# Measurement of the Wrong-Sign Decays $D^{0} \rightarrow K^{+} \pi^{-} \boldsymbol{\pi}^{0}$ and $\boldsymbol{D}^{\mathbf{0}} \rightarrow \boldsymbol{K}^{+} \boldsymbol{\pi}^{-} \boldsymbol{\pi}^{+} \boldsymbol{\pi}^{-}$, and Search for $\boldsymbol{C P}$ Violation 

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Using $281 \mathrm{fb}^{-1}$ of data from the Belle experiment recorded at or near the $Y(4 S)$ resonance, we have measured the rates of the "wrong-sign" decays $D^{0} \rightarrow K^{+} \pi^{-} \pi^{0}$ and $D^{0} \rightarrow K^{+} \pi^{-} \pi^{+} \pi^{-}$relative to those of the Cabibbo-favored decays $D^{0} \rightarrow K^{-} \pi^{+} \pi^{0}$ and $D^{0} \rightarrow K^{-} \pi^{+} \pi^{+} \pi^{-}$. These wrong-sign decays proceed via a doubly Cabibbo-suppressed amplitude or via $D^{0}-\bar{D}^{0}$ mixing; the latter has not yet been observed. We obtain $R_{\mathrm{WS}}\left(K \pi \pi^{0}\right)=\left[0.229 \pm 0.015(\mathrm{stat})_{-0.009}^{+0.013}(\mathrm{syst})\right] \%$ and $R_{\mathrm{WS}}(K 3 \pi)=[0.320 \pm$ $\left.0.018(\text { stat })_{-0.013}^{+0.018}(\mathrm{syst})\right] \%$. The $C P$ asymmetries are measured to be $-0.006 \pm 0.053$ and $-0.018 \pm$ 0.044 for the $K^{+} \pi^{-} \pi^{0}$ and $K^{+} \pi^{-} \pi^{+} \pi^{-}$final states, respectively.

DOI: 10.1103/PhysRevLett.95.231801
PACS numbers: $13.25 . \mathrm{Ft}, 11.30 . \mathrm{Er}, 12.15 . \mathrm{Ff}$

Studies of mixing in the $K^{0}-\bar{K}^{0}$ and $B^{0}-\bar{B}^{0}$ meson systems [1] have had an important impact on the development of the standard model (SM). The latter allowed the top quark mass to be predicted prior to its direct observation. In contrast, the $D^{0}-\bar{D}^{0}$ mixing rate is strongly suppressed by Cabibbo-Kobayashi-Maskawa (CKM) factors and the Glashow-Iliopoulos-Maiani mechanism [2]; the SM predicted rate is far below current experimental upper limits. Observation of mixing significantly larger than this prediction could indicate new physics [3]. Previously, $D^{0}-\bar{D}^{0}$ mixing has been searched for in "wrong-sign" (WS) $D^{0} \rightarrow$ $K^{+} \pi^{-}$decays [4-6], in WS $D^{0} \rightarrow K^{+} \pi^{-} \pi^{0}$ and $D^{0} \rightarrow$ $K^{+} \pi^{-} \pi^{+} \pi^{-}$decays [4,7,8], and in Dalitz-plot analyses of $D^{0} \rightarrow K_{s}^{0} \pi^{+} \pi^{-}$decays [9]. Here we investigate the WS multibody modes $D^{0} \rightarrow K^{+} \pi^{-}(n \pi)$ [10] with a data sample more than 30 times larger than that of previous studies. These modes can arise from a $D^{0}$ mixing into $\bar{D}^{0}$ and subsequently decaying via the "right-sign" (RS) Cabibbofavored (CF) decay $\bar{D}^{0} \rightarrow K^{+} \pi^{-}(n \pi)$. The final states can also arise from a doubly Cabibbo-suppressed (DCS) amplitude; the ratio of DCS decays to CF decays can be used to measure the CKM phase $\phi_{3}$ in $B^{+} \rightarrow D^{0} K^{+}$[11].

