# Measurement of $\left|V_{u b}\right|$ Using Inclusive $B \rightarrow X_{u} \ell \nu$ Decays with a Novel $X_{u}$-Reconstruction Method 

H. Kakuno, ${ }^{41}$ K. Abe, ${ }^{6}$ K. Abe, ${ }^{38}$ I. Adachi, ${ }^{6}$ H. Aihara, ${ }^{40}$ Y. Asano, ${ }^{45}$ T. Aso, ${ }^{44}$ V. Aulchenko, ${ }^{1}$ T. Aushev, ${ }^{10}$ A. M. Bakich, ${ }^{35}$ Y. Ban, ${ }^{29}$ S. Banerjee, ${ }^{36}$ I. Bizjak, ${ }^{11}$ A. Bondar, ${ }^{1}$ A. Bozek, ${ }^{23}$ M. Bračko, ${ }^{17,11}$ T. E. Browder, ${ }^{5}$ Y. Chao, ${ }^{22}$ B. G. Cheon, ${ }^{34}$ R. Chistov, ${ }^{10}$ S.-K. Choi, ${ }^{4}$ Y. Choi, ${ }^{34}$ Y. K. Choi, ${ }^{34}$ A. Chuvikov, ${ }^{30}$ S. Cole, ${ }^{35}$ M. Danilov, ${ }^{10}$ M. Dash, ${ }^{46}$ L. Y. Dong, ${ }^{8}$ J. Dragic, ${ }^{18}$ A. Drutskoy, ${ }^{10}$ S. Eidelman, ${ }^{1}$ V. Eiges, ${ }^{10}$ N. Gabyshev, ${ }^{6}$ A. Garmash, ${ }^{30}$ T. Gershon, ${ }^{6}$ G. Gokhroo, ${ }^{36}$ B. Golob, ${ }^{16,11}$ J. Haba, ${ }^{6}$ C. Hagner, ${ }^{46}$ T. Hara, ${ }^{27}$ M. Hazumi, ${ }^{6}$ I. Higuchi, ${ }^{39}$ L. Hinz, ${ }^{15}$ T. Hokuue, ${ }^{19}$ Y. Hoshi, ${ }^{38}$ W.-S. Hou, ${ }^{22}$ H.-C. Huang, ${ }^{22}$ T. Iijima, ${ }^{19}$ K. Inami, ${ }^{19}$ A. Ishikawa, ${ }^{6}$ R. Itoh, ${ }^{6}$ H. Iwasaki, ${ }^{6}$ Y. Iwasaki, ${ }^{6}$ J. H. Kang, ${ }^{47}$ J. S. Kang, ${ }^{13}$ P. Kapusta, ${ }^{23}$ N. Katayama, ${ }^{6}$ H. Kawai, ${ }^{2}$ T. Kawasaki, ${ }^{25}$ H. Kichimi, ${ }^{6}$ H. J. Kim, ${ }^{47}$ J. H. Kim, ${ }^{34}$ K. Kinoshita, ${ }^{3}$ S. Korpar, ${ }^{17,11}$ P. Križan, ${ }^{16,11}$ P. Krokovny, ${ }^{1}$ Y.-J. Kwon, ${ }^{47}$ G. Leder, ${ }^{9}$ S. H. Lee, ${ }^{33}$ T. Lesiak, ${ }^{23}$ J. Li, ${ }^{32}$ A. Limosani, ${ }^{18}$ S.-W. Lin, ${ }^{22}$ J. MacNaughton, ${ }^{9}$ F. Mandl, ${ }^{9}$ A. Matyja, ${ }^{23}$ Y. Mikami, ${ }^{39}$ W. Mitaroff, ${ }^{9}$ K. Miyabayashi, ${ }^{20}$ H. Miyata, ${ }^{25}$ G. R. Moloney, ${ }^{18}$ T. Mori, ${ }^{41}$ T. Nagamine, ${ }^{39}$ Y. Nagasaka, ${ }^{7}$ T. Nakadaira, ${ }^{40}$ E. Nakano, ${ }^{26}$ M. Nakao, ${ }^{6}$ H. Nakazawa, ${ }^{6}$ Z. Natkaniec, ${ }^{23}$ S. Nishida, ${ }^{6}$ O. Nitoh, ${ }^{43}$ T. Nozaki, ${ }^{6}$ S. Ogawa, ${ }^{37}$ T. Ohshima, ${ }^{19}$ S. Okuno, ${ }^{12}$ S. L. Olsen, ${ }^{5}$ W. Ostrowicz, ${ }^{23}$ H. Ozaki, ${ }^{6}$ P. Pakhlov, ${ }^{10}$ C.W. Park, ${ }^{13}$ H. Park, ${ }^{14}$ K. S. Park, ${ }^{34}$ N. Parslow, ${ }^{35}$ L. S. Peak, ${ }^{35}$<br>L. E. Piilonen, ${ }^{46}$ H. Sagawa, ${ }^{6}$ S. Saitoh, ${ }^{6}$ Y. Sakai, ${ }^{6}$ O. Schneider, ${ }^{15}$ A. J. Schwartz, ${ }^{3}$ S. Semenov, ${ }^{10}$ K. Senyo, ${ }^{19}$ M. E. Sevior, ${ }^{18}$ H. Shibuya, ${ }^{37}$ B. Shwartz, ${ }^{1}$ V. Sidorov, ${ }^{1}$ J. B. Singh,,${ }^{28}$ N. Soni, ${ }^{28}$ S. Stanič, ${ }^{45, *}$ M. Starič, ${ }^{11}$ A. Sugiyama, ${ }^{31}$ T. Sumiyoshi, ${ }^{42}$ S. Y. Suzuki, ${ }^{6}$ O. Tajima, ${ }^{39}$ F. Takasaki, ${ }^{6}$ K. Tamai, ${ }^{6}$ M. Tanaka, ${ }^{6}$ Y. Teramoto, ${ }^{26}$ T. Tomura, ${ }^{40}$ T. Tsuboyama, ${ }^{6}$ T. Tsukamoto, ${ }^{6}$ S. Uehara, ${ }^{6}$ K. Ueno, ${ }^{22}$ T. Uglov, ${ }^{10}$ Y. Unno, ${ }^{2}$ S. Uno, ${ }^{6}$ G. Varner, ${ }^{5}$ K. E. Varvell, ${ }^{35}$ C. C. Wang, ${ }^{22}$ C. H. Wang, ${ }^{21}$ J. G. Wang, ${ }^{46}$ M.-Z. Wang, ${ }^{22}$ M. Watanabe, ${ }^{25}$ Y. Watanabe, ${ }^{41}$ B. D. Yabsley, ${ }^{46}$ Y. Yamada, ${ }^{6}$ A. Yamaguchi, ${ }^{39}$ H. Yamamoto, ${ }^{39}$ Y. Yamashita, ${ }^{24}$ M. Yamauchi, ${ }^{6}$ H. Yanai, ${ }^{25}$ Heyoung Yang, ${ }^{33}$ Y. Yuan, ${ }^{8}$ Y. Yusa, ${ }^{39}$ J. Zhang, ${ }^{6}$ Z. P. Zhang, ${ }^{32}$ V. Zhilich, ${ }^{1}$ and D. Žontar ${ }^{16,11}$

