On Fundamental Limits of Slow-Fluid Antenna Multiple Access for Unsourced Random Access

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***Abstract*—Massive connectivity in future networks demands fundamental solutions for novel multiple access techniques. Notably, unsourced random access (URA) and fluid antenna multiple access (FAMA) offer significant capacity improvements through coding gain and spatial diversity gain. In this work, the slow-FAMA (s-FAMA) URA model is presented, with detailed propositions on achievable bounds and performance floor for URA in a multi-input and multi-output (MIMO) system. Both achievable results and performance floors are provided in closed-form expressions. Compared to existing MIMO URA bounds, the system capacity is substantially improved in both single-user and multi-user scenarios. Numerical results demonstrate the promising enhancement from the presented s-FAMA URA.**

***Index Terms*—Fluid antenna system (FAS), slow/fast-fluid antenna multiple access (s/f-FAMA), unsourced random access (URA), multi-input and multi-output (MIMO) system.**

I. Introduction

1) Coding-Oriented Massive Connectivity Approach: Massive machine-type communication (mMTC) is anticipated to be one of the most critical application scenarios for future networks [1]. The unprecedented equipment density and demand for low latency require fundamental solutions for multiple access. Notably, unsourced random access (URA) [2], [3] reshapes massive connectivity into a theoretical coding problem, offering elegant capacity derivations under finite block length (FBL) [4], [5]. URA prophets a random concatenated coding approach, which projects information bits onto a common codebook, resulting in highly favorable spectral and energy efficiency [6], [7]. The established capacity bounds have become theoretical benchmarks for designing practical codes that approach achievable bound [8]. Furthermore, [9] and [10] explore the achievability and error floor for URA within the multi-input and multi-output (MIMO) model.

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2) Great Diversity in a Small Space: The fluid antenna system (FAS) [11], [12] refers to any software-controllable fluidic, conductive, or dielectric structure capable of dynamically altering its radiation characteristics based on its requirements [13], [14]. FAS has also found applications in the rethinking of multiple access, namely fluid antenna multiple access (FAMA) [15]. The most attractive feature of FAS and FAMA is their ability to generate significant spatial diversity within a compact antenna array [16]. FAMA is a novel technology in which a single fluid antenna can adjust its physical position within a predefined area [17]. It leverages deep fading of interference to achieve an optimal channel state for the desired signal, eliminating the need for complex signal processing. Based on port selection, FAMA can be classified into two types: slow and fast (s/f-FAMA). In f-FAMA [18], the port is reselected whenever the instantaneous interference signal changes. In s-FAMA [19], the selected port remains unchanged until the channel conditions evolve.

3) Contributions: In this work, we explore the integration of s-FAMA and URA to support massive connectivity. We present propositions regarding the achievable performance bounds and performance floor for the s-FAMA URA system. For the achievable results, information bits are mapped onto the common codebook and transmitted over the MIMO FAS channel. Without the knowledge of the channel conditions, the URA system performance is shown to be inversely proportional to the channel gain provided by the fluid antenna. The achievable results are derived following the Chernoff inequality train-of-thought and are explicitly presented with closed-form expressions. For the performance floor, we consider two cases of activated ports: one with correlated array and the other with no correlation.

4) Content Structures: In Sec.II, the system configurations and channel model for s-FAMA URA are described. In Sec.III, the approximate achievable results are presented in mathematical expressions. Sec.IV provides a detailed proof of the achievable bound, while the performance floor is approximated in Sec.V. Numerical results are provided in Sec.VI, and conclusions are drawn in Sec.VII.

II. s-FAMA URA System Description

1) System Configurations: There are K_a single-antenna active devices, each with B bits of information to transmit over n channel uses. The power constraint is termed as energy-per-bit $\frac{E_b}{n_0}$, where

$$\frac{E_b}{n_0} = \frac{Pn}{B\sigma_n^2}, \tag{1}$$

with P representing the energy per channel use and σ_n^2 being the noise variance. Let $\mathbf{h}_k \in \mathbb{C}^M$ denote the channel coefficients between the k-th user and the receiver. In the context of URA, the information bits are passed through a concatenated encoder and projected onto a common codebook $\mathbf{A} \in \mathbb{C}^{2^B \times n}$, which contains 2^B codewords. Each codeword has length n and follows a complex Gaussian distribution with zero mean and variance, i.e., $\mathcal{CN}(0, P)$. The received signals $\mathbf{Y} \in \mathbb{C}^{M\times n}$ at the receiver side can be written as:

