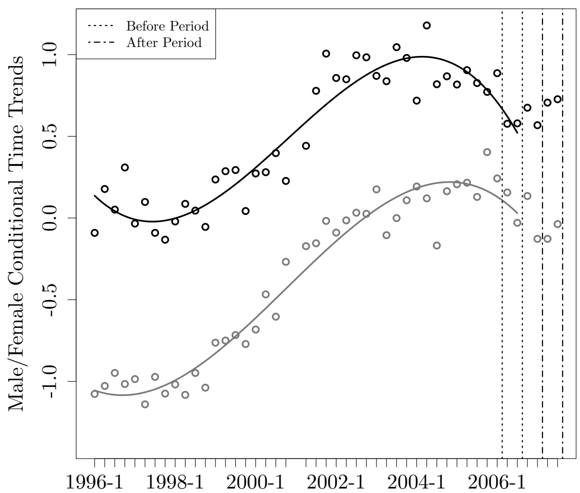
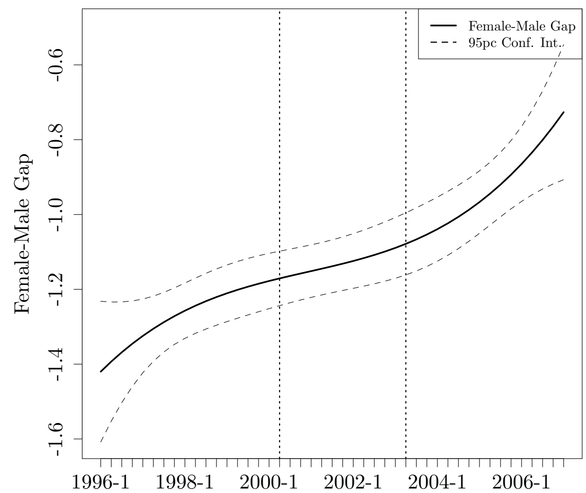


Figure 5.4: Common trend analysis - hourly wages

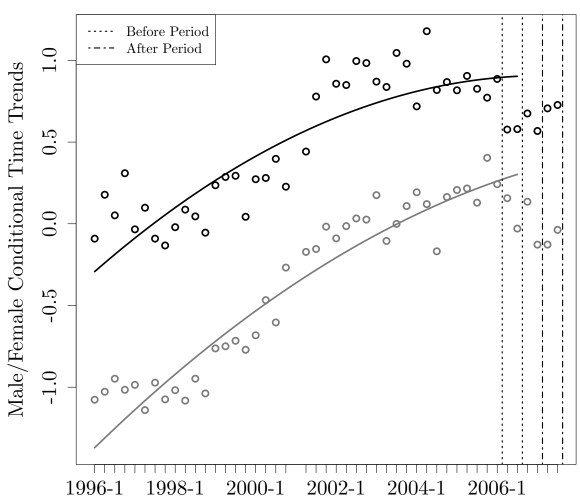
(a) Hourly wages residuals age 25-34 (Cubic)



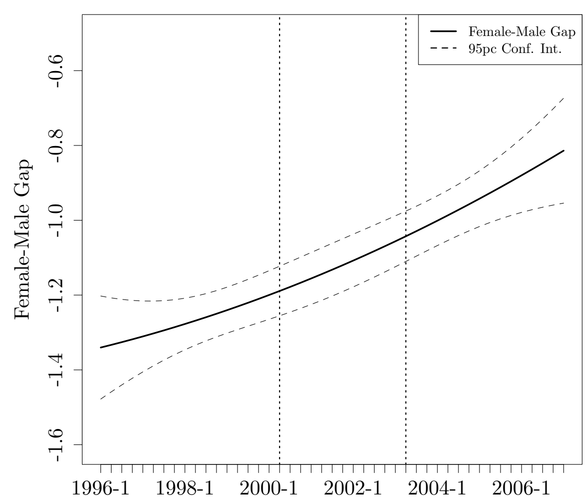
(b) Hourly wage gap age 25-34 (Cubic)



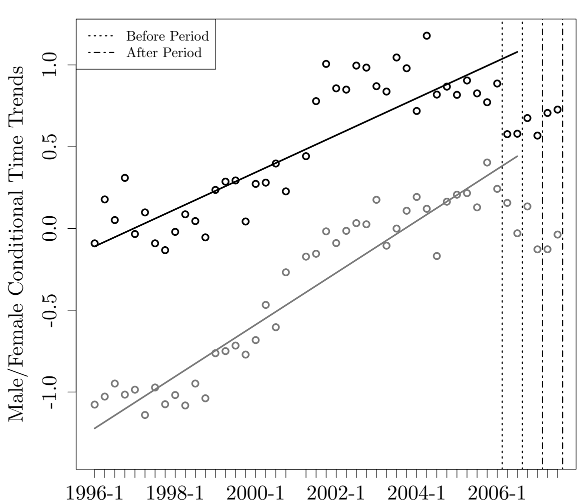
(c) Hourly wages residuals age 25-34 (Quadratic)



(d) Hourly wage gap age 25-34 (Quadratic)



(e) Hourly wages residuals age 25-34 (Linear)



(f) Hourly wage gap age 25-34 (Linear)

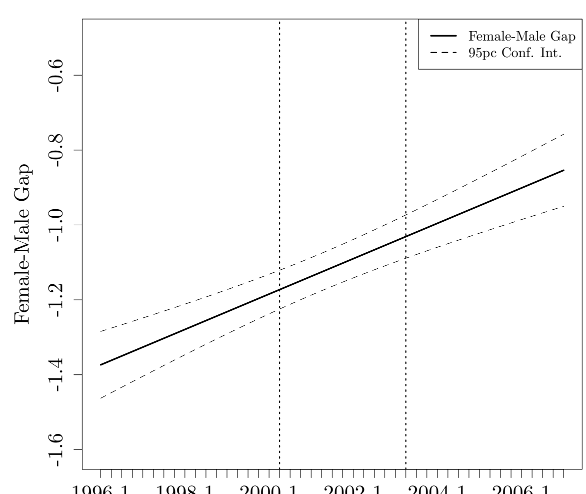


Table 5.6: Hourly wages 2006-2007 - No Children

VARIABLES	(1) 16-24	(2) 25-34	(3) 35-44	(4) 45-64
female	-0.084	-0.571***	-0.668***	-1.645***
	(0.106)	(0.135)	(0.253)	(0.164)
year	0.064	0.068	-0.070	-0.216*
	(0.130)	(0.134)	(0.235)	(0.116)
femaleyear	-0.068	-0.233	0.175	0.086
	(0.153)	(0.193)	(0.322)	(0.162)
Constant	1.018	0.880*	-3.424***	-5.940***
	(0.636)	(0.521)	(0.945)	(0.542)
Observations	4,248	6,526	4,527	15,433
R-squared	0.257	0.279	0.304	0.322
Female Bound 25-34		[-0.233, 0.000]		
Male Bound 25-34		[0.000, 0.233]		

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(1) Other controls include age finished education, region-industry fixed effects, a dummy for whether born in the UK, a dummy for whether working part time and a dummy for whether working in the public or private sector

(2) year = 0 if observation year is 2006, year = 1 if observation year is 2007

(3) Cluster robust standard errors with clustering at the region-industry level are reported

Table 5.7: Labour market participation 2006-2007 - Logit Model

VARIABLES	(1) 16-24	(2) 25-34	(3) 25-34	(4) 45-64
female	-0.984***	-1.564***	-1.306***	-0.131***
	(0.052)	(0.045)	(0.037)	(0.021)
year	-0.044	0.072	-0.016	-0.013
	(0.059)	(0.057)	(0.045)	(0.021)
femaleyear	0.020	-0.108*	-0.003	0.040
	(0.074)	(0.065)	(0.052)	(0.030)
Constant	-2.553***	-2.135***	-0.639***	-1.592***
	(0.221)	(0.135)	(0.125)	(0.080)
Observations	24,698	47,522	64,070	103,859
Impact on R.O.R.	0.008	-0.021*	-0.001	0.036
	(0.028)	(0.013)	(0.014)	(0.027)

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

(1) Other controls include age finished education, GOR region and dummy for whether born in UK

(2) year = 0 if observation year is 2006, 1 if observation year is 2007

(3) Estimation is by quasi-maximum likelihood.

(4) The impact on R.O.R. measures the change in the odds ratio of a female participating in the labour market relative to a male

Table 5.8: Labour market participation 2006-2007 - LPM Model

VARIABLES	(1) 16-24	(2) 25-34	(3) 35-44	(4) 45-64
female	-0.127***	-0.171***	-0.144***	-0.021***
	(0.006)	(0.004)	(0.004)	(0.004)
year	-0.004	0.004	-0.001	-0.002
	(0.006)	(0.003)	(0.003)	(0.003)
femaleyear	0.001	-0.010	-0.001	0.007
	(0.009)	(0.006)	(0.005)	(0.005)
Constant	0.323***	0.402***	0.572***	0.342***
	(0.025)	(0.014)	(0.013)	(0.012)
Observations	24,698	47,522	64,070	103,859
R-squared	0.056	0.100	0.061	0.024

Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

(1) Other controls include age finished education, GOR region and dummy for whether born in UK

(2) year = 0 if observation year is 2006, 1 if observation year is 2007

Table 5.9: Employment conditional on participation 2006-2007 - Logit model

VARIABLES	(1) 16-24	(2) 25-34	(3) 35-44	(4) 45-64
female	0.188***	-0.006	-0.152**	0.415***
	(0.057)	(0.066)	(0.064)	(0.061)
year	0.024	0.169**	0.102	0.008
	(0.053)	(0.067)	(0.068)	(0.053)
femaleyear	0.025	-0.150	0.003	-0.047
	(0.081)	(0.096)	(0.094)	(0.086)
Constant	-0.962***	-0.317	0.312	0.789***
	(0.242)	(0.242)	(0.235)	(0.196)
Observations	20,588	40,063	54,555	80,988
Impact on R.O.R.	0.030	-0.139	0.003	-0.070
	(0.100)	(0.088)	(0.081)	(0.128)
Female Bound 25-34		[-0.006, 0.000]		
Male Bound 25-34		[0.000, 0.006]		

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

(1) Other controls include age finished education, GOR region and dummy for whether born in UK

(2) year = 0 if observation year is 2006, year = 1 if observation year is 2007

(3) Estimation is by quasi-maximum likelihood.

(4) The impact on R.O.R. measures the change in the odds ratio of a female being employed relative to a male

Figure 5.5: Placebo analysis - Conditional employment age 25-34 - Logit model

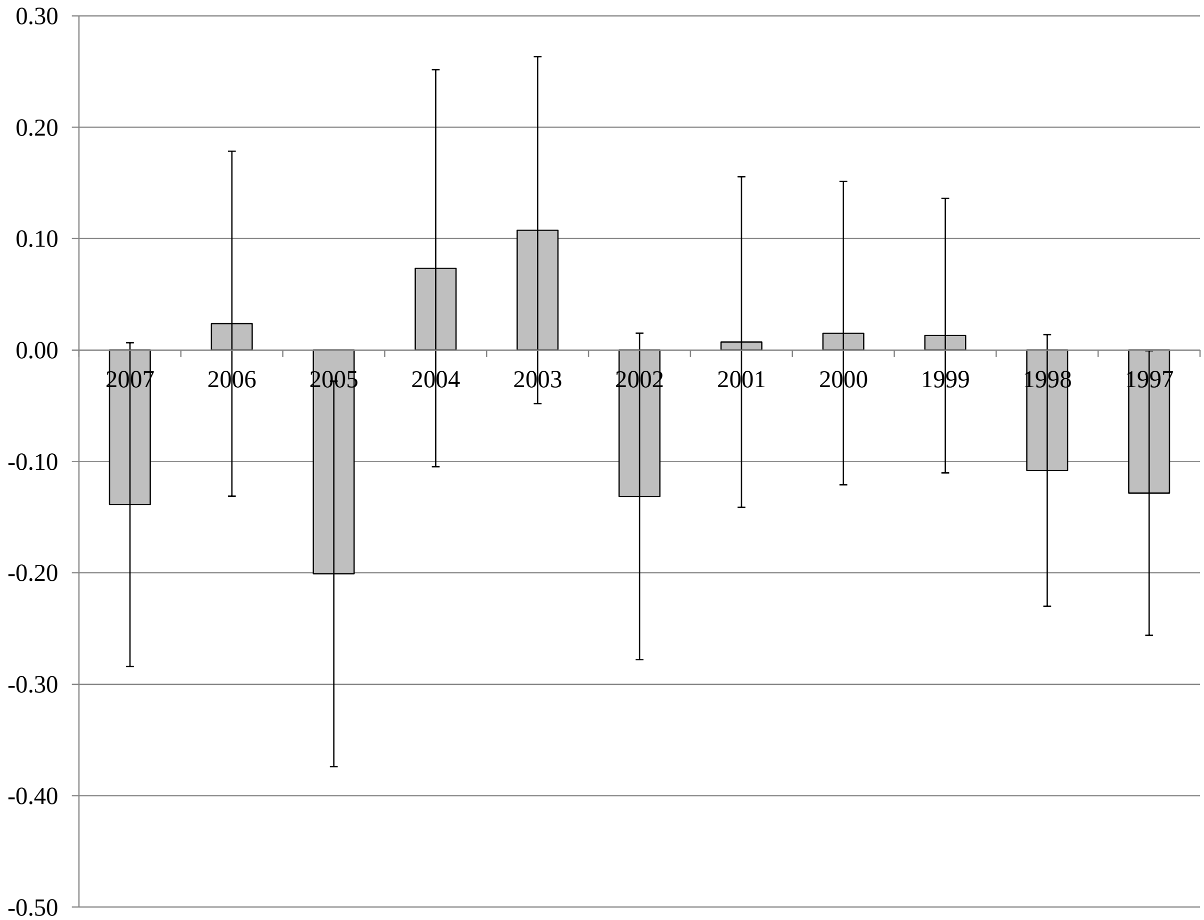
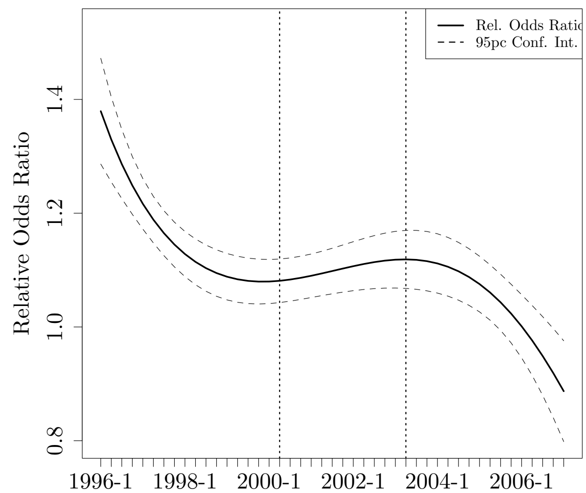


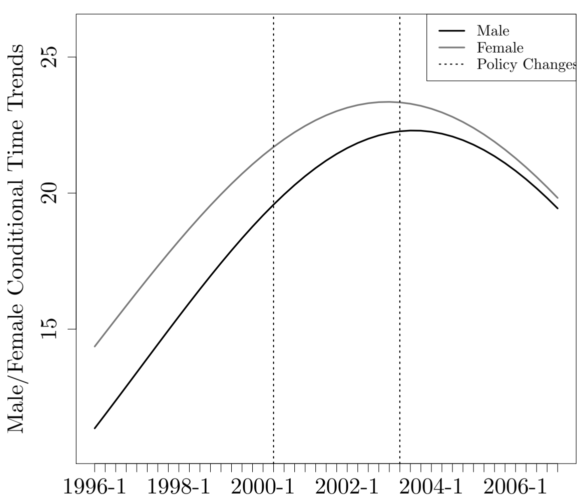
Figure 5.6: Common trend analysis - conditional employment - Logit model

(a) Conditional empl. age 25-34 (Cubic)

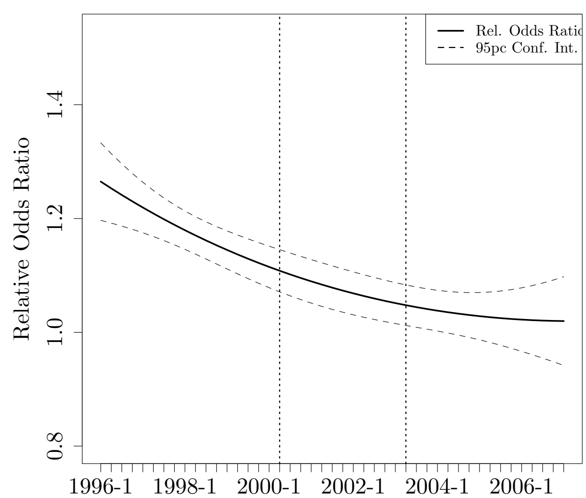
(b) Conditional empl. gap age 25-34 (Cubic)