In this Letter, we present measurements of the ratio of rates for WS to RS decays, $R_{\mathrm{WS}}^{\left(K^{+} \pi^{-} \pi^{0}\right)} \equiv \Gamma\left(D^{0} \rightarrow\right.$ $\left.K^{+} \pi^{-} \pi^{0}\right) / \Gamma\left(D^{0} \rightarrow K^{-} \pi^{+} \pi^{0}\right) \quad$ and $\quad R_{\mathrm{WS}}^{\left(K^{+} \pi^{-} \pi^{+} \pi^{-}\right)} \equiv$ $\Gamma\left(D^{0} \rightarrow K^{+} \pi^{-} \pi^{+} \pi^{-}\right) / \Gamma\left(D^{0} \rightarrow K^{-} \pi^{+} \pi^{+} \pi^{-}\right)$. Assum-
ing negligible $C P$ violation, this ratio is given by [12]

$$
\begin{equation*}
R_{\mathrm{WS}}=R_{D}+\sqrt{R_{D}} y^{\prime}+\frac{1}{2}\left(x^{\prime 2}+y^{\prime 2}\right) \tag{1}
\end{equation*}
$$

where $R_{D}$ is the ratio of the magnitudes squared of the DCS to CF amplitudes, and $x^{\prime}$ and $y^{\prime}$ are "rotated" versions of the mixing parameters $x \equiv \Delta m / \bar{\Gamma}$ and $y \equiv \Delta \Gamma / 2 \bar{\Gamma}: x^{\prime}=$ $x \cos \delta+y \sin \delta$ and $y^{\prime}=y \cos \delta-x \sin \delta$, where $\delta$ is an effective strong phase difference between the DCS and CF amplitudes [13]. The parameters $x$ and $y$ are modeindependent, depending only on the differences in mass $(\Delta m)$ and decay width $(\Delta \Gamma)$ between the two $D^{0}-\bar{D}^{0}$ mass eigenstates and on their mean decay width $(\bar{\Gamma})$.

The data sample consists of $281 \mathrm{fb}^{-1}$ recorded by the Belle experiment at KEKB [14], an asymmetric $e^{+} e^{-}$ collider operating at or near the $Y(4 S)$ resonance. The Belle detector is a large-solid-angle magnetic spectrometer consisting of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Čerenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL), all located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux return outside the coil is instrumented to detect $K_{L}^{0}$ mesons and to identify muons (KLM). The detector is described in detail elsewhere $[15,16]$.

