Three Essays on Financial Development, Inequality, and Growth

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Contents

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Abstract

This thesis presents three papers that to contribute to our theoretical knowledge on economic growth and secular stagnation.

Chapter 2 (which constituted my Job Market paper) presents a theory in which economic development manifests itself primarily as a process of sectoral differentiation. As the variety of sectors expands, the allocation of heterogeneously talented individuals improves. The paper shows that, in addition to increasing the average productivity of the matches between agents and sectors, this process also mitigates informational frictions affecting the functioning of financial markets. The positive impact of sectoral variety on the efficiency of financial markets gives rise to a novel feedback between financial development and horizontal innovations, which may yield different types of dynamics. A successful economy typically exhibits a continuous increase in the variety of productive activities, which in turn leads to lower frictions in the financial markets. However, a poverty-trap may also arise. This situation is characterised by a rudimentary productive structure with poor matching of skills to activities, and where the operation of financial markets is severely affected by the talent mismatching.

Chapter 3 proposes a theory of insurance market imperfections along the path of development based on the endogenous emergence of informational asymmetries during development. The source of the inefficiency in the insurance market is private information regarding entrepreneurial skills. Development is driven by the action of the entrepreneurs, and materialises when the agents best suited for undertaking entrepreneurial activities fully exercise their skills. Yet, due to private information, an adverse selection problem endogenously arises when the prospective entrepreneurs intend to diversify away their idiosyncratic risks. The adverse selection problem prevents the provision of first-best insurance contracts against entrepreneurial risks, which may discourage entrepreneurial investment and halt the process of development.

Chapter 4 (written in collaboration with Vincenzo Merella, from Birkbeck College) turns the attention towards a world economy. The past literature on trade has explored conditions under which international trade might be a factor magnifying income disparities between the advanced North and the backward South. No attention has yet been placed on the effect of trade on countries that do not display substantial dissimilarities concerning capital endowments and income per head. The paper shows that even when no single country is technologically more advanced than any other one and productivity changes are uniform and identical in all countries, international trade may still be the source of income divergence. Divergence will be experienced when comparative advantages induce patterns of specialisation that, although optimal for each country at some initial point in time, do not offer the same scope for improvements in terms of subsequent quality upgrading of final products.

Chapter 1

Introduction

The standard Neoclassical growth model - i.e. Solow (1956) and Cass (1965) - predicts that poor economies should catch up with rich economies. As a result, income disparities across countries should tend to vanish over time. However, except for a handful of successful experiences in East Asia, in the past fifty years income disparities across countries have not in general substantially shrunk, and more remarkably they have actually widened between the top and bottom end of the distribution - see, for instance, Jones (1997) and Azariadis and Stachurski (2005). In fact, evidence on distribution dynamics in Quah (1993, 1996) suggests that the world income distribution has been converging towards a bimodal distribution. The observation that income differences in the world have proven long-lasting has spawned the concept of *poverty-traps* in the economic development literature. In other words, poor countries might not be catching up with rich countries because they are stuck in a low-development long-run equilibrium.

The first theoretical attempts to explain why poverty-traps may emerge have mainly focused on the existence of technological non-convexities and the presence of spillovers which could lead to coordination failures in the aggregate economy. Among the most prominent papers in this vein are: Murphy, Shleifer and Vishny (1989), Aziariadis and Drazen (1990), Matsuyama (1991), Durlauf (1993), and Zilibotti (1995).

A subsequent strand of literature initiated by the papers by Banerjee and Newman (1993) and Galor and Zeira (1993) has turned to investigate the effects of financial markets imperfections arising from informational asymmetries on the process of development.[1] As a general feature, this literature predicts that the presence of imperfections in financial markets prevents the poor from starting up investment projects or from accumulating human capital, which would be optimal in a first-best environment. As a consequence, the initial wealth distribution will play a key role in determining the development path

[1] Aghion and Bolton (1997), Piketty (1997), and Lloyd-Ellis and Berhardt (2000), are among the first papers that followed this analytical approach.

followed by different economies. In particular, this literature shows that when the fraction of poor agents is large enough, an economy might get stuck in an under-development trap, where typically output is low, social mobility is constrained, and the operation of financial markets is highly inefficient. The next two chapters in this thesis (Chapter 2 and 3) mainly contribute to this latter strand of literature by two separate angles.[2]

Chapter 2 presents a theory in which the variety of productive activities in the economy expands during the process of development. The main claim of this theory states that this process of sectoral differentiation leads to improvements in the allocation of skills, and, as a by-product, helps to mitigate informational frictions affecting the operation of financial markets. This idea relies on the presumption that skills are subject to private information. As a result, an adverse selection problem linked to the allocation of talent emerges in case agents need to obtain credit in order to exercise their skills (i.e., in case they need credit to start up their entrepreneurial investment projects). In that sense, a larger variety of sectors leads to better operation of the credit market because, by facilitating the sorting of heterogeneous skills, it in turn raises the average quality of the pool of credit applicants.

In this theory, the variety of activities is itself endogenous. In particular, it is the result of the optimal behaviour of specific agents (inventors) who seek to produce new ideas (inventions) to sell to the entrepreneurs. Since inventors appropriate part of the surplus generated by their ideas, and since this surplus is increasing in the amount of entrepreneurial investment, a source of positive interaction between financial development and innovation activities arises. More precisely, on the one hand, more efficient operation of financial markets spurs the incentives to invest in R&D, because it fosters entrepreneurial investment. On the other hand, higher investment in R&D contributes to financial development, because it expands the variety of sectors helping to alleviate the adverse selection problem. This positive interaction can give rise to multiple long-run dynamics in the economy. In same cases, the economy grows and develops into a diversified productive structure, enjoying good allocation of skills and efficient operation of credit markets. In other cases, the economy falls into a poverty-trap, characterised by a very rudimentary productive structure, poor sorting of talent, and highly inefficient operation of credit markets.

In Chapter 3, I study the long-run consequences of imperfect risk-sharing in an economy populated with heterogeneously talented entrepreneurs. In this chapter, entrepreneurial talent is again private information. The ensuing adverse selection problem prevents the provision of first-best insurance contracts, which are needed to support risky

[2]Chapter 2 has constituted my Job Market paper.

entrepreneurial activities. Lack of full insurance will naturally discourage first-best risk-taking. Furthermore, under the (realistic) assumption that risk aversion is decreasing in income, the poor refrain from undertaking risky projects more strongly than the rich do. This chapter shows that when entrepreneurial projects are highly productive but very risky, insurance under-provision may discourage entrepreneurship so drastically that a poverty-trap may arise, even if the production technology is convex.

Besides its own theoretical appeal, the ultimate intention of Chapter 3 is to start filling the current gap in the development literature in the context of heterogeneous agents and asymmetric information. This literature has so far extensively studied the long-run effects of credit constrains. But, surprisingly, it has placed very little attention on the dynamic implications of insurance under-provision in poor economies. In that regard, the findings presented in Chapter 3 add further theoretical arguments to the already well established view that imperfect risk-sharing is in itself a really serious issue, and can severely affect the efficiency of economies in the long run.

Chapter 4 (written in collaboration with Vincenzo Merella) turns the attention towards a world economy where countries are linked by international trade of consumption goods. This essay's starting point goes back to the traditional Ricardian view of international specialisation, where trade patterns are governed by countries' comparative advantages. Within such a framework, we build a model with non-homothetic preferences in which different economies might follow divergent income dynamic paths as a consequence of the (either favorable or unfavourable) evolution of their terms of trade. Our model's main results are in line with the old hypothesis by Prebish (1950) and Singer (1950); i.e., that international trade exerts a negative impact on poorer economies, because these economies tend to specialise in commodities with low income demand elasticity, and hence experience a secular tendency of declining terms of trade.

A particularly interesting feature of the model is that, by relying on a quality-ladder framework a-la Grossman and Helpman (1991), it allows for divergent income paths without exogenously imposing any sort of initial absolute advantages for any specific country. In fact, the world-economy featured in this chapter undergoes a long initial phase of "primitive accumulation", during which all countries display identical income. Only once world's productivity surpasses some threshold do income disparities across countries start to arise. This particular type of dynamics not only contrasts with previous literature in the subject – e.g. Flam and Helpman (1987), Stokey (1991), and Matsuyama (2000) –, but also seems to be in line with the world income distribution dynamics before and after the Industrial Revolution – see Galor (2005).

Chapter 2

Sectoral Differentiation, Allocation of Talent, and Financial Development

2.1 Introduction

Over the course of development, the variety of productive activities in the economy tends to increase in conjunction with the aggregate stock of capital and output. This observation implies that economic development partly manifests itself as a process of sectoral diversification. Such a dynamic pattern had initially been suggested by Adam Smith (1776) in his discussion on the division of labour and its relation with the size of the market (*The Wealth of Nations*, chapter 3). I propose here a theory in which this process of sectoral diversification helps to mitigate frictions affecting the operation of financial markets and, thereby, fosters financial development. The degree of sectoral differentiation is itself endogenous, and it is in fact positively influenced by the level of financial development. As a result, sectoral differentiation and financial markets efficiency interplay with each other, and this positive interaction becomes a fundamental ingredient that shapes the pattern of development followed by different economies.

The paper studies the evolution of an economy populated by heterogeneous individuals in terms of entrepreneurial skills. More precisely, individuals in this economy are characterized by distinct comparative advantages concerning entrepreneurial activities. Entrepreneurial skills are, however, private information. This feature gives rise to an adverse selection problem linked to the allocation of entrepreneurial talent and generates the main friction that contaminates the operation of the economy.

The paper argues that this informational friction does not remain constant along the process of development. In particular, given skills heterogeneity, sectoral variety permits better sorting of agents to activities. Consequently, if individuals need credit to start up their projects, this fact would then raise the quality of the pool of credit applicants. In that regard, adverse selection here stems from an underlying problem of scarcity of sectors, because this hinders the efficient sorting of (unobservable) talent. When the variety of sectors is small, a large number of agents have no other choice but specializing in activities for which they might not be exceptionally talented. Asymmetric information concerning skills, in turn, spreads the consequences of talent mismatching to other sectors in the economy, since it prevents the efficient (*ex-ante*) screening of heterogeneous agents in the credit market. As a result, those agents who are not able to exploit their comparative advantages inflict a negative externality (through the adverse selection problem) on those who, in principle, would be able to fully exercise their intrinsic skills.

I model an economy constituted by a large number of potential sectors. Each sector represents a different industry or productive activity, and requires the application of some specific type of entrepreneurial skill. At a particular moment in time, only a fraction among those potential sectors are available to agents (i.e., only a fraction of sectors actually *exist*). The appearance of new sectors is assumed to be the result of innovations; this reflects the idea that carrying out new industrial activities or producing new types of goods requires first an increase in the stock of knowledge in the society. The key point in this paper lies on the hypothesis that sectors variety facilitates the allocation of individuals' talent. This fact reduces the severity of the adverse selection problem in the credit market, enabling the provision of more satisfactory credit contracts, which fosters entrepreneurial investment. The impact of sectoral variety on the credit market efficiency, in turn, gives rise to a novel positive feedback between financial development and innovation activities. Entrepreneurs are the agents who put innovations into practise in the economy. This means that the level of entrepreneurial investment is what ultimately determines the size of the market for innovations. As a result, better operation of financial markets spurs the incentives to invest in R&D (by fostering entrepreneurial investment) and, at the same time, higher investment in R&D contributes to financial development (by expanding the number of sectors available in the economy).

From a dynamic perspective, the development path followed by a successful economy is characterised by a continuous process of capital differentiation (sectoral diversification). In addition to that, the allocation of talent improves and financial institutions become increasingly efficient, as adverse selection problems tend to vanish away concomitantly with sectoral diversification. Nevertheless, this model may also generate a peculiar type of poverty-trap. In this undesirable situation, economies exhibit a rudimentary productive structure, with few active industries, poor allocation of individuals' talent, and highly inefficient financial institutions. In that sense, this poverty-trap is the result of a general organisational failure in the economy, leading to the collapse of several markets.

Historically, sectoral differentiation has been considered to increase productivity by either permitting the exploitation of *economies of scale* (e.g., Smith (1776), Young (1928), Yang and Borland (1991)) or enabling heterogeneous agents to obtain a better *match* (e.g., Rosen (1978), Miller (1984), Kim (1989)). The contribution of this paper is to show that sectoral horizontal expansion brings about an additional positive effect on growth, because a larger variety of activities helps to lessen adverse selection problems associated to the allocation of skills.

The possibility that credit markets efficiency might be influenced by agents' payoffs in other markets of the economy has been suggested by De Meza and Webb (2000). Yet, in their model these payoffs are exogenously set. Ghatak, Morelli and Sjöström (2006) follow this idea, but they explicitly endogenise agents' payoffs, exploiting an interesting "two-way" interaction between the credit market and the labour market. In their model, when the economy is able to provide high wages, *low-quality* entrepreneurs find themselves better-off selling their labour in the market. As a result, high wages help to "clean" the pool of credit applicants, reducing informational frictions and enabling better operation of the credit market.

This paper differs from Ghatak *et al* in that it studies the sorting of talent within a multi-sectoral endogenous growth model. Innovation and the creation of new productive activities become thus key features of the model, since they lead to improved sorting of skills to sectors. Two main novel findings result from my model compared to Ghatak *et al.* First, it shows that innovation improves the assignment of skills, which in turn feeds back on innovation by increasing the returns to R&D. Second, it highlights a new role for the innovation process, very different from the one traditionally stressed in the growth literature. Innovations are not only desirable because they *directly* augment the productivity of inputs. They are also desirable because they help to mitigate informational frictions hindering the operation of financial markets. From that perspective, this paper contributes to the literature on horizontal innovation and growth initiated by Romer (1990), proposing an additional channel whereby variety expansion promotes development.

Acemoglu and Zilibotti (1999) also build a theory in which agents' intrinsic performance improves during the process of development. However, they focus on how a society endeavours to provide correct incentives to agents, and why incentives become more effective as an economy grows. They do not study how the allocation of heterogeneous skills evolves during development. Furthermore, they do not incorporate innovation decisions into their theory, which precludes the variety of activities from expanding over time. In another paper, Acemoglu and Zilibotti (1997) construct a model where the degree of market incompleteness tends to disappear with capital accumulation, and this leads to financial development (in particular, it improves risk-sharing). Nonetheless, neither this model deals with the issue of skills allocation and adverse selection. Financial markets are enhanced with sectoral differentiation, simply because this allows better pooling of sector-specific shocks. In contrast, in my model financial development is the consequence of the alleviation of informational failures due to improvements in the sorting of skills.

The present paper is also closely related to the literature about financial market imperfections and poverty – e.g., Banerjee and Newman (1993, 1994), Galor and Zeira (1993), Piketty (1997), Aghion and Bolton (1997), Lloyds-Ellis and Bernhardt (2000), and Ghatak and Jiang (2002). These articles stress the influence of the wealth distribution on the dynamic behaviour of the economy when agency-costs lead to credit rationing. As a general result, their models commonly lead to poverty-traps when the number of poor agents is large enough. This paper contributes to this literature by different channels. It first provides a fully micro-founded explanation of why agency-costs may arise in a developing economy. Secondly, it is able to generate dynamics where these agency-costs go down as an economy develops. As a result, *rationing* is not just solved because people become rich enough (so that they can afford better credit or insurance contracts), but mainly because financial markets' operation itself becomes more efficient along development.

Section 2.1.1 presents some evidence about sectoral diversification along the path of development; this section could be skipped if the reader wants to proceed immediately to the model. Section 2.2 describes the basic set-up of the model. Section 2.3 studies the static equilibrium of the economy; in particular it analyses the entrepreneurs' optimal choice in the presence of adverse selection. Section 2.4 introduces the innovation activities into the model, which endogenises the variety of sectors in the economy. Section 2.5 proceeds to the dynamic study of this economy. Section 2.6 discusses some extensions to the basic model. Section 2.7 concludes. Appendix A presents some cross-country data consistent with the main predictions of the model. Omitted proofs are provided in the Appendix B.

2.1.1 Sectoral Diversification and Development in the Data

Sectoral diversification is a feature recurrently observed during the process of development. Allyn Young (1928) writes "industrial differentiation has been and remains the type of change characteristically associated with the growth of production" (p. 537). Landes (1969) claims that the most evident effects brought about by the Industrial Revolution were both the gains in productivity and the increase in the variety of products and occupations (p. 5). In a passage of his book he writes "the whole tendency of industrialization and urbanization was to specialize labour ever farther and break down the versatility of the household", proceeding to enumerate a long list of new occupations (ranging from bakers, butchers, manufacturers of candles, soap and polish, to others like carpenters, masons, plumbers, and plasterers) which started to appear and expand with the Industrial Revolution (p. 119). Kubo *et al* (1986) show evidence that the share of intermediate goods substantially increased along with output per-capita for a sample of nine semi-industrialised countries in the post-war period.[1] This suggests that industry differentiation took place in conjunction with growth in those economies.

Econometric evidence also gives support to the premise that sectoral diversification is experienced over the path of development. For a panel of 67 countries, Imbs and Wacziarg (2003) show that *sectoral concentration* (the opposite of *sectoral diversification*) drastically falls at early stages of development, following a "U-shape" relation with respect to income per-head.[2] They conclude that, during development, economies initially experience a long process of sectoral diversification which eventually reaches a maximum beyond where the process begins to revert. Given the implications of my paper, two observations need to be stressed here: (i) the "turning-point" in the diversification process tends to occur at relatively high income per-capita levels (the authors argue that this point is located roughly at the income per-head reached by Ireland in 1992); (ii) the eventual re-concentration process does only partly offset the effect of the initial diversification phase - see figures 1, 2 and 3 in their paper, p. 69.

FIGURE 2.1 provides an overview of the association between sectoral diversification and income per-head found by Imbs and Wacziarg. Sectoral concentration is measured by the Imbs and Wacziarg's Gini coefficients for employment shares based on the UNIDO 3-digit dataset (a smaller Gini coefficient would thus reflect a more diversified economy in terms of manufacturing industries).[3] Income per-head is measured by GDP per-capita

[1] These countries are: Norway, Israel, Taiwan, Korea, Japan, Yugoslavia, Turkey, Mexico, and Colombia.

[2] Imbs and Wacziarg (2003) use the non-parametric technique *lowess* (*locally weighted scatterplot smoothing*) to capture the association between sectoral concentration and income per-capita. They build five different concentration indices based on employment shares (Gini coefficient, Herfindahl index, log-variance of sector shares, coefficient of variation, and the max-min spread). These indices are constructed for three different datasets: 1-digit level (9 sectors) from the International Labor Office (ILO), 3-digit level (28 sectors) from the United Nations Industrial Development Organisation (UNIDO), and 2-digit level (20 sectors) from the OECD. For the UNIDO and OECD datasets, value-added per sector is also available and utilised. All their results are robust to the use of different indices and datasets.

[3] I am indebted to Jean Imbs for kindly providing me with this data.

in thousands of PPP 1985 constant US dollars (from Summers and Heston (1991)). To allow for the possibility of a non-monotonic relation between sectoral concentration and GDP per-capita, I run a fifth-order polynomial regression. I also show in FIGURE 2.1 the results of a quadratic regression on the GDP per-head. Both regressions additionally control for country fixed-effects.[4] We can observe the pattern described in Imbs and Wacziarg: sectoral concentration initially decreases with income per-capita, eventually reaching a turning-point beyond which the relation partially reverts.

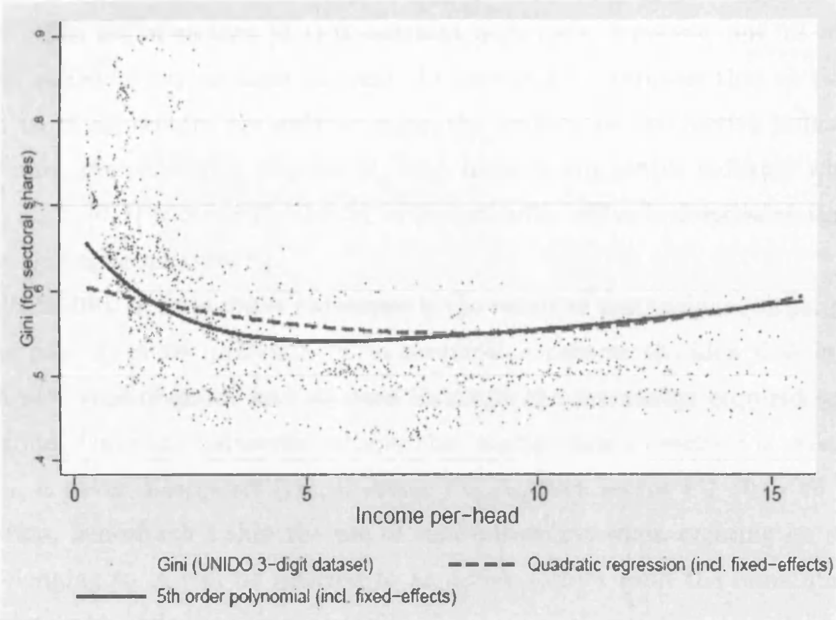

FIGURE 2.1: Sectoral Concentration and Income Per-Head.

The Gini coefficient in the picture measures the degree of sectoral concentration in the economy in terms of employment shares across 28 manufacturing sectors (3-digit level disaggregation, UNIDO dataset).

This paper will focus strictly on the initial stages of development, where sectoral diversification and income per-head increase together. The eventual *re-specialisation* pattern may presumably be partly explained by the joint effect of increasing returns to scale (both static and dynamic) and regional specialisation; a phenomenon which will be completely neglected in my theory. Despite this omission, in Section 2.6 and Section 2.7, I discuss a possible extension to my benchmark model that could make my theory still consistent with the non-monotonic relation between income per-head and sectoral diversification displayed in FIGURE 2.1.

[4] Country fixed-effects are very important at determining the productive structure of economies. Since in this paper we are following individual economies over their own path of development, it is then strictly necessary to control for fixed-effects to obtain a consistent picture of this process.

2.2 Environment

The paper considers a small economy enjoying full access to international credit markets. Life evolves over a discrete-time infinite horizon $t = \{0, 1, ..., \infty\}$. In each period t a single-period lived continuum of agents with mass normalised to 2 is alive.

The economy contains a continuum of *sectors* indexed by the letter $i \in [0, 1]$. Each sector i represents a particular industry in the economy (nevertheless, for analytical simplicity, I will assume that in each sector i the same single final good is actually produced). The set of sectors $[0, 1]$ is constant over time; however, not all sectors are necessarily *active* at any moment in time. In particular, I suppose that at time t only a fraction n_t of all sectors are able to enjoy the activity of productive industries. At the same time, the remaining fraction $(1 - n_t)$ lacks of any active industry whatsoever. Hereafter, $\mathcal{A}_t \subset [0, 1]$ will denote the set of sectors with active industries at time t. The set $\mathcal{A}_t$ has Lesbegue measure n_t.

The availability of productive industries is the result of innovations (either generated during the past or in the present). This assumption reflects the idea that in order to produce a new type of good, we first need to create the knowledge required to produce this new good. Once the industrial activity that corresponds to sector i is *created* by an innovation, it never disappears (i.e., if sector $i \in \mathcal{A}_t$, then sector $i \in \mathcal{A}_{t+\delta}$ $\forall \delta \geq 0$). To ease notation, henceforth I skip the use of time-subscripts when creating no confusion. Sectors belonging to $\mathcal{A}$ will be referred to as *active sectors* (and the remaining sectors will accordingly be called *inactive sectors*).

A sector $i \in \mathcal{A}$ provides the agents in the economy the chance to invest in an entrepreneurial project called Project-i. The return of Project-i is random, subject to an idiosyncratic shock. Project-i's return also depends on the application of some specific entrepreneurial skills, and on the amount of capital (k) invested in the project. A full description of Project-i is provided in the following subsection (equations (2.1) and (2.2) ahead in the text).

Each generation-t of agents comprises two different groups of individuals, each one with mass equal to 1; namely:

1. *Entrepreneurs*: These agents are endowed with entrepreneurial skills which are needed to organise and undertake the production of final goods.

2. *Inventors*: They carry out R&D in order to generate new ideas that can be used by the entrepreneurs in the production of new final goods.[5]

[5]To illustrate this distinction, take the Pharmaceutical Industry as an example. The innovator would be represented by a biochemist whose task consists in designing a formula to produce a new drug. On

2.2.1 Entrepreneurs

At any time t, there exists a continuum of (prospective) entrepreneurs who are indexed by the letter $i \in [0, 1]$. The index i denotes the entrepreneur's *type*. The entrepreneur i will be referred to as the Type-i.

The cohort-t of entrepreneurs is alive during period t. A new cohort is born just at the end of the previous cohort's lifespan. Each (dying) entrepreneur procreates one (new) entrepreneur. For the moment, I assume agents are *non-altruistic* and are born with zero initial wealth (in Section 2.6 this assumption is relaxed).

All entrepreneurs are risk-neutral, sharing identical preferences over the single consumption good. Accordingly, they all seek to maximise their expected consumption. The ex-post level of consumption will be determined by their ex-post investment net returns. Since entrepreneurs are born with zero initial wealth, the only way in which they can provide themselves with future consumption is by borrowing capital from credit markets and investing it in an entrepreneurial project. Diversification among entrepreneurial projects is not feasible; in other words, these agents must *specialise* in one particular project.

Entrepreneurs are heterogeneous with respect to their entrepreneurial skills. More precisely, if a Type-$j \in [0, 1]$ invests k units of capital in Project-$i \in \mathcal{A}$, then his Project-i's gross return ($y_{i,j}$) is given by:

$$y_{i,j} = \theta_{i,j} \, f(k_{i,j}).$$

The function $f(k)$ is strictly increasing, strictly concave, twice continuously differentiable, and satisfies Inada conditions. The variable $k_{i,j}$ represents the amount of capital invested in Project-i by the Type-j. Capital fully depreciates during the process of production. Finally, $\theta_{i,j}$ denotes the realisation of a random-variable with support $\{0, 1\}$. The value taken by $\theta_{i,j}$ is governed by the following distribution function:

$$\theta_{i,j} = \begin{cases} 1 & \text{with probability} \quad p_{i,j} \\ 0 & \text{with probability} \quad 1 - p_{i,j}. \end{cases}$$

Where,

$$\begin{aligned} p_{i,j} &= 1 && \text{for all } i, j \in [0, 1] \text{ if } j = i, \\ p_{i,j} &= p \in (0, 1) && \text{for all } i, j \in [0, 1] \text{ if } j \neq i. \end{aligned}$$

the other hand, the pharmaceutical company would represent the entrepreneur. This agent organises the production process of the drug and takes it to the market, turning the (abstract) formula into a final good ready for consumption.

In short, a Type-i is an agent with intrinsic comparative advantage in Project-i.[6] Gross returns of Project-i are thus given by:

$$y_{i,i} = f(k_{i,i}) \tag{2.1}$$

$$y_{i,j}(\theta_{i,j}) = \begin{cases} f(k_{i,j}) & \text{with probability} \quad p \\ 0 & \text{with probability} \quad 1-p \end{cases}, \text{ where } j \neq i \tag{2.2}$$

Concerning the informational structure in the economy, types are private information. In other words, there is asymmetric information regarding entrepreneurial skills. In addition to that, I assume types are intergenerationally uncorrelated, implying that parents' historical outcomes provide no information whatsoever about the type of a child.

Lastly, I assume that everybody has access to a "backyard" activity which requires no initial investment and yields net return equal to v with certainty. Without any loss of generality, I set $v = 0$ (implying that the activities participation constraint will never bind).[7]

2.2.2 Inventors

In addition to the set of entrepreneurs, in any period t there is also a continuum of agents with mass 1 (the *inventors*) who are born with the particular skill to be able to produce *new* ideas. New ideas, in turn, materialise in *innovations* and thus expand set of active sectors available in the economy in period t (i.e., the set $\mathcal{A}_t$). This means that, in any period t, the set $\mathcal{A}_t$ is the result of the stock of innovations generated during the set of periods $\{0, 1, ...t\}$; that is, during the history of the economy up to t.

The presentation of the inventors' optimisation problem will be postponed until Section 2.4. For the time being (in Section 2.3) the exposition of model can be perfectly carried out without its explicit incorporation.

2.2.3 Credit Markets

Since agents in the economy are born with zero wealth, they will need to rely on credit markets in order to undertake their investment projects. The rest of the world will provide local agents with the needed liquidity. All credit market transactions with the rest of the world are mediated by some firms called *financial intermediaries.* The credit market

[6] The concept of comparative advantage is defined in terms of average productivity (the average productivity of Type-i in Project-i is higher than the average productivity of Type-$j \neq i$ in Project-i).

[7] If $v > 0$, agents would have access to an outside option with positive payoff, hence their participation constraint may bind in equilibrium. This might have some minor implications on the type of credit contracts observed in equilibrium, however, none the main results and insights of the paper would be altered by letting $v > 0$.

is characterised by free-entry and absence of set-up or sunk costs. Since the economy is small and there is perfect international capital mobility, financial intermediaries are able to draw liquid funds from international credit markets facing a perfectly elastic supply at the international (net) interest rate R^f. For algebraic simplicity, I set $R^f = 0$.

Financial intermediaries offer credit contracts stipulating fixed interest rates (i.e., interest rates that are not contingent upon *ex-post* output). When the borrower cannot pay back the amount agreed in the credit contract, he goes bankrupt and loses all rights on his final output, which goes entirely to the financial intermediary. These contracts are standard loan contracts (Gale and Hellwig [1985]). A credit contract offered to an entrepreneur can thus be specified as a pair $(l_j, r_j) \in \mathbb{R} \times \mathbb{R}$; where l_j represents the loan extended to the Type-j and r_j stands for the (net) interest rate charged on the loan l_j. Individuals are protected by *limited-liability*, meaning that their consumption cannot fall below a lower-bound which I set equal to zero. As a result, $l_j(1+r_j)$ will be (in principle) paid back to financial intermediaries only in the event of success. On the other hand, when the project fails, the entrepreneur goes bankrupt and the financial intermediary recovers 0 income.

2.3 Static Equilibrium Analysis

Throughout this section the set of active sectors $\mathcal{A}_t$ is taken as exogenously given. Thus, the paper focuses on the entrepreneurs' optimal behaviour, and on the set of credit contracts offered by financial intermediaries, given $\mathcal{A}_t$. This course of action will yield the equilibrium solution of the model at some specific period of time t. In the next sections I proceed to study the dynamic evolution of the economy. This will require explicitly incorporating the inventors' optimisation problem, which endogenises the set $\mathcal{A}_t$.

Let $\mathfrak{Q} \subset \mathbb{R} \times \mathbb{R}$ denote the set of all feasible credit contracts (l, r), and $\mathcal{C}_t \subseteq \mathfrak{Q}$ denote the subset of feasible credit contracts offered by financial intermediaries in period t. An entrepreneur $j \in [0,1]$ alive during t will choose an allocation $[(r_j, l_j)^*, k^*_{i,j} : i \in \mathcal{A}_t]$, solving the following two-stage optimisation problem:

- **First-Stage (specialisation decision)**: $j \in [0,1]$ selects sector $i \in \mathcal{A}_t$ in which to invest.

- **Second-Stage (optimal investment in sector i):**[8]

$$\max_{k_{i,j},(r_j,l_j)} \; : \; E_i(U_j) = p_{i,j} \max\{0\,,\; f(k_{i,j}) - (1+r_j)l_j + (l_j - k_{i,j})\} + (1-p_{i,j}) \max\{0\,,\; -(1+r_j)l_j + (l_j - k_{i,j})\} \quad \text{(I)}$$

$$\begin{aligned} \text{subject to:}\quad & k_{i,j} \le l_j \quad \text{(budget constraint)},\\ & k_{i,j} \ge 0 \quad \text{(feasibility constraint)},\\ & (r_j, l_j) \in \mathcal{C}_t \quad \text{(set of offered credit contracts)}. \end{aligned}$$

Definition 1 (Equilibrium at time t) *Given the set $\mathcal{A}_t$, an equilibrium at time $t \in \mathbb{Z}^+$ is a set of entrepreneurial allocations $[(r_j, l_j)^*, k^*_{i,j} : i \in \mathcal{A}_t]_{j\in[0,1]}$ and a set of offered entrepreneurial credit contracts $\mathcal{C}_t$, such that the following two conditions are satisfied:*

1) Entrepreneurs' optimal allocation: *Given the set $\mathcal{C}_t$, $\forall j \in [0,1]$ alive in period t, the allocation $[(r_j, l_j)^*, k^*_{i,j} : i \in \mathcal{A}_t]$ solves the two-stage optimisation problem (I).*

2) Credit markets (competitive) equilibrium: *(i) No credit contract belonging to $\mathcal{C}_t$ makes negative expected profits; and (ii) there exists no other credit contract $\mathfrak{z} \in \mathfrak{Q}$, such that $\mathfrak{z} \notin \mathcal{C}_t$, and which, if offered in addition to $\mathcal{C}_t$, would make positive expected profits.*

2.3.1 Credit Market Equilibrium Contracts

Following the literature on adverse selection in financial markets (e.g. Rothschild and Stiglitz (1976), Wilson (1977), and Milde and Riley (1988)), one would reasonably expect two different kinds of equilibria to possibly arise in this model's credit market: 1) a *pooling equilibrium*, in which all types receive an identical credit contracts; 2) a *separating equilibrium*, in which types are *screened*, receiving distinctive contracts which induce truthful self-revelation of their (unobservable) skills.