$$\mathbf{Y} = \sum_{j=1}^{K_a} \mathbf{h}_j \mathbf{a}_j + \mathbf{N}, \tag{2}$$

where $\mathbf{N} \in \mathbb{C}^{M\times n}$ is the additive white Gaussian noise (AWGN) with a complex Gaussian distribution with zero mean and variance σ_n^2, i.e., $\mathcal{CN}(0, \sigma_n^2)$, and $\mathbf{a}_j \in \mathbb{C}^n$ is the j-th active codewords selected from $\mathbf{A}$. In the sequel, we use $\mathbf{A}_d$ and $\mathbf{H}_d$ to denote all the active codewords to be detected and their corresponding channel coefficients respectively.

Overall, the receiver performs two main tasks: *1) detecting the presence of the transmitted codeword*, and *2) decoding the codeword using an appropriate decoder*. Intuitively, and without referring to specific methods, the first task is typically performed through multi-user detection, activity detection, and similar approaches. The second task is generally accomplished using techniques such as channel decoding, interference cancellation, etc. The achievable bounds are upper-bounded by errors arising from both of these major tasks.

As is typical in URA, the per-user probability of error (PUPE), denoted P_e, is adopted as the systematic performance error, defined as:

$$P_e = \frac{\mathbb{E}\left\{|\mathcal{L}_{md}| + |\mathcal{L}_{dec}| + |\mathcal{L}_{colli}|\right\}}{|\mathcal{M}|}, \tag{3}$$

where $|\mathcal{L}_{md}|$ denotes the list of codewords that are not detected, $|\mathcal{L}_{dec}|$ denotes the list of codewords that are correctly detected but encounter potential decoding failure (inherent error under finite block length), and $|\mathcal{L}_{colli}|$ denotes the codewords that are in collision and the list of all active codewords is denoted by $\mathcal{M}$, i.e., set cardinality is $|\mathcal{M}| = K_a$.

2) Fluid Antenna Channel Model: In this work, a $W\lambda$-length fluid antenna under planar propagation model with finite scatterers [12] is considered:

$$h_{k,m} = \underbrace{\sigma_{k,0} e^{-j\frac{2\pi(m-1)d}{\lambda}\cos\theta_{k,0}}}_{\text{LOS component}} + \underbrace{\sum_{l=1}^{L_s} \sigma_{k,l} e^{-j\frac{2\pi(m-1)d}{\lambda}\cos\theta_{k,l}}}_{\text{scatterers component}}. \tag{4}$$

There are N_f configurable ports, with only M ports activated. The model includes one line-of-sight (LOS) propagation path and $L_s$scatterer paths. For the LOS component, $\sigma_{k,0} = \sqrt{\frac{K}{K+1}} e^{j\alpha_k}$, where K is the Rice factor, α_k is the arbitrary LOS phase and $\theta_{k,0}$ represents the azimuth angle-of-arrival (AoA) of the specular component for the k-th user. For scatterers component, $\sigma_{k,l}$ is the complex coefficient of the l-th path for the k-th user, and $\theta_{k,l}, l \in [1 : L_s]$ represents the azimuth AoAs of the l-th path of k-th user. The coefficients $\sigma_{k,l}$ satisfy the condition $\sum_{l=1}^{L_s} |\sigma_{k,l}|^2 = \frac{1}{K+1}$, implying that $\sum_{l=0}^{L_s} |\sigma_{k,l}|^2 = 1$. The ports/array element gap distance is $d = \frac{W\lambda}{N_f - 1}$. Due to the randomized overlapping of path components, the fading envelope may fluctuate drastically.

3) S-FAMA: The general concept of s-FAMA is to select the M ports out of N_f ports with the maximum signal-to-interference plus noise ratio (SINR). Without resorting to any specific methods but following user-centric principle, the SINR at m-th ports of k-th user is written as:

$$\gamma_{k,m} = \frac{P|h_{k,m}|^2}{\sigma_n^2 + \sum_{i=1, i\neq k}^{K_a} P|h_{k,m}|^2}. \tag{5}$$

For s-FAMA, M ports with largest $\gamma_{k,m}$ are selected when decoding the data of the k-th user during the coherence time of channel.

