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Optimizing investment period length and strategies for later stage venture capital staged financing portfolio

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ABSTRACT

In this paper, we analyze the investment and exit decisions in late-stage Venture Capital (VC) rounds of financing portfolios. We utilize a stochastic programming framework to minimize the investment period and find the optimal investment strategy for the VC portfolio under a predefined payoff. Validation of the model is conducted using the US later stage rounds of financing deals from PitchBook. Numerical results of the model reveal an 'S'-shaped relationship between the portfolio's payoff and investment period length, demonstrating the importance of timely termination for maximizing returns. Furthermore, a longer period with stricter exit multiples leads to a higher DPI payoff due to increased selectivity. However, excessively high exit multiples may reduce exits, hampering the portfolio's overall payoff. Finally, portfolios with positive correlation perform better than the uncorrelated ones. These findings shed light on VC portfolio dynamics, providing insights for informed decision-making in staged financing investments.

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1. Introduction

Venture capital (VC) plays a significant role as a major financing source for startup and high-growth firms (Chen, Baierl, and Kaplan 2012; Standaert, Knockaert, and Manigart 2022). This form of financing is pivotal in supporting the development of startup firms, providing the necessary capital for the innovation of projects and expansion development. Dahiya and Ray (2012) and Gompers (2022) show that staged financing allows the investors to efficiently assess and support the evolving needs of the startups, fostering sustainable growth and maximizing the chances of success for ventures while mitigating the risk for VCs.

The lack of cash flow in the early period of a VC fund makes it an illiquid asset class which sets it apart from liquid portfolios of stocks or bonds. In a VC-staged financing portfolio lifecycle, the investment period denotes the timeframe during which VCs invest in selected companies until the exit event, such as Initial Public Offerings (IPOs) and Mergers & Acquisitions (M&A), occurs (Espenlaub, Khurshed, and Mohamed 2015). It involves investment and exit strategies that directly impact both the overall payoff of the portfolio and the success of individual portfolio companies (Gompers 2022). However, despite its pivotal role, the formulation of optimal strategies to minimize the investment period length and achieve a satisfying payoff has remained relatively understudied. Given the illiquid nature of VC funds, understanding investment period length is particularly critical, as investors cannot exit freely as in public markets. The extended holding periods introduce challenges. Specifically, late-stage investments pose a significant trade-off: exiting too early may miss the peak

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valuation of a company, while holding too long can lead to declining returns and growing operational risks. The optimal investment period length is therefore a key strategic decision that directly influences fund performance, but remains underexplored in existing research. With our paper, we want to bridge this gap and to do so, we will focus only on the monetary aspect of the payoffs for a VC fund: its returns. The investigation of non-monetary and strategic dimensions, such as reputational issues, signaling and style drifting, though extremely important for understanding the characteristics and dynamics of private capital, are beyond the scope of this paper and worthy of independent further investigation.

Scholars have studied the topic of VC-staged financing through various lenses and methodologies. Some research focuses on investment decision-making and exit strategies, considering both early-stage and late-stage investments. Lukas, Mölls, and Welling (2016) examine the impact of multi-staged financing, economic and technological uncertainty on optimal contracting. They develop a dynamic model of entrepreneurial venture financing based on option exercise games between an entrepreneur and a VC. Ferreira and Pereira (2021) explore a dynamic model studying the entry and exit decision for a VC investing in a startup. Their model provides the expected cash multiple for the VC and introduces a time-adjusted version of the cash multiple to assess expected investment performance. Guo, Lou, and Pérez-Castrillo (2015) propose a model analyzing investment decisions, duration, and exit strategies for VC-backed startups, incorporating uncertainty, information asymmetry, and discount rates. Their findings suggest that longer investment durations increase the likelihood of an acquisition exit, while larger investments raise the probability of an IPO exit. Li (2008) applies a real options perspective to VC staging, revealing that market uncertainty delays investment decisions, while competition, project-specific uncertainty, and agency concerns accelerate them. Studies also examine additional factors influencing investment and exit decisions. Adkins and Paxson (2017) develop an analytical framework for sequential investment opportunities with stage-specific risks and drifts. Bergemann and Hege (1998) investigate optimal contracts between venture capitalists and entrepreneurs.