We consider the decay chain $D^{*+} \rightarrow D^{0} \pi_{s}^{+} \rightarrow$ $K \pi(n \pi) \pi_{s}^{+}$, where the "slow" pion $\pi_{s}^{+}$has a characteristic soft momentum spectrum. The charge of $\pi_{s}$ is used to identify whether a $D^{0}$ or $\bar{D}^{0}$ was initially produced. We require that all tracks have at least two SVD hits in both $r-\phi$ and $z$ coordinates. We use information from the TOF, ACC, and CDC to select kaons (pions) with momentum dependent efficiencies of $80 \%-95 \%(90 \%-95 \%)$ and pion (kaon) misidentification probabilities of 5\%-20\% (15\%$20 \%$ ). To suppress background from semileptonic decays, we remove tracks identified as electrons (muons) based on ECL (KLM) information. We select $\pi^{0}$ candidates that satisfy $118 \mathrm{MeV} / c^{2}<M_{\gamma \gamma}<150 \mathrm{MeV} / c^{2}( \pm 3 \sigma$ in resolution); we then apply a mass constrained fit for the photons. We require photon energies to be larger than 60 (120) MeV in the barrel (end cap) region.
$D^{0} \rightarrow K^{+} \pi^{-} \pi^{0}$ candidates are reconstructed by combining two oppositely charged tracks with a $\pi^{0}$ candidate having $p>310 \mathrm{MeV} / c$ in the center-of-mass (c.m.) frame. The $K^{+} \pi^{-} \pi^{0}$ invariant mass is required to be in the range $1.78-1.92 \mathrm{GeV} / c^{2}$ ( $\pm 6 \sigma$ in resolution). To reject background from $D^{0} \rightarrow K^{-} \pi^{+} \pi^{0}$ in which the $K$ is misidentified as $\pi$ and the $\pi$ as $K$, we calculate $m_{K \pi \pi^{0}}$ with the $K$ and $\pi$ assignments swapped and reject events having $m_{K \pi \pi^{0} \text { (swapped) }}$ in the range $1.78-1.90 \mathrm{GeV} / c^{2}$.
$D^{0} \rightarrow K^{+} \pi^{-} \pi^{+} \pi^{-}$candidates are formed from combinations of four charged tracks; $m_{K 3 \pi}$ is required to be in the range $1.81-1.91 \mathrm{GeV} / c^{2}( \pm 7 \sigma)$. To reject background due to misidentification of $D^{0} \rightarrow K^{-} \pi^{+} \pi^{-} \pi^{+}$, we calculate $m_{K 3 \pi}$ with the $K$ and $\pi$ assignments swapped and reject events satisfying $\left|m_{K 3 \pi(\text { swapped })}-m_{D^{0}}\right|<$ $20 \mathrm{MeV} / c^{2}$. The Cabibbo-suppressed decay $D^{0} \rightarrow$ $\bar{K}^{0} K^{+} \pi^{-}$followed by $\bar{K}^{0} \rightarrow \pi^{+} \pi^{-}$can also mimic the WS signal; to reject this background, we calculate $m_{\pi^{+}} \pi^{-}$ for both oppositely charged pion combinations and reject events satisfying $\left|m_{\pi^{+} \pi^{-}}-m_{K^{0}}\right|<16 \mathrm{MeV} / c^{2}$.

The charged $D^{0}$ daughters are required to originate from a common vertex. The $D^{0}$ momentum vector is extrapolated back to the interaction point (IP) profile, and a production vertex is determined. The $D^{*+}$ candidate is then formed by combining the $D^{0}$ candidate with a $\pi_{s}^{+}$. We refit the $\pi_{s}^{+}$track, requiring that it intersect the $D^{0}$ production point; this greatly suppresses combinatorial background and improves the resolution on the energy released in the $D^{*}$ decay, $Q \equiv M_{\pi_{s}^{+} K^{+} \pi^{-}(n \pi)}-M_{K^{+} \pi^{-}(n \pi)}-m_{\pi_{s}^{+}}$. For $D^{*+} \rightarrow D^{0} \pi_{s}^{+}$decays, $Q$ is only 5.85 MeV (slightly above threshold) and provides substantial background rejection. We subsequently require $Q<12 \mathrm{MeV}$, which is $>99 \%$ efficient.

To eliminate $D$ mesons produced in $B \bar{B}$ events and further suppress combinatorial background, the reconstructed $D^{*+}$ momentum in the c.m. frame is required to be greater than $2.5 \mathrm{GeV} / c$. Finally, we require that the $\chi^{2}$ per degree of freedom resulting from the $D^{0}$ vertex fit, the IP vertex fit, and the $\pi_{s}$ track refit be satisfactory. The
fraction of events containing multiple signal candidates is less than $3 \%$ for both modes (and is the same for RS and WS decays); multiple signal candidates are retained for subsequent analysis.

We determine the RS and WS signal yields by performing binned maximum likelihood fits in $M-Q$ space with $M=M_{K \pi(n \pi)}$. The signal and background distributions are determined using a large Monte Carlo (MC) sample [17]. The backgrounds can be divided into three categories: (a) "random $\pi_{s}$ " background, in which a random $\pi^{+}$is combined with a true $\bar{D}^{0} \rightarrow K^{+} \pi^{-}(n \pi)$ decay, (b) charm decay background other than (a), and (c) background from continuum $e^{+} e^{-} \rightarrow u \bar{u}, d \bar{d}$, or $s \bar{s}$ production.

