Exploiting Known Interference as Green Signal Power for Downlink Beamforming Optimization

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Abstract—We propose a data-aided transmit beamforming scheme for the multi-user multiple-input-single-output (MISO) downlink channel. While conventional beamforming schemes aim at the minimization of the transmit power subject to suppressing interference to guarantee quality of service (QoS) constraints, here we use the knowledge of both data and channel state information (CSI) at the transmitter to exploit, rather than suppress, constructive interference. More specifically, we design a new precoding scheme for the MISO downlink that minimizes the transmit power for generic phase shift keying (PSK) modulated signals. The proposed precoder reduces the transmit power compared to conventional schemes, by adapting the QoS constraints to accommodate constructive interference as a source of useful signal power. By exploiting the power of constructively interfering symbols, the proposed scheme achieves the required QoS at lower transmit power. We extend this concept to the signal to interference plus noise ratio (SINR) balancing problem, where higher SINR values compared to the conventional SINR balancing optimization are achieved for given transmit power budgets. In addition, we derive equivalent virtual multicast formulations for both optimizations, both of which provide insights of the optimal solution and facilitate the design of a more efficient solver. Finally, we propose a robust beamforming technique to deal with imperfect CSI, that also reduces the transmit power over conventional techniques, while guaranteeing the required QoS. Our simulation and analysis show significant power savings for small scale MISO downlink channels with the proposed data-aided optimization compared to conventional beamforming optimization.

Index Terms—Constructive interference, convex optimization, downlink beamforming, power minimization, SINR balancing.

I. INTRODUCTION

THE power efficiency of wireless transmission links has recently attracted considerable research interest, in-line with the global initiative to reduce the CO_2 emissions and operational expenses of communication systems. Accordingly, transmit beamforming schemes for power minimization

have become particularly relevant for the downlink channel. Capacity achieving non-linear dirty paper coding (DPC) techniques [1], [2] have been proposed for pre-subtracting interference prior to transmission. The DPC methods developed so far are in general complex as they require sophisticated sphere-search algorithms [3] to be employed at the transmitter and assume codewords with infinite length for the encoding of the data. Their suboptimal counterparts [4]–[6] offer a complexity reduction at a comparable performance. Still however, the associated complexity is prohibitive for their deployment in current communication standards. On the other hand, linear precoding schemes based on channel inversion [7]–[13] offer the least complexity, but their performance is far from achieving the optimum maximum likelihood bound for small scale MIMO systems, while the contrary has been shown for large scale MIMO [14]–[16]. Their non-linear adaptation, namely vector perturbation (VP) precoding [17]–[21] provides a performance improvement at the expense of an increased complexity since the search for the optimal perturbation vectors is an NP-hard problem, typically solved by complex sphere search algorithms at the transmitter.

More relevant to this work, optimization based techniques that directly minimize the transmit power subject to quality of service (QoS) constraints—most commonly to interference-plus-noise ratio (SINR)—have been studied for broadcast channels in [22], where convex optimization problems of such nature were proposed. Recent works have focused on the utility maximization in MIMO interfering broadcast channels [23], [24] and full duplex radio [25]. For the cases of channel state information (CSI) errors, robust versions of these optimizations have been studied in [26]–[35]. In [30], a robust max-min approach was developed for a single-user MIMO system based on convex optimization. Later in [36], the robust transmission schemes to maximize the compound capacity for single and multiuser rank-one Ricean MIMO channels were addressed, based on the uncertainty set in [31]. Robust beamforming for multiuser multiple-input single-output (MISO) downlink channels with individual QoS constraints under an imperfect channel covariance matrix was studied in [22], [32]. Recently in [33], the optimal power allocation over fixed beamforming vectors was obtained in the presence of errors in CSI matrices. Most recently, efficient numerical solutions to find conservative robust beamforming for multiuser MISO systems with mean-square-error (MSE) and SINR constraints and different bounded CSI errors have been developed in [34], [35]. Moreover, SINR balancing optimizations have been proposed in [37] where the minimum achievable SINR is maximized, subject to a total transmit power constraint.

This paper is based on the concept of interference exploitation, first introduced in [8], [9], [38], [39] where analytical interference classification criteria and low-complexity precoders based on channel inversion were derived. The analysis showed how the knowledge of both CSI and data at the base station can be used to predict and classify interference into constructive and destructive. The closed-form precoders in [8], [9] showed that by exploiting the constructive part of interference, higher receive SNRs can be provided without increasing the per user transmit power. However, when precoders are fully optimized, less is understood about the performance gain of the constructive interference approach over the conventional optimization, e.g., [22]. Recent work has explored the application of the concept of constructive interference [8], [9] in downlink beamforming based on symbol level optimization [40], [41]. The work has focused on allowing constructive interference in the received symbols by means of perfect CSI at the transmitter and by constraining the resulting angle in the received PSK constellation to be strictly equal to the angle of the nominal constellation symbol. Accordingly, in this work we aim to improve such optimization-based precoders by exploiting constructive interference as a source of signal power, and allowing for a more relaxed optimization in received symbols. By doing so, we further reduce the transmit power required for guaranteeing the SNR constraints in the optimization, thus improving the power efficiency of transmission. We further derive CSI-robust precoders for interference exploitation, which have not been studied before in the relevant literature. For clarity, we list the contributions of this paper as follows:

1) We introduce a new linear precoder design for PSK that a) reduces the transmit power for given performance compared to existing precoders based on the proposed constructive interference regions, and b) as opposed to conventional precoders, applies to scenarios with higher numbers of users than transmit antennas,

2) We further adapt this concept to the SINR balancing problem, where higher SINR values compared to the conventional SINR balancing optimization are achieved for given transmit power budgets,

3) We re-cast the optimization problem as a virtual multicast optimization problem based on which we derive the structure of the optimal solution and develop an efficient gradient projection algorithm to solve it,

4) We use the multicast formulation to derive a robust precoding scheme suitable for imperfect CSI with bounded CSI errors.

We note that, while the following analysis focuses on PSK modulation, the above concept and relevant optimizations can be extended to other modulation formats such as quadrature amplitude modulation (QAM) by adapting the decision thresholds of the constellation to accommodate for constructive interference [38]. It should be stressed however, that the proposed schemes are most useful in high interference scenarios where low order modulation such as BPSK and QPSK are used in the communication standards to ensure reliability [42]. In addition, constant envelope modulation such as PSK has received particular interest recently with the emergence of large scale MIMO systems [43]. All the above motivate our focus on PSK constellations.

Finally, we note that, in line with the relevant literature we assume a time division duplex (TDD) transmission here, where the base station directly estimates the downlink channel using uplink pilot symbols and uplink-downlink channel reciprocity.

The rest of the paper is organized as follows. Section II introduces the system model considered in this paper and briefly describes the two conventional optimizations of interest: *power minimization* and *SINR balancing*. In Section III the proposed beamforming optimization based on constructive interference is detailed for the case of power minimization, while its multicasting equivalent optimization is shown in Section IV. Section V presents the constructive interference optimization for the SINR balancing problem. The CSI robust versions of these optimizations are examined in Section VI for both power minimization and SINR balancing, for the case of bounded CSI errors. Finally numerical results are illustrated and discussed in Section VII and concluding remarks are given in Section VIII. In the following, we use the terms beamforming and precoding interchangeably, in-line with the most relevant literature.

