## Evidence for $B \rightarrow \boldsymbol{\phi} \boldsymbol{\phi} K$

H.-C. Huang, ${ }^{25}$ K. Abe, ${ }^{7}$ K. Abe, ${ }^{40}$ T. Abe, ${ }^{41}$ I. Adachi, ${ }^{7}$ H. Aihara,,${ }^{42}$ M. Akatsu, ${ }^{21}$ T. Aso, ${ }^{46}$ V. Aulchenko, ${ }^{2}$ T. Aushev, ${ }^{11}$ A. M. Bakich, ${ }^{37}$ Y. Ban, ${ }^{32}$ E. Banas, ${ }^{26}$ A. Bay, ${ }^{17}$ I. Bedny, ${ }^{2}$ P. K. Behera, ${ }^{47}$ I. Bizjak, ${ }^{12}$ A. Bondar, ${ }^{2}$ A. Bozek, ${ }^{26}$ M. Bračko, ${ }^{19,12}$ T. E. Browder, ${ }^{6}$ B. C. K. Casey, ${ }^{6}$ P. Chang, ${ }^{25}$ Y. Chao, ${ }^{25}$ K.-F. Chen, ${ }^{25}$ B. G. Cheon, ${ }^{36}$ R. Chistov, ${ }^{11}$ Y. Choi, ${ }^{36}$ Y. K. Choi, ${ }^{36}$ M. Danilov, ${ }^{11}$ L. Y. Dong, ${ }^{9}$ A. Drutskoy, ${ }^{11}$ S. Eidelman, ${ }^{2}$ V. Eiges, ${ }^{11}$ Y. Enari, ${ }^{21}$ C. Fukunaga, ${ }^{44}$ N. Gabyshev, ${ }^{7}$ A. Garmash,,${ }^{2,7}$ T. Gershon, ${ }^{7}$ B. Golob, ${ }^{18,12}$ R. Guo, ${ }^{23}$ J. Haba, ${ }^{7}$ F. Handa, ${ }^{41}$ T. Hara, ${ }^{30}$ N. C. Hastings, ${ }^{7}$ H. Hayashii,,$^{22}$ M. Hazumi, ${ }^{7}$ L. Hinz, ${ }^{17}$ T. Hokuue, ${ }^{21}$ Y. Hoshi, ${ }^{40}$ W.-S. Hou, ${ }^{25}$ Y. B. Hsiung, ${ }^{25, *}$ Y. Igarashi, ${ }^{7}$ T. Iijima, ${ }^{21}$ K. Inami, ${ }^{21}$ A. Ishikawa, ${ }^{21}$ R. Itoh, ${ }^{7}$ Y. Iwasaki, ${ }^{7}$ H. K. Jang, ${ }^{35}$ J. H. Kang, ${ }^{50}$ J. S. Kang, ${ }^{14}$ P. Kapusta, ${ }^{26}$ N. Katayama, ${ }^{7}$ H. Kawai, ${ }^{3}$ N. Kawamura, ${ }^{1}$ T. Kawasaki, ${ }^{28}$ H. Kichimi, ${ }^{7}$ D.W. Kim, ${ }^{36}$ H. J. Kim, ${ }^{50}$ Hyunwoo Kim, ${ }^{14}$ J. H. Kim, ${ }^{36}$ K. Kinoshita, ${ }^{5}$ P. Koppenburg, ${ }^{7}$ S. Korpar, ${ }^{19,12}$ P. Križan, ${ }^{18,12}$ P. Krokovny, ${ }^{2}$ R. Kulasiri, ${ }^{5}$ S. Kumar, ${ }^{31}$ Y.-J. Kwon, ${ }^{50}$ G. Leder, ${ }^{10}$ S. H. Lee, ${ }^{35}$ T. Lesiak, ${ }^{26}$ J. Li, ${ }^{34}$ A. Limosani, ${ }^{20}$ S.-W. Lin, ${ }^{25}$ D. Liventsev, ${ }^{11}$ J. MacNaughton, ${ }^{10}$ G. Majumder, ${ }^{38}$ F. Mandl, ${ }^{10}$ D. Marlow, ${ }^{33}$ H. Matsumoto, ${ }^{28}$ T. Matsumoto, ${ }^{44}$ W. Mitaroff, ${ }^{10}$ H. Miyata, ${ }^{28}$ G. R. Moloney, ${ }^{20}$ T. Mori, ${ }^{4}$ T. Nagamine, ${ }^{41}$ Y. Nagasaka, ${ }^{8}$ T. Nakadaira, ${ }^{42}$ E. Nakano, ${ }^{29}$ M. Nakao, ${ }^{7}$ H. Nakazawa, ${ }^{7}$ J.W. Nam, ${ }^{36}$ Z. Natkaniec,,${ }^{26}$ S. Nishida, ${ }^{15}$ O. Nitoh, ${ }^{45}$ T. Nozaki, ${ }^{7}$ S. Ogawa, ${ }^{39}$ T. Ohshima, ${ }^{21}$ T. Okabe, ${ }^{21}$ S. Okuno, ${ }^{13}$ S. L. Olsen, ${ }^{6}$ W. Ostrowicz, ${ }^{26}$ H. Ozaki, ${ }^{7}$ H. Palka, ${ }^{26}$ C.W. Park, ${ }^{14}$ H. Park, ${ }^{16}$ K. S. Park, ${ }^{36}$ N. Parslow, ${ }^{37}$ J.-P. Perroud, ${ }^{17}$ M. Peters, ${ }^{6}$ L. E. Piilonen, ${ }^{48}$ M. Rozanska, ${ }^{26}$ H. Sagawa, ${ }^{7}$ S. Saitoh, ${ }^{7}$ Y. Sakai, ${ }^{7}$ T. R. Sarangi, ${ }^{47}$ A. Satpathy, ${ }^{7,5}$ O. Schneider, ${ }^{17}$ J. Schümann, ${ }^{25}$ C. Schwanda, ${ }^{7,10}$ A. J. Schwartz, ${ }^{5}$ T. Seki, ${ }^{44}$ S. Semenov, ${ }^{11}$ M. E. Sevior, ${ }^{20}$ T. Shibata, ${ }^{28}$ H. Shibuya, ${ }^{39}$ V. Sidorov, ${ }^{2}$ J. B. Singh,,${ }^{31}$ S. Stanič ${ }^{7, \dagger}$ M. Starič, ${ }^{12}$ A. Sugi, ${ }^{21}$ K. Sumisawa, ${ }^{7}$ T. Sumiyoshi, ${ }^{44}$ S. Suzuki, ${ }^{49}$ S. Y. Suzuki, ${ }^{7}$ T. Takahashi,,$^{29}$ F. Takasaki, ${ }^{7}$ K. Tamai, ${ }^{7}$ N. Tamura, ${ }^{28}$ J. Tanaka, ${ }^{42}$ M. Tanaka, ${ }^{7}$ G. N. Taylor, ${ }^{20}$ Y. Teramoto, ${ }^{29}$ T. Tomura, ${ }^{42}$ S. N. Tovey, ${ }^{20}$ K. Trabelsi, ${ }^{6}$ T. Tsuboyama, ${ }^{7}$ T. Tsukamoto, ${ }^{7}$ S. Uehara, ${ }^{7}$ S. Uno, ${ }^{7}$ G. Varner, ${ }^{6}$ K. E. Varvell, ${ }^{37}$ C. C. Wang, ${ }^{25}$ C. H. Wang, ${ }^{24}$ J. G. Wang, ${ }^{48}$ M.-Z. Wang, ${ }^{25}$ Y. Watanabe, ${ }^{43}$ E. Won, ${ }^{14}$ B. D. Yabsley, ${ }^{48}$ Y. Yamada, ${ }^{7}$ A. Yamaguchi, ${ }^{41}$ Y. Yamashita, ${ }^{27}$ M. Yamauchi, ${ }^{7}$ H. Yanai, ${ }^{28}$ Heyoung Yang, ${ }^{35}$ Y. Yusa, ${ }^{41}$ C. C. Zhang, ${ }^{9}$ Z. P. Zhang, ${ }^{34}$ Y. Zheng, ${ }^{6}$ V. Zhilich, ${ }^{2}$ and D. Žontar ${ }^{18,12}$
(Belle Collaboration)