(Belle Collaboration)
${ }^{1}$ Budker Institute of Nuclear Physics, Novosibirsk
${ }^{2}$ Chiba University, Chiba
${ }^{3}$ University of Cincinnati, Cincinnati, Ohio 45221
${ }^{4}$ Gyeongsang National University, Chinju
${ }^{5}$ University of Hawaii, Honolulu, Hawaii 96822
${ }^{6}$ High Energy Accelerator Research Organization $($ KEK), Tsukuba
${ }^{7}$ Hiroshima Institute of Technology, Hiroshima
${ }^{8}$ Institute of High Energy Physics, Chinese Academy of Sciences, Beijing
9 Institute of High Energy Physics, Vienna
${ }^{10}$ Institute for Theoretical and Experimental Physics, Moscow
${ }_{11}^{11}$ J. Stefan Institute, Ljubljana
${ }^{12}$ Kanagawa University, Yokohama
${ }^{13}$ Korea University, Seoul
${ }^{14}$ Kyungpook National University, Taegu
${ }^{15}$ Swiss Federal Institute of Technology of Lausanne, EPFL, Lausanne
${ }^{16}$ University of Ljubljana, Ljubljana
${ }^{17}$ University of Maribor, Maribor
${ }^{18}$ University of Melbourne, Victoria
${ }^{19}$ Nagoya University, Nagoya
${ }^{20}$ Nara Women's University, Nara
${ }^{21}$ National Lien-Ho Institute of Technology, Miao Li
${ }^{22}$ Department of Physics, National Taiwan University, Taipei
${ }^{23}$ H. Niewodniczanski Institute of Nuclear Physics, Krakow
${ }^{24}$ Nihon Dental College, Niigata
${ }^{25}$ Niigata University, Niigata
${ }^{26}$ Osaka City University, Osaka
${ }^{27}$ Osaka University, Osaka
${ }^{28}$ Panjab University, Chandigarh
${ }^{29}$ Peking University, Beijing
${ }^{30}$ Princeton University, Princeton, New Jersey 08545