II. SYSTEM MODEL AND BEAMFORMING OPTIMIZATION

Consider a K−user Gaussian broadcast channel where an N-antenna BS transmits signals to K single-antenna users. Channel vector, precoding vector, received noise, data and SINR constraints for the i–th user are denoted as $\mathbf{h}_i^\dagger$, $\mathbf{t}_i$, n_i, $d_i = d e^{j\phi_i}$ and Γ_i respectively with d denoting the constant amplitude of the PSK modulated symbols and $n_i \sim \mathcal{CN}(0, N_0), \forall i$ where $\mathcal{CN}(\mu, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean μ and variance σ^2. The received signal at user i is

$$y_i = \mathbf{h}_i^T \sum_{k=1}^K \mathbf{t}_k d_k + n_i$$
$$= \mathbf{h}_i^T \sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k - \phi_i)} d_i + n_i. \tag{1}$$

The receive SINR at the i-th receiver for this scenario is given as

$$\gamma_i = \frac{|\mathbf{h}_i^T \mathbf{t}_i|^2}{\sum_{k=1, k \neq i}^K |\mathbf{h}_i^T \mathbf{t}_k|^2 + N_0}, \forall i. \tag{2}$$

The transmit signal is

$$\sum_{k=1}^K \mathbf{t}_k d_k = \sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k - \phi_i)} d_i, \tag{3}$$

where any of the users' symbols $d_i = d e^{j\phi_i}$ can be taken as reference. Without loss of generality, let us use $d_1 = d e^{j\phi_1}$ as reference hereof. Accordingly, the instantaneous transmit power for a signal with envelope $d = 1$ is defined as

$$P_T = \left\| \sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right\|^2. \tag{4}$$
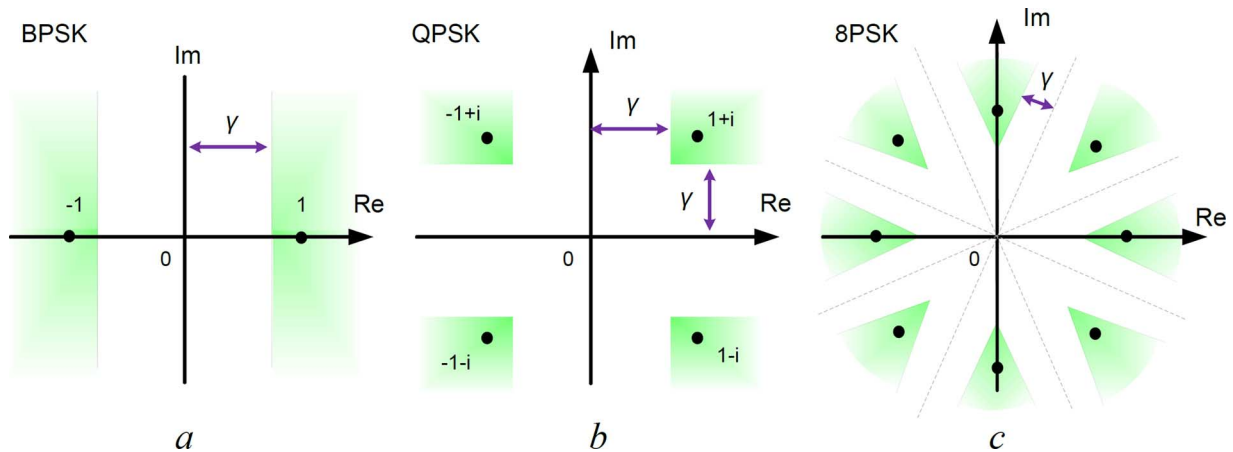
Fig. 1.   Constructive interference in a) BPSK, b) QPSK and c) 8PSK.

A. Power Minimization

The conventional power minimization precoder, treating all interference as harmful, aims to minimize the average transmit power, subject to interference constraints as shown below [22]

$$\min_{\{\mathbf{t}_i\}} \quad \sum_{i=1}^{K} \|\mathbf{t}_i\|^2$$
$$\text{s.t.} \quad \frac{|\mathbf{h}_i^T \mathbf{t}_i|^2}{\sum_{k=1, k\neq i} |\mathbf{h}_i^T \mathbf{t}_k|^2 + N_0} \geq \Gamma_i, \forall i. \tag{5}$$

While optimal from a stochastic viewpoint, the above precoder ignores the fact that, instantaneously, interference can contribute constructively to the received signal power [9], and therefore from an instantaneous point of view we later show that it is suboptimal.

B. SINR Balancing

As regards the second optimization that is of interest in this work, SINR balancing maximizes the minimum achievable SINR subject to a transmit power budget, in the form

$$\max_{\mathbf{t}_i} \quad \Gamma_t$$
$$\text{s.t.} \quad \frac{\|\mathbf{h}_i^T \mathbf{t}_i\|^2}{\sum_{k=1, k\neq i} \|\mathbf{h}_i^T \mathbf{t}_k\|^2 + N_0} \geq \Gamma_t, \forall i.$$
$$\sum_{i=1}^{K} \|\mathbf{t}_i\|^2 \leq P \tag{6}$$

where P denotes the total transmit power budget. We note that the above optimization is non-convex and the solution involves more complex iterative solutions [37].

III. PROPOSED OPTIMIZATION BASED ON CONSTRUCTIVE INTERFERENCE AND PHASE CONSTRAINTS

With the knowledge of the downlink channel and all user's data readily available at the transmitter, and with the aim of exploiting the instantaneous interference, the interference for PSK modulation can be classified to constructive and destructive based on known criteria [9], [38]. For clarity, these are summarized schematically in Fig. 1 for the basic PSK constellations. Here the scalar γ denotes the threshold distance to the decision

variables of the constellation, that relates to the SNR constraint, as detailed in the following. In brief, constructive interference is defined as the interference that moves the received symbols away from the decision thresholds of the constellation. This represents the green areas in the constellations of Fig. 1 where these are taken with reference to a minimum distance from the decision thresholds as per the SNR constraints. Note that these need not center on the nominal constellation points (the black dots in the figure) for generic SNR constraints. We refer the reader to [8], [9], [38] for further details on this topic.

As per the above classification and discussion, the optimizations in (5), (6) can be modified to take the constructive interference into account in the power minimization. This can be done by imposing interference constraints, not in terms of suppressing the stochastic interference, but rather optimizing instantaneous interference to contribute to the received signal power, thus reducing the required transmit power accordingly. For the case when interference has been aligned by means of precoding vectors $\mathbf{t}_k$ to overlap constructively with the signal of interest, all interference in (1) contributes constructively to the useful signal. Therefore all interference terms form components of the useful signal energy, which is given by the squared magnitude of the full sum in (1). Accordingly, it has been shown in [9] that the receive SNR is given as

$$\gamma_i = \frac{\left| \mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k d_k \right|^2}{N_0} \tag{7}$$

where all interference contributes in the useful received signal power. Accordingly, and based on the classification criteria detailed in [8] and Fig. 1 for M-PSK based on constructive interference, where the received interference is aligned to the symbols of interest, the problem can be reformulated as

$$\min_{\{\mathbf{t}_i\}} \quad \left\| \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right\|^2$$
$$\text{s.t.} \quad \angle \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k d_k \right) = \angle(d_i), \forall i$$
$$\text{Re} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) \geq \sqrt{\Gamma_i N_0}, \forall i, \tag{8}$$
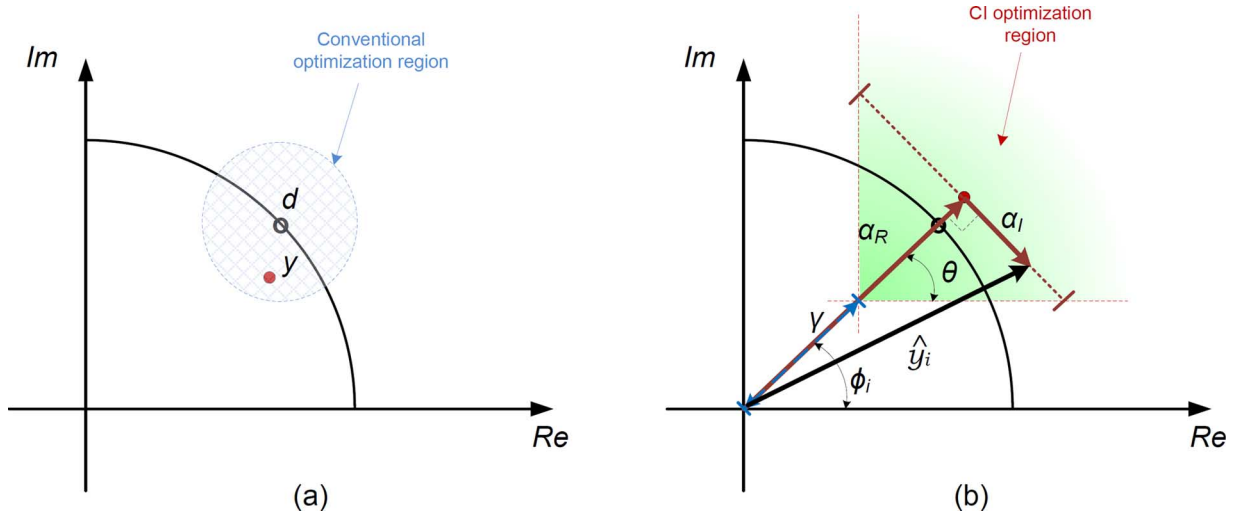
Fig. 2. Optimization regions for (a) conventional precoding and (b) precoding for interference exploitation and generic optimization constraints (QPSK example).