III. Approximate Achievable Result

For the s-FAMA URA model, the PUPE is bounded by:

$$P_e \le P_{cons} + P_{colli} + P_{md} + P_{dec}, \tag{6}$$

where P_{cons} represents the probability that at least one user surpassed the power constraints; P_{colli} denotes the codeword collision error probability; P_{md} denotes the codeword detection error probability of active codewords $\mathcal{M}$ over common codebook where the transmitted codeword is wrongly detected or omitted; P_{dec} represents the inherent finite block length error probability. The above quantities are given by:

$$P_{cons} = 1 - F_{\chi^2}\left(\frac{2n\bar{P}}{P}, 2n\right)^{|\mathcal{M}|}, \tag{7a}$$

$$P_{colli} = \frac{1}{K_a} \textstyle\sum_{k=2}^{K_a} k \cdot \frac{\binom{K_a}{k}}{N^{k-1}}, \tag{7b}$$

$$P_{md} = \sum_{K_e=0}^{K_a} \frac{K_e}{K_a} e^{L_e - nM\log\left(1+\frac{0.25\sigma_e^2}{\sigma_n^2}\right)}, \tag{7c}$$

$$P_{dec} = \sum_{K_e=0}^{K_a-1} \left(1 - e^{L_e - nM\log\left(1+\frac{0.25\sigma_e^2}{\sigma_n^2}\right)}\right) \mathbb{P}\left(\mathcal{K}_0\right), \tag{7d}$$

$$\mathbb{P}\left(\mathcal{K}_0\right) = \sum_{k=1}^{K_0} \mathbb{P}(\mathcal{D}_{s,k}) \frac{K_0 - k + 1}{K_0}, \tag{7e}$$

$$\mathbb{P}\left(\mathcal{D}_{s,k}\right) = \mathbb{P}(\mathcal{D}_{n,k}) \prod_{j=1}^{k-1} \left(1 - \mathbb{P}(\mathcal{D}_{n,j})\right), \tag{7f}$$

$$\mathbb{P}(\mathcal{D}_{n,k}) = Q\left(\frac{C - R_c}{\sqrt{V_{dis}/n_0}}\right)^{(K_0-k+1)}, \tag{7g}$$

$$\sigma_e^2 = \hat{g} K_e P, \tag{7h}$$

$$K_0 = K_a - K_e, \tag{7i}$$

$$L_e = \sum_{i=0}^{K_e-1} \left(\log\left(\frac{K_a - i}{K_e - i}\right) + \log\left(\frac{2^B - i}{K_e - i}\right)\right), \tag{7j}$$

$$C = 0.5\log_2(1 + \alpha_k), \tag{7k}$$

$$V_{dis} = \frac{\alpha_k}{2}\frac{\alpha_k + 2}{(\alpha_k + 1)^2} \log_e^2(2), \tag{7l}$$

$$\alpha_k = M\hat{g}P/\sigma_n^2, \tag{7m}$$

$$R_c = B/n. \tag{7n}$$

In (7h), (7l) (7m), $\hat{g}$ would be the average variance of channel coefficients after ports selection, affecting both the detection (7c) and decoding error (7d). It's generated via Monte Carlo due to the so-far unpredictable overlapping of channel responses from finite scatterer paths.

IV. Achievable Bound Proof

A. Power Constraint and Collision Error, P_{cons}&P_{colli}

1) P_{cons}: The error P_{cons} arises from the statistical perspective of the power constraint, where a randomized codebook may not maintain identical power levels for all codewords (i.e., fluctuations in power), potentially causing errors to some extent. Error P_{cons} is defined as:

$$P_{cons} = 1 - \prod_{j\in\mathcal{M}} \mathbb{P}\left(\|\mathbf{a}_j\|^2/n < \bar{P}\right). \tag{8}$$

Considering $\|\mathbf{a}_j\|^2 = \sum_{i=1}^{n} a_{j,i}^2,\ a_{j,i}^2 \in \mathcal{CN}(0,P)$, the chi-squared distribution of the random variable can be obtained as $\frac{2}{P}\|\mathbf{a}_j\|^2 \sim \chi^2_{2n}$. Thus, the error probability of this component can be calculated as $P_{cons} = 1 - F_{\chi^2}\left(\frac{2n\bar{P}}{P}, 2n\right)^{|\mathcal{M}|}$.