${ }^{31}$ Saga University, Saga<br>${ }^{32}$ University of Science and Technology of China, Hefei<br>${ }^{33}$ Seoul National University, Seoul<br>${ }^{34}$ Sungkyunkwan University, Suwon<br>${ }^{35}$ University of Sydney, Sydney New South Wales<br>${ }^{36}$ Tata Institute of Fundamental Research, Bombay<br>${ }^{37}$ Toho University, Funabashi<br>${ }^{38}$ Tohoku Gakuin University, Tagajo<br>${ }^{39}$ Tohoku University, Sendai<br>${ }^{40}$ Department of Physics, University of Tokyo, Tokyo<br>${ }^{41}$ Tokyo Institute of Technology, Tokyo<br>${ }^{42}$ Tokyo Metropolitan University, Tokyo<br>${ }^{43}$ Tokyo University of Agriculture and Technology, Tokyo<br>${ }^{44}$ Toyama National College of Maritime Technology, Toyama<br>${ }^{45}$ University of Tsukuba, Tsukuba<br>${ }^{46}$ Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061<br>${ }^{47}$ Yonsei University, Seoul

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#### Abstract

We report the measurement of an inclusive partial branching fraction for charmless semileptonic $B$ decay and the extraction of $\left|V_{u b}\right|$. Candidates for $B \rightarrow X_{u} \ell \nu$ are identified with a novel $X_{u}$ reconstruction method based on neutrino reconstruction via missing 4-momentum and a technique called "simulated annealing." Based on $86.9 \mathrm{fb}^{-1}$ of data taken with the Belle detector, we obtain $\Delta \mathcal{B}\left(B \rightarrow X_{u} \ell \nu ; M_{X}<1.7 \mathrm{GeV} / c^{2}, q^{2}>8.0 \mathrm{GeV}^{2} / c^{2}\right)=[7.37 \pm 0.89$ (stat) $\pm 1.12$ (syst) $\pm$ $0.55(b \rightarrow c) \pm 0.24(b \rightarrow u)] \times 10^{-4}$ and determine $\left|V_{u b}\right|=[4.66 \pm 0.28($ stat $) \pm 0.35($ syst $) \pm 0.17(b \rightarrow$ c) $\pm 0.08(b \rightarrow u) \pm 0.58($ theory $)] \times 10^{-3}$.