${ }^{1}$ Aomori University, Aomori<br>${ }^{2}$ Budker Institute of Nuclear Physics, Novosibirsk<br>${ }^{3}$ Chiba University, Chiba<br>${ }^{4}$ Chuo University, Tokyo<br>${ }^{5}$ University of Cincinnati, Cincinnati, Ohio 45221<br>${ }^{6}$ University of Hawaii, Honolulu, Hawaii 96822<br>${ }^{7}$ High Energy Accelerator Research Organization (KEK), Tsukuba<br>${ }^{8}$ Hiroshima Institute of Technology, Hiroshima<br>${ }^{9}$ Institute of High Energy Physics, Chinese Academy of Sciences, Beijing<br>${ }^{10}$ Institute of High Energy Physics, Vienna<br>${ }^{11}$ Institute for Theoretical and Experimental Physics, Moscow<br>${ }^{12}$ J. Stefan Institute, Ljubljana<br>${ }^{13}$ Kanagawa University, Yokohama<br>${ }^{14}$ Korea University, Seoul<br>${ }^{15}$ Kyoto University, Kyoto<br>${ }^{16}$ Kyungpook National University, Taegu<br>${ }^{17}$ Institut de Physique des Hautes Énergies, Université de Lausanne, Lausanne<br>${ }^{18}$ University of Ljubljana, Ljubljana<br>${ }^{19}$ University of Maribor, Maribor<br>${ }^{20}$ University of Melbourne, Victoria<br>${ }^{21}$ Nagoya University, Nagoya<br>${ }^{22}$ Nara Women's University, Nara<br>${ }^{23}$ National Kaohsiung Normal University, Kaohsiung<br>${ }^{24}$ National Lien-Ho Institute of Technology, Miao Li<br>${ }^{25}$ Department of Physics, National Taiwan University, Taipei<br>${ }^{26}$ H. Niewodniczanski Institute of Nuclear Physics, Krakow<br>${ }^{27}$ Nihon Dental College, Niigata<br>${ }^{28}$ Niigata University, Niigata