Proposition 1 *Assume the set of inactive sectors at time t is non-empty (i.e. $\mathcal{A}_t \neq [0,1]$). Take any sector $i \in \mathcal{A}_t$ and any sector $j \notin \mathcal{A}_t$. Then, there can never exist an equilibrium at t in which the Type-i and the Type-j are offered different credit contracts.*

Proposition 1 implies there cannot exist a separating equilibrium in this model. As a consequence, if an equilibrium is to exist at all, it should entail pooling credit contracts. The result in Proposition 1 stems from the conjunction of five different assumptions: 1) agents displaying risk-neutrality, 2) the particular form of the production functions in (2.1) and (2.2), 3) the limited-liability constraint, 4) agents being born with zero initial wealth (so they have no collateral), and 5) the outside option yields $v = 0$. Intuitively,

[8] $E_i(U_j)$ denotes the expected utility of Type-j when he invests in Project-i (recall that the success probability $p_{i,j}$ depends on the match between the type and the sector).

given a set of credit contracts, any contract that maximises net returns for (2.1) must also necessarily maximise expected net returns for (2.2) (since, in the presence of limited-liability and no collateral, expected net returns when (2.2) holds are proportional to net returns when (2.1) prevails).[9]

Given the set of active sectors at time t, $\mathcal{A}_t \subset [0,1]$, we may split the population of entrepreneurs alive during t in two disjoint subsets. Firstly, we may gather all those entrepreneurs of type-$i \in [0,1]$, such that sector $i \in \mathcal{A}_t$. Secondly, we may bunch together all those entrepreneurs of type-$j \in [0,1]$, such that sector $j \notin \mathcal{A}_t$. The first group of agents would be able to fully exploit their comparative skills, whereas the second one have to specialise in a sector for which they are not (exceptionally) talented. Abusing a bit of the language utilised in the adverse selection literature, I will call the first group the *good-types,* while the second group will be denoted as the *bad-types.*[10]

In a pooling equilibrium, all entrepreneurs receive an identical credit contract (l,r). Notice then that $\mathcal{C}_t$ must comprise one single element; namely $\mathcal{C}_t = (l,r)$. Additionally, in any (competitive) pooling equilibrium, credit contracts must necessarily verify the following two properties. First, the contract must make non-negative expected profits; otherwise this contract would simply be withdrawn. Second, the contract must maximise the expected utility of the good-types; otherwise financial intermediaries could offer a different contract such that it makes non-negative profits and, at the same time, it makes these agents better-off.[11]

Assume for the moment that the Type-i chooses to specialise in sector $i \in \mathcal{A}$ (as it will become clear later on, this will necessarily be true in equilibrium). Then, given $\mathcal{C}_t = (l,r)$, his optimisation problem boils down to:

$$\begin{aligned} \max_{k_{i,i}\geq 0} \ &: \ \max\{0\,,\, f(k_{i,i}) - (1+r)l + (l - k_{i,i})\} \qquad \text{(I')} \\ s.t \ &: \ k_{i,i} \leq l \quad \text{(budget constraint).} \end{aligned}$$

[9]See Ghatak, Morelli and Sjöström (2006), and also Grüner (2003), for models that obtain pooling contracts in a similar fashion.

[10]More rigorously: *good-types*$_t$ $= \{h \in [0,1] \mid$ *sector* $h \in \mathcal{A}_t\}$ and *bad-types*$_t$ $= \{h \in [0,1] \mid$*sector* $h \notin \mathcal{A}_t\}$. Notice that in this paper whether a particular Type-$h \in [0,1]$ is a good-type or a bad-type is not fixed, but it is contingent on the set $\mathcal{A}_t$. In that sense, from a dynamic point of view, everyone could eventually become a good-type, if the set of active sectors constantly expands over time.

[11]It must be also clear that $\mathcal{C}_t = (l,r) \in \mathbb{R}^+ \times \mathbb{R}^+$. Although nothing precludes the fact that l could be in principle negative (i.e., entrepreneurs could lend capital to financial intermediaries), this possibility will never arise in equilibrium, as entrepreneurs are born with zero initial wealth. In addition to that, in equilibrium, financial intermediaries would never offer loan contracts with $r < 0$, as these contracts would entail (expected) losses.

Note now that entrepreneurs borrow from financial intermediaries only to invest in an entrepreneurial project. As a consequence, credit contracts offered in equilibrium must necessarily satisfy the condition $f'(l) \geq 1$ (because no entrepreneur would ever invest in his project beyond the point in which the marginal productivity of capital falls below 1, since the *net* interest on deposits equals $R^f = 0$). The budget constraint for the Type-i will thus bind; in other words, $k_{i,i} = l$ will hold in the optimum. Problem (I') will then yield the following (standard) first-order condition:

$$f'(k^*) = (1 + r) \tag{2.3}$$

From (2.3), we can then obtain the optimal amount of capital invested in the project, given the interest rate r. That is, $k^*(r)$; where $k'(r) < 0$ since $f''(\cdot) < 0$. An equilibrium pooling contract will, therefore, display the following structure: $(l, r) = (k^*(r), r)$. (So that it maximises the expected utility of the good-types.)

2.3.2 The Equilibrium Interest Rate (on Entrepreneurial Loans)

The pair $(k^*(r), r)$ characterises the equilibrium credit contract, given the interest rate r. Therefore, in order to determine the exact credit contract that holds in period t, it still remains to find the equilibrium value of r in t. Let us denote this variable by r_t^*. Perfect competition in the credit market naturally implies that financial intermediaries must make zero profits in equilibrium; hence r_t^* will be pinned down by the respective zero-profit condition.

Consider sector $i \in \mathcal{A}_t$ and sector $j \notin \mathcal{A}_t$. Take the Type-i alive in t, and imagine he decides to invest in Project-i. Then, given r, his consumption $(c_{i,i})$ would be determined by:

$$c_{i,i} = f(k^*(r)) - (1 + r)k^*(r). \tag{2.4}$$

Now, imagine this Type-i chooses to invest in Project-$x \in \mathcal{A}_t$, where $x \neq i$. In that case, his consumption $(c_{x,i})$ would amount $c_{x,i} = p\left[f(k^*(r)) - (1 + r)k^*(r)\right]$. From these two expressions, it becomes straightforward that $c_{i,i} > c_{x,i}$, no matter the value of r. Therefore, as long as sector $i \in \mathcal{A}_t$, the Type-i (alive in t) will specialise in Project-i.

Take now the type-j entrepreneur alive during t. This agent could invest in Project-i (or in any Project-x, such that sector $x \in \mathcal{A}_t$), obtaining as expected consumption $(c_{i,j})$:

$$c_{i,j} = p\left[f(k^*(r)) - (1 + r)k^*(r)\right]. \tag{2.5}$$

Notice that because $f(k)$ satisfies Inada conditions (in particular, because $\lim_{k \to 0} f'(k) = \infty$) the expression in (2.5) yields $c_{i,j} > 0$, irrespective of the value taken by r. This means it will always be desirable for the Type-j to invest $k^*(r)$ in Project-i (or in Project-x indifferently).

From the previous discussion, we can then deduce that a fraction n_t of the population of entrepreneurs (the good-types) will always pay back the financial intermediaries the agreed amount $(1+r)k^*(r)$. On the other hand, the remaining fraction $1-n_t$ (the bad-types) will go bankrupt with probability $1-p$. Being protected by limited-liability, the bad-types are *expected* to pay back financiers only the amount $p(1+r)k^*(r)$. Then, the *zero-profit condition* reads thus as follows: $n_t\,(1+r_t^*)k^*(r_t^*) + (1-n_t)\,p(1+r_t^*)k^*(r_t^*) = (1+R^f)k^*(r_t^*)$ (where, recall that for algebraic simplicity, $R^f = 0$ will be assumed).

Proposition 2 *The equilibrium interest rate charged on credit contracts offered to entrepreneurs is a decreasing function of the fraction of active sectors. More precisely,*

$$r_t^* = r^*(n_t) = \frac{(1-n_t)(1-p)}{n_t + (1-n_t)p}. \tag{2.6}$$

From (2.6), it can also be noted that: $r^*(0) = (1-p)/p$, $r^*(1) = 0$, and $r''(n_t) > 0$.

Proposition 2 reflects one the most important insights of this paper. A larger number of active sectors leads to a more efficient operation of credit markets; this is the case because a higher value of n_t improves the sorting of entrepreneurial skills, alleviating the adverse selection problem in the credit market. Intuitively, as the set $\mathcal{A}_t$ expands, a higher fraction of agents find it feasible to specialise in the sector they are most talented at. This fact reduces the *average default rate* in the economy, enabling financiers to charge a lower interest rate on the loans they extend to entrepreneurs, without incurring in expected losses.[12]

2.3.3 Entrepreneurial Consumption Level / Net Returns

Take again some Type-$i \in [0,1]$, such that sector $i \in \mathcal{A}$ (a good-type representative). His consumption level will be dictated by (2.4). Denote by $U_g(r)$ the utility level achieved by an entrepreneur who belongs to the subset of good-types. Differentiating (2.4) with respect to r, and taking (2.3) into account, we get:

$$U_g'(r) = -k^*(r). \tag{2.7}$$

Select now some Type-$j \in [0,1]$, such that sector $j \notin \mathcal{A}$ (a bad-types representative). His expected consumption will be given by (2.5). Hence, letting $U_b(r)$ denote the level of expected utility reached by a bad-type, we obtain:

$$U_b'(r) = -p\,k^*(r), \tag{2.8}$$

[12] The reader might actually prefer to call r^* the *risk-premium*. In that sense, it is the risk-premium on entrepreneurial loans what diminishes as n goes up due to the better sorting of talent.

where derivation of (2.8) also makes use of (2.3).

Lemma 1 *Let* $\Delta(r) \equiv U_g(r) - U_b(r)$. *Then,* $\Delta(r) > 0$ *and* $\Delta'(r) < 0$, *for all possible values* r *may take in equilibrium.*

The proof of Lemma 1 is straightforward from inspection of (2.7) and (2.8). The derivative $\Delta'(r) < 0$ means that good-types benefit from a fall in the interest rate r more than bad-types do. The reason for this result lies on the fact that good-types never go bankrupt, thus they will appropriate the full cost-reduction induced by a lower r. On the other hand, since bad-types go bankrupt with probability $(1 - p)$, they will profit from a smaller r only with probability $p < 1$. Lemma 1 will play a key role in the inventors' optimisation problem (next section).

So far, the set $\mathcal{A}_t$ has been taken as exogenously determined. In this way, the model has managed to characterise the entrepreneurs' equilibrium choices in some specific period t. In order to endogenise the set $\mathcal{A}_t$ and study the dynamics of this model, the inventors' behaviour needs to be explicitly incorporated. I proceed now to do so.

2.4 Inventors, Market for Ideas, and Innovations

I model the appearance of *new* active sectors as the result of innovations. Following the Endogenous Growth Theory paradigm,[13] innovations result from deliberate profit-maximising R&D policies undertaken by private agents which I refer to as *inventors*. I will focus only on horizontal innovations, as those are the kind of innovations that will lead to improvements in the allocation of agents' talent; the key mechanism at work in this paper.

In each period t there is a continuum of single-period lived inventors with mass 1. Inventors are able to generate *new* ideas (this is their specific skill) – think of an idea as a blueprint or design, which contains the information needed to produce new types of goods. Like previously done with sectors and entrepreneurs, let inventors be indexed by $i \in [0, 1]$. Inventors are also assumed non-altruistic and risk-neutral. Each (dying) inventor gives birth to a (new) inventor. Except for their particular index i, all inventors within the same cohort are *ex-ante* identical. I suppose the inventor i can only possibly innovate for sector i. Since vertical innovations are assumed away, the subset of inventors who (would) innovate for sectors which were already *active* in period $t - 1$ will thus not play any relevant role during t.

In order to come up with a new idea, an inventor needs first to carry out R&D, which is costly. A new idea, however, does not per-se modify the technological frontier of the

[13] E.g. Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992).

economy; for that to happen, the idea must by *applied* by an entrepreneur. In other words, there is a strong intrinsic complementarity between inventors and entrepreneurs, and this requires both agents' specific skills to be implemented for the technological frontier of the economy to expand.[14] When the *idea* designed by inventor i is put into practise by some entrepreneur $j \in [0,1]$, this idea becomes *technology*, and materialises as Project-i (turning sector i into an active sector). Technology is a pure public-good; that is, its use is non-rival and non-excludable. More precisely, once some particular entrepreneur $j \in [0,1]$ applies a new idea, the underlying knowledge becomes readily (and instantly) available to all the other entrepreneurs from t onwards. On the contrary, an idea is excludable, since the inventor who has generated it can keep his idea *undisclosed* as long as he wants, simply by not spelling it out to any other agent.

An inventor who comes up with a new idea, will then try to sell it to an entrepreneur. I assume entrepreneurs pay the inventors after production takes place and that the transaction between an inventor and an entrepreneur is not observed by the financiers. Given the public nature of technology, only the Type-i would be willing to pay a positive price to obtain the idea generated by inventor i. To see this, recall from Lemma 1 that $\Delta(r) > 0$ for any possible value that r may take in equilibrium. This $\Delta(r)$ equals the increment in (expected) utility that the Type-i would get by applying the idea generated by inventor i (were this idea given to him for free!). Notice $\Delta(r)$ is a surplus resulting from a *bilateral-monopoly* relation between the Type-i and the inventor i. In principle, the surplus $\Delta(r)$ could be distributed between the two parties according to various different rules. For simplicity, I will assume that the whole surplus $\Delta(r)$ is appropriated by the inventor, leaving the entrepreneur just indifferent between buying or not the new idea (in other words, the inventor makes a take-it-or-leave-it-offer to the entrepreneur for the transfer of the idea).[15]

[14]This is in line with the view of economic development by Joseph A. Schumpeter (1934); he writes *"Entrepreneurship must be distinguished from 'invention'. As long as they are not carried out into practice, inventions are economically irrelevant. And to carry any improvement into effect is a task entirely different from the inventing of it, and requiring different kinds of aptitudes. Although entrepreneurs of course may be inventors, it would not be by nature of their function but by coincidence."*, pp. 88-89. Relatedly, Hobsbawm (1977) claims it was not scientific supremacy what explains why the Industrial Revolution occurred first in UK; in fact, he asserts that both France and Germany were notably above UK in terms of scientific knowledge at that time (pp. 29-30).

[15]Nonetheless, as long as it is assumed that the inventor's income is increasing in the total surplus $\Delta(r)$, none of the main findings of this paper would be affected if the entrepreneur could actually appropriate part of $\Delta(r)$ (for instance, if the surplus were split following a Nash-bargaining rule).

2.4.1 Inventors' Optimisation Problem

Suppose inventors must expend effort in order to generate new ideas. Effort generates disutility. Let $\iota_{i,t}$ denote the effort-cost (measured in units of consumption-good) spent in R&D activities by the inventor i alive during period t. Additionally, denote by $\Pr(I_i = 1)$ the probability that the inventor i will generate a new idea. Consider sector $i \notin \mathcal{A}_{t-1}$; the probability that inventor i generates an idea for sector i in period t is given by (henceforth, I skip the use of time subscripts on $\iota_{i,t}$ to ease notation):

$$\Pr(I_i = 1) = \beta(\iota_i), \tag{2.9}$$

where: $\beta'(\iota) > 0$, $\beta''(\iota) < 0$, $\beta(0) = 0$, $\lim_{\iota\to\infty} \beta(\iota) \leq 1$, and $\lim_{\iota\to 0} \beta'(\iota)$ is finite.

Since sector $i \notin \mathcal{A}_{t-1}$, the inventor i alive in period t would (in principle) be able to generate a new idea. This inventor will thus choose the value of ι_i so as to maximise his expected profits function derived from the *generation* and *sale* of new ideas.[16] Denote by $\overline{\iota}_t$ the level of R&D effort chosen by all the inventors belonging to the subset $-\mathcal{A}_{t-1}^{-i}$, where $-\mathcal{A}_{t-1}^{-i} = \{\ j \in [0,1] \mid j \neq i$ and sector $j \notin \mathcal{A}_{t-1}\}$.[17] Having managed to produce a new idea, inventor i would optimally charge a price $\Delta(r^*(n_t))$ when selling this idea to the Type-i. Notice that, assuming that all new ideas are sold to entrepreneurs (which will be true in equilibrium), $n_t = n_{t-1} + \beta(\overline{\iota}_t)(1 - n_{t-1})$.[18] Hence, we can rewrite $\Delta(r^*(n_t)) = \Psi(n_{t-1}, \overline{\iota}_t)$. Lemma 2 characterises the optimisation problem faced by inventor i.

Lemma 2 *Consider sector $i \notin \mathcal{A}_{t-1}$, and take the inventor i alive during t. He solves:*

$$\max_{\iota_i \geq 0} : \ \Pi_{i,t}(\iota_i, n_{t-1}, \overline{\iota}_t) = \beta(\iota_i) \cdot \Psi(n_{t-1}, \overline{\iota}_t) - \iota_i \tag{II}$$

Where the function $\Psi(n_{t-1}, \overline{\iota}_t) : [0,1] \times \mathbb{R}^+ \to \mathbb{R}^+$ is increasing in both of its arguments. More precisely: (i) $\Psi'_n(\cdot) > 0, \forall n_{t-1} \in [0,1]$ and $\overline{\iota}_t \geq 0$; and (ii.a) $\Psi'_{\overline{\iota}}(\cdot) > 0, \forall n_{t-1} \in [0,1)$ and $\overline{\iota}_t \geq 0$, (ii.b) $\Psi'_{\overline{\iota}}(\cdot) = 0$ if $n_{t-1} = 1$.

From Lemma 2 it follows that $\Pi_{i,t}(\iota_i, n_{t-1}, \overline{\iota}_t)$ must be increasing in both n_{t-1} and $\overline{\iota}_t$. To grasp some intuition, notice that, since active sectors do not ever revert to inactive, the higher n_{t-1} is, the higher n_t is expected to be. As a result, relatively high values of

[16] If sector $i \in \mathcal{A}_{t-1}$, then the inventor i alive in t trivially chooses $\iota_i = 0$.

[17] This $\overline{\iota}_t$ should actually be a mapping $\overline{\iota}_t : -\mathcal{A}_t^{-i} \to [0, \infty)$, summarising the choice of ι for each inventor belonging to $-\mathcal{A}_t^{-i}$. However, in the optimum, all these inventors will select the same value of ι. Hence, a *singleton* $\overline{\iota}_t$ turns out to be sufficient to represent their aggregate behaviour.

[18] This is because: 1) the sectors that were already active in $t-1$ remain active in t, and 2) a fraction $\beta(\overline{\iota}_t)$ among the inactive sectors in $t-1$ become active in t.

n_{t-1} will tend to be associated with relatively low levels of r^* in t (Proposition 2). This, in turn, implies that the surplus generated by new innovations, $\Delta(r^*)$, is expected to be large (Lemma 1), allowing inventors to charge a relatively high price for their ideas. Similarly, larger values of $\bar{\iota}_t$ are also associated with less severe adverse selection leading to lower r^* and higher $\Delta(r^*)$. In this case, the reason for this positive effect is that a larger $\bar{\iota}_t$ means more innovations will actually be produced, raising thus the value of n_t (from the given n_{t-1}). In addition to that, note $\Psi'_{\bar{\iota}}(\cdot) > 0$ implies that there exists a positive externality across inventors. This externality arises because when an inventor $j \in [0,1]$ comes up with a new idea, this may turn sector j into an active sector, increasing the value of n_t (something which all inventors will benefit from).

Problem (II) leads to the following first-order condition:

$$\beta'(\iota_i^*) \cdot \Psi(n_{t-1}, \bar{\iota}_t) \leq 1 \quad \text{and} \quad \iota_i^* \left[\beta'(\iota_i^*) \cdot \Psi(n_{t-1}, \bar{\iota}_t) - 1\right] = 0 \tag{2.10}$$

Proposition 3 *Let* $\iota_i^* \equiv \arg\max_{\iota_i} \left\{\Pi_{i,t}(\iota_i, n_{t-1}, \bar{\iota}_t)\right\}$. *Then,* $\iota_i^* = \iota_i^*(n_{t-1}, \bar{\iota}_t) : [0,1] \times \mathbb{R}^+ \to \mathbb{R}^+$, *and it exhibits the following two properties:* 1) $\iota_i^*(n_{t-1}, \bar{\iota}_t)$ *is (weakly) increasing in* n_{t-1}; 2) $\iota_i^*(n_{t-1}, \bar{\iota}_t)$ *is (weakly) increasing in* $\bar{\iota}_t$.

Results in Proposition 3 are straightforward implications of Lemma 2 and (2.10). Intuitively, as $\partial\Pi_{i,t}(\cdot)/\partial\iota_i$ is increasing in both n_{t-1} and $\bar{\iota}_t$, larger values of either variables will induce inventors to increase the optimal amount of effort spent in R&D.

The positive impact of n_{t-1} on ι_i^* represents the main result of this section. This feature is the underlying force generating the novel positive feedback between financial development and innovation activities proposed in this paper. Essentially, a larger n_{t-1} is associated with weaker distortions in the credit market, thereby leading to higher entrepreneurial investment which raises profit to inventors. This induces higher R&D effort which, in turn, leads to a faster rate of innovations, feeding back on n_t. This positive feedback gives rise to the possibility of non-ergodic dynamics in the model, as it will be discussed in detail in Section 2.5.

For the remainder of the paper, it proves convenient to restrict the parameters configuration such that the following two conditions hold:

Assumption 1. $\exists \bar{n} \in (0,1)$, such that: $\beta'(0)\,\Psi(\bar{n}, 0) = 1$.
Assumption 2. $\exists \underline{n} \in (0,1)$, such that: $\beta'(0)\left[\lim_{\bar{\iota}\to\infty} \Psi(\underline{n}, \bar{\iota})\right] = 1$.

Corollary 1 *If Assumption 1 holds, then: (i)* $\forall n_{t-1} \leq \bar{n}$: *if* $\bar{\iota}_t = 0 \Rightarrow \iota_i^* = 0$; *(ii)* $\forall n_{t-1} > \bar{n} : \iota_i^* > 0$, *regardless of the value taken by* $\bar{\iota}_t$.

Corollary 2 *If Assumption 2 holds, then:* $\forall n_{t-1} \leq \underline{n} : \iota_i^* = 0$, *regardless of the value taken by* $\bar{\iota}_t$. *(Notice Lemma 2 implies* $\underline{n} < \bar{n}$.*)*

FIGURE 2.2 provides a visual description of the results stated in Proposition 3. The left panel plots ι_i^* against n_{t-1}, given four different values of $\bar{\iota}_t$ (these values are: $0 < \bar{\iota}_B < \bar{\iota}_A < \infty$). Analogously, the right panel plots ι_i^* against $\bar{\iota}_t$, given five different values of n_{t-1} ($n_A < n_B < \bar{n} < n_C < 1$). Notice that the notation in both panels is consistent with each other (i.e., the value $\bar{\iota}_A$ in panel (a) corresponds to the value $\bar{\iota}_A$ in panel (b), and so on and so forth). Additionally, in FIGURE 2.2.b (although not plotted) for $n_{t-1} = \underline{n}$ we should have $\iota_i^*(n_{t-1}, \bar{\iota}_t) = 0$ for all values of $\bar{\iota}_t$. (The 45° line is just plotted for future reference.)

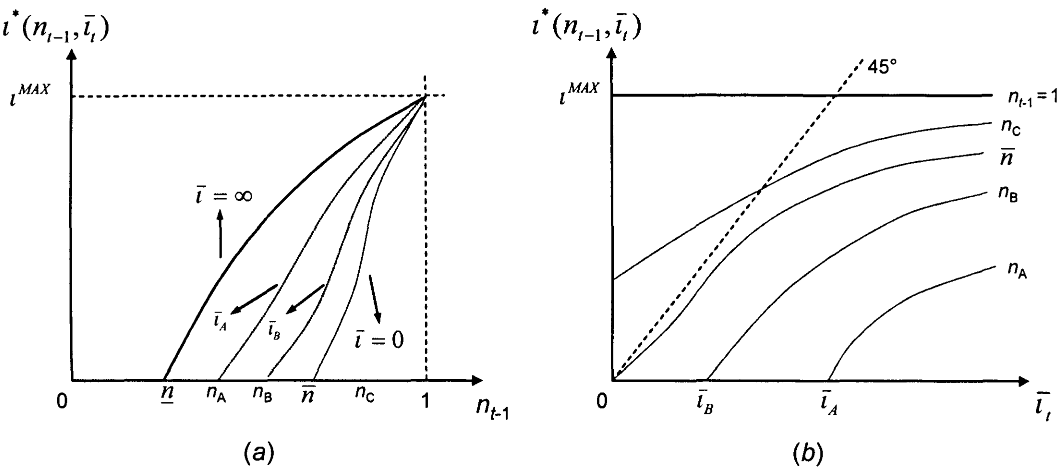

FIGURE 2.2: Optimal R&D effort as a function of n_{t-1} and $\bar{\iota}_t$.

2.4.2 Inventors Nash Equilibrium Solution

FIGURE 2.2 characterises the result of the optimisation problem faced by inventor i alive in period t when sector $i \notin \mathcal{A}_{t-1}$, given n_{t-1} and the (expected) behaviour of the rest of the inventors. Nevertheless, I haven't yet discussed whether inventors' expectations, summarised by $\bar{\iota}_t$, are indeed *correct*. In fact, expectations play an important role in the model because R&D effort by a particular inventor exerts a positive externality on the others. More specifically, as stated in Proposition 3, the optimal policy of an inventor positively depends the value of $\bar{\iota}_t$. As a result, we must restrict the attention only to those solutions of Problem (II) which also represent a *Nash Equilibrium* (NE) when we consider the whole set of inventors.

Given the structure of the model, any NE will be *symmetric* (SNE) - see Vives (2005), p. 441. The SNE are determined by the intersections between the 45° line and the curves plotted in FIGURE 2.b. For some ranges of $n_{t-1} \in (\underline{n}, 1)$, the model might lead to *multiple* SNE.[19] Equilibrium multiplicity may arise because inventors are subject to *strategic complementarities* (Cooper and John (1988)). FIGURE 2.3 shows two possible SNE schedules as a function of n_{t-1} (only the SNE schedule for an inventor i alive in t such that sector $i \notin \mathcal{A}_{t-1}$ is plotted). In FIGURE 2.2.(*b*) and 2.3.(*b*) the parameters configuration leads always (i.e., for all values of n_{t-1}) to unique SNE.[20] On the other hand, in FIGURE 2.3.(*a*) multiple equilibria emerge for values of $n_{t-1} \in (\widehat{n}, \bar{n})$ - two equilibria are possible in this case; one where $\iota_t^* = 0$, and another one in which $\iota_t^* > 0$. Bear in mind that, as it can be deduced from Corollary 2, for any $n_{t-1} \leq \underline{n}$, the SNE must necessarily be unique and encompass $\iota_t^* = 0$. Furthermore, for values of n_{t-1} sufficiently close to 1, the SNE must also necessarily be unique (since $\lim_{n \to 1} \Psi'_{\bar{\iota}} = 0$); but comprising $\iota_t^* > 0$ (because $0 < \bar{n} < 1$).

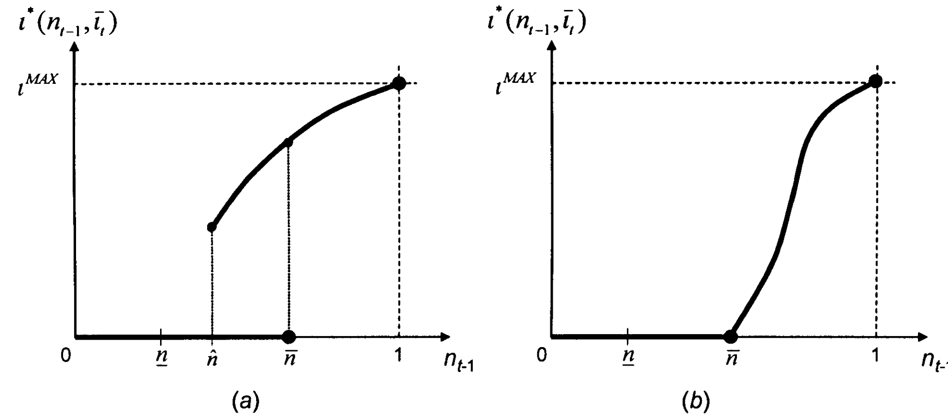

FIGURE 2.3: Inventors' Symmetric Nash Equilibrium.

Remark. Since the optimal R&D effort is a function of the bilateral-surplus, $\Delta(r_t^*)$, which has been assumed to be fully appropriated by the inventor, all the previous results of this section in terms of $\iota_i^* = \iota_i^*(n_{t-1}, \bar{\iota}_t)$ will remain unchanged if inventor i and entrepreneur i were in fact the *same* agent. All that is needed in that case is to reinterpret $\iota_{i,t}$ as the R&D effort-cost by entrepreneur i alive in period t.

2.5 Aggregate Dynamic Analysis

The analysis in Section 2.3 has been conducted within a static framework (the set $\mathcal{A}_t$ was taken as given). Section 2.4 provides the bridge between the static and the dynamic

[19] In what follows I restrict the analysis only to *stable* SNE though.

[20] A sufficient condition for *uniqueness* of SNE is that: $\frac{\partial \iota^*}{\partial \bar{\iota}} = -\frac{\beta'(\iota^*)\Psi'_{\bar{\iota}}}{\beta''(\iota^*)\Psi(n,\bar{\iota})} < 1, \forall n \in [0,1]$ and $\bar{\iota} \geq 0$. Generally speaking, *uniqueness* requires innovators' external effects not to be too strong, so that the curves plotted on FIGURE 2.2.*b* do not ever cross the 45° line from below – see Cooper and John (1988).

analysis of the economy, since the inventors' behaviour determines the evolution of the set $\mathcal{A}_t$ which, in turn, dictates the exact equilibrium that holds at any time t according to Definition 1. In this section, I present the dynamics of $\mathcal{A}_t$. Since agents are born with zero initial wealth and all sectors are (ex-ante) symmetric, n_t turns out to be the only variable whose behaviour we need study in order to keep track of the dynamics of the economy.

Definition 2 (Dynamic Equilibrium) *A dynamic equilibrium is a sequence of static equilibria, linked together across time by the "law of motion" of n_t specified in (2.11).*

$$\textbf{\textit{Law of Motion:}} \qquad n_t = n_{t-1} + \beta(\iota_t^*)(1 - n_{t-1}); \tag{2.11}$$

where ι_t^* represents the R&D effort by inventor $h \in [0,1]$ alive in period t when sector $h \notin \mathcal{A}_{t-1}$, resulting from the SNE described in Section 2.4.2. More precisely, $\forall h \notin \mathcal{A}_{t-1}$: $\iota_t^* \in \mathbb{R}^+$ solves Problem (II), given the function Υ^*: $[0,1]_{\neq h} \to \mathbb{R}^+$, that summarises $\iota_{k,t}^*$ for all $k \neq h \in [0,1]$ in period t.

2.5.1 Stagnation vs. Development (multiple dynamic equilibria)

This subsection investigates the characteristics of the dynamic paths followed by economies that differ in terms of their initial conditions. In particular, it studies whether economies that differ in terms of n_0 may follow divergent dynamic paths, reaching different long-run equilibria. For this reason, I impose here the following condition on the parameters configuration (so that the inventors' SNE will always be unique, leading to a situation as the one in FIGURE 2.3.*b*).

Assumption 3 (sufficient condition for uniqueness of SNE).

$$\frac{\partial \iota^*}{\partial \bar{\iota}} = -\frac{\beta'(\iota^*)\,\Psi'_{\bar{\iota}}}{\beta''(\iota^*)\,\Psi(n,\bar{\iota})} < 1, \text{ for all } n \in [0,1] \text{ and } \bar{\iota} \geq 0.$$

Proposition 4 (Stagnation vs. Development) *Suppose Assumptions 1 and 3 hold. Then:*

(i) Any economy that starts off with $n_0 \leq \bar{n}$ remains forever at n_0 and displays no innovation activities. That is, if $n_0 \leq \bar{n}$, then: $n_t = n_0$ for all $t \geq 0$, while $\iota_t^* = 0$ for all $t > 0$.

(ii) In any economy in which $n_0 > \bar{n}$, n_t will continuously grow over time, converging monotonically to $n_\infty = 1$.