2) P_{colli}: For any K_a active users with a N-size common codebook, the averaged number of codeword collision from k users (a subset of users) on one codeword is given by $\mathbb{E}(k,K_a,N) = \frac{\binom{K_a}{k}\binom{N}{1}}{N^k} = \frac{\binom{K_a}{k}}{N^{k-1}}$, producing the probability of collision error equal to $\mathbb{P}_{colli}(k,K_a,N) = \frac{\mathbb{E}(k,K_a,N)}{K_a}$. Accumulating the error in all $k \geq 2$ users, the collision error is upperbounded by $P_{colli} \leq \frac{1}{K_a}\sum_{k=2}^{K_a} k \cdot \frac{\binom{K_a}{k}}{N^{k-1}}$.

B. Approximate Detection and Decoding error, P_{md}&P_{dec}

1) P_{md}: Before analyzing detection and decoding error probability P_{md}, the priority is to properly define the detection model. To estimate active codewords matrix $\mathbf{A}_d$, the target function is formulated as:

$$\tilde{\mathbf{A}}_d = \arg\min_{\mathbf{A}_d} \|\mathbf{Y} f_d(\mathbf{A}_d)\|^2, \tag{9}$$

where $f_d(\cdot)$ is the detection function, defined as:

$$f_p(\mathbf{A}_d) = \mathbf{I}_n - \mathbf{A}_d\mathrm{H}(\mathbf{A}_d\mathbf{A}_d\mathrm{H})^{-1}\mathbf{A}_d. \tag{10}$$

(9) finds the codewords with the smallest residuals, i.e. $\mathbf{Y} f_d(\mathbf{A}_d)$, and notably, *in this work, no prior knowledge of channel state information is required*. Since $\mathbf{A}_d f_d(\mathbf{A}_d) = \mathbf{0} \preceq \mathbf{I}_n$ and $f_d(\mathbf{A}_d) f_d(\mathbf{A}_d)^{\mathrm{H}} = f_d(\mathbf{A}_d)$, one can have:

$$\|\mathbf{Y} f_d(\mathbf{A}_d)\|^2 \leq \|\mathbf{N}\|^2. \tag{11}$$

Next, define detection error event $\mathcal{D}_e$ as verdicting $\mathbf{A}_d$ into $\mathbf{A}_e = \begin{bmatrix}\mathbf{A}_d' \\ \mathbf{A}_e'\end{bmatrix}$, where $\mathbf{A}_e$ has identical size to $\mathbf{A}_d$ but contains potential errors. $\mathbf{A}_d' \subseteq \mathbf{A}_d$ denote a subset of correct codewords and $\mathbf{A}_e' \in \mathbb{C}^{K_e\times n}$ contains K_e incorrectly detected codewords. In summary, error event $\mathcal{D}_e$ can be expressed as:

$$\mathcal{D}_e = \left\{\|\mathbf{Y} f_d(\mathbf{A}_e)\|^2 \leq \|\mathbf{Y} f_d(\mathbf{A}_d)\|^2\right\}. \tag{12}$$

Furthermore, let's define the span space of $\mathbf{A}_d$ and $\mathbf{A}_e$ as set $\mathcal{F}_d$ and set $\mathcal{F}_e$ respectively. Then, the cardinality of span space set are $|\mathcal{F}_d| = \binom{2^B}{K_a}$ and $|\mathcal{F}_e| = \binom{2^B}{K_e}\binom{K_a}{K_e}$.

Subsequently, the detection error probability is declared:

$$\begin{aligned} P_{\mathbf{A}_d,\mathbf{A}_e} &= \mathbb{P}\left(\bigcup_{\mathbf{A}_e\in\mathcal{F}_e}\bigcap_{\mathbf{A}_d'\in\mathcal{F}_d}\{\mathcal{D}_e\}\right), \\ &\leq |\mathcal{F}_e|\mathbb{P}(\mathcal{D}_e). \end{aligned} \tag{13}$$

Here are the explanations for construction on union-intersect event '$\bigcup\bigcap\{\mathcal{D}_e\}$' and the inequity conversion '$|\mathcal{F}_e|\mathbb{P}(\mathcal{D}_e)$':