Another key dimension of VC-staged financing explored in the literature is the investment horizon. Cumming and Johan (2010), Cumming and MacIntosh (2001) examine cross-country differences in VC investment duration, formulating a theory that VCs exit when the expected marginal cost of maintaining an investment exceeds the expected marginal benefit. This theory is supported by data from Canada and the United States. Guo, Lou, and Pérez-Castrillo (2015) consider uncertainty, information asymmetry, and discount rate for investment duration and exit strategies of VC-backed companies. Li (2008) further explores the factors influencing VC investment delays using a real options perspective. Wong (2024) investigates how correlated multi-stage funding affects the speed of venture exits. Félix, Pires, and Gulamhussen (2014) analyze the investment period length of individual VC-backed portfolio companies in Europe using a competing risks model, highlighting how investor characteristics, investments, and contracting variables impact investment duration and exit decisions. However, these studies primarily examine staged financing at the individual entrepreneurial firm level, differing from our focus on portfolio-level decision-making. Lee (2012), Bekkers, Doeswijk, and Lam (2009), Ennis and Sebastian (2005) attempt to optimize the returns of VC and private equity (PE) fund portfolios by modeling cash flows and adapting the traditional mean-variance approach into a multistage optimization model. Lee (2012) demonstrates that the Markowitz mean-variance method can be applied to VC fund assets and validate their model with VC portfolio cash flow data. While these studies acknowledge the multistage dynamic nature of VC investments, they overlook firm-level perspectives and the heterogeneity of investment operations. Therefore, a framework is needed to model investment and exit strategies for VC-staged financing portfolios.

Stochastic programming models have long been applied to investment decision-making. Aouni, Colapinto, and La Torre (2013) propose a stochastic goal programming model using satisfaction functions for single-time investments. They assume expected revenue and volatility in the Italian IT and communication industries, evaluating multiple scenarios. Colapinto and Torre (2015) develop scenario-based stochastic goal programming models and fuzzy goal programming formulations, demonstrating their practical implementation using real data from Italian VC funds. Colapinto and Torre (2015) introduce a cardinality constrained fuzzy goal programming model for a single-time VC portfolio investment design. While these studies all focus on early-stage and single-time VC investment decisions, stochastic programming has the potential to be extended to multi-stage portfolio decision-making. This is demonstrated by Kettunen and Lejeune (2022), who apply it to project selection problems, offering a framework that can be adapted to the VC stage financing portfolio setting.

Therefore, in this paper, we build a quantitative model, the Sequential Investment Allocation Model (SIAM), to provide optimal investment (exit) strategies for VC portfolios. We particularly focus on later-stage financing, specifically after series C, because earlier stages typically entail more unquantifiable factors and are characterized by instability in investment evaluation (Bernstein, Korteweg, and Laws 2017; Carter et al. 2003). In our model, we integrate insights from VC valuation methods with the multistage selection modeling proposed by Kettunen and Lejeune (2022). The valuation model forms the basis to quantify portfolio company performance (Brandon, Hoesli, and Shan 2022; Gornall and Strebulaev 2021), while Kettunen and Lejeune (2022)'s framework provides the modeling structure to determine optimal decisions within a minimal investment period. The model is validated using the data from US VC-backed rounds of financing venture companies provided by PitchBook.

With our model, we find an 'S'-shaped relationship between the expected payoff and investment period, with peak payoffs occurring around 8–10 years, matching the average exit time for successful exit companies' data provided by PitchBook. Additionally, a high exit multiple increases the payoff, while an excessively high multiple may result in a negative effect on the maximum payoff reached due to zero company exits. Lastly, our study reveals that positively correlated portfolios exhibit significantly higher Distributions to Paid-in payoffs compared to uncorrelated diversified funds. This corresponds to one of the VC portfolio construction strategies, where specialized knowledge and expertise in specific industries help portfolio companies achieve higher returns and improve overall performance. These findings enhance understanding of VC portfolio dynamics and offer guidance for informed decision-making regarding optimal investment strategies in the private sector.

This paper is structured as follows. Section 2 introduces the Sequential Investment Allocation Model (SIAM) for the VC portfolio during rounds of financing. The model's overall structure is explained, followed by the definition and modeling of its parameters. Additionally, the section includes a validation of the model using data from PitchBook. In Section 3, we conduct simulations of a VC portfolio and examine the impact of different assumptions on the investment period length. Factors such as investment decisions and expected payoff are analyzed to provide insights on strategies that yield higher payoff within the minimum investment period.

2. The Sequential Investment Allocation Model (SIAM)

In this section, we aim to build the Sequential Investment Allocation Model (SIAM) based on the stochastic programming approach (Kettunen and Lejeune 2022), to find the minimum investment period τ under investment strategies $\mathbf{y}$ for a VC portfolio with rounds of financing while ensuring its payoff higher than a predefined threshold R.

2.1. Model framework

Suppose a VC fund has a portfolio containing $\mathbf{I} = \{1, \cdots, I\}$ companies that will go through rounds of financing at each time $t \in \mathbf{T} = \{0, 1, \cdots, T\}$, with T denoting the maximum acceptable terminated date for the fund that is established earlier by VC. In each round t, the VC fund portfolio manager re-evaluates each business i's past performance and prospects, then determines investment strategies including whether to continue financing the business or not (Félix, Pires, and Gulamhussen 2014), the investment amount and investment timing.

In the study of Kettunen and Lejeune (2022), a multistage decision-making model **A-M** is proposed for the project portfolio selection problem for startup companies with a similar aim – minimizing the amount of investment period while ensuring the probabilistic payoff higher than a certain level. The model takes a stochastic programming form that can be expressed as a disjunctive integer nonlinear and dynamic chance-constrained problem.