Secular Stagnation: Take an economy for which $n_0 \leq \bar{n}$. Then, for this economy, the equilibrium in $t = 1$ encompasses $\iota_1^* = 0$. In addition to zero R&D effort and absence of innovation, this economy will exhibit highly inefficient credit provision and low levels of entrepreneurial investment. The credit market inefficiency is the consequence of severe adverse selection problems, which derive from the high degree of sector incompleteness. On the other hand, repressed entrepreneurship is the result of both lack of opportunities (few active sectors) and inadequate credit provision.

From (2.11), since $\iota_1^* = 0$, then $n_1 = n_0$. This implies that $\iota_2^* = 0$ will hold again at $t = 2$, in turn leading to $n_2 = n_1 = n_0$. Furthermore, in the absence of any substantial exogenous shock, this stagnant equilibrium will perpetuate itself for all $t \in \{0, 1, ...\infty\}$.

Prosperity and Development: Consider now an economy in which n_0 is large enough; more specifically, $n_0 > \bar{n}$. In this case, the equilibrium at $t = 1$ displays $\iota_1^* > 0$. Intuitively, since n is relatively large, the adverse selection problem associated to the allocation of talent does not become too serious, and the operation of the economy does not turn out to be severely distorted (in particular, innovation activities do not completely disappear).

From (2.11), $\iota_1^* > 0$ implies that some additional sectors become active during $t = 1$. As a result, $n_1 > n_0 > \bar{n}$, and $\iota_2^* > \iota_1^* > 0$. Moreover, this prosperous dynamics will perpetuate *ad infinitum*, and this economy will eventually reach a long-run equilibrium characterised by complete sectors ($n_\infty = 1$). During the transition period, the economy experiences development and growth; this manifests itself as a continuous process of sectoral horizontal expansion (capital differentiation) and better sorting of entrepreneurial skills. At the same time, financial market operation concomitantly improves, as adverse selection problems tend to vanish as n_t rises.

2.5.2 History vs. Expectations (multiple static equilibria)

Section 2.4.2 has shown that, within the range of $n_{t-1} \in (\underline{n}, 1)$, for some set of parameters configurations the model might display multiple SNE in the inventors game. As a particular example, in FIGURE 2.3.*a*, for $n_{t-1} \in [\hat{n}, \bar{n}]$, where $\hat{n} \in (\underline{n}, \bar{n})$, we find two possible (stable) SNE. Multiplicity of the inventors' SNE will lead to multiplicity of static equilibria in this model. It is beyond the scope of this paper to study this sort of equilibrium multiplicity, as the main intention here is to analise how dynamic paths may depend on the initial conditions. Nevertheless, I provide below a brief discussion of the equilibrium characteristics of an economy whose parameters configuration leads to a situation as the one depicted in FIGURE 2.3.*a*.

When parameters in the model lead to a situation as the one plotted in FIGURE 2.3.*a*, then if the value of $n_0 \in [\hat{n}, \bar{n}]$, this economy will be subject to multiple static equilibria. Equilibrium multiplicity will be driven by inventors' expectations. In particular, if expectations coordinate in $\bar{\iota}_1 = 0$, then $\iota_1^* = 0$ will prevail. Besides this "bad" equilibrium, we can observe that there also exists some specific value $\bar{\iota}_1^* > 0$, which would lead to a "better" equilibrium comprising $\iota_1^* = \bar{\iota}_1^* > 0$. More importantly, from a dynamic perspective, whether expectations in $t = 1$ lead to $\iota_1^* = 0$ or $\iota_1^* > 0$ may carry dramatic future consequences. Dynamically, $\iota_1^* = 0$ entails that n_t stays stagnant during period $t = 1$; as a result, initial conditions in $t = 2$ would identically replicate those faced in $t = 1$, with the economy still at risk of suffering from coordination failures. On the other hand, $\iota_1^* > 0$ means that $n_1 > n_0$ and, consequently, this could possibly shoot up n_1 above $\bar{n}$, and ignite a process of continuous prosperity and development thereafter. For an economy with $n_{t-1} \in [\hat{n}, \bar{n}]$, the larger n_{t-1} is, the higher the chances that $n_t > \bar{n}$ will hold if $\iota_t^* > 0$. Hence, within $[\hat{n}, \bar{n}]$, both *history* and *expectations* matter in the sense of Krugman (1991), and the economy might display periods of growth and technical change, followed by periods of stagnation.

2.6 Incorporating Wealth into the Model

So far it has been supposed that all individuals are born with zero initial wealth. In many aspects this assumption might seem far too extreme. Nevertheless, the zero initial wealth assumption has allowed the model to completely isolate the impact of the fraction of active sectors on the operation of the economy. In this section, I let agents be born with positive initial wealth; furthermore, I allow initial wealth to differ across individuals of the same cohort. In particular, this section features individuals who are *warm-glow* altruistic and, accordingly, bequeath a fraction of their *net life-time income* to their offspring (this bequest will constitute the next generation's initial wealth) – see Andreoni (1989). In short, this section shows that none of the main results and insights presented in this paper will be altered when we permit agents' initial wealth to be positive, stemming from parental bequests.

Let $w_{i,t}$ denote the initial wealth of the Type-i alive in period t. Initial-wealth is assumed publicly observable, and is distributed in the population of entrepreneurs according to the cumulative distribution function $\Omega_t(w)$.[21] Since types are assumed to be intergenerationally uncorrelated, then, in a steady state, initial wealth and types will turn out to be uncorrelated as well (accordingly, the specific value of $w_{i,t}$ will provide no information about the i's type).

[21] The presence of positive initial wealth will only affect the equilibrium in the economy through its effect on the entrepreneurs. Accordingly, without any loss of generality, we can restrict the attention here only to the initial wealth distribution among the population of entrepreneurs.

2.6.1 The Participation Constraint

When initial wealth is positive we need to take care of the *participation constraint* (PC) in the credit market. In particular, when $w > 0$ a bad-type might prefer not to engage in any credit market transaction, and behave as if he were in complete *autarky*, since he may now invest a positive amount of capital ($k \leq w$) in a project, without the need to borrow.

Suppose a bad-type with initial wealth w must choose his portfolio allocation in autarky. In such case, he will solve:

$$\max_{0 \leq k \leq w} : \quad p\, f(k) + (w - k).$$

This optimisation problem yields the following investment policies:

$$\begin{aligned} k^* &= w \qquad && \text{if} \qquad w \leq k_B^*, \\ k^* &= k_B^* \qquad && \text{if} \qquad w > k_B^*. \end{aligned}$$

Where $f'(k_B^*) = p^{-1}$ (i.e., k_B^* is the *first-best* investment level of bad-types).

Imagine now that this bad-type decides to participate in the credit market. In this case, he will invest $k_P^*(r)$ units of capital in the project, paying an interest rate r on the borrowed amount $(k_P^*(r) - w)$; where r corresponds to the interest rate that would hold in a pooling equilibrium. The function $k_P^*(r)$ stems form the first-order condition $f'(k_P^*) = 1 + r$; analogous to (2.3) in the main model. Notice that $1 + r \leq p^{-1}$, hence $k_P^*(r) \geq k_B^*$. A bad-type will participate in the credit market only if his PC is not violated; this requires that: $p\,[f(k_P^*(r)) - (1+r)(k_P^*(r) - w)] \geq p f(k_B^*) + (w - k_B^*)$, for $w > k_B^*$.[22] From this condition, it follows that a bad-type will participate in the credit market if and only if his initial wealth does not surpass the threshold $\widehat{w}(r) \in (k_B^*, k_P^*(r))$; that is, if and only if $w < \widehat{w}(r)$, where

$$\widehat{w}(r) \equiv \frac{p\,[f(k_P^*(r)) - f(k_B^*) - (1+r)k_P^*(r)] + k_B^*}{1 - p(1+r)}.$$

2.6.2 The Incentive Compatibility Constraint

Take an entrepreneur whose $w \geq \widehat{w}(r)$. If he is a good-type, he must get a *separating* credit contract (paying an interest rate equal to $R^f = 0$), as no bad-type with $w \geq \widehat{w}(r)$ desires to participate in the credit market at the interest rate r. Despite that, a good-type with $w \geq \widehat{w}(r)$ will not necessarily obtain a *first-best* credit contract. For this to happen, an equally rich bad-type should find no incentives to imitate the good-type

[22] The participation constraint also requires that: $p\,[f(k_P^*(r)) - (1+r)(k_P^*(r) - w)] \geq p f(w)$, for all $w \leq k_B^*$. Nevertheless, this last condition never binds.

first-best behaviour. Denote with k_G^* the result deriving from the first-order condition $f'(k_G^*) = 1$; i.e., k_G^* designates the first-best entrepreneurial investment level of the good-types. Notice that $k_G^* \geq k_P^*(r)$, since $1 + r \geq 1$. A good-type will thus receive a first-best credit contract if and only if: $p\,[f(k_G^*) - (k_G^* - w)] < pf(k_B^*) + (w - k_B^*)$. This last condition requires that his initial wealth is larger than the threshold $\widetilde{w} \in (\widehat{w}(r), k_G^*)$; that is, it calls for $w > \widetilde{w}$, where

$$\widetilde{w} \equiv \frac{p\,[f(k_G^*) - f(k_B^*) - k_G^*] + k_B^*}{1-p}.$$

What happens to a good-type whose $w \in [\widehat{w}(r), \widetilde{w}]$? This agent will certainly receive a separating contract. However, he won't be able to get a first-best contract, as this would violate the *incentive-compatibility constraint* (IC) of the bad-types with identical w. In fact, the IC will bind for those entrepreneurs whose $w \in [\widehat{w}(r), \widetilde{w}]$. As a result, the credit contract received by a good-type with $w \in [\widehat{w}(r), \widetilde{w}]$ derives from:

$$p\,[f(k_S^*) - (k_S^* - w)] = pf(k_B^*) + (w - k_B^*). \tag{2.12}$$

Equation (2.12) (implicitly) yields a function $k_S^*(w)$; which displays the following properties: (*i*) $dk_S^*/dw = \frac{1-p}{p}\left(f'(k_S^*) - 1\right)^{-1} > 0$, (*ii*) $d^2k_S^*/(dw)^2 > 0$, and (*iii*) $\lim_{w \to \widetilde{w}} k_S^*(w) = k_G^*$. TABLE 2.A summarises the main features displayed by the credit contracts offered to entrepreneurs.[23]

TABLE 2.A: Equilibrium Contracts (main features)

	$w < \widehat{w}(r)$	$w \in [\widehat{w}(r), \widetilde{w}]$	$w > \widetilde{w}$
type of credit contract	pooling	sub-optimal separating	first-best separating
investment by *good-types*	$k_P^*(r)$	$k_S^*(w)$	k_G^*
interest rate (on credit)	$0 < r < \frac{1-p}{p}$	0	0

2.6.3 Entrepreneurial Consumption and Sketch of Dynamics

As in Section 2.3.3, denote by U_g (U_b) the expected utility level achieved by a good-type (bad-type). When initial wealth is incorporated into the model, it will naturally be the case that (expected) utility will depend on w as well – i.e., $U_g = U_g(r, w)$ and $U_b = U_b(r, w)$. TABLE 2.B summarises how entrepreneurial expected utility depends on w (and r).

[23] The underlying reason why richer agents receive more favourable credit contracts is the same as in the papers on financial markets imperfections and poverty cited in the Introduction. Namely, since richer agents have more of their own wealth at stake in the projects, their incentives are more closely aligned to those of lenders.

TABLE 2.B: Entrepreneurial Consumption – $U_g(r,w)$ and $U_b(r,w)$.

	$w < \widehat{w}(r)$	$w \in [\widehat{w}(r), \widetilde{w}]$	$w > \widetilde{w}$
good-types	$f(k_P^*(r)) - (1+r)(k_P^*(r) - w)$	$f(k_S^*(w)) - (k_S^*(w) - w)$	$f(k_G^*) - (k_G^* - w)$
bad-types	$p[f(k_P^*(r)) - (1+r)(k_P^*(r) - w)]$	$pf(k_B^*) + (w - k_B^*)$	$pf(k_B^*) + (w - k_B^*)$

From the results presented in TABLE 2.B, this lemma follows, (a formal proof of this lemma is available from the author upon request).

Lemma 3 *Let* $\Delta(r,w) \equiv U_g(r,w) - U_b(r,w)$. *Then: (i)* $\Delta(\cdot) > 0$, $\forall w, r \geq 0$; *(ii)* $\Delta_r'(\cdot) < 0$, $\forall r \geq 0$ *and* $w \in [0, \widehat{w}(r)]$; *(iii) a)* $\Delta_w'(\cdot) > 0$, $\forall w \in [0, \widetilde{w})$ *and* $r \geq 0$; *b)* $\Delta_w'(\cdot) = 0$, $\forall w \geq \widetilde{w}$.

Lemma 3 represents the counterpart of Lemma 1, when entrepreneurs start their lives with positive wealth. On the one hand, Lemma 3 shows that Lemma 1's key result $\Delta_r'(\cdot) < 0$ holds as well when $w > 0$. On the other hand, it shows that the gap $\Delta(\cdot)$ is (weakly) increasing in w, which implies that richer entrepreneurs benefit from a larger n_t more than poorer entrepreneurs do. Furthermore, recall that the larger $\Delta(\cdot)$ is, the higher the incentives for inventors to undertake R&D – Lemma 2 and Proposition 3. Therefore, $\Delta_w'(\cdot) > 0$ entails that, for a given value of n_t –which, following Proposition 2, will determine $r^*(n_t)$–, the aggregate distortions generated by the adverse selection problem in the credit market will become less severe the wealthier the economy is. FIGURE 2.4 plots the gap $\Delta(r,w)$ against w at four different values of r (namely: $1/p > r_H > r_L > 0$), as a visual description of results in Lemma 3.[24]

From a dynamic perspective, notice finally that economies exhibiting a larger n_t tend to be richer as well. This is the case because the larger the fraction of active sectors, the higher the average productivity in the economy. As a result, introducing wealth dynamics into the model (by means of bequests, or any other reason that would still generate positive serial correlation in w_t) will not invalidate any of the main findings of this paper. In fact, as n_t and wealth affect the economy's performance in the same direction, the presence of bequests will actually reinforce the dynamics previously discussed in Section 2.5.

[24]Recall $r = p^{-1}$ when $n = 0$, and $r = 0$ when $n = 1$. Additionally, notice $\widehat{w}'(r) < 0$, where $\lim_{r \to p^{-1}} \widehat{w}(r) = k_B^*$ and $\lim_{r \to 0} \widehat{w}(r) = \widetilde{w}$.

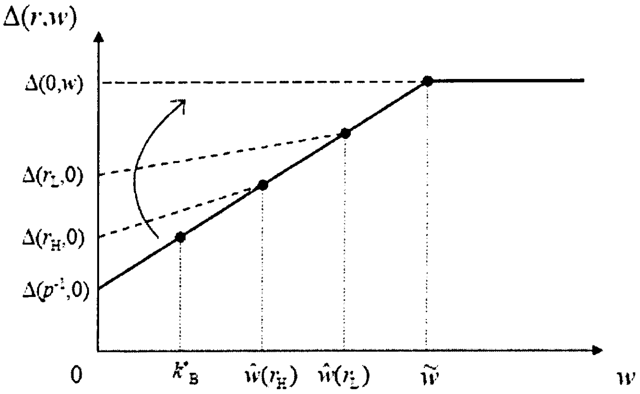

FIGURE 2.4: $\Delta(r,w)$ against w at four different levels of r.

2.6.4 Dynamics with Positive Bequests

Suppose preferences are given by $U_{i,t} = c_{i,t}^{1-\delta}\, b_{i,t}^{\delta}$; where $c_{i,t}$ denotes the consumption of agent i alive in t, $b_{i,t}$ represents the bequest left to his offspring, and $\delta \in (0,1)$. Given those preferences, individuals will optimally bequeath a fraction δ of their lifetime income to their offspring. The amount $b_{i,t}$ will in turn fully determine the initial wealth of i's son; i.e., $w_{i,t+1} = b_{i,t}$. Henceforth, we split the population of entrepreneurs in *lineages* indexed by the letter $i \in [0,1]$. Since types are intergenerationally uncorrelated, the initial wealth transition equations for any lineage i of entrepreneurs are given by:

$$w_{i,t+1} = \begin{cases} \delta\left[f(k_P^*(r_t)) - (1+r_t)(k_P^*(r_t) - w_{i,t})\right] & \text{with } \Pr = n_t + p(1-n_t) \\ 0 & \text{with } \Pr = (1-p)(1-n_t) \end{cases} \quad \text{if } w_{i,t} < \widehat{w}(r_t)$$

$$w_{i,t+1} = \begin{cases} \delta\left[f(k_S^*(w_{i,t})) - k_S^*(w_{i,t}) + w_{i,t}\right] & \text{with } \Pr = n_t \\ \delta\left[f(k_B^*) - k_B^* + w_{i,t}\right] & \text{with } \Pr = p(1-n_t) \\ \delta\left[w_{i,t} - k_B^*\right] & \text{with } \Pr = (1-p)(1-n_t) \end{cases} \quad \text{if } w_{i,t} \in [\widehat{w}(r_t), \widetilde{w}]$$

$$w_{i,t+1} = \begin{cases} \delta\left[f(k_G^*) - k_G^* + w_{i,t}\right] & \text{with } \Pr = n_t \\ \delta\left[f(k_B^*) - k_B^* + w_{i,t}\right] & \text{with } \Pr = p(1-n_t) \\ \delta\left[w_{i,t} - k_B^*\right] & \text{with } \Pr = (1-p)(1-n_t) \end{cases} \quad \text{if } w_{i,t} > \widetilde{w}$$

When w is linked across generations by bequests, the dynamics of the economy can no longer be solely determined by the value of n_t but also depend on the initial wealth distribution $\Omega_t(w)$. In particular, the economy's dynamic path is now dictated by the following system:

$$n_t = n_{t-1} + \beta(\iota_t^*)(1 - n_{t-1}) \tag{2.13}$$

$$\Omega_{t+1}(w) = \Gamma_t\left[\Omega_t(w)\right]. \tag{2.14}$$

Where:

$$\iota_t^* = \arg\max_{\iota} \left\{ \beta(\iota) \int_{\Omega_t(w)} \Delta(r_t, w)\, d\Omega_t(w) - \iota \right\}. \tag{2.15}$$

Remark. For this section we continue assuming that the NE of the inventors' game is always unique (or, alternatively, that coordination failures, even if possible, do not arise). Accordingly, from (2.15), we can write $\iota_t^* = \iota_t^*(n_{t-1}, \Omega_t(w))$, as the function that pins down the optimal ι_t, given n_{t-1} and the initial wealth distribution $\Omega_t(w)$. (Recall, once again, that n_{t-1} determines n_t which in turn determines r_t; hence we can write r_t as a function of n_{t-1}).

The operator $\Gamma_t\ [\cdot]$ maps the initial wealth distribution prevailing in period t into the initial wealth distribution holding in $t+1$, based on the transition equations specified above. Notice that this operator changes over time, since the transition equations and their associated occurrence probabilities both depend on the value of n_t. Additionally, the dynamic behaviour of n_t is affected by $\Omega_t(w)$ through (2.15). These two features of the dynamic system described by (2.13) and (2.14) make it non-stationary and highly complicated to study. Some interesting general results are however not difficult to prove.

Lemma 4 *(i) Consider two different initial wealth distributions $\Omega_t(w)$ and $\Omega'_t(w)$, and suppose $\Omega_t(w)$ first-order stochastically dominates $\Omega'_t(w)$ – henceforth denoted as $\Omega_t(w) \succeq \Omega'_t(w)$. Then: $\iota_t^*(n_{t-1}, \Omega_t(w)) \geq \iota_t^*(n_{t-1}, \Omega'_t(w))$.*
(ii) Consider two economies (A and B) with identical initial wealth distribution, i.e. $\Omega_t^A(w) = \Omega_t^B(w) = \Omega_t(w)$. Suppose also that $n_t^A > n_t^B$. Then: $\Omega_{t+1}^A(w) \succeq \Omega_{t+1}^B(w)$.

Lemma 4 (i) states that, all other things equal, wealthier economies tend to spend more in R&D, and its underlying intuition is straightforward from Lemma 3.[25] On the other hand, Lemma 4 (ii) says that economies with a larger fraction of active sectors tend to be richer too. The reason for this result lies in two combined effects: first, a higher n_t means that more agents are able to find a sector in which they have a comparative advantage, increasing the average productivity in the economy; second, a higher n_t leads to the provision of better credit contracts, spurring entrepreneurial investment. Lemma 4 thus formally proves that introducing wealth dynamics into the model (through bequests motives) reinforces the dynamics that have been described before in Section 2.5.

Proposition 5 *Suppose Assumption 1 holds, where we should now interpret $\Psi(n_{t-1}, \iota_t^*) = \Delta(r_t^*(n_{t-1}, \iota_t^*), 0)$, and let $\Omega_{\widetilde{w}}$ (Ω_0) denote the degenerate distribution function in which*

[25] Notice that given the shape of $\Delta(r, w)$ as plotted in FIGURE 2.4, we cannot say much about the effect of higher moments of $\Omega_t(w)$ on ι_t^*. In particular, since $\Delta(r, w)$ has initially a convex segment (with respect to w), followed by a concave segment, the effect on ι_t^* of subjecting $\Omega_t(w)$ to a *mean-preserving spread* is ambiguous.

$w_i = \widetilde{w}$ ($w_i = 0$) for all $i \in [0,1]$. Then:

(i) If $n_{t-1} > \bar{n}$, n_t will converge monotonically to $n_\infty = 1$, regardless of $\Omega_t(w)$.

(ii) Suppose $\Omega_t(w) = \Omega_{\widetilde{w}}$. Then, there exists $\bar{n}_{\widetilde{w}} < \bar{n}$ such that if $n_{t-1} > \bar{n}_{\widetilde{w}}$, n_t will converge monotonically to $n_\infty = 1$

(iii) Suppose $\Omega_0 \preceq \Omega_t(w) \preceq \Omega_{\widetilde{w}}$. Then, $\exists\, \bar{n}_{\Omega(w)} \in [\bar{n}_{\widetilde{w}}, \bar{n}]$ such that if $n_{t-1} > \bar{n}_{\Omega(w)}$, n_t will converge monotonically to $n_\infty = 1$. Furthermore, consider $\Omega_t(w) \succeq \Omega_t'(w)$, then $\bar{n}_{\Omega(w)} \leq \bar{n}_{\Omega'(w)}$.

Proposition 5 firstly shows that the main result in Proposition 4 still holds true when we incorporate standard wealth dynamics into the model – when n_t is sufficiently large, the economy embarks in a sustainable process of development, regardless of the wealth distribution in t. Secondly, it shows that initial wealth acts as a partial "substitute" for n_t. This last result stems from the fact that both n_t and w_t contribute to alleviate adverse selection problems in the credit market. Notice that Proposition 5 (*ii*) and (*iii*) imply that the minimum degree of sectoral variety needed to guarantee long-run growth is smaller the richer the economy is. This result can be interpreted as saying that the importance of sectoral diversification as a factor improving the operation of financial markets is relatively higher at initial stages of development, and tends to decrease as the economy develops and becomes wealthier.

2.7 Concluding Remarks

This paper has proposed a theory in which financial markets efficiency is a key condition for growth and development. I have suggested that an expanding variety of activities available in the economy may account for a very important factor leading to financial development. In particular, this theory has stressed a side effect associated to the innovation process that had not been explored before, but which could exert significant impact on development. I have argued that innovation activities can lead to a reduction of frictions in financial markets and foster financial development, because by expanding the variety of productive activities in the economy, they concomitantly facilitate the allocation of skills, alleviating adverse selection problems.

The core model that illustrates this theory has made use of several simplifying assumptions. One assumption that may seem particularly worrying is the fact that individuals are born with no initial wealth. In that regard, Section 2.6 has shown that none of the model's main findings would be affected if we let agents be born with positive wealth. Despite not altering its main results, introducing wealth may carry some additional interesting implications within a more general model. Imagine we gave room for increasing returns to scale and international trade. If sectoral diversification really matters as a mechanism to solve adverse selection only at early stages of development (as suggested by Section 2.6), then in the presence of increasing returns and trade, at some point in the development path economies might find it worthwhile to revert the diversification tendency and embark in some sort of re-specialisation process. This feature would be in fact consistent with the evidence found in Imbs and Wacziarg (2003), providing a sound explanation for the *non-monotonic* relation between sectoral diversification and income per-head shown initially in FIGURE 2.1.

Another feature of the model that deserves further discussion is the financial intermediaries behaviour. In this model financiers respond "passively" to the environment. However, it can be argued that the financial system operation improves along development not only because frictions are alleviated, but also because the screening capacity of the financiers gets better. The paper has abstracted from the latter mechanism. One remark concerning this omission is worth noting, though. The amount of screening effort is an endogenous choice, and will be certainly influenced by the cost of screening. This paper states that screening effort is eased by sectoral variety, as this allows heterogeneous agents to self-select better. Yet, this is not necessarily implying that richer economies should do less credit screening than poorer ones. In fact, as sectoral variety decreases the cost of screening, in some cases more screening effort could be the optimal response by lenders to the new environment, rather than simply deny credit so as to avoid the screening cost fully.

From a policy perspective, an important implication of this theory concerns poverty-alleviation programmes. Section 2.5 has shown that some economies might get stuck in a peculiar type of *poverty-trap*. This is the result of a "deep-rooted" organisational failure, affecting several markets at the same time. Underdevelopment is characterised by few sectors in which individuals can specialise, inefficient financial markets, and scant innovation activities. The market failure contaminating the operation of the economy stems from the incapacity of some individuals to find an activity for which they are comparatively talented. Most theories on poverty-traps imply that economies can be easily rescued from poverty by receiving a sufficiently large wealth transfer. Instead, my theory suggests that foreign-aid should presumably also include important transfers of technology and know-how, as standard wealth transfers alone might not suffice to suppress the adverse selection problem (at least in a reasonably short time frame).

Appendix A: Financial Development and Sectoral Diversification in the Data

TABLE A.1: Summary Statistics

Dependent Variable: Log(Cred/GDP) - 1187 observations

Variable	Mean	Median	Std. Dev.	Min	Max
Log(Credit/GDP)	-1.355	-1.277	1.202	-15.12	0.554
Log Income per-head	1.30	1.38	0.98	-1.24	3.00
Herfindahl Index	0.119	0.087	0.099	0.060	0.877

Dependent Variable: Log(SMVK /GDP) - 471 observations

Variable	Mean	Median	Std. Dev.	Min	Max
Log(SMKT/GDP)	-2.198	-2.122	1.379	-7.13	0.339
Log Income per-head	1.70	1.78	0.81	-0.15	2.90
Herfindahl Index	0.092	0.079	0.056	0.060	0.450

Dependent Variable: Log(SMVT/GDP) - 517 observations

	Mean	Median	Std. Dev.	Min	Max
Log(SMVT/GDP)	-4.145	-3.809	2.092	-10.78	1.867
Log Income per-head	1.72	1.79	0.79	-0.21	2.90
Herfindahl Index	0.087	0.078	0.043	0.060	0.423

Note: Log income per-head equal to -1.24 corresponds to income per head 290 in 1985 PPP US dollars. This is the income per head (in PPP) of Ethiopia in 1967. Log income per-head equal to 2.90 corresponds tc income per head 18,095 in 1985 PPP US dollars. This is the income per head (in PPP) of US in 1989.

TABLE A.2: Sectoral Diversification and Financial Development

Independent Variable	Dependent Variable					
	Log(Credit/GDP)		Log(SMK/GDP)		Log(SMVT/GDP)	
	(1)	(2)	(3)	(4)	(5)	(6)
Sectoral Concentration (Herfindahl)	-0.813** (-2.51)	-1.178*** (-3.56)	-3.47** (-2.35)	-4.25*** (-2.88)	5.83 (1.47)	-1.41 (-0.32)
Log Income per-head (Y)	0.806*** (16.37)	0.575*** (8.27)	2.88*** (16.62)	2.05*** (6.87)	4.634*** (13.58)	2.72*** (4.15)
$Y \times$ Herfindahl (interaction term)		2.062*** (4.65)		8.26*** (3.42)		20.23*** (3.42)
R squared (within)	0.21	0.22	0.41	0.43	0.28	0.30
Obs. / Countries	1187 / 56	1187 / 56	471 / 39	471 / 39	517 / 40	517 / 40

Note: t-statistic in parentheses. All regressions include an intercept and country fixed-effects.
Regressions are run on an unbalanced panel of countries during years 1975 - 92.
Log(Cred/GDP) is the logarithm of Total Private Credit to GDP. Log(SMK/GDP) is the log of Stock Market Capitalisation to GDP.
Log(SMVT/GDP) is the log of Stock Market Value Traded to GDP.
Log Income per-head is the log of GDP per-head in PPP in 1,000 of 1985 US dollars from Summers and Heston (1991).
The Herfindahl coefficients are based on the UNIDO 3-digit employment dataset from Imbs and Wacziarg (2003)
* significant at 10% level, ** significant at 5% level, *** significant at 1% level.

One of the main predictions of the paper is the positive feedback between the level of sectoral diversification and the size of the financial system. This feedback implies that those two variables should display positive correlation in cross-country data. TABLE A.2 presents some evidence of this correlation for an unbalanced panel of countries during 1975-92 (TABLE A.1 shows the summary statistics for the regressions in TABLE A.2). To quantify the level of financial development, I take three different indicators traditionally used in the literature of financial development and growth: 1) the logarithm of the ratio of private credit by financial institutions to GDP, Log(Credit/GDP); 2) the logarithm of the ratio of stock market capitalisation to GDP, Log(SMK/GDP); 3) the logarithm of the ratio of stock market value traded to GDP, Log(SMVT/GDP).[26] To measure the degree of sectoral concentration, I use the Herfindahl indices for the employment shares across the 28 manufacturing sectors in the UNIDO 3-digit dataset; this measure of sectoral concentration is also used in Imbs and Wacziarg (2003), from which I take the data.[27]

In columns (1), (3) and (5) of TABLE A.2, the financial development indicators are regressed against the sectoral concentration index, after controlling for country fixed-effects and including GDP per-capita as an additional regressor. Country fixed-effects are included because the intention of the paper is to follow individual economies over their own path of development. On the other hand, including GDP per-head controls for the fact that financial indicators and sectoral diversification might be moving together just as consequence of income shocks affecting both variables simultaneously. The equations estimated in the odd columns in TABLE A.2 display thus the following structure: $FD_{i,t} = \alpha + \beta Y_{i,t} + \gamma H_{i,t} + \zeta_i + \nu_{i,t}$, where $FD_{i,t}$ denotes the level of financial development of country i in year t, $Y_{i,t}$ stands for logarithm of income per-head of i in t, $H_{i,t}$ is the Herfindahl for the sectoral employment-shares of i in t, ζ_i is a country fixed-effect, and $\nu_{i,t}$ is an idiosyncratic shock. From columns (1) and (3) we can observe that the estimated γ exhibits the expected sign and is also highly significant. According to those two regressions, sectoral diversification is positively and significantly correlated with financial development *within* each country over the years of the sample, even after controlling for the possibility of common income shocks. When the stock market value traded to GDP ratio is used as a proxy for financial development, the estimate turns out

[26] All data on financial indicators is taken from Beck *et al* (1999). Refer to this paper for a detailed description of those indicators.

[27] The reason why I am using here the Herfindahl instead of the Gini to measure sectoral concentration is that the former displays more variability than the latter, so it permits a more precise estimation of the coefficients in TABLE A.2. In particular, the coefficient associated to the *interaction term* in TABLE A.2. cannot be precisely estimated if using the Gini, while this is not the case if using the Herfindahl. To have an idea of the problem, the correlation between ($Y\times$ Gini) and Y is 0.98, while the correlation between ($Y\times$ Herfindahl) and Y is 0.74.

to be insignificant and displays the opposite sign. About this issue, credit to GDP and stock market capitalisation to GDP can be argued to be better proxies for the level of financial development than the ratio of stock market value traded to GDP, which seems more to account for the liquidity of the stock market rather than for the size of that market. In that regard, the supportive evidence in columns (1) and (3) is presumably more compelling than the ambiguous result in column (5).

Section 2.6 of this paper suggests that sectoral diversification should play a more important role in poorer economies compared to richer ones. In short, that section has shown that wealth-effects operate in the same direction as the variety of sectors. Therefore, given the variety of sectors, richer economies would tend to suffer from less severe adverse selection, displaying accordingly higher financial development. In order to test for the presence of this effect, regressions (2), (4) and (6) in TABLE A.2 include an *interaction term* between the log of GDP per-head (Y) and the degree of sectoral concentration (Herfindahl). The estimated equations in the even columns are then: $FD_{i.t} = \alpha + \beta Y_{i,t} + \gamma H_{i,t} + \lambda[Y_{i,t} \times H_{i,t}] + \zeta_i + \nu_{i,t}$. As predicted by Section 2.6, all the estimated λ have the expected positive sign, being also highly significant. Furthermore, in column (6), including the interaction term makes the coefficient associated to $H_{i,t}$ become negative as predicted by the paper (although it still remains statistically insignificant).