- 1) In terms of the *intersection of events*, for a erroneous matrix to be verdicted as final results by (9), $\mathbf{A}_e$ should generate the smallest residuals among all $\mathbf{A}_d$; And the *union of events* derives from there are many possible combinations of $\mathbf{A}_e$;
- 2) Properties $\mathbb{P}\left(\bigcup_{i\in\mathcal{S}} S_i\right) \leq \sum_{i\in\mathcal{S}}\mathbb{P}(S_j)$ and $\mathbb{P}\left(\bigcap_{i\in\mathcal{S}} S_i\right) \leq \mathbb{P}(S_j), j\in\mathcal{S}$ are used to do the *inequity conversion*.

Hereafter, probability $\mathbb{P}(\mathcal{D}_e)$ in (13) is upper-bounded. Denoting the channel coefficients of $\mathbf{A}_d'$ and $\mathbf{A}_e'$ as $\mathbf{H}_d'$ and $\mathbf{H}_e'$ respectively. The received signal can be written as:

$$\mathbf{Y} = [\mathbf{H}_d', \mathbf{0}]\,\mathbf{A}_e + \mathbf{N}' + \mathbf{N}, \tag{14}$$

where $\mathbf{N}' = \mathbf{H}_e'\mathbf{A}_e'$. Assuming independence among channels and codewords of each user, the elements of $\mathbf{N}'$ approximately follow distribution of $\mathcal{CN}(0,\sigma_e^2)$, where $\sigma_e^2 = \hat{g}K_eP$ and $\hat{g}$ would be *the average variance of channel coefficients after ports selection*. Assuming weak law of large numbers, we have $\mathbf{N}'\mathbf{A}_e^{\mathrm{H}} \approx \mathbf{N}\mathbf{A}_e^{\mathrm{H}} \approx \mathbf{0}$, i.e., $\mathbf{N}' f_d(\mathbf{A}_e) \approx \mathbf{N}'$ and $\mathbf{N} f_d(\mathbf{A}_e) \approx \mathbf{N}$. Along with the approximation of $\mathbf{A}_d f_d(\mathbf{A}_d) = \mathbf{0}$ below (10), one can get:

$$\|\mathbf{Y} f_d(\mathbf{A}_e)\|^2 \approx \|\mathbf{N} + \mathbf{N}'\|^2. \tag{15}$$

With (11) and (15), the probability $\mathbb{P}(\mathcal{D}_e)$ can be approximated in (16). In (16a), $\mathbf{n}'$ and $\mathbf{n}$ are the vectorized version of matrix $\mathbf{N}'$ and $\mathbf{N}$ and vectorization does not affect Frobenius norm calculation. Meanwhile, if K_e codewords are incorrectly detected, the PUPE would be $\frac{K_e}{K_a}$. Therefore, the overall detection error probability can be approximated as $P_{md} \leq \sum_{K_e=0}^{K_a} \frac{K_e}{K_a} e^{L_e - nM\log\left(1+0.25\sigma_e^2/\sigma_n^2\right)}$, where $L_e = \sum_{i=0}^{K_e-1}\left(\log\left(\frac{K_a-i}{K_e-i}\right) + \log\left(\frac{2^B-i}{K_e-i}\right)\right)$.

2) P_{dec}: This subsection approximates the inherent error probability [4] under FBL when decoding the k-th active codeword after detection, *which can be trivial under suitable conditions, but this consideration is necessary for the approximation of the upper bound*. Assuming, in decoding the k-th active codeword, there remain $K_0 - k + 1$ users that need to be decoded. We also assume that the decoding stops if the decoded codeword is invalid.

Two major error events are defined: Let $\mathcal{D}_{s,k}$ be the event that the iterative algorithm is stopped at decoding the k-th codeword, and $\mathcal{D}_{n,k}$ denote the event that none of the users are decoded in the k-th round. $\mathcal{D}_{s,k}$ occurs if the algorithm is not stopped in the previous $k-1$ decoding, and it is stopped in the current round. Assuming the decoding process in different iterations to be independent of each other, the probability that $\mathcal{D}_{s,k}$ occurs is given by $\mathbb{P}(\mathcal{D}_{s,k}) \approx$