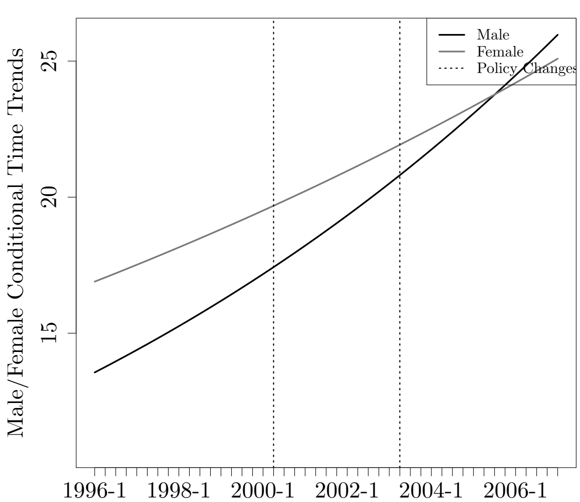
(c) Conditional empl. age 25-34 (Quadratic)



(d) Conditional empl. gap age 25-34 (Quadratic)



(e) Conditional empl. age 25-34 (Linear)



(f) Conditional empl. gap age 25-34 (Linear)

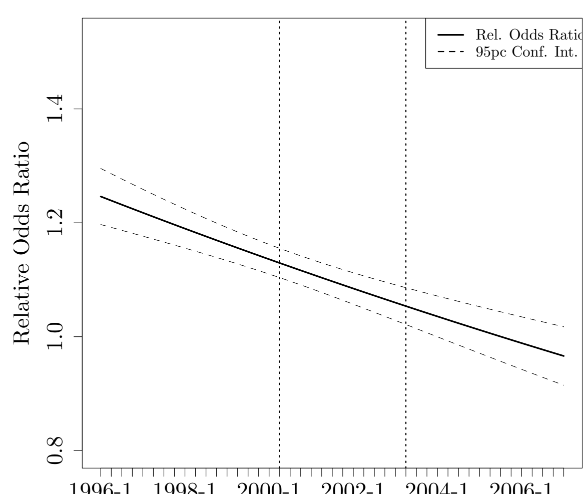


Table 5.10: Employment conditional on participation 2006-2007 - LPM Model

VARIABLES	(1) 16-24	(2) 25-34	(3) 35-44	(4) 45-64
female	0.023***	-0.000	-0.005**	0.011***
	(0.007)	(0.003)	(0.002)	(0.002)
year	0.004	0.007**	0.003	0.000
	(0.007)	(0.003)	(0.002)	(0.002)
femaleyear	0.002	-0.006	0.000	-0.001
	(0.010)	(0.004)	(0.003)	(0.002)
Constant	0.550***	0.814***	0.859***	0.892***
	(0.026)	(0.010)	(0.008)	(0.006)
Observations	20,588	40,063	54,555	80,988
R-squared	0.015	0.009	0.008	0.005
Female Bound 25-34		[-0.006, 0.000]		
Male Bound 25-34		[0.000, 0.006]		

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(1) Other controls include age finished education, GOR region and dummy for whether born in UK

(2) year = 0 if observation year is 2006, year = 1 if observation year is 2007

Figure 5.7: Placebo analysis - Conditional employment age 25-34 - LPM Model

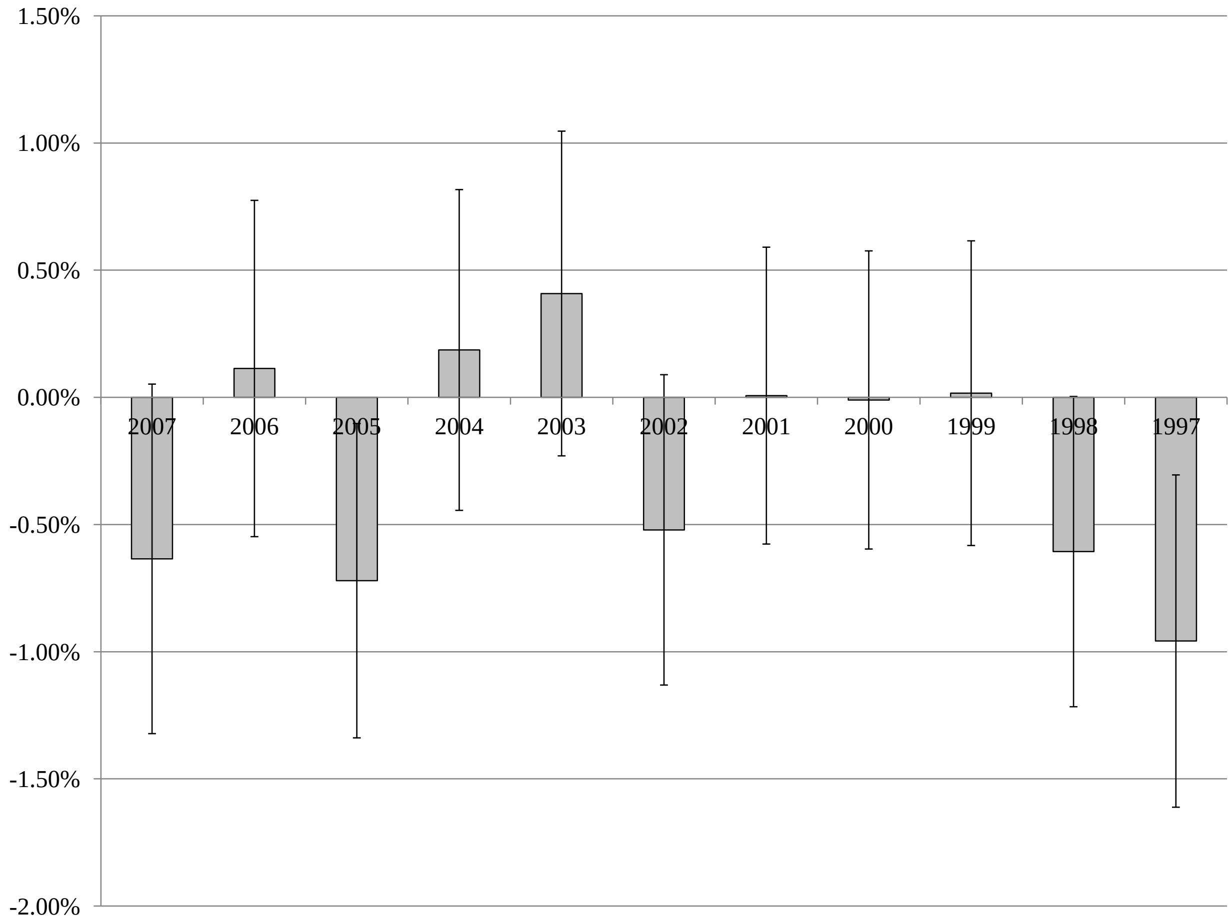
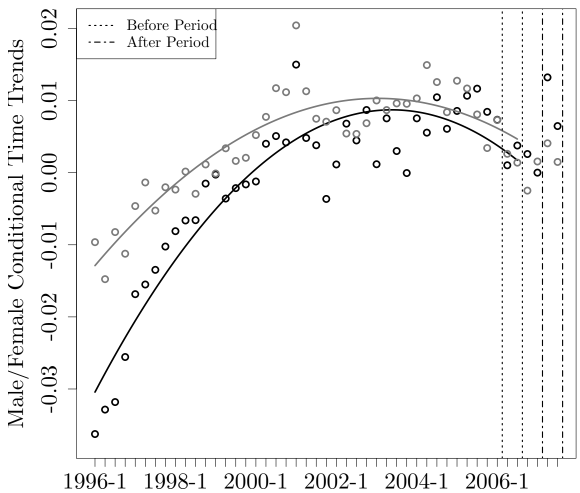
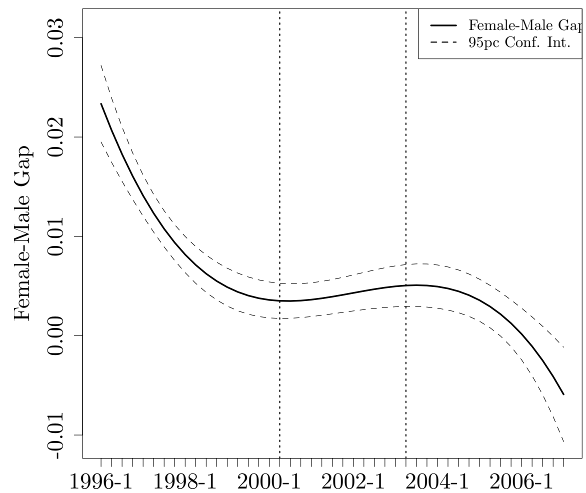


Figure 5.8: Common trend analysis - conditional employment - LPM Model

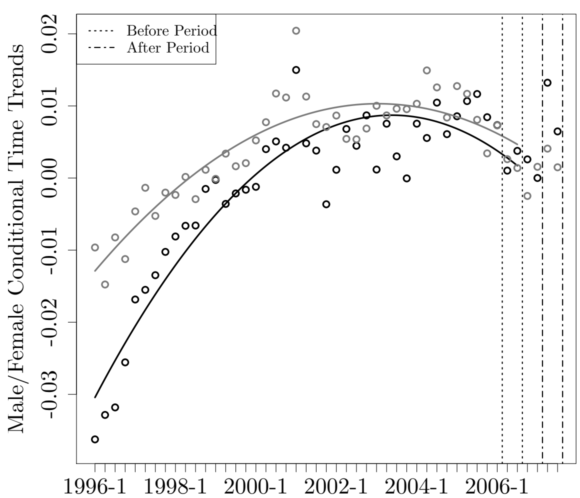
(a) Conditional empl. age 25-34 (Cubic)



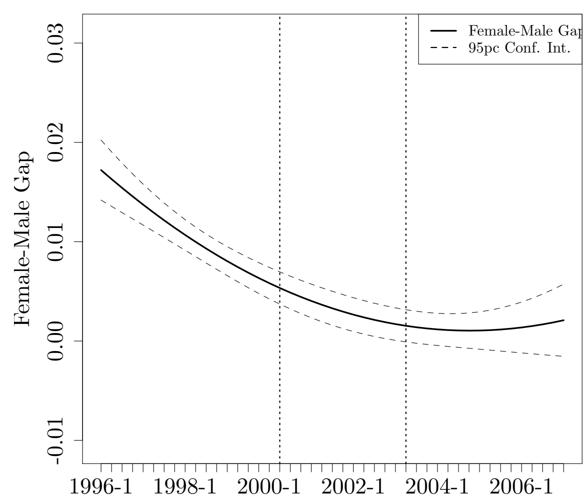
(b) Conditional empl. gap age 25-34 (Cubic)



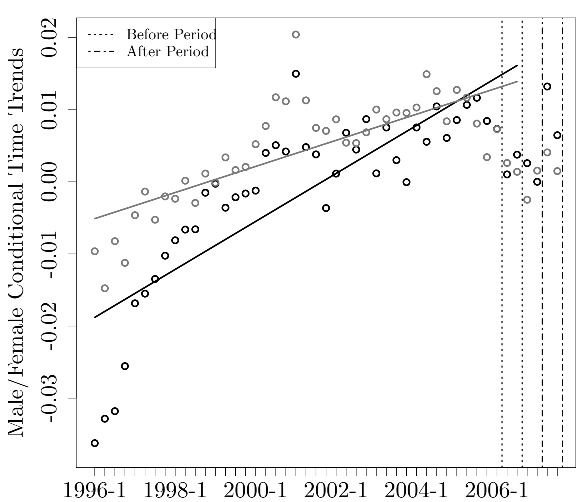
(c) Conditional empl. age 25-34 (Quadratic)



(d) Conditional empl. gap age 25-34 (Quadratic)



(e) Conditional empl. age 25-34 (Linear)



(f) Conditional empl. gap age 25-34 (Linear)

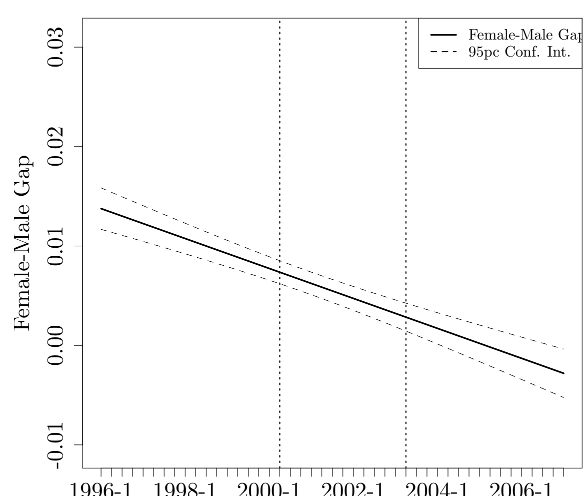


Table 5.11: Experienced redundancy in last 3 months 2006-2007 - Logit model

VARIABLES	(1) 16-24	(2) 25-34	(3) 35-44	(4) 45-64
female	0.074	-0.330*	-0.289	-0.153
	(0.209)	(0.196)	(0.179)	(0.153)
year	0.082	-0.318*	-0.233	-0.071
	(0.181)	(0.171)	(0.151)	(0.126)
femaleyear	-0.163	0.596**	0.308	0.071
	(0.280)	(0.268)	(0.237)	(0.205)
Constant	-0.737	-4.494***	-4.634***	-4.767***
	(1.167)	(0.950)	(0.923)	(0.875)
Observations	18,191	38,764	53,149	79,506
Impact on R.O.R.	-0.162	0.586**	0.270	0.064
	(0.278)	(0.284)	(0.212)	(0.182)
Female Bound 25-34		[0.000, 0.002**]		
Male Bound 25-34		[-0.003*, 0.000]		

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

(1) Other controls include age finished education, GOR region and dummy for whether born in UK and industry

(2) year = 0 if observation year is 2006, year = 1 if observation year is 2007

(3) Estimation is by quasi-maximum likelihood.

(4) The impact on R.O.R. measures the change in the odds ratio of a female experiencing redundancy relative to a male

Figure 5.9: Placebo analysis - Redundancy age 25-34 - Logit model

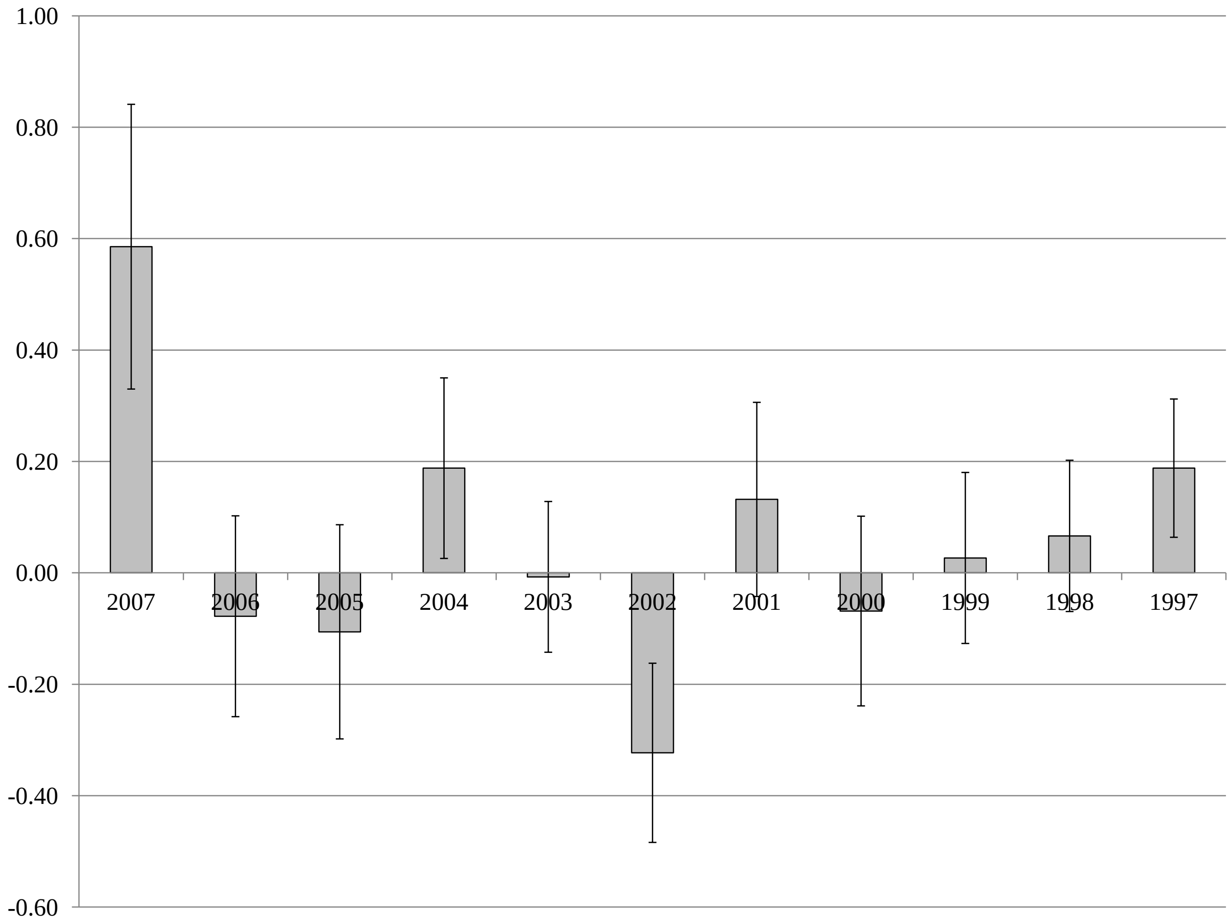
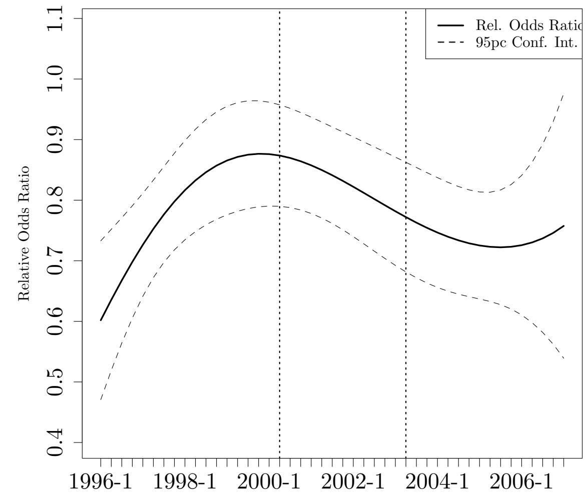