Appendix B: Proofs

Proof of Proposition 1. Take two different credit contracts $(l^*, r^*) \in [0, \infty) \times [0, \infty)$ and $(\tilde{l}, \tilde{r}) \in [0, \infty) \times [0, \infty)$, such that $f'(k = l^*) \geq 1$ and $f'(k = \tilde{l}) \geq 1$.[28] Hence, in equilibrium, all the amount that is borrowed will be invested in the entrepreneurial projects. Accordingly, let's denote: $k^* = l^*$ and $\tilde{k} = \tilde{l}$. Assume that:

$$f(k^*) - (1 + r^*)k^* > f(\tilde{k}) - (1 + \tilde{r})\tilde{k} \tag{P.1.1}$$

Then, from (P.1.1), if the Type-i decides to specialise in sector $i \in \mathcal{A}$, he will prefer contract (k^*, r^*) to contract $(\tilde{k}, \tilde{r})$.

Take now the Type-j. Since sector $j \notin \mathcal{A}$, he will specialise (indifferently) in any sector $h \in [0, 1]$, such that sector $h \in \mathcal{A}$. Given *limited-liability*, the Type-j will (weakly) prefer contract $(\tilde{k}, \tilde{r})$ to contract (k^*, r^*), if and only if:

$$p[f(\tilde{k}) - (1 + \tilde{r})\tilde{k}] \geq p\left[f(k^*) - (1 + r^*)k^*\right] \tag{P.1.2}$$

[28]It must be straightforward to notice that entrepreneurs only borrow in order to finance entrepreneurial investment. Therefore, in equilibrium, they would never borrow beyond the point $f'(k) = 1$.

But, since $p > 0$, (P.1.2) contradicts (P.1.1). Hence, it cannot be true that, while the Type-i prefers contract (k^*, r^*) to contract $(\tilde{k}, \tilde{r})$, the Type-j prefers $(\tilde{k}, \tilde{r})$ to (k^*, r^*) instead. Finally, since $(\tilde{k}, \tilde{r})$ and (k^*, r^*) can be any credit contracts; whenever the Type-i prefers (k^*, r^*) to $(\tilde{k}, \tilde{r})$, then the Type-j also prefers (k^*, r^*) to $(\tilde{k}, \tilde{r})$, and no equilibrium can possibly encompass separating credit contracts among those two types. ■

Proof of Proposition 2. Differentiating (2.6) with respect to n_t yields:

$$dr_t^*/dn_t = -(1-p)\left[n_t + (1-n_t)p\right]^{-2} < 0. \quad \blacksquare$$

Proof of Lemma 2. Assume that the inventor $i \in [0,1]$ alive in t expends ι_i units of effort. If he manages to generate a new idea, then from Lemma 1 it should be straightforward that he will optimally charge a price equal to $\Delta(r_t^*)$ to transfer the idea (to the Type-i) - this is the maximum price the inventor i could charge, while the Type-i is still willing to buy the new idea. Making use of Proposition 2, we can write $\Delta(r_t^*) = \Delta(r^*(n_t)) \equiv \widetilde{\Delta}(n_t)$, where $\widetilde{\Delta}'(n_t) = \Delta'(\cdot)\frac{dr_t^*}{dn_t} > 0$ (from Proposition 2 and Lemma 1). How is the value n_t determined? Suppose all inventors belonging to $-\mathcal{A}_{t-1}^{-i}$ choose $\bar{\iota}_t$. Since active sectors in $t-1$ never revert to inactive in t, and recalling (2.9), then:

$$n_t = n_{t-1} + (1 - n_{t-1})\beta(\bar{\iota}_t) \equiv \Phi(n_{t-1}, \bar{\iota}_t) \tag{L.2.1}$$

Notice that, because $\beta(\bar{\iota}_t)$ is bounded away from 1, (L.2.1) implies $\Phi(\cdot)$ is increasing in both n_{t-1} and $\bar{\iota}_t$. Now, plugging $\Phi(\cdot)$ from (L.2.1) into $\widetilde{\Delta}(n_t)$, we can rewrite $\widetilde{\Delta}(\Phi(n_{t-1}, \bar{\iota}_t)) \equiv \Psi(n_{t-1}, \bar{\iota}_t)$. From where it follows that: (i) $\Psi_n' = \widetilde{\Delta}'(n_t)\,(1-\beta(\bar{\iota}_t))n_{t-1} > 0$; (ii) $\Psi_{\bar{\iota}}' = \widetilde{\Delta}'(n_t)\,(1-n_{t-1})\beta'(\bar{\iota}_t)$, which leads to $\Psi_{\bar{\iota}}' > 0$ if $n_{t-1} \in [0,1)$ and $\Psi_{\bar{\iota}}' = 0$ if $n_{t-1} = 1$.

Finally, noting that having exerted effort ι_i, inventor i will succeed in generating a new idea with probability $\beta(\iota_i)$, we may write: $\Pi_{i,t}(\iota_i, n_{t-1}, \bar{\iota}_t) = \beta(\iota_i) \cdot \Psi(n_{t-1}, \bar{\iota}_t) - \iota_i$; which is the expression stipulated in Lemma 2. ■

Proof of Proposition 3. Part 1). Consider two values of n_{t-1}; $n_0, n_1 \in [0,1]$, such that $n_0 < n_1$. Denote: $\iota_0^* \equiv \iota_i^*(n_0, \bar{\iota}_t)$ and $\iota_1^* \equiv \iota_i^*(n_1, \bar{\iota}_t)$; where $\bar{\iota}_t \geq 0$. Finally, suppose $\iota_0^* > \iota_1^*$. Thus, from (2.10), it follows that:

$$\beta'(\iota_1^*)\Psi(n_1, \bar{\iota}_t) \leq \beta'(\iota_0^*)\Psi(n_0, \bar{\iota}_t). \tag{P.3.1}$$

Since, $\beta''(\iota) < 0$, when $\iota_0^* > \iota_1^*$, $\beta'(\iota_0^*) < \beta'(\iota_1^*)$ must then hold. As a result, (P.3.1) necessarily requires that: $\Psi(n_0, \bar{\iota}_t) > \Psi(n_1, \bar{\iota}_t)$, which contradicts $\Psi_{n_{t-1}}' > 0$ for all $\bar{\iota}_t \geq 0$ proved in Lemma 2. Consequently, $n_0 < n_1 \Rightarrow \iota_0^* \leq \iota_1^*$.

Remark. Notice that if $\iota_0^* > 0$, then, given that $\iota_0^* \leq \iota_1^*$, (2.10) leads to: $\beta'(\iota_1^*)\Psi(n_1, \overline{\iota}_t) = \beta'(\iota_0^*)\Psi(n_0, \overline{\iota}_t)$. From where we obtain: $\iota_1^* > \iota_0^*$ (since $\Psi(n_1, \overline{\iota}_t) > \Psi(n_0, \overline{\iota}_t)$, due to Lemma 2).

Part 2). Take two values of $\overline{\iota}$; $\overline{\iota}_a, \overline{\iota}_b \in \mathbb{R}^+$, such that $\overline{\iota}_a > \overline{\iota}_b$. Denote: $\iota_a^* \equiv \iota_i^*(n_{t-1}, \overline{\iota}_a)$ and $\iota_b^* \equiv \iota_i^*(n_{t-1}, \overline{\iota}_b)$; where $n_{t-1} \in [0,1]$. Finally, suppose $\iota_a^* < \iota_b^*$. Then, from (2.10), it follows that:

$$\beta'(\iota_a^*)\Psi(n_{t-1}, \overline{\iota}_a) \leq \beta'(\iota_b^*)\Psi(n_{t-1}, \overline{\iota}_b). \qquad \text{(P.3.2)}$$

In addition to that, $\beta''(\iota) < 0$ implies that, if $\iota_a^* < \iota_b^*$, then $\beta'(\iota_a^*) > \beta'(\iota_b^*)$. As a result, (P.3.2) necessarily requires: $\Psi(n_{t-1}, \overline{\iota}_a) < \Psi(n_{t-1}, \overline{\iota}_b)$, which contradicts $\Psi'_{\overline{\iota}} > 0$ for all $n_t \in [0,1)$ (and $\Psi'_{\overline{\iota}} = 0$ when $n_t = 1$), proved in Lemma 2. Therefore, $\overline{\iota}_a > \overline{\iota}_b \Rightarrow \iota_a^* \geq \iota_b^*$. ■

Remark. Notice that if $\iota_b^* > 0$ and $n_{t-1} \in [0,1)$, then, bearing in mind that $\iota_b^* \leq \iota_a^*$, (2.10) leads to: $\beta'(\iota_b^*)\Psi(n_{t-1}, \overline{\iota}_b) = \beta'(\iota_a^*)\Psi(n_{t-1}, \overline{\iota}_a)$. From where it follows: $\iota_a^* > \iota_b^*$ (because $\Psi(n_{t-1}, \overline{\iota}_a) > \Psi(n_{t-1}, \overline{\iota}_b)$, due to results in Lemma 2).

Proof of Corollary 1. (*i*) Since, from Lemma 2, $\Psi'_n(\cdot) > 0$, setting $\overline{\iota}_t = 0$ we obtain:

$$\beta'(0)\Psi(n_{t-1}, 0) \leq \beta'(0)\Psi(\bar{n}, 0) = 1, \quad \forall n_{t-1} \leq \bar{n}. \qquad \text{(C.1.1)}$$

Thus, given $\beta''(\iota) < 0$ and the conditions stated in (2.10), (C.1.1) entails that $\iota_i^* = 0$ must necessarily prevail for any value of $n_{t-1} \leq \bar{n}$ when $\overline{\iota}_t = 0$.
(*ii*) Since $\Psi'_n(\cdot) > 0$, it follows that:

$$\beta'(0)\Psi(n_{t-1}, 0) > \beta'(0)\,\Psi(\bar{n}, 0) = 1, \quad \forall n_{t-1} > \bar{n}. \qquad \text{(C.1.2)}$$

Therefore, given $\beta''(\iota) < 0$, (C.1.2) implies that $\iota_i^* > 0$ must necessarily hold for any $n_{t-1} > \bar{n}$ when $\overline{\iota}_t = 0$, so that to comply with (2.10). Finally, since $\Psi'_{\overline{\iota}}(\cdot) \geq 0$,

$$\beta'(0)\Psi(n_{t-1}, \overline{\iota}_t) \geq \beta'(0)\Psi(n_{t-1}, 0) > \beta'(0)\,\Psi(\bar{n}, 0) = 1, \quad \forall n_{t-1} > \bar{n} \text{ and } \overline{\iota}_t > 0.$$

Hence, in order to comply with (2.10), $\iota_i^* > 0$ must hold for all $n_{t-1} > \bar{n}$ and $\overline{\iota}_t \geq 0$. ■

Proof of Corollary 2. Since $\Psi'_{\overline{\iota}}(\cdot) \geq 0$, then: $\beta'(0)\,\Psi(n_{t-1}, \infty) \geq \beta'(0)\,\Psi(n_{t-1}, \overline{\iota}_t)$, for all values of $\overline{\iota}_t \geq 0$ and $n_{t-1} \in [0,1]$. As a result, if $\beta'(0)\,\Psi(\underline{n}, \infty) = 1$, it must be the case that:

$$\beta'(\iota_i)\,\Psi(n_{t-1}, \overline{\iota}_t) \leq \beta'(0)\,\Psi(n_{t-1}, \infty) \leq 1, \quad \forall n_{t-1} \leq \bar{n}, \text{ and } \iota_i, \overline{\iota}_t > 0. \qquad \text{(C.2.1)}$$

Thus, given (2.10), from (C.2.1) it follows $\iota_i^* = 0$ must hold for all $n_{t-1} \leq \underline{n}$ and $\overline{\iota}_t \geq 0$. ■

Proof of Proposition 4. (i) Take an economy in which $n_0 \leq \bar{n}$ and focus on equilibrium $t = 1$. Given Assumption 1, Corollary 1 implies there must exist a SNE for the inventors game in which $\iota_1^* = 0$. On the other hand, Assumption 3 entails that this SNE is unique. Since $\beta(0) = 0$, then (2.11) implies that $n_1 = n_0 \leq \bar{n}$. As a result, in $t = 2$ conditions for the inventors game remain identical as they were at $t = 1$; thus, $\iota_2^* = 0$ represents again the unique SNE in $t = 2$. Repeating the same argument *ad infinitum*, it follows that: $n_t = n_0\ \forall t \geq 0$ and $\iota_t^* = 0\ \forall t > 0$.

(ii) Take an economy where $n_0 > \bar{n}$ and focus on $t = 1$. Given Assumption 1, Corollary 1 implies that $\iota_1^*(n_0, 0) > 0$. As a result, there must necessarily exist a SNE for the inventors game in $t = 1$ in which $\iota_1^* > 0$. Given Assumption 3, then this $\iota_1^* > 0$ represents the unique SNE. Since $\iota_1^* > 0$, from (2.11) it follows that $n_1 = n_0 + \beta(\iota_1^*)(1 - n_0)$; hence, $n_1 > n_0$. In particular, this leads to $n_1 > n_0 > \bar{n}$. Proposition 3 then implies that $\iota_2^* > \iota_1^* > 0$. As a result of this, $n_2 > n_1$. Repeating this argument *ad infinitum,* we can observe that: $\bar{n} < n_0 < n_1 < n_2 < ... < n_\infty$. Furthermore, since $\beta(\iota_t^*)(1 - n_{t-1}) \to 0$ as $n_t \to 1$, and because $\beta(\iota_t^*)(1 - n_{t-1})$ is bounded away from zero for any $n_{t-1} \in [0, 1)$ and $\iota_t^* > 0$; then it follows that $\lim_{t\to\infty} n_t = 1$. ■

Proof of Lemma 4. (i) The expression in (2.15) yields the first-order condition:

$$\beta'(\iota_t^*) \int_{\Omega_t(w)} \Delta(r_t, w)\, d\Omega_t(w) = 1. \tag{L.4.1}$$

Since r_t is a decreasing function of n_t, and n_t is an increasing function of n_{t-1} for all $n_{t-1} \in [0, 1)$; restating $\Delta(r_t, w) \equiv \Lambda(n_{t-1}, w)$, (L.4.1) can be rewritten as:

$$\beta'(\iota_t^*) \int_{\Omega_t(w)} \Lambda(n_{t-1}, w)\, d\Omega_t(w) = 1, \tag{L.4.2}$$

where $\partial\Lambda/\partial n_{t-1} > 0$ for all $n_{t-1} \in [0, 1)$, and $\partial\Lambda/\partial w = \partial\Delta/\partial w \geq 0$ (Lemma 3). As a result, from (L.4.2) it follows that if $\Omega_t(w) \succeq \Omega_t'(w)$, then: $\int_{\Omega_t(w)} \Lambda(n_{t-1}, w)\, d\Omega_t(w) \geq \int_{\Omega_t'(w)} \Lambda(n_{t-1}, w)\, d\Omega_t'(w)$, which in turn implies $\iota_t^*(n_{t-1}, \Omega_t(w)) \geq \iota_t^*(n_{t-1}, \Omega_t'(w))$. ♦

(ii) We need to prove the following: for all $w \geq 0$, and for all $n^A, n^B \in [0, 1]$, such that $n^A > n^B$: then, $\forall x \geq 0$, $P\left(w, [0, x] \mid n^B\right) \geq P\left(w, [0, x] \mid n^A\right)$; where $P(w, [0, x] \mid n)$ denotes the probability that when $w_t = w$, then $w_{t+1} \in [0, x]$, conditional on $n_t = n$.

Step 1: Suppose $w \in [0, \widehat{w}(r))$. Let $y(n_t, w_t) \equiv \delta\left[f(k_P^*(r_t)) - (1 + r_t)(k_P^*(r_t) - w_{i,t})\right]$; where the fact that $r_t^* = r(n_t)$ is taken into account when defining $y(\cdot)$. Notice that $\partial y/\partial n_t > 0$ and $\partial y/\partial w_t = (1 + r_t) > 0$. Additionally, define the following index-function:

$$\mathbb{I}_{y(n,w)<x} = \begin{cases} 1 & \text{if } y(n, w) < x \\ 0 & \text{otherwise} \end{cases}. \tag{L.4.3}$$

Notice that, because $\partial y/\partial n > 0$, then the following two properties hold: 1) $\mathbb{I}_{y(n^A,w)<x} = 1 \Rightarrow \mathbb{I}_{y(n^B,w)<x} = 1$; 2) $\mathbb{I}_{y(n^B,w)<x} = 0 \Rightarrow \mathbb{I}_{y(n^A,w)<x} = 0$. Hence, if $\mathbb{I}_{y(n^B,w)<x} \neq \mathbb{I}_{y(n^A,w)<x}$, it must be the case that $\mathbb{I}_{y(n^B,w)<x} = 1$ while $\mathbb{I}_{y(n^A,w)<x} = 0$. Using (L.4.3), thus:

$$\begin{aligned} P\left(w,[0,x] \mid n^B\right) - P\left(w,[0,x] \mid n^A\right) &= \left[(1-p)n^B + p\right] \mathbb{I}_{y(n^B,w)<x} \\ &\quad - \left[(1-p)n^A + p\right] \mathbb{I}_{y(n^A,w)<x} + (1-p)(n^A - n^B). \end{aligned} \tag{L.4.4}$$

Hence, if $\mathbb{I}_{y(n^A,w)<x} = 0$, the right-hand side in (L.4.4) yields a strictly positive number. Alternatively, if $\mathbb{I}_{y(n^A,w)<x} = 1$, then the right-hand side of (L.4.4) equals zero. Therefore, $P\left(w,[0,x] \mid n^B\right) \geq P\left(w,[0,x] \mid n^A\right)$ for all $w \in [0, \widehat{w}(r))$.

Step 2: Suppose $w \geq \widehat{w}(r)$. First, note that either if $\delta\left[f(k_S^*(w)) - k_S^*(w) + w\right] < x$ when $w \in [\widehat{w}(r), \widetilde{w}]$, or if $\delta\left[f(k_G^*) - k_G^*(w) + w\right] < x$ when $w > \widetilde{w}$; then in both cases: $P\left(w,[0,x] \mid n^B\right) = P\left(w,[0,x] \mid n^A\right) = 1$. Second, when the opposite results hold, three different cases may arise:

Case 1: $\delta\left(w - k_B^*\right) > x$. Then, $P\left(w,[0,x] \mid n^B\right) = P\left(w,[0,x] \mid n^A\right) = 0$.

Case 2: $\delta\left[f(k_B^*) - k_B^* + w\right] > x$ and $\delta\left(w - k_B^*\right) < x$. Now, $P\left(w,[0,x] \mid n\right) = (1-p)(1-n)$; thus: $P\left(w,[0,x] \mid n^B\right) - P\left(w,[0,x] \mid n^A\right) = (1-p)(n^A - n^B) > 0$.

Case 3: $\delta\left[f(k_B^*) - k_B^* + w\right] < x$ and $\delta\left(w - k_B^*\right) < x$. Now, $P\left(w,[0,x] \mid n\right) = (1-n)$; hence: $P\left(w,[0,x] \mid n^B\right) - P\left(w,[0,x] \mid n^A\right) = (n^A - n^B) > 0$.

Therefore, as a result of all these four possible cases, we can deduce that: $P\left(w,[0,x] \mid n^B\right) \geq P\left(w,[0,x] \mid n^A\right)$ for all $w \geq \widehat{w}(r))$ as well. ■

Proof of Proposition 5. (i) Let Θ denote the set of all feasible distribution functions $\Omega(w)$. Suppose $\Omega_t(w) = \Omega_0$. Since $n_{t-1} > \bar{n}$, then $\iota_t^* > 0$. Furthermore, since $\Omega_t(w) \succeq \Omega_0$ for any $\Omega_t(w) \in \Theta$, then from Lemma 4 (i) it follows that: $\iota_t^* > 0$ for any $\Omega_t(w) \in \Theta$. Therefore, $n_t > n_{t-1} > \bar{n}$, implying, in turn, that $\iota_{t+1}^* > 0$ for any $\Omega_{t+1}(w) \in \Theta$. Repeating the same argument *ad infinitum,* the claimed result obtains. ♦

(ii) When $n_{t-1} = \bar{n}$, we have that $\beta'(0)\Delta(\bar{r}, 0) = 1$; where $\bar{r} = r^*(\bar{n})$. Thus, $\iota_t^*(\bar{n}, \Omega_0) = 0$. Furthermore, from (2.15) notice that $\iota_t^*(\bar{n}, \Omega_{\widetilde{w}})$ is the solution to:

$$\beta'(\iota_t^*(\bar{n}, \Omega_{\widetilde{w}}))\, \Delta(r^*(n_t), \widetilde{w}) = 1, \quad \text{where } n_t = \bar{n} + (1-\bar{n})\, \beta(\iota_t^*(\bar{n}, \Omega_{\widetilde{w}})) \tag{P.5.1}$$

From Lemma 3, and the fact that $r^*(n_t) \geq \bar{r}$, it follows that $\Delta(r^*(n_t), \widetilde{w}) > \Delta(\bar{r}, 0)$. Therefore, to comply with (P.5.1), $\iota_t^*(\bar{n}, \Omega_{\widetilde{w}}) > 0$ must hold. As a result, there must exist $\bar{n}_{\widetilde{w}} < \bar{n}$, such that $\iota_t^*(\bar{n}_{\widetilde{w}}, \Omega_{\widetilde{w}}) > 0$ and $\bar{n} = \bar{n}_{\widetilde{w}} + (1 - \bar{n}_{\widetilde{w}})\beta(\iota_t^*(\bar{n}_{\widetilde{w}}, \Omega_{\widetilde{w}}))$; from which it follows that if $n_{t-1} > \bar{n}_{\widetilde{w}}$ when $\Omega_t(w) = \Omega_{\widetilde{w}}$, then n_t will grow over time, converging monotonically to $n_\infty = 1$. ♦

(iii) From Lemma 3 (i), it follows that: $\iota_t^*(\bar{n}, \Omega_t(w)) \geq 0$ and $\iota_t^*(\bar{n}_{\widetilde{w}}, \Omega_t(w)) \leq \iota_t^*(\bar{n}_{\widetilde{w}}, \Omega_{\widetilde{w}})$. As a result, there must exist $\bar{n}_{\Omega(w)} \in [\bar{n}_{\widetilde{w}}, \bar{n}]$, such that $\iota_t^*(\bar{n}_{\Omega(w)}, \Omega_t(w)) \geq 0$ and $\bar{n} =$

$\bar{n}_{\Omega(w)} + (1 - \bar{n}_{\Omega(w)})\, \beta \left[\iota_t^*(\bar{n}_{\Omega(w)}, \Omega_t(w))\right]$; from which it follows that if $n_{t-1} > \bar{n}_{\Omega(w)}$ when $\Omega_t(w)$ holds, then n_t will grow over time, converging monotonically to $n_\infty = 1$. Finally, applying Lemma 3 (i) again $\iota_t^*(\bar{n}_{\Omega(w)}, \Omega_t(w)) \geq \iota_t^*(\bar{n}_{\Omega(w)}, \Omega_t'(w))$ obtains, from where $\bar{n}_{\Omega(w)} \leq \bar{n}_{\Omega'(w)}$ if $\Omega_t(w) \succeq \Omega_t'(w)$ immediately follows. ■

Chapter 3

Adverse Selection and Entrepreneurship in a Model of Development

3.1 Introduction

Since the seminal papers by Rothschild and Stiglitz (1976) and Wilson (1977), the consequences of adverse selection on the operation of insurance markets have been widely acknowledged and understood. However, no serious attempts have been yet taken to apply the Rothschild-Stiglitz-Wilson framework to the theory entrepreneurship, risk-taking and development. This omission by the development literature seems surprising for two main reasons. First, the literature that has so far explored the effects of imperfect risk-sharing on development has always relied on the presence of exogenous fixed costs [Greenwood and Jovanovic (1990) and Saint-Paul (1992)] or technological non-convexities limiting the scope for diversification in poor economies [Acemoglu and Zilibotti (1997)]. Thus, a fully micro-founded theory of the emergence and evolution of insurance market imperfections along the path of development is still missing. Second, informational asymmetries have long been recognised a serious impediment to development because they restrict the access to credit markets to the poor.[1] Yet, despite the contractual similarities between credit and insurance, not much effort has been spent on formally linking informational asymmetries and the efficiency of risk-sharing markets within a theory of development.[2]

[1]E.g., Banerjee and Newman (1993), Galor and Zeira (1993), Aghion and Bolton (1997), Piketty (1997), Lloyd-Ellis and Berhardt (2000), Ghatak and Jiang (2002), and Mookherjee and Ray (2002).

[2]There is a small literature that has studied entrepreneurial decisions under imperfect risk-sharing due to a moral hazard problem related to effort unobservability [Banerjee and Newman (1991) and Newman (2007)]. Those papers have led to results that are at odds with the reality, namely: the poor become entrepreneurs and bear the entrepreneurial uninsurable risks, while the rich choose to work as employees, remaining then fully insured by receiving a fixed wage. This seems another important reason why to explore the implications of alternative sources of asymmetric information (such as adverse selection) for risk-sharing efficiency and development.

This paper tackles the issues raised above by introducing informational asymmetries concerning entrepreneurial skills into an overlapping-generations model of development driven by entrepreneurial choice. A successful process of development requires the agents best suited for undertaking entrepreneurial activities to fully exercise their skills. However, because entrepreneurial skills are private information, an adverse selection problem endogenously arises when the prospective entrepreneurs intend to diversify away their idiosyncratic risks. The adverse selection problem prevents the provision of first-best insurance contracts against entrepreneurial risks, which may in turn depress the amount of entrepreneurial investment and slow down growth.

In order to isolate the effects brought about by the insurance market imperfection, we let all agents enjoy perfect access to the credit market. In the model, the old generation may undertake entrepreneurial projects that are subject to idiosyncratic risks. The young generation supplies labour, which is used as an input by the entrepreneurs. Imperfect insurance provision discourages entrepreneurial investment and, thus, diminishes labour demand, pushing down wages which represent the income of the young. Individuals' preferences in the model display decreasing absolute risk aversion, hence the poorer the agents are, the more strongly risk-taking is deterred by the presence of uninsured risk.[3] As a result, if the old generation is poor, entrepreneurial investment will be low, and this will be carried over to the next generation by the low wages prevailing in the labour market. This feedback between entrepreneurial investment and wages implies that income displays persistence across generations. Furthermore, when risk aversion is sufficiently responsive to income and the entrepreneurial projects are sufficiently risky, the feedback between investment and wages becomes so strong that it may lead to the appearance of multiple long-run equilibria.

Empirical evidence in Townsend (1994) and Udry (1994) indicates that risk-sharing is an important concern in poor economies and that those economies have in fact managed to prevent consumption swings due to idiosyncratic risks quite efficiently. Yet, it seems to be the case that low-income countries fulfil a large portion of their insurance needs by means of rather informal arrangements.[4] This contrasts sharply with the case of developed economies in which most of risk-sharing is managed by formal institutions

[3]The model makes use of a utility function that actually displays a stronger property than decreasing absolute risk aversion (DARA), namely: decreasing relative risk aversion (DRRA). It can be easily proved that DRRA implies DARA. In the end of Section 3.4, I discuss the consequences of using instead a utility function with DARA, but without DRRA.

[4]E.g., sharecropping policies (Bardhan 1977), intra-family transfers (Kotlikoff and Spivak (1981) and Rosenzweig (1988)), or buying and selling durable productive assets as a mean to provide self-insurance (Rosenzweig and Wolpin (1993)).

established with that purpose (e.g., stock markets, credit institutions, insurance companies, options and future markets, welfare states). This paper will explicitly focus on the latter set of institutions, since those seem to be the relevant types of social arrangements for supporting entrepreneurial activities and long-run growth. Informal schemes have proven relatively efficient at preventing consumption drops, mainly due to intrinsic weather and health risks within poorer agricultural economies. However, the activities that spur long-run growth and ignite the process of modernisation to a manufacturing economy imply taking on additional entrepreneurial risks, and as such they need to be supported by institutions deliberately devised to share those risks. In that regard, the adverse selection problem presented in this paper only arises when an economy intends to switch from an agricultural village economy to a modern industrial economy. The reason is that skills heterogeneity becomes an important issue to deal with only when there is a large manufacturing sector.[5]

Lastly, from a policy perspective, disentangling the underpinnings of insurance versus credit market imperfections appears also as a relevant issue for investigation because some of their implications are in conflict (Banerjee, 2000). For example, if insurance markets imperfections are viewed as a serious hindrance to long-run growth, an immediate policy recommendation would be to increase the protection to the poor when bad states of nature realise, in order to enhance their willingness to take on risky entrepreneurial activities. However, credit constraints typically arise because, in the presence of limited liability, the poor have literally "nothing to lose" and, thus, cannot truthfully commit to repay their debt or to exert optimal effort. As a result, if access to credit is the main concern, strengthening the protection to the poor in the bad states of nature may be a wrong policy to follow, as it would further aggravate their incentives problem.

The rest of the paper is organised as follows. Section 3.2 describes the set up of the model. Section 3.3 characterises the static equilibrium under imperfect risk-sharing due to the adverse selection problem. Section 3.4 analyses the dynamics of the economy, specifying the conditions under which multiple long-run equilibria may coexist. Section 3.5 concludes.

[5] Another possible justification for the claim that adverse selection becomes more severe in modern economies is that information about peers flows better within village economies. This is the type of argument usually put forward by the theories proposing group lending in village economies; e.g., Ghatak (1999) and Van Tassel (1999). Since the focus of this paper is on the process of development and industrialisation, and not on how to improve efficiency within the rural economy, I do not attempt to model this point.

3.2 Environment

Consider an overlapping-generations economy in which life evolves over a discrete-time infinite horizon, $t = \{0, 1, ...\infty\}$. The economy is small and enjoys perfect access to international credit markets at the fixed international (net) interest rate $r = 0$.

Individuals in the economy live for two periods. In every period t a continuum of individuals with mass normalised to 1 is born. As a result, in every period t the economy is populated by two different generations, each one with unit mass: those who were born in $t-1$ (the *old* in period t), and those born in period t (the *young* in period t). All individuals are born with an identical endowment of 1 unit of time, which they use entirely to work while they are young. In the second period of life, when individuals are old, they can choose either to retire or to become entrepreneurs. Retiring yields zero income.

Young agents may choose to work in two different occupations: they can work in the agricultural sector, becoming independent labourers working in a communal plot of land; alternatively they can work in the manufacturing sector as employees for old entrepreneurs, earning there a fixed wage v.

Any old agent may decide to become an entrepreneur. However, not all them would be equally good as entrepreneurs. In particular, there exist two *types* (or *qualities*) of entrepreneur indexed by $T \in \{B, G\}$, where B (G) stands for *bad-types* (*good-types*). The good-types represent a fraction $\eta \in (0,1)$ of the population and possess higher expected productivity as entrepreneurs than the bad-types do, who comprise the remaining fraction $(1-\eta)$. The fractions of good- and bad-types (η and $1-\eta$) are constant over time. Regarding the informational structure in the economy, the *type* is assumed private information. In other words, entrepreneurial ability is subject to asymmetric information.

Individuals are risk averse. Furthermore, there is a subsistence level of consumption, which I normalise to 1, below which utility falls to $-\infty$. To simplify the analysis, I assume individuals care only about consumption in the second period (hence, all the income they earn while young will be saved and invested to provide future consumption). In particular, I assume the Bernoulli utility function of individual i born in t is given by:

$$u_{i,t} = \begin{cases} \ln(c_{i,t+1} - 1) & \text{if } c_{i,t+1} > 1, \\ -\infty & \text{otherwise;} \end{cases} \tag{3.1}$$

where $c_{i,t+1}$ denotes the consumption in $t+1$ by agent i born in t.

3.2.1 Agricultural Sector Technology

Aggregate production in the agricultural sector (Y) depends on the total amount of communal land (X), and on the mass of young agents working in the agricultural sector (L), following a Cobb-Douglas production function. There are no property rights over land, thus each agricultural labourer obtains as income the average output $y(L) \equiv Y(L)/L$. The amount of land is fixed at $\bar{X} > 0$. Hence, Y can be written as follows:[6]

$$Y(L) = L^{\alpha}, \quad \text{where } \alpha \in (0,1). \tag{3.2}$$

3.2.2 Manufacturing Sector Technology

Production in the manufacturing sector requires 1 unit of entrepreneurial skill (coming from the *old* generation) and raw labour (coming from the *young* generation). The return of the entrepreneurial projects is random, subject to an idiosyncratic shock. I suppose there are only two possible outcomes for the projects: *success* or *failure.* Imagine an old agent hires l units of young labour at the beginning of period t; then, in the event of success, the project yields $\rho\, l$ units of output at the end of t, where $\rho > 0$. On the other hand, in the event of failure, the project yields 0 output regardless of l. A good-type undertaking an entrepreneurial project fails with probability $\phi \in (0,1)$, whereas a bad-type fails with probability equal to 1.