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The off-diagonal element $V_{u b}$ in the Cabibbo-Kobayashi-Maskawa (CKM) matrix plays an important role in $C P$ violation and rare decays of the $B$ meson. It is an important ingredient in overconstraining the unitarity triangle by measuring its sides and angles. In the experiments on the $Y(4 S)$ resonance, its magnitude is extracted from measurements of the $B \rightarrow X_{u} \ell \nu$ process in the limited region of lepton momentum [1] or the hadronic recoil mass $M_{X}$ [2] where the contribution of background from the $B \rightarrow X_{c} \ell \nu$ process is suppressed. These experiments achieve more precise measurements than Large Electron Positron (LEP) collider experiments [3] due to higher signal purity; however, the need to extrapolate measured rates from such limited regions results in large theoretical uncertainties on $\left|V_{u b}\right|$. A recent theoretical development suggests that one can significantly reduce the theoretical uncertainty on the extrapolation by applying simultaneous cuts on $M_{X}$ and the invariant mass squared of the lepton-neutrino system ( $q^{2}$ ) in inclusive $B \rightarrow X_{u} \ell \nu$ [4]. We report here the first result with simultaneous requirements on $M_{X}$ and $q^{2}$. The result is obtained with a novel $X_{u}$ reconstruction method based on a combination of neutrino reconstruction and a technique called simulated annealing [5] to separate the two $B$ meson decays. This method allows us to measure $M_{X}$ and $q^{2}$ with good efficiency so that it achieves good statistical precision and small theoretical uncertainty with a modest integrated luminosity. This analysis is based on $78.1 \mathrm{fb}^{-1}$, corresponding to $85 \times 10^{6} B \bar{B}$ pairs, taken at the $Y(4 S)$
resonance, and $8.8 \mathrm{fb}^{-1}$ taken at an energy 60 MeV below the resonance, by the Belle detector [6] at the energyasymmetric $e^{+} e^{-}$collider KEKB [7].

We select hadronic events containing one lepton candidate (electron or muon) having momentum above $1.2 \mathrm{GeV} / c$ in the center-of-mass system (c.m. system) of the $Y(4 S)$. To remove events with more than one neutrino, we exclude events containing additional lepton candidates $\left(p_{e}^{*}>0.5 \mathrm{GeV} / c\right.$ and $p_{\mu}^{*}>0.8 \mathrm{GeV} / c$ ). The neutrino is reconstructed from the missing 4-momentum in the event $\left(\vec{p}_{\nu} \equiv \vec{p}_{\Upsilon(4 S)}-\Sigma_{i} \vec{p}_{i}, E_{\nu} \equiv E_{\Upsilon(4 S)}-\Sigma_{i} E_{i}\right)$. The net observed momentum ( $\Sigma_{i} \vec{p}_{i}$ ) and energy ( $\Sigma_{i} E_{i}$ ) are calculated using particles surviving track quality cuts based on the impact parameter to the interaction point and a shower energy cut. Pairs of pions and electrons passing secondary vertex criteria are treated as $K_{S}^{0}$ and photons, respectively. All remaining charged tracks are classified as kaons, pions, or protons, based on particle identification information. $K_{L}^{0}$ candidates are identified from isolated clusters of hits in the detector for $K_{L}^{0} \mathrm{~s}$ and muons [8]. The energy of each particle candidate is calculated based on its momentum and mass assignment. We then compute the missing mass of the event, defined as $M M^{2} \equiv E_{\nu}^{2} / c^{4}-\left|\vec{p}_{\nu}\right|^{2} / c^{2}$, where the sign of $E_{\nu}^{2}$ is reversed when $E_{\nu}<0$. We require $-1.5<M M^{2}<$ $1.5 \mathrm{GeV}^{2} / c^{4}$ to suppress events with missing particles and with particles removed due to poor reconstruction quality. For events that pass this requirement, we add back tracks and clusters rejected earlier due to reconstruction
quality, selecting the combination that gives the smallest value of $\left|M M^{2}\right|$. This determines the set of particles that are used in the subsequent analysis. Events are further required to have a net charge of 0 or $\pm 1$, and a polar angle for the missing momentum within the barrel region ( $32^{\circ}<\theta<128^{\circ}$ ). To suppress beam-gas events, we demand that the net charge of all proton candidates be zero. Requiring that the cosine of the angle of $K_{L}^{0}$ candidates with respect to the missing momentum be less than 0.8 rejects events where the neutrino candidate is actually a $K_{L}^{0}$ meson.