where $\text{Re}(x)$, $\text{Im}(x)$, $sign(x)$ and $\angle x$ denote the real and imaginary part, the sign and angle of x respectively. Here the angle of interference is strictly constrained to equal the angle of the symbol of interest. The problem can be equivalently formulated as

$$\min_{\{\mathbf{t}_i\}} \quad \left\| \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right\|^2$$

$$\text{s.t.} \quad \text{Im}\left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) = 0, \forall i$$

$$\text{Re}\left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) \geq \sqrt{\Gamma_i N_0}, \forall i. \quad (9)$$

We note the use of the sum of phase shifted (by the phase of the symbol of interest ϕ_i) interfering symbols plus the symbol of interest in the above expressions. This serves to isolate the received amplitude and phase shift in the symbol of interest due to interference. Here, the first constraint requires that the interference perfectly aligns with the phase of the symbol of interest, to ensure that it overlaps constructively to the useful symbol [9], [38]. The second constraint requires that this constructive interference is enough to satisfy the receive SNR threshold. Note that the above two conditions contain K equations and K inequalities, while there are $2N$ real variables. Therefore, for the case $N \geq K$ there are always sufficient degrees of freedom to satisfy these two sets of constraints, while our results in the following show that the proposed can be feasible with high probability even for cases with $N < K$. Still, it can be seen that due to the strict angle constraint, the formulation (9) is more constrained than the constructive interference regions in Fig. 1 where the strict angle constraints do not exist. To obtain a more relaxed optimization for M–PSK, let us observe the constellation example shown in the diagram of Fig. 2 for QPSK. Here, we have used the definitions $\alpha_R = \text{Re}\left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right)$ and $\alpha_I = \text{Im}\left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right)$ which are the real and imaginary components of $\hat{y}_i \triangleq \mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)}$ in the figure, the received symbol ignoring noise, phase shifted by the phase

of the desired symbol. We also define $\tilde{\gamma} = \sqrt{\Gamma_i N_0}$. By means of their definition, α_R and α_I essentially shift the observation of received symbol onto the axis from the origin of the constellation diagram to the constellation symbol of interest. Clearly, α_R provides a measure of the amplification of the received constellation point along the axis of the nominal constellation point due to constructive interference and α_I provides a measure of the angle shift from the original constellation point, i.e., the deviation from the axis of the nominal constellation point with phase ϕ_i.

At this point, we should emphasize the key idea in the proposed optimization which is the central strength of the proposed scheme that relaxes the optimization and allows additional reduction in the transmit power. In conventional optimizations, the signal power is optimized subject to SINR constraints. This is equivalent to constraining the interference each user experiences, so that the received symbol is within a certain distance from the nominal constellation symbol. From the view point of multiple users this essentially constrains the transmit vectors such that the received symbol y is contained within a circle (or a hyper-sphere for multidimensional optimizations) around the nominal constellation point d, so that the interference caused by the other symbols is limited. This is denoted by the dashed circle around the constellation point in Fig. 2(a). In contrast to this, here by use of the concept of constructive interference we allow a relaxation of α_R and α_I for all transmit symbols, under the condition that the interference caused is constructive, as secured by the constraints of the optimization. This gives rise to the constructive interference sector denoted by the green shaded sector in Fig. 2(b) [9], [38]. It can be seen that α_R and α_I are allowed to grow infinitely, as long as their ratio is such that the received symbol is contained within the constructive area of the constellation, i.e., the distances from the decision thresholds, as set by the SNR constraints Γ_i, are not violated. It is clear that the region in Fig. 2(b) is only constrained along the vicinity of the decision thresholds, it therefore extends infinitely to the directions away from the decision thresholds and hence provides a more relaxed optimization with respect to the conventional region of Fig. 2(a).

As regards the constructive area in the constellation, with respect to (9), it can be seen that the angle of interference need not strictly align with the angle of the useful signal, as long as it falls within the constructive area of the constellation. For a given modulation order M the maximum angle shift in the constructive interference area is given by $\theta = \pm \pi/M$. Accordingly, to relax the optimization, α_I is allowed to be non-zero as long as the resulting symbol lies within the constructive area of the constellation. Using basic geometry and the constructive interference regions derived for generic M-PSK in [13], we arrive at the optimization problem expressed as in (10), shown at the bottom of the page.

By noting that the last constraint actually incorporates the one for the real part of the received symbols, the optimization can be further reduced to the compact form of (11).

Clearly, the above optimization is equivalent to

$$
\min_{\{\mathbf{t}_i\}} \quad \left\| \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right\|^2
$$

$$
\text{s.t.} \quad \left| \angle \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k d_k \right) - \angle(d_i) \right| \leq \vartheta, \forall i
$$

$$
\operatorname{Re} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) \geq \sqrt{\Gamma_i N_0}, \forall i \quad (12)
$$

where for ϑ we have

$$
\tan \vartheta = \tan \theta \left(1 - \frac{\sqrt{\Gamma_i N_0}}{\operatorname{Re} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right)} \right). \quad (13)
$$

It can be seen that the above optimization in (11) is more relaxed than the zero-angle-shift optimization (8), which results in a smaller minimum in the transmit power. Moreover, it contains a number of K inequalities which result in an increased feasibility region compared to the conventional optimization, as will be shown in the following. Problem (11) is a standard second-order cone program (SOCP), thus can be optimally solved using numerical software, such as Semudi. However, in the following

section, we derive a more computationally efficient algorithm to solve it. For the illustrative example of BPSK used in the following results, the optimization can be modified to

$$
\min_{\{\mathbf{t}_i\}} \quad \left\| \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right\|^2
$$

$$
\text{s.t.} \quad \operatorname{Re} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k d_k \right) sign(d_i) \geq \sqrt{\Gamma_i N_0}, \forall i. \quad (14)
$$

In (14) the constraint requires that the interference on each user's symbol has the same sign as the symbol of interest (and is therefore constructive) and that this constructive interference has enough power to satisfy the SNR threshold. For the case of QPSK the same principle needs to be applied separately to the real and imaginary part of the received symbol (see Fig. 1(b)) and hence we have

$$
\min_{\{\mathbf{t}_i\}} \quad \left\| \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right\|^2
$$

$$
\text{s.t.} \quad \operatorname{Re} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k d_k \right) sign(\operatorname{Re}(d_i)) \geq \sqrt{\frac{\Gamma_i N_0}{2}}, \forall i.
$$

$$
\operatorname{Im} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k d_k \right) sign(\operatorname{Im}(d_i)) \geq \sqrt{\frac{\Gamma_i N_0}{2}} \forall i. \quad (15)
$$

IV. A VIRTUAL MULTICAST FORMULATION AND A NEW EFFICIENT ALGORITHM

A. A Virtual Multicast Formulation of (11)

The optimization in (11) can be re-cast as a virtual multicasting problem [44] according to the following theorem.

Theorem 1: The broadcast problem (11) with constructive interference is equivalent to the multicast problem below

$$
\min_{\mathbf{w}} \quad \|\mathbf{w}\|^2
$$

$$
\text{s.t.} \quad \left| \operatorname{Im} \left(\tilde{\mathbf{h}}_i^T \mathbf{w} \right) \right| \leq \left(\operatorname{Re} \left(\tilde{\mathbf{h}}_i^T \mathbf{w} \right) - \sqrt{\Gamma_i N_0} \right) \tan \theta, \forall i, \quad (16)
$$

$$
\min_{\{\mathbf{t}_i\}} \quad \left\| \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right\|^2
$$

$$
\text{s.t.} \quad \operatorname{Re} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) \geq \sqrt{\Gamma_i N_0}, \forall i
$$

$$
\left| \operatorname{Im} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) \right| \leq \left(\operatorname{Re} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) - \sqrt{\Gamma_i N_0} \right) \tan \theta, \forall i \quad (10)
$$

$$
\min_{\{\mathbf{t}_i\}} \quad \left\| \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right\|^2
$$

$$
\text{s.t.} \quad \left| \operatorname{Im} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) \right| \leq \left(\operatorname{Re} \left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) - \sqrt{\Gamma_i N_0} \right) \tan \theta, \forall i. \quad (11)
$$

where the modified channel is defined as $\tilde{\mathbf{h}}_i \triangleq \mathbf{h}_i e^{j(\phi_1 - \phi_i)}$. To be more specific, the optimal solutions to (11) and (16), $\{\mathbf{t}_i^*\}$ and $\mathbf{w}^*$ respectively, have the following relation

$$\mathbf{t}_1^* = \frac{\mathbf{w}^*}{K}, \tag{17}$$

$$\mathbf{t}_k = \mathbf{t}_1^* e^{j(\phi_1 - \phi_k)} = \frac{\mathbf{w}^* e^{j(\phi_1 - \phi_k)}}{K}, k = 2, \ldots, K. \tag{18}$$

Proof: First we re-write the constraint in (11) as (19) at the bottom of the page.