Each entrepreneur is a price taker and must thus pay the market wage v_t for each unit of labour hired. I assume entrepreneurs must pay workers' wages at the beginning of the production process. As a result, the amount $l_i\, v_t$ equals the total investment by entrepreneur i.

3.2.3 Financial Markets

Credit Market: All credit market transactions between natives and with the rest of the world are mediated by *banks.* The local credit market is characterised by free entry and absence of set-up or sunk costs. As a result, banks must make zero profit in equilibrium. All individuals in the economy enjoy perfect access to the credit market. I assume individuals always choose to honour their debts, should they be able to pay back lenders the agreed amount specified in the credit contract. In other words, there are no moral hazard problems contaminating the operation of credit markets.

[6] The expression in (3.2) is a reduced form of $Y(X,L) = AX^{1-\alpha}L^{\alpha}$, under the assumption that $\bar{X} = A^{-1/(1-\alpha)}$. The fact that $y(1) = 1$ guarantees that this economy can always meet the subsistence level of consumption (and, consequently, the individuals' optimisation problem is always well-defined).

Insurance Market: The insurance market is run by a continuum of insurance companies $j \in \mathcal{I}_t$ which offer contracts that protect against entrepreneurial failure. There exist no sunk or set-up costs in the insurance industry. I suppose insurance contracts cannot be negotiated in advance; in other words, a contract agreed in period t only covers events occurring during t. Throughout this paper, when referring to the insurance market, I will make use of the equilibrium concept defined in Rothschild and Stiglitz (1976) – hereafter, this equilibrium concept will be referred to as *RS*. Because of the well-known equilibrium (non-)existence problem when using *RS*, the fraction of bad-types $(1-\eta)$ will accordingly be assumed large enough so as to ensure the existence of an *RS*.

Without any loss of generality, I assume that each insurance company offers at most one contract to each entrepreneur i. An insurance contract offered by j to i in period t can be written as follows: $\mathcal{C}_{i,j,t}=[q_{i,t}, \mathcal{P}_{i,t}(q_{i,t}; l_{i,t}, \omega_{i,t-1})] \in \mathbb{R}^+ \times \mathbb{R}$. This contract specifies the payment $\mathcal{P}_{i,t}(q_{i,t};\cdot)$ that i must make in order to buy $q_{i,t}$ units of an Arrow-Debreu commodity that pays back 1 unit of income in the event of entrepreneurial failure, conditional on the amount of labour hired $l_{i,t}$ (i.e. the *size* of his entrepreneurial project) and the income generated in the first period $\omega_{i,t-1}$.

Entrepreneurs will receive (in principle) contract offers from several insurance companies. Accordingly, let $\mathfrak{Q}_{i,t} = \{\mathcal{C}_{i,j,t}\}_{j\in\mathcal{I}_t}$ denote the set of all insurance contracts offered to entrepreneur i in period t.

An important feature of the insurance contracts observed in the model is that they condition on the amount of labour hired by the entrepreneurs. In other words, l_i is publicly verifiable and, furthermore, insurance companies make use of the implicit information conveyed by this variable. The assumption that l_i is publicly verifiable is necessary for the existence of an *RS* in this particular model.[7]

3.3 Static Equilibrium Analysis

Fix the time in period t and consider the problem faced by the agent i born in $t-1$. Suppose this agent has earned income equal to $\omega_{i,t-1}$ while he was young. Additionally, let his type be $T=\{B,G\}$, and let $\phi_T = 1$ if $T=B$ and $\phi_T=\phi$ if $T=G$. Then, given

[7] To see this, recall that bad-types fail with probability equal to 1. As a result, no matter what value of l the bad-types announced they would choose, in case l were unverifiable, they would (ex-post) always optimally choose $l=0$. This deviation by the bad-types from their (unverifiable) announcements would always destroy the Rothschild and Stiglitz (separating) equilibrium in this model.

v_t, this agent solves:

$$\max_{s_{i,t},l_{i,t},[q_{i,t},\mathcal{P}_{i,t}(\cdot)]} : E(u_{i,t-1}) = \phi_T \ln(s_{i,t} + q_{i,t} - 1) + (1-\phi_T)\ln(s_{i,t} + \rho\, l_{i,t} - 1) \quad (3.3)$$

$$\text{subject to: } s_{i,t} + \mathcal{P}_{i,t}(q_{i,t};\, l_{i,t}, \omega_{i,t-1}) + v_t\, l_{i,t} = \omega_{i,t-1}, \quad (3.4)$$

$$[q_{i,t}, \mathcal{P}_{i,t}(q_{i,t};\, l_{i,t}, \omega_{i,t-1})] \in \mathfrak{Q}_{i,t}, \quad \text{and} \quad l_{i,t} \geq 0. \quad (3.5)$$

Where s_i denotes the amount lent to (or borrowed from, if negative) banks at the interest rate $r = 0$.[8]

Let Υ_τ denote the set of *young* agents in period τ, and Θ_τ denote the set of *old* agents in period τ. Define $\Omega_{t-1} : \Upsilon_{t-1} \to \mathbb{R}^+$ as the function that summarises the income earned by each agent in Υ_{t-1} during his youth. Then, given Ω_{t-1}, an equilibrium in period t is a collection $\{s_{i,t}, l_{i,t}, [q_{i,t}, \mathcal{P}_{i,t}(q_{i,t};\cdot)], \mathfrak{Q}_{i,t}\}_{i\in\Theta_t}$ and a market wage v_t, such that:

1. The allocation $(s_{i,t}, l_{i,t}, [q_{i,t}, \mathcal{P}_{i,t}(q_{i,t};\cdot)])$, solves (3.3) subject to (3.4) and (3.5) for each $i \in \Theta_t$.

2. Given the set of contracts $\mathfrak{Q}_{i,t}$ offered to each $i \in \Theta_t$: (*i*) No contract belonging to $\mathfrak{Q}_{i,t}$ makes negative expected profits, and (*ii*) there exists no other feasible contract $\mathfrak{z} \notin \mathfrak{Q}_{i,t}$, which, if offered in addition to $\mathfrak{Q}_{i,t}$, would make positive expected profits.

3. Each agent in the set Υ_t selects the occupation in t to maximise (3.1).

4. The labour market clears; i.e. $\int_{\Theta_t} l_{i,t}\, di = 1 - L_t$.

Young agents will choose the occupation (agricultural labourers vs. manufacturing employees) that yields higher income. Therefore, in equilibrium, $\omega_{i,t} = \max\{v_t, L_t^{\alpha-1}\}$ will hold for all $i \in \Upsilon_t$ and all $t \geq 0$. From this expression we can first observe that all individuals of the same generation will earn identical incomes when young, i.e., $\omega_{i,t} = \omega_t$ for all $i \in \Upsilon_t$ and $t \geq 0$.

Second, when the young are indifferent between occupations, the wage in the manufacturing sector must thus be equal to the average productivity in the agricultural sector. Alternatively, when all the young agents specialise in the agricultural sector, $\omega_t = 1 \geq v_t$. Similarly, if all young agents specialise in the manufacturing sector, $\omega_t = v_t \geq 0^{\alpha-1}$. Notice that since $0^{\alpha-1} \to \infty$, a situation in which $L_t = 0$ (i.e., full manufacturing specialisation) will never hold in equilibrium, as it would require $v_t \to \infty$, which is incompatible with non-negative entrepreneurial profits. As a result, it turns out that $\omega_t = L_t^{\alpha-1} \geq v_t$ will always prevail in equilibrium.

[8] Notice that the agent i may optimally set $l_i = 0$. We can interpret this decision of i as *retiring* when old.

3.3.1 Insurance Contracts and Entrepreneurial Investment

Incentive-Compatible Contracts

Let $\mathbb{G}_\tau$ and $\mathbb{B}_\tau$ denote, respectively, the subset of good- and bad-types born in τ. If insurance contracts intend to condition on the agent's type, bad-types must be induced to truthfully reveal their (unobservable) type. Insurance companies will *screen* agents by restricting the maximum amount of insurance that old individuals are allowed to purchase, conditional on their first-period earnings and their choices of l. More precisely, given ω_{t-1}, the level of q_t will be set low enough so as to dissuade any $h \in \mathbb{B}_{t-1}$ from deviating from his *outside option* and mimic the behaviour of an agent $i \in \mathbb{G}_{t-1}$. These sorts of contract are incentive-compatible, screening out the bad-types. The drawback of this screening policy is that when limiting insurance provision below first-best levels, insurers might also end up discouraging first-best risk-taking by good-types.

Perfect competition in the insurance market implies that in an equilibrium where types are screened, any $i \in \mathbb{G}_{t-1}$ should face $\mathcal{P}_{\mathbb{G}_{t-1}}(q_{\mathbb{G}_{t-1}};\cdot)/q_{\mathbb{G}_{t-1}} = \phi$. Denote by $l^*_{\mathbb{G}_{t-1}}$ the level of $l_{i,t}$ that solves (3.3) - (3.5) when $i \in \mathbb{G}_{t-1}$. A bad-type trying to "disguise" himself as a good-type should also hire $l^*_{\mathbb{G}_{t-1}}$ workers (otherwise, he would be assessed as a bad-type). Therefore, incentive-compatibility for any $h \in \mathbb{B}_{t-1}$ requires the following condition to hold:

$$\ln(\omega_{t-1} - 1) \geq \ln[\omega_{t-1} - \upsilon_t\, l^*_{\mathbb{G}_{t-1}} + (1-\phi)\widehat{q}_t - 1]; \tag{3.6}$$

where $\widehat{q}_t$ denotes the maximum level of insurance that old agents in period t are allowed to buy at a unit price ϕ, if they hire $l^*_{\mathbb{G}_{t-1}}$ workers. The right-hand side of (3.6) shows the level of utility achieved by any $h \in \mathbb{B}_{t-1}$ when he replicates the portfolio allocation chosen by a member of $\mathbb{G}_{t-1}$ (given $\widehat{q}_t$). On the other hand, the left-hand side equals the utility that any member of $\mathbb{B}_{t-1}$ would achieve by investing all his first-period earnings in the safe asset at $r = 0$ (that is, by setting $s_t = \omega_{t-1}$); this investment policy represents the outside option available to the bad-types. Thus, (3.6) wipes out the existence of a profitable deviation available to agents in the set $\mathbb{B}_{t-1}$.

Optimal Risk-Taking under Imperfect Insurance Markets

Following the former discussion on incentive-compatible insurance contracts, the optimisation problem (3.3) - (3.5) for any good-type born it $t-1$ can thus be rewritten as follows (to reduce notation, for the rest of the paper $\omega_{t-1} \geq 1$ will always be implicitly assumed unless otherwise explicitly noted):[9]

$$\max_{l_t \geq 0, q_t \geq 0} : E(u_{t-1}) = \phi \ln [\omega_{t-1} + (1-\phi)q_t - \upsilon_t l_t - 1] + (1-\phi) \ln [\omega_{t-1} - \phi q_t + (\rho - \upsilon_t) l_t - 1] \tag{3.7}$$

$$\text{subject to: } q_t \leq \widehat{q}_t. \tag{3.8}$$

The solution of the optimisation problem (3.3) - (3.5), together with the incentive compatibility constraint (3.6), yields the following result (the full derivation of (3.9) is provided in the Appendix):

$$l^*_{\mathbb{G}_{t-1}} = \begin{cases} \dfrac{1-\phi}{\phi}\dfrac{1}{\upsilon_t}(\omega_{t-1} - 1) & \text{if } (1-\phi)\,\rho > \upsilon_t, \\ \left[0, \dfrac{1-\phi}{\phi}\dfrac{1}{\upsilon_t}(\omega_{t-1} - 1)\right] & \text{if } (1-\phi)\,\rho = \upsilon_t, \\ 0 & \text{if } (1-\phi)\,\rho < \upsilon_t. \end{cases} \tag{3.9}$$

The expression in (3.9) summarises the risk-taking behaviour of the good-types born in $t-1$ when adverse selection in the insurance market prevents full risk-sharing. A key property of (3.9) is that –whenever $(1-\phi)\,\rho > \upsilon_t$– entrepreneurial investment by the good-types (i.e., $\upsilon_t \, l^*_{\mathbb{G}_{t-1}}$) is an increasing and convex function of their initial income. In particular, its elasticity with respect to ω_{t-1} is strictly larger than 1.[10] This convex response with respect to ω_{t-1} is due to the fact that, given the specification in (3.1), individuals display *decreasing relative risk aversion* (DRRA). When preferences exhibit DRRA, the fraction of initial income invested in riskier assets is increasing in the individual's initial income – see Mas-Colell *et al* (1995), pp. 185-194. Since in this model insurance is imperfectly provided, investing in the entrepreneurial projects entails a risky decision, and will thus increase convexly with the initial income of the good-types.

The equation (3.9) can alternatively be seen as the individual labour demand function. As it is the usual case, we can observe that labour demand is decreasing in the wage υ_t.[11]

3.3.2 Equilibrium in the Labour Market

The last variable that remains to be determined in order to characterise fully the equilibrium in period t is the market wage υ_t. This variable is pinned down in the labour

[9] In any case, as it will be formally proved in Section 3.4, $\omega_{t-1} \geq 1$ will always hold in equilibrium within a full dynamic setting – see Lemma 5.

[10] This elasticity equals $\omega_{t-1}/(\omega_{t-1} - 1)$, for all $(1-\phi)\,\rho > \upsilon_t$.

[11] There is, though, a difference between (3.9) and the standard neoclassical labour demand function. In the neoclassical case, labour demand is decreasing in the wage because firms need to adjust the (decreasing) marginal productivity of labour to the higher wage. In this model, the production function ρl is linear then, provided $(1-\phi)\,\rho > \upsilon_t$, labour demand should (in principle) not respond to higher wages. However, because of imperfect insurance provision, when υ_t rises, entrepreneurs need to reduce their labour demand in order to achieve better consumption smoothing across states of nature.

market, where the labour supply derives from the optimal occupational choice of the young generation, while the labour demand results from adding up (3.9) across all good-types born in $t-1$. To avoid the trivial case in which no manufacturing sector ever arises in equilibrium, I impose the following condition:

Assumption 1 $(1-\phi)\,\rho > 1$.

The equilibrium in the labour market in period t is determined by the intersection of the labour demand (l_t^D) and labour supply (l_t^S) correspondences, where:

$$l_t^D = \begin{cases} \eta \dfrac{1-\phi}{\phi} \dfrac{1}{\upsilon_t} (\omega_{t-1} - 1) & \text{if } (1-\phi)\,\rho > \upsilon_t, \\ \left[0, \eta \dfrac{1-\phi}{\phi} \dfrac{1}{\upsilon_t} (\omega_{t-1} - 1)\right] & \text{if } (1-\phi)\,\rho = \upsilon_t, \\ 0 & \text{if } (1-\phi)\,\rho < \upsilon_t. \end{cases} \tag{3.10}$$

$$l_t^S = \begin{cases} 0 & \text{if } \upsilon_t < 1, \\ 1 - \left(\dfrac{1}{\upsilon_t}\right)^{\frac{1}{1-\alpha}} & \text{if } \upsilon_t \geq 1. \end{cases} \tag{3.11}$$

Notice that when $\upsilon_t \geq 1$, $l_t^S = 1 - y^{-1}(\upsilon_t)$, where $y^{-1}(\cdot)$ is the *inverse* function of the average agricultural output $y(L)$. This is the case because when $\upsilon_t \geq 1$, the young must be indifferent between working in the agricultural or in the manufacturing sector, hence l_t^S must be such that $\upsilon_t = y\left(1 - l_t^S\right)$.

Let l_t^* and υ_t^* denote henceforth the labour market equilibrium values of l and υ, and define $\hat{\omega} \equiv 1 + \frac{\rho\phi}{\eta}\left[1 - \left(\frac{1}{(1-\phi)\rho}\right)^{\frac{1}{1-\alpha}}\right]$, where notice that $\hat{\omega} > 1$.

Proposition 6 (Labour Market Equilibrium)
(i) Whenever $\omega_{t-1} > 1$, the equilibrium wage υ_t^* is a non-decreasing function of ω_{t-1}. In particular, if $\omega_{t-1} > 1$, $\upsilon_t^*(\omega_{t-1}) : (1,\infty) \rightarrow (1,(1-\phi)\rho]$, such that: a) for all $\omega_{t-1} \in (1,\hat{\omega})$, $\upsilon_t^* < (1-\phi)\rho$ and υ_t^* is strictly increasing in ω_{t-1}; b) for all $\omega_{t-1} \geq \hat{\omega}$, $\upsilon_t^* = (1-\phi)\rho$. Furthermore, whenever $\omega_{t-1} > 1$, $l_t^* = 1 - (1/\upsilon_t^*(\omega_{t-1}))^{1/(1-\alpha)}$, thus $l_t^* \in (0,1)$.
(ii) If $\omega_{t-1} \in [0,1]$, then $\upsilon_t^* \in [0,1]$ and $l_t^* = 0$.

Proof. *In Appendix.* ■

FIGURE 3.1 provides a visual illustration of the equilibrium in the labour market for four different levels of ω_{t-1}, namely: $\omega_a, \omega_b, \hat{\omega}$ and ω_c (where, $1 < \omega_a < \omega_b < \hat{\omega} < \omega_c$).[12]

[12] Although not drawn in FIGURE 3.1, when $\omega_{t-1} \in [0,1]$ the labour demand is a straight line along $l_t = 0$ (i.e., $l_t^D(\cdot)$ coincides with the vertical axis). As a result, for all $\omega_{t-1} \in [0,1]$, $l_t^D(\omega_{t-1}, \upsilon_t)$ and $l_t^S(\upsilon_t)$ intersect each other at $l_t = 0$, along the whole segment $\upsilon_t \in [0,1]$; which is the result (ii) in Proposition 6.

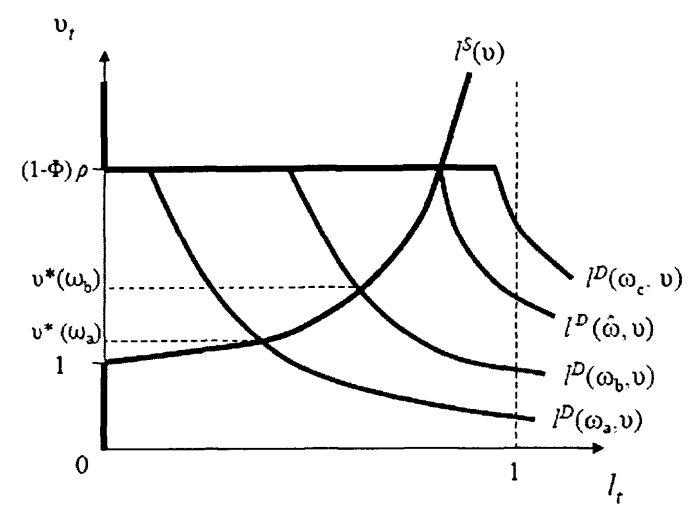

FIGURE 3.1: Labour Market Equilibrium.

Labour market equilibrium for four different levels of ω_{t-1}.

Proposition 6 describes how υ_t^* responds positively to the previous generation first-period income (ω_{t-1}). Since a larger ω_{t-1} leads to higher risk-taking by the good-types, labour demand turns out to be (weakly) increasing in ω_{t-1}. As labour demand increases with ω_{t-1}, the equilibrium wage υ_t^* must rise to attract some additional young agents from the agricultural sector to the manufacturing sector. This positive impact of ω_{t-1} and υ_t^* represents the key mechanism that may give rise to multiple long-run equilibria.

3.4 Dynamic Analysis

In order to characterise the dynamic behaviour of the economy, it proves convenient to state the following preliminary result:

Lemma 5 *$\omega_\tau \in [1, (1-\phi)\,\rho]$, regardless of the value of $\omega_{\tau-1}$, for all $\tau \in \{1, 2, ...\infty\}$.*

Proof. *Firstly, notice that the minimum value ω_τ can take in equilibrium is 1, as this is the average productivity of the agricultural sector when $L_\tau = 1$. Secondly, observe from (3.10) that if $\upsilon_\tau > (1-\phi)\,\rho$, then $l_\tau^D = 0$. As a result, all the young population alive in τ should work in the agricultural sector, whose average productivity would then equal 1. Therefore, $\omega_\tau > (1-\phi)\,\rho$ cannot hold in equilibrium either.* ■

From Lemma 5, it follows that we can restrict the state space of ω_{t-1} to the interval $[1, (1-\phi)\,\rho]$. When $\omega_{t-1} \in (1, (1-\phi)\,\rho]$, the equilibrium in the (manufacturing sector) labour market encompasses $l_t^* \in (0, 1)$. Therefore, young agents alive in t must be indifferent between the two occupations, earning $\omega_t = v_t = y(1 - l_t^*)$. On the other hand, when $\omega_{t-1} = 1$, labour demand by entrepreneurs falls to zero, and all the young generation must thus go to the agricultural sector, earning income $\omega_t = y(1) = 1$. Define $\bar{\omega} \equiv \min\{\hat{\omega}, (1-\phi)\rho\}$. We can thus write down the *law of motion* for ω_t as follows:

$$\text{Law of Motion:} \quad \begin{cases} \Psi(\omega_{t-1}, \omega_t) \equiv \dfrac{1-\phi}{\phi}\dfrac{\eta}{\omega_t}(\omega_{t-1} - 1) + \left(\dfrac{1}{\omega_t}\right)^{\frac{1}{1-\alpha}} - 1 = 0, & \text{if } \omega_{t-1} \in [1, \bar{\omega}]; \\ \omega_t = (1-\phi)\rho, \quad \text{if } \omega_{t-1} \in (\bar{\omega}, (1-\phi)\rho] \text{ and } (\bar{\omega}, (1-\phi)\rho] \neq \emptyset. & \end{cases} \tag{3.12}$$

If $\hat{\omega} \geq (1-\phi)\rho$, then the implicit function $\Psi(\omega_{t-1}, \omega_t) = 0$ alone depicts the dynamic behaviour of ω_t. Alternatively, if $\hat{\omega} < (1-\phi)\rho$, the dynamics of ω_t are determined by $\Psi(\omega_{t-1}, \omega_t) = 0$ when $\omega_{t-1} \in [1, \hat{\omega}]$, while $\omega_t = (1-\phi)\rho$ when $\omega_{t-1} \in (\hat{\omega}, (1-\phi)\rho]$.

Lemma 6 *$\Psi(\omega_{t-1}, \omega_t) = 0$ yields a mapping $\omega_t(\omega_{t-1}) : [1, \bar{\omega}] \to [1, (1-\phi)\rho]$, which is strictly increasing and strictly convex in ω_{t-1}.*

Proof. *In Appendix.* ■

Given the specific parametric configuration of the model, we can find three different types of dynamics in terms of their qualitative features and their long-run equilibria.

Proposition 7 (Long-Run Equilibria)
(i) Suppose $\phi/[\eta(1-\phi)] \in (1-\alpha, 1)$. Then, there exists a threshold level $\bar{\rho}(\alpha) > 1/(1-\phi)$, where $\bar{\rho}'(\alpha) > 0$, such that: $\forall \rho > \bar{\rho}(\alpha)$, there exist two (locally) stable stationary equilibria, namely, $\omega = 1$ and $\omega = (1-\phi)\rho$.
(ii) Suppose $\phi/[\eta(1-\phi)] \geq 1$. Then, the only stable stationary equilibrium in the economy is $\omega = 1$. In addition, if $\phi/[\eta(1-\phi)] \in (1-\alpha, 1)$ holds, but $\rho \leq \bar{\rho}(\alpha)$, then the only stable stationary equilibrium in the economy is $\omega = 1$.[13]
(iii) Suppose $\phi/[\eta(1-\phi)] \leq 1-\alpha$. Then, the only stable stationary equilibrium in the economy is $\omega = (1-\phi)\rho$.

Proof. *In Appendix.* ■

Proposition 7 shows that when $\phi/[\eta(1-\phi)] \in (1-\alpha, 1)$, two (locally) stable long-run equilibria may coexist in the economy. First, we have a *poverty trap* in which $\omega = 1$ and $l = 0$; in other words, an equilibrium where the economy is poor (it just affords subsistence consumption) and fully agricultural. Second, there might be a *higher-income* long-run equilibrium in which $\omega = (1-\phi)\rho$ and $l \in (0,1)$, (so part of the economy works in the manufacturing sector). This higher-income equilibrium arises when ρ is large enough;

[13] In the specific situation $\phi/[\eta(1-\phi)] \in (1-\alpha, 1)$ and $\rho = \bar{\rho}(\alpha)$, the point $\omega = (1-\phi)\rho$ becomes another stationary point, but in this case it is *unstable*.

in other words, when the manufacturing sector is sufficiently productive. This last result seems quite intuitive. Proposition 6 shows that, within a certain range, a larger ω_{t-1} leads to higher wages in period t; when ρ is sufficiently large, the entrepreneurial projects are so productive that the positive impact of ω_{t-1} on v_t^* extends over such a long interval that a higher-income stable stationary point arises in the model.

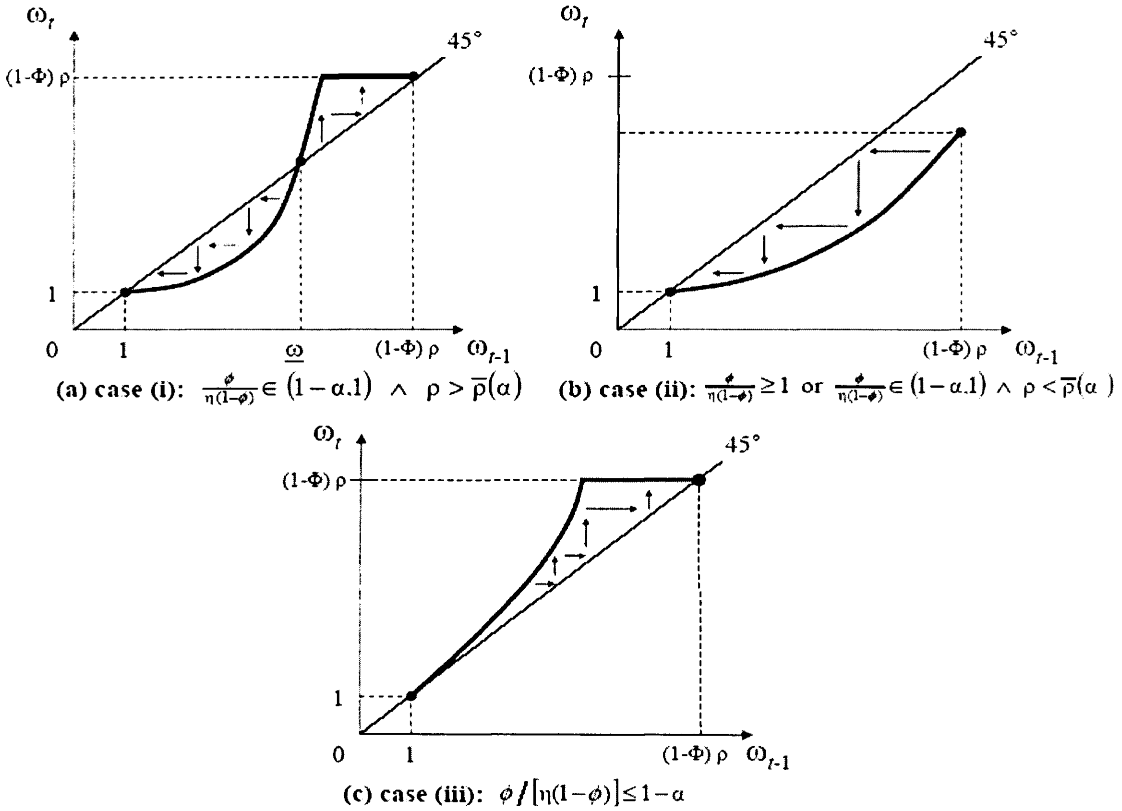

FIGURE 3.2: Initial income dynamics.

FIGURE 3.2 displays examples of the three distinct cases discussed in Proposition 7. In (a), a situation leading to multiple long-run equilibria is shown. Whenever $\omega_0 > \underline{\omega}$, ω_t will be continuously growing over time, converging monotonically towards $\omega = (1-\phi)\rho$. During this process, l_t^* will also be rising, meaning both that the manufacturing sector is expanding and that risk-taking by the entrepreneurs is increasing. On the other hand, if $\omega_0 < \underline{\omega}$, the economy will converge over time towards $\omega = 1$ (a poverty trap), where $l_t^* = 0$. Essentially, in $\omega = 1$ individuals are so poor that they completely refrain from risk-taking as a way to avoid the (dramatically) low levels of consumption that would prevail in the event of failure. This, in turn, implies that labour demand in the manufacturing sector falls to zero; thus, the entire young generation must resort to agricultural production, driving down its average productivity to $y(1) = 1$. As a result, $\omega = 1$ becomes self-sustaining.[14]

In FIGURE 3.2.(b) the poverty trap represents the unique long-run equilibrium. This situation arises when the failure probability ϕ is sufficiently large. In other words, when entrepreneurial projects are sufficiently risky, inefficient insurance provision prevents the economy from breaking away from the poverty trap in $\omega = 1$.

Finally, in FIGURE 3.2.(c) a case in which, for any $\omega_0 > 1$, the economy converges to $\omega = \rho(1 - \phi)$ in the long run is plotted. In contrast with the example in FIGURE 3.2.(b), this situation appears when ϕ is small enough. Intuitively, when the failure risk is sufficiently low, inefficient insurance provision will not discourage entrepreneurial investment too severely, and the economy will thus move over time towards $\omega = \rho(1-\phi)$.

Further Discussion: relaxing *decreasing relative risk aversion*

The possibility that the model displays multiple long-run equilibria crucially depends on the positive impact of ω_{t-1} on v_t^*. This effect requires that individuals display decreasing *absolute* risk aversion (DARA), so that the richer they are, the more they invest in the (risky) entrepreneurial projects. Yet, this model has assumed that individuals exhibit a stronger property, namely decreasing *relative* risk aversion (DRRA). One might wonder how sensitive the results are to this last assumption.

Given the set-up of the model (in particular, given that the technology in all sectors in the economy is convex), DRRA is a necessary feature for multiple long-run equilibria to *coexist*. In other words, situations like the one depicted in FIGURE 3.2.(a) can only arise when DRRA is assumed. Alternative utility functions that drop DRRA but maintain DARA may still give rise to dynamics with poverty traps similar to the one plotted in FIGURE 3.2.(b), or convergence to high income similar to the one in FIGURE 3.2.(c), depending on the specific parametric configurations. However, they cannot generate dynamics where both types of long-run equilibria coexist.[15] The key reason why DRRA is required for generating non-ergodic dynamics is that risk-taking must be sufficiently *responsive* to income variations, so that the curvature of the schedule generated by (3.12) is sufficiently pronounced (hence it crosses the 45° line at least once from below).

3.5 Concluding Remarks

This paper has presented a model in which, along the path of development, the economy evolves from a small-scale rural economy to an entrepreneurial manufacturing one.

[14]The point $\omega = \underline{\omega}$ is also a stationary equilibrium in FIGURE 3.2.(a), but it is unstable.

[15]A formal exposition of these results is available from the author upon request.

Following such a development path is however not guaranteed because an adverse selection problem prevents the provision of first-best insurance contracts which are needed to support entrepreneurial manufacturing activities. Development to a manufacturing economy tends to fail to take place when entrepreneurial activities encompass very high risks, since those are the cases in which efficient insurance provision matters most for encouraging entrepreneurship.

In terms of risk-bearing, some results of the model are in contrast with those of Banerjee and Newman (1991). In their paper, poorer agents bear the risks, while richer agents choose safer activities (they become "rentiers"); see also Newman (2007) for similar results. Their results are driven by the fact that riskier activities require agents to exert (unobservable) effort. Since effort is assumed to enter linearly in a separable utility function, whereas marginal utility of consumption is decreasing, the marginal rate of substitution of leisure for consumption is increasing in initial wealth. As a result, it turns out to be easier to incentivize poorer agents to exert high effort if they bear some risk. In my model, this incentives problem does not arise, as I disregard moral hazard issues. Similarly, Banerjee and Newman (1991) disregard adverse selection problems by assuming that all agents are intrinsically identical in terms of skills. From an empirical point of view, it is clear that initial wealth represents a key determinant of entrepreneurial choice due to the presence of financial markets imperfections – see, for example, Evans and Jovanovic (1989). In that sense, this paper contributes to the literature on informational asymmetries and development by providing a mechanism that relies on adverse selection to generate a market failure that keeps the poor away from entrepreneurial activities.[16]

Lastly, the model could yield a reasonable explanation for the phenomenon of under-migration from small villages to the city, similarly as proposed by Banerjee and Newman (1998), though they look at credit rather than insurance. In that regard, migrating to the city could be interpreted as investing in a risky asset with higher expected income. The local village, on the other hand, provides its inhabitants with deep social networks that protect them from idiosyncratic shocks (Das Gupta (1987) and Hugo (1982)). This interpretation seems also consistent with the view that information inside the villages flows better; hence adverse selection there would be less troublesome than in the cities.