Similar to the **A-M** model, we set the objective function to minimize the investment period length τ for a VC fund portfolio. For the dynamic chance constraints controlling the payoff, we introduce a different decision factor $\mathbf{y}$ and the payoff measure to accommodate the dynamic nature of the investment period within the VC portfolio. We name the model as shown in Definition 2.1.

Definition 2.1: (SIAM). The minimum investment period τ for the VC fund portfolio while ensuring the payoff is higher than a threshold R with probability $1-p$ can be calculated through the following Sequential Investment Allocation Model (SIAM):

$$\begin{aligned} \min \quad & \tau \\ \text{s.t.} \quad & (\boldsymbol{\tau}, \mathbf{y}) \in \bigvee_{t \in \{0,\cdots,\tau\}} \mathcal{G}_t \\ & \mathrm{y} \in \mathcal{Y} \\ & \tau \in \mathbf{T} \subset \mathbb{Z}_+ \end{aligned} \tag{1}$$

where

$$\mathcal{G}_t := \{\mathbf{y} : \mathbb{P}(Re_t(\mathbf{x}, \mathbf{y}) \geq R) \geq 1-p\} \tag{2}$$

Therefore, the SIAM involves two decision factors – the investment period of the fund that we hope to minimize and the investment strategies, a vector of factors $\mathbf{y}$ controlled by the VC fund managers. The payoff of the portfolio can be written as a function $Re(\mathbf{x}, \mathbf{y})$ where $Re(\cdot)$ represent the measure of the payoff, $\mathbf{x}$ stands for the vector of factors showing portfolio companies' performance and $\mathbf{y}$ is the investment strategies, like investment timing and amount.

The model takes the form of a disjunctive integer nonlinear chance-constrained algorithm (Shapiro, Dentcheva, and Ruszczynski 2021) that has an objective function that minimizes the stopping time τ with three constraints. The first constraint ensures the payoff of the portfolio reaches the target level R at time t with a reliability level at least equal to $1-p$, which takes a disjunctive normal form.[1] The significance level p here can also be seen as a measure of risk acceptability for VC portfolio managers. Specifically, as p increases, managers face a greater risk (lower probability) of achieving the expected payoff level. Consequently, a higher value of p implies a greater level of risk acceptability for the manager.

The second constraint $\mathcal{Y}$ controls the investment strategies $\mathbf{y}$ including the investment timing and investment amount, the exact form will be shown in the later section, after a thorough explanation of the vector of factors $\mathbf{y}$. The final constraint regulates the timing of the financing round, which must be a positive integer with units in years and less than the termination time T. As a result, the next step involves understanding the intricate composition of $Re(\cdot)$ and $\mathcal{Y}$, as well as the vectors of factors $\mathbf{x}$ and $\mathbf{y}$.

2.2. Measuring the payoff

As pointed out in section 2.1, we need to define a measure of the payoff $Re(\cdot)$ within the SIAM. The Distributions to Paid-in (DPI) measure represents the cumulative value of distributions paid to the VC relative to the investment amount. When evaluating the payoff of the VC portfolio, DPI stands out by considering the relative return to investment amount (Phalippou and Gottschalg 2009; Wiltbank, Dew, and Read 2015). We use the current discounted value of DPI as Equation 3 shows at time T. It is calculated by summing the cash distributions made to VCs, over time t and discounted to present value using a discount rate r, and then dividing by the sum of the invested capital In_t, also discounted to present value.

$$\mathrm{DPI}_T = \frac{\sum_{t=1}^{T} \frac{D(t)}{(1+r)^t}}{\sum_{t=1}^{T} \frac{In(t)}{(1+r)^t}} \tag{3}$$

In this work, we focus on studying the investment period at the VC portfolio's level, often referred to as the gross rate, rather than from the perspective of Limited Partners (LP) also denoted as net rate[2] (Gompers, Kaplan, and Mukharlyamov 2016). Therefore, the cumulative value of distributions can be expressed as the sum of the value generated from the divestment or exit event of companies in that portfolio. For simplicity, we use the firm value immediately before the company exits. The corresponding paid-in is the total investment amount that the VC invested in the portfolio companies.