${ }^{29}$ Osaka City University, Osaka<br>${ }^{30}$ Osaka University, Osaka<br>${ }^{31}$ Panjab University, Chandigarh<br>${ }^{32}$ Peking University, Beijing<br>${ }^{33}$ Princeton University, Princeton, New Jersey 08545<br>${ }^{34}$ University of Science and Technology of China, Hefei<br>${ }^{35}$ Seoul National University, Seoul<br>${ }^{36}$ Sungkyunkwan University, Suwon<br>${ }^{37}$ University of Sydney, Sydney New South Wales<br>${ }^{38}$ Tata Institute of Fundamental Research, Bombay<br>${ }^{39}$ Toho University, Funabashi<br>${ }^{40}$ Tohoku Gakuin University, Tagajo<br>${ }^{41}$ Tohoku University, Sendai<br>${ }^{42}$ Department of Physics, University of Tokyo, Tokyo<br>${ }^{43}$ Tokyo Institute of Technology, Tokyo<br>${ }^{44}$ Tokyo Metropolitan University, Tokyo<br>${ }^{45}$ Tokyo University of Agriculture and Technology, Tokyo<br>${ }^{46}$ Toyama National College of Maritime Technology, Toyama<br>${ }^{47}$ Utkal University, Bhubaneswer<br>${ }^{48}$ Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061<br>${ }^{49}$ Yokkaichi University, Yokkaichi<br>${ }^{50}$ Yonsei University, Seoul

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We report evidence for the decay mode $B \rightarrow \phi \phi K$ based on an analysis of $78 \mathrm{fb}^{-1}$ of data collected with the Belle detector at KEKB. This is the first example of a $b \rightarrow s \bar{s} s \bar{s} s$ transition. The branching fraction for this decay is measured to be $\mathcal{B}\left(B^{ \pm} \rightarrow \phi \phi K^{ \pm}\right)=\left(2.6_{-0.9}^{+1.1} \pm 0.3\right) \times 10^{-6}$ for a $\phi \phi$ invariant mass below $2.85 \mathrm{GeV} / c^{2}$. Results for other related charmonium decay modes are also reported.

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We report evidence for the decay mode $B \rightarrow \phi \phi K$, the first example of a $b \rightarrow s \bar{s} s \bar{s} s$ transition. In the standard model (SM), this decay channel requires the creation of an additional final $s \bar{s}$ quark pair than in $b \rightarrow s \bar{s} s$ processes, which have been previously observed in modes such as $B \rightarrow \phi K$. In addition to improving our understanding of charmless $B$ decays, the $\phi \phi K$ state may be sensitive to glueball production in $B$ decays, where the glueball decays to $\phi \phi$ [1]. Furthermore, with sufficient statistics, the decay $B \rightarrow \phi \phi K$ could be used to search for a possible non-SM $C P$-violating phase in the $b \rightarrow s$ transition [2]. Direct $C P$ violation could be enhanced to as high as the $40 \%$ level if there is sizable interference between transitions due to non-SM physics and decays via the $\eta_{c}$ resonance.

We use a $78 \mathrm{fb}^{-1}$ data sample collected with the Belle detector at the KEKB asymmetric-energy $e^{+} e^{-}$(3.5 on 8 GeV ) collider [3] operating at the $\mathrm{Y}(4 S)$ resonance $(\sqrt{s}=10.58 \mathrm{GeV})$. The sample contains $85.0 \times 10^{6}$ produced $B \bar{B}$ pairs. The Belle detector is a large-solid-angle magnetic spectrometer consisting of a three-layer silicon vertex detector, a 50-layer central drift chamber (CDC), a system of aerogel threshold Čerenkov counters (ACC), time-of-flight scintillation counters (TOF), and an array of $\mathrm{CsI}(\mathrm{Tl})$ crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to
identify $K_{L}^{0}$ and muons. The detector is described in detail elsewhere [4].

We select well measured charged tracks that have impact parameters with respect to the nominal interaction point (IP) that are less than 0.2 cm in the radial direction and less than 2 cm along the beam direction $(z)$. Each track is identified as a kaon or a pion according to a $K / \pi$ likelihood ratio, $\mathcal{L}_{K} /\left(\mathcal{L}_{\pi}+\mathcal{L}_{K}\right)$, where $\mathcal{L}_{K(\pi)}$ are likelihoods derived from responses of the TOF and ACC systems and $d E / d x$ measurements in the CDC. We select kaon candidates by requiring $\mathcal{L}_{K} /\left(\mathcal{L}_{\pi}+\mathcal{L}_{K}\right)>0.6$. This requirement has a kaon efficiency varying from $89.6 \pm 1.0 \%$ for momentum of 500 MeV to $86.9 \pm 0.4 \%$ for momentum of 3 GeV and a misidentification rate from pions of $8.5 \%$. Kaon candidates that are electronlike according to the information recorded in the $\operatorname{CsI}(\mathrm{Tl})$ calorimeter are rejected.