[16] A passage in Newman (2007) is worth mentioning here. He states "Since embedding the Knightian theory [of entrepreneurship] into a standard moral hazard framework reveals the fragility of its predictions [regarding risk-bearing], it is natural to ask what happens in the presence of other causes of imperfect insurance."

The results of my paper should not be understood as Knightian, though. Adverse selection prevents efficient risk-sharing; hence the rich, who are less risk-averse, take on larger risks. Yet, entrepreneurs here are undertaking a productive task (for which they are particularly talented), and not providing insurance to other individuals (workers) through fixed wages, which seems to be the essence of the Knightian theory.

Appendix

Derivation of Equation (3.9). I proceed here to derive each one of the expressions in (3.9). It proves convenient to first state the following preliminary results:

Lemma A.1. If $(1-\phi)\rho > v_t$, the incentive-compatible insurance quantity $\widehat{q}_t$ must be strictly smaller than the full-information insurance provision level.

Proof. Suppose instead that $\widehat{q}_t$ is equal or larger than the insurance provision under a full-information when $(1-\phi)\rho > v_t$. Then, in the optimum, the constraint (3.8) should not bind, and the first-order conditions for problem (3.7) - (3.8) would yield: $l_t > 0$ and $q_t = \rho\, l_t$. Replacing these values into (3.6) yields that to comply with the incentive compatibility constraint $[(1-\phi)\rho - v_t]\; l_t \leq 0$ must hold; which contradicts the facts that $(1-\phi)\rho > v_t$ and $l_t > 0$. ||

Lemma A.2. Suppose $(1-\phi)\,\rho > v_t$. Then, in the optimum, the problem (3.7) – (3.8) yields: $q_t = \widehat{q}_t$ and

$$l^*_{\mathbb{G}_{t-1}} = \frac{1}{v_t}\left[\frac{(1-\phi)\,\rho - v_t}{\rho - v_t}\,(\omega_{t-1}-1) + \frac{(1-\phi)^2\,\rho + 2\,\phi\, v_t - v_t}{\rho - v_t}\,\widehat{q}_t\right] \tag{3.13}$$

Proof. Whenever $\widehat{q}_t$ is below the full-information insurance provision level, the constraint (3.8) will bind, and thus $q_t = \widehat{q}_t$ will apply in the solution of problem (3.7) - (3.8). As a result, when $(1-\phi)\rho > v_t$, the following first-order condition for l_t obtains:

$$\frac{(1-\phi)\,(\rho - v_t)}{(\omega_{t-1}-1) - \phi\widehat{q}_t + (\rho - v_t)\, l^*_{\mathbb{G}_{t-1}}} - \frac{\phi v_t}{(\omega_{t-1}-1) + (1-\phi)\,\widehat{q}_t - v_t l^*_{\mathbb{G}_{t-1}}} = 0.$$

Finally, from this expression, (3.13) immediately follows after some simple algebra. ||

Lemma A.3. Suppose $(1-\phi)\,\rho > v_t$. Then, in equilibrium, $\widehat{q}_t = v_t\, l^*_{\mathbb{G}_{t-1}}/(1-\phi)$.

Proof. Firstly, notice that from (3.6) $\widehat{q}_t \leq v_t\, l^*_{\mathbb{G}_{t-1}}/(1-\phi)$ can be readily obtained. Secondly, suppose that (3.6) does not bind. In that case, insurance companies could actually offer a contract carrying $q_t > \widehat{q}_t$, which would still screen out the bad-types and that will make all the good-types better off. Hence, in equilibrium, $\widehat{q}_t = v_t\, l^*_{\mathbb{G}_{t-1}}/(1-\phi)$ must apply. ||

By using the results in Lemmas A.2 and A.3, we can replace $\widehat{q}_t = v_t\, l^*_{\mathbb{G}_{t-1}}/(1-\phi)$ into (3.13), to obtain $l^*_{\mathbb{G}_{t-1}} = \frac{1-\phi}{\phi} v_t^{-1}\,(\omega_{t-1}-1)$ when $(1-\phi)\,\rho > v_t$.

Suppose now $v_t = (1-\phi)\,\rho$. Replacing v_t by $(1-\phi)\,\rho$ into (3.6), yields $\widehat{q}_t \leq \rho\, l^*_{\mathbb{G}_{t-1}}$. In equilibrium, $\widehat{q}_t = \rho\, l^*_{\mathbb{G}_{t-1}}$ will hold, for a similar argument as in Lemma A.3. Then, the agent $i \in \mathbb{G}_{t-1}$ will optimally set $q_t = \rho\, l_t$ (which in fact represents the same solution that would apply under full information). As a result, his optimisation problem can be simplified to: $\max_{l_t \geq 0}$: $\{\ln(\omega_{t-1} - 1)\}$. This last problem can be trivially maximised by any feasible value of l_t. In particular, any $l_t \in \left[0, \frac{1-\phi}{\phi} v_t^{-1} (\omega_{t-1} - 1)\right]$, may solve the previous optimisation problem.

Finally, when $(1 - \phi)\rho < v_t$, $l^*_{\mathbb{G}_{t-1}}$ must equal zero, since by investing all their first-period income in the safe-asset, good-types can obtain a higher expected return without bearing any risks.

Proof of Proposition 6. Part (i). Inspecting (3.10) and (3.11) we can observe that, for all $\omega_{t-1} \in (1, \hat{\omega})$, v_t^* is pinned down by the following equation:

$$\eta \frac{1-\phi}{\phi} \frac{1}{v_t^*} (\omega_{t-1} - 1) = 1 - \left(\frac{1}{v_t^*}\right)^{\frac{1}{1-\alpha}}; \tag{3.14}$$

as equation (3.14) yields indeed $v_t^* \in (1, (1-\phi)\rho)$, $\forall \omega_{t-1} \in (1, \hat{\omega})$. Next, totally differentiating (3.14), we obtain:

$$\frac{dv_t^*}{d\omega_{t-1}} = \eta \frac{1-\phi}{\phi} \left[\eta \frac{1-\phi}{\phi} \frac{\omega_{t-1} - 1}{v_t^*} + \frac{1}{1-\alpha} \left(\frac{1}{v_t^*}\right)^{\frac{1}{1-\alpha}}\right]^{-1} > 0.$$

In addition, since $v_t^* \in (1, (1-\phi)\rho)$, from (3.11) it follows that $l_t^* = 1 - (1/v_t^*(\omega_{t-1}))^{1/(1-\alpha)}$, for all $\omega_{t-1} \in (1, \hat{\omega})$. Hence, $l_t^* \in (0, 1)$.

Now, let $\omega_{t-1} = \hat{\omega}$ and note that $l_t^S((1-\phi)\rho) = 1 - \left[\frac{1}{(1-\phi)\rho}\right]^{\frac{1}{1-\alpha}} = (\phi\rho)^{-1}\eta(\hat{\omega} - 1)$. Furthermore, observe thus that: $l_t^S((1-\phi)\rho) < (\phi\rho)^{-1}\eta(\omega_{t-1} - 1)$ for any $\omega_{t-1} > \hat{\omega}$. Therefore, since $l_t^D = 0$ for all $v_t > (1-\phi)\rho$, and $l_t^D = [0, (\phi\rho)^{-1}\eta(\omega_{t-1} - 1)]$ for $v_t = (1-\phi)\rho$; then, for any $\omega_{t-1} \geq \hat{\omega}$, the labour market equilibrium yields $v_t^* = (1-\phi)\rho$ and $l_t^* = (\phi\rho)^{-1}\eta(\hat{\omega} - 1)$. ||

Part (ii). For all $\omega_{t-1} \in [0, 1]$, labour demand equals zero. Therefore, in equilibrium, l_t^S must equal zero too; which requires $v_t^* \in [0, 1]$. ■

Proof of Lemma 6. Differentiating $\Psi(\omega_{t-1}, \omega_t) = 0$ in (3.12) by using the rule of derivation for implicit functions yields:

$$\frac{d\omega_t}{d\omega_{t-1}} = \eta \frac{1-\phi}{\phi} \left[\eta \frac{1-\phi}{\phi} \frac{\omega_{t-1} - 1}{\omega_t} + \frac{1}{1-\alpha} \left(\frac{1}{\omega_t}\right)^{\frac{1}{1-\alpha}}\right]^{-1} > 0, \quad \forall \omega_{t-1} \in [1, \bar{\omega}]. \tag{3.15}$$

Next, differentiating (3.15) with respect to ω_{t-1} yields:

$$\frac{d^2\omega_t}{(d\omega_{t-1})^2} = \frac{d\omega_t/d\omega_{t-1}}{\Lambda\,\omega_t} \left\{ \left(\frac{d\omega_t}{d\omega_{t-1}}\right) \left[\frac{1}{(1-\alpha)^2\,\omega_t^{1/(1-\alpha)}} + \frac{\eta(1-\phi)}{\phi} \frac{(\omega_{t-1} - 1)}{\omega_t}\right] - \frac{\eta(1-\phi)}{\phi} \right\},$$

$$\forall \omega_{t-1} \in [1, \bar{\omega}]; \quad \text{where: } \Lambda \equiv \left[\frac{1-\phi}{\phi} \frac{\omega_{t-1} - 1}{\omega_t} \eta + \frac{1}{1-\alpha} \left(\frac{1}{\omega_t}\right)^{1/(1-\alpha)}\right] > 0. \tag{3.16}$$

Hence, the sign of $d^2\omega_t/(d\omega_{t-1})^2$ is determined by the sign of the expression within braces in the right-hand side of (3.16). Using then (3.15) and some simple algebra, we can observe that:

$$\frac{d^2\omega_t}{(d\omega_{t-1})^2} > 0, \forall\, \omega_{t-1} \in [1, \bar{\omega}] \;\Leftrightarrow\; \alpha > 0. \quad \blacksquare$$

Proof of Proposition 7. Part (i). First of all, notice that the point $\omega_t = 1$ represents *always* a stationary point of (3.12), since $\Psi(1,1) = 0$. Next, given the statement in Lemma 6, it follows that a necessary and sufficient condition for $\omega = 1$ to be locally stable is that the first derivative in (3.15) computed at $\omega_{t-1} = 1$ is strictly smaller than 1. Thus, replacing $\omega_{t-1} = \omega_t = 1$ into (3.15), we get:

$$\left.\frac{d\omega_t}{d\omega_{t-1}}\right|_{\omega_{t-1}=1} = \eta\frac{(1-\phi)}{\phi}(1-\alpha).$$

Therefore, $\phi/[\eta(1-\phi)] > 1-\alpha$ implies $\left.\frac{d\omega_t}{d\omega_{t-1}}\right|_{\omega_{t-1}=1} < 1$.

Second, since $\omega_t = (1-\phi)\rho$ for all $\omega_{t-1} \in (\bar{\omega}, (1-\phi)\rho]$, whenever this interval is non-empty; in order to show that $\omega_t = (1-\phi)\rho$ is also a locally stable stationary equilibrium, it suffices to prove that, under the stipulated conditions, $\hat{\omega} < (1-\phi)\rho$. From the expressions in (3.10) and (3.11), we can observe that:

$$\hat{\omega} < (1-\phi)\rho \quad \Leftrightarrow \quad \underbrace{\eta\frac{1-\phi}{\phi}\frac{(1-\phi)\rho - 1}{(1-\phi)\rho}}_{M(\rho)} > \underbrace{1 - \left[\frac{1}{(1-\phi)\rho}\right]^{\frac{1}{1-\alpha}}}_{N(\rho,\alpha)}. \tag{3.17}$$

From (3.17), it follows that:

$$\lim_{\rho\to 1/(1-\phi)} M(\rho) = \lim_{\rho\to 1/(1-\phi)} N(\rho,\alpha) = 0, \tag{3.18}$$

$$\lim_{\rho\to\infty} M(\rho) = \tfrac{\eta(1-\phi)}{\phi} > \lim_{\rho\to\infty} N(\rho,\alpha) = 1. \tag{3.19}$$

Differentiating $M(\rho)$ and $N(\rho,\alpha)$ with respect to ρ, we obtain: $dM/d\rho = \eta/\phi\rho^2$, and $\partial N/\partial\rho = [(1-\alpha)\rho]^{-1}\left[\frac{1}{(1-\phi)\rho}\right]^{\frac{1}{1-\alpha}}$. Therefore:

$$\frac{dM}{d\rho} \gtrless \frac{\partial N}{\partial \rho} \quad \Leftrightarrow \quad \frac{\eta(1-\phi)}{\phi}(1-\alpha) \gtrless \left[\frac{1}{(1-\phi)\rho}\right]^{\frac{\alpha}{1-\alpha}}. \tag{3.20}$$

Denote by $\hat{\rho}(\alpha)$ the value of ρ that solves the second expression in (3.20) with strict equality; that is:

$$\eta\frac{(1-\phi)}{\phi}(1-\alpha) \equiv \left[\frac{1}{(1-\phi)\hat{\rho}}\right]^{\frac{\alpha}{1-\alpha}}, \tag{3.21}$$

where it can be observed that $\hat{\rho}(\alpha) > 1/(1-\phi)$. Then, the expression in (3.20), together with (3.21) and the fact that $\phi/[\eta(1-\phi)] > 1-\alpha$, imply:

$$\text{1) for all } \rho \in (1/(1-\phi), \hat{\rho}(\alpha)):\; dM/d\rho < \partial N/\partial\rho \tag{3.22}$$

$$\text{2) for all } \rho > \hat{\rho}(\alpha):\; dM/d\rho > \partial N/\partial\rho \tag{3.23}$$

$$\text{3) when } \rho = \hat{\rho}(\alpha):\; dM/d\rho = \partial N/\partial\rho. \tag{3.24}$$

As a result, combining (3.22) and (3.24) with the result in (3.18), we can deduce that $M(\rho) < N(\rho, \alpha)$ for all $\rho \in (1/(1-\phi), \hat{\rho}(\alpha)]$. Furthermore, because of (3.23) and the result in (3.19), we can observe that: $\exists\, \bar{\rho} > \hat{\rho}(\alpha)$, such that $M(\bar{\rho}) = N(\bar{\rho}, \alpha)$, and $M(\rho) > N(\rho, \alpha)$ for all $\rho > \bar{\rho}$, while $M(\rho) < N(\rho, \alpha)$ for all $\rho < \bar{\rho}$. Using again (3.17), we can observe that $\bar{\rho}$ must solve:

$$\eta \frac{1-\phi}{\phi} \frac{(1-\phi)\bar{\rho}-1}{(1-\phi)\bar{\rho}} = 1 - \left[\frac{1}{(1-\phi)\bar{\rho}}\right]^{\frac{1}{1-\alpha}}; \tag{3.25}$$

from where it follows that $\bar{\rho} = \bar{\rho}(\alpha)$. This completes the proof that $\exists\, \bar{\rho}(\alpha) > 1/(1-\phi)$, such that for all $\rho > \bar{\rho}(\alpha)$ there exists another locally stable stationary point at $\omega = (1-\phi)\,\rho$.

Finally, totally differentiating (3.25), we get:

$$\frac{d\bar{\rho}}{d\alpha} = \frac{\frac{1}{(1-\alpha)^2}\left[\frac{1}{(1-\phi)\bar{\rho}}\right]^{\frac{1}{1-\alpha}} \ln\left[(1-\phi)\bar{\rho}\right]}{\dfrac{\eta}{\phi\bar{\rho}^2} - \dfrac{1}{(1-\alpha)\,\bar{\rho}}\left[\dfrac{1}{(1-\phi)\bar{\rho}}\right]^{\frac{1}{1-\alpha}}}. \tag{3.26}$$

Given that at $\rho = \bar{\rho}$, $dM/d\rho > \partial N/\partial\rho$, the denominator in the right-hand side of (3.26) must thus be positive. Furthermore, the numerator in the right-hand side of (3.26) is also positive, because $\bar{\rho} > 1/(1-\phi)$. As a result, it follows that $d\bar{\rho}/d\alpha > 0$. ||

Part (*ii*). Note first that $\partial N/\partial\alpha > 0$. As a result, if (3.17) does not hold for $\alpha \to 0$, it will then not hold for any $\alpha \in (0,1)$ either. Taking the limit on the expressions in (3.17) as $\alpha \to 0$, we obtain:

$$\text{if } \alpha \to 0\text{:}\;\; \hat{\omega} < (1-\phi)\,\rho \quad \Leftrightarrow \quad \eta\frac{1-\phi}{\phi}\left[1 - \frac{1}{(1-\phi)\rho}\right] > 1 - \frac{1}{(1-\phi)\rho} \tag{3.27}$$

Therefore, if $\phi/[\eta\,(1-\phi)] \geq 1$, (3.27) implies that $\hat{\omega} \geq (1-\phi)\,\rho$ when $\alpha \to 0$, and thus the only stable stationary point is $\omega = 1$.

Lastly, the proof that if $\phi/[\eta\,(1-\phi)] \in (1-\alpha, 1)$ holds, but $\rho \leq \bar{\rho}(\alpha)$, then the only stable stationary equilibrium is the point $\omega = 1$, is implicit in the previous proof of Part (*i*) of this proposition. ||

Part (*iii*). If $\phi/[\eta\,(1-\phi)] \leq 1-\alpha$, then: $\left.\frac{d\omega_t}{d\omega_{t-1}}\right|_{\omega_{t-1}=1} \geq 1$. As a consequence, the fixed point $\omega = 1$ is locally unstable. Moreover, because $\Psi(\omega_t, \omega_{t-1}) = 0$ yields an increasing an convex function in ω_{t-1}, it follows that:

$$\frac{d\omega_t}{d\omega_{t-1}} > 1, \;\; \forall\, \omega_{t-1} \in [1, \bar{\omega}]. \tag{3.28}$$

Given (3.28), and given that $\omega = 1$ is a fixed point, it follows that $\omega_t > \omega_{t-1}$ for all $\omega_{t-1} \in (1, \bar{\omega}]$. Therefore, $\hat{\omega} < \bar{\omega}$, and $\omega = (1-\phi)\rho$ thus represents the unique *stable* fixed point of (3.12). ■

Chapter 4

Income Divergence in a Ricardian Model of International Trade without Absolute Advantages

4.1 Introduction

In the past two decades a number of articles on international trade have acknowledged the importance of using non-homothetic preferences to capture some relevant features of North-South trade – e.g., Matsuyama (2000), Flam and Helpman (1987), and Stokey (1991). These papers have developed tractable models that predict patterns of specialisation where richer countries produce and export goods with high value added and high income demand elasticity. One of the main predictions of those models is that the impact of international trade on growth may be uneven across countries which are at different stages in the process of development. More precisely, trade would tend to be more beneficial to developed economies, and it may even be detrimental to underdeveloped countries. The key mechanism at work is the one originally argued by Prebish (1950) and Singer (1950). That is, the fact that as the world income rises, world aggregate demand deviates towards the goods produced by richer economies (the North), which improves their terms of trade and, thereby, magnifies initial income disparities between South and North.

The papers mentioned above have thus restricted the attention to a world economy where some countries (the North) have somehow historically accumulated larger amounts of human and physical capital than others (the South), and show conditions under which trade magnifies initial income disparities resulting from those capital differences. However, the pattern of international specialisation and trade might also be the source of income differentials between countries that do not display any substantial dissimilarity regarding their levels of human and physical capital. In this paper, we look at economies that start off with similar capital endowments and income per head, and propose a theory of uneven growth induced by trade based on non-homothetic preferences and productive specialisation driven by comparative advantages.

Our paper is based on four fundamental elements. First, each consumption good in the economy is present in several degrees of quality, while the higher the quality of the good, the more costly it is to produce it. Second, individuals care about the quality of the goods they consume and, moreover, their willingness to pay for higher quality of consumption increases with their income. Third, some goods offer larger scope for quality upgrading than others, in the sense that it is cheaper to increase their quality. Fourth, countries which are similar in terms of their *average* productivities specialise in the production of different varieties of goods according to their intrinsic comparative advantages.

The first three elements above give room for non-homothetic demand schedules, where the income demand elasticity of different goods is tied to the specific degree of quality in which each particular good is (optimally) traded in the market. The last element yields patterns of regional specialisation which, combined with non-homothetic demand schedules, may lead to divergent dynamics among countries that are initially similar concerning capital endowments and income per head. In such a framework, we show that international trade may induce income divergence across countries characterised by similar initial income levels and with no clear *absolute* advantages over one another. In particular, income divergence will be experienced when comparative advantages dictate patterns of specialisation that, although optimal for each specific country at a given point in time, do not offer the same scope for technological improvements in terms of subsequent quality upgrading of final goods.

To convey some preliminary intuition of how non-homothetic demand schedules arise as an equilibrium result of our model, it is worth discussing in further detail some of the specificities of the commodity space. In that respect, we closely follow the quality ladder structure featured in Grossman and Helpman (1991) – that is, in a continuum of horizontally differentiated varieties of goods, an infinite number of qualities for each variety are available in the market. Our commodity space is thus bidimensional, with the horizontal axis indexing the variety of the good and the vertical axis indexing the quality level of a specific variety. Unlike Grossman and Helpman, however, in our framework the optimal expenditure shares across varieties do not remain constant as income changes. In particular, we postulate that the additional utility the individual derives from a marginal increase in the quality of the goods he consumes increases with the quantity of consumption, hence with the individual's income (in other words, the individual's *taste for quality* increases with his income). As a result, as individuals become richer they will optimally shift resources towards those varieties whose quality can be set at relatively higher levels. The budget constraint, in turn, implies that the extent by which quality can be raised for any given variety is related to its specific cost of quality upgrading. Thus, the distribution of quality upgrading across varieties results from the interaction between the underlying technological structure and the response of the consumers' *taste for quality* to income variations. If the cost of quality upgrading differs across varieties, then the shift towards higher-quality goods with rising income will (optimally) occur at different speeds across varieties. More precisely, the lower the cost of quality upgrading for a specific variety, the faster the speed of quality upgrading for this variety. This uneven climbing-up-the-quality-ladder will in turn lead to non-homothetic demand schedules, where the fraction of income spent in different varieties depends on the level of income.

To get a glimpse of how the interaction between quality upgrading and comparative advantages may lead to divergent income paths through its effect on the evolution of trade, take some hypothetical country (call it country Z) that specialises in the variety x, which exhibits high cost of quality upgrading.[1] According to the mechanism proposed in this paper, the speed of quality upgrading for x will be relatively slow in an equilibrium in which world income is growing. Hence, in such an equilibrium, the world expenditure share on x will decrease over time. As a result, as the world income rises, Z will experience a decline in its terms of trade, because the types of goods it produces display low income demand elasticity.

4.1.1 An Illustrative Historical Example – colonial Jamaica and pre-industrial Argentina

Situations where the mechanism proposed in this paper may have played an important role include the cases of economies for which exogenous initial geographical conditions greatly influenced their specialisation in the world economy during some period in history. As an illustrative example, take the case of colonial Jamaica and compare to that of pre-industrial Argentina.

From the second half of the XVII century until the first half of the XIX century, the Jamaican economy grew mainly based on the exploitation and export of sugar from sugarcane. This is not surprising given the excellent climate conditions this tropical island offered for that type of crop. By 1805, Jamaica was the largest sugar exporter in the world (Higman, 2005). Given the value attributed to sugar by European consumers, during that period Jamaica was deemed probably the most important British colony

[1]This specialisation decision might be related, for instance, to geographical conditions influencing the technology available in Z. The following historical example will illustrate this point in further detail.

in the Americas (Hall, 1959). Although sugar was indeed a very valuable consumption good at that time, it clearly was a type of good with very limited scope for undergoing subsequent improvements in quality. As such, according to our model, sugar was bound to eventually lose its status of luxury among consumers. In fact, by the second half of the XIX century, sugar began to lose its economic preeminence in the world markets and started experiencing a long phase of declining prices.[2]

In Argentina, geographical conditions made this country exceptionally apt for the breeding of cattle and growing cereals, which constituted the main engines of its economy until 1920. The commercial exploitation of cattle started in the late second half of the XVIII century with the appearance of the *saladeros*, where salt-cured beef would be produced. Salt-cured beef was a rather unsophisticated product that was mostly exported to Cuba and Brazil to feed the slaves there. In fact, the industry of the *saladeros* did not mean a big push to the Argentinean economy, which was at that time still a very marginal country within the world economy. The *big boom* for the cattle industry in Argentina actually came about much later with the introduction of the cold storage by the end of the XIX century. This technology permitted selling chilled and frozen beef in Europe, attracting thus well-to-do consumers from that continent.[3] During this period, Argentina grew on average at rate of 5% yearly, attracted millions of immigrants from Europe and became one of the richest countries in the world.[4] The exportation of chilled and frozen beef was undoubtedly one of the main activities that spurred this phase of fast and steady economic growth in Argentina between 1880-1920.[5]

The previous example illustrates how exogenous productive conditions greatly influenced the path of GDP growth in Jamaica and in Argentina via the evolution of their exports, in the way as our model would predict. Jamaica was comparatively efficient at producing sugar, while Argentina enjoyed a comparative advantage in beef production. Sugar offered very limited scope for quality improvements, which is analogous to assuming that the cost of quality upgrading for sugar products is extremely steep. On the contrary, beef did offer a lot more scope for quality upgrading than sugar, which mate-

[2]The decline of the sugarcane industry in Jamaica at that time was also importantly helped by the abolition of slavery in 1838, which led to higher labour costs to landowners (Hall, 1959). It can be argued, though, that the abolition of slavery was itself an endogenous outcome, and that lower sugar prices created an environment more prone to it by diminishing the profits from slavery.

[3]The main market for Argentinean chilled and frozen beef at that time was by far the prosperous Great Britain of end of XIX and beginning of XX century (in 1914, 83.5% of the total Argentinean exports of chilled and frozen beef was sent to UK).

[4]By 1913, the GDP per head in Argentina was similar to that of France and Germany - Blanchard and Perez Enrri (2002).

[5]The main source of historical information used for this paragraph was Rapaport (1988).

rialised as the switch from salt-cured beef production (low-quality good) to chilled and frozen beef (high-quality good; at least by the end of the XIX century). As predicted by our model, sugar exports initially sustained high growth in Jamaica, until rising income in the world shifted aggregate demand towards varieties of goods which could be offered in higher quality degrees, such as chilled and frozen beef from Argentina.

The paper is organised as follows. Section 4.2 describes the set-up of the model. Section 4.3 solves the consumer's problem in a partial equilibrium set-up, illustrating the specificities of the non-homotheticity of demand in our model. Section 4.4 computes the general equilibrium in the world economy, examining the effects of uniform aggregate productivity growth. Section 4.5 concludes. The appendices contain the omitted proofs and some additional algebraic derivations used in the main text.

4.2 Structure of the Model

We consider a world composed by two countries: the *Home* country and the *Foreign* country. For brevity, hereafter we refer to the former as H and to the latter as F. These two economies share a common commodity space, defined along two distinct dimensions: *horizontal* and *vertical*. The first dimension (*horizontal*) designates the *variety* of the good – e.g., food, TV, etc. Different varieties are indexed by the letter v along the variety space $\mathbb{V} \subset \mathbb{R} : v \in [0, 1]$. The second dimension (*vertical*) refers to the intrinsic *quality* of the good of a particular variety v – e.g., organic vs. non-organic food, LCD TV vs. cathode ray tube TV, etc. Within each variety $v \in \mathbb{V}$, commodities are *vertically* ordered by the quality-index q belonging to the set $\mathbb{Q} \subset \mathbb{R} : q \in [1, \infty)$, where a higher q denotes a higher quality level.[6] The commodity space is then given by the set $\mathbb{V} \times \mathbb{Q} = [0, 1] \times [1, \infty)$, and each commodity is identified by a pair $(v, q) \in \mathbb{V} \times \mathbb{Q}$.[7]

We assume that all commodities are tradable. Additionally, we assume there are no transport cost and no tariffs affecting international trade.

[6]None of the main results of the model depend on the fact that the lower-bound on q is normalised at one. Yet, adjusting the model so as this lower-bound is set at zero would greatly increase the algebraic complexity of the model.

[7]In our setup, different varieties should be then understood as groups of commodities that aim at satisfying different *needs*. On the other hand, different qualities for a particular variety refer to the *extent* (or *degree*) in which the *need* is actually satisfied by the commodity. In that regard, food satisfies a different need when compared to TVs (physiological nutrition vs. visual entertainment), but an LCD TV satisfies the need for visual entertainment (objectively!) *better* than a cathode ray tube TV.

4.2.1 Preferences and Budget Constraint

Both H and F are inhabited by a continuum of individuals with identical preferences defined over the commodity space $\mathbb{V} \times \mathbb{Q}$. Whenever it proves needed, hereafter we adopt the following notation: unstarred symbols refer to H, starred ones to F.

Denote by $x_{vq} \in \mathbb{R}_+$ the consumed *quantity* of commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$ (i.e., the consumed quantity of variety v in quality q) by a representative individual from H. His preferences are summarised by the following utility function:

$$U = \int_{\mathbb{V}} \ln \left[\int_{\mathbb{Q}} \max \{x_{vq}, (x_{vq})^q\} \, dq \right] dv \tag{4.1}$$

This utility function captures the notion that quality is a desirable feature. However, notice that according to specification in (4.1), although quality is never bad, it only *magnifies* the utility derived from (physical) consumption when $x_{vq} > 1$. This last property of (4.1) intends to capture the idea that individuals first seek to satisfy their basic consumption needs, and just after these basic needs are met, do they start paying attention to the quality dimension of the goods they consume.

Some additional properties about the utility function specified in (4.1) are worth noting. First, within each variety v, marginal utility is unbounded above as consumption approaches zero, implying that all varieties will be actively consumed in an optimum. Second, convexity in quantities of the inner integrals of U means that individuals will optimally consume only *one* type of quality for each variety v. Third, considering two different levels of the quality-index $\underline{q} < \overline{q}$ for the same variety v, the marginal rate of substitution of $x_{v\overline{q}}$ for $x_{v\underline{q}}$ is non-decreasing along a *proportional expansion path* of $x_{v\overline{q}}$ and $x_{v\underline{q}}$.[8] This last property of (4.1) will allow demand functions to display non-homothetic behaviour, where the rich spend a larger fraction of their income in high-qualities than the poor.

Each individual is endowed with one unit of *effective* labour. Labour is immobile across countries. As a result, each individual in H supplies inelastically his entire labour endowment to domestic firms in return of a wage $w \in \mathbb{R}_{++}$ (hereafter, all prices are measured in a common *numeraire*). This wage represents the only source of income for the individual. Therefore, his budget constraint reads as follows:

$$\int_{\mathbb{V}} \int_{\mathbb{Q}} p_{vq} \, x_{vq} \, dq \, dv \leq w \tag{4.2}$$

[8]To see this, note the $MRS(x_{v\overline{q}}, x_{v\underline{q}})$ is defined by $(\partial U/\partial x_{v\overline{q}}) \,/\, (\partial U/\partial x_{v\underline{q}})$, and along a proportional expansion path $x_{v\overline{q}} = k \, x_{v\underline{q}}$, where $k > 0$. Then, for $x_{v\underline{q}}, x_{v\overline{q}} > 1$:

$$MRS(k \, x_{v\underline{q}}, x_{v\underline{q}}) = \frac{\overline{q}}{\underline{q}} k^{\overline{q}-1} \left(x_{v\underline{q}}\right)^{\overline{q}-\underline{q}},$$

from where it is clear that, along the *ray* $x_{v\overline{q}} = k \, x_{v\underline{q}}$, $MRS(x_{v\overline{q}}, x_{v\underline{q}})$ is increasing in $x_{v\underline{q}}$.

where $p_{vq} \in \mathbb{R}_{++}$ denotes the (international) price of each unit of commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$.

We define $\beta_v \equiv w^{-1} \int_{\mathbb{Q}} p_{vq} x_{vq} dq$ as the *demand intensity* of variety $v \in \mathbb{V}$.[9] In the optimum, given the specification in (4.1), the budget constraint (4.2) will naturally bind. It is thus straightforward to notice that demand intensities will sum up to one across varieties (i.e., $\int_{\mathbb{V}} \beta_v dv = 1$).

All individuals in the world face the same prices for the reproducible commodities. As a result, the analogous expressions in (4.1) and (4.2) corresponding to F read, respectively, as follows: $U^* = \int_{\mathbb{V}} \ln \left[\int_{\mathbb{Q}} \max \left\{ x^*_{vq}, \left(x^*_{vq} \right)^q \right\} dq \right] dv$ and $\int_{\mathbb{V}} \int_{\mathbb{Q}} p_{vq} x^*_{vq} dq dv \leq w^*$; where w^* denotes the wage in F in terms of the common *numeraire* (clearly, since labour is immobile, w and w^* need not be equal).