The RS signal shape as predicted by MC simulation is parametrized in $M$, with a sum of a double Gaussian and a double bifurcated Gaussian with common mean, and in $Q$, with a bifurcated Student's $t$ function. Background distributions are parametrized with similar empirical expressions determined from MC simulation. In the RS sample fit, the mean and width of the signal distribution are left free to vary, while other parameters are fixed to MC values. The relative normalizations of individual background categories are fixed to MC values for the $D^{0} \rightarrow K^{+} \pi^{-} \pi^{0}$ fit and left free for the $D^{0} \rightarrow K^{+} \pi^{-} \pi^{+} \pi^{-}$fit. In the WS sample fit, the mean and width of the signal are fixed to the values obtained from the RS fit; the normalizations of the backgrounds are left free to vary.

The RS sample fit obtains a signal yield of $(8.683 \pm$ $0.002) \times 10^{5}$ for $D^{0} \rightarrow K^{-} \pi^{+} \pi^{0}$ and (5.259 $\left.\pm 0.002\right) \times$ $10^{5}$ for $D^{0} \rightarrow K^{-} \pi^{+} \pi^{+} \pi^{-}$. The WS fit finds $1978 \pm 104$ for $D^{0} \rightarrow K^{+} \pi^{-} \pi^{0} \quad$ and $1721 \pm 75$ for $D^{0} \rightarrow$ $K^{+} \pi^{-} \pi^{+} \pi^{-}$. The fit results are projected onto the $M$ and $Q$ distributions in Fig. 1 for $D^{0} \rightarrow K^{+} \pi^{-} \pi^{0}$ and in Fig. 2 for $D^{0} \rightarrow K^{+} \pi^{-} \pi^{+} \pi^{-}$. The hatched histograms show the fit results, and the points with error bars show the data.

In $D^{0} \rightarrow K \pi(n \pi)$ decays, intermediate resonances dominate the decay rate and cause a nonuniform event distribution in phase space. Since RS and WS decays may have different resonant substructure, their acceptances may differ. We correct the event yields for acceptance and reconstruction efficiency as follows. For $D^{0} \rightarrow K^{ \pm} \pi^{\mp} \pi^{0}$, we determine efficiencies using MC simulation in bins of ( $M_{K \pi}^{2}, M_{\pi \pi^{0}}^{2}$ ); for $D^{0} \rightarrow K^{ \pm} \pi^{\mp} \pi^{+} \pi^{-}$, we use bins in a five-dimensional space comprised of the invariant mass squared for various $K, \pi$ combinations. We then calculate efficiency-corrected signal yields in each bin for the RS and WS samples. The background is taken to be the overall background yield multiplied by the fraction falling in that bin; the distribution of background among the bins is taken from the sideband $|Q-5.85 \mathrm{MeV}|>2.0 \mathrm{MeV}$. The resulting signal yields are summed over all bins, and the ratio of the total signal yields gives $R_{\mathrm{WS}}$. The results are $R_{\mathrm{WS}}^{K^{+} \pi^{-} \pi^{0}}=(2.29 \pm 0.15) \times 10^{-3}$ and $R_{\mathrm{WS}}^{K^{+}} \pi^{-} \pi^{+} \pi^{-}=$ $(3.20 \pm 0.18) \times 10^{-3}$, where the errors are statistical only.