Figure 1 demonstrates the investment period and decisions throughout the whole investment period. We assume round t as the time period $t \in [t-1, t)$ and denote the investment at the beginning of each round as

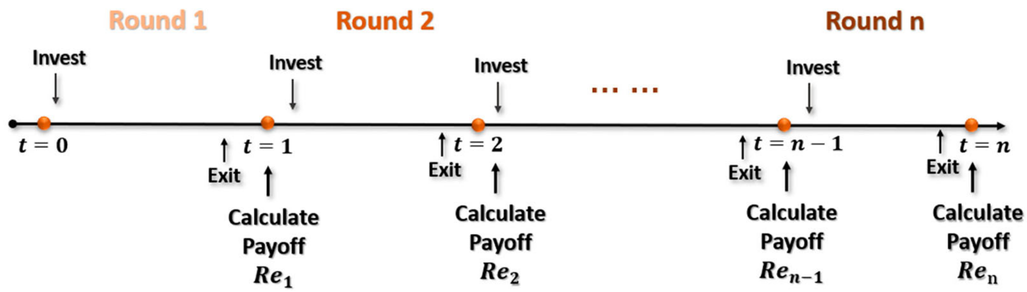


Figure 1. Investment period for rounds of financing portfolios.

In_{t-1}. Then, as the divestment decision is made at the end of round t, we denote the firm value at exit as ξ_{t-1^+}. While the firm value is a continuous variable, so ξ_{t+} can be seen as equal to the firm value at the beginning of the next round (before the investment), where we have $\xi_{t-1^+} = \xi_t$.

Based on the above assumptions, we can further define the vector of factors $\mathbf{x}, \mathbf{y}$. Specifically, the company's performance can be expressed by firm value variable $\mathbf{x} = \boldsymbol{\xi}$, and VC's investment strategies vector is determined by two random variables $\mathbf{y} = [\mathbf{d}, \mathbf{In}]$ – the time to exit or investment period length d for each company and the investment decision In for each company at each round.

Therefore, the payoff measured by the DPI is shown as the Equation (4),

$$Re_t^{\mathrm{DPI}}(\mathbf{d}, \boldsymbol{\xi}, \mathbf{In}) = \frac{\sum_{t'=1}^{t+1} \sum_{i \in C_{t'}(\mathbf{d})} \frac{\omega_{i,t'} \xi_{i,t'}}{(1+r)^{t'}}}{\sum_{t'=1}^{t} \frac{In_{i,t'}}{(1+r)^{t'}}} \tag{4}$$

where $\mathbf{d} = \{d_i\}_i, i \in I$ and d_i is the investment period for company i, $C_{t'}(\mathbf{d})$ is a set of indexes for companies that can exit at round t and $In_{t'}(\mathbf{d})$ is a set of indexes for companies that can exit at round t and the VC's cumulative percentage of share for company i is ω_i. It is calculated through the VC's total share of the company divided by the company's total shares. The total share equals to the earlier share amount plus the new share amount, which is the investment amount multiplied by the earlier round share price. In this paper, we use the firm value divided by the total number of shares to approximate the share price.

As for the construction of constraints for investment decisions, following the convention of later-stage VC, we need to ensure the portfolio companies that give up by VC or exit from the fund cannot be reinvested later. Specifically, defining $y_{i,t}$ as the status showing whether the portfolio company is in the portfolio or has been divested. Suppose the status of the company equals 1 until exit or is given up by the fund and $y_{i,0} = 1$, so the constraint $\mathcal{Y}$ can be written as:

$$\mathcal{Y} = \{y \in \{0,1\} : y_{i,t} \geq y_{i,t+1}, t \in \mathbf{T}, i \in \mathbf{I}_t(\mathbf{d})\} \tag{5}$$

Therefore, the SIAM can be further written as:

$$\begin{aligned} \min \quad & \tau \\ \text{s.t.} \quad & (\mathbf{d}, \boldsymbol{\xi}, \mathbf{In}) \in \bigvee_{\mathrm{t} \in \{0, \cdots, \tau\}} \mathcal{G}_{\mathrm{t}} \\ & \mathrm{y} \in \mathcal{Y} \\ & \tau \in \mathbf{T} \subset \mathbb{Z}_+ \\ & \text{where} \\ & \mathcal{G}_\tau := \{\mathbf{d}, \boldsymbol{\xi}, \mathbf{In} : \mathbb{P}(\mathrm{Re}_\tau(\mathbf{d}, \boldsymbol{\xi}, \mathbf{In}) \geq \mathrm{R}) \geq 1 - \mathrm{p}\} \\ & \mathcal{Y} = \{\mathrm{y} \in \{0,1\} : \mathrm{y}_{\mathrm{i,t}} \geq \mathrm{y}_{\mathrm{i,t+1}}, \mathrm{t} \in \mathbf{T}, \mathrm{i} \in \mathbf{I}_{\mathrm{t}}(\mathbf{d})\} \end{aligned} \tag{6}$$

Table 1. Notation for the SIAM model.