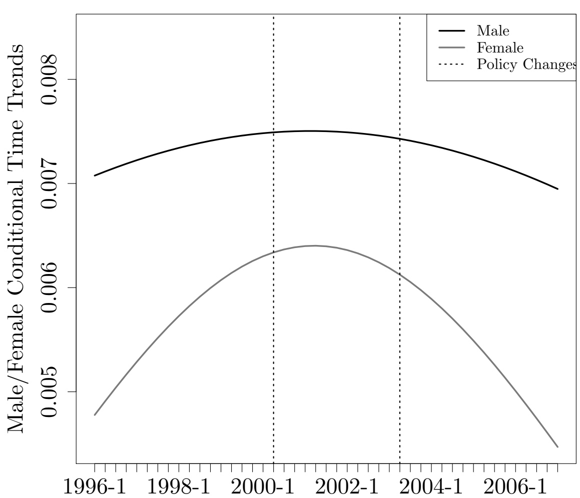
Figure 5.10: Common trend analysis - redundancy - Logit model

(a) Redundancy age 25-34 (Cubic)

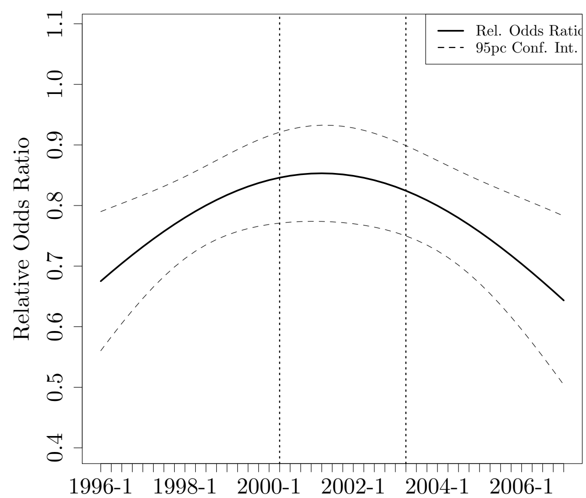
(b) Redundancy gap age 25-34 (Cubic)



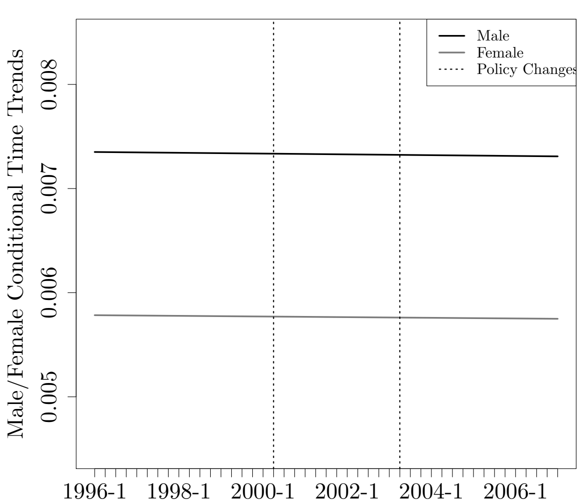
(c) Redundancy age 25-34 (Quadratic)



(d) Redundancy gap age 25-34 (Quadratic)



(e) Redundancy age 25-34 (Linear)



(f) Redundancy gap age 25-34 (Linear)

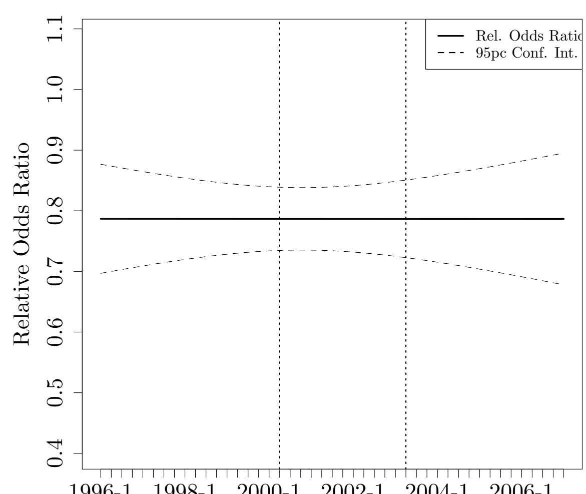


Table 5.12: Experienced redundancy in last 3 months 2006-2007 - LPM Model

VARIABLES	(1) 16-24	(2) 25-34	(3) 35-44	(4) 45-64
female	0.001	-0.002*	-0.002*	-0.001
	(0.002)	(0.001)	(0.001)	(0.001)
year	0.001	-0.002*	-0.002	-0.000
	(0.002)	(0.001)	(0.001)	(0.001)
femaleyear	-0.002	0.004**	0.002	0.000
	(0.003)	(0.002)	(0.001)	(0.001)
Constant	0.044***	0.010**	0.009**	0.010***
	(0.010)	(0.005)	(0.004)	(0.003)
Observations	18,191	38,764	53,149	79,506
R-squared	0.003	0.003	0.002	0.002
Female Bound 25-34		[0.000, 0.004**]		
Male Bound 25-34		[-0.004**, 0.000]		

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(1) Other controls include age finished education, GOR region and dummy for whether born in UK and industry

(2) year = 0 if observation year is 2006, year = 1 if observation year is 2007

Figure 5.11: Placebo analysis - Redundancy age 25-34 - LPM Model

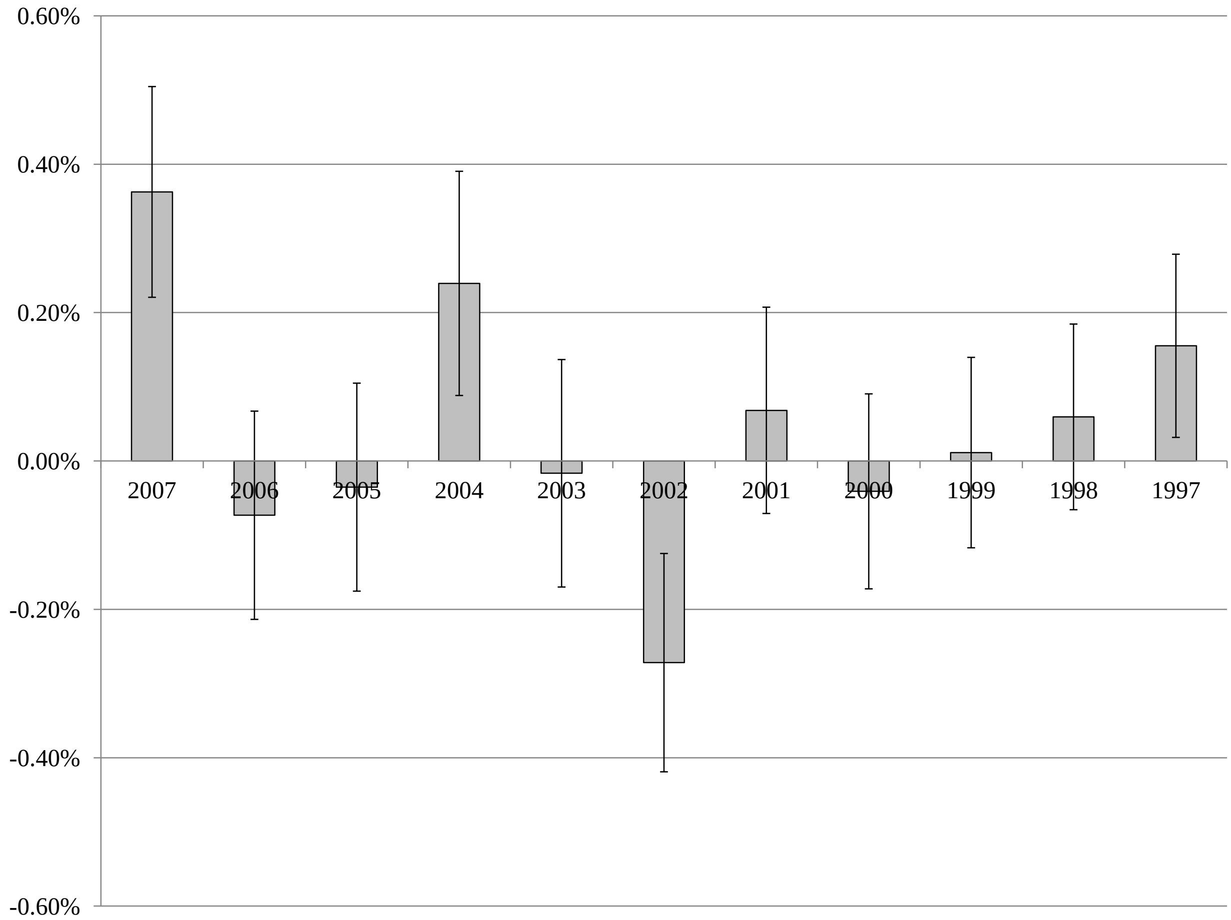
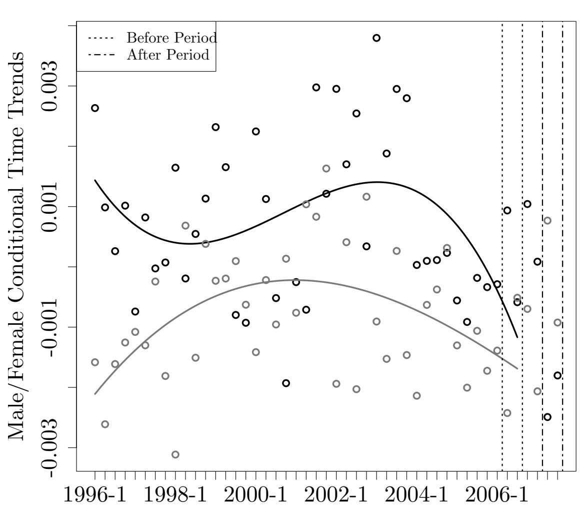
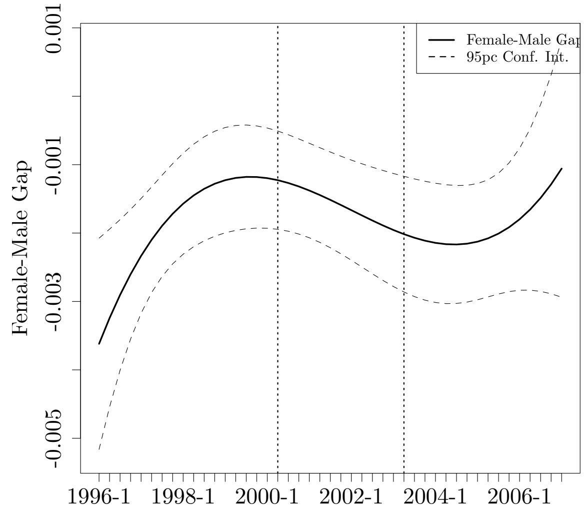


Figure 5.12: Common trend analysis - redundancy - LPM Model

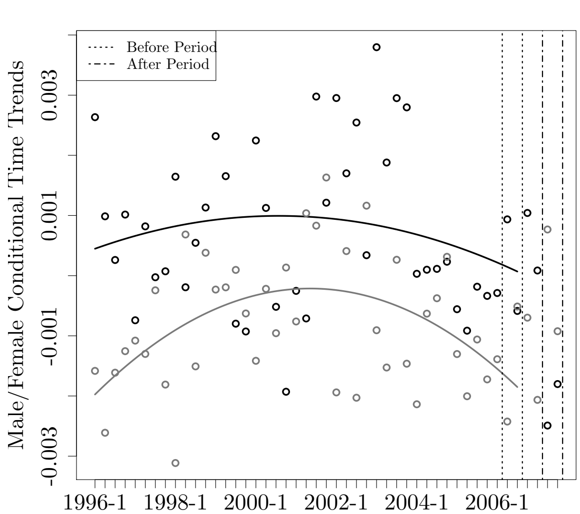
(a) Redundancy residuals age 25-34 (Cubic)



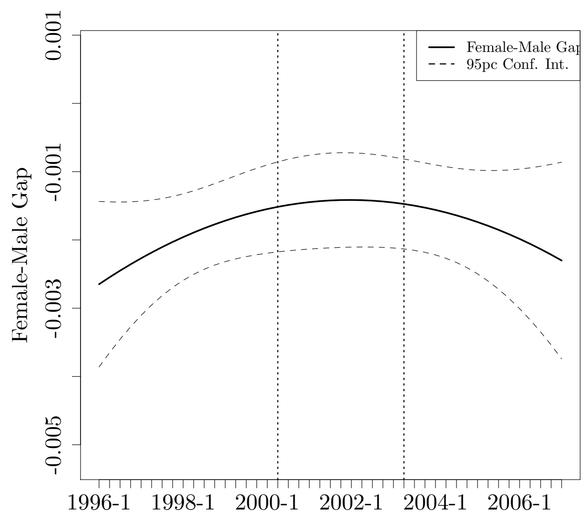
(b) Redundancy gap age 25-34 (Cubic)



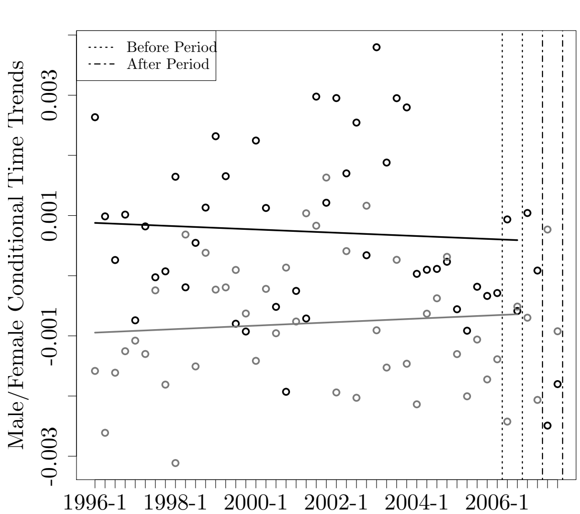
(c) Redundancy residuals age 25-34 (Quadratic)



(d) Redundancy gap age 25-34 (Quadratic)



(e) Redundancy residuals age 25-34 (Linear)



(f) Redundancy gap age 25-34 (Linear)

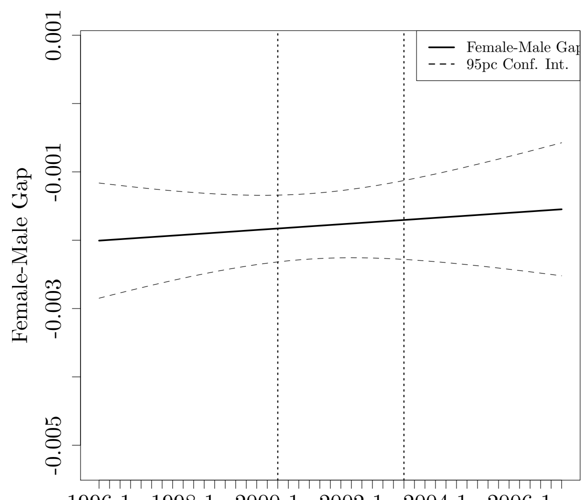


Table 5.13: Whether hired/changed job in last 3 months 2006-2007 - Logit model

	(1)	(2)	(3)	(4)
VARIABLES	16-24	25-34	25-34	25-34
female	0.068	0.118**	-0.009	0.062
	(0.059)	(0.059)	(0.063)	(0.062)
year	0.073	0.086	0.074	-0.005
	(0.055)	(0.056)	(0.058)	(0.058)
femaleyear	-0.018	-0.041	0.051	0.015
	(0.079)	(0.080)	(0.084)	(0.084)
Constant	-1.801***	-2.074***	-2.252***	-3.587***
	(0.270)	(0.271)	(0.298)	(0.290)
Observations	19,113	39,558	53,911	80,172
Impact on R.O.R.	-0.019	-0.045	0.052	0.017
	(0.084)	(0.088)	(0.086)	(0.090)
Female Bound 25-34		[-0.003, 0.000]		
Male Bound 25-34		[0.000, 0.002]		

Robust standard errors in parentheses
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

(1) Other controls include age finished education, GOR region and dummy for whether born in UK and industry

(2) Those hired and fired within 3 months treated as not hired

(3) year = 0 if observation year is 2006, year = 1 if observation year is 2007

(4) Estimation is by quasi-maximum likelihood.