We then seek the most likely combination of particles belonging to $X \ell \nu$, the remainder being from the associated $B$ meson ( $B_{\text {opp }}$ ). Six discriminant variables are used: the momentum, energy, and polar angle of $B_{\text {opp }}\left(p_{B}^{*}, E_{B}^{*}\right.$, $\left.\cos \theta_{B}^{*}\right)$ in the c.m. system, its charge multiplicity ( $N_{\mathrm{ch}}$ ) and net charge times the lepton charge ( $Q_{B} \times Q_{\ell}$ ), and the missing mass squared recalculated with the energy and mass of $B_{\text {opp }}$ constrained to the known values ( $M M_{X(\nu}^{2}$ ). Using Monte Carlo (MC) simulation events for $\mathrm{Y}(4 S) \rightarrow B \bar{B}$ where at least one $B$ decays into $X \ell \nu$, we determine probability density functions (PDFs) for correct $X \ell \nu$ combinations and for random $X \ell \nu$ combinations. Random candidates for $X \ell \nu$ consist of the lepton and neutrino candidates plus particles from the remainder of the event, selected randomly so that the relative multiplicities between $X$ and the remainder of the event matches that at the generator level. From the PDFs we calculate two likelihoods, $\mathcal{L}$ (correct) and $\mathcal{L}$ (random). The most likely candidate combination in each event is found by minimizing the parameter $W \equiv$ $\mathcal{L}($ random $) /[\mathcal{L}($ random $)+\mathcal{L}($ correct $)]$.

To minimize $W$, we have developed an approximate iterative algorithm based on simulated annealing. We start from the initial candidate for $X \ell \nu$ that consists of the lepton and neutrino plus approximately one-third of the remaining particles, selected randomly. We move a


FIG. 1. Distributions for three discriminants and $W$ before (dashed histogram) and after (solid histogram) simulated annealing, for real data. Distributions for three discriminants for correct combination of particles for MC events (solid curve).
random particle (other than the lepton or neutrino) between the $X$ and $B_{\text {opp }}$ sides in an iterative way, where in one iteration we cross all particles at least once, and search for the combination that gives the minimum $W$ with 50 iterations. During the iteration process we take special care to reduce the chance of convergence to a local minimum of $W$. For instance, after every fifth iteration we compare all combinations that can be constructed by crossing one particle and use the combination that gives the largest value of $W$ to seed a new cycle. We repeat this iteration process 10 times, starting each time with a different initial candidate, and select the case with the smallest $W$. Figure 1 shows the distributions in three of the six discriminant variables and $W$, before and after simulated annealing. Also shown are the distributions for the correct combination in signal MC events.

The final candidate is required to satisfy (i) $W<0.1$; (ii) $5.1<E_{B}^{*}<5.4 \mathrm{GeV}$; (iii) $0.25<p_{B}<0.42 \mathrm{GeV} / c$; (iv) $-2<Q_{\ell} \times Q_{B}<+1$; and (v) $-0.2<M M_{X \ell_{\nu}}^{2}<$ $0.4 \mathrm{GeV}^{2} / c^{4}$. Contamination from the continuum is reduced by demanding $\left|\cos \theta_{B \ell}\right|<0.8$, where $\theta_{B \ell}$ is the angle between the thrust axis of $B_{\text {opp }}$ and the lepton momentum. Figure 2 shows the resolutions in $M_{X}$ and $q^{2}$ for $B \rightarrow X_{u} \ell \nu \mathrm{MC}$ events. Also shown in the figure are the resolutions for the correct combination of particles.

The validity of the method is checked with two data samples, one containing 38600 fully reconstructed $B \rightarrow$ $D^{*} \ell \nu$ decays and the other containing $84100 B \rightarrow J / \psi X$, $J / \psi \rightarrow \ell^{+} \ell^{-}$decays. These data samples are also used to calibrate the detection efficiency. For the $J / \psi X$ sample we treat one of the two leptons from $J / \psi$ as a neutrino, to emulate $B \rightarrow X \ell \nu$. Corresponding MC events are generated with the QQ event generator [9] and the detector response is simulated using GEANT3 [10]. Figure 3 shows the $M_{X}$ distribution for the $D^{*} \ell \nu$ sample and the $q^{2}$ distribution for the $J / \psi X$ sample, with MC distributions scaled to the number of background-subtracted events in the respective data samples. The peaks are at the $D^{*}$ mass and $J / \psi$ mass squared, as expected, and the shapes are in good agreement between data and MC. Although these results verify that the simulated annealing method works


FIG. 2. $M_{X}$ (a) and $q^{2}$ (b) resolution distributions for $B \rightarrow$ $X_{u} \ell \nu$ MC events. Histograms (curves) show the results with the simulated annealing method (with correct particle assignment to $X_{u}$ ).