$$\mathbb{P}\left(\mathcal{D}_e\right) \leq \mathbb{P}\left(\|\mathbf{n}+\mathbf{n}'\|^2 < \|\mathbf{n}\|^2\right) \xrightarrow[\mathbb{P}(x>0)\leq\mathbb{E}(e^{\lambda_1 x})]{\text{Chernoff Inequity}} \leq \mathbb{E}\left\{e^{-\lambda_1\|\mathbf{n}+\mathbf{n}'\|^2+\lambda_1\|\mathbf{n}\|^2}\right\}, \tag{16a}$$

$$\xrightarrow[\mathbb{E}\left\{e^{x\|\mathbf{a}+\mathbf{b}\|^2}\right\}=\frac{e^{\frac{x\|\mathbf{b}\|^2}{(1-x\sigma_a^2)}}}{(1-x\sigma_a^2)^n}]{\text{Using Identity:}} = \mathbb{E}\left\{\frac{e^{\lambda_1\|\mathbf{n}\|^2}e^{\frac{-\lambda_1\|\mathbf{n}\|^2}{(1+\lambda_1\sigma_e^2)}}}{(1+\lambda_1\sigma_e^2)^{nM}}\right\} = \mathbb{E}\left\{\frac{e^{\left(\frac{-\lambda_1}{(1+\lambda_1\sigma_e^2)}+\lambda_1\right)\|\mathbf{n}\|^2}}{(1+\lambda_1\sigma_e^2)^{nM}}\right\} = \mathbb{E}\left\{\frac{e^{\left(\frac{\lambda_1^2\sigma_e^2}{(1+\lambda_1\sigma_e^2)}\right)\|\mathbf{n}\|^2}}{(1+\lambda_1\sigma_e^2)^{nM}}\right\} \tag{16b}$$

$$= \frac{1}{(1+\lambda_1\sigma_e^2)^{nM}}\frac{1}{\left(1-\frac{\lambda_1^2\sigma_e^2\sigma_n^2}{(1+\lambda_1\sigma_e^2)}\right)^{nM}} = \frac{1}{(1+\lambda_1\sigma_e^2-\lambda_1^2\sigma_e^2\sigma_n^2)^{nM}} \xrightarrow[\lambda_1=0.5/\sigma_n^2]{\text{Fraction Minimization}} = e^{-nM\log\left(1+0.25\sigma_e^2/\sigma_n^2\right)}. \tag{16c}$$

$\mathbb{P}(\mathcal{D}_{n,k})\prod_{j=1}^{k-1}(1-\mathbb{P}(\mathcal{D}_{n,j}))$. Besides, considering FBL error, $\mathbb{P}(\mathcal{D}_{n,k})$ can be approximated as:

$$\mathbb{P}(\mathcal{D}_{n,k}) \approx Q\left(\frac{C-R_c}{\sqrt{V_{dis}/n_0}}\right)^{(K_0-k+1)}, \tag{17}$$

where $C = 0.5\log_2(1+\alpha_k)$, $V_{dis} = \frac{\alpha_k}{2}\frac{\alpha_k+2}{(\alpha_k+1)^2}\log_e^2(2)$, and $R_c = B_0/n_0$, and $\alpha_k = \bar{\gamma}$, where $\bar{\gamma} = M\hat{g}P/\sigma_n^2$ is the effective SINR when decoding the k-th codewords. Here is an explanation on $\bar{\gamma}$, to decode the detected $K_a - K_e + 1$ codewords:

$$\mathbf{Y} = \underbrace{\underbrace{\mathbf{h}_i\mathbf{a}_i}_{\text{to be decoded}} + \sum_{j\neq i}^{K_a-K_e}\mathbf{h}_j\mathbf{a}_j}_{K_a-K_e+1 \text{ correctly detected codeword}} + \mathbf{N}' + \mathbf{N}, \tag{18}$$

then, the signal $\mathbf{a}_i$ can be decoded as:

$$\tilde{\mathbf{a}}_i = \|\mathbf{h}_i\|^2\mathbf{a}_i + \underbrace{\sum_{j\neq i}^{K_a-K_e}\mathbf{h}_i^{\mathrm{H}}\mathbf{h}_j\mathbf{a}_j + \mathbf{h}_i^{\mathrm{H}}\mathbf{N}'}_{\text{independent channel coefficients}} + \mathbf{h}_i^{\mathrm{H}}\mathbf{N}. \tag{19}$$