(5) The impact on R.O.R. measures the change in the odds ratio of a female being hired/changing jobs relative to a male

Figure 5.13: Placebo analysis - Hiring age 25-34 - Logit model

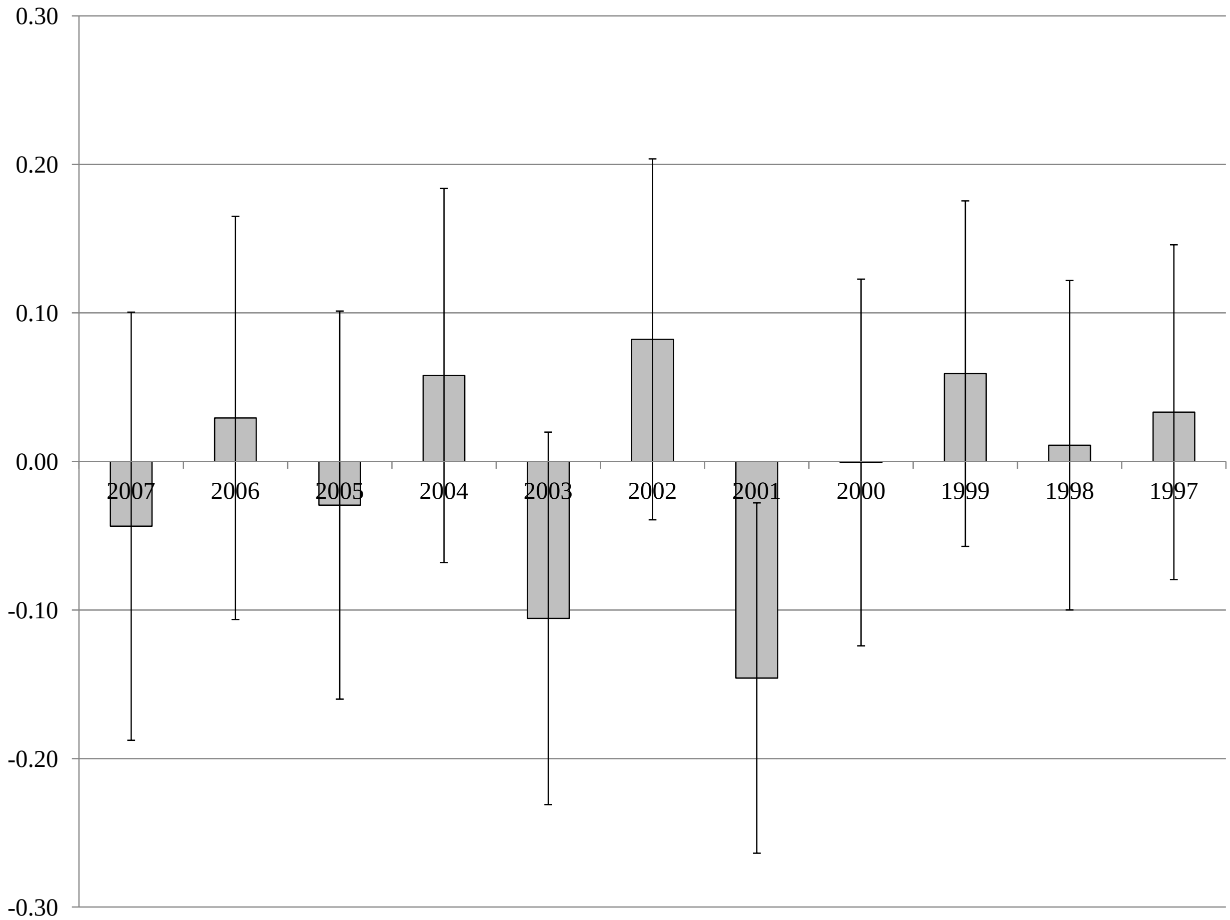
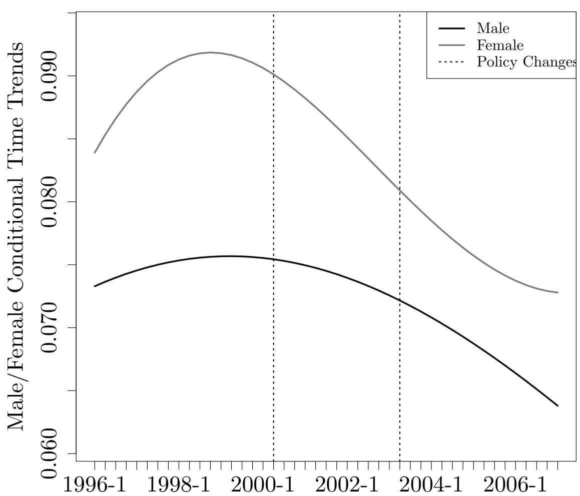
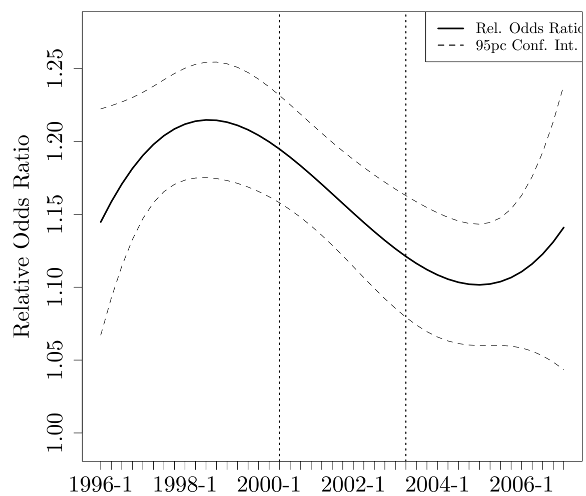


Figure 5.14: Common trend analysis - hiring - Logit model

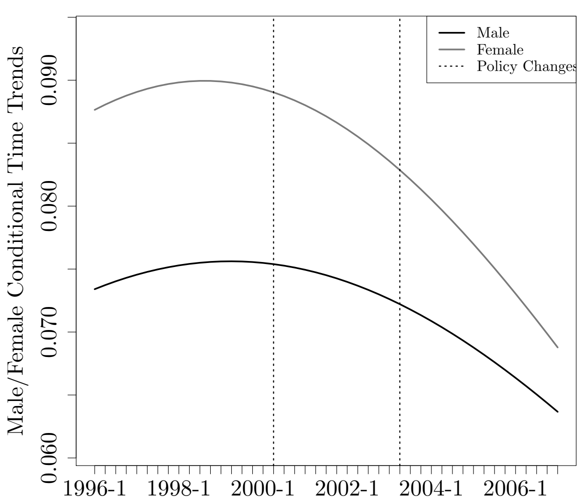
(a) Hiring age 25-34 (Cubic)



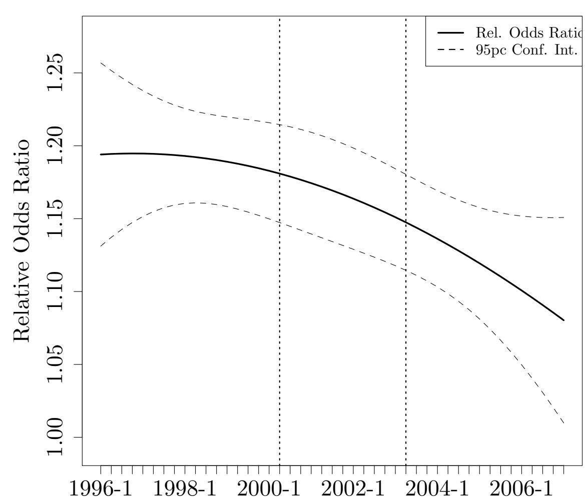
(b) Hiring gap age 25-34 (Cubic)



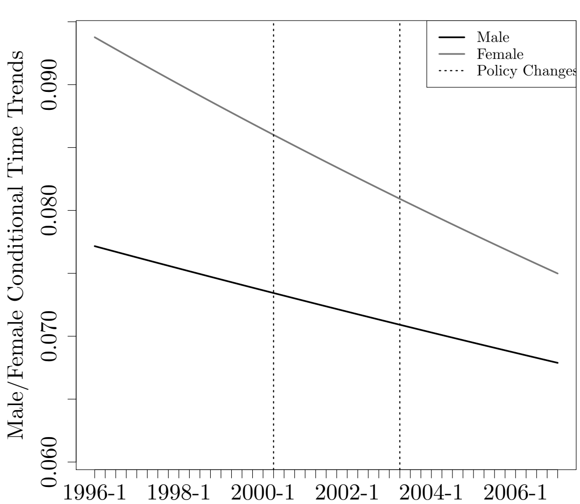
(c) Hiring age 25-34 (Quadratic)



(d) Hiring gap age 25-34 (Quadratic)



(e) Hiring age 25-34 (Linear)



(f) Hiring gap age 25-34 (Linear)

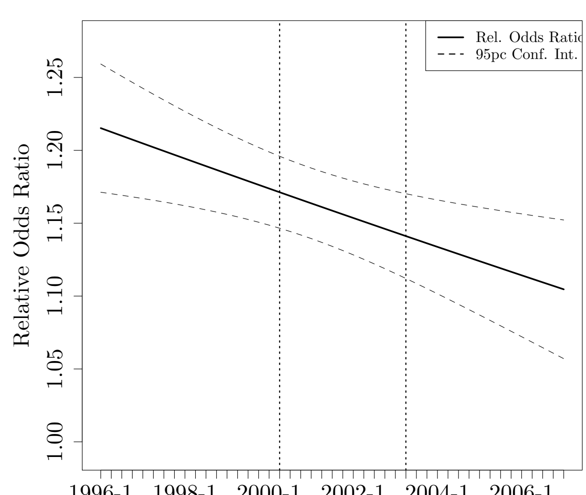


Table 5.14: Whether hired/changed job in last 3 months 2006-2007 - LPM model

VARIABLES	(1) 16-24	(2) 25-34	(3) 35-44	(4) 45-64
female	0.008	0.007*	-0.000	0.002
	(0.007)	(0.004)	(0.003)	(0.002)
year	0.009	0.005	0.003	-0.000
	(0.007)	(0.004)	(0.002)	(0.002)
femaleyear	-0.001	-0.002	0.002	0.001
	(0.010)	(0.005)	(0.004)	(0.002)
Constant	0.152***	0.109***	0.081***	0.030***
	(0.038)	(0.017)	(0.012)	(0.007)
Observations	20,426	39,880	54,322	80,637
R-squared	0.018	0.007	0.004	0.002
Female Bound 25-34		[-0.002, 0.000]		
Male Bound 25-34		[0.000, 0.002]		

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(1) Other controls include age finished education, GOR region and dummy for whether born in UK and industry

(2) Those hired and fired within 3 months treated as not hired

(3) year = 0 if observation year is 2006, year = 1 if observation year is 2007

Figure 5.15: Placebo analysis - Hiring age 25-34 - LPM model

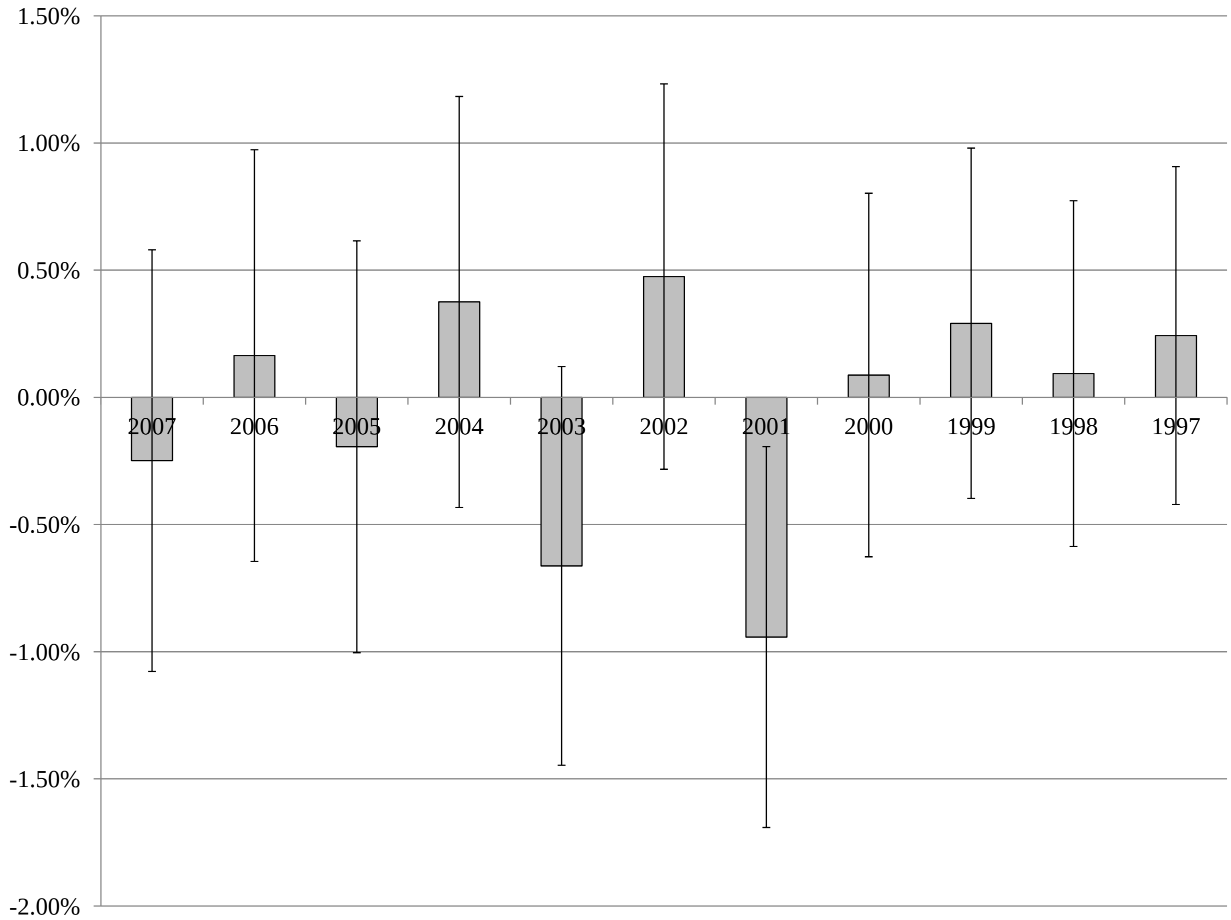
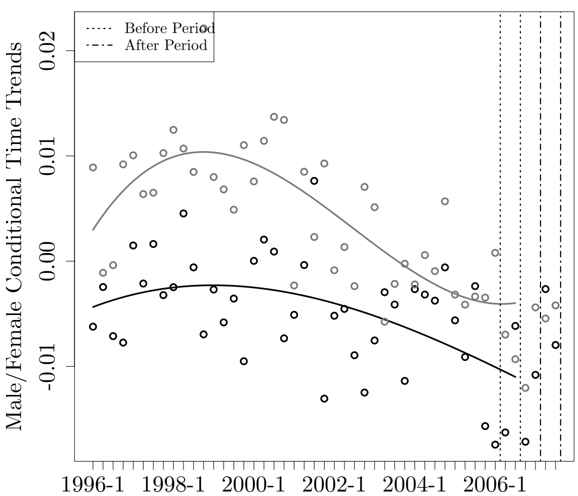
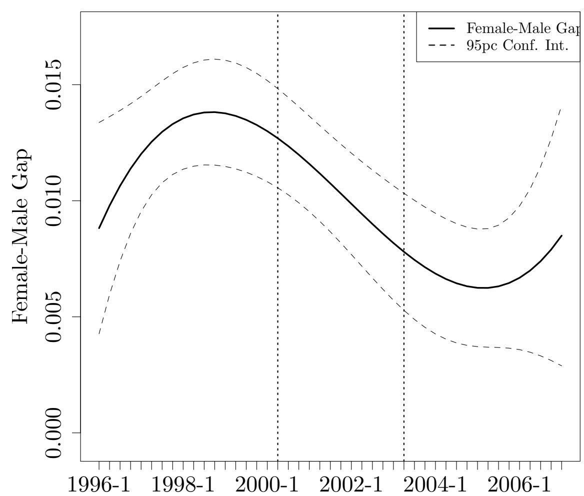


