Gradient Based Optimization of Multi-Step-Index Fibers for Low Differential Mode Delay

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Abstract: *In this paper, we investigate the design of multi-step-index multimode fibers with a cladding trench for 45 spatial modes at 1550nm using an efficient, gradient based radial fiber mode solver.*
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I. INTRODUCTION

Spatial division multiplexing (SDM) has been proposed as a solution to enhance network capacity, as current infrastructure systems based on wavelength-division multiplexing (WDM) with coherent detection near their theoretical limits [1]. SDM systems based on multimode fibers (MMFs), multiplex information over multiple orthogonal transmission modes referred to as Linearly Polarized (LP) modes [2]. To undo modal crosstalk, multiple-input output (MIMO) digital signal processing (DSP) is used. A key challenge in implementing full-scale SDM systems is minimizing intermodal dispersion, as the complexity of DSP increases with dispersion. The MIMO equalizer must be designed with sufficient duration to compensate for the intermodal spread [3]. To reduce the intermodal spread, MMFs are designed with parabolic, graded index profiles and a cladding trench which reduces the macro-bend loss of the higher order modes [4]. The specific measurement of the different arrival times of each mode group is called the differential mode delay (DMD). For a graded index MMF, the optimization process is trivial, where the maximum DMD has a convex dependency on the fiber core grading exponent α and Δn_{tr} difference of the index of the fiber trench w.r.t. the cladding index [4]. However, perfect parabolic graded index fibers are difficult to manufacture as they require precise control over the dopant concentrations in the core where any fluctuations in the temperature, gas flow or chemical composition can lead to a non-ideal index profile [5]. In contrast, multi-step index (MSI) fibers are easier to manufacture as the profile consists a few of discrete layers, each with a constant index – the fewer the number of steps the easier to manufacture. However, they often exhibit higher differential mode delay (DMD) compared to graded-index fibers. Optimizing the design of these fibers is challenging because of the increased number of parameters and the nonconvex nature of the problem. This nonconvexity stems from the nonlinear relationship between the parameters defining the step index heights and widths and the resulting modal effective indices, which determine the propagation speeds. Calculating these effective indices involves solving an eigenvalue problem, which is inherently nonconvex [6].

Machine learning (ML), gradient-based methods have been shown to excel at nonconvex problems especially when using the adaptive gradient optimizers such as RMSProp, Adam and AdaDelta [8]. These optimizers use adaptive learning rates for each parameter based on past gradients and momentum terms assuring proper adjustment ensuring resilience to saddle points and stagnation in flat areas.

Previous research on MSI MMF optimization has utilized non-gradient-based algorithms, such as the genetic algorithm [4] and a numerical approach based on the characteristic matrix method [7]. However, these methods come at the cost of efficiency and are unlikely to converge to the global minimum.

At the heart of this optimization problem lies the mode solver where an eigensolver calculates the effective indices which determine the DMD. By employing implicit differentiation of the eigenvalue decomposition, gradients with respect to the input parameters can be computed efficiently, thereby enabling end-to-end backpropagation through the eigensolver. In fact, previous differentiable mode solvers have been proposed to find the waveguide dispersion for second harmonic generation with maximized phase-matching bandwidth [9]. The authors showed that the cost of the gradient backpropagation is independent of the number parameters required for the generation of complex waveguide structures.

In this paper we present optimization of a MSI MMF using a gradient based solver. We calculate the optimum index profiles resulting in the lowest maximum DMD for 45 modes for 9 to 19 step fibers. We compare the results to a parabolic graded index fiber and show that we can reduce the max DMD provided by the optimized parabolic profile using gradient based optimization.