Candidate $\phi$ mesons are reconstructed via the $\phi \rightarrow$ $K^{+} K^{-}$decay mode; we require the $K^{+} K^{-}$invariant mass to be within $\pm 20 \mathrm{MeV} / c^{2}( \pm 4.5$ times the full width) of the $\phi$ mass [5]. For the $B^{0}\left(\bar{B}^{0}\right) \rightarrow \phi \phi K_{S}^{0}$ decay mode, we use $K_{S}^{0} \rightarrow \pi^{+} \pi^{-}$candidates in the mass window $482 \mathrm{MeV} / c^{2}<M\left(\pi^{+} \pi^{-}\right)<514 \mathrm{MeV} / c^{2}( \pm 4 \sigma)$, where the distance of closest approach between the two daughter tracks is less than 2.4 cm , the magnitude of the impact parameter of each track in the radial direction exceeds 0.02 cm , and the flight length is greater than
0.22 cm . The difference in the angle between the pionpair vertex direction from the IP and its reconstructed flight direction in the $x-y$ plane is required to be less than 0.03 radians.

To isolate the signal, we form the beam-constrained mass, $M_{\mathrm{bc}}=\sqrt{E_{\text {beam }}^{2}-\left|\vec{P}_{\text {recon }}\right|^{2}}$, and the energy difference $\Delta E=E_{\text {recon }}-E_{\text {beam }}$. Here $E_{\text {beam }}$ is the beam energy, and $E_{\text {recon }}$ and $\vec{P}_{\text {recon }}$ are the reconstructed energy and momentum of the signal candidate, in the $\mathrm{Y}(4 S)$ center-of-mass frame. The signal region for $\Delta E$ is $\pm 30 \mathrm{MeV}$ which corresponds to $\pm 3.1 \sigma$, where $\sigma$ is the resolution determined from a Gaussian fit to the Monte Carlo (MC) simulation and verified using the decay of $B^{+} \rightarrow$ $\bar{D}^{0} \pi^{+}$and $\bar{D}^{0} \rightarrow K^{+} \pi^{-} \pi^{+} \pi^{-}$. The signal region for $M_{\mathrm{bc}}$ is $5.27 \mathrm{GeV} / c^{2}<M_{\mathrm{bc}}<5.29 \mathrm{GeV} / c^{2}$. The beamconstrained mass resolution is $2.8 \mathrm{MeV} / c^{2}$, which is mostly due to the beam energy spread of KEKB.

The major background for the $B \rightarrow \phi \phi K$ process is from continuum $e^{+} e^{-} \rightarrow q \bar{q}$ production, where $q$ is a light quark ( $u, d, s$, or $c$ ). Several event topology variables are used to discriminate the continuum background, which tends to be collimated along the original quark direction, from the $B \bar{B}$ events, which are more isotropic than the former. Five modified Fox-Wolfram moments, the $S_{\perp}$ variable [6], and the cosine of the thrust angle are combined into a Fisher discriminant [7]. We form signal and background probability density functions (PDFs) for this Fisher discriminant and for the cosine of the $B$ decay angle with respect to the $z$ axis $\left(\cos \theta_{B}\right)$ for the signal MC and sideband ( $5.20 \mathrm{GeV} / c^{2}<M_{\mathrm{bc}}<5.26 \mathrm{GeV} / c^{2}$ and $0.1<|\Delta E|<0.2 \mathrm{GeV}$ ) data. The PDFs are multiplied together to form signal and background likelihoods, $\mathcal{L}_{S}$ and $\mathcal{L}_{B G}$. The likelihood ratio $\mathcal{L} \mathcal{R} \equiv \mathcal{L}_{S} /\left(\mathcal{L}_{S}+\mathcal{L}_{B G}\right)$ is then required to be greater than 0.1 . This requirement retains $97 \%$ of the signal while removing $55 \%$ of the continuum background.

Figure 1(a) shows the $\phi \phi$ invariant mass spectrum for events in the $B^{ \pm} \rightarrow \phi \phi K^{ \pm}$signal region, where a clear $\eta_{c}$ peak and some excess in the lower mass region are evident.

To extract signal yields, we apply an unbinned, extended maximum likelihood (ML) fit to the events with $|\Delta E|<0.2 \mathrm{GeV}$ and $M_{\mathrm{bc}}>5.2 \mathrm{GeV} / c^{2}$. The extended likelihood for a sample of $N$ events is $\mathcal{L}=$ $e^{-\left(N_{S}+N_{B}\right)} \prod_{i=1}^{N}\left(N_{S} \mathcal{P}_{i}^{S}+N_{B} \mathcal{P}_{i}^{B}\right)$, where $\mathcal{P}_{i}^{S(B)}$ describes the probability for candidate event $i$ to belong to the signal (background), based on its measured $M_{\mathrm{bc}}$ and $\Delta E$ values. The exponential factor in the likelihood accounts for Poisson fluctuations in the total number of observed events $N$. The signal yield $N_{S}$ and the number of background events $N_{B}$ are obtained by maximizing $\mathcal{L}$. The statistical errors correspond to unit changes in the quantity $\chi^{2}=-2 \ln \mathcal{L}$ around its minimum value. The significance of the signal is defined as the square root of the change in $\chi^{2}$ when constraining the number of signal