Figure 5.16: Common trend analysis - hiring - LPM model

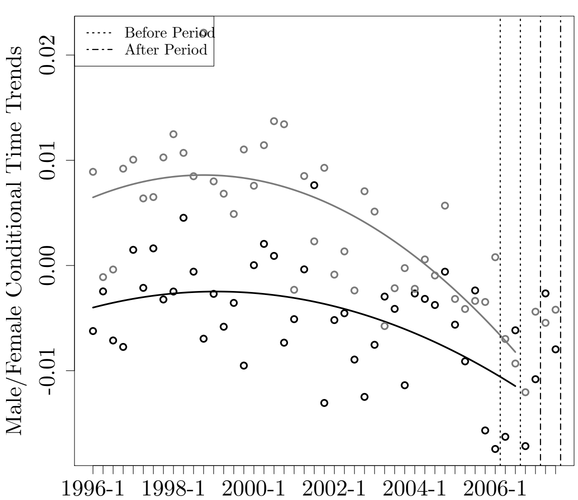
(a) Hiring residuals age 25-34 (Cubic)



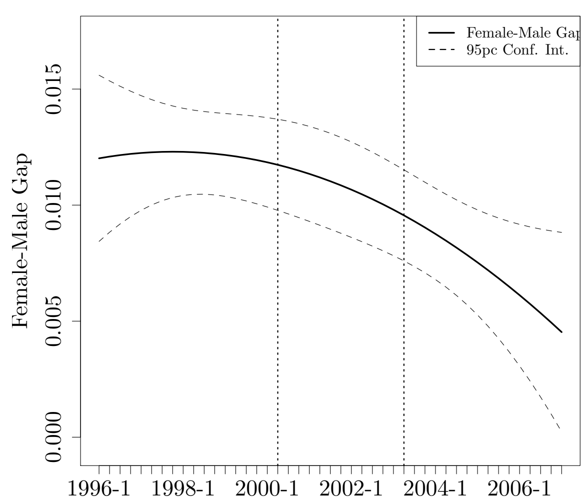
(b) Hiring gap age 25-34 (Cubic)



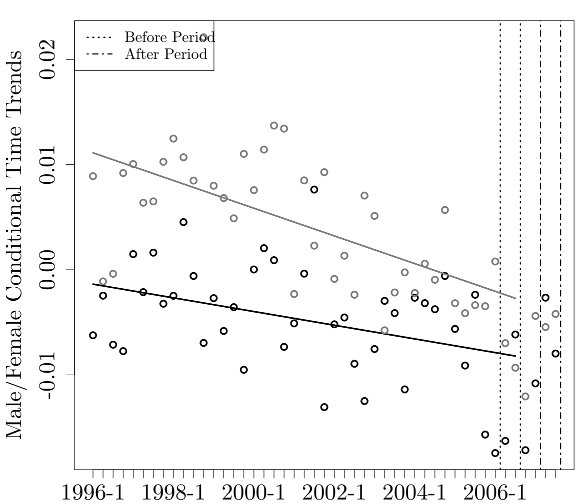
(c) Hiring residuals age 25-34 (Quadratic)



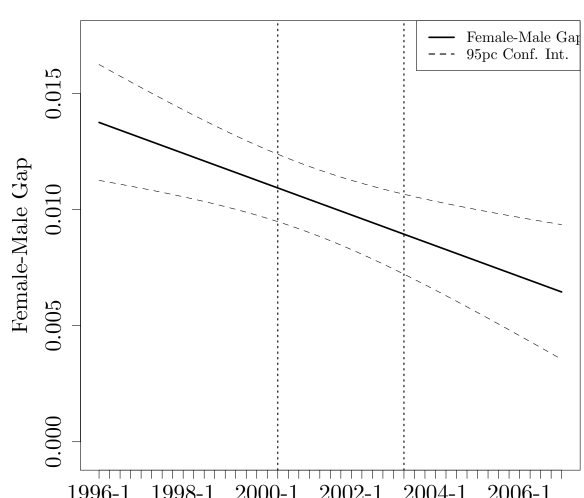
(d) Hiring gap age 25-34 (Quadratic)



(e) Hiring residuals age 25-34 (Linear)



(f) Hiring gap age 25-34 (Linear)



5.A Appendix

Appendix A

Male-Female Wage Gap

The theoretical model discussed in Section 5.3 in chapter 5 implies there will be male-female wage gap in equilibrium. Solving for equilibrium wages:

$$w_m^* = \frac{\beta}{1+\beta} \frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))} - \frac{c+t}{1+\beta}$$

$$w_f^* = \frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))} \frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))} - \frac{c+t}{1+\beta(1-\delta(\theta))}$$

Therefore the model implies that in equilibrium there will be a male-female wage gap, since:

$$\begin{aligned}
w_m^* - w_f^* =& \frac{\beta(1+\beta(1-\delta(\theta)))}{(1+\beta)(1+\beta(1-\delta(\theta)))} \frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))} - \frac{(c+t)(1+\beta(1-\delta(\theta)))}{(1+\beta)(1+\beta(1-\delta(\theta)))} \\
& - \left[\frac{\beta(1-\delta(\theta))(1+\beta)}{(1+\beta)(1+\beta(1-\delta(\theta)))} \frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))} - \frac{(c+t)(1+\beta)}{(1+\beta)(1+\beta(1-\delta(\theta)))} \right] \\
=& \frac{\beta\delta(\theta)}{(1+\beta)(1+\beta(1-\delta(\theta)))} \left(\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))} + (c+t) \right)
\end{aligned}$$

$$\Rightarrow w_m^* - w_f^* = \frac{\beta\delta(\theta)}{(1+\beta)(1+\beta(1-\delta(\theta)))} \left(\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))} + (c+t) \right)$$

which is greater than zero since $F' \geq 0$, $0 < \delta(\theta) < 1$ and $0 < \beta < 1$.

Male-Female Wage Gap and Increases in Maternity Leave Duration

Since male/female equilibrium wages and the male-female equilibrium wage gap can not be expressed as a function of θ, implicit functions are used to estimate the impact of an increase in maternity leave on the male-female wage gap. Interest is in $d(w_m^* - w_f^*)/d\theta$; the change in the male-female wage gap as maternity leave generosity increases.
The following two implicit functions are defined

$$g_m(w_m^*, w_f^*, \theta) = w_m^* - \frac{\beta}{1+\beta}\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))} + \frac{c+t}{1+\beta} = 0$$
$$g_f(w_m^*, w_f^*, \theta) = w_f^* - \frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))} + \frac{c+t}{1+\beta(1-\delta(\theta))} = 0$$

Taking total derivatives

$$\frac{\partial g_m(w_m^*, w_f^*, \theta)}{\partial w_m^*}dw_m^* + \frac{\partial g_m(w_m^*, w_f^*, \theta)}{\partial w_f^*}dw_f^* + \frac{\partial g_m(w_m^*, w_f^*, \theta)}{\partial \theta}d\theta = 0$$

$$\frac{\partial g_f(w_m^*, w_f^*, \theta)}{\partial w_m^*}dw_m^* + \frac{\partial g_f(w_m^*, w_f^*, \theta)}{\partial w_f^*}dw_f^* + \frac{\partial g_f(w_m^*, w_f^*, \theta)}{\partial \theta}d\theta = 0$$

Solving the set of simultaneous equations for $dw_m^*/d\theta$ and $dw_f^*/d\theta$ (and using the following shorthand notation: $g_m(w_m^*, w_f^*, \theta) = g_m$, $g_f(w_m^*, w_f^*, \theta) = g_f$)

$$\frac{dw_m^*}{d\theta} = -\frac{\left[\frac{\partial g_m}{\partial \theta} - \frac{\frac{\partial g_m}{\partial w_f^*}}{\frac{\partial g_f}{\partial w_f^*}}\frac{\partial g_f}{\partial \theta}\right]}{\left[\frac{\partial g_m}{\partial w_m^*} - \frac{\frac{\partial g_m}{\partial w_f^*}}{\frac{\partial g_f}{\partial w_f^*}}\frac{\partial g_f}{\partial w_m^*}\right]}$$

$$\frac{dw_f^*}{d\theta} = -\frac{\left[\frac{\partial g_m}{\partial \theta} - \frac{\frac{\partial g_m}{\partial w_m^*}}{\frac{\partial g_f}{\partial w_m^*}}\frac{\partial g_f}{\partial \theta}\right]}{\left[\frac{\partial g_m}{\partial w_f^*} - \frac{\frac{\partial g_m}{\partial w_m^*}}{\frac{\partial g_f}{\partial w_m^*}}\frac{\partial g_f}{\partial w_f^*}\right]}$$

Therefore, the male-female wage gap changes with θ as shown below

$$\frac{dw_m^*}{d\theta} - \frac{dw_f^*}{d\theta} = -\frac{\left[\frac{\partial g_m}{\partial \theta}\frac{\partial g_f}{\partial w_f^*} - \frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial \theta}\right]}{\left[\frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial w_f^*} - \frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial w_m^*}\right]} + \frac{\left[\frac{\partial g_m}{\partial \theta}\frac{\partial g_f}{\partial w_m^*} - \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial \theta}\right]}{\left[\frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial w_m^*} - \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial w_f^*}\right]}$$

Noticing the denominators are the same except for sign

$$\frac{dw_m^*}{d\theta} - \frac{dw_f^*}{d\theta} = \frac{\left[\frac{\partial g_m}{\partial \theta}\frac{\partial g_f}{\partial w_f^*} - \frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial \theta} + \frac{\partial g_m}{\partial \theta}\frac{\partial g_f}{\partial w_m^*} - \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial \theta}\right]}{\left[\frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial w_m^*} - \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial w_f^*}\right]}$$

Writing the relevant partial derivatives separately

$$\frac{\partial g_m}{\partial w_m^*} = 1 - \frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}$$

$$\frac{\partial g_m}{\partial w_f^*} = -\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}$$

$$\frac{\partial g_m}{\partial \theta} = -\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} *$$
$$\left[\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \delta(\theta)}\frac{\partial \delta(\theta)}{\partial \theta}\right]$$

$$\frac{\partial g_f}{\partial w_m^*} = -\frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}$$

$$\frac{\partial g_f}{\partial w_f} = 1 - \frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}$$

$$
\begin{aligned}
\frac{\partial g_f}{\partial \theta} =& \frac{\beta \frac{\partial \delta(\theta)}{\partial \theta}}{(1+\beta(1-\delta(\theta)))^2}\left[\frac{\partial F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}+(c+t)\right] \\
&- \frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))} * \\
&\left[\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial \theta}+\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial \delta(\theta)}\frac{\partial \delta(\theta)}{\partial \theta}\right]
\end{aligned}
$$

Therefore, the denominator of the impact of a change in θ on the male-female wage gap is

$$
\begin{aligned}
\frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial w_m^*}-\frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial w_f^*} =& \left[-\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}\right] * \\
&\left[-\frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m}\right] - \\
&\left[1-\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}\right] * \\
&\left[1-\frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}\right]
\end{aligned}
$$

Which simplifies to

$$
\begin{aligned}
\frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial w_m^*}-\frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial w_f^*} =& -1+ \\
&\left[\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}\right] + \\
&\left[\frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}\right]
\end{aligned}
$$

Which is negative, since it is the sum of three negative terms. This is since

$$
\begin{gathered}
\frac{\beta}{1+\beta} \geq 0 \\
\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))} \leq 0 \\
\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)} \geq 0 \\
\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*} \geq 0 \\
\frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))} \geq 0 \\
\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_m(w_m^*)} \geq 0 \\
\frac{\partial S_m(w_m^*)}{\partial w_m^*} \geq 0
\end{gathered}
$$

Therefore the denominator is always negative in this model. Now consider the numerator - if this is also negative it can be concluded that an increase in maternity leave length increases the male-female wage gap.

$$\frac{\partial g_m}{\partial \theta}\frac{\partial g_f}{\partial w_f^*} - \frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial \theta} + \frac{\partial g_m}{\partial \theta}\frac{\partial g_f}{\partial w_m^*} - \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial \theta} = \frac{\partial g_m}{\partial \theta}\left[\frac{\partial g_f}{\partial w_f^*} + \frac{\partial g_f}{\partial w_m^*}\right] - \frac{\partial g_f}{\partial \theta}\left[\frac{\partial g_m}{\partial w_f^*} + \frac{\partial g_m}{\partial w_m^*}\right]$$

$$\begin{aligned}
\text{num} = &\left\{ -\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} * \right.\\
&\left.\left[\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \delta(\theta)}\frac{\partial \delta(\theta)}{\partial \theta}\right]\right\} *\\
&\left[\left(1 - \frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}\right) + \right.\\
&\left.\left(-\frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}\right)\right]\\
&-\\
&\left\{\frac{\beta\frac{\partial \delta(\theta)}{\partial \theta}}{(1+\beta(1-\delta(\theta)))^2}\left[\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} + (c+t)\right]\right.\\
&-\frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} *\\
&\left.\left[\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \delta(\theta)}\frac{\partial \delta(\theta)}{\partial \theta}\right]\right\} *\\
&\left[\left(-\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}\right) + \right.\\
&\left.\left(1 - \frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}\right)\right]
\end{aligned}$$

Simplifying the above expression

$$\text{num} = -\frac{\beta\delta(\theta)}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} *$$
$$\left[\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \delta(\theta)}\frac{\partial \delta(\theta)}{\partial \theta}\right]$$

$$-$$

$$\frac{\beta\frac{\partial\delta(\theta)}{\partial\theta}}{(1+\beta(1-\delta(\theta)))^2}\left[\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} + (c+t)\right] *$$
$$\left[1 - \frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}\right.$$
$$\left. - \frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}\right]$$

The second half of this term is negative, since

$$\frac{\beta\frac{\partial\delta(\theta)}{\partial\theta}}{(1+\beta(1-\delta(\theta)))^2} \geq 0$$
$$\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} \geq 0$$
$$\frac{\beta}{1+\beta} \geq 0$$
$$\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} \leq 0$$
$$\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*)} \geq 0$$
$$\frac{\partial S_m(w_m^*)}{\partial w_m^*} \geq 0$$
$$\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)} \geq 0$$
$$\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*} \geq 0$$

The first half of this term is negative if $\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial\theta} \leq 0$, since

$$\frac{\beta\delta(\theta)}{1+\beta(1-\delta(\theta))} \geq 0$$
$$\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))} \leq 0$$

Male Wages and Increases in Maternity Leave Duration

From the previous section there is:

$$\frac{dw_m^*}{d\theta} = \frac{\left[\frac{\partial g_m}{\partial\theta}\frac{\partial g_f}{\partial w_f^*} - \frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial\theta}\right]}{\left[\frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial w_m^*} - \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial w_f^*}\right]}$$

It has already been shown that the denominator is negative.
Consider the numerator

$$\begin{aligned}
\text{num} &= \frac{\partial g_m}{\partial\theta}\frac{\partial g_f}{\partial w_f^*} - \frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial\theta} \\
&= \left\{-\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))} * \right. \\
&\left.\left[\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial\theta} + \frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial\delta(\theta)}\frac{\partial\delta(\theta)}{\partial\theta}\right]\right\} * \\
&\left\{1 - \frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}\right\} \\
&\qquad - \\
&\left\{-\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}\right\} * \\
&\left\{\frac{\beta\frac{\partial\delta(\theta)}{\partial\theta}}{(1+\beta(1-\delta(\theta)))^2}\left[\frac{\partial F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))} + (c+t)\right]\right. \\
&- \frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))} * \\
&\left.\left[\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial\theta} + \frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial\delta(\theta)}\frac{\partial\delta(\theta)}{\partial\theta}\right]\right\}
\end{aligned}$$

Simplifying the above

$$\text{num} = \frac{\partial g_m}{\partial \theta}\frac{\partial g_f}{\partial w_f^*} - \frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial \theta}$$
$$= \left\{\frac{\beta}{1+\beta}\frac{\beta\frac{\partial\delta(\theta)}{\partial\theta}}{(1+\beta(1-\delta(\theta)))^2}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial w_f^*}\right\}*$$
$$\left[\frac{\partial F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))} + (c+t)\right]$$

$$+$$

$$\left\{-\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta)))}{\partial L_2^2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}*\right.$$
$$\left.\left[\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial\theta} + \frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial\delta(\theta)}\frac{\partial\delta(\theta)}{\partial\theta}\right]\right\}$$

The first term of the above expression is negative. The second term is negative if $\frac{\partial L_2(S_m(w_m^*),S_f(w_f^*,\theta),\delta(\theta))}{\partial\theta} \leq 0$.
Therefore, if this condition holds then the numerator is surely negative, and therefore since the denominator is also negative, this model implies that an increase in maternity leave length will lead to an increase in male wages.
To provide some intuition for what the mechanism is here, the first term captures the impact of the increasing maternity leave on female productivity. Increasing maternity leave length decreases female productivity, reducing their wages. Therefore, since male and female labour are substitutes, employers substitute away from female labour towards male labour, pushing up male wages. The second term captures the impact of the change in maternity leave on period two labour, holding equilibrium wages constant. An increase in maternity length is assumed to decrease period two labour, which increases marginal productivity and hence increases male wages.