Observe that with (19), the composite precoding term $\sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k - \phi_1)}$ in (11) can be treated as a single vector $\mathbf{w}$ precoder, i.e., $\sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k - \phi_1)} = \mathbf{w}$. Therefore the multicast reformulation (16) follows immediately.

Suppose the optimal solutions of user 1's precoding vector in (11) is $\mathbf{t}_1^*$. Without loss of optimality, the other users' precoding vectors can be simple rotated versions of $\mathbf{t}_1^*$, i.e., $\mathbf{t}_i^* = \mathbf{t}_1 e^{j(\phi_1 - \phi_i)}, i = 2, \ldots, K$. As a special case, we have $\frac{\mathbf{w}^*}{K} = \mathbf{t}_1^*$. This completes the proof. ∎

Note that different from the classical multicast beamforming design, which is non-convex and difficult to solve [44], (16) is a convex problem with a quadratic objective function and $2K$ linear constraints thus easily solvable. In a similar fashion, the problem (12) will have a similar multicast formulation.

Theorem 1 provides useful insight into the structure of the precoding vectors. It tells us that the original broadcast problem now becomes a multicast problem if interference is utilized constructively. This is understandable because we take into account the correlation of the originally independent data streams, therefore the transmission can be re-formulated as sending a common data stream to all users, and re-shaping the channel and the resulting signal overlap, so that the intended data is delivered to each receiver. More importantly, the multicast problem contains only a single vector $\mathbf{w}$ and is therefore more easily solved than the original broadcast problem. The rest of this section is devoted to deriving an efficient algorithm to solve (16).

B. Real Representation of the Problem

For convenience, we separate the real and imaginary parts of each complex notation as follows

$$\tilde{\mathbf{h}}_i = \tilde{\mathbf{h}}_{Ri} + j\tilde{\mathbf{h}}_{Ii}, \mathbf{w} = \mathbf{w}_R + j\mathbf{w}_I, \tag{20}$$

where $\tilde{\mathbf{h}}_{Ri} = \mathrm{Re}(\tilde{\mathbf{h}}_i)$, $\tilde{\mathbf{h}}_{Ii} = \mathrm{Im}(\tilde{\mathbf{h}}_i)$, $\mathbf{w}_R = \mathrm{Re}(\mathbf{w})$, $\mathbf{w}_I = \mathrm{Im}(\mathbf{w})$, $j = \sqrt{-1}$.

Further define real-valued vectors

$$\mathbf{f}_i = [\tilde{\mathbf{h}}_{Ri}; \tilde{\mathbf{h}}_{Ii}], \mathbf{w}_1 \triangleq [\mathbf{w}_I; \mathbf{w}_R], \mathbf{w}_2 = [\mathbf{w}_R; -\mathbf{w}_I]. \tag{21}$$

It is easy to verify that $\mathbf{w}_1 = \Pi \mathbf{w}_2$, where $\Pi \triangleq [\mathbf{0}_N \ \mathbf{I}_N; -\mathbf{I}_N \ \mathbf{0}_N]$. $\mathbf{0}_N$ and $\mathbf{I}_N$ denote $N \times N$ all-zero matrix and identity matrix, respectively. With the new notations, we can express the real and imaginary parts in (17) as follows

$$\mathrm{Re}(\tilde{\mathbf{h}}_i^T \mathbf{w}) = \mathbf{f}_i^T \mathbf{w}_2, \mathrm{Im}(\tilde{\mathbf{h}}_i^T \mathbf{w}) = \mathbf{f}_i^T \mathbf{w}_1 = \mathbf{f}_i^T \Pi \mathbf{w}_2. \tag{22}$$

As a consequence, the constraint in (17) can be rewritten as

$$|\mathbf{f}_i^T \Pi \mathbf{w}_2| \le \mathbf{f}_i^T \mathbf{w}_2 \tan\theta - \sqrt{\Gamma_i N_0} \tan\theta, \forall i. \tag{23}$$

Define $\mathbf{g}_i \triangleq \Pi^T \mathbf{f}_i$, and we rewrite (16) as

$$\min_{\{\mathbf{w}_2\}} \quad \|\mathbf{w}_2\|^2$$
$$\text{s.t.} \quad \mathbf{g}_i^T \mathbf{w}_2 - \mathbf{f}_i^T \mathbf{w}_2 \tan\theta + \sqrt{\Gamma_i N_0} \tan\theta \le 0, \forall i, \tag{24}$$
$$-\mathbf{g}_i^T \mathbf{w}_2 - \mathbf{f}_i^T \mathbf{w}_2 \tan\theta + \sqrt{\Gamma_i N_0} \tan\theta \le 0, \forall i. \tag{25}$$

C. The Dual Problem

To further simplify the optimization let us formulate its dual problem. We let $\boldsymbol{\mu}, \boldsymbol{\nu} \ge \mathbf{0}$ be the dual vector variables associated with the two sets of constraints in (24) and (25) respectively, and consider the Lagrangian of (24) as (26) at the bottom of the next page.

The dual objective is thus given by $\min_{\mathbf{w}_2} \mathcal{L}(\mathbf{w}_2, \boldsymbol{\mu}, \boldsymbol{\nu})$. Setting $\frac{\partial \mathcal{L}(\mathbf{w}_{\in}, \boldsymbol{\mu}, \boldsymbol{\nu})}{\partial \mathbf{w}_2} = \mathbf{0}$, we obtain the structure of the optimal $\mathbf{w}_2$ below:

$$\mathbf{w}_2^* = \frac{\sum_{i=1}^K (\mu_i - \nu_i) \mathbf{g}_i^T - \sum_{i=1}^K (\mu_i + \nu_i) \mathbf{f}_i^T}{2}. \tag{27}$$

It is not surprising to see that the optimal $\bar{\mathbf{w}}_2$ is the linear combination of the channel coefficients.

Substituting (27) into $\mathcal{L}(\mathbf{w}_2, \boldsymbol{\mu}, \boldsymbol{\nu})$ leads to

$$\mathcal{L}(\boldsymbol{\mu}, \boldsymbol{\nu}) = -\frac{\|\sum_{i=1}^K (\mu_i - \nu_i) \mathbf{g}_i^T - \sum_{i=1}^K (\mu_i + \nu_i) \mathbf{f}_i^T\|^2}{4}$$
$$+ \sqrt{\Gamma_i N_0} \tan\theta \sum_{i=1}^K (\mu_i + \nu_i). \tag{28}$$

Define $\boldsymbol{\lambda} = [\boldsymbol{\mu}; \boldsymbol{\nu}]$ and rearrange the terms, and we obtain the following dual problem:

$$\max_{\boldsymbol{\lambda} \ge \mathbf{0}} -\frac{\|\mathbf{A}\boldsymbol{\lambda}\|^2}{4} + \sqrt{\Gamma_i N_0} \mathbf{1}^T \boldsymbol{\lambda}, \tag{29}$$

where $\mathbf{A} \triangleq [\mathbf{f} - \mathbf{g} \ \mathbf{f} + \mathbf{g}]$ is an $2N \times 2K$ real matrix, and the i-th column of f and g are defined in (21) and after (23).

$$\left| \mathrm{Im}\left(\mathbf{h}_i^T e^{j(\phi_1 - \phi_i)} \sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right) \right| \le \left(\mathrm{Re}\left(\mathbf{h}_i^T e^{j(\phi_1 - \phi_i)} \sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right) - \sqrt{\Gamma_i N_0} \right) \tan\theta, \forall i. \tag{19}$$

The problem (29) is a non-negative least-squares problem. It has wide applications but is known to be a challenging problem [46]. Without the non-negative constraint, its solution is given by

$$\boldsymbol{\lambda}^* = 2\sqrt{\Gamma N_0}(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{1}. \tag{30}$$

Based on the above results, we have the following corollary to provide useful insight.

