Bound States in a Quasi-Two-Dimensional Fermi Gas

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We consider the problem of N identical fermions of mass $m_\uparrow$ and one distinguishable particle of mass $m_\downarrow$ interacting via short-range interactions in a confined quasi-two-dimensional (quasi-2D) geometry. For $N = 2$ and mass ratios $m_\uparrow/m_\downarrow < 13.6$, we find non-Efimov trimers that smoothly evolve from 2D to 3D. In the limit of strong 2D confinement, we show that the energy of the $N + 1$ system can be approximated by an effective two-channel model. We use this approximation to solve the $3 + 1$ problem and we find that a bound tetramer can exist for mass ratios $m_\uparrow/m_\downarrow$ as low as 5 for strong confinement, thus providing the first example of a universal, non-Efimov tetramer involving three identical fermions.

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An understanding of the few-body problem can be important for gaining insight into the many-body system. In dimensions higher than one, few-body bound states can, for instance, impact the statistics of the many-body quasiparticle excitations. Indeed, for fermionic systems, the two-body bound state is fundamental to the understanding of the BCS-BEC crossover [1–4], while the existence of three-body bound states of fermions [5,6] with unequal masses can lead to dressed trimer quasiparticles in the highly polarized Fermi gas [7]. Even in one dimension (1D), few-body bound states can impact the many-body phase: It has already been shown that one can have a Luttinger liquid of trimers [8].

In general, attractively interacting bosons readily form bound clusters, with the celebrated example being the Efimov effect in 3D [9]. Here, there is a universal hierarchy of trimer states for resonant short-range interactions, while clusters of four or more bosons can also form [10–13]. Even in the limit of a 2D geometry, where the Efimov effect is absent, both trimers [14] and tetramers [15] have been predicted. On the other hand, bound states of identical fermions are constrained to have odd angular momentum owing to Pauli exclusion and thus, even for attractive interactions, identical fermions are subject to a centrifugal barrier. For short-range s-wave [16] interactions in 3D, non-Efimov trimers consisting of two identical fermions with mass $m_\uparrow$ and one distinguishable particle with mass $m_\downarrow$ can only exist above the critical mass ratio $m_\uparrow/m_\downarrow \simeq 8.2$ [5], while Efimov trimers only appear once $m_\uparrow/m_\downarrow \gtrsim 13.6$ [17]. However, the existence of larger $(N + 1)$-body bound states involving $N > 2$ identical fermions remains largely unknown—it has only recently been shown that Efimov tetramers exist in 3D [18].

In this Letter, we investigate the problem of N identical fermions interacting with one distinguishable particle in a confined quasi-2D geometry, where the centrifugal barrier is reduced and the binding of fermions should be favored. Such 2D geometries have recently been realised in ultracold atomic Fermi gases [19–23], where the fermions are confined to 2D with an effective harmonic potential. In addition to allowing one to explore the 2D-3D crossover, the harmonic confinement can strongly modify the scattering properties of atoms via confinement-induced resonances [6,24,25]. It has already been demonstrated that stable non-Efimov trimers can exist for lower mass ratios $m_\uparrow/m_\downarrow$ in quasi-2D [6,26]. Here we show that tetramers involving $N = 3$ identical fermions can appear for $m_\uparrow/m_\downarrow$ as low as 5 in quasi-2D (see Fig. 1), thus putting it within reach of current cold-atom experiments.

We construct the general equations for the bound state of the $N + 1$ system in quasi-2D and we reveal how to simplify the problem in the case of the trimer ($N = 2$). In the limit of strong 2D confinement, we show that the $N + 1$ problem can be described by an effective two-channel model, analogous to that used for Feshbach resonances.