Female Wages and Increases in Maternity Leave Duration

From the previous section there is

$$\frac{dw_m^*}{d\theta} = -\frac{\left[\frac{\partial g_m}{\partial\theta}\frac{\partial g_f}{\partial w_m^*} - \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial\theta}\right]}{\left[\frac{\partial g_m}{\partial w_f^*}\frac{\partial g_f}{\partial w_m^*} - \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial w_f^*}\right]}$$

It has already been shown that the denominator is negative.

Consider the numerator

$$\begin{aligned}
\text{num} &= \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial \theta} - \frac{\partial g_m}{\partial \theta}\frac{\partial g_f}{\partial w_m^*} \\
&= \left\{1 - \frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}\right\} * \\
&\left\{\frac{\beta\frac{\partial \delta(\theta)}{\partial \theta}}{(1+\beta(1-\delta(\theta)))^2}\left[\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} + (c+t)\right]\right. \\
&- \frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} * \\
&\left.\left[\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \delta(\theta)}\frac{\partial \delta(\theta)}{\partial \theta}\right]\right\}
\end{aligned}$$

$$-$$

$$\begin{aligned}
&\left\{-\frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} * \right. \\
&\left.\left[\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \delta(\theta)}\frac{\partial \delta(\theta)}{\partial \theta}\right]\right\} * \\
&\left\{-\frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}\right\}
\end{aligned}$$

Simplifying the above

$$\begin{aligned}
\text{num} &= \frac{\partial g_m}{\partial w_m^*}\frac{\partial g_f}{\partial \theta} - \frac{\partial g_m}{\partial \theta}\frac{\partial g_f}{\partial w_m^*} \\
&= \left\{1 - \frac{\beta}{1+\beta}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_m(w_m^*)}\frac{\partial S_m(w_m^*)}{\partial w_m^*}\right\} * \\
&\left\{\frac{\beta\frac{\partial \delta(\theta)}{\partial \theta}}{(1+\beta(1-\delta(\theta)))^2}\left[\frac{\partial F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} + (c+t)\right]\right\}
\end{aligned}$$

$$-$$

$$\begin{aligned}
&\left\{\frac{\beta(1-\delta(\theta))}{1+\beta(1-\delta(\theta))}\frac{\partial^2 F(L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta)))}{\partial L_2^2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))} * \right. \\
&\left.\left[\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial S_f(w_f^*,\theta)}\frac{\partial S_f(w_f^*,\theta)}{\partial \theta} + \frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \delta(\theta)}\frac{\partial \delta(\theta)}{\partial \theta}\right]\right\}
\end{aligned}$$

The first term of the above expression is positive. The second term is negative if $\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*,\theta), \delta(\theta))}{\partial \theta} \leq 0$.
Therefore, it is not clear what the impact of an increase in maternity length will be on female wages due to these opposing effects.
To provide some intuition for what the mechanism is here, the first term captures the

impact of the increasing maternity leave on female productivity. Increasing maternity leave length decreases female productivity, reducing their wages. The second term captures the impact of the change in maternity leave on period two labour, holding equilibrium wages constant. An increase in maternity length is assumed to decrease period two labour, which increases marginal productivity and hence applies an opposing upwards pressure on female wages.

Male and Female Employment and Increases in Maternity Leave Duration

The model also allows us to analyse what happens to male and female employment in response to a increase in maternity leave length, θ. Suppose that an increase in maternity length is assumed to decrease period two labour, holding equilibrium wages constant. Therefore, increasing maternity leave length will result in increased male wages. Therefore since in equilibrium labour supply = labour demand, what will happen to male employment can be estimated from the following;
Taking the total derivative of male equilibrium labour supply

$$\begin{aligned} d(S_m(w_m^*, \theta)) &= \frac{\partial S_m(w_m^*, \theta)}{\partial \theta} d\theta + \frac{\partial S_m(w_m^*, \theta)}{\partial w_m^*} dw_m^* \\ \rightarrow \frac{d(S_m(w_m^*, \theta))}{d\theta} &= \frac{\partial S_m(w_m^*, \theta)}{\partial \theta} + \frac{\partial S_m(w_m^*, \theta)}{\partial w_m^*} \frac{dw_m^*}{d\theta} \end{aligned}$$

The second term of the above is positive, since $\frac{\partial S_m(w_m^*,\theta)}{\partial w_m^*} \geq 0$, and given the assumption decreasing period 2 labour $\frac{dw_m^*}{d\theta} \geq 0$.
The first term may be negative, given the labour substitution argument forwarded by cite Ruhm (1998). If the male labour supply response is sufficiently small, or zero then the impact of an increase in maternity leave length will be to increase male wages and male employment.
In conclusion, increases in maternity leave increase male employment, unless there are large negative male supply responses.
Similarly, for females there is

$$\begin{aligned} d(S_f(w_f^*, \theta)) &= \frac{\partial S_f(w_f^*, \theta)}{\partial \theta} d\theta + \frac{\partial S_f(w_f^*, \theta)}{\partial w_f^*} dw_f^* \\ \rightarrow \frac{d(S_f(w_f^*, \theta))}{d\theta} &= \frac{\partial S_f(w_f^*, \theta)}{\partial \theta} + \frac{\partial S_f(w_f^*, \theta)}{\partial w_f^*} \frac{dw_f^*}{d\theta} \end{aligned}$$

The first term in the above is presumably positive; holding equilibrium wages constant, increasing maternity length generosity increases female labour supply. On the other hand, the know the sign of the second component is not known. There were opposing forces acting on female wages when $\frac{\partial L_2(S_m(w_m^*), S_f(w_f^*, \theta), \delta(\theta))}{\partial \theta} \leq 0$. If female equilibrium wages are reduced as a result of increasing maternity leave duration then depending on which component has the larger absolute size the impact on equilibrium female employment may be negative or positive. On the other hand, if increasing maternity leave length actually increases female wages, then it can be concluded that the impact of increasing maternity leave length will also increase female employment.

In conclusion, if increases in maternity leave decrease female wages, then there is an ambiguous effect on female employment. If increases in maternity leave increase female wages, then there is a positive effect on female employment.

Appendix B

Legislative History

Legislated provision of maternity benefits in the UK originated with the *National Insurance Act* (1911). Some of the key legislative changes impacting upon maternity benefits from this date onwards are discussed in the following. Maternity grants, maternity allowances and statutory payment rates varied frequently over the time period in question, and will not be covered exhaustively. Focus is on key changes to qualifying conditions, or fundamental changes to the benefits allowed.

Some form of support for working mothers has existed in the United Kingdom since the introduction of the *National Insurance Act* (1911). This Act introduced maternity benefit for insured people. All workers earning under £160 a year were required to become insured and to make national insurance contributions.[52] £160 would have had the same purchasing power as £16,445 would have in 2013.[53] The weekly contribution was 4 pence (£1.71 in 2013).[54] To qualify for insurance you need to have made 26 weekly national insurance contributions since entry into insurance (so you must have been employed for at least 6 months, but not necessarily continuously). Insured women, the wife of an insured man, or the widow of an insured man having a posthumous child were eligible for a maternity grant of 30 shillings upon confinement (£154.17 in 2013).

The *National Insurance Act* (1946) introduced a maternity allowance for up to 13 weeks. The Act required all individuals living in Great Britain and who were between school leaving and pensionable age to be insured.[55] Employed, self-employed people and non-employed people made different levels of contributions. There was provisions for low income individuals, full-time students and unemployed to be exempt from contributing. Employed/self-employed contributions were counted towards eligibility for maternity benefits. Insured women, or the wives of insured men received a £4 maternity grant for each baby (£146.18 in 2013). If the women was insured and gainfully employed she also received a maternity allowance of 36 shillings a week for 13 weeks, beginning with the 6th week before the expected week of confinement (£43.85 in 2013), if she was insured herself, or the wife of an insured man, but not employed she received an attendance allowance of 20 shillings a week for 4 weeks beginning with the day of confinement (£24.36 in 2013). In order to qualify for the grant/attendance allowance, at least 26 national insurance contributions had to be paid since entry into insurance, and at least 26 contributions had to be

[52] Excluded self-employed

[53] All conversions in this section use inflation data complied by the House of Commons (Allen, 2012) combined with the all item RPI annual figures (2011-2013) from the ONS (Office for National Statistics, 2013)

[54] There was 240 pennies in a pound

[55] Northern Ireland passed a separate National Insurance Act in 1946.

paid or credited in the contribution year before the year in which confinement occurred. In order to qualify for the maternity allowance, at least 45 national insurance contributions had to be paid or credited in the 52 weeks immediately preceding the payment period, and at least 26 have to have been actually paid.

The *National Insurance Act* (1953) extended the duration of maternity allowance for insured, employed women to 18 weeks beginning with the 11th week before the expected week of confinement. In addition, in 1953 the attendance allowance and the maternity grant (for insured women or the wives of insured men) were amalgamated into one maternity grant payment. At this stage the maternity grant was increased to £9 (£219.26 in 2013) and the weekly maternity allowance was 32 shillings and 6 pence (£26.60 in 2013). In order to qualify for the maternity allowance this act now stated that at least 50 national insurance contributions have to have been paid or credited in the 52 weeks immediately preceding the 13th week before the expected week of confinement.

In the *Social Security Act* (1973) a supplementary earnings related component of the maternity allowance was introduced that covered most of the maternity leave period (the earnings related component began on the 13th day of maternity leave and could be paid for up to a maximum of 156 days) for all women who qualified for the maternity allowance.[56] The flat maternity allowance at this point was £6.75 (£71.23 in 2013) and the maternity grant was £25 (£263.82 in 2013). The 1973 Act covered Northern Ireland also. In the 1973 Social Security Act in order to qualify for maternity allowance (grant) you were required to have paid or been credited with national insurance contributions of the relevant class, and the earnings factor derived from these contributions had to be at least equal to the lower earnings limit (LEL) multiplied by a factor of 50 (25 for maternity grant) in the relevant past year (the benefit year before the benefit year of interruption).[57] In addition, in any other year they must have actually *paid* contributions of a relevant class, with an earnings factor of that year's lower earnings limit multiplied by a factor of 25. This qualifying condition was also invoked in the *Social Security Act* (1975) and the *Social Security Act* (1986).

The *Employment Protection Act* (1975) made it illegal to sack a woman because she was pregnant - unless she is incapable of doing the work she was employed to do (if this is the case then if there is a suitable alternative vacancy this has to be offered to her). In addition, if a woman was employed until immediately before the start of the 11th week

[56] The earnings related component of maternity allowance was calculated as the lower of (a) the sum of $\frac{1}{3}$ of the amount of weekly reckonable earnings that exceeds the LEL but does not exceed £30 and 15% of the earnings that do exceed £30 but do not exceed the upper earnings limit (a level above which national insurance contributions rates change), or (b) the amount by which the fixed maternity allowance falls short of 85% of weekly reckonable earnings. Reckonable weekly earnings were defined as the earnings factor for the benefit year immediately preceding the benefit year of interuption divided by 50. See www.niconsultancy.co.uk/article0402.htm for discussion on earnings factors.

[57] The lower earnings limit is the level below which no national insurance contributions are paid

before the expected confinement week, and at that point had been continuously employed by her employer for at least two years, then her employer had to pay her maternity pay for up to six weeks at a rate equal to 90% of her weekly pay less the maternity allowance payable. The maternity pay period covered the first 6 weeks of absence starting on or after the beginning of the 11th week before the expected week of confinement. This payment was reclaimable in full by the employer through a maternity pay rebate. In the same year, the *Social Security Act* (1975) removed the earnings related component of maternity allowance, meaning that the only earnings related maternity benefit was the maternity pay received from employers. In addition, if the woman met these qualifying conditions she had the right to return, at any time up until the end of the 29th week beginning with the week in which confinement occurred, to the job in which she was originally employed. Terms and conditions of employment were to be *as if the employment period before absence due to maternity leave and the employment after returning from maternity leave were continuous*. Since maternity leave could typically begin 11 weeks before the expected week of confinement, this meant that the legislation gave (expected) maternity leave of up to 40 weeks, 18 weeks of which either maternity pay or maternity allowance were payable to qualifying women. In 1975 the maternity grant was set at £25 (£71.73 in 2013) and the weekly maternity allowance was £9.80 (£182.99 in 2013). The 1975 Act covered Northern Ireland also.

In addition, in 1975 the *Sex Discrimination Act* (1975) was passed, which prohibited differential treatment for male and female workers. Case law has since ruled that since only a women can become pregnant, a refusal to employ her, or decision to dismiss her because of pregnancy is equivalent to sex discrimination. Furthermore, the *Employment Equality (Sex Discrimination) Regulations* (2005) amend the *Sex Discrimination Act* (1975) to explicitly cover differential treatment due to pregnancy as discriminatory behaviour.

In the *Social Security Act* (1986) the term “Statutory Maternity Pay” was defined, and referred to the maternity benefit paid by employers to qualifying employees. Previously this had been referred to as maternity pay. The amount of time a female needed to be employed by an employee in order to qualify for this Statutory Maternity Pay (SNP) was reduced to a period of 26 weeks running up until the 14th week before the expected week of confinement (from the previously stipulated 2 year period). An earnings qualifying rule for maternity pay was introduced; her normal weekly earnings (calculated using the 8 week period up until the 14th week before the expected week of confinement) had to be above the lower earnings limit.[58] The maternity pay period was for a period of 18 weeks. The period began on whichever was earlier: the week following the week an employee stopped work, given she gave notice to her employer that she intended to stop work, and that week is after the 12th week before the expected week of confinement, *or*, the 6th week before

[58]If she did not meet this rule she might still qualify for state maternity allowance

the expected week of confinement.[59][60][61] The first six weeks were typically paid at a higher rate of 90% of average weekly earnings, with the remaining 12 weeks paid at a lower weekly rate. In order to qualify for the higher rate you had to be continuously employed with the same employer for a period of 2 years immediately preceding the 14th week before the expected week of confinement. Women working for less than 16 hours a week did not qualify for the higher rate of SMP, unless they worked between 8-16 hours a week and had been employed by their employer for a period of at least 5 years. The lower weekly rate in 1987 was set at £32.85 (£83.99 in 2013) (see Smith (2010) for historical SMP flat rates from 1987-2006). SMP paid by employers was deductible from contribution payments. Women receiving SMP were no longer eligible for maternity allowance in the same week. This Act also abolished the Maternity Grant, but established a social fund, which paid out funds for maternity expenses in some circumstances. Social fund officers determined in which cases payment should be made, and how much to be awarded in respect of an application for payment. The *Social Security (Northern Ireland) Order* (1986) brought about the same changes in Northern Ireland.

The *Social Security Act* (1989) stipulated that employment rights or accrual of any benefits (e.g. pension benefits/gym membership/health insurance) of an employed woman earner (or who was an employed earner immediately before a paid maternity leave period) during any period of paid maternity leave (statutory or contractual) must be conferred *as if she were still employed*. The *Social Security (Northern Ireland) Order* (1989) brought about the same changes in Northern Ireland.

The *Social Security Contributions and Benefits Act* (1992) changed the qualifying conditions for the state maternity allowance from the *Social Security Act* (1973). Now the qualifying conditions stipulated that an employee had to have made national insurance contributions of a relevant class for at least 26 weeks in the 52 weeks immediately before the 14th week before the expected week of confinement (note these contributions had to be made by the claimant, her husband's contributions no longer insured her). In the case of Class 1 contributions (payable by employees), they must not have been secondary rate or reduced rate contributions.[62] The flat rate of maternity allowance was £42.25 (£76.29

[59] If she is confined before the 11th week before the expected week of confinement or between the 12th and 6th week before expected week of confinement and this time precedes the date given by her in notice to her employer, then the period begins the week after the week in which she is confined

[60] The state maternity allowance period was stipulated to be the period which if the employee had been eligible for statutory maternity pay would have been her maternity pay period

[61] The *Social Security, Statutory Maternity Pay and Statutory Sick Pay (Miscellaneous Amendments) Regulations* (2002) amended the start of the maternity pay period to be the earlier of: the week following the week an employee stopped work, given she gave notice to her employer that she intended to stop work, and that week is after the 12th week before the expected week of confinement, *or*, the week following the week of confinement. If the employee was absent from work for a partial/full day due to pregnancy on any day on or after the 4th week before the expected week of confinement, the first week of maternity pay is the week beginning the day after the absence.