Corollary 1: If $\boldsymbol{\lambda}^*$ is a positive vector, the optimal solution to the original problem (11) with constructive interference is achieved when $\text{Im}\left(\mathbf{h}_i^T\sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k-\phi_i)}\right) = 0, \forall i$. The optimal precoding solution can be obtained from (30) and (27). This is can be explained by the fact that $\boldsymbol{\lambda}$ is the dual variable and needs to satisfy the complementary slackness conditions in (24):

$$\mu_i(\mathbf{g}_i^T\mathbf{w}_2 - \mathbf{f}_i^T\mathbf{w}_2\tan\theta + \sqrt{\Gamma_i N_0}\tan\theta) = 0, \forall i, \tag{31}$$
$$\nu_i(-\mathbf{g}_i^T\mathbf{w}_2 - \mathbf{f}_i^T\mathbf{w}_2\tan\theta + \sqrt{\Gamma_i N_0}\tan\theta) = 0, \forall i. \tag{32}$$

When $\mu_i > 0, \nu_i > 0$, this implies that

$$\mathbf{g}_i^T\mathbf{w}_2 - \mathbf{f}_i^T\mathbf{w}_2\tan\theta + \sqrt{\Gamma_i N_0}\tan\theta =$$
$$-\mathbf{g}_i^T\mathbf{w}_2 - \mathbf{f}_i^T\mathbf{w}_2\tan\theta + \sqrt{\Gamma_i N_0}\tan\theta = 0, \tag{33}$$

and it follows that $\mathbf{g}_i^T\mathbf{w}_2 = 0$ or $\text{Im}\left(\mathbf{h}_i^T\sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k-\phi_i)}\right) = 0, \forall i$.

D. A Gradient Projection Algorithm to Solve (29)

The general solution to (29) is difficult to derive. Here we propose a gradient projection algorithm to solve it. The gradient projection algorithm is the natural extension of the unconstrained steepest descent algorithm to bound constrained problems [45]. To this end, we first rewrite (29) as a standard convex problem:

$$\min_{\boldsymbol{\lambda}\geq 0} \quad f(\boldsymbol{\lambda}) \triangleq \frac{\|\mathbf{A}\boldsymbol{\lambda}\|^2}{4} - \sqrt{\boldsymbol{\Gamma}^T N_0}\boldsymbol{\lambda}, \tag{34}$$

where $\boldsymbol{\Gamma} = [\Gamma_1, \ldots, \Gamma_K]^T$. It is easy to verify that the gradient of $f(\boldsymbol{\lambda})$ is given by

$$\nabla f(\boldsymbol{\lambda}) = \frac{\mathbf{A}^T\mathbf{A}\boldsymbol{\lambda}}{2} - \sqrt{\boldsymbol{\Gamma}^T N_0}\mathbf{1}. \tag{35}$$

Algorithm 1: Efficient Gradient Projection Algorithm to solve (34)

1: **Input:** $\mathbf{h}, \mathbf{d}, \Gamma, N_0$

2: **begin**

   3: Initialize arbitrarily $\boldsymbol{\lambda}^{(0)} \geq \mathbf{0}$.

   4: In the n-th iteration, update $\boldsymbol{\lambda}$:

$$\boldsymbol{\lambda}^{(n)} = \max\left(\boldsymbol{\lambda}^{(n-1)} - a_n\left(\frac{\mathbf{A}^T\mathbf{A}\boldsymbol{\lambda}^{(n-1)}}{2} - \sqrt{\boldsymbol{\Gamma}^T N_0}\mathbf{1}\right), \mathbf{0}\right), \tag{36}$$

   where the step size a_n can be chosen according to the Armijo rule or some other line search scheme.

   5: Go back to line 4 until convergence.

6: **end**

7: **Output:** $\boldsymbol{\lambda}_{\text{opt}}$.

We then propose the following Algorithm 1 to solve (29). Once the optimal dual solution $\boldsymbol{\lambda}_{\text{opt}}$ is found, the original precoding solution can be obtained from (27).

V. CONSTRUCTIVE INTERFERENCE OPTIMIZATION FOR SINR BALANCING

The above concept of constructive interference can be applied to the SINR balancing problem of (6). The problem for the case of constructive interference can be reformulated as

$$\max_{\{\mathbf{t}_i, \Gamma_t\}} \quad \Gamma_t$$

$$\text{s.t.} \quad \left|\angle\left(\mathbf{h}_i^T\sum_{k=1}^K \mathbf{t}_k d_k\right) - \angle(d_i)\right| \leq \vartheta, \forall i$$

$$\text{Re}\left(\mathbf{h}_i^T\sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k-\phi_i)}\right) \geq \sqrt{\Gamma_t N_0}, \forall i$$

$$\left\|\sum_{k=1}^K \mathbf{t}_k e^{j(\phi_k-\phi_1)}\right\|^2 \leq P \tag{37}$$

$$\mathcal{L}(\mathbf{w}_2, \boldsymbol{\mu}, \boldsymbol{\nu}) = \|\mathbf{w}_2\|^2 + \sum_{i=1}^K \mu_i(\mathbf{g}_i^T\mathbf{w}_2 - \mathbf{f}_i^T\mathbf{w}_2\tan\theta - \sqrt{\Gamma_i N_0}\tan\theta)$$

$$+ \sum_{i=1}^K \nu_i\left(-\mathbf{g}_i^T\mathbf{w}_2 - \mathbf{f}_i^T\mathbf{w}_2\tan\theta - \sqrt{\Gamma_i N_0}\tan\theta\right)$$

$$= \|\mathbf{w}_2\|^2 + \left(\sum_{i=1}^K(\mu_i - \nu_i)\mathbf{g}_i^T - \sum_{i=1}^K(\mu_i + \nu_i)\mathbf{f}_i^T\tan\theta\right)\mathbf{w}_2 + \sqrt{\Gamma_i N_0}\tan\theta\sum_{i=1}^K(\mu_i + \nu_i)$$

$$= \|\mathbf{w}_2\|^2 + \sum_{i=1}^K \mu_i\left((\mathbf{g}_i^T - \mathbf{f}_i^T\tan\theta)\mathbf{w}_2 + \sqrt{\Gamma_i N_0}\tan\theta\right) + \sum_{i=1}^K \nu_i\left(-(\mathbf{g}_i^T + \mathbf{f}_i^T\tan\theta)\mathbf{w}_2 + \sqrt{\Gamma_i N_0}\tan\theta\right). \tag{26}$$

which is not a convex problem because $\sqrt{\Gamma_t}$ is concave. However, this can be simply resolved by replacing it with a new variable, i.e., $\Gamma_{t2} = \sqrt{\Gamma_t}$. Then we obtain an equivalent problem of (38) at the bottom of the page.

It can be seen that, as opposed to the conventional SINR balancing problem (6), which is non-convex, the formulation in (38) is convex and can be solved by standard convex optimization techniques.

A. Multicast Formulation of (37)

In a similar fashion to the power minimization problem, the optimization in (37) can be re-cast as a multicasting problem according to the following theorem.

Theorem 2: Problem (38) is equivalent to the multicast problem below:

$$\max_{\{\mathbf{w},\Gamma_{t2}\}} \quad \Gamma_{t2}$$
$$\left| \mathrm{Im}\left(\tilde{\mathbf{h}}_i^T \mathbf{w} \right) \right| \leq \left(\mathrm{Re}\left(\tilde{\mathbf{h}}_i^T \mathbf{w} \right) - \Gamma_{t2}\sqrt{N_0} \right) \tan\theta, \forall i,$$
$$\|\mathbf{w}\|^2 \leq P \qquad (39)$$

where $\tilde{\mathbf{h}}_i = \mathbf{h}_i e^{j(\phi_1 - \phi_i)}$. To be more specific, if the optimal solutions to (37) and (16) are $\{\mathbf{t}_i^*\}$ and $\mathbf{w}^*$, respectively, then they have the following relation:

$$\mathbf{t}_1^* = \frac{\mathbf{w}^*}{K}, \qquad (40)$$

$$\mathbf{t}_k = \mathbf{t}_1^* e^{j(\phi_1 - \phi_k)} = \frac{\mathbf{w}^* e^{j(\phi_1 - \phi_k)}}{K}, k = 2, \ldots, K. \quad (41)$$

Proof: The proof follows the one for Theorem 1. ∎

The above is a standard SOCP problem which can be efficiently solved using known approaches.

VI. ROBUST POWER MINIMIZATION WITH BOUNDED CSI ERRORS

A. Channel Error Model and Problem Formulation

In this section, we study the robust procoding design when the CSI is imperfectly known. We model user i's actual channel as

$$\mathbf{h}_i = \hat{\mathbf{h}}_i + \mathbf{e}_i, \forall k, \qquad (42)$$

where $\hat{\mathbf{h}}_i$ denotes the CSI estimates known to the BS, and $\mathbf{e}_i$ represents the CSI uncertainty within the spherical set $\mathcal{U}_i = \{\mathbf{e}_i : \|\mathbf{e}_i\|^2 \leq \delta_i^2\}$.