FIG. 3. (a) $M_{X}$ distributions for $B \rightarrow D^{*} \ell \nu$ control samples. (b) $q^{2}$ distributions for $B \rightarrow J / \psi X$ control samples. Points are data and the histogram is MC.
as expected, we observe a small difference of efficiency between the data and MC. By averaging the results of the two data samples, we obtain the efficiency ratio $r_{\text {eff }}=$ $0.891 \pm 0.043$ between the data and MC.

We observe 8910 events in the $X_{u} \ell \nu$ "signal" region, defined as $M_{X}<1.7 \mathrm{GeV} / c^{2}$ and $q^{2}>8.0 \mathrm{GeV}^{2} / c^{2}$. These consist of semileptonic decays, $B \rightarrow X_{c, u} \ell \nu$, other $B \bar{B}$ background, and residual continuum events. The continuum contribution is estimated from off-resonance data to be $251 \pm 48$ events and is subtracted directly from the analyzed distributions. The contributions from $B \rightarrow X_{c} \ell \nu$ and other $B \bar{B}$ backgrounds are estimated via MC in the "background" region $M_{X}>1.8 \mathrm{GeV} / c^{2}$, where $X_{c} \ell \nu$ dominates, and extrapolated to the signal region. We estimate them by fitting the $M_{X}$ and $q^{2}$ distributions from MC events to those from the data using a twodimensional $\chi^{2}$ fit method. Contributions to $X_{c} \ell \nu$ come
from $D^{(*)} \ell \nu, D^{* *} \ell \nu$, and $D^{(*)} \pi \ell \nu$. Their branching fractions are floated in the fit. The total rate for other $B \bar{B}$ backgrounds, arising from sources such as $b \rightarrow c \rightarrow s \ell \nu$ and fake leptons, which amounts to less than $1 \%$ of events in the signal region, is floated. The small $X_{u} \ell \nu$ contribution is estimated iteratively and is found to be $(0.94 \pm$ $0.04) \%$ of events in the background region. The obtained branching fractions for the exclusive $B \rightarrow X_{c} \ell \nu$ modes are consistent with the Particle Data Group values [11]. The $B \bar{B}$ background in the signal region is estimated to be $7283 \pm 130 \pm 63$ events, where the first and second errors come from fit and MC statistics, respectively. The upper plots in Fig. 4 show the $M_{X}$ distribution for $q^{2}>8.0 \mathrm{GeV}^{2} / c^{2}$ and the $q^{2}$ distribution for $M_{X}<$ $1.7 \mathrm{GeV} / c^{2}$ after continuum subtraction. After subtracting the $B \bar{B}$ backgrounds, we obtain distributions for the $X_{u} \ell \nu$ signal, shown in the lower plots. The net


FIG. 4. (a) $M_{X}$ distribution for $q^{2}>8.0 \mathrm{GeV}^{2} / c^{2}$. (b) $q^{2}$ distribution for $M_{X}<1.7 \mathrm{GeV} / c^{2}$. Points are the data and histograms are backgrounds from $D^{*} \ell \nu$ (dotted line), $D \ell \nu$ (short dashed line), others (long dashed line), and total background contribution (solid line). Lower plots show the data after background subtraction. Solid curves show the inclusive MC predictions for $B \rightarrow X_{u} \ell \nu$.
signal is $N_{\text {obs }}=1376 \pm 167$ events where the error is statistical only.