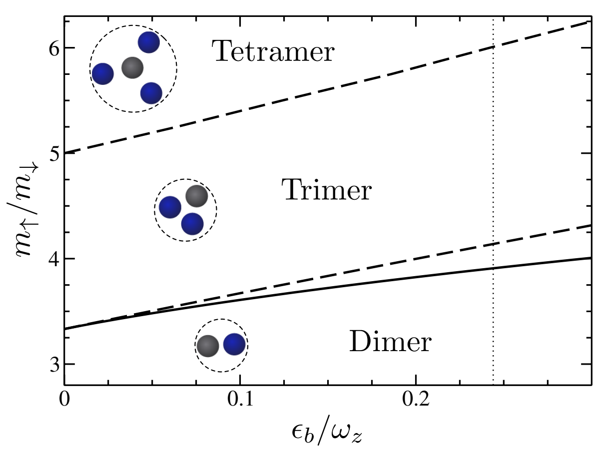

FIG. 1 (color online). Critical mass ratio for the appearance of trimers and tetramers in quasi-2D, where the 2D limit corresponds to $\epsilon_b/\omega_z \to 0$. The solid line follows from the solution of the full three-body quasi-2D problem, Eq. (7). Dashed lines follow from an effective two-channel model. The vertical dotted line marks unitarity, where the 3D scattering length diverges.

This important simplification allows us to solve the aforementioned $N = 3$ problem in quasi-2D.

In the following, we assume the two atomic species $\{\uparrow, \downarrow\}$ to be confined to a quasi-2D geometry by an approximately harmonic potential along the z direction, $V_{\uparrow,\downarrow}(z) = \frac{1}{2} m_{\uparrow,\downarrow}\omega_z^2 z^2$. Here, we restrict ourselves to equal confinement frequencies for the two species since it allows a separation of the relative and center of mass motion along the z direction, as we discuss below. Such a scenario can, in principle, be engineered experimentally using spin-dependent optical lattices. However, even in the case where the confinement frequency is species dependent, regimes exist in which the few-body properties are only weakly affected by this dependence. For instance, for large mass ratios and on the molecular side of the Feshbach resonance, once the dimer is smaller than the light atom oscillator length, $l_z^{\downarrow} = \sqrt{\hbar/m_{\downarrow}\omega_z}$, the light atom is essentially confined by its interaction with the heavy atoms [6].

The starting point of our analysis is the T matrix describing the repeated two-body interspecies interaction. In the ultracold gases, the interaction is described by a zero-range model as the Van der Waals range of the interatomic potential is much smaller than all other length scales in the problem, including the confinement lengths. The T matrix may be considered in the basis of the individual motion of a spin-$\downarrow$ and $\uparrow$ atom. However, due to the restriction to equal confinement frequencies for the two species, the center of mass and relative motion separate and it is advantageous to work in this basis. In the center of mass frame of the harmonic oscillator potential, at energy ϵ below the two-body threshold ω_z (we set $\hbar = 1$) and at total 2D momentum $\mathbf{q}$, the T matrix takes the form [27]

$$\mathcal{T}(\mathbf{q}, \epsilon) = \frac{\sqrt{2\pi}}{m_r}\left\{\frac{l_z^r}{a_s} - \mathcal{F}\left(\frac{-\epsilon + \mathbf{q}^2/2(m_\uparrow + m_\downarrow)}{\omega_z}\right)\right\}^{-1}, \quad (1)$$

where the zero-range interaction is renormalized by the use of the 3D scattering length, a_s. Here, $m_r = m_\uparrow m_\downarrow/(m_\uparrow + m_\downarrow)$ is the reduced mass and $l_z^r = \sqrt{1/2m_r\omega_z}$ is the confinement length corresponding to the relative motion. We use the definition of $\mathcal{F}$ [28]

$$\mathcal{F}(x) = \int_0^\infty \frac{du}{\sqrt{4\pi u^3}}\left(1 - \frac{e^{-xu}}{\sqrt{[1-\exp(-2u)]/2u}}\right). \quad (2)$$

The two-dimensional scattering always admits a two-body bound state of mass $M = m_\uparrow + m_\downarrow$ and binding energy $\epsilon_b > 0$ satisfying $l_z^r/a_s = \mathcal{F}(\epsilon_b/\omega_z)$.