FIG. 1 (color online). (a) $\phi \phi$ invariant mass spectrum. The open histogram corresponds to events from the $B^{ \pm} \rightarrow \phi \phi K^{ \pm}$ signal region and the shaded histogram corresponds to events from the $\Delta E$ sidebands. (b) $M_{K^{+} K^{-}}$of one $\phi$ meson candidate versus $M_{K^{+} K^{-}}$of the other for the events satisfying $M_{\phi \phi}<$ $2.85 \mathrm{GeV} / c^{2}$. Dots are for $\phi \phi K^{ \pm}$and squares for $\phi \phi K_{S}^{0}$. Each event is plotted twice for combinations. The dashed box shows the selected signal region.
events to zero in the likelihood fit; it reflects the probability for the background to fluctuate to the observed event yield.

The probability $\mathcal{P}$ for a given event $i$ is calculated as the product of independent PDFs for $M_{\mathrm{bc}}$ and $\Delta E$. The signal PDFs are represented by a Gaussian for $M_{\mathrm{bc}}$ and a double Gaussian for $\Delta E$. The background PDF for $\Delta E$ is a linear function; for the $M_{\mathrm{bc}}$ background we use a phase-space-like function with an empirical shape [8]. The parameters of the PDFs are determined from highstatistics MC samples for the signal and sideband data for the background.

For $M(\phi \phi)<2.85 \mathrm{GeV} / c^{2}$, the region below the charm threshold, the ML fit gives an event yield of $7.3_{-2.5}^{+3.2}$ with a significance of 5.1 standard deviations $(\sigma)$. Projections of the $\Delta E$ distribution (with $5.27 \mathrm{GeV} / c^{2}<M_{\mathrm{bc}}<5.29 \mathrm{GeV} / c^{2}$ ) and of the $M_{\mathrm{bc}}$ distribution (with $|\Delta E|<30 \mathrm{MeV}$ ) are shown in Figs. 2(a) and 2(b). As a consistency check, a ML fit to the projected $\Delta E$ distribution [Fig. 2(b)] gives a signal yield of $7.5_{-2.7}^{+3.3}$ with a $4.8 \sigma$ statistical significance. Figure 1(b) shows a scatter plot of the two $K^{+} K^{-}$invariant masses for events in the $B$ meson signal region with the $\phi$ mass requirements relaxed. Here there is a clear concentration in the overlap region of the two $\phi$ bands. To confirm that the observed signal is from $B^{ \pm} \rightarrow \phi \phi K^{ \pm}$, we apply a tighter $\phi$ mass requirement $\left( \pm 10 \mathrm{MeV} / c^{2}\right)$, which reduces the signal efficiency by $15 \%$, and obtain a signal yield of 5.6 with $4.6 \sigma$ statistical significance. Using a signal efficiency of $3.3 \%$, obtained from a large-statistics MC that uses three-body phase space to model the $B^{ \pm} \rightarrow$ $\phi \phi K^{ \pm}$decays, we determine the branching fraction for charmless $B^{ \pm} \rightarrow \phi \phi K^{ \pm}$with $M_{\phi \phi}<2.85 \mathrm{GeV} / c^{2}$ to be

$$
\mathcal{B}\left(B^{ \pm} \rightarrow \phi \phi K^{ \pm}\right)=\left(2.6_{-0.9}^{+1.1} \pm 0.3\right) \times 10^{-6}
$$

where the first error is statistical and the second is systematic.


FIG. 2 (color online). Projections of $M_{\mathrm{bc}}$ and $\Delta E$ overlaid with the fitted curves for (a),(b) $B^{ \pm} \rightarrow \phi \phi K^{ \pm}$with $M_{\phi \phi}<$ $2.85 \mathrm{GeV} / c^{2}$, (c),(d) $B^{ \pm} \rightarrow \eta_{c} K^{ \pm}$and $\eta_{c} \rightarrow \phi \phi$, (e),(f) $B^{ \pm} \rightarrow$ $\eta_{c} K^{ \pm}$and $\eta_{c} \rightarrow 2\left(K^{+} K^{-}\right)$, and (g),(h) $B^{ \pm} \rightarrow J / \psi K^{ \pm}$and $J / \psi \rightarrow 2\left(K^{+} K^{-}\right)$.