Decision variables	Explanation
τ	Time (in years) at which targeted payoff is attained
Random variables	Explanation
$\xi_{i,t}$	Firm value for company *i* at round *t*
d_i	Investment period for company *i*
$In_{t,i}$	Investment amount in companies at round *t* for company *i*
Other parameters	Explanation
$y_{i,t}$	Investment status representing whether company *i* is within the portfolio at round *t*
R	Expected payoff ratio of the portfolio
T	Maximum investment period of the portfolio
p	Probability, significance level
r	Discount factor
Sets	Explanation
$\mathbf{T} = \{1, \cdots, T\} \subset \mathbf{Z}^+$	Set of round (in years) throughout the portfolio lifetime
$\mathbf{T}_i$	Set of rounds which company *i* maintain in the portfolio
$\mathbf{I} = \{1, \cdots, I\}$	Set of companies' index targeted by VC fund
$\mathbf{I}_t(d)$	Set of companies' index that remained in the portfolio at round *t*
$C_{t'}(d)$	Set of companies' index that successfully exit from the fund at round *t*

A conclusion of parameters used so far is collected in Table 1. Now we further look into the random variables in the SIAM.

2.3. Model assumptions

2.3.1. Investment timing and amount

Before the beginning of the investment period, a contract related to the investment plan is usually signed between the VC and portfolio companies (Wright et al. 2019). Within the contract, VC portfolio managers decide the total investment amount, and the investment amount or strategies for each company in the fund based on their screening and prediction. Therefore, the investment amount is usually a fixed number or a fixed percentage of the current firm value, while the time of investment rounds can be either predefined or conditioned on companies' performance. Conditioning on performance is a way to delay investment, as investing too soon destroys the value of the illiquid portfolio (Brandon, Hoesli, and Shan 2022). Thus, by conditioning investment on companies' payoff, we can enhance the overall portfolio payoff. Li (2008), Lukas, Mölls, and Welling (2016) incorporate the Real Options approach to find the investment threshold that decides whether or not to invest.

In this study, we assume the investment amount In for each company i in each investment round t is a fixed portion of its firm value $q \times \xi_{i,t}$. Following the intuition of Brandon, Hoesli, and Shan (2022), we condition the investment on the firm value increment between rounds (discounted to the present round). Specifically, we may only invest in companies whose firm value increments are higher than a percentage, θ.

Suppose the investment for company i is paid at round $T_{invest} = \{f_1, f_2, \cdots\}$, where f_i depend on the increment of firm value and θ. Then, the sum of investment for N companies at round t can be written as:

$$\sum_{i=1}^{N} In_{i,t} = \sum_{i=1}^{N} q \times \xi_{i,t} 1_{\{f_i = t\}}$$

Note that, Gornall and Strebulaev (2021) assume the investment amount In_t divided by the firm value at round t follows a log-normal distribution and is related to the previous rounds. Therefore, a log-normal distribution will also be fitted to the data in the later validation process and compared with the above approach.

2.3.2. Firm value at exit

The firm value of each company throughout the investment period can be seen as a sequence of random variables with the index having the interpretation of time, which forms a stochastic process (Joshi 2003). As most financial instruments like stocks in the public market, the change in firm value exhibits a general trend while containing random fluctuation over time. Therefore, the Geometric Brownian motion (GBM) and Jump Diffusion are suitable models to predict the firm value (Gornall and Strebulaev 2021). A GBM is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion with a drift (Joshi 2003). The Jump Diffusion is similar to GBM but adds a discrete component that accounts for sudden and abrupt changes or jumps in the underlying financial assets price (Hainaut and Deelstra 2019).

Gornall and Strebulaev (2021) used the jump-diffusion process to model the firm value ξ during rounds of financing until company exit. The stochastic differential equation form of the model is shown in Equation (8):

$$d\xi_t = \xi_t(r_f + \alpha)dt + \xi_t\sigma\, dW_t + \xi_t I_i dN_t \tag{8}$$

α is the drift parameter that represents the average rate of return of the asset over time. It can also be seen as the expected growth rate, σ measures the variability of the firm value around its average. It quantifies the uncertainty or risk associated with the asset's price movement and r_f is the risk-free interest rate. N_t, W_t are independent where W_t is the Wiener process and N_t is a Poisson process with jump intensity λ. I_i represents the investment amount at round t.

In the meantime, Majd and Pindyck (1987) and Brandon, Hoesli, and Shan (2022) proposed a mixed jump-diffusion process shown in Equation (9) to model the valuation of underlying private companies invested by PE funds.

$$d\xi_t = \xi_t(\mu - \sigma)dt + \sigma\xi_t dW_t + \xi_t\phi^1 dN_t^1 + \xi_t\phi^2 dN_t^2 \tag{9}$$

They use the two jump components as the PE funds often invest in projects that are risky and may either experience a significant increase in value (positive jump in ξ) or fail before completion (negative jump in ξ).