Considering independent channel coefficients among users, the SINR can be calculated by:

$$\bar{\gamma} = \frac{\frac{1}{n}\mathbb{E}\left\{(\|\mathbf{h}_i\|^2\mathbf{a}_i)(\|\mathbf{h}_i\|^2\mathbf{a}_i)^{\mathrm{H}}\right\}}{\frac{1}{n}\mathbb{E}\left\{(\mathbf{h}_i^{\mathrm{H}}\mathbf{N})(\mathbf{h}_i^{\mathrm{H}}\mathbf{N})^{\mathrm{H}}\right\}} = \frac{P\|\mathbf{h}_i\|^4}{\sigma_n^2\|\mathbf{h}_k\|^2} = \frac{M\hat{g}P}{\sigma_n^2}, \tag{20}$$

where $\hat{g}$ will be generated via Monte Carlo due to the unpredictable overlapping of the steering vectors. The ports selection would be simplified into maximizing (20) by ports selection and investigate the averaged variance $\hat{g}$ of channel coefficients after ports selection, which is assumed as *optimal ports selection*.

Moreover, if $\mathcal{D}_{s,k}$ happens, the PUPE is calculated as $(K_0-k+1)/K_0$. Therefore, if K_0 codewords are correctly detected, the error probability of event $\mathcal{K}_0$ defined by correct detection on K_0 codewords would be

$$\mathbb{P}\left(\mathcal{K}_0\right) = \sum_{k=1}^{K_0}\mathbb{P}(\mathcal{D}_{s,k})\frac{K_0-k+1}{K_0},\ K_0 = K_a - K_e. \tag{21}$$

Also considering at least one codeword is correctly decoded, the decoding error probability can be approximated as $P_{dec} = \sum_{K_e=0}^{K_a-1}\left(1-e^{L_e-nM\log\left(1+0.25\sigma_e^2/\sigma_n^2\right)}\right)\mathbb{P}\left(\mathcal{K}_0\right)$.

V. Performance Floor: Optimistic Bound

In this section, we obtain the lower bound for the s-FAMA URA system. The lower bound of the performance is termed as the *optimistic bound* due to ideal assumptions to obtain the potential minimum error. Under certain level of energy-per-bit E_b/n_0 representing the performance lower bound, we have the system metric in (22a) and constraint of (22b):

$$P_e \leq \frac{1}{K_a}\sum_{k=2}^{K_a}k\cdot\frac{\binom{K_a}{k}}{N^{k-1}}, \tag{22a}$$

$$\mathbb{E}\left\{\log_2\left(\det\left(\mathbf{I}_M + \frac{P}{\sigma^2}\mathbf{H}\mathbf{H}^{\mathrm{H}}\right)\right)\right\} \to \frac{BK_a}{n}, \tag{22b}$$

where $\mathbf{H} \in \mathbb{C}^{M\times K_a}$ is the channel matrix of all users with optimum ports selection in (20). Constraint (22b) is the averaged channel capacity/sum-rate at the multi-antenna system. According to Shannon's theorem, error-free transmission is possible when the rate stays below the averaged capacity. PUPE in (22a) incorporates only collision-derived error.

VI. Numerical Results

In this section, the single-user achievable bound, multi-user achievable bound, and performance floor of the approximated s-FAMA URA are presented. The benchmarks include existing influential works such as the projection-based URA bound (achievable) in [9], the ML-based URA bounds (achievable and floor) in [10], and the FBL single-input and multi-output (SIMO) single-user bound (floor) in [5]. The number of active users, K_a, is a known parameter in all the works. The universal system configurations are as follows: the number of channel uses is $n = 3200$, the number of receiving antennas is $M = 50$, and the number of information bits is $B = 100$, which is a prevailing setups among the existing URA works. The targeted PUPE is 0.025.