The T matrix in the basis of individual motion is related to $\mathcal{T}$ by the change of basis

$$T_{n_0 n_1}^{n_0' n_1'}(\mathbf{q}, \epsilon) = \sum_{n n_r n_r'} C_{n n_r}^{n_0 n_1}(m_\downarrow, m_\uparrow) C_{n n_r'}^{n_0' n_1'}(m_\downarrow, m_\uparrow) \times \psi_{n_r}(0)\psi_{n_r'}(0)\mathcal{T}(\mathbf{q}, \epsilon - n\omega_z). \quad (3)$$

Here, n_0 and n_1 are the quantum numbers labelling the eigenstates of the single-particle Hamiltonians $\mathcal{H}_{\downarrow\uparrow} = -\frac{\nabla_{0,1}^2}{2m_{\downarrow\uparrow}} + \frac{1}{2} m_{\downarrow\uparrow}\omega_z^2 z_{0,1}^2$ while n_r and n are the quantum numbers in the basis of relative, $z_{01} = z_0 - z_1$, and center of mass, $Z_{01} = (m_\downarrow z_0 + m_\uparrow z_1)/M$, coordinates. The wave function of the relative motion takes the value $\psi_{n_r}(0) = (-1)^{n_r/2}\sqrt{(n_r - 1)!!/n_r!!}$ if n_r is even, and 0 otherwise. The Clebsch-Gordan coefficients $C_{n n_r}^{n_0 n_1}(m_\downarrow, m_\uparrow) \equiv \langle n_0 n_1 | n n_r\rangle$ were obtained in Ref. [29] and vanish unless $n_0 + n_1 = n + n_r$.

We now turn to the question of the existence of bound states consisting of N spin-$\uparrow$ atoms and a single spin-$\downarrow$ atom. To this end, we construct the sum of connected diagrams with $N + 1$ incoming atoms (Fig. 2). The $\uparrow$ atoms are considered on shell with 2D momenta $\mathbf{k}_i$, harmonic oscillator quantum numbers n_i, and corresponding single-particle energies $\epsilon_{\mathbf{k}_i n_i \uparrow} = k_i^2/2m_\uparrow + n_i\omega_z$ for $i = 1, \ldots, N$. We consider scattering in the centre of mass frame of the 2D motion and at a total energy E below the $N + 1$ atom threshold $(N + 1)\omega_z/2$. Thus, the $\downarrow$ atom has 2D momentum $\mathbf{k}_0 \equiv -\sum_{i=1}^N \mathbf{k}_i$, harmonic oscillator quantum number n_0, and energy $E_0 \equiv E - \sum_{i=1}^N \epsilon_{\mathbf{k}_i n_i \uparrow}$. The sum of diagrams with $N + 1$ incoming particles in which the $\downarrow$ atom interacts first with the $\uparrow$ atom numbered 1 is denoted $f_{\mathbf{k}_2 \ldots \mathbf{k}_N}^{n_0 \ldots n_N}$. Note that there is no dependence on $\mathbf{k}_1$ as the initial interaction depends only on the total momentum of the two atoms.

The occurrence of a bound state corresponds to a singularity of f at its binding energy. This singularity results from the summation of an infinite number of diagrams and, at the pole, f satisfies the homogeneous integral equation illustrated in Fig. 2: The initial interaction is described by a T matrix, and then the spin-$\downarrow$ atom subsequently interacts with another of the $\uparrow$ atoms. Thus, the right hand side contains $N - 1$ terms and the integral equation satisfied by the bound state energy is (setting the volume to 1):