Therefore, based on these earlier works, we model the firm value as a Jump Diffusion process with two Poisson jump terms as shown in Equation (9). The drift term within the model contains two variables: α represents the yearly growth rate of firm value and μ is the risk-free interest rate. The volatility of firm value change is expressed by variable σ. For the jump terms, the first jump represents the change in the company's firm value right after receiving the investment and the second jump represents the random downturn of the company's growth. Note that, if the investment decision is predefined for the company, the first jump term will be replaced by an increment in firm value at the predefined time. The decreases in firm value arrive at a Poisson rate λ_1 with a decrease value $\phi^1 < 0$ and the increase in firm value arrives at a Poisson rate λ_2 with an increment percentage $\phi^2 > 0$ of the investment amount In.

$$d\xi_t \quad = \xi_t(\mu + \alpha)dt + \sigma\xi_t dW_t + \xi_t\phi^1 dN_t^1 + \xi_t\phi^2 In_t dN_t^2 \tag{10}$$

The exit value of a company can vary depending on numerous factors, such as market conditions, the potential buyer's interest, and negotiations between the parties, we here assume fair market conditions and the exit value fairly depends on the company's financial performance, firm value, at the exit.

2.3.3. Exit condition for portfolio companies

The investment period is controlled by the portfolio manager and like the investment amount, it can either be predefined as a fixed time length or condition on the company's performance. For earlier stage VC investment, Félix, Pires, and Gulamhussen (2014) proposed a sophisticated model considering several different factors to determine the exit timing of each company. However, as Dai, Chapman, and Shen (2022) points out, for later-stage VCPE rounds of financing targets there are only two key dimensions of firm performance: innovation and financial performance.

Following the standard practice of later-stage VC investments, we apply the firm value multiples, i.e. the exit amount divided by the initial value, to evaluate the portfolio company's performance. We assume the portfolio manager initially predefines an exit multiple for each company and will exit the company when the firm value reaches that level.

Finally, as Lukas, Mölls, and Welling (2016), Tian (2011) point out, a timely divestment in a company is one of the most important drivers of VC portfolio payoff, therefore, we assume the model will terminate the investment in companies if their firm value shrinks below a percentage of its initial value. The exact percentage is set in the simulation Section 4.1 using the actual data in Section 3.1. In the meantime, we incorporate a maximum investment period constraint into our model, requiring all companies to exit or give up before reaching this threshold. This constraint reflects the common practice among VC funds, which typically aim to exit all investments within a predefined period. We determine the maximum investment period length based on deal data retrieved from PitchBook, which will be discussed in detail in Section 4.1.

3. Model validation

In this section, we first analyze a dataset with successful exit VC-backed later-stage financing companies using deals data from PitchBook. Next, we combine this dataset with empirical findings from Guo, Lou, and Pérez-Castrillo (2015), Cumming and MacIntosh (2001), and Sethuram, Taussig, and Gaur (2021) to validate the assumptions in Section 2.3 and determine appropriate parameters for the models described in those assumptions.

3.1. Data collection

As we focus on VC-backed later-stage deals, we first collect all deals after Series C. Since we are studying late-stage financing rounds, we follow PitchBook's definition, which classifies Series C or later as late-stage, regardless of the time since the company was founded (PitchBook 2025). Due to data availability, we collect the deals for company exits through IPOs and M&As in the US between 01.01.2002 and 30.12.2021. The data is retrieved from PitchBook and contains information on a total of 8500 companies and 16,625 rounds of financing deals.

The dataset is then filtered and aggregated at the company level.[3] As shown in Table 2, out of the 8500 companies, 5213 are still receiving VC funds and have recently completed rounds of financing deals. Additionally, 1959 companies have gone through rounds of financing and successfully exited through either an IPO or M&A, while 916 companies ceased their operations. As we are studying VC later stage rounds of financing and we use the first round post-valuation information as the initial firm value, only companies that had undergone more than one round were considered. Companies with missing investment amount or post-money valuation data are excluded. Additionally, we trimmed our sample to within two standard deviations from the mean value to smooth the effect of outliers. Our dataset includes data on later-stage financing rounds for 710 companies that have achieved successful exits. To address potential selection bias, we also conducted two comparison tests: one using the full dataset, including all rounds of financing (not only successful exits), and another using the untrimmed sample. The results of these tests are presented in the Appendix 4. Due to the privacy concerns of VC and PE, it is common for companies to only disclose information about successfully exited ventures while withholding information about unsuccessful ones. As a result, the failure exit rate, as found in the data (10.8%), is believed to be lower than the actual rate, which is typically around 50% (Capizzi, Croce, and Tenca 2022; Quintero 2017). Therefore, we will apply bi-model for parameters when modeling the firm value and this will be further discussed in Section 4.1.

A summary of statistical details for this dataset can be found in Table 3. We assume that the initial round for each company represents the time the investment period starts. The investment period length is defined as the time between the first investment rounds and exit rounds, while the exit multiple is calculated as the firm valuation at exit divided by the initial valuation, as described in Section 2.3.

For Table 3, we observe that the typical investment period for later-stage ventures lasts approximately 8 years. This is due to the fact that we only include muti-rounds of financing data for later-stage ventures. Furthermore,

Table 2. Business status for companies backed by VC later stage RoF.