For the performance floor of the s-FAMA URA, two important cases are discussed based on whether the correlation of channel coefficients is considered. Specifically, if the activated ports are extremely close, they may become coupled. In the case of uncorrelated channels, the channel matrix in (22b) is generated using a complex Gaussian distribution with zero mean and a variance equal to the channel gain $\hat{g}$. For the potentially correlated case, channel coefficients are simulated using the model in (4). *It must be emphasized that the eventually activated ports are not necessarily close to each other in a practical system. Whether they are theoretically or empirically correlated is entirely determined by the channel* *model and simulation randomness. In our case, considering a simple planar propagation model, the selected ports may be correlated. However, constructing an accurate channel model [14] is a challenging task, which falls outside the scope of this work.* To estimate the channel gain at $M = 50$ antennas, the recommended setups after extensive trials for channel generations are: $L_s = 14$, $K = 0.3$, $N_f = 4,000$. For the performance floor under the correlated channel model, $N_f = 180$ is recommended. Notably, only achievable bound and performance floor (uncorrelated) involve $\hat{g}$. The potentially correlated channel matrix is fed into (22b) after ports selection.

TABLE I: Single User Achievable Results

Single User Achievable Results For $P_e \leq 0.025$	
Bound Name	Minimum-Required E_b/n_0 (dB)
FBL SIMO[5]	-13.75
ML-based URA[10]	-14.5
s-FAMA URA	-17.25

The s-FAMA URA has a better capacity than FBL SIMO (3.5 dB) and ML-based URA (2.75 dB).

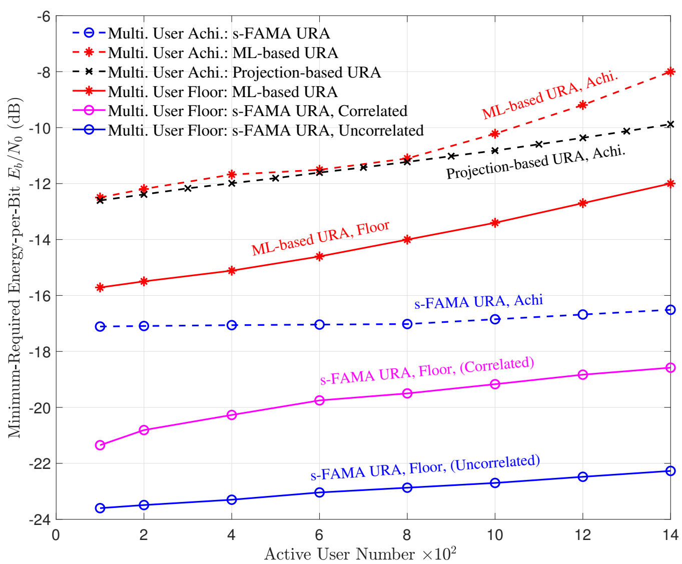

Fig. 1: Illustrations on the multi-user achievable bound and performance floor of the proposed scheme under $P_e \leq 0.025$, $M = 50$, $n = 3200$. The benchmarks are the projection-based URA [9], the ML-based URA [10]. *All benchmarks considers no correlation among antennas, i.e., independent and uncorrelated antennas.*

In Tab. I, the single-user achievable results for the proposed bound, the ML-based URA bound [10], and the FBL SIMO bound [5] are presented. It can be observed that the URA aided by s-FAMA achieves significantly better energy efficiency, offering improvements of approximately 3.5 dB and 2.75 dB in the minimum-required E_b/n_0 when compared to the FBL SIMO bound and the ML-based URA bound, respectively.

Fig. 1 shows the multi-user achievable bounds and performance floors. The s-FAMA URA model achieves nearly 6 dB capacity gain as users increase from 100 to 1,400, compared to ML-based and Projection-based URA achievable bounds. The performance floor requires 8 dB less energy-per-bit at 100 users and 10 dB less at 1,400 users than the ML-based floor. With antenna correlation, the s-FAMA URA performance floor still requires 6 dB less energy-per-bit than existing bounds, though with a 2-3 dB capacity loss compared to the uncorrelated case. Overall, s-FAMA significantly improves both achievable capacity and performance floor, with greater resilience to user density, benefiting future mMTC networks.

VII. Conclusion

This work presents propositions on the achievable performance bounds and performance floor for s-FAMA URA in a MIMO system. Both single-user and multi-user scenarios are analyzed and compared with existing MIMO URA bounds. s-FAMA URA demonstrates a significant performance improvement, with an 8-10 dB capacity gain varying from 100 to 1,400 users, and a 2.75 dB gain for single-user case. Additionally, s-FAMA URA exhibits better tolerance for increased user access, benefiting multiple access for future networks.

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