$$f_{\mathbf{k}_2 \ldots \mathbf{k}_N}^{n_0 \ldots n_N} = -\sum_{\mathbf{k}_1', n_0' n_1'} \frac{T_{n_0 n_1}^{n_0' n_1'}(\mathbf{k}_0 + \mathbf{k}_1, E_0 + \epsilon_{\mathbf{k}_1 n_1 \uparrow})}{E_0 + \epsilon_{\mathbf{k}_1 n_1 \uparrow} - \epsilon_{\mathbf{k}_0' n_0' \downarrow} - \epsilon_{\mathbf{k}_1' n_1' \uparrow}} \times \{f_{\mathbf{k}_1' \mathbf{k}_3 \ldots \mathbf{k}_N}^{n_0' n_2 n_1' n_3 \ldots n_N} + \cdots + f_{\mathbf{k}_2 \ldots \mathbf{k}_{N-1} \mathbf{k}_1'}^{n_0' n_N n_2 \ldots n_{N-1} n_1'}\}, \quad (4)$$

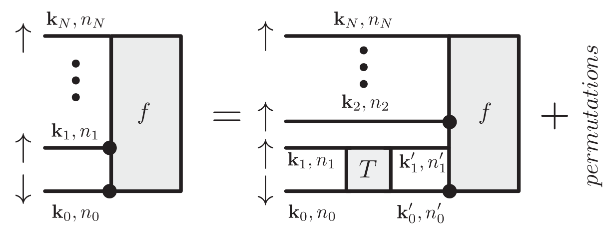

FIG. 2. The diagrams which give the binding energy of the $N + 1$ bound state in quasi-2D. Black dots indicate the initial interaction inside f.

where $\mathbf{k}_0' = \mathbf{k}_0 + \mathbf{k}_1 - \mathbf{k}_1'$, and the minus sign on the right-hand side appears because f is antisymmetric under the exchange of incoming fermions. Equation (4) embodies a simple and generic formulation for the $(N+1)$-body problem in quasi-2D, which in principle allows us to capture the crossover from 2D to 3D. Indeed, for the case of $N = 2$, it is a generalization of the Skorniakov—Ter-Martirosian equation for atom-dimer scattering [30], while for $N = 1$, Eq. (4) simply reduces to the condition for the two-body binding energy. Finally, we note that Ref. [31] derived an expression similar to our Eq. (4) for the 3D $N + 1$ problem.

An important simplification to Eq. (4) becomes possible in the limit of strong quasi-2D confinement, $\omega_z \gg \epsilon_b$. Here, the function $\mathcal{F}$ can be expanded as

$$\mathcal{F}(x) \approx \frac{1}{\sqrt{2\pi}} \ln(\pi x/B) + \frac{\ln 2}{\sqrt{2\pi}} x + \mathcal{O}(x^2), \qquad (5)$$

with $B \approx 0.905$ [27,28]. On the other hand, consider the denominator on the right-hand side of Eq. (4) which we shall write for simplicity as $\epsilon - n\omega_z$. Here, the typical energy scale $\epsilon \sim \epsilon_b$ since, for bound states, the function f is strongly peaked at momenta $\sim\sqrt{2m_r\epsilon_b}$, while it quickly decays for large momenta. Now, if we expand the denominator in powers of ϵ_b/ω_z (assuming $n \neq 0$), then the lowest order term vanishes when integrated over momentum due to the antisymmetry of $f_{\mathbf{k}_1'\ldots}$. Consequently, the lowest nonvanishing contribution from the denominator is of order $(\epsilon_b/\omega_z)^2$ when the harmonic oscillator index n is nonzero. We conclude that to linear order in ϵ_b/ω_z the integral equation for the $N + 1$ bound state reduces to