We assume that the BS has no knowledge about $\mathbf{e}_i$ except for its error bound δ_i^2 thus we take a worst-case approach for the transmit precoding design to guarantee the resulting solution is robust to all possible channel uncertainties within $\mathcal{U}_i$. The specific robust design problem is to minimize the overall transmit power P_T for ensuring the users' individual SINR constraints by optimizing the precoding vector $\{\mathbf{t}_i\}$, i.e.,

$$\min_{\mathbf{w}} \quad P_T \quad \text{s.t.} \quad \mathrm{SINR}_i \geq \Gamma_i, \forall \mathbf{e}_i \in \mathcal{U}_i, \forall i. \qquad (43)$$

B. Conventional Robust Precoding

In the conventional multiuser MISO systems, the total transmit power is $P_T = \sum_{i=1}^{K} \|\mathbf{t}_i\|^2$ and robust design problem can be expressed as

$$\min_{\{\mathbf{t}_i\}} \quad \sum_{i=1}^{K} \|\mathbf{t}_i\|^2$$

$$\text{s.t.} \quad \frac{\left| \mathbf{h}_i^T \mathbf{t}_i \right|^2}{\sum_{\substack{k=1 \\ k \neq i}}^{K} \left| \mathbf{h}_i^T \mathbf{t}_k \right|^2 + N_0} \geq \Gamma_i, \forall \mathbf{e}_i \in \mathcal{U}_i, \forall k. \quad (44)$$

The robust precoding design is characterized by the following theorem [35].

Theorem 3: The robust beamforming problem (44) can be relaxed to the following semi-definite programming (SDP) problem

$$\min_{\{\mathbf{T}_i \succeq \mathbf{0}, s_i \geq 0\}} \quad \sum_{k=1}^{K} \mathrm{trace}(\mathbf{T}_i)$$

$$\text{s.t.} \begin{bmatrix} \hat{\mathbf{h}}_i^* \mathbf{Q}_i \hat{\mathbf{h}}_i^T - \gamma_i N_0 - s_i \delta_i^2 & \hat{\mathbf{h}}_i^* \mathbf{Q}_i \\ \mathbf{Q}_i \hat{\mathbf{h}}_i^T & \mathbf{Q}_i + \delta_i^2 \mathbf{I} \end{bmatrix} \succeq \mathbf{0} \quad \forall k, \quad (45)$$

where

$$\mathbf{Q}_i \triangleq \mathbf{T}_i - \Gamma_i \sum_{\substack{k=1 \\ k \neq i}}^{M} \mathbf{T}_n \quad \forall k.$$

The problem (45) is convex and hence can be optimally solved. The resulting objective value of (45) provides a lower bound for the conventional power minimization.

$$\max_{\{\mathbf{t}_i, \Gamma_{t2}\}} \quad \Gamma_{t2}$$

$$\text{s.t.} \quad \left| \mathrm{Im}\left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) \right| \leq \left(\mathrm{Re}\left(\mathbf{h}_i^T \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_i)} \right) - \Gamma_{t2}\sqrt{N_0} \right) \tan\theta, \forall i$$

$$\left\| \sum_{k=1}^{K} \mathbf{t}_k e^{j(\phi_k - \phi_1)} \right\|^2 \leq P. \qquad (38)$$

Remark: When the SDP relaxation is tight, or (45) returns all rank-1 solution $\{\mathbf{T}_i\}$, then the optimal solution $\{\mathbf{t}_i\}$ to solve (44) can be obtained by matrix decomposition such that $\mathbf{T}_i = \mathbf{t}_i \mathbf{t}_i^{\dagger}, \forall i$; otherwise, the required power in (44) is always higher than that in (45).

C. Robust Precoding Based on Constructive Interference

Based on the multicast formulation of the power minimization problem (16), the worst-case robust design for the case of constructive interference becomes

$$
\begin{aligned}
\min_{\mathbf{w}} \quad & \|\mathbf{w}\|^2 \\
\text{s.t.} \quad & \left| \text{Im}\left(\tilde{\mathbf{h}}_i^T \mathbf{w} \right) \right| - \left(\text{Re}\left(\tilde{\mathbf{h}}_i^T \mathbf{w} \right) - \sqrt{\Gamma_i N_0} \right) \tan\theta \leq 0, \\
& \forall \|\mathbf{e}_i\|^2 \leq \delta_i^2, \forall i.
\end{aligned}
\tag{46}
$$

The constraint in (46) is intractable due to the infinite number of error vectors. Below we show how to tackle it using convex optimization techniques. For ease of composition, we omit the user index.

First notice that robust constraint (46) can be guaranteed by the modified constraint below:

$$
\max_{\|\mathbf{e}\|^2 \leq \delta^2} \left(\left| \text{Im}\left(\tilde{\mathbf{h}}^T \mathbf{w} \right) \right| - \left(\text{Re}\left(\tilde{\mathbf{h}}^T \mathbf{w} \right) - \sqrt{\Gamma N_0} \right) \tan\theta \right) \leq 0.
\tag{47}
$$

We again separate the real and imaginary parts of each complex notation as follows

$$
\begin{aligned}
\tilde{\mathbf{h}} &= \tilde{\mathbf{h}}_R + j\tilde{\mathbf{h}}_I \\
&= \hat{\mathbf{h}}_R + j\hat{\mathbf{h}}_I + \mathbf{e}_R + j\mathbf{e}_I.
\end{aligned}
\tag{48}
$$

Further define real-valued error vector and channel estimation vector

$$
\bar{\mathbf{e}} \triangleq [\mathbf{e}_R; \mathbf{e}_T], \hat{\mathbf{f}} = [\hat{\mathbf{h}}_R; \hat{\mathbf{h}}_I].
\tag{49}
$$

Apparently $\|\bar{\mathbf{e}}\|^2 \leq \delta^2$. With the new notations, we can re-express the real and imaginary parts in (47) as follows

$$
\begin{aligned}
\text{Im}(\tilde{\mathbf{h}}^T \mathbf{w}) &= \hat{\mathbf{h}}_R^T \mathbf{w}_I + \hat{\mathbf{h}}_I^T \mathbf{w}_R + \mathbf{e}_R^T \mathbf{w}_I + \mathbf{e}_I^T \mathbf{w}_R, \\
&= \hat{\mathbf{f}}^T \mathbf{w}_1 + \bar{\mathbf{e}}^T \mathbf{w}_1;
\end{aligned}
\tag{50}
$$
$$
\begin{aligned}
\text{Re}(\tilde{\mathbf{h}}^T \mathbf{w}) &= \hat{\mathbf{h}}_R^T \mathbf{w}_R - \hat{\mathbf{h}}_I^T \mathbf{w}_I + \mathbf{e}_R^T \mathbf{w}_R - \mathbf{e}_I^T \mathbf{w}_I, \\
&= \hat{\mathbf{f}}^T \mathbf{w}_2 + \bar{\mathbf{e}}^T \mathbf{w}_2.
\end{aligned}
\tag{51}
$$

As a consequence, (47) can be rewritten as

$$
\max_{\|\bar{\mathbf{e}}\|^2 \leq \delta^2} |\hat{\mathbf{h}}^T \mathbf{w}_1 + \bar{\mathbf{e}}^T \mathbf{w}_1| - (\hat{\mathbf{h}}^T \mathbf{w}_2 + \bar{\mathbf{e}}^T \mathbf{w}_2) \tan\theta \\
+ \sqrt{\Gamma N_0} \tan\theta \leq 0.
\tag{52}
$$

The above constraint is equivalent to the following two constraints:

$$
\max_{\|\bar{\mathbf{e}}\|^2 \leq \delta^2} \hat{\mathbf{h}}^T \mathbf{w}_1 + \bar{\mathbf{e}}^T \mathbf{w}_1 - (\hat{\mathbf{h}}^T \mathbf{w}_2 + \bar{\mathbf{e}}^T \mathbf{w}_2) \tan\theta \\
+ \sqrt{\Gamma N_0} \tan\theta \leq 0,
\tag{53}
$$
$$
\max_{\|\bar{\mathbf{e}}\|^2 \leq \delta^2} -\hat{\mathbf{h}}^T \mathbf{w}_1 - \bar{\mathbf{e}}^T \mathbf{w}_1 - (\hat{\mathbf{h}}^T \mathbf{w}_2 + \bar{\mathbf{e}}^T \mathbf{w}_2) \tan\theta \\
+ \sqrt{\Gamma N_0} \tan\theta \leq 0,
\tag{54}
$$

whose robust formulations are given by

$$
\hat{\mathbf{h}}^T \mathbf{w}_1 - \hat{\mathbf{h}}^T \mathbf{w}_2 \tan\theta + \delta\|\mathbf{w}_1 - \mathbf{w}_2 \tan\theta\| \\
+ \sqrt{\Gamma N_0} \tan\theta \leq 0,
\tag{55}
$$
$$
-\hat{\mathbf{h}}^T \mathbf{w}_1 - \hat{\mathbf{h}}^T \mathbf{w}_2 \tan\theta + \delta\|\mathbf{w}_1 + \mathbf{w}_2 \tan\theta\| \\
+ \sqrt{\Gamma N_0} \tan\theta \leq 0.
\tag{56}
$$