Contributions to the systematic error include the uncertainties due to the tracking efficiency ( $5.4 \%$ ), particle identification efficiency (5\%), and the modeling of the likelihood ratio cut (2\%). The error due to the modeling of the likelihood ratio cut is determined using $B^{-} \rightarrow$ $D^{0}\left(\rightarrow K^{-} \pi^{+} \pi^{-} \pi^{+}\right) \pi^{-}$events in the same data sample; these events have the same number of final-state particles and an event topology that is similar to the $B^{ \pm} \rightarrow \phi \phi K^{ \pm}$ signal. The uncertainty due to the MC $M_{\phi \phi}$ modeling (4\%) accounts for the $M_{\phi \phi}$ dependence of the detection efficiency. The systematic error in the signal yield (6\%) is determined by varying the means and $\sigma$ of the signal and the shape parameters of the background. We determine an upper limit of $5 \%$ on the possible contamination by nonresonant $B^{ \pm} \rightarrow \phi\left(K^{+} K^{-}\right)_{\mathrm{NR}} K^{ \pm}$or $B^{ \pm} \rightarrow 2\left(K^{+} K^{-}\right)_{\mathrm{NR}} K^{ \pm}$ decays by redoing the fits with the $\phi$ mass requirement relaxed. The sources of systematic error are combined in quadrature to obtain the final systematic error of $12 \%$.

For the $B^{0}\left(\bar{B}^{0}\right) \rightarrow \phi \phi K_{S}^{0}$ mode, there are only four signal candidates. We combine the $B^{ \pm} \rightarrow \phi \phi K^{ \pm}$and $B^{0}\left(\bar{B}^{0}\right) \rightarrow \phi \phi K_{S}^{0}$ modes and perform a ML fit and obtain a signal event yield of $8.7_{-2.9}^{+3.6}$ with $5.3 \sigma$ statistical sig-
nificance. Assuming isospin symmetry, we obtain

$$
\mathcal{B}(B \rightarrow \phi \phi K)=\left(2.3_{-0.8}^{+0.9} \pm 0.3\right) \times 10^{-6},
$$

for $M_{\phi \phi}<2.85 \mathrm{GeV} / c^{2}$.
No enhancement is observed in the $M_{\phi \phi}$ region corresponding to the $f_{J}(2220)$ glueball candidate [5], also refered to as $\xi$. Assuming the mass and width of $f_{J}(2220)$ to be $2230 \mathrm{MeV} / c^{2}$ and $20 \mathrm{MeV} / c^{2}$, we define a signal region of $2.19 \mathrm{GeV} / c^{2}<M_{\phi \phi}<2.27 \mathrm{GeV} / c^{2}$, $5.27 \mathrm{GeV} / c^{2}<M_{\mathrm{bc}}<5.29 \mathrm{GeV} / c^{2}$, and $|\Delta E|<30 \mathrm{MeV}$. One event is observed in this region with an expected background, estimated from the sideband, of 0.5 . Using an extended Cousins-Highland method that uses the the Feldman-Cousins ordering scheme and takes systematic uncertainties into account [9], we obtain a $90 \%$ confidence level (C.L.) upper limit of 3.7 signal events, which corresponds to

$$
\mathcal{B}\left(B^{ \pm} \rightarrow f_{J}(2220) K^{ \pm}\right) \times \mathcal{B}\left(f_{J}(2220) \rightarrow \phi \phi\right)<1.2 \times 10^{-6}
$$

We select $B^{ \pm} \rightarrow \eta_{c} K^{ \pm}, \eta_{c} \rightarrow \phi \phi$ candidates by requiring 2.94 $\mathrm{GeV} / c^{2}<M_{\phi \phi}<3.02 \mathrm{GeV} / c^{2}$. This decay has been searched by previous experiments [10]. A clear signal is evident in Figs. 2(c) and 2(d), and the fitted yield of $N_{S}=7.0_{-2.3}^{+3.0}$ events has a significance of $8.8 \sigma$. The corresponding product branching fraction is

$$
\mathcal{B}\left(B^{ \pm} \rightarrow \eta_{c} K^{ \pm}\right) \times \mathcal{B}\left(\eta_{c} \rightarrow \phi \phi\right)=\left(2.2_{-0.7}^{+1.0} \pm 0.5\right) \times 10^{-6}
$$

In addition to the previously listed source of systematic errors, here the error also includes the possible contamination from charmless $B^{ \pm} \rightarrow \phi \phi K^{ \pm}$decays, which is estimated to be less than 1.2 events. Using the measured branching fraction $\mathcal{B}\left(B^{ \pm} \rightarrow \eta_{c} K^{ \pm}\right)=(1.25 \pm 0.42) \times$ $10^{-3}$ [11], we determine the $\eta_{c} \rightarrow \phi \phi$ branching fraction to be

$$
\mathcal{B}\left(\eta_{c} \rightarrow \phi \phi\right)=\left(1.8_{-0.6}^{+0.8} \pm 0.7\right) \times 10^{-3}
$$

which is smaller than the current world average value of $(7.1 \pm 2.8) \times 10^{-3}[5]$.


FIG. 3 (color online). (a) $2\left(K^{+} K^{-}\right)$and (b) $\phi K^{+} K^{-}$invariant mass spectra in the $\eta_{c}$ and $J / \psi$ regions. The open histograms correspond to events from the $B$ signal region, and the shaded histograms correspond to events from the $M_{\mathrm{bc}}-\Delta E$ sidebands.