FIG. 1. Results of the $M-Q$ fit for $D^{0} \rightarrow K^{+} \pi^{-} \pi^{0}$, in projections onto (a) RS $M_{K \pi \pi^{0}}$ with $0 \mathrm{MeV}<Q<12.0 \mathrm{MeV}$; (b) RS $Q$ with $1.780 \mathrm{GeV} / c^{2}<M_{K \pi \pi^{0}}<1.920 \mathrm{GeV} / c^{2}$; (c) WS $M_{K \pi \pi^{0}}$ with $5.31 \mathrm{MeV}<Q<6.42 \mathrm{MeV}$; and (d) WS $Q$ with $1.844 \mathrm{GeV} / c^{2}<M_{K \pi \pi^{0}}<1.887 \mathrm{GeV} / c^{2}$.

The average efficiency for a mode is obtained by dividing the signal yield from the $M-Q$ fit by the total efficiencycorrected signal yield; the ratio of average efficiencies $\left\langle\varepsilon_{\mathrm{RS}}\right\rangle /\left\langle\varepsilon_{\mathrm{WS}}\right\rangle$ is $1.01 \pm 0.05$ for $D^{0} \rightarrow K^{ \pm} \pi^{\mp} \pi^{0}$ and $0.98 \pm 0.04$ for $D^{0} \rightarrow K^{ \pm} \pi^{\mp} \pi^{+} \pi^{-}$.

Contributions to the systematic uncertainty on $R_{\text {WS }}$ are listed in Table I: The size of each term is assessed by varying the analysis as described below and repeating the fits. Many effects cancel in the ratio due to the similar kinematics of the RS and WS modes; one distinction is the significant background contribution to the WS sample. We vary the selection criteria over reasonable ranges (the WS yield changes by $\sim 10 \%$ ); the largest positive and negative variations in $R_{\text {WS }}$ are assigned as systematic errors. We check the parametrization of the signal shape by varying the means and widths in $M$ and $Q$ by $\pm 1 \sigma$. We check background fractions and parametrizations by varying individual fractions and distribution parameters by $\pm 1 \sigma$; we also try alternative functional forms. We investigate possible fit bias by fitting a large MC RS sample; the small difference between the fitted yield and the true number of RS events is taken as an additional systematic error. The total systematic error is obtained by combining the individual terms in quadrature.

Assuming a value for $x^{\prime}$, Eq. (1) can be used to constrain $R_{D}$ as a function of $y^{\prime}$. This constraint is shown in Fig. 3 for $x^{\prime}=0$ and $\left|x^{\prime}\right|=0.028$; the latter value is the $95 \%$ C.L. upper limit on $\left|x^{\prime}\right|$ obtained from our previous analysis of $D^{0} \rightarrow K^{+} \pi^{-}$decays [6]. Values of $\left(x^{\prime}, y^{\prime}\right)$ for different decay modes would be equivalent if the strong phase


FIG. 2. Results of the $M-Q$ fit for $D^{0} \rightarrow K^{+} \pi^{-} \pi^{+} \pi^{-}$, in projections onto (a) RS $M_{K 3 \pi}$ with $0 \mathrm{MeV}<Q<12.0 \mathrm{MeV}$; (b) RS $Q$ with $1.810 \mathrm{GeV} / c^{2}<M_{K 3 \pi}<1.910 \mathrm{GeV} / c^{2}$; (c) WS $M_{K 3 \pi}$ with $5.47 \mathrm{MeV}<Q<6.28 \mathrm{MeV}$; and (d) WS $Q$ with $1.852 \mathrm{GeV} / c^{2}<M_{K 3 \pi}<1.878 \mathrm{GeV} / c^{2}$.
differences $(\delta)$ for the modes were equal. In the absence of mixing (i.e., $x=y=0$ ), our measurements give $R_{D}\left(K \pi \pi^{0}\right)=\left(0.85_{-0.07}^{+0.08}\right) \tan ^{4} \theta_{C} \quad$ and $\quad R_{D}(K 3 \pi)=$ $\left(1.18_{-0.09}^{+0.10}\right) \tan ^{4} \theta_{C}\left(\theta_{C}\right.$ is the Cabibbo angle), consistent with theoretical expectations [18].