Finally we reach the following robust problem formulation

$$
\begin{aligned}
\min_{\mathbf{w}_1, \mathbf{w}_2} \quad & \|\mathbf{w}_1\|^2 \\
\text{s.t.} \quad & \text{Constraints (55) and (56)}, \forall i, \\
& \mathbf{w}_1 = \Pi \mathbf{w}_2.
\end{aligned}
\tag{57}
$$

Note that (57) is standard SOCP problem therefore can be efficiently solved. After we obtain the optimal $\mathbf{w}_1^*, \mathbf{w}_2^*$, the robust solution $\mathbf{w}^*$ can be determined using the relation in (21).

D. Robust SINR Balancing

For completeness, we also study the robust SINR balancing given total BS power constraint. Similar procedures can be taken as the above to derive it, therefore we give the problem formulation directly below

$$
\begin{aligned}
\min_{\mathbf{w}_1, \mathbf{w}_2, \Gamma_{t2}} \quad & \Gamma_{t2} \\
\text{s.t.} \quad & \text{Constraints (55) and (56)}, \forall i, \\
& \mathbf{w}_1 = \Pi \mathbf{w}_2, \\
& \|\mathbf{w}_1\|^2 \leq P_T.
\end{aligned}
\tag{58}
$$

The above is a typical SOCP problem that can be solved using standard approaches. Suppose the optimal Γ_{t2} is Γ_{t2}^*, then the maximum SINR value becomes $\Gamma_{t2}^{*}{}^2$.

VII. NUMERICAL RESULTS

This section presents numerical results based on Monte Carlo simulations of the proposed optimization techniques, termed as CI in the following, and conventional precoding based on optimization for the frequency flat Rayleigh fading statistically uncorrelated multiuser MISO channel for both cases of perfect and erroneous CSI. Systems with BPSK, QPSK and 8PSK modulation are considered while it is clear that the benefits of the proposed technique extend to higher order modulation. To focus on the proposed concept, we compare the proposed CI optimization solely to the conventional optimization of [22], [37], while it is clear the benefits of the proposed concept extend to modified optimization designs in the literature, by direct application of the constructive interference concept and the adaptation of the relevant QoS constraints. We use '$N \times K$' to denote a mutiuser MISO system with N transmit antennas and K single-antenna terminals. Unless otherwise specified, QPSK is the default modulation scheme.

First, in Fig. 3 we compare the average transmit power with the conventional optimization and the proposed optimization problems (14) and (15) in the 5×4 scenario for BPSK and QPSK, respectively. Power savings of up to 50% can be seen. It can also be observed that the relaxed optimization (11) results in significant power gains compared to the strict angle constraints in (9). The same trend can be observed for the 4×4 systems
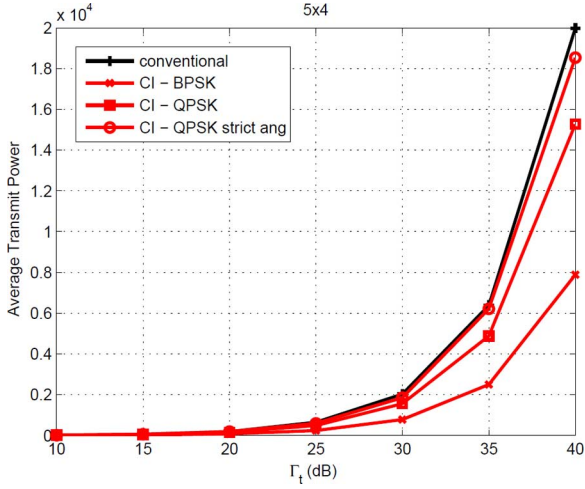
Fig. 3. Transmit power vs. Γ_t for conventional and CI, $K = 4$, $N = 5$.
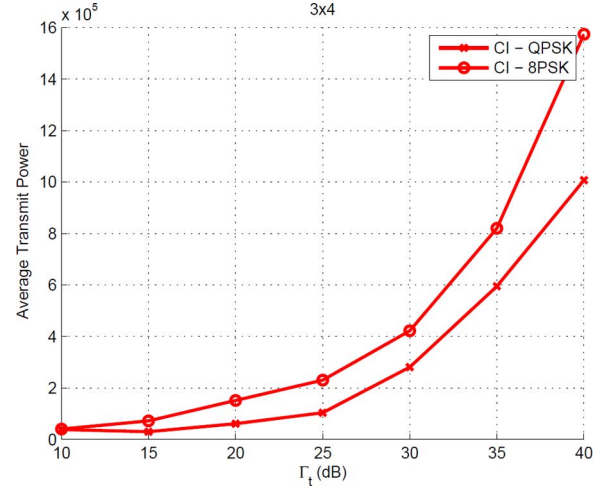
Fig. 5. Transmit power vs. Γ_t for conventional and CI, $K = 4$, $N = 3$.
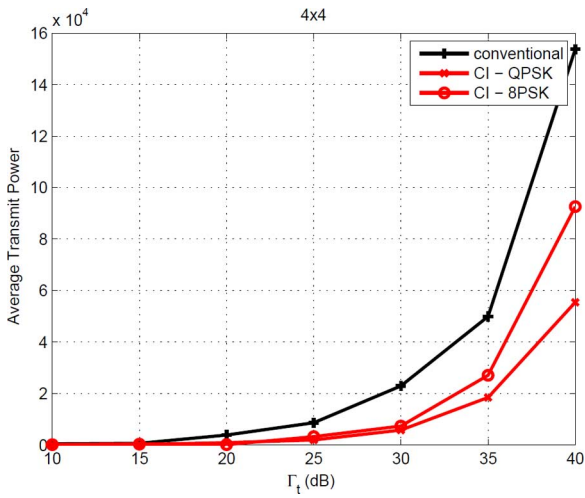
Fig. 4. Transmit power vs. Γ_t for conventional and CI, $K = 4$, $N = 4$.
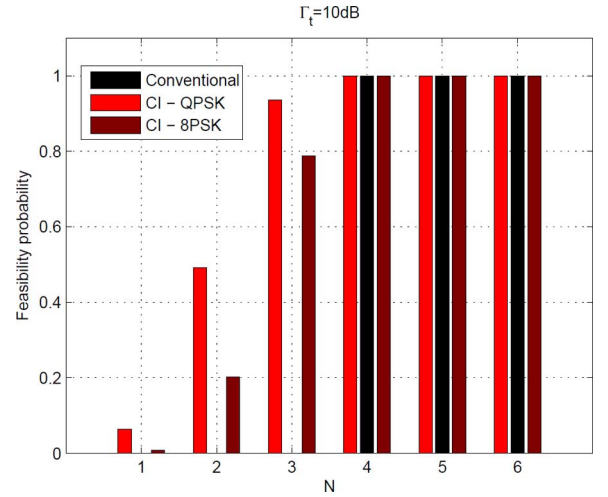
Fig. 6. Feasibility probability vs. N for conventional and CI, $K = 4$, $\Gamma_t = 10$ dB.

case of Fig. 4 where the conventional optimization results in a solution with increased transmit power, because less transmit antennas are available at the BS. The gains in this case are amplified for the proposed relaxed optimization. The transmit power is also shown in Fig. 5 for the 3×4 scenario where the conventional precoder is inapplicable, as the optimization in (5) is infeasible for $N < K$. Remarkably, the proposed optimization problem is feasible in the 92.6% of the cases. The relaxed nature of the problem indeed leads to larger feasibility regions. To illustrate the extended feasibility region for the proposed optimization problems, Fig. 6 shows the feasibility probability of a $K = 4$ user system with respect to the number of transmit antennas. It can be seen that while the conventional optimization is only feasible for $N \geq K$, the proposed can be feasible for lower values of N.