[62] Married women/widows used to be able to elect to pay a reduced rate of national insurance contri-

in 2013) The *Social Security Contributions and Benefits (Northern Ireland) Act* (1992) brought about the same changes in Northern Ireland.

The *Maternity (Compulsory Leave) Regulations* (1994)[63] stated that an employee can not work from the day of childbirth for a two week period. In addition, in the same year, The *Statutory Maternity Pay (Compensation of Employers) and Miscellaneous Amendment Regulations* (1994)[64] made a distinction between small employers and other employers in terms of the amount of SMP that was reclaimable. Small employers (those whose contribution payments for the qualifying tax year do not exceed £20,000), received a payment of 104% of the amount paid out in statutory maternity pay.[65] Other employers from this date only received a payment of 92% of the amount paid out in statutory maternity pay, implying that 8% of statutory maternity pay was now covered by employers, with the exception of small employers.

The *Employment Rights Act* (1996)[66], which implemented in part the *EU Council Directive 92/85/EEC* (1992), gave all employees the right to 14 weeks of maternity leave from the date of commencement (or the birth of the child if later). An employee could choose when to begin her maternity leave from the beginning of the 11th week before the expected week of childbirth, by giving notice to her employee. If earlier, the maternity leave period automatically starts on the first day after the first whole or partial day of absence due to her pregnancy from the beginning of the sixth week before the expected week of childbirth, or, if earlier, the day on which childbirth occurs. This Act did not change SMP/maternity allowance qualifying conditions. If an employee is made redundant during the maternity leave period then the employee is entitled employment if there is an alternative suitable vacancy with her employer/successor or associated employer, under

butions which meant they were not eligible for contributory state benefits. Secondary rate contributions are contributions made on your behalf, but not paid by you

[63]and the *Maternity (Compulsory Leave) Regulations (Northern Ireland)* (1994)

[64]and the *Statutory Maternity Pay (Compensation of Employers) and Miscellaneous Amendment Regulations (Northern Ireland)* (1994)

[65]This was increased to 105% by The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations* (1995) and The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations (Northern Ireland)* (1995), to 105.5% by The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations* (1996) and The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations (Northern Ireland)* (1996), to 106.5% by The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations* (1997) and The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations (Northern Ireland)* (1996), to 107% by The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations* (1998) and The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations (Northern Ireland)* (1998), back to 105% by The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations* (1999) and *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations (Northern Ireland)* (1999), to 104.5% by The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations* (2002), and to 103% by The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations* (2011). The contribution payments cutoff threshold for a small employer was increase to £45,000 by The *Statutory Maternity Pay (Compensation of Employers) Amendment Regulations* (2004)

[66]and the *Employment Rights (Northern Ireland) Order* (1996)

conditions not substantially less favourable. Furthermore, this act stipulated the right to return after this period of maternity leave for work with terms and conditions not less favourable than *if she had not been absent on maternity leave*. For employees qualifying for the additional leave period of up to 29 weeks after birth had the right to return to her employment with terms and conditions as to remuneration not less favourable than *if she had not been absent on maternity leave*, with seniority, pension rights and similar rights *as if there had been no interruption between the periods of her employment*, and otherwise with terms and conditions not less favourable than if she had not been absent from the end of her 14 week standard maternity leave period.[67]

The *Employment Relations Act* (1999)[68] increased the maternity leave period from 14 weeks to 18 weeks for all employers regardless of tenure. The *Employment Relations Act* (1999) also formally defined Compulsory Maternity Leave (CML), Ordinary Maternity Leave (OML) and Additional Maternity Leave (AML). An employee fulfilling conditions which may be prescribed (in subsequent regulations) qualifies for OML, and has the right to return from leave to the *same job*. CML falls within an OML period. An employee fulfilling conditions which may be prescribed (in subsequent regulations) qualifies for AML, and has the right to return from leave to a job *fulfilling certain conditions*. Parental leave was also defined in this Act, and the right to return to a job fulfilling certain conditions. Some of the conditions are left vague in this Act, in order for future Acts to be able to change certain conditions without having to implement a new Act. The *Maternity and Parental Leave etc. Regulations* (1999)[69] outlined some of the necessary conditions; for instance, if requested to do so by her employer, an employee applying for OML must produce proof of expected date of childbirth, however from this point on essentially all employees qualified for OML. Furthermore, this Act states that the tenure requirements for Additional Maternity Leave is one year of continuous employment with employer at the beginning of the 11th week before the expected week of childbirth. Previously to this Act, employees who had been continuously employed for 2 years had the right to maternity leave up until the 29th week, so the Act essentially reduced the tenure requirement from 2 years of continuous employment to one. Compulsory Maternity Leave was the period of two weeks commencing on the day of childbirth. Ordinary Maternity Leave lasted for a duration of 18 weeks (or until the end of the compulsory maternity leave period if later), and can not begin earlier than the 11th week before the expected week of childbirth. Additional Maternity Leave commmenced on the day after the last day of her Ordinary Maternity Leave Period. The Additional Maternity Leave period continues up until the end of the 29th week after the

[67]Since all terms and conditions were protected during this initial 14 week leave period *as if she had not been absent on maternity leave* this essentially meant all of these other terms and conditions were protected as if she had not been absent from the beginning of her leave period.

[68]and the *Employment Relations (Northern Ireland) Order* (1999)

[69]and the *Maternity and Parental Leave etc. Regulations (Northern Ireland)* (1999)

week of childbirth. Subject to providing 21 days of notice an employee has the right to return to work before the end of her OML/AML. The *Employment Relations Act* (1999) stipulated that the right to return after ordinary maternity leave meant with all terms and conditions *as if she had not been absent*. The Maternity and Parental Leave etc. Regulations 1999 stated the right to the same terms and conditions for women taking additional maternity leave (or parental leave immediately after ordinary maternity leave, and for employees taking parental leave) as to remuneration as if the employment *interruption had not happened*. Seniority, pension rights and other rights were to be *as if the period of employment before and after the leave periods were continuous* (subject to the *Social Security Act* (1989) pension rights for women on paid maternity leave). Other terms and conditions are not to be less favourable than *had she not been absent from the commencement of her ordinary leave period/parental leave period*.[70] These regulations also stated that the changes applied for employees whose expected week of childbirth began on or after 30th April 2000. The *Employment Relations Act* (1999) also introduced time off for dependants - this includes, for instance, an allowance for an employee to take a reasonable amount of time off when a dependant (partner/child) gives birth/falls ill/is injured.

The *Maternity and Parental Leave etc. Regulations* (1999) also specified conditions surrounding parental leave. Any employee who has been employed continuously for at least a year and has parental responsibilities for a child under the age of 5 who was born on or after the 15th December 1999 had the right to 13 weeks of parental leave in respect of that child (there are some exceptions, for instance in the case of a child entitled to disability allowance the age cut off is 18).[71] The maximum amount that can be taken in any one year is four weeks. If you leave a job you can carry over any unused parental leave entitlement, but will not be able to use it until after a year of continuous employment with the new employer. If the parental leave period taken was less than 4 weeks the employee has the right to return to the same job, if the leave period is longer they have the right to return to the same job unless not practical for the employer, in which case they are entitled to return to another suitable job. Parental leave must be taken in multiples of a full week. These regulations also introduced an additional exemption for small employers - employees dismissed for being pregnant, or for any reason connected with her pregnancy or for taking maternity leave, and whose employer who had fewer than 5 employees, was not considered to have been unfairly dismissed.

[70] Since all terms and conditions were protected during the initial leave period *as if the employee had not been absent* this essentially meant all of these other terms and conditions were protected as if the employee had not been absent from the beginning of the leave period.

[71] The *Maternity and Parental Leave (Amendment) Regulations* (2001) (and The *Maternity and Parental Leave etc. (Amendment) Regulations (Northern Ireland)* (2002)) extended the qualifying birth date by five years, and allowed parents of these children to take up their parental leave allowance up until the 31st March 2005. The parental leave allowance for a disabled child was increased to 18 weeks.

The *Welfare Reform and Pensions Act* (1999) (which also covered Northern Ireland) amended the earnings qualifying rule for maternity allowance. A woman needed to have worked for 26 weeks of the 66 weeks immediately preceding the expected week of confinement ("the test period"), and her average weekly earnings within a specified period (to be defined in subsequent regulations) had to be above the maternity allowance threshold which was set at £30 (£45.36 in 2013). This threshold has not changed to date. If her average weekly earnings were above the lower earnings limit for the relevant tax year, then she received maternity allowance at the lower SMP rate (£59.55 which was £90.05 in 2013). If her average weekly earnings were lower than the lower earnings limit for the relevant tax year but above the maternity allowance threshold, she received maternity allowance at a rate equal to 90% of her average weekly earnings, or the lower SMP rate, whichever was lower. The specified period was defined in the *Social Security (Maternity Allowance) (Earnings) Regulations* (2000)[72], to be the period of 13 consecutive weeks falling within the 66 week period immediately preceding the expected week of confinement for which the woman's average earnings were highest. The *Social Security (Maternity Allowance) (Earnings) (Amendment) Regulations* (2003)[73] changed the period to be any (not necessarily consecutive) 13 weeks in the 66 week test period.

The *Maternity and Parental Leave (Amendment) Regulations* (2002)[74] extended the period of Ordinary Maternity Leave from 18 to 26 weeks. Therefore, all employees could take maternity leave of up to 26 weeks, beginning the earliest at 11 weeks before the expected week of childbirth. The Additional Leave Period was changed from covering the period up until the 29th week after childbirth to covering the period from the end of the Ordinary Leave Period and ending 26 weeks later. In addition, the tenure requirement for qualifying for AML was further reduced from 1 year of continuous employment at the beginning of the 11th week before the expected week of confinement to being 26 weeks of continuous employment at the beginning of the 14th week before the expected week of confinement. Therefore, qualifying women could take up to 52 weeks of maternity leave at this stage. To coincide with the extension of the Ordinary Maternity Leave period, the *Social Security, Statutory Maternity Pay and Statutory Sick Pay (Miscellaneous Amendments) Regulations* (2002)[75] extended the Statutory Maternity Pay period from 18 weeks to 26 weeks. The state maternity allowance period was also extended from 18 weeks to 26 weeks. These changes applied to women whose expected week of childbirth began on or after 6th April

[72] or The *Social Security (Maternity Allowance) (Earnings) Regulations (Northern Ireland)* (2000) for Northern Ireland

[73] or The *Social Security (Maternity Allowance) (Earnings) (Amendment) Regulations (Northern Ireland)* (2003) for Northern Ireland

[74] and the *Maternity and Parental Leave etc. (Amendment No. 3) Regulations (Northern Ireland)* (2002)

[75] The *Social Security, Statutory Maternity Pay and Statutory Sick Pay (Miscellaneous Amendments) Regulations (Northern Ireland)* (2002)

2003.

The *Employment Act* (2002)[76] introduced paternity leave for birth and adoption, statutory paternity pay for birth and adoption, in addition to general adoption leave and additional adoption leave for parents of children born on or after 6th April 2003, or whose expected week of birth begins on or after that date (for parents of adopted children they must have been notified of the match on or after that date, or the child must have been placed for adoption on or after that date). The *Paternity and Adoption Leave Regulations* (2002) and the *Statutory Paternity Pay and Statutory Adoption Pay (General) Regulations* (2002)[77] stipulate a number of conditions referred to in the *Employment Act* (2002) that needed to be met to qualify for the leave/statutory pay benefits. These legislative changes introduced 2 weeks of paternity leave for birth/adoption. Statutory parental leave could taken either as one week, or as two consecutive weeks. Statutory paternity pay was payable for a maximum of two weeks. Statutory paternity pay was paid at the lower of £100 or 90% of normal weekly earnings, as stipulated in the *Statutory Paternity Pay and Statutory Adoption Pay(Weekly Rates) Regulations* (2002)[78]. Employees had to have been employed for a period of at least 26 weeks continuously ending with the week immediately before the 14th week before the expected week of child birth for birth, or ending with the week in which notification of adoption match was made. The employee had to satisfy an earnings rule; his normal weekly earnings for the period of 8 weeks immediately before the qualifying week (the 14th week immediately preceding the expected week of childbirth/date of notification of adopter match) had to be above the lower earnings limit. In addition, the employee had to be the father of the child or married to or the partner of the child's mother (or in the case of adoption leave had to be either married to or the partner of the child's adopter) and has or expects to have responsibility for the upbringing of the child. Similar job protection applied as that existing for employees taking maternity/parental leave as set out in the *Employment Relations Act* (1999) and the *Maternity and Parental Leave etc. Regulations* (1999) (terms and conditions were to be as if the employee had not been absent). General adoption leave and statutory adoption pay for employees was pretty much in line with the legislation surrounding maternity leave and statutory maternity pay - although there was no tenure requirement for additional adoption leave as there was at this time for additional maternity leave. The *Employment Act* (2002) also introduced the right to request flexible working if they have children under the age of six (or 18 if disabled).[79]

[76]and the *Employment (Northern Ireland) Order* (2002)

[77]and the *Paternity and Adoption Leave Regulations (Northern Ireland)* (2002) *and* the *Statutory Paternity Pay and Statutory Adoption Pay (General) Regulations (Northern Ireland)* (2002)

[78]and the *Statutory Paternity Pay and Statutory Adoption Pay (Weekly Rates) Regulations (Northern Ireland)* (2002)

[79]This was extended to carers of some adults and parents of children under the age of 18 by the *Work*

The two legislative changes that are used as a natural experiment in the empirical analysis are the *Maternity and Parental Leave etc. and the Paternity and Adoption Leave (Amendment) Regulations* (2006)[80], and *Statutory Maternity Pay, Social Security (Maternity Allowance) and Social Security (Overlapping Benefits) (Amendment) Regulations* (2006)[81]. Both sets of regulations came into force on the 1st October 2006. The relevant changes in maternity leave/statutory maternity pay applied for women whose expected week of childbirth fell on or after 1st April 2007. The *Maternity and Parental Leave etc. and the Paternity and Adoption Leave (Amendment) Regulations* (2006) removed the tenure qualifying condition for Additional Maternity Leave, implying that essentially all employees were entitled to 26 weeks of Ordinary Maternity Leave plus 26 weeks of Additional Maternity Leave. Furthermore, these regulations removed the exemption of small employees that was introduced in The *Maternity and Parental Leave etc. Regulations* (1999), which meant that all employees had the right to return to the same or a similar job regardless of the size of her employee's firm, and any dismissal for reasons relating to her pregnancy or taking up of maternity leave would be treated as unfair dismissal. The *Statutory Maternity Pay, Social Security (Maternity Allowance) and Social Security (Overlapping Benefits) (Amendment) Regulations* (2006) also introduced "keeping in touch" days, which allowed an employee on maternity leave to work for up to 10 days during her maternity leave period without that period coming to an end. The *Statutory Maternity Pay, Social Security (Maternity Allowance) and Social Security (Overlapping Benefits) (Amendment) Regulations* (2006) extended the period of Statutory Maternity Pay to 39 weeks. The state maternity allowance period was also extended from 26 weeks to 39 weeks.

Since the 2006 legislative changes there have been a number of subsequent changes. The *Maternity and Parental Leave etc. and the Paternity and Adoption Leave (Amendment) Regulations* (2008)[82] upgraded the rights of employees taking additional maternity leave or additional adoption leave, so that all their terms and conditions were to be treated *as if they had not been absent*. The *Additional Statutory Paternity Pay (General) Regulations* (2010)[83] allowed fathers or partners of mothers or adopters to receive an additional period of statutory paternity pay as long as they satisfied the tenure condition, the earnings condition, and where the mother/adopter was eligible for maternity allowance or statutory maternity pay and had returned to work with at least two weeks of their maternity allowance period/maternity pay period unexpired. From the 20th week of birth

and Families Act (2006)

[80]and the *Maternity and Parental Leave etc. (Amendment) Regulations (Northern Ireland)* (2006)

[81]and the *Statutory Maternity Pay, Social Security (Maternity Allowance) and Social Security (Overlapping Benefits) (Amendment) Regulations (Northern Ireland)* (2006)

[82]and the *Maternity and Parental Leave etc. and the Paternity and Adoption Leave (Amendment) Regulations (Northern Ireland)* (2008)

[83]and the *Additional Statutory Paternity Pay (General) Regulations (Northern Ireland)* (2010)

a maximum of 26 weeks of additional statutory paternity pay (within the first 52 weeks of birth/adoption placement) could be transferred from the mother/adopter's entitlement of statutory maternity pay to the mother/adopter's partner. These regulations also cover early births and entitlement in the event of the death of the mother/adopter.