Category	Number of companies	Percentage of total companies
Total numbers of companies	8, 500	100.0%
Generating revenue with last deal rounds of financing	5, 213	61.3%
Successful exit through merger and acquisition after rounds of financing	1, 959	23.1%
Out of business or bankrupted	916	10.8%

Data retrieved from PitchBook.

Table 3. Summary statistics of successful exit companies.

Variables	Count	Mean	Std	Median
Initial pre-money valuation (Millions $)	710	90.96	259.59	39.75
Initial investment time (Relative to 01.01.2002 in years)	710	6.87	3.78	6.97
Sum of investment amount (Millions $)	710	174.50	670.84	74.96
Investment increment average (Millions $)	710	20.41	101.33	5.25
Number of financing round	710	4.01	1.33	3.00
exit value (Millions $)	710	1349.88	5363.98	206.01
Exit multiples	710	19.80	45.18	5.56
Investment period length (Years)	710	8.18	3.64	8.00

Data from PitchBookand only for companies with two or more last stage RoF and exit through IPO and M&A.

the average initial firm value is approximately 90 million, and the exit multiple stands at 19.80. On average, these companies undergo four financing rounds before their eventual exit from the fund, with a total investment received in the neighborhood of 175 million.

It is also worth noting that as identified by Cumming and Johan (2013), Harris, Jenkinson, and Kaplan (2014), the VC dataset faces a crucial issue of selection bias. However, in this study, historical transaction data are solely employed for parameter modeling and providing insights into model usage. Other users such as VC portfolio managers utilizing this model can customize their parameters based on their knowledge of the companies within their portfolio, thus circumventing this problem.

3.2. Validation at the single company level

In the dataset we collected and processed from PitchBook for successful exit companies, each company provides the pre-money valuation at each funding round, investment amount and timing, exit multiples and investment period length. We use the pre-money valuation for each company to approximate the firm value immediately before each round. As the data is at the firm level instead of the portfolio level, we first validate the SIAM for one company with the choice of parameter models in Section 2.2.

The model for investment amount and timing is first tested. As shown in Table 4, the mean of the investment amount to firm value at each round is equal to 0.35 and the change (average increment) in this percentage between rounds is −0.1. Therefore, the empirical result supports the assumption of the investment amount as a fixed percentage of the firm value.[4] Meanwhile, as the exit multiples for companies are available, we assume the exit conditions are predefined based on this multiple.

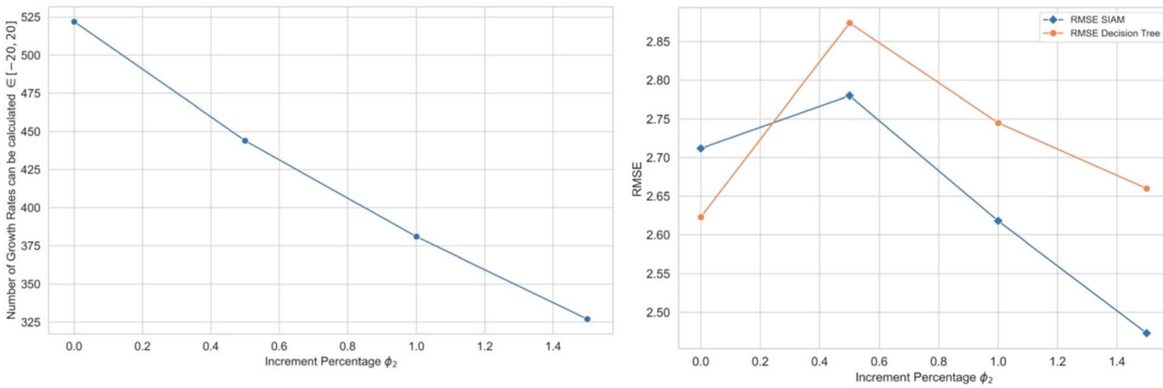
Then, we can compare the predicted investment period length using the SIAM with the actual length to validate the model. In Section 2.3.2, we model the firm value as a jump-diffusion process with two Poisson jump terms and the exit condition only depends on exit multiples γ . Therefore, the investment period length τ for each company is the period from when the company enters the portfolio until its firm value ξ achieves the exit value shown as Equation (11). Also note that we suppose the investment strategy including the investment amount and timing is predefined following the data, so the increase in firm value can be replaced by a fixed increment at the investment time. Thus, the parameters we need to define[5] for firm value are the yearly growth rate α, the firm value decrease rate ϕ^1 and increment percentage ϕ^2.

$$\tau = \inf\{t > 0 : \xi_t > \xi_0 \times \gamma\} \tag{11}$$

Table 4. Investment increment between rounds.

(In Millions $)	Mean	Std
Investment increment	20.15	229.38
Investment amount to pre-money valuation	0.35	0.59
The logarithm of investment amount to pre-money valuation	−1.62	1.08
The increment of the investment amount to pre-money valuation	−0.10	0.74
The increment of the logarithm of investment amount to pre-money valuation	−0.23	1.25

Data from PitchBook after prepossessing steps.