$$f_{\mathbf{k}_2\ldots\mathbf{k}_N} = \tilde{\mathcal{T}}(\mathbf{k}_0 + \mathbf{k}_1, \tilde{E}_0 + \epsilon_{\mathbf{k}_1\uparrow}) \times \sum_{\mathbf{k}_1'} \frac{f_{\mathbf{k}_1'\mathbf{k}_3\ldots\mathbf{k}_N} + \ldots + f_{\mathbf{k}_2\ldots\mathbf{k}_{N-1}\mathbf{k}_1'}}{\tilde{E}_0 + \epsilon_{\mathbf{k}_1\uparrow} - \epsilon_{\mathbf{k}_0'\downarrow} - \epsilon_{\mathbf{k}_1'\uparrow}}, \qquad (6)$$

with the single particle energies $\epsilon_{\mathbf{k}} \equiv \epsilon_{\mathbf{k}0}$, $\tilde{E}_0 = E - \sum_{i=1}^{N} \epsilon_{\mathbf{k}_i\uparrow}$, and $\tilde{\mathcal{T}}$ obtained from Eq. (1) using the linear expansion of $\mathcal{F}$, Eq. (5). As the effects of confinement in this limit are contained solely within the linearized T matrix, Eq. (6) may be obtained through a strictly 2D 2-channel model [32], where the closed channel corresponds to excited harmonic oscillator modes. Thus, the confinement length l_z^r plays the role of an effective range in this model, with the 2D limit $l_z^r/a_s \to 0$ corresponding to a single-channel model. This simplification crucially depends on the antisymmetry resulting from Fermi statistics and it thus does not apply to bound clusters involving bosons confined to 2D, as considered in Refs. [33,34]. Finally, we note that a similar simplification was recently obtained for quasi-1D atom-dimer scattering [35] (see also Ref. [36]).

We now proceed to solve the three-body problem using the above methods. First, note that using the zero-range condition and removing the center of mass generally allows one to reduce the number of harmonic oscillator quantum numbers by two in Eq. (4) [6]. For the three-body problem, this is achieved by changing coordinates to the relative motion of the two atoms initially interacting, z_{01}, the relative motion of the pair and the third atom, $z_2^{01} = (m_\downarrow z_0 + m_\uparrow z_1)/(m_\downarrow + m_\uparrow) - z_2$, and the center of mass $Z_{012} = (m_\downarrow z_0 + m_\uparrow z_1 + m_\uparrow z_2)/(m_\downarrow + 2m_\uparrow)$. Defining the corresponding quantum numbers n_{01}, n_2^{01}, and N_{012}, we adopt the new basis $\chi_{\mathbf{k}_2}^{n_2^{01}} = \frac{1}{\psi_{n_{01}}(0)} \sum_{n_0 n_1 n_2} \times \langle N_{012} n_2^{01} n_{01} | n_0 n_1 n_2 \rangle f_{\mathbf{k}_2}^{n_0 n_1 n_2}$ [37]. Then, Eq. (4) for the trimer becomes

$$\chi_{\mathbf{k}_2}^{n_2^{01}} = \mathcal{T}(\mathbf{k}_2, E - \epsilon_{\mathbf{k}_2\uparrow} - n_2^{01}\omega_z) \times \sum_{\mathbf{k}_1', n_1^{02} n_{02} n_{01}'} \frac{\psi_{n_{02}}(0)\psi_{n_{01}'}(0)\langle n_2^{01} n_{01}' | n_1^{02} n_{02}\rangle \chi_{\mathbf{k}_1'}^{n_1^{02}}}{E - \epsilon_{\mathbf{k}_1'\uparrow} - \epsilon_{\mathbf{k}_2\uparrow} - \epsilon_{\mathbf{k}_1'+\mathbf{k}_2\downarrow} - (n_1^{02} + n_{02})\omega_z}. \qquad (7)$$

The matrix element in Eq. (7) may be evaluated by a series of coordinate transformations:

$$\langle n_2^{01} n_{01} | n_1^{02} n_{02} \rangle = \sum_{n_0 n_1 n_2 N_{01} N_{02}} C_{0 n_2^{01}}^{N_{01} n_2}(M, m_\uparrow) C_{N_{01} n_{01}}^{n_0 n_1}(m_\downarrow, m_\uparrow) \times C_{N_{02} n_{02}}^{n_0 n_2}(m_\downarrow, m_\uparrow) C_{0 n_1^{02}}^{N_{02} n_1}(M, m_\uparrow),$$

where several sums can be dropped due to the constraints on the Clebsch-Gordan coefficients.