4.2.2 Technology

In both countries competitive firms produce commodities based on linear production functions in which labour represents their only input. We let unit labour requirements vary both across varieties and across qualities of each variety. Also, we let unit labour requirements differ across countries. In particular, in H the unit labour requirement for commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$ is given by $c_{vq} = a(v) q^{\eta(v)} / \kappa$, while in F is given by $c^*_{vq} = a^*(v) q^{\eta(v)} / \kappa$; where $\kappa > 0$ denotes a *world aggregate-productivity* parameter, $a(v)$ and $a^*(v)$ represent *variety-specific* technological parameters which may differ between countries, and $\eta(v)$ summarises the *cost elasticity of quality upgrading* for each variety v which is assumed to be the same for H and F. We suppose that $a(v) : [0, 1] \to \mathbb{R}_{++}$, where $a'(\cdot) \geq 0$; analogously, $a^*(v) : [0, 1] \to \mathbb{R}_{++}$, where $a^{*\prime}(\cdot) \geq 0$. We also assume that $\eta(v) : [0, 1] \to \mathbb{R}_{++}$, where $\eta'(\cdot) > 0$ and $\eta(0) > 1$.[10]

The next assumption dictates the pattern of *comparative* advantages across countries.

Assumption 1 *Let $A(v) \equiv a^*(v) / a(v)$. We suppose: $A(0.5) = 1$ and $A'(v) < 0$.*

Assumption 1 represents the *only* source of heterogeneity across countries in our model. In particular, this last assumption implies that H enjoys a comparative advantage in lower-indexed commodities, while F has a comparative advantage in the upper-indexed commodities.

[9] We borrow this nomenclature from Horvath (2000).

[10] From the labour requirements functions it is apparent that qualitative upgrade is costly, which seems a natural assumption to make. Additionally, from our assumptions it follows that $\eta(v) > 1$ for all $v \in \mathbb{V}$, which implies that the marginal cost of improving quality is, for each variety, increasing along the quality-space. In that sense, this assumption also seems quite natural, as it reflects the fact that subsequent quality improvements become increasingly costly. Finally, note that $\eta'(\cdot) > 0$ –coupled with $a'(\cdot) \geq 0$– implies that varieties are sorted by their cost of quality upgrading.

Note that given the cost functions c_{vq} and c^*_{vq} specified above, we are allowing countries to possibly display identical income per-head in equilibrium, since we are not imposing any direct source of *absolute* advantage in the model. Furthermore, notice that because $\eta(v)$ is the same for H and F, the nature of the comparative advantages does not change as we move up in the quality-ladder. In that sense, in the model comparative advantages always refer to particular varieties of goods, irrespective of the quality at which this variety is actually produced (for example, a country that has a comparative advantage in producing foodstuff, will have this advantage both in organic and in non-organic food products).

In our world economy, each country will naturally specialise in those commodities which they can produce more cheaply. As a result, the *international* price of each commodity will be given by $p_{vq} = \min\{c_{vq}w, c^*_{vq}w^*\}$. Given Assumptions 1, we can write the international price of each commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$ as follows:

$$p_{vq} = \kappa^{-1}\alpha(v)\, q^{\eta(v)}, \tag{4.3}$$

where $\alpha(v) \equiv \min\{a(v)\, w, a^*(v)\, w^*\}$. In addition, from (4.3) we can determine the marginal variety m (that is, the variety that can be produced by both countries at the same cost) as:

$$w/w^* = A(m). \tag{4.4}$$

Equation (4.4) implies that, given the relative wage w/w^*, H will produce all the varieties in the interval $[0, m]$ and F will produce all the varieties within $[m, 1]$.

4.2.3 Brief Remark about the Timing in the Model

In the rest of this paper we study the equilibrium solution of the model and perform some comparative statics experiments to see how these shocks affect the equilibrium. The model is presented as a static one, where the specific equilibria are computed for various parametric configurations. However, this static representation of the model should be seen just as concise simplification of a dynamic sequence of static problems. Therefore, the comparative statics experiments should be accordingly interpreted through a dynamic spectacle, where changes in key parameters are to be understood as changes over time in the parameters' values.

Reducing a dynamic model to a static one where agents maximise (4.1) subject to (4.2), while the technology available in the world determines the prices by (4.3), equals to either assuming that agents live for one period and are non-altruistic, or that they are myopic in some sense. In the former case, we should take the single-period life as a "condensed" version of an intertemporal problem faced by non-altruistic agents, where *one* period is the relevant unit of time under which to analyse processes of technical change of potential importance to our paper. In the latter case, myopia could reflect the fact that the scope in terms of future quality upgrading corresponding to different varieties of goods is not known ex ante.

4.3 The Individual's Optimal Consumption Choice

In this section we present the optimal consumption choice of a representative individual from H, given the set of prices in the world economy. The results so obtained can be easily extended to an individual from F, which is done in Appendix .2.

An individual from H chooses the quantities $x_{vq} \in \mathbb{R}_+$ to consume of each commodity $(v, q) \in \mathbb{V} \times \mathbb{Q}$ to solve the following problem:

$$\begin{aligned} \max_{\{x_{vq}\}_{(v,q)\in\mathbb{V}\times\mathbb{Q}}} \quad & U = \int_{\mathbb{V}} \ln \left[\int_{\mathbb{Q}} \max\left\{x_{vq}, (x_{vq})^q\right\} dq \right] dv, \\ \text{subject to:} \quad & \textstyle\int_{\mathbb{V}} \beta_v dv = 1, \\ & p_{vq} = \kappa^{-1} \alpha(v)\, q^{\eta(v)}, \ \forall (v, q) \in \mathbb{V} \times \mathbb{Q}. \end{aligned} \tag{4.5}$$

In order to solve (4.5), it proves convenient to state the following preliminary results.

Lemma 7 (Preliminary Results)

(i) For each variety $v \in \mathbb{V}$, at most one quality, denoted henceforth by $q_v \in \mathbb{Q}$, is consumed in strictly positive amount in an optimum; formally: $x_{vq_v} \geq 0$, $x_{vq} = 0$, $\forall q \neq q_v$.

(ii) Take x_{vq_v}, $\forall v \in \mathbb{V}$. Then: $q_v > 1 \Rightarrow x_{vq_v} > 1$.

Proof. See Appendix .3. ■

From Lemma 7, Part (i), it immediately follows that the income devoted to purchasing commodities of variety v is entirely spent on quality q_v. Hence, for each $v \in \mathbb{V}$ the consumed quantity of the optimal quality q_v is given by $x_{vq_v} = \beta_v w / p_{vq_v}$. In addition, from Part (ii), it follows that we may replace the inner integral $\int_{\mathbb{Q}} \max\left\{x^*_{vq}, \left(x^*_{vq}\right)^q\right\} dq$ in (4.5) by the simpler expression $\int_{\mathbb{Q}} \left(x^*_{vq}\right)^q dq$, without altering any of the final results of that problem.[11]

Given Lemma 7 the individual's optimisation problem in (4.5) can be thus restated in a simpler form in terms of two sets of control variables $\{\beta_v, q_v\}_{v\in\mathbb{V}}$ replacing the set

[11]To this more clearly, notice that (keeping in mind the physical constraint $x_{vq} \geq 0$) $x_{vq_v} > (x_{vq_v})^{q_v}$ if and only if $x_{vq_v} < 1$ and $q_v > 1$, which according to Lemma 7, Part (ii), cannot be true.

of physical quantities $\{x_{vq}\}_{(v,q)\in\mathbb{V}\times\mathbb{Q}}$. In particular, (4.5) can be restated in the following *reduced-form*:

$$\begin{aligned} \max_{\{q_v,\beta_v\}_{v\in\mathbb{V}}} \quad & U = \int_{\mathbb{V}} q_v \ln\left(\frac{w\beta_v}{p_{vq_v}}\right) dv, \\ \textit{subject to:} \quad & \int_{\mathbb{V}} \beta_v dv = 1, \\ & q_v \geq 1, \ \forall v \in \mathbb{V}, \\ & p_{vq_v} = \kappa^{-1}\alpha(v)\, q_v^{\eta(v)}, \ \forall v \in \mathbb{V}. \end{aligned} \tag{4.6}$$

The first-order conditions corresponding to (4.6) are stated in the Appendix .1. From those first-order conditions we may obtain the following expression for each β_v in the optimum:

$$\beta_v = \frac{q_v}{\int_{\mathbb{V}} q_z dz}, \ \forall v \in \mathbb{V}. \tag{4.7}$$

The denominator of the right-hand side of (4.7) can be regarded as an aggregate index measuring the optimal consumption bundle's *average quality*, and is henceforth denoted by $Q \equiv \int_{\mathbb{V}} q_z dz$. Notice that, according to (4.7), the fraction of income spent on variety v is determined by its optimal quality relative to the average quality of consumption. In that regard, if all varieties were optimally consumed at identical quality degrees (i.e., if $q_v = Q$, $\forall v \in \mathbb{V}$), then $\beta_v = 1$ would hold for all $v \in \mathbb{V}$, and our model would behave exactly as the one by Dornbusch, Fischer and Samuelson (1977).

4.3.1 Distribution of Qualities and Demand Intensities across Varieties

Given the technology in the world economy –summarised by κ, $\alpha(\cdot)$ and $\eta(\cdot)$– it is possible to characterise the distribution of the optimal qualities across varieties according to their position within the set $\mathbb{V}$. Lemma 8 provides the first result in that direction.

Lemma 8 *Consider two varieties $\underline{v}, \bar{v} \in \mathbb{V}$, such that $\underline{v} < \bar{v}$. Then: $q_{\underline{v}} \geq q_{\bar{v}}$, with strict inequality if and only if $q_{\underline{v}} > 1$.*

Proof. See Appendix .3. ■

Lemma 8, implies that the consumed quality q_v is non-increasing in the variety-index v. The underlying intuition for Lemma 8 is straightforward – those varieties which can be more cheaply upgraded tend to be optimally consumed in higher quality levels.

The monotonicity of q_v implied by Lemma 8 allows us to split the variety-space in two disjoint subsets. The first subset containing varieties that are bound to be consumed at the baseline quality level (i.e. with $q_v = 1$) – these are the higher-indexed varieties. The second one comprising the varieties for which the constraint $q_v \geq 1$ in (4.6) does not bind – these are the lower-indexed varieties. Let us denote by $\mathbb{L} \subseteq \mathbb{V}$ the latter subset.

Definition 3 *Let* $\mathbb{L} = \{v \in \mathbb{V} : \lambda_v = 0\}$*, where* λ_v *is the Lagrange multiplier associated to the constraint* $q_v \geq 1$.

Remark. Both $\mathbb{L} = \emptyset$ and $\mathbb{L} = \mathbb{V}$ are in principle possible. In fact, $\mathbb{L} = \emptyset$ will hold if κ is sufficiently small, while $\mathbb{L} = \mathbb{V}$ will hold if κ is sufficiently large. (See Lemma 9 ahead.)

Lastly, regarding the distribution of the demand intensities, from the condition in (4.7) we can observe that, in the optimum, demand intensities are set proportional to the optimal qualities. As a result, the distribution of β_v across varieties will qualitatively mirror that one of q_v.

4.3.2 Effects of Aggregate Productivity Shocks on Demand

When the technology is subject to changes, both *substitution-effects* (due to adjustments in relative prices) and *income-effects* (due to the overall effect of variations in productivity) arise. Here we focus our attention solely on income-effects. In order to isolate income-effects from substitution-effects, we let the parameter κ vary, while we keep constant the functions $a(\cdot)$, $a^*(\cdot)$ and $\eta(\cdot)$.

Lemma 9 *Let* $\underline{\kappa} \equiv a(0)\exp[\eta(0)]$*. Then:*
(i) for all $\kappa \in (0, \underline{\kappa}) : \mathbb{L} = \emptyset$*;*
(ii) for all $\kappa \geq \underline{\kappa} : \mathbb{L} = [0, \tilde{v}(\kappa)]$*, where* $\tilde{v}(\kappa) : [\underline{\kappa}, \infty) \to [0,1]$*,* $\tilde{v}(\underline{\kappa}) = 0$*, and* $\tilde{v}'(\kappa) > 0$ *whenever* $\tilde{v}(\kappa) < 1$.

Proof. See Appendix .3. ■

In short, Lemma 9 implies that the subset of varieties consumed at the *baseline* quality level initially comprises the entire set $\mathbb{V}$, and eventually starts narrowing as world aggregate productivity increases beyond the threshold $\underline{\kappa}$. The next lemma describes in further detail how optimal qualities evolve as the parameter κ changes.

Lemma 10

i) If $\kappa \in (0, \underline{\kappa})$*:* $\partial q_v / \partial \kappa = 0$ *for all* $v \in \mathbb{V}$*;*

ii) If $\kappa \geq \underline{\kappa}$*: a) for all* $v \in \mathbb{L}$*,* $\partial q_v / \partial \kappa > 0$*; b) for all* $v \notin \mathbb{L}$*,* $\partial q_v / \partial \kappa = 0$*; c) for all* $\underline{v}, \overline{v} \in \mathbb{V}$*, such that* $\underline{v} < \overline{v}$*,* $\partial q_{\underline{v}} / \partial \kappa \geq \partial q_{\overline{v}} / \partial \kappa$*, with strict inequality if and only if* $\underline{v} \in \mathbb{L}$.

Proof. See Appendix .3. ■

Lemma 10 shows that, for all varieties belonging to $\mathbb{L}$, quality increases when the aggregate productivity in the world rises. Furthermore, this effect is stronger for those varieties whose quality can be more cheaply upgraded – i.e., those varieties carrying a lower $\eta(v)$. On the other hand, we can observe that the optimal quality of varieties that do not belong to $\mathbb{L}$ does not respond to (infinitesimal!) changes in κ.

Based on Lemma 9 and Lemma 10, we can accordingly identify two distinct regimes depending on the level of κ that prevails. First, we refer to an economy such that $\kappa \leq \underline{\kappa}$ as a *subsistence economy* – in a subsistence economy all varieties are consumed at the baseline quality level. Second, we refer to an economy with $\kappa > \underline{\kappa}$ as a *modern economy* – in a modern economy some varieties (and possibly all of them) are consumed strictly above the baseline quality level. In what follows we proceed to further characterise these two regimes.

Subsistence Economy – $\kappa \leq \underline{\kappa}$

In this regime $q_v = 1$ holds for all $v \in \mathbb{V}$. This in turn means that $Q = 1$ and $\beta_v = 1$ must hold for all $v \in \mathbb{V}$ as well. Thus, in a subsistence economy demand intensities remain constant and equal to one for all varieties as κ increases.[12] In that regard, a subsistence economy displays analogous behaviour to the economy discussed in Dornbusch *et al* (1977), where demand schedules are homothetic.

Modern Economy – $\kappa > \underline{\kappa}$

This regime is characterised by $q_v > 1$ for all $v \in [0, \tilde{v}(\kappa))$. Hence, the average quality can be written as $Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_z \, dz$, from where it follows that $\partial Q/\partial\kappa = \int_0^{\tilde{v}(\kappa)} (\partial q_z/\partial\kappa) \, dz > 0$. Since $\partial q_v/\partial\kappa = 0$ for all $v \notin \mathbb{L}$, then because of (4.7), $\partial\beta_v/\partial\kappa < 0$ must hold for all $v \notin \mathbb{L}$. As a result, given that $\int_{\mathbb{V}} \beta_v \, dv = 1$, it must thus be the case that the demand intensities of some (and possibly all) $v \in \mathbb{L}$ will increase as κ rises. Let $\mathbb{J} \subset \mathbb{V}$ denote the subset of $\mathbb{V}$ comprising all those varieties for which $\partial\beta_v/\partial\kappa > 0$.

Definition 4 *Let* $\mathbb{J} = \{v \in \mathbb{V} : \partial\beta_v/\partial\kappa > 0\}$.

In a subsistence economy $\mathbb{J} = \emptyset$, while in a modern economy $\mathbb{J} \neq \emptyset$. In other words, in a modern economy the homotheticity of demand intensities no longer holds, as a subset of varieties whose income demand elasticity is larger than one shows up. Notice finally that $\mathbb{J} \subset \mathbb{L}$, since $\partial q_v/\partial\kappa > 0$ is a necessary condition for $\partial\beta_v/\partial\kappa > 0$ to hold.

The next proposition further characterises the behaviour of the demand intensity β_v of a generic variety v, in relation to those of the other varieties, as κ rises.

[12] It must be noted that this result applies only if $\kappa \leq \underline{\kappa}$ holds *after* performing the comparative statics exercise.

Proposition 8 *Consider any two varieties* $\underline{v}, \overline{v} \in \mathbb{V}$*, such that* $\underline{v} < \overline{v}$*. Then:*

i) If $\underline{v} \in \mathbb{J}$*:* $\partial\beta_{\underline{v}}/\partial\kappa > \partial\beta_{\overline{v}}/\partial\kappa$*;*

ii) If $\underline{v} \notin \mathbb{J}$*:* $\partial\beta_{\overline{v}}/\partial\kappa \leq 0$*.*

Proof. See Appendix .3. ∎

To interpret our previous results more clearly, notice that $\mathbb{J}$ may be understood as the set of *luxury goods*, where by luxury goods we refer to those varieties whose income demand elasticity is larger than 1. Since the set $\mathbb{J}$ always comprises lower-indexed varieties, the luxury goods are exactly those varieties whose quality degree q_v is relatively high compared to the average quality Q. In that sense, in our model it is the (relative) *quality* what determines whether or not a particular variety is *luxurious.* When individuals are still poor (i.e., when $\kappa \leq \underline{\kappa}$), satisfying all basic needs constitutes their main goal, leading them to keep the quality of all goods at the baseline level and setting accordingly equal expenditure shares for all varieties. As individuals become rich enough some (and eventually all) varieties start being consumed in higher quality degrees. Additionally, the varieties whose quality degree is relatively higher attract increasingly larger income shares, as given the preference specification in (4.1) individuals tend to *value* high-quality commodities relatively more as they become wealthier. This last point becomes more apparent in the following corollary:

Corollary 3 *Let* $\vartheta(v) \equiv \int_0^v \beta_z \, dz$*. Then:*

(i) If $\kappa < \underline{\kappa} : \partial\vartheta(v)/\partial\kappa = 0, \ \forall v \in \mathbb{V}$*;*

(ii) If $\kappa \geq \underline{\kappa} : \partial\vartheta(v)/\partial\kappa > 0, \ \forall v \in [0, 1)$*.*

Proof. See Appendix .3. ∎

Corollary 3 synthesizes the eventual non-homothetic behaviour of the demand schedules implied by our model. In particular, whenever $\kappa < \underline{\kappa}$, demand schedules are homothetic across varieties. However, when κ lies above the threshold $\underline{\kappa}$, income starts being spent in growing proportion on lower-indexed varieties.

4.4 General Equilibrium in the World Economy

In Section 4.3, we have studied the optimal consumption choice of an individual from H, taking the wages in H and in F, w and w^*, as exogenously given. (In Appendix .2, we do the same for the case of an individual from F.) These wages in turn determine the prices of all reproducible commodities in the world economy through equation (4.3). Our former analysis has therefore yielded only partial equilibrium results.

The present section computes the general equilibrium in this world economy. This requires endogenising wages and, thereby, the prices of all reproducible commodities. Given that in a general equilibrium only relative prices are determined, we henceforth take the wage in F as the *numeraire*, by setting $w^* = 1$.

In order to disregard the effects of heterogeneous population size in different countries, we suppose that both H and F are inhabited by a continuum of individuals with identical mass, which we normalise to one. (We explore the general equilibrium effects of heterogenous population size and population growth later on in Section 4.4.2.)

A representative individual from H will then solve:

$$\begin{aligned} &\max_{\{q_v,\beta_v\}_{v\in\mathbb{V}}} \quad U = \int_0^m q_v \ln\left(\frac{\beta_v \kappa}{a(v)\, q_v^{\eta(v)}}\right) dv + \int_m^1 q_v \ln\left(\frac{\beta_v \kappa w}{a^*(v)\, q_v^{\eta(v)}}\right) dv, \\ &\text{subject to:} \quad \int_{\mathbb{V}} \beta_v \, dv = 1; \text{ and } q_v \geq 1, \ \forall v \in \mathbb{V}. \end{aligned} \tag{4.8}$$

On the other hand, a representative individual from F solves:

$$\begin{aligned} &\max_{\{q_v^*,\beta_v^*\}_{v\in\mathbb{V}}} \quad U^* = \int_0^m q_v^* \ln\left(\frac{\beta_v^* \kappa}{a(v)\, q_v^{\eta(v)} w}\right) dv + \int_m^1 q_v^* \ln\left(\frac{\beta_v^* \kappa}{a^*(v)\, q_v^{\eta(v)}}\right) dv, \\ &\text{subject to:} \quad \int_{\mathbb{V}} \beta_v^* \, dv = 1; \text{ and } q_v^* \geq 1, \ \forall v \in \mathbb{V}. \end{aligned} \tag{4.9}$$

The solution of (4.8) and (4.9) yields the demand functions of each variety $v \in \mathbb{V}$ by H and F, respectively. By using $\vartheta(v) \equiv \int_0^v \beta_z \, dz$ –as defined in Corollary 3– and $\vartheta^*(v) \equiv \int_0^v \beta_z^* dz$ –see Corollary 3 (Foreign) in Appendix .2–, we can write the equilibrium condition for the market of goods produced in H as follows:

$$\vartheta(m)\, w + \vartheta^*(m) = w. \tag{4.10}$$

Condition (4.10) essentially says that the aggregate amount of income spent by the world in goods produced in H must be equal to the aggregate income of H. This condition can also be understood as the equilibrium condition for the labour market in H.[13]

The world economy general equilibrium is determined by (4.4), (4.8), (4.9), and (4.10). We will henceforth focus our attention on the equilibrium values of w and m, and to how these two variables respond to some simple comparative statics exercises.

4.4.1 Worldwide Uniform Aggregate Productivity Growth

In this subsection, we look at the impact of changes in κ on the equilibrium values of w and m. We can split the results in two different cases.

[13]Because of the Walras' Law, an analogous condition can be derived for the equilibrium in the labour market in F.

Subsistence economies – $\kappa \leq \underline{\kappa}$

From our previous discussion, we can observe that when $\kappa \leq \underline{\kappa}$, the optimal demand intensities are set at $\beta_v = \beta_v^* = 1$ for all $v \in \mathbb{V}$. This result in turn implies that $\vartheta(m) = \vartheta^*(m) = m$. Therefore, (4.10) simplifies to:

$$w = m/(1-m). \tag{4.11}$$

Combining then (4.4) with (4.11), leads to $m/(1-m) = A(m)$, from where it follows that, for all $\kappa \leq \underline{\kappa}$, in equilibrium: $w = 1$ and $m = 0.5$. That is, H and F exhibit the same level of income, and the pattern of regional specialisation is accordingly dictated by the "natural" comparative advantage of each country without relative-wage bias (i.e., the comparative advantage that derives purely from the heterogeneity in the technological structure implied by Assumption 1).[14]

Modern economies – $\kappa > \underline{\kappa}$

When aggregate productivity is sufficiently high, the income equality between H and F no longer holds. In particular, as κ rises above the threshold $\underline{\kappa}$, the terms of trade start moving in favour of H, and thus H becomes relatively richer than F. Furthermore, the income disparity between H and F increases as κ keeps rising.

Proposition 9 *Suppose Assumptions 1 holds. In addition, suppose $\kappa > \underline{\kappa}$. Then, in equilibrium:*

(i) $w > 1$ and $m < 0.5$.

(ii) $\partial w/\partial\kappa > 0$ and $\partial m/\partial\kappa < 0$.

Proof. Part (i). When $\kappa > \underline{\kappa}$, from Corollary 3 it follows that $\vartheta(m) > m$ and $\vartheta^*(m) > m$. As a result, by using (4.10), we can obtain:

$$w = \frac{\vartheta^*(m)}{1-\vartheta(m)} > \frac{m}{1-m}. \tag{4.12}$$

Combining next (4.12) with (4.4), and recalling Assumption 1 leads to:

$$A(m) = \frac{\vartheta^*(m)}{1-\vartheta(m)} > \frac{m}{1-m} \quad \Leftrightarrow \quad m < 0.5.$$

Finally, since $m < 0.5$, (4.4) implies that $w > 1$.

[14] Notice that, since $w = 1$ for all $\kappa \leq \underline{\kappa}$, in fact $\underline{\kappa} = \underline{\kappa}^*$ (that is, the threshold on κ that divides a subsistence-economy from a modern economy happens to be the same for both H and F). As a consequence, we can refer to *both* thresholds simply as $\underline{\kappa}$.

Part (ii). Next, to study how w and m vary as κ keeps rising above $\underline{\kappa}$, we differentiate the equilibrium conditions (4.4) and (4.10). This leads to:

$$\frac{\partial w}{\partial \kappa} = A'(m)\frac{\partial m}{\partial \kappa} \tag{4.13}$$

and

$$(w\beta_m + \beta^*_m)\frac{\partial m}{\partial \kappa} + \left(w\frac{\partial \vartheta(m)}{\partial w} + \vartheta(m) + \frac{\partial \vartheta^*(m)}{\partial w}\right)\frac{\partial w}{\partial \kappa} + \left(\frac{\partial \vartheta(m)}{\partial \kappa} + \frac{\partial \vartheta^*(m)}{\partial \kappa}\right) = \frac{\partial w}{\partial \kappa}, \tag{4.14}$$

where the first term in (4.14) uses the fact that $\partial\vartheta(m)/\partial m = \beta_m$ and $\partial\vartheta^*(m)/\partial m = \beta^*_m$. Plugging (4.13) into (4.14), we can obtain:

$$\frac{\partial m}{\partial \kappa} = \frac{\partial\vartheta(m)/\partial\kappa + \partial\vartheta^*(m)/\partial\kappa}{[1 - \vartheta(m) - w\,\partial\vartheta(m)/\partial w - \partial\vartheta^*(m)/\partial w]\,A'(m) - (w\beta_m + \beta^*_m)}. \tag{4.15}$$

For determining the sign of (4.15), we can use the following two results: first, Corollary 3 states that both $\partial\vartheta(m)/\partial\kappa > 0$ and $\partial\vartheta^*(m)/\partial\kappa > 0$; second, as shown in Appendix .4, $\partial\vartheta(m)/\partial w \leq 0$ and $\partial\vartheta^*(m)/\partial w < 0$. Therefore, since $1 - \vartheta(m) > 0$ and $A'(m) < 0$, then $\partial m/\partial\kappa < 0$ obtains from the right-hand side of (4.15). Finally, from (4.13) it then follows that $\partial w/\partial\kappa > 0$. ■

Proposition 9 shows that as the worldwide productivity parameter, κ, increases, the income in H eventually begins diverging away from the income in F. The reason for the divergence lies in the fact that H enjoys a comparative advantage in lower-indexed varieties, which tend to be consumed in relatively higher quality levels and display accordingly higher income demand elasticity. As a consequence, as the world economy grows uniformly above $\underline{\kappa}$, aggregate world expenditure shifts towards the set of commodities produced by H. The ensuing excess demand for commodities produced in H causes excess labour demand in H and w thus goes up. In turn, as w rises, the marginal variety moves to the left (i.e., m falls), and some of the varieties that used to be produced by H start being now produced by F, restoring the equilibrium in the labour markets.

4.4.2 Some Other Comparative Statics Exercises

This subsection briefly studies the results of two other comparative statics experiments commonly explored by the previous literature on international trade with non-homothetic preferences. First, we look at the case of income inequality within countries. Second, we analyse the effects of unequal population growth across countries. It turns out that the results of both exercises are in line with those of the literature on North-South trade. On the one hand, income inequality within countries tends to improve the terms of trade and the relative income of the economy that specialises in varieties that display higher income demand elasticity. On the other hand, the country in which population grows faster tends to experience a decline in its terms of trade and relative income.

Heterogeneous Population: the effects of income inequality

So far we have assumed that within each country all individuals are identical regarding all relevant features for our model. In this section we examine the general equilibrium consequences of introducing some degree of income heterogeneity within countries.

To keep the analysis short and concise, we first introduce income inequality only in F, while we maintain the assumption that the population in H is homogeneous. In particular, we assume that F is inhabited by two types of individuals: p and r, where the p stands for *poor* and r stands for *rich*. Each sub-group of foreigners has mass equal to 0.5. A type-p is endowed with $1-\iota$ units of effective labour, while a type-r is endowed with $1+\iota$ units of it; where $\iota \in (0,1)$. In H everyone is endowed with 1 unit of effective labour.

Introducing income inequality in F leads to interesting results when the types-p are so poor that, in equilibrium, they consume all varieties at the baseline quality level, whereas in contrast the types-r can afford consuming some of the varieties strictly above that level. To focus on such case, we accordingly assume that $\kappa = \underline{\kappa}$.

Proposition 10 *Suppose the population in F is split in two groups, p and r, of equal mass; individuals in p are endowed with $1-\iota$ units of effective labour and individuals in r are endowed with $1+\iota$ units of it, where $\iota > 0$. Additionally, suppose that $\kappa = \underline{\kappa}$. Then, in equilibrium:*

(i) $w > 1$ and $m < 0.5$.

(ii) $\partial w/\partial \iota > 0$ and $\partial m/\partial \iota < 0$.

Proof. Part (i). When $\kappa = \underline{\kappa}$, in F, $\vartheta_p^*(m) = m$ and $\vartheta_r^*(m) > m$; where $\vartheta_j^*(m)$ denotes the fraction of income that types $j \in \{p, r\}$ spend on varieties belonging to $[0, m)$. On the other hand, in H, $\vartheta(m) = m$; since when $\kappa = \underline{\kappa}$ all individuals from H optimally consume all varieties at the baseline level.[15] As a result, the equilibrium condition in the labour market in H reads as follows:

$$m\,w + \tfrac{1}{2}\left[(1-\iota)\,m + (1+\iota)\vartheta_r^*(m)\right] = w. \tag{4.16}$$

From (4.16), since $\vartheta_r^*(m) > m$, it immediately follows that $w > 1$, which in turn implies $m < 0.5$.

[15] Recall from Lemma 9 that $\underline{\kappa} \equiv a\,(0) \exp\left[\eta\,(0)\right]$, which is independent of w/w^*.

Part (ii). Totally differentiating (4.16), and using the fact that in equilibrium $\partial w/\partial\iota = A'(m)\,(\partial m/\partial\iota)$ must be verified, leads to:

$$\frac{\partial w}{\partial\iota} = \frac{(1+\iota)\,(\partial\vartheta_r^*/\partial\iota) + (\vartheta_r^*(m) - m)}{2(1-m) - (1+\iota)\,(\partial\vartheta_r^*/\partial w) - \left[2w + (1-\iota) + (1+\iota)\beta_{r,m}^*\right]/A'(m)} > 0. \quad (4.17)$$

The positive sign in (4.17) stems from the fact that $\partial\vartheta_r^*(m)/\partial\iota > 0$, $\partial\vartheta_r^*(m)/\partial w < 0$, and $A'(m) < 0$. ■

When $\kappa = \underline{\kappa}$, introducing income inequality in F raises the relative wage in H. This result is owing to the non-homotheticity of the demand schedules of the rich foreigners. More precisely, increasing ι transfers income from the poor foreigners who spend a fraction m of it in goods from H, to the rich foreigners who spend a fraction $\vartheta_r^*(m) > m$ of their income on those commodities. As a result, aggregate demand for goods produced in H goes up leading to higher w.

Incorporating inequality in H in an analogous manner as done before in F would carry similar consequences on w and m. This is the case because the rich locals would tend to shift aggregate demand towards the goods produced in H, exactly as it occurred in Proposition 10 with the rich foreigners. The next proposition states this result more formally.

Proposition 11 *Suppose the population in H is split in two groups, p and r, of equal mass; individuals in p are endowed with $1-\iota$ units of effective labour and individuals in r are endowed with $1+\iota$ units of it, where $\iota > 0$. Additionally, suppose that $\kappa = \underline{\kappa}$. Then, in equilibrium:*

(i) $w > 1$ and $m < 0.5$.

(ii) $\partial w/\partial\iota > 0$ and $\partial m/\partial\iota < 0$.

Proof. The proof is analogous to the proof of Proposition 10. See Appendix .3. ■

Population Growth

In this section we return to a situation in which all individuals in the world are homogeneous (hence, we disregard again inequality issues). However, we let the population size in F be larger than in H.

Let the total mass of individuals in F equal $L > 1$. Then the labour market equilibrium condition in H will be given by:

$$\vartheta(m)w + L\vartheta^*(m) = w. \quad (4.18)$$

Visual inspection on (4.18) and (4.10) –combined with (4.4)– immediately implies that the equilibrium value of w that is delivered by (4.18) will be strictly larger than that yielded by (4.10). In particular, in equilibrium $w > 1$, regardless of the value of κ. The next proposition shows that this source of income divergence between F and H is stronger the larger the value of L.