In order to extract the partial branching fraction $\Delta \mathcal{B}$ for $B \rightarrow X_{u} \ell \nu$ in the signal region, a Monte Carlo simulation is used to convert $N_{\text {obs }}$ to the true number of signal events produced in this region, $N_{\text {true }}$, and to estimate the efficiency for these events to be observed anywhere, $\boldsymbol{\epsilon}_{\text {signal }}$. In the MC simulation, $B \rightarrow X_{u} \ell \nu$ decays are simulated based on the prescription of Ref. [12]. That analytic result gives $\mathcal{O}\left(\alpha_{s}\right)$ corrections to leading order in the heavy-quark expansion for the triple differential $B \rightarrow$ $X_{u} \ell \nu$ rate and includes the effect of the $b$-quark's Fermi motion. Two parameters therein, the $b$-quark pole mass, $m_{b}$, and the average momentum squared of the $b$ quark inside the $B$ meson, $\mu_{\pi}^{2}$, are derived from the CLEO measurements of the hadronic mass moments in inclusive $B \rightarrow X_{c} \ell \nu$ and photon energy spectrum in $B \rightarrow X_{s} \gamma$ [13]. We use $m_{b}=4.80 \pm 0.12 \mathrm{GeV} / c^{2}$ and $\mu_{\pi}^{2}=0.30 \pm$ $0.11 \mathrm{GeV}^{2} / c^{2}$, which differs from CLEO's evaluation in that terms proportional to $1 / m_{b}^{3}$ and $\alpha_{s}^{2}$ have been removed from the relation between the measured observables and $m_{b}$ and $\mu_{\pi}^{2}$. The MC events are generated with the EVTGEN generator [14]. $N_{\text {true }}$ is estimated by $N_{\text {true }}=$ $N_{\text {obs }} \times F\left(F=1+N_{2} / N_{1}-N_{3} / N_{1}\right)$. Here $N_{1}$ is the number of events observed in the signal region and $N_{2}$ $\left(N_{3}\right)$ is the number of events generated inside (outside) the signal region and observed outside (inside) the signal region. We find $F=0.938$, and thus $N_{\text {true }}=1291 \pm 157$. The efficiency $\boldsymbol{\epsilon}_{\text {signal }}$ is predicted to be $0.578 \%$. We determine $\Delta \mathcal{B}$ by $0.5 \times N_{\text {true }} /\left(\epsilon_{\text {signal }} \times r_{\text {eff }}\right) /\left(2 N_{B}\right)$, where $r_{\text {eff }}$ is the efficiency correction factor described earlier, $N_{B}$ is the number of $B \bar{B}$ events and the factor 0.5 is needed to take into account the electron and muon data:

$$
\Delta \mathcal{B}=(7.37 \pm 0.89 \pm 1.12 \pm 0.55 \pm 0.24) \times 10^{-4}
$$

The errors are statistical, systematic, from $B \rightarrow X_{c} \ell \nu$ model dependence, and $B \rightarrow X_{u} \ell \nu$ model dependence, respectively. Sources of systematic uncertainty include signal MC statistics (1.8\%), lepton identification (2.6\%), uncertainty of $F$ due to imperfect detector simulation ( $1.2 \%$ ), selection and reconstruction efficiency ( $4.9 \%$ ), and $B \bar{B}$ background estimation ( $14.0 \%$ ). The uncertainty in the $B \bar{B}$ background estimation is from MC statistics (4.6\%) and distortion of the $M_{X}$ and $q^{2}$ distributions due to imperfect detector simulation (13.2\%). Major contributions to the last error are from $K_{L}^{0}$ contamination (8.6\%), electromagnetic cluster finding efficiency ( $8.2 \%$ ), lepton efficiency ( $3.2 \%$ ), clusters produced by charged tracks ( $2.9 \%$ ), lepton fake rate ( $2.7 \%$ ), and $K / \pi$ separation ( $2.7 \%$ ). The error from $K_{L}^{0}$ contamination is estimated using inclusive $K_{S}^{0}$ events where we discard $K_{S}^{0} \mathrm{~s}$ to emulate inclusive $K_{L}^{0}$ events. The error from the cluster finding efficiency is estimated by reducing the photon-finding efficiency within its uncertainty. The model dependence of $X_{c} \ell \nu$ is estimated to be $7.4 \%$ by
varying the $D_{1} \ell \nu$ plus $D_{2}^{*} \ell \nu$ fraction in the $D^{* *} \ell \nu$ by $25 \%$ and by varying the slope parameters of the form factors for $D \ell \nu$ and $D^{*} \ell \nu, \rho_{D}^{2}=1.19 \pm 0.19$ and $\rho^{2}=1.51 \pm$ 0.13 [11], within their errors. $B \rightarrow X_{u} \ell \nu$ model dependence (3.4\%) is estimated by varying the parameters of the inclusive model within their errors and by comparing to a simulation with a full exclusive implementation of the ISGW2 model [15].