Since the trimer consists of identical fermions, it must necessarily have odd angular momentum L in the x-y plane of the 2D layer. Thus, the lowest-energy trimer has $L = 1$, and it can be regarded as a p-wave pairing of $\uparrow$ fermions mediated by their s-wave interactions with the light $\downarrow$ particle. In this case, we have $\chi_{\mathbf{k}_2}^{n_2^{01}} = \tilde{\chi}_{k_2}^{n_2^{01}} e^{i\phi_2}$, where ϕ_2 is the angle of $\mathbf{k}_2$ with respect to the x axis and $\tilde{\chi}$ is a function of $k_2 \equiv |\mathbf{k}_2|$. Integrating over ϕ_2 in Eq. (7) then leaves an integral equation that only depends on k_2 and n_2^{01}. The same applies for the two-channel model Eq. (6) with $N = 2$, where now there is only a dependence on k_2.

We have calculated the trimer binding energy as a function of confinement for a range of mass ratios, as depicted in Fig. 3. We see that the binding energy decreases as we perturb away from 2D and the centrifugal barrier is increased. Correspondingly, we find that the critical mass ratio $m_\uparrow/m_\downarrow$ for the trimer binding increases as we perturb away from the 2D limit, as shown in Fig. 1, and smoothly evolves towards the 3D result of 8.2 [5]. For the special case of ^{6}Li-^{40}K mixtures, where $m_\uparrow/m_\downarrow = 6.64$, we have checked that our results agree with Ref. [6]. In the limit of strong 2D confinement, we see that the two-channel model

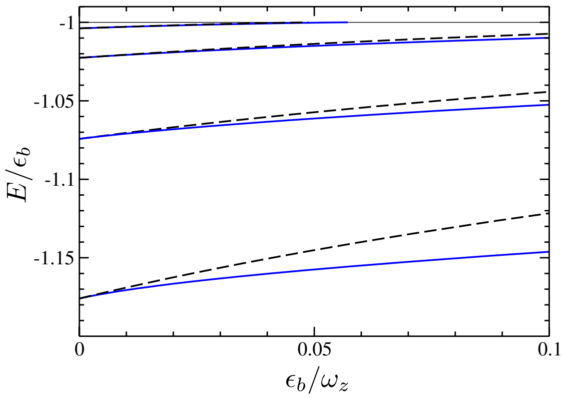

FIG. 3 (color online). Energy of the trimer in quasi-2D for mass ratios $m_\uparrow/m_\downarrow = 3.5$, 4, 5, 6.64 (from top to bottom). The solid lines correspond to the full calculation, while the dashed lines are derived from the effective two-channel model, Eq. (6).

captures the lowest-order dependence on ϵ_b/ω_z of the trimer energy and critical mass ratio.

We can exploit the two-channel model (6) to solve the more complicated four-body ($N = 3$) problem in quasi-2D. Once again, the presence of identical fermions requires us to consider total angular momentum $L = 1$. Thus, we have for the tetramer

$$f_{\mathbf{k}_2\mathbf{k}_3} = \tilde{f}(k_2, k_3, \Delta\phi_{32})e^{i\phi_2} = -\tilde{f}(k_3, k_2, -\Delta\phi_{32})e^{i\phi_3},$$

where $\Delta\phi_{32} = \phi_3 - \phi_2$. We note that a similar equation for the tetramer energy was obtained for the 3D problem in Ref. [18].