II. METHODS

A. *Mode Background*

LP modes are solutions to the radially symmetric wave equation (1) that satisfy the boundary conditions and which the spatial profile does not change with propagation [2].

$$\frac{d^2F}{d\rho^2} + \frac{1}{\rho}\frac{dF}{d\rho} + \left(n^2k_0^2 - \beta^2 - \frac{m^2}{\rho^2}\right)F = 0 \qquad (1)$$

Where $F_m(\rho)$ is the radial component of the field, m is the azimuth index of the mode group, ρ being the radius, n is the index profile, k_0 is the wavenumber and β is the propagation constant. The constants β are solved for using a finite difference numerical method. The radius ρ is discretized into a 1D grid, the derivatives are approximated by finite difference approximations or central differences. With the first derivative approximated as:

$$\frac{dF(\rho)}{d\rho} \approx \frac{F_{j+1} - F_{j-1}}{2\Delta\rho} \qquad (2)$$

Where $\Delta\rho$ is the spacing, and the second derivative is approximated as:

$$\frac{d^2F(\rho)}{d\rho^2} \approx \frac{F_{j+1} - 2F_j + F_{j-1}}{\Delta\rho^2} \qquad (3)$$

The appropriate boundary conditions for the field need to be set. We set for, at $m = 0$: $\frac{dF}{d\rho} = 0$ at $\rho = 0$ and for $m \neq 0$: $F = 0$, in accordance with the LP mode spatial profiles. A small offset is provided at $\rho = 0$ to prevent infinite values. All terms which do not involve β are used to build a sparse matrix operator where the diagonal entries collect $-2/\Delta\rho^2$ from the second derivative term and the potential term: $\left(n^2k_0^2 - \frac{m^2}{\rho^2}\right)$. The ± 1 off-diagonal terms collect the first derivative $1/\Delta\rho^2$ and the $\pm 1/2\rho\Delta\rho$. The eigenvectors F_m corresponding to the field profile and eigenvalues β corresponding to the propagation constants are then calculated.

The effective indices n_{eff} are then calculated by β/k_0. The group index n_g is calculated by $n_{eff} - 2\pi\, f_0 \frac{dn_{eff}}{d\omega}$ Where $\frac{dn_{eff}}{d\omega}$ is approximated by a finite difference approach $\frac{n_{eff}(\lambda-\mu) - n_{eff}(\lambda+\mu)}{2d\omega}$ where μ is a wavelength offset and $d\omega$ is defined as $2\pi(f_0\ (f_0 - df)$ where df is the frequency offset associated with the wavelength offset.

B. Implementation

We write the eigenvalue solver using PyTorch, a ML library with support for accelerated operation on CUDA enabled graphics processing units. The matrices are constructed for different azimuthal indices can be constructed and solved for in parallel using the torch.linalg.eig function and the resulting eigenvalues are sorted from largest to smallest.

To construct the step profiles, we utilize custom sigmoid functions defined as:

$$\sigma = \frac{1}{1+\exp(-kx)} \qquad (4)$$

Where k is a sharpness parameter determining the derivative at $x = 0$. For sharp step generation we set k to be 1e11. However, its derivative will give 0 across the whole domain. To allow gradient propagation, we set k to be 1e4 on the backwards pass. A simple scheme of the optimization process is shown in Fig. 1.

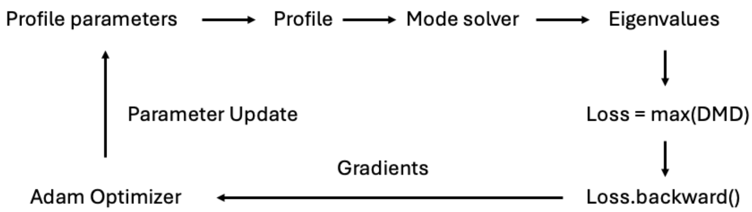

Fig. 1. Algorithm outline.

C. Fiber profile initialization

For all simulation settings we start with a fiber initialization of core radius r of 25 μm, a core index of n_{core} 1.4585, a cladding index n_{cl} of 1.444, with a radius step size $\Delta\rho$ of 0.25 μm, a trench offset from core of 2 μm with a trench width of 6 μm and an initial depth of 1.4425. We use a wavelength of 1550 nm and a frequency offset of 1 MHz. The step profiles are initialized by naively discretizing the parabolic fiber profile shown in Fig. 2.