The complexity of the proposed power minimization problem is addressed in Fig. 7 for the system with $N = 5$. While analytical complexity expressions are hard to derive for optimization-based precoding, here the complexity of the solvers with the broadcast (BC) formulation of (11), multicast (MC) formulation of (16) and gradient projection solver in Algorithm

1 is shown in terms of the average execution time of the optimization algorithm, with increasing numbers of users. While the BC and MC show a similar computational complexity, the gradient projection solver in Algorithm 1 offers a significant reduction in the involved complexity down to less than 15%, which further motivates the multicast formulation of the optimization problem. Still, we note that the proposed optimization needs to be performed on a symbol-by-symbol basis. Accordingly, the proposed may involve excess complexity compared to conventional precoding optimization especially for slow fading scenarios, where the convectional channel-dependent precoding may not require the optimization to be performed frequently. Accordingly, for the slow fading scenario where the channel is constant over a transmission frame and for the example of an LTE frame with 14 symbol time slots, the proposed precoding with the gradient projection solver translates to a doubling of complexity per frame ($14 \times 15\% = 210\%$) for the proposed scheme w.r.t. conventional precoding optimization. However, for the obtained transmit-power benefits as shown in our results, the end complexity increase is definitely worthwhile the performance benefits offered. In fact, in terms of the ultimate
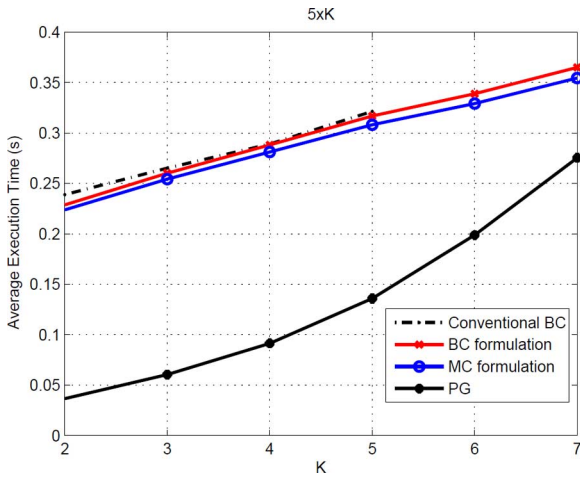
Fig. 7. Average execution time vs. K for CI optimization with the BC formulation of (11), MC formulation of (16) and gradient projection solver in Algorithm 1, $N = 5$, QPSK.
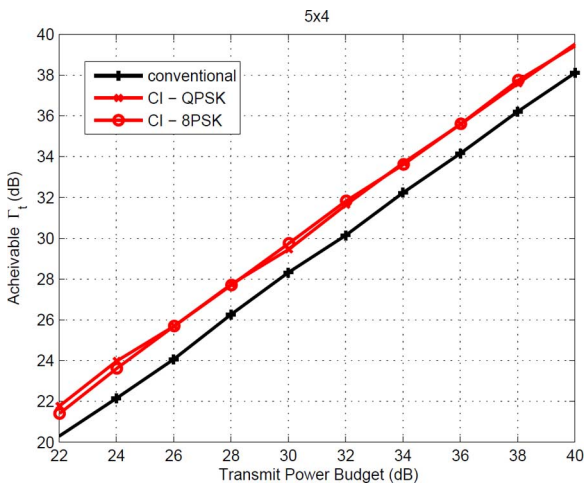

Fig. 9. Achievable Γ_t vs. Transmit power budget for conventional and CI, $K = 4$, $N = 4$.

Fig. 8. Achievable Γ_t vs. Transmit power budget for conventional and CI, $K = 4$, $N = 5$.

Fig. 10. Transmit power vs. error bound for conventional and CI, $K = 4$, $N = 4$, $\Gamma = 20$ dB.
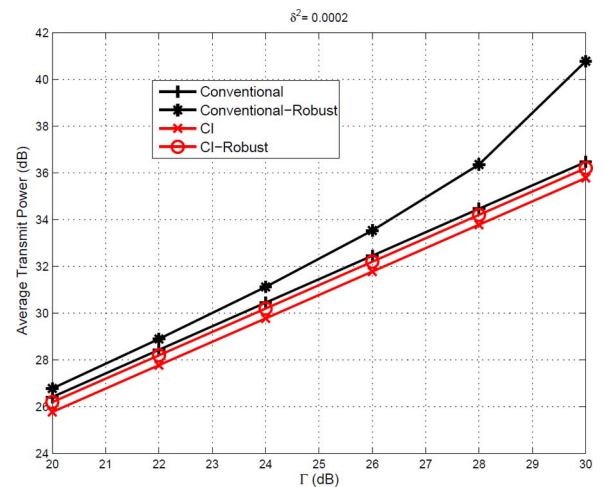
Fig. 11. Transmit power vs. SINR for conventional and CI, $K = 4$, $N = 4$, $\delta^2 = 10^{-4}$.

metric of power efficiency of the transmitter, we note that for an LTE base station the transmit power is measured in the order of around 20Watts, while the power consumption of the DSP processing is typically of the order of hundreds of milliWatts. Since with the proposed precoding our results show a halving of the transmit power at roughly double the DSP power, the gains in the power efficiency for the proposed scheme w.r.t. conventional precoding are therefore significant.

Figs. 8 and 9 compare the achievable SNR for the SINR balancing problems (6) and (37). In Fig. 8 the achievable Γ_t is shown for the 5×4 MISO where an SNR gain of about 2dB can be observed. The same trend is observed in Fig. 9 for the 5×4 MISO with an SNR gain of about 3dB. It is worth noting that for the case of constructive interference these SNR gains are due to the effect of constructive interference.

Finally Figs. 10 and 11 compare the performance of the proposed CSI-robust CI precoder with the conventional CSI-robust precoder of [35] for the 4×4 MISO. Fig. 10 shows the obtained transmit power with increasing CSI error bounds δ where it can be seen that for values in the region of $\delta^2 = 10^{-3}$ the
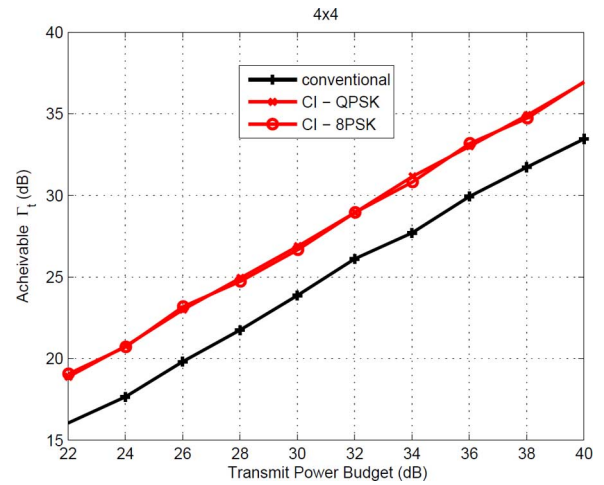
transmit power increases significantly. On the contrary, the proposed optimization shows a modest increase in transmit power

for high values of δ thanks to the relaxed optimization obtained by exploiting constructive interference. This is also shown in Fig. 11 where the average transmit power is shown with increasing SNR thresholds, for $\delta^2 = 10^{-4}$. Again the proposed shows a constant loss of less than 1dB compared to the perfect CSI case, while conventional precoding shows an increased sensitivity to the CSI errors.

VIII. CONCLUSION

A number of improved optimization-based precoding designs have been proposed for the multi-user MISO downlink channel. By taking advantage of the constructive interference, the proposed schemes deliver reduced transmit power for given QoS constraints, or equivalently increased minimum SNRs for a given transmit power budget. Both optimizations were adapted to virtual multicast formulations which were shown to be more efficiently solvable. The proposed concept was further extended to robust designs for imperfect CSI with bounded CSI errors. In all cases the proposed schemes provide superior performance over the conventional precoding schemes, which confirms the benefit of constructive interference when the precoders are fully optimized.

The concept of constructive interference opens up new opportunities for future work in the optimization of precoding designs. Firstly, the extension to QAM constellation is non-trivial, and it needs a careful redesign of constructive interference sectors for determining the relevant optimization constraints. Secondly, the robust design in Section VI takes a conservative approach while the optimal robust precoder design is an important but challenging problem to be studied. Thirdly, as this work considers a single-cell system, and as multi-cell cooperation is made practical in systems such as cloud-RAN [47], it is worth investigating the potential of the proposed scheme in multi-cell environments. Another promising application scenario is multi-beam satellite systems where both CSI and data for different beams are available at the gateway station where forward link beamforming is designed [48].

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