Beginning with the 2D limit ($\epsilon_b/\omega_z = 0$), we determine the energy of the tetramer compared to the trimer and dimer energies (see Fig. 4). Following the transition from

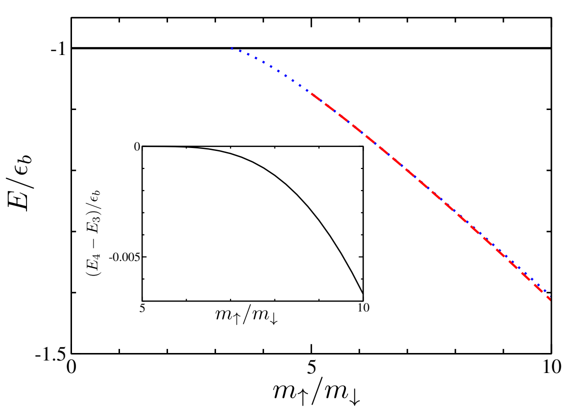

FIG. 4 (color online). Energy of the dimer (solid line), trimer (dotted line), and tetramer (dashed line) in 2D as a function of mass ratio. The trimer binds when $m_\uparrow/m_\downarrow > 3.33$, consistent with Ref. [26], while the trimer-tetramer transition occurs at $m_\uparrow/m_\downarrow = 5.0$. *Inset:* The difference between trimer and tetramer energies, $E_3 - E_4$.

a dimer to a trimer at mass ratio $m_\uparrow/m_\downarrow \simeq 3.33$, we find a trimer-tetramer transition at $m_\uparrow/m_\downarrow \simeq 5.0$. In principle, we can use Eq. (4) to consider bound states of even larger N, but the problem quickly becomes intractable numerically for $N > 3$. However, we conjecture that composite bound states of larger N become possible as $m_\uparrow/m_\downarrow$ is increased, since the relative importance of the centrifugal barrier between heavy particles (which goes as $1/m_\uparrow$) diminishes compared with the effective attractive potential induced by the light particle ($\sim 1/m_\downarrow$).

Perturbing away from the 2D limit, we find that the trimer-tetramer transition shifts to larger $m_\uparrow/m_\downarrow$ with increasing ϵ_b/ω_z, as shown in Fig. 1. Eventually, we expect to encounter the four-body Efimov effect in 3D for $m_\uparrow/m_\downarrow > 13.4$ [18]. However, it remains an open question whether our quasi-2D tetramers exist in 3D below the critical mass ratio for Efimov physics.

To conclude, we have provided the first example of a universal, non-Efimov tetramer involving three identical fermions. Since this quasi-2D tetramer exists for mass ratios $m_\uparrow/m_\downarrow$ as low as 5, it could potentially be probed with ultracold ^{6}Li-^{40}K mixtures. Its small binding energy (Fig. 4) suggests that it could appear as a resonance in atom-trimer interactions. For instance, in the collision of a cloud of atoms and a cloud of trimers under strong quasi-2D confinement, we expect the resonance to be observable as a highly asymmetric density profile of scattered atoms. This is similar to the proposal of Ref. [38] for detecting an atom-dimer resonance. In addition, the presence of trimers and tetramers has implications for the many-body phases in quasi-2D, particularly for the highly polarized Fermi gas [39,40].

We emphasize that although we have focussed on the $N + 1$ problem in quasi-2D, the form of Eq. (4) is completely general and may be extended to other shapes of the confining potential and/or different dimensionalities. For instance, in quasi-1D, one would use the T matrix derived in Refs. [24,28], along with appropriately redefined harmonic oscillator and momentum indices. Furthermore, the problem may be studied close to narrow Feshbach resonances, characterized by a large effective range, by using an energy-dependent scattering length [41]. Finally, our work suggests that a two-channel model may be used to model strongly confined quasi-2D Fermi systems in general.

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Note added.—After the submission of this manuscript, a similar non-Efimov tetramer was predicted to exist in a 3D geometry [42].

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