TABLE I. Signal yields, efficiencies including secondary branching fractions, statistical significances, and branching fractions (or branching fraction products) of $B \rightarrow \phi \phi K$ and the related decays. The branching fractions for modes with $K^{+} K^{-}$pairs include contributions from $\phi \rightarrow K^{+} K^{-}$.

| Mode | Yield | Efficiency $(\%)$ | Significance $(\sigma)$ | $\mathcal{B}\left(\times 10^{-6}\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| $B^{ \pm} \rightarrow \phi \phi K^{ \pm}\left(M_{\phi \phi}<2.85 \mathrm{GeV} / c^{2}\right)$ | $7.3_{-2.5}^{+3.2}$ | $3.3 \pm 0.3$ | 5.1 | $2.6_{-0.9}^{+1.1} \pm 0.3$ |
| $B \rightarrow \phi \phi K\left(M_{\phi \phi}<2.85 \mathrm{GeV} / c^{2}\right)$ | $8.7_{-2.9}^{+3.6}$ | $2.2 \pm 0.2$ | $2.3_{-0.8}^{+0.9} \pm 0.3$ |  |
| $B^{ \pm} \rightarrow f_{J}(2220) K^{ \pm}, f_{J}(2220) \rightarrow \phi \phi$ | $<3.7$ | $3.6 \pm 0.3$ | $<1.2$ |  |
| $B^{ \pm} \rightarrow \eta_{c} K^{ \pm}, \eta_{c} \rightarrow \phi \phi$ | $7.0_{-2.3}^{+3.0}$ | $3.7 \pm 0.3$ |  | $2.2_{-0.7}^{+1.0} \pm 0.5$ |
| $B^{ \pm} \rightarrow \eta_{c} K^{ \pm}, \eta_{c} \rightarrow \phi K^{+} K^{-}$ | $14.1_{-3.7}^{+4.7}$ | $4.6 \pm 0.4$ | $3.6_{-0.9}^{+1.1} \pm 0.8$ |  |
| $B^{ \pm} \rightarrow \eta_{c} K^{ \pm}, \eta_{c} \rightarrow 2\left(K^{+} K^{-}\right)$ | $14.6_{-3.9}^{+4.6}$ | $9.6 \pm 0.9$ | 7.7 | $1.8_{-0.5}^{+0.6} \pm 0.4$ |
| $B^{ \pm} \rightarrow J / \psi K^{ \pm}, J / \psi \rightarrow \phi K^{+} K^{-}$ | $9.0_{-3.0}^{+3.7}$ | $4.4 \pm 0.4$ | 6.6 | $2.4_{-0.8}^{+1.0} \pm 0.3$ |
| $B^{ \pm} \rightarrow J / \psi K^{ \pm}, J / \psi \rightarrow 2\left(K^{+} K^{-}\right)$ | $11.0_{-3.5}^{+4.3}$ | $9.2 \pm 0.9$ | 5.3 | $1.4_{-0.4}^{+0.6} \pm 0.2$ |

Since the $J / \psi$ and $\eta_{c}$ charmonium resonances decay to $2\left(K^{+} K^{-}\right)$, we also measure branching fractions of the decays $B \rightarrow$ charmonium $+K$ with charmonium $\rightarrow$ $2\left(K^{+} K^{-}\right)$. To select $B \rightarrow 2\left(K^{+} K^{-}\right) K$ candidates, we apply tighter particle identification and continuum suppression requirements than in the case of $B \rightarrow \phi \phi K$ in order to reduce the larger combinatorial background. Figure 3(a) shows the invariant mass distribution of any two pairs of $K^{+} K^{-}, M_{4 K}$, between $2.8 \mathrm{GeV} / c^{2}$ and $3.2 \mathrm{GeV} / c^{2}$ for the events in the $B$ signal region. Significant contributions from both $\eta_{c}$ and $J / \psi$ intermediate states are seen.

To identify the signals from $\eta_{c}$ and $J / \psi$ intermediate states, we require that the invariant mass of $2\left(K^{+} K^{-}\right)$satisfy $2.94 \mathrm{GeV} / c^{2}<M_{4 K}<3.02 \mathrm{GeV} / c^{2}$ and $3.06 \mathrm{GeV} / c^{2}<M_{4 K}<3.14 \mathrm{GeV} / c^{2}$, respectively. We use signal yields from ML fits to determine branching fractions. Figures $2(\mathrm{e})-2(\mathrm{~h})$ show the $M_{\mathrm{bc}}$ and $\Delta E$ projection plots with the fitted curves superimposed. Table I summarizes the signal yields, efficiencies, statistical significances, and the branching-fraction products. By requiring the invariant mass of one of the $K^{+} K^{-}$pairs to correspond to a $\phi$ meson, we also measure the decays of $B^{ \pm} \rightarrow \eta_{c}(J / \psi) K^{ \pm}$and $\eta_{c}(J / \psi) \rightarrow \phi K^{+} K^{-}$. The results are included in Table I.