Proposition 12 *The relative wage in H is increasing in the size of the population in F.*

Proof. Totally differentiating (4.18), and bearing in mind that $\partial w/\partial L = A'(m)\ (\partial m/\partial L)$ must hold in equilibrium, leads to:

$$\frac{\partial w}{\partial L} = \frac{w}{L} \frac{1 - \vartheta(m)}{\left[1 - \vartheta(m) - w\frac{\partial \vartheta(m)}{\partial w} - L\frac{\partial \vartheta^*(m)}{\partial w} - \frac{1}{A'(m)}\left(\beta_m w + \beta_m^* L\right)\right]} > 0. \tag{4.19}$$

where the positive in (4.19) obtains from $\partial\vartheta(m)/\partial w \leq 0$, $\partial\vartheta^*(m)/\partial w \leq 0$, and $A'(m) < 0$. ■

When the labour supply in F increases, the relative wage w must go up so as to accommodate the excess supply of labour in F. More precisely, a larger L requires more goods to be produced by F in order to keep full employment there; this is accomplished by letting w go up, which in turn shifts the marginal variety m to the left, helping restore the equilibrium in the labour markets.

4.5 Conclusion

We have provided a model of international trade based on comparative advantages that are *unrelated* to the relative stage in the process of development in which countries are. This is the main point of departure with respect to the past literature on North-South trade, where comparative advantages originate from the fact that some countries have historically accumulated larger amounts of capital than others. We show instead that even when no single country enjoys a clear absolute advantage over any other country and when productivity changes are uniform and identical in all countries, international trade may still be the source of income divergence in the world economy. Income divergence will be experienced when comparative advantages induce patterns of specialisation that, although optimal for each country at early stages in the process of development, do not offer the same scope for improvements in terms of quality upgrading of final products in the long run. We have argued that our model may shed light on historical cases where comparative advantages emerged as a result of heterogeneous geographical conditions.

Appendices

.1 First-Order Conditions for Consumption Choice in H

The optimisation problem in (4.6) yields the following first-order conditions (where μ represents the Lagrange multiplier associated to the budget constraint and $\{\lambda_v\}_{v\in\mathbb{V}}$ denote those associated to the constraints $\{q_v \geq 1\}_{v\in\mathbb{V}}$):

$$\ln\left(\frac{\beta_v w}{\kappa^{-1}\alpha(v)\, q_v^{\eta(v)}}\right) - \eta(v) + \lambda_v = 0,\ \forall v \in \mathbb{V}; \tag{20}$$

$$\frac{q_v}{\beta_v} - \mu = 0,\ \forall v \in \mathbb{V}; \tag{21}$$

$$q_v - 1 \geq 0,\ \lambda_v \geq 0,\ \text{and}\ \lambda_v (q_v - 1) = 0,\ \forall v \in \mathbb{V}; \tag{22}$$

$$1 - \int_{\mathbb{V}} \beta_v\, dv = 0. \tag{23}$$

From (21), it follows that $\beta_v = q_v/\mu$. Then, replacing this last expression into (23) leads to $\int_{\mathbb{V}} q_z\, dz = \mu$, from where the condition (4.7) immediately obtains by using again (21).

By using the condition (4.7), we can rewrite (20) as:

$$\lambda_v = \eta(v) + \ln[\alpha(v)/w] - \ln\kappa + \ln Q + [\eta(v) - 1]\ln q_v. \tag{24}$$

The expression in (24) will be used in many of the following proofs.

.2 Optimal Consumption Choice in F

Bearing in mind Assumption 1, we can write down the optimisation problem faced by a representative individual from F as follows:

$$\max_{\{x^*_{vq}\}_{(v,q)\in\mathbb{V}\times\mathbb{Q}}} \quad U^* = \int_{\mathbb{V}} \ln\left[\int_{\mathbb{Q}} \max\left\{x^*_{vq}, \left(x^*_{vq}\right)^q\right\} dq\right] dv,$$

$$\text{subject to:} \quad \int_{\mathbb{V}} \beta^*_v dv = 1,$$

$$p_{vq} = \kappa^{-1} q^{\eta(v)} \alpha(v),\ \forall (v,q) \in \mathbb{V}\times\mathbb{Q}.$$

Lemma 7 holds for x^*_{vq} in a similar fashion as for x_{vq}. Hence, we can re-state the problem specified above in terms of q^*_v and β^*_v, as it was previously done for H (where q^*_v now denotes the quality of variety v consumed, in the optimum, in F). This way, we can obtain the following first-order conditions, which constitute the analogous versions for F of (4.7) and (24), respectively:

$$\beta^*_v = \frac{q^*_v}{\int_{\mathbb{V}} q^*_z\, dz},\ \forall v \in \mathbb{V}, \tag{25}$$

$$\lambda^*_v = \eta(v) + \ln[\alpha(v)/w^*] - \ln\kappa + \ln Q^* + [\eta(v) - 1]\ln q^*_v. \tag{26}$$

Given the first-order conditions in (25) and (26), all the ensuing results found in Section 4.3 follow through in qualitative terms. In particular, we can derive functions $\{q_v^*\}_{v\in\mathbb{V}}$ and $\{\beta_v^*\}_{v\in\mathbb{V}}$ displaying identical qualitative properties as their *counterparts* in H, that is $\{q_v\}_{v\in\mathbb{V}}$ and $\{\beta_v\}_{v\in\mathbb{V}}$, in terms of Lemmas 8 - 10 and Proposition 8. Furthermore, we can similarly find the threshold $\underline{\kappa}^*$ for the worldwide aggregate-productivity parameter, which splits F in the regimes of *subsistence-economy* and *modern-economy*; both exhibiting analogous properties as described for H.[16] Finally, likewise for H in Corollary 3, for F the following holds:

Corollary 3 (Foreign) Let $\vartheta^*(v) \equiv \int_0^v \beta_z^*\, dz$. Then:
(i) If $\kappa < \underline{\kappa}^* : \partial\vartheta^*(v)/\partial\kappa = 0$, $\forall\, v \in \mathbb{V}$;
(ii) If $\kappa \geq \underline{\kappa}^* : \partial\vartheta^*(v)/\partial\kappa > 0$, $\forall\, v \in [0,1)$.

.3 Omitted Proofs

Proof of Lemma 7.

Part (i). First, notice that utility is given by an additive function over logarithms. Optimization can thus be split in two stages: (a) maximise U with respect to the logarithms; (b) maximise those logarithms with respect to x_{vq}. About (b), notice that the logarithms are defined on the integral over convex functions of x_{vq}, and therefore are themselves convex functions. It follows that (b) optimally requires corner solutions, so the result claimed obtains.

Part (ii). The proof follows immediately from noting that, for all $v \in \mathbb{V}$, utility derived from consuming $x_{vq} \in (0,1]$ is independent of the quality-index q, while according to (4.3) the price of commodity $(v,q) \in \mathbb{V} \times \mathbb{Q}$ is strictly increasing along the quality space.

■

Proof of Lemma 8.

First, suppose $q_{\underline{v}} < q_{\overline{v}}$. Since $q_{\underline{v}} \geq 1$, then $q_{\overline{v}} > 1$, hence (24) paired with (22) yield: $\eta(\underline{v}) + \ln[\alpha(\underline{v})/w] - \ln(\kappa/Q) \geq 0$, while $\eta(\overline{v}) + \ln[\alpha(\overline{v})/w] - \ln(\kappa/Q) + [\eta(\overline{v}) - 1]\ln q_{\overline{v}} = 0$. Thus:

$$\eta(\underline{v}) + \ln\alpha(\underline{v}) \geq \eta(\overline{v}) + \ln\alpha(\overline{v}) + [\eta(\overline{v}) - 1]\ln q_{\overline{v}}.$$

This last equality in turn leads to:

$$[\eta(\overline{v}) - \eta(\underline{v})] + \ln[\alpha(\overline{v})/\alpha(\underline{v})] + [\eta(\overline{v}) - 1]\ln q_{\overline{v}} \leq 0,$$

which cannot possibly hold if $q_{\overline{v}} > 1$, as its left-hand side would then be strictly positive. Therefore, it must necessarily be the case that $q_{\underline{v}} \geq q_{\overline{v}}$.

[16]From Section 4.4, it is straightforward to observe that, given Assumption 1, $\underline{\kappa}^* = \underline{\kappa}$.

Next, suppose $q_{\underline{v}} = q_{\overline{v}} > 1$. In this case, (24) in conjunction with (22) yield:

$$\eta\left(\underline{v}\right) + \ln\alpha\left(\underline{v}\right) + \left[\eta\left(\underline{v}\right) - 1\right]\ln q_{\overline{v}} = \eta\left(\overline{v}\right) + \ln\alpha\left(\overline{v}\right) + \left[\eta\left(\overline{v}\right) - 1\right]\ln q_{\overline{v}} = 0.$$

This last equality in turn leads to:

$$-\left[\eta\left(\overline{v}\right) - \eta\left(\underline{v}\right)\right]\left(1 + \ln q_{\overline{v}}\right) = \ln\left[\alpha\left(\overline{v}\right)/\alpha\left(\underline{v}\right)\right].$$

However, this last equality cannot possibly hold since its right-hand side is strictly positive, while the left-hand side is negative. As a result, $q_{\underline{v}} > q_{\overline{v}}$ must necessarily hold when $q_{\underline{v}} > 1$. ■

Proof of Lemma 9.

In order to prove this lemma it proves convenient to state first the following result:

Claim 1 The optimal quality q_v of any variety $v \in \mathbb{V}$ can be written as follows:

$$q_v = \max\left\{\Phi_{0,v}\left(q_0\right)^{\Upsilon_{0,v}}, 1\right\}; \tag{27}$$

where:

$$\Phi_{0,v} \equiv \left[\frac{e^{\eta(0)}\alpha\left(0\right)}{e^{\eta(v)}\alpha\left(v\right)}\right]^{\frac{1}{\eta(v)-1}} > 0, \text{ and } \Upsilon_{0,v} \equiv \frac{\eta\left(0\right) - 1}{\eta\left(v\right) - 1} > 0.$$

Proof. See Appendix .4.

Next, notice that, from (27), $\partial\Phi_{0,v}\left(v\right)/\partial v < 0$ and $\partial\Upsilon_{0,v}\left(v\right)/\partial v < 0$ since $a'\left(v\right) \geq 0$ and $\eta'\left(v\right) > 0$, hence the set $\mathbb{L} \subseteq \mathbb{V}$ comprises the lower-indexed varieties in $\mathbb{V}$, with $\tilde{v}(\underline{\kappa})$ representing its upper bound.

Part (i). When $\kappa \in (0, \underline{\kappa})$, conditions stipulated in (22) and (24) applied on $v = 0$ entail that: $q_0 = 1$ and $\lambda_0 > 0$. As a result, from Lemma 8 it follows that $q_v = 1$, $\forall v \in \mathbb{V}$. Therefore, since $a'\left(v\right) \geq 0$ and $\eta'\left(v\right) > 0$, again from (24), $\lambda_v > 0$ for all $v \in \mathbb{V}$ obtains, and thus $\mathbb{L} = \emptyset$.

Part (ii). Note that (24) applied on $v = 0$, in conjunction Lemma 8, implies that when $\kappa = \underline{\kappa}$, then $\lambda_0 = 0$ and $q_0 = 1$. Then, Lemma 8 implies $Q = 1$. Using these results in (24) yields: $\lambda_v = \eta\left(v\right) + \ln\left[\alpha\left(v\right)/w\right] - \ln\underline{\kappa}$, implying that $\lambda_v > 0$ for all $v \in (0, 1]$. As a result, the set $\mathbb{L} = 0$, meaning that $\tilde{v}(\underline{\kappa}) = 0$.

Claim 2 If $\tilde{v}(\kappa) < 1$, then $q_{\tilde{v}(\kappa)} = 1$.

Proof. See Appendix .4.

Given Claim 2 and Lemma 8, the aggregate quality index can be written as follows: $Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_v \, dv$. Furthermore, observe that, whenever $\tilde{v}(\kappa) < 1$, $\ln\left(\kappa/Q\right) = \eta\left(\tilde{v}(\kappa)\right) + \ln\left[\alpha\left(\tilde{v}(\kappa)\right)/w\right]$ must hold in equilibrium. This last condition yields, after some simple algebra, $Q = \kappa w \exp\left[-\eta\left(\tilde{v}\right)\right]/\alpha\left(\tilde{v}\right)$. In addition to that, because of Lemma 8, in equilibrium, $[\eta(v)-1]\ln q_v = \ln(\kappa/Q) - \eta(v) - \ln[\alpha(v)/w]$ must hold for any $v \leq \tilde{v}(\kappa)$. By using the former in the latter, after some algebra, we may obtain:

$$q_v = q_v(\tilde{v}(\kappa)) \equiv \left[\frac{\alpha(\tilde{v}(\kappa))}{\alpha(v)}\right]^{\frac{1}{\eta(v)-1}} \exp\left[\frac{\eta(\tilde{v}(\kappa)) - \eta(v)}{\eta(v)-1}\right], \ \forall v \in [0, \tilde{v}(\kappa)]. \tag{28}$$

In equilibrium, it must be the case that:

$$\kappa w \exp[-\eta(\tilde{v}(\kappa))] / a(\tilde{v}(\kappa)) = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_v(\tilde{v}(\kappa))\, dv, \tag{29}$$

where the right hand-side of (29) uses (28). Computing the total differentiation of (29), yields after some algebra:[17]

$$\frac{Q}{\kappa} d\kappa = \left[\frac{\alpha'(\tilde{v}(\kappa))}{\alpha(\tilde{v}(\kappa))} + \eta'(\tilde{v}(\kappa))\right]\left[Q + \int_0^{\tilde{v}(\kappa)} \frac{q_v}{\eta(v)-1} dv\right] d\tilde{v},$$

leading finally to:

$$\frac{d\tilde{v}}{d\kappa} = \left[\frac{\kappa}{Q}\left(\frac{\alpha'(\tilde{v}(\kappa))}{\alpha(\tilde{v}(\kappa))} + \eta'(\tilde{v}(\kappa))\right)\left(1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} \frac{\eta(v)}{\eta(v)-1} q_v dv\right)\right]^{-1} > 0.$$

where the last inequality follows from the properties of the functions $\alpha(\cdot)$ and $\eta(\cdot)$ ■

Proof of Lemma 10.

Part (i). Proof follows immediately from noting that Lemma 9 implies that, whenever $\kappa \in (0, \underline{\kappa})$, $q_v = 1$ must hold for all $v \in \mathbb{V}$. Thus, whenever $\kappa \in (0, \underline{\kappa})$, $\partial q_v / \partial \kappa = 0$ for all $v \in \mathbb{V}$.

Part (ii.a). Differentiating (27), computed for any $v \in \mathbb{L}$, with respect to κ yields:

$$\frac{dq_v}{d\kappa} = \frac{\eta(0)-1}{\eta(v)-1}\left[\frac{e^{\eta(0)}\alpha(0)}{e^{\eta(v)}\alpha(v)}\right]^{\frac{1}{\eta(v)-1}} (q_0)^{\frac{\eta(0)-\eta(v)}{\eta(v)-1}} \frac{dq_0}{d\kappa}, \ \forall v \in \mathbb{L}.$$

Using again (27), the equation above can be written:

$$\frac{dq_v}{d\kappa} = \frac{\eta(0)-1}{\eta(v)-1}\frac{q_v}{q_0}\frac{dq_0}{d\kappa}, \ \forall v \in \mathbb{L}. \tag{30}$$

(Since $\eta(\cdot) > 1$, notice that $dq_v/d\kappa$ and $dq_0/d\kappa$ must then share the same sign, for all $v \in \mathbb{L}$). Given that $Q = 1 - \tilde{v}(\kappa) + \int_0^{\tilde{v}(\kappa)} q_z\, dz$, it follows that:

$$\frac{dQ}{d\kappa} = \int_0^{\tilde{v}(\kappa)} \frac{dq_z}{d\kappa} dz = \frac{1}{q_0}\left(\int_0^{\tilde{v}(\kappa)} \frac{\eta(0)-1}{\eta(z)-1} q_z dz\right)\frac{dq_0}{d\kappa}.$$

[17] One subtle caveat applies here. Even if both $wa(v)$ and $w^* a^*(v)$ are differentiable functions over the whole domain of v, the envelope function $\alpha(v)$ will not necessarily be so. In particular, $\alpha(v)$ may not be differentiable at the point $v = m$. As a result, if $\tilde{v} = m$, $\alpha'(\tilde{v})$ may not exist. In the very specific case where this "anomaly" holds, we take that $\alpha'(v) = \lim_{\Delta v \to 0^+} \frac{\Delta\alpha(v)}{\Delta v}$.

Applying (24) to $v = 0$ when $\lambda_0 = 0$ yields: $q_0 = \left[a(0)\, e^{\eta(0)} Q\right]^{-\frac{1}{\eta(0)-1}} \kappa^{\frac{1}{\eta(0)-1}}$. Thus:

$$\frac{dq_0}{d\kappa} = \frac{q_0}{\eta(0)-1}\frac{Q}{\kappa}\left(1 - \tilde{v} + \int_0^{\tilde{v}(\kappa)} \frac{\eta(z)}{\eta(z)-1} q_z\, dz\right)^{-1} > 0.$$

Therefore, from (30) it follows that $dq_v/d\kappa > 0$, $\forall v \in \mathbb{L}$ must also hold.

Part (ii.b). Since $q_v = 1$ must hold for all $v \notin \mathbb{L}$. Proof is analogous to that of Part (i) of this Proposition.

Part (ii.c). Part (ii.a) and (ii.b) of this Proposition, taken together, imply that $dq_{\underline{v}}/d\kappa = dq_{\overline{v}}/d\kappa = 0$ if $\underline{v}, \overline{v} \notin \mathbb{L}$, and $dq_{\underline{v}}/d\kappa > dq_{\overline{v}}/d\kappa = 0$ if $\underline{v} \in \mathbb{L}$ and $\overline{v} \notin \mathbb{L}$. For $\underline{v}, \overline{v} \in \mathbb{L}$, such that $\underline{v} < \overline{v}$, (30) leads to:

$$\frac{dq_{\underline{v}}}{d\kappa} = \frac{\eta(0)-1}{\eta(\underline{v})-1}\frac{q_{\underline{v}}}{q_0}\frac{dq_0}{d\kappa} > \frac{\eta(0)-1}{\eta(\overline{v})-1}\frac{q_{\overline{v}}}{q_0}\frac{dq_0}{d\kappa} = \frac{dq_{\overline{v}}}{d\kappa},$$

since by assumption $\eta(\underline{v}) < \eta(\overline{v})$ and, from Lemma 8, $q_{\underline{v}} > q_{\overline{v}}$. ■

Proof of Proposition 8.

Firstly, considering the definition of average quality, taking logarithms and differentiating (4.7) with respect to κ yields: $(d\beta_v/d\kappa)/\beta_v = (dq_v/d\kappa)/q_v - (dQ/d\kappa)/Q$. Using (30), we can write:

$$\frac{dq_{\underline{v}}}{d\kappa}\frac{1}{q_{\underline{v}}} = \frac{\eta(0)-1}{\eta(\underline{v})-1}\frac{dq_0}{d\kappa}\frac{1}{q_0} > \frac{\eta(0)-1}{\eta(\overline{v})-1}\frac{dq_0}{d\kappa}\frac{1}{q_0} = \frac{dq_{\overline{v}}}{d\kappa}\frac{1}{q_{\overline{v}}}.$$

Hence:

$$\frac{d\beta_{\underline{v}}}{d\kappa}\frac{1}{\beta_{\underline{v}}} > \frac{d\beta_{\overline{v}}}{d\kappa}\frac{1}{\beta_{\overline{v}}}. \tag{31}$$

Part (i). Using (31), the claim trivially follows by noting that, from Lemma 8 in conjunction with (4.7), $\beta_{\underline{v}} > \beta_{\overline{v}}$ must always hold.

Part (ii). Suppose instead that $\partial\beta_{\overline{v}}/\partial\kappa > 0$ when $\partial\beta_{\underline{v}}/\partial\kappa \leq 0$. Using (31), it follows that:

$$\frac{d\beta_{\overline{v}}}{d\kappa} < \frac{\beta_{\overline{v}}}{\beta_{\underline{v}}}\frac{d\beta_{\underline{v}}}{d\kappa} \leq 0;$$

which contradicts the fact that $\partial\beta_{\overline{v}}/\partial\kappa > 0$ when $\partial\beta_{\underline{v}}/\partial\kappa \leq 0$. As a result, if $\underline{v} \notin \mathbb{J}$, then $\partial\beta_{\overline{v}}/\partial\kappa \leq 0$ must hold. ■

Proof of Corollary 3.

Preliminarily, recall $\int_{z\in\mathbb{V}} \beta_z\, dz = 1$, which implies $\int_0^1 (\partial\beta_z/\partial\kappa)\, dz = 0$.[18]

Part (i). Claim immediately follows since, whenever $\kappa < \underline{\kappa}$, $\partial\beta_z/\partial\kappa = 0$ for all $z \in \mathbb{V}$.

Part (ii). Note first that when $\kappa \geq \underline{\kappa}$, the set $\mathbb{J} \neq \emptyset$. As a result, from Proposition 8,

[18]Note that it is then trivial to observe that $\partial\vartheta(1)/\partial\kappa = 0$, $\forall\kappa > 0$.

Part (i), it follows that $\int_0^v (\partial\beta_z/\partial\kappa)\,dz > \int_v^1 (\partial\beta_z/\partial\kappa)\,dz$. Then, since $\int_0^v (\partial\beta_z/\partial\kappa)\,dz + \int_v^1 (\partial\beta_z/\partial\kappa)\,dz = 0$, we must necessarily have that $\int_0^v (\partial\beta_z/\partial\kappa)\,dz > 0$. ■

Proof of Proposition 11.

Part (i). The equilibrium condition for the labour market in H is as follows:

$$\tfrac{1}{2}\left[(1-\iota)\,m + (1+\iota)\vartheta_r(m)\right] + \frac{m}{w} = 1. \tag{32}$$

From where $w > 1$ and $m < 0.5$ immediately obtain since $\vartheta_r(m) > m$.

Part (ii). Totally differentiating (32), and using the fact that in equilibrium $\partial w/\partial\iota = A'(m)\,(\partial m/\partial\iota)$ must be hold, leads to:

$$\frac{\partial w}{\partial\iota} = \frac{(1+\iota)\,(\partial\vartheta_r/\partial\iota) + (\vartheta_r(m) - m)}{2\,m\,w^{-2} - (1+\iota)\,(\partial\vartheta_r/\partial w) - \left[2\,w^{-1} + (1-\iota) + (1+\iota)\beta_{r,m}\right]/A'(m)} > 0. \tag{33}$$

The positive sign in (33) stems from the fact that $\partial\vartheta_r(m)/\partial\iota > 0$, $\partial\vartheta_r(m)/\partial w \leq 0$, and $A'(m) < 0$. ■

.4 Auxiliary Derivations and Proofs

Proof of Claim 1

Recall that $q_v = 1$, $\forall v \notin \mathbb{L}$. For all other varieties, (24) in conjunction with (22) yield:

$$\eta(v) + \ln\alpha(v) + [\eta(v) - 1]\ln q_v = \eta(0) + \ln\alpha(0) + [\eta(0) - 1]\ln q_0,\ \forall v \in \mathbb{L}.$$

Isolating $[\eta(v) - 1]\ln q_v$, and applying exponentials to both sides gives:

$$(q_v)^{\eta(v)-1} = \frac{e^{\eta(0)}}{e^{\eta(v)}}\frac{\alpha(0)}{\alpha(v)}(q_0)^{\eta(0)-1},\ \forall v \in \mathbb{L}.$$

Finally, raising both sides to the power $[\eta(v) - 1]^{-1}$, and considering Lemma 8, (27) obtains.

Proof of Claim 2

By definition of $\mathbb{L}$, $\lambda_{\tilde{v}(\kappa)} = 0$. Thus, the condition (24) applied on $\tilde{v}(\kappa)$ yields:

$$\eta(\tilde{v}(\kappa)) + \ln\left[\alpha(\tilde{v}(\kappa))/w\right] - \ln\kappa + \ln Q = -\left[\eta(\tilde{v}(\kappa)) - 1\right]\ln q_{\tilde{v}(\kappa)} \tag{34}$$

Suppose now that $q_{\tilde{v}(\kappa)} > 1$, and take some $\varepsilon \in (0, 1-\tilde{v}(\kappa)]$. Then, since $v = \tilde{v}(\kappa)+\varepsilon \notin \mathbb{L}$, it must be the case that:

$$\eta(\tilde{v}(\kappa)+\varepsilon) + \ln\left[\alpha(\tilde{v}(\kappa)+\varepsilon)/w\right] - \ln\kappa + \ln Q = \lambda_{\tilde{v}(\kappa)+\varepsilon}. \tag{35}$$

Then, by continuity of $\eta(\cdot)$ and $\alpha(\cdot)$, and using the result in (34), we must have:

$$\lim_{\varepsilon\to 0}\left\{\eta(\tilde{v}(\kappa)+\varepsilon) + \ln\left[\alpha(\tilde{v}(\kappa)+\varepsilon)/w\right] - \ln\kappa + \ln Q\right\} = -\left[\eta(\tilde{v}(\kappa)) - 1\right]\ln q_{\tilde{v}(\kappa)} < 0.$$

Hence, $q_{\tilde{v}(\kappa)} > 1$ cannot possibly hold when $\tilde{v}(\kappa) < 1$ as it would imply that $\lambda_{\tilde{v}(\kappa)+\varepsilon} < 0$ in (35) for $\varepsilon \to 0$, violating (22).

Proof of $\partial\vartheta(m)/\partial w \leq 0$.

Suppose first that $\tilde{v} < m$. Then, $\mathbb{L} \subset [0, m)$. Differentiating (24) with respect to w yields:

$$\frac{\eta(v) - 1}{q_v}\frac{\partial q_v}{\partial w} + \frac{1}{Q}\frac{\partial Q}{\partial w} = 0, \ \forall v \in \mathbb{L}. \tag{36}$$

Furthermore, from (27) it follows that:

$$\frac{\partial q_v}{\partial w} = \frac{\eta(0) - 1}{\eta(v) - 1}\frac{q_v}{q_0}\frac{\partial q_0}{\partial w}, \ \forall v \in \mathbb{L}. \tag{37}$$

Since $\partial Q/\partial w = \int_0^{\tilde{v}} (\partial q_z/\partial w)\, dz$, combining (36) and (37) yields:

$$\left(1 - \tilde{v} + \int_0^{\tilde{v}} \frac{\eta(z)}{\eta(z) - 1} q_z dz\right) \frac{\eta(0) - 1}{q_0} \frac{1}{Q} \frac{\partial q_0}{\partial w} = 0 \ \Rightarrow \ \frac{\partial q_0}{\partial w} = 0,$$

Therefore, using again (37), $\partial q_v/\partial w = 0$ for all $v \in [0, \tilde{v}]$ obtains. In addition, because of Lemma 8, it must thus be the case that $\partial q_v/\partial w = 0$ holds as well for all $v \in (\tilde{v}, 1]$. Finally, recalling (4.7) it then follows that $\partial\beta_v/\partial w = 0$ for all $v \in \mathbb{V}$, which in turn implies that $\partial\vartheta(m)/\partial w = 0$.

Suppose now that $\tilde{v} \geq m$. Differentiating (24) with respect to w now yields:

$$\frac{\eta(v) - 1}{q_v}\frac{\partial q_v}{\partial w} + \frac{1}{Q}\frac{\partial Q}{\partial w} = \begin{cases} 0, & \forall v \in [0, m) \\ 1/w, & \forall v \in [m, \tilde{v}] \end{cases} \tag{38}$$

From (38) it follows that a necessary condition for $\partial\vartheta(m)/\partial w > 0$ to hold is that $\partial Q/\partial w < 0$.[19] However, (38) means that if $\partial Q/\partial w < 0$, then $\partial q_v/\partial w > 0$ should hold for all $v \in [m, \tilde{v}]$. If $\tilde{v} = 1$, it must be straightforward to observe that $\partial Q/\partial w < 0$ cannot thus hold. Alternatively, if $\tilde{v} < 1$, then $\partial Q/\partial w < 0$ would require that $\partial q_v/\partial w < 0$ prevails for some $v \in (\tilde{v}, 1]$ which is not feasible either since it would lead to violating the constraint $q_v \leq 1$. As a result, $\partial Q/\partial w \geq 0$ must hold, which in turn implies $\partial\vartheta(m)/\partial w \leq 0$. ∎

Proof of $\partial\vartheta^*(m)/\partial w < 0$.

Suppose first that $\tilde{v}^* < m$. Then, $\mathbb{L}^* \subset [0, m)$. Differentiating (24) – adjusted for representing an individual from F – with respect to w yields:

$$\frac{\eta(v) - 1}{q_v^*}\frac{\partial q_v^*}{\partial w} + \frac{1}{Q^*}\frac{\partial Q^*}{\partial w} = -\frac{1}{w}, \ \forall v \in \mathbb{L}^*. \tag{39}$$

[19]Otherwise, if $\partial Q/\partial w \geq 0$, (38) would imply that $\partial q_v/\partial w \leq 0$ for all $v \in [0, m)$. Recalling (4.7), it is then straightforward to observe that $\partial Q/\partial w \geq 0$ would mean $\partial\beta_v/\partial w \leq 0$ for all $v \in [0, m)$, which in turn leads to $\partial\vartheta(m)/\partial m \leq 0$.

In addition, from (27) it follows that:

$$\frac{\partial q_v^*}{\partial w} = \frac{\eta(0) - 1}{\eta(v) - 1} \frac{q_v^*}{q_0^*} \frac{\partial q_0^*}{\partial w}, \ \forall v \in \mathbb{L}^*. \tag{40}$$

Combining (39) and (40) leads to:

$$\left(1 - \tilde{v}^* + \int_0^{\tilde{v}^*} \frac{\eta(z)}{\eta(z) - 1} q_z dz\right) \frac{\eta(0) - 1}{q_0^*} \frac{1}{Q^*} \frac{\partial q_0^*}{\partial w} = -\frac{1}{w} \quad \Rightarrow \quad \frac{\partial q_0^*}{\partial w} < 0.$$

Hence, using again (40), $\partial q_v^* / \partial w < 0$ for all $v \in [0, \tilde{v}^*]$ obtains, which in turn implies $\partial Q^* / \partial w < 0$. Next, since for all $v \geq \tilde{v}^*$ the constraint $q_v^* \geq 1$ is binding, it must be the case that $\partial q_v^* / \partial w \geq 0$, $\forall v \in (\tilde{v}^*, 1]$. As a result, because of (4.7), $\partial \beta_v^* / \partial w > 0$ for all $v \in [m, 1]$ follows, which in turn implies $\partial \vartheta^*(m) / \partial w < 0$.

Suppose now $\tilde{v}^* \geq m$. Differentiating (24) with respect to w now yields:

$$\frac{\eta(v) - 1}{q_v^*} \frac{\partial q_v^*}{\partial w} + \frac{1}{Q^*} \frac{\partial Q^*}{\partial w} = \begin{cases} -1/w, & \forall v \in [0, m) \\ 0, & \forall v \in [m, \tilde{v}^*] \end{cases} \tag{41}$$

Suppose $\partial Q^* / \partial w \geq 0$. From (41) it follows that $\partial q_v^* / \partial w < 0$ for all $v \in [0, \tilde{v}^*)$. Furthermore, Lemma 8 then implies that $\partial q_v^* / \partial w \leq 0$ for all $v \in [\tilde{v}^*, 1]$; as a result, $\partial Q^* / \partial w < 0$ must necessarily hold. Now, notice that if $\partial Q^* / \partial w < 0$, then (41) implies $\partial q_v^* / \partial w > 0$ for all $v \in [m, \tilde{v}^*]$. Moreover, in case $\tilde{v}^* < 1$, since $\forall v \in (\tilde{v}^*, 1]$ the constraint $q_v^* \geq 1$ is binding, $\partial q_v^* / \partial w \geq 0$ must necessarily hold for all $v \in (\tilde{v}^*, 1]$. As a result, if $\partial Q^* / \partial w < 0$, then $\partial \beta_v^* / \partial w > 0$ for all $v \in [m, 1]$, which in turn leads to $\partial \vartheta^*(m) / \partial w < 0$. ■

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Declaration

I, Esteban Jaimovich, confirm that the work presented in this thesis is my own. All chapters in this dissertation have been written entirely on my own, except for Chapter 4 which has been written in collaboration with Vincenzo Merella (Birkbeck College). No part of this dissertation contains material previously submitted to examiners of any other university. Where information has been derived from other sources, I confirm that this has been duly indicated in the thesis.