By separately fitting the $D^{0}$ and $\bar{D}^{0}$ samples, we measure the $C P$ asymmetry

$$
A_{C P}=\frac{R_{\mathrm{WS}}^{D^{0} \rightarrow K^{+}} \pi^{-}(n \pi)}{}-R_{\mathrm{WS}}^{\bar{D}^{0} \rightarrow K^{-}} \pi^{+}(n \pi) .
$$

We obtain $A_{C P}\left(K \pi \pi^{0}\right)=-0.006 \pm 0.053$ and $A_{C P}(K 3 \pi)=-0.018 \pm 0.044$, which are both consistent with zero. The systematic uncertainties are $<0.01$ (much smaller than the statistical errors) and are neglected. The first value represents a large improvement over the previously published result [7]; the second value has not been previously measured.

TABLE I. Systematic uncertainties for $R_{\mathrm{WS}}$, in percentage.

| Source | $D^{0} \rightarrow K^{+} \pi^{-} \pi^{0}$ |  | $D^{0} \rightarrow K^{+} \pi^{-} \pi^{+} \pi^{-}$ |  |
| :--- | :--- | :--- | :--- | :--- |
| Selection criteria | +5.22 | -2.38 | +5.25 | -3.78 |
| Signal shape param. | +0.09 | -0.10 | +0.10 | -0.10 |
| Background fraction | +0.00 | -0.07 | +0.01 | -0.01 |
| Background param. | +0.42 | -2.89 | +0.34 | -0.59 |
| Possible fit bias | +2.23 | -0.94 | +0.91 | -0.88 |
| Total | +5.7 | -3.9 | +5.4 | -4.0 |



FIG. 3. $68.3 \%$ C.L. bands for $R_{D}$ as a function of $y^{\prime}$ for $x^{\prime}=0$ and $\left|x^{\prime}\right|=0.028$. The latter value is the upper limit obtained from our analysis of $D^{0} \rightarrow K^{+} \pi^{-}$decays assuming no $C P$ violation [6]. The point with $1 \sigma$ error bars is the result from the $D^{0} \rightarrow K^{+} \pi^{-}$analysis for $x^{\prime}=0$ (and no $C P$ violation). Note that $\delta$ and, thus, $x^{\prime}, y^{\prime}$ may differ for the three modes.

In summary, using $281 \mathrm{fb}^{-1}$ of data, we measure the ratio of WS to RS decay rates for $D^{0} \rightarrow K^{ \pm} \pi^{\mp} \pi^{0}$ and $D^{0} \rightarrow K^{ \pm} \pi^{\mp} \pi^{+} \pi^{-}$to be

$$
\begin{gathered}
R_{\mathrm{WS}}^{K^{+} \pi^{-} \pi^{0}}=\left[2.29 \pm 0.15(\mathrm{stat})_{-0.09}^{+0.13}(\mathrm{syst})\right] \times 10^{-3} \\
R_{\mathrm{WS}}^{K^{+} \pi^{-} \pi^{+} \pi^{-}}=\left[3.20 \pm 0.18(\mathrm{stat})_{-0.13}^{+0.18}(\mathrm{syst})\right] \times 10^{-3} .
\end{gathered}
$$

These results are much more precise than previously published results $[4,7,8]$. The $C P$ asymmetries measured are consistent with zero.

We thank the KEKB group for the excellent operation of the accelerator, the KEK cryogenics group for the efficient operation of the solenoid, and the KEK computer group and the NII for valuable computing and Super-SINET network support. We acknowledge support from MEXT and JSPS (Japan); ARC and DEST (Australia); NSFC (Contract No. 10175071, China); DST (India); the BK21 program of MOEHRD and the CHEP SRC program of KOSEF (Korea); KBN (Contract No. 2P03B 01324,

Poland); MIST (Russia); MHEST (Slovenia); SNSF (Switzerland); NSC and MOE (Taiwan); and DOE (USA).
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