In the context of heavy quark effective theory (HQET) and operator product expansion (OPE) the partial branching fraction $\Delta \mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)$ is related to $\left|V_{u b}\right|[4,16,17]$,

$$
\left|V_{u b}\right|=0.00444\left(\frac{\Delta \mathcal{B}\left(B \rightarrow X_{u} \ell \nu\right)}{0.002 \times 1.21 G\left(q_{\mathrm{cut}}^{2}, m_{\mathrm{cut}}\right)} \frac{1.55 \mathrm{ps}}{\tau_{B}}\right)^{1 / 2}
$$

where $1.21 G\left(q_{\text {cut }}^{2}, m_{\text {cut }}\right)=f_{\mathrm{HQET}} \times\left[m_{b}^{1 S} /\left(4.7 \mathrm{GeV} / c^{2}\right)\right]^{5}$, $f_{\mathrm{HQET}}$ represents the fraction of events with $q^{2}>q_{\text {cut }}^{2}$ and $M_{X}<m_{\text {cut }}$, and $m_{b}^{1 S}$ is one-half of the perturbative contribution to the mass of the $\Upsilon(1 S) . G\left(q_{\mathrm{cut}}^{2}, m_{\text {cut }}\right)$ is calculated to $\mathcal{O}\left(\alpha_{s}^{2}\right)$ and $\mathcal{O}\left(1 / m_{b}^{2}\right)$ in [4], including the effect of the Fermi motion of the $b$ quark, which is expressed in terms of $m_{b}^{1 S}$. We use $m_{b}^{1 S}=4.70 \pm 0.12 \mathrm{GeV} / c^{2}[4,18]$, which gives $G\left(q_{\text {cut }}^{2}, m_{\text {cut }}\right)=0.268$ [4,17]. The theoretical uncertainty on $\left|V_{u b}\right|$ is determined only by the uncertainty on $G\left(q_{\mathrm{cut}}^{2}, m_{\mathrm{cut}}\right)$. The uncertainty on $G\left(q_{\mathrm{cut}}^{2}, m_{\mathrm{cut}}\right)$, in total $25 \%$, consists of $6 \%$ for perturbative terms, $8 \%$ for nonperturbative terms (dominated by the weak annihilation contribution), and $23 \%$ from the uncertainty on $m_{b}^{1 S}$ [4,19]. The $23 \%$ error contains $10 \%$ for $f_{\text {HQET }}$ and $13 \%$ for $\left(m_{b}^{1 S}\right)^{5}$. These uncertainties are positively correlated, so we add them linearly, whereas they have been given separately in conventional analyses. Using $\tau_{B}=$ $1.604 \pm 0.012 \mathrm{ps}$ [11], we obtain

$$
\left|V_{u b}\right|=(4.66 \pm 0.28 \pm 0.35 \pm 0.17 \pm 0.08 \pm 0.58) \times 10^{-3}
$$

where the errors are statistical, systematic, $b \rightarrow c$ model dependence, $b \rightarrow u$ model dependence, and theoretical uncertainty for OPE, respectively.

To summarize, we have performed the first measurement of $\left|V_{u b}\right|$ with simultaneous requirements on $M_{X}$ and $q^{2}$ using a novel $X_{u}$ reconstruction method. The result of $\left|V_{u b}\right|=(4.66 \pm 0.76) \times 10^{-3}$ is consistent with the previous inclusive measurements [1-3], and the total error is comparable with those of the previous measurements on $\Upsilon(4 S)[1,2]$. Because of simultaneous requirements on $M_{X}$ and $q^{2}$, the $f_{\mathrm{HQET}}$ error is much smaller than those of the previous measurements on $Y(4 S)$ [1,2].

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