Using the known branching fractions $\mathcal{B}\left(B^{ \pm} \rightarrow\right.$ $\left.J / \psi K^{ \pm}\right)=(1.01 \pm 0.05) \times 10^{-3}[5]$ and $\mathcal{B}\left(B^{ \pm} \rightarrow \eta_{c} K^{ \pm}\right)$, we obtain the secondary branching fractions for $J / \psi$ and $\eta_{c}$ decays to $2\left(K^{+} K^{-}\right)$and $\phi K^{+} K^{-}$listed in Table II.

Our measured branching fractions for $\eta_{c} \rightarrow \phi \phi$ and $\eta_{c} \rightarrow 2\left(K^{+} K^{-}\right)$are smaller than those of previous experiments [5], while those for $J / \psi$ decays are consistent. The decay $\eta_{c} \rightarrow 2\left(K^{+} K^{-}\right)$proceeds dominantly through $\eta_{c} \rightarrow \phi K^{+} K^{-}$with $\phi \rightarrow K^{+} K^{-}$. This is the first measurement of $\eta_{c} \rightarrow \phi K^{+} K^{-}$. The decay of $\eta_{c} \rightarrow \phi \phi$ with $\phi \rightarrow K^{+} K^{-}$makes up approximately $1 / 3$ of the branching fraction of $\eta_{c} \rightarrow \phi K^{+} K^{-}$.

In summary, we have observed evidence for the charmless three-body decay $B \rightarrow \phi \phi K$, which is the first example of a $b \rightarrow s \bar{s} s \bar{s} s$ transition. The branching fraction
$\mathcal{B}\left(B^{ \pm} \rightarrow \phi \phi K^{ \pm}\right)=\left(2.6_{-0.9}^{+1.1} \pm 0.3\right) \times 10^{-6}$ for $M_{\phi \phi}<$ $2.85 \mathrm{GeV} / c^{2}$, is measured with a significance of $5.1 \sigma$. No signal is observed for the decay $B \rightarrow f_{J}(2220) K$ with $f_{J}(2220) \rightarrow \phi \phi$. The corresponding upper limit at $90 \%$ C.L. is $\mathcal{B}\left(B^{ \pm} \rightarrow f_{J}(2220) K^{ \pm}\right) \times \mathcal{B}\left(f_{J}(2220) \rightarrow \phi \phi\right)<$ $1.2 \times 10^{-6}$. We have also observed significant signals for $B^{ \pm} \rightarrow \eta_{c} K^{ \pm}$with $\eta_{c} \rightarrow \phi \phi$, with $\eta_{c} \rightarrow \phi K^{+} K^{-}$, and with $\eta_{c} \rightarrow 2\left(K^{+} K^{-}\right)$, as well as a signal for $B^{ \pm} \rightarrow$ $J / \psi K^{ \pm}$with $J / \psi \rightarrow \phi K^{+} K^{-}$. We report the first measurement of $\eta_{c} \rightarrow \phi K^{+} K^{-}$with a branching fraction of $\mathcal{B}\left(\eta_{c} \rightarrow \phi K^{+} K^{-}\right)=\left(2.9_{-0.8}^{+0.9} \pm 1.1\right) \times 10^{-3}$. Our measured branching fractions for $\eta_{c} \rightarrow \phi \phi$ and $2\left(K^{+} K^{-}\right)$ are smaller than those of previous experiments.

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TABLE II. Measured branching fractions of secondary charmonium decays and the world averages [5]. The branching fractions for modes with $K^{+} K^{-}$pairs include contributions from $\phi \rightarrow K^{+} K^{-}$.

| Decay mode | $\mathcal{B}$ (this work) | $\mathcal{B}(\mathrm{PDG})$ |
| :---: | :---: | :---: |
| $\eta_{c} \rightarrow \phi \phi$ | $\left(1.8_{-0.6}^{+0.8} \pm 0.7\right) \times 10^{-3}$ | $(7.1 \pm 2.8) \times 10^{-3}$ |
| $\eta_{c} \rightarrow \phi K^{+} K^{-}$ | $\left(2.9_{-0.8}^{+0.9} \pm 1.1\right) \times 10^{-3}$ |  |
| $\eta_{c} \rightarrow 2\left(K^{+} K^{-}\right)$ | $\left(1.4_{-0.4}^{+0.5} \pm 0.6\right) \times 10^{-3}$ | $(2.1 \pm 1.2) \%$ |
| $J / \psi \rightarrow \phi K^{+} K^{-}$ | $\left(2.4_{-0.8}^{+1.0} \pm 0.3\right) \times 10^{-3}$ | $(7.4 \pm 1.1) \times 10^{-4}$ |
| $J / \psi \rightarrow 2\left(K^{+} K^{-}\right)$ | $\left(1.4_{-0.4}^{+0.5} \pm 0.2\right) \times 10^{-3}$ | $(7.0 \pm 3.0) \times 10^{-4}$ |

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*On leave from Fermi National Accelerator Laboratory, Batavia, IL 60510.
${ }^{\dagger}$ On leave from Nova Gorica Polytechnic, Nova Gorica.
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