(a) Number of Growth Rates for Different Increment Percentages

(b) Comapring the RMSE for Growth Rates calculated through SIAM and Decision Tree

Figure 2. Comparing the growth rate α for different investment increment percentage ϕ^2.

Since the dataset only includes successfully exited companies, we assume $\phi^1 = 0$. This rate will be modified based on former research on the failure rate of VC-backed rounds of financing and will be further explained in Section 4.1. As for the increment percentage ϕ^2, we test $\phi^2 = 1.5, 1, 0.5, 0$ based on the investment and pre-money value increment summary statistics in Table 4. Then the growth rate is fitted with the actual investment period using the bisection method.

Figure 2 shows the number of companies whose growth rate can be fitted within range $[-20, 20]$. We chose this range as the yearly growth rate increases or decreases larger than 2000% are generally impractical and inconsistent with the assumptions of empirical research (Cumming and Johan 2010; Gornall and Strebulaev 2021). Through the figure, we find that as the increment percentage ϕ^2 increases, the number can be fitted decreased. This is due to a large ϕ^2 resulting in an excessive increase in the firm value which makes the growth rate impossible to calculate.

In the meantime, we also investigate the influence of varying growth rates on prediction accuracy for the investment period. For most companies backed by VC funds, the information for validating the venture's future performance like the growth rate is limited during rounds of financing. Therefore, predictions are usually made using deal information for comparable companies in VC's past funds. Following a similar setting, we split the dataset into training and testing sets and predicted the growth rate for testing sets using the training set. The rooted mean square errors (RMSE) measure is used to compare the prediction accuracy and we also use a Decision Tree Model (Maimon and Rokach 2014) as a benchmark (Further details are provided in Appendix 1). The RMSE for the two models is shown in Figure 2(b), we observe that the predictive performance of SIAM surpasses that of the Decision Tree when investment percentage ϕ^2 increases, as a lower RMSE indicates better model accuracy. As a result, balancing stability and accuracy, we select a moderate investment jump rate of $\phi^2 = 1$ for our model, aligning with the choice of Gornall and Strebulaev (2021).

3.3. Validation at the portfolio level

Following the validation for the single company's investment period, we further validate the SIAM at the portfolio level by grouping companies in our dataset. We select companies with investment periods shorter than 15 years[6] and starting in the same year as a portfolio. Following a similar validation step as Section 3.2, we fitted the investment period for each company in the portfolio and compare it with the actual investment period of the portfolio. The outcomes are illustrated in Figure 3, and the average prediction error RMSE is 2.65 years. Given that our time unit is in years and the variance of the investment period is 3.6, this prediction result is considered within an acceptable range.

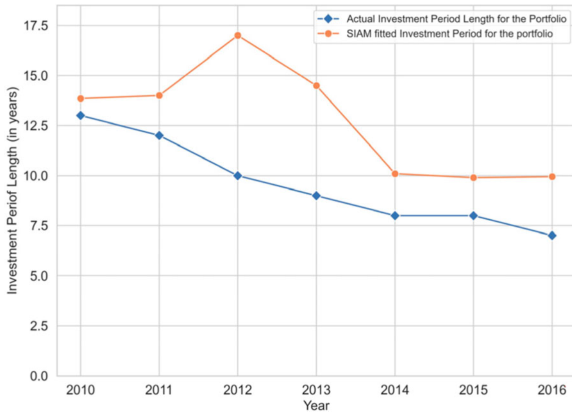


Figure 3. Evaluating the investment period fitted by SIAM for portfolios.

4. Numerical illustration and management insights

This section provides an example of how our model can be applied to the determination of the optimal investment periods for VC fund portfolios. We first explain the model setup and illustrate the minimum investment period for different expected payoffs derived from the medium value of the dataset. Then, by comparing various investment and exit strategies, valuable insights are provided for the VC industry.

4.1. Model setup for the VC portfolio

In line with the assumptions and model validation (see Section 3), we constructed a portfolio comprising 20 companies with each investment amount as a fixed percentage of the predicted firm value, constrained to a maximum amount of 300 million dollars. The size of the VC fund is relatively large compared with the market average, as we are targeting a later-stage portfolio with large initial firm values.

Based on the median value for the data collected from PitchBook and earlier research (Ferreira and Pereira 2021), the initial firm value is set as $40 million, the maximum investment period is set to 20 years and the exit multiple as 6. The investment timings are first assumed to be conditional on the firm value increment larger than the investment amount and the investment rounds happen on a yearly basis. Both the investment and exit conditions will be further studied and compared in the later sections.

The VC investment entails inherent high risks, especially when it comes to investment in growing companies. According to the empirical result from Burns (2016) and Kalyanasundaram, Subrahmanya, and Ramachandrula (2021), around 40% of VC-backed growth-stage companies that go through rounds of financing fail to achieve a successful exit. Therefore, we consider this factor by