Other Welfare Reforms

Free early education

In September 1998 the government introduced free early education for all 4 year olds. They were entitled to five $2\frac{1}{2}$ hour sessions for 33 weeks of the year. There have been a number of expansions of this program since. In April 2004 the program was expanded to cover 3 year olds. In April 2006 the entitlement was increased to 38 weeks. In September 2010 the entitlement was expanded to 15 hours per week for 38 weeks, with more flexibility (for instance the entitlement could be taken as five 3 hour sessions for 38 weeks, three 5 hour sessions for 38 weeks, or fewer hours spread over more weeks (National Audit Office, 2012). In 2013 the entitlement was expanded to all disadvantaged 2 year olds (Department for Education, 2011).[84]

New Deal Programmes

There was a number of active labour market policies introduced into the UK in recent years (Jarvis, 1997). The New Deal for Young People was rolled out nationally in April 1998. Those aged 18-24 and who have been unemployed for 6 months were automatically enrolled into the programme, which included careers advice, help with jobsearch techniques and basic skills courses if needed. If still unemployed after 4 months, the participant could choose a number of options; they could try to get subsidised employment, they could choose to work with the Environmental Taskforce or in the voluntary sector (they received a weekly grant of approximately £15 plus benefits) or if eligible they could enrol in certain training courses (while enrolled they received their usual benefits). The New Deal for the Long-term Unemployed was rolled out in June 1998, which had two strands. First, subsidised employment of up to 6 months for people aged over 25 and who had been unemployed for over 2 years, and secondly, those unemployed for over two years could enrol in full time education without jeopardising their job-seekers benefits. The New Deal for Lone Parents was rolled out in October 1998, which was a voluntary programme that included careers advice and provided support for organising childcare.

Sure Start grants

A Sure Start Maternity Grant of £200 was introduced in 1999. This grant was payable to claimants who were recipients (or partners of recipients) of a number of income support/tax

[84] There was an initial pilot phase between 2006-2008 in 32 local authorities, then from September 2009 all local authorities were expected to fund free early education for disadvantaged 2 year olds. 23,000 places were funded from 2009 (expected to increase to 130,000 with the 2013 expansion).

credits. Claimants also received advice on maternal health and advice on the health and welfare of the new baby by a health professional. The level of the grant was increased to £300 in 2000 and to £500 in 2002. Changes were made in 2011 such that the Sure Start Maternity Grant was only payable for the first child (at least if there were no other children under 16 living in the household. Multiple births each qualified for a grant as long as there were no other children under 16 living in the household) (Kennedy, 2011).

Minimum wage

The National Minimum Wage Act 1998 introduced a national minimum wage for the first time in the UK, which came into affect on the 1st April 1999. There are different rates depending on age, and the rates have been updated every October. The minimum wage increased at a rate slightly above inflation until about 2006, then it flattened until 2009 at which point inflation was slightly above the growth in the minimum wage (Low Pay Commission, 2013).

Tax changes

There were also a number of changes to support for low income families in the UK recently. In 1999 the Working Family Tax Credit replaced the Family Credit. See Brewer and Browne (2006) for a detailed review of the different benefit structures. The main differences were the higher generosity of the WFTC, in terms of tax credits and formal childcare support. In 2001 a children's tax credit was introduced. In 2003 the Working Tax Credit and the Child Tax Credit replaced the Working Family Tax credit and the children's tax credit. See Blundell et al. (2004a) for a detailed review. The main differences were increased benefits for low income families with children.

Appendix C

Continuous Outcome Difference in Differences

The policy change occurs between time period T=0 and time period T=1. Therefore, since in time period T=0 the policy had not been changed (P=0), observed outcomes for all individuals in the first period are $Y_i^0(0)$. Similarly, in time period T=1 the policy had been changed (P=1), and so observed outcomes for all individuals in the second period are $Y_i^1(1)$. $Y_i^1(0)$ and $Y_i^0(1)$ are never observed for any individual. Therefore, there is

$$Y_i(T) = Y_i^0(0) + (Y_i^1(1) - Y_i^0(0))T$$

where $Y_i(T)$ is the observed outcome in period T.
Notice from the above it can be written:

$$\begin{aligned} Y_i(1) &= Y_i^0(0) + Y_i^1(1) - Y_i^0(0) \\ &= Y_i^0(0) \quad + \quad \underbrace{(Y_i^1(1) - Y_i^0(1))}_{\text{treatment effect in period T=1 for i}} \quad + \quad \underbrace{(Y_i^0(1) - Y_i^0(0))}_{\text{time trend for i}} \end{aligned} \tag{5.1}$$

where the second term on the RHS is the difference in potential outcomes for individual i in time period T=1 with and without the policy change (i.e. the treatment effect of the change in policy in period T=1 for individual i). The third term is the difference in potential outcomes for individual i between time period 1 and 0 if there had been no policy change (i.e. the time trend for that individual).
Also

$$Y_i(0) = Y_i^0(0) \tag{5.2}$$

Let $W_{iT} = 1$ if an individual i is observed in the data at time T, and 0 otherwise. X is a set of observable characteristics. Consider the following four conditional expectations (dropping i subscripts for convenience):

$$E[Y(T)|T = 0, F = 1, W_{T=0} = 1, X] = E[Y^0(0)|F = 1, W_0 = 1, X]$$

$$E[Y(T)|T = 0, F = 0, W_{T=0} = 1, X] = E[Y^0(0)|F = 0, W_0 = 1, X]$$

$$\begin{aligned} E[Y(T)|T = 1, F = 1, W_{T=1} = 1, X] =& E[Y^0(0)|F = 1, W_1 = 1, X] \\ &+ E[Y^1(1) - Y^0(1)|F = 1, W_1 = 1, X] \\ &+ E[Y^0(1) - Y^0(0)|F = 1, W_1 = 1, X] \end{aligned}$$

$$
\begin{aligned}
E[Y(T)|T=1,F=0,W_{T=1}=1,X] =& E[Y^0(0)|F=0,W_1=1,X] \\
&+ E[Y^1(1)-Y^0(1)|F=0,W_1=1,X] \\
&+ E[Y^0(1)-Y^0(0)|F=0,W_1=1,X]
\end{aligned}
$$

Therefore, taking the difference in the average outcomes for females before and after the policy change there is:

$$
\begin{aligned}
E[Y(T)|T=1,F=1,W_{T=1}=1,X] - E[Y(T)|T=0,F=1,W_{T=0}=1,X] = \\
E[Y^0(0)|F=1,W_1=1,X] - E[Y^0(0)|F=1,W_0=1,X]+ \\
E[Y^1(1)-Y^0(1)|F=1,W_1=1,X]+ \\
E[Y^0(1)-Y^0(0)|F=1,W_1=1,X]
\end{aligned}
$$

And taking the difference in the average outcomes for males before and after the policy change there is:

$$
\begin{aligned}
E[Y(T)|T=1,F=0,W_{T=1}=1,X] - E[Y(T)|T=0,F=0,W_{T=0}=1,X] = \\
E[Y^0(0)|F=0,W_1=1,X] - E[Y^0(0)|F=0,W_0=1,X]+ \\
E[Y^1(1)-Y^0(1)|F=0,W_1=1,X]+ \\
E[Y^0(1)-Y^0(0)|F=0,W_1=1,X]
\end{aligned}
$$

Therefore, the DiD estimator, which takes the difference of these differences gives:

$$
\begin{gathered}
E[Y(T)|T=1,F=1,W_{T=1}=1,X] - E[Y(T)|T=0,F=1,W_{T=0}=1,X]- \\
(E[Y(T)|T=1,F=0,W_{T=1}=1,X] - E[Y(T)|T=0,F=0,W_{T=0}=1,X]) \\
= E[Y^0(0)|F=1,W_1=1,X] - E[Y^0(0)|F=1,W_0=1,X]- \\
\big(E[Y^0(0)|F=0,W_1=1,X] - E[Y^0(0)|F=0,W_0=1,X]\big) + \\
E[Y^1(1)-Y^0(1)|F=1,W_1=1,X] - E[Y^1(1)-Y^0(1)|F=0,W_1=1,X]+ \\
E[Y^0(1)-Y^0(0)|F=1,W_1=1,X] - E[Y^0(1)-Y^0(0)|F=0,W_1=1,X]
\end{gathered}
$$

Assumptions:

1. Conditional Common Time Trend Assumption
   $E[Y^0(1)-Y^0(0)|F=1,W_1=1,X] - E[Y^0(1)-Y^0(0)|F=0,W_1=1,X] = 0$
   The average growth in outcomes for females who are observed in time period T=1 ($W_1=1$) would have been the same as the average growth for males who are observed

in time period T=1 if there had been no policy change conditional on some set of specified covariates X.

2. No Composition Effects conditionally
$E[Y^0(0)|F = 1, W_1 = 1, X] - E[Y^0(0)|F = 1, W_0 = 1, X] = 0$
and $E[Y^0(0)|F = 0, W_1 = 1, X] - E[Y^0(0)|F = 0, W_0 = 1, X] = 0$
Therefore, $E[Y^0(0)|F = 1, W_1 = 1, X] = E[Y^0(0)|F = 1, W_0 = 1, X] = E[Y^0(0)|F = 1, X]$,
and $E[Y^0(0)|F = 0, W_1 = 1, X] = E[Y^0(0)|F = 0, W_0 = 1, X] = E[Y^0(0)|F = 0, X]$
The average non-treated outcome in time period T=0 is the same for females observed in the survey in T=0 and T=1 conditional on X. The same applies for males.

Under these assumptions, the conditional DiD estimator estimates:

$$
\begin{aligned}
&E[Y(T)|T = 1, F = 1, W_{T=1} = 1, X] - E[Y(T)|T = 0, F = 1, W_{T=0} = 1, X] - \\
&(E[Y(T)|T = 1, F = 0, W_{T=1} = 1, X] - E[Y(T)|T = 0, F = 0, W_{T=0} = 1, X]) \\
&= E[Y^1(1) - Y^0(1)|F = 1, W_1 = 1, X] - E[Y^1(1) - Y^0(1)|F = 0, W_1 = 1, X]
\end{aligned}
$$

which gives the extent to which females were affected more than males by the policy change conditional on X.
If the set of covariates to be conditioned on are discrete, then the conditional means can be estimated by simply taking cell means. If covariates are continuous, conditional means can be estimated using kernel methods. However, in the empirical work linear index restrictions are additionally imposed to avoid the curse of dimensionality, and heterogeneity is restricted in the following ways;

Imposing index restriction on all conditional means:

$$
\begin{aligned}
E[Y(T)|F = 0, T = 0, X] =& E[Y^0(0)|F = 0, X] \\
\equiv& \alpha_{00} + \alpha_{01}X \\
E[Y(T)|F = 1, T = 0, X] =& E[Y^0(0)|F = 1, X] \\
\equiv& \alpha_{10} + \alpha_{11}X \\
E[Y(T)|F = 0, T = 1, X] =& E[Y^0(0)|F = 0, X] + E[Y^1(1) - Y^0(1)|F = 0, W_1 = 1, X] + E[Y^0(1) - Y^0(0)|F = 0, W_1 = 1, X] \\
\equiv& \alpha_{00} + \alpha_{01}X \quad + \quad \beta_{00} + \beta_{01}X \quad + \quad \delta_{00} + \delta_{01}X \\
E[Y(T)|F = 1, T = 1, X] =& E[Y^0(0)|F = 1, X] + E[Y^1(1) - Y^0(1)|F = 1, W_1 = 1, X] + E[Y^0(1) - Y^0(0)|F = 1, W_1 = 1, X] \\
\equiv& \alpha_{10} + \alpha_{11}X \quad + \quad \beta_{10} + \beta_{11}X \quad + \quad \delta_{10} + \delta_{11}X
\end{aligned}
$$

Therefore,

$$E[Y(T)|F,T,X] = a_1 + a_2F + a_3X + a_4XF + b_1T + b_2FT + b_3XT + b_4XFT + c_1T + c_2FT + c_3XT + c_4XFT$$

And,

$$Y_i(T) = a_1 + a_2F_i + a_3X_i + a_4X_iF_i + (b_1+c_1)T_i + (b_2+c_2)F_iT_i + (b_3+c_3)X_iT_i + (b_4+c_4)X_iF_iT_i + \varepsilon_{it}$$

Where $E[\varepsilon_{it}|F,T,X] = 0$

And where

$$\begin{aligned} a_1 &= \alpha_{00} & b_1 &= \beta_{00} & c_1 &= \delta_{00} \\ a_2 &= \alpha_{10} - \alpha_{00} & b_2 &= \beta_{10} - \beta_{00} & c_2 &= \delta_{10} - \delta_{00} \\ a_3 &= \alpha_{01} & b_3 &= \beta_{01} & c_3 &= \delta_{01} \\ a_4 &= \alpha_{11} - \alpha_{01} & b_4 &= \beta_{11} - \beta_{01} & c_4 &= \delta_{11} - \delta_{01} \end{aligned}$$

The conditional common time trends assumption implies:
$\delta_{00} + \delta_{01}X = \delta_{10} + \delta_{11}X$ for all X
$\rightarrow \delta_{00} = \delta_{10}$ and $\delta_{01} = \delta_{11}$
$\rightarrow c_2 = 0$ and $c_4 = 0$

Therefore the above equation simplifies to:

$$Y_i(T) = a_1 + a_2F_i + a_3X_i + a_4X_iF_i + (b_1 + c_1)T_i + b_2F_iT_i + (b_3 + c_3)X_iT_i + b_4X_iF_iT_i + \varepsilon_{it}$$

Where $(b_1 + c_1), (b_3 + c_3)$ are jointly but not separably identified but the other coefficients are identified. Note

$$\begin{aligned} b_2 + b_4X &= \beta_{10} + \beta_{11}X - (\beta_{00} + \beta_{01}X) \\ &= E[Y^1(1) - Y^0(1)|F = 1, W_1 = 1, X] - E[Y^1(1) - Y^0(1)|F = 0, W_1 = 1, X] \end{aligned}$$

which is the estimate of the impact of the policy change on females relative to males for individuals with the observable characteristics X.
With the additional assumptions that restrict heterogeneity (which are often implicitly imposed in similar empirical work):

- No heterogeneity in mean conditional treatment effects, therefore $b_3 = b_4 = 0$
- No heterogeneity in mean conditional time trends, therefore $c_3 = 0$
- Covariates enter the same way into male and female conditional mean non-treated outcomes in period T=0, therefore $a_4 = 0$

the above equation simplifies to:

$$Y_i(T) = a_1 + a_2 F_i + a_3 X_i + (b_1 + c_1) T_i + b_2 F_i T_i + \varepsilon_{it}$$

Where

$$b_2 = E[Y^1(1) - Y^0(1)|F = 1, W_1 = 1] - E[Y^1(1) - Y^0(1)|F = 0, W_1 = 1]$$

is the estimate of the impact of the policy change on females compared to males.

Chapter 6

Conclusion

This thesis applies and extends microeconometric methods to analyse economic questions of interest related to education choice and fertility choice, and their interaction with the labour market.

Chapter 2 extends the literature on non-parametric bounds on the returns from education by allowing for non-random selection into both education and labour market participation simultaneously. The key finding from this research is that allowing for both forms of non-random selection leads to very wide bounds on the returns from education, typically straddling zero unless assumptions impose otherwise. This finding highlights both the role of parametric assumptions in pinpointing the magnitude of these effects, but also the importance of rigorously validating the assumptions leading to point estimates in such a wide identified set.

Chapter 3 overviews the marginal treatment effect approach. This chapter outlines how the MTE model can be used to estimate the selection effect, and to estimate whether an advantage exists for the treated group in either potential outcome. Furthermore, this chapter discusses how these effects can be decomposed into the component due to observable characteristics and the component due to unobservable characteristics. Finally, the chapter rigorously discusses the comparison of ATE, OLS and IV estimates, and discusses what can be inferred from these comparisons.

Chapter 4 applies the MTE model to estimate heterogeneity in the returns to higher education in the UK. While no heterogeneity owing to unobservables was found, significant heterogeneity was found owing to observable characteristics, in particular with individuals with higher levels of observed ability receiving lower returns to higher education than lower ability individuals. Since graduates have higher mean levels of ability than non-graduates this led to the finding of negative selection; with individuals who do not attend higher education standing to gain more from participation than those who do attend.

Finally, chapter 5 found that extensions in the duration of paid maternity leave led to deteriorating female labour market conditions, with female employees receiving lower pay and experiencing higher levels of redundancy relative to males as a result of an expansion in the duration of paid maternity leave. Future research directions related to this analysis include analysing how these labour market effects evolve over time, and a rigorous analysis of the role played by heterogeneous selection (e.g. into labour market participation) in driving these results.

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