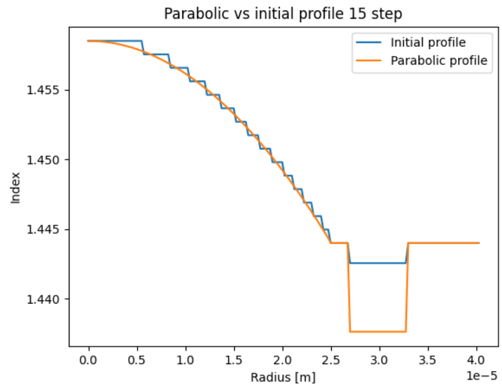

Fig. 2. Initialization of 15-step index fiber profile with an optimized parabolic profile.

We allow the step widths and heights to be free floating, constrained by the n_{core} and r respectively. We also constrain a descending step pattern to ensure efficient convergence. To ensure valid comparisons, we consider strongly guided modes where the condition is defined as:

$$\frac{n_{eff} - n_{cl}}{n_{core} - n_{cl}} > 0.01 \qquad (5)$$

The fiber initialization settings allow for a minimum of 45 modes to be strongly guided. The Adam optimizer is used with default settings and a learning rate of 1e-7 for 5000 iterations. All step radii, all step heights excluding the first height at the center are optimized for. The value of the trench index is also optimized and allowed to take values from 1.416 to 1.446.

III. Results

To act as a comparison, we determine the minimum max DMD for a parabolic profile with a sweep of the α from 1.94 to 2 and obtain a minimum of 30 ps/km determined by max(DMD) – min(DMD) for a value of α = 1.987 for 45 modes. The max DMD of a 19-step fiber at initialization was determined to be 2338 ps/km and after optimization was determined to be at 108 ps/km. The complete results are shown in Fig. 3 A) with an example of a 15-step initial vs optimized fiber profiles in Fig 3. B).

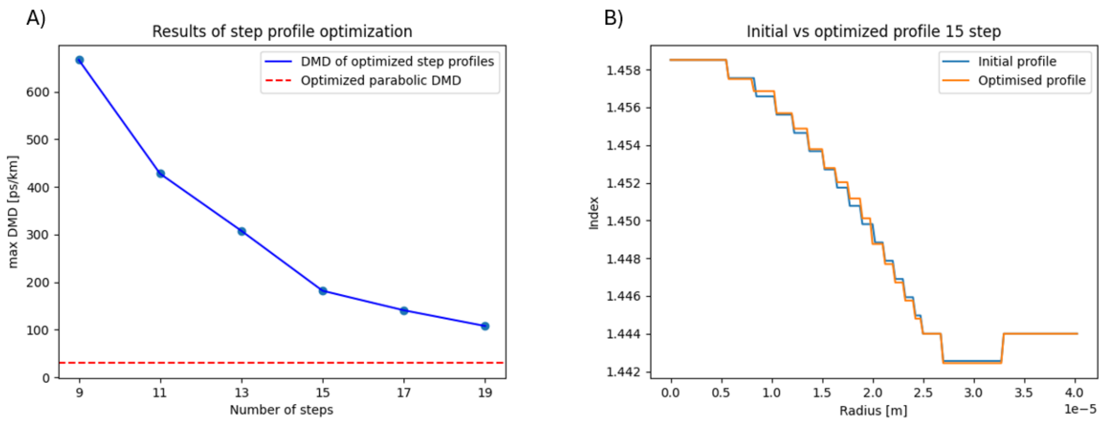

Fig. 3 A). Results of step profile optimization for 5 – 17 step fiber profiles. 3 B) Initial vs optimized 9 step fiber profiles.

IV. Conclusions

In this work we investigated the design of multi-step index (MSI) fibers using a gradient based solver allowing for efficient learning. 9 to 19 step designs were investigated and were compared to an optimized parabolic graded index fiber. While our approach does not surpass the max DMD of a parabolic graded index fiber, our results demonstrate the potential of gradient based fiber profile design. For future work, we will extend our optimization to higher step counts and conduct optimizations across the entire C-band to improve broadband performance. Additional constraints, such as enforcing a constant step radius or height, could be introduced to enhance manufacturability. Furthermore, relaxing the descending staircase condition would allow for the formation of trenches within the core, potentially unlocking new index profiles with improved dispersion and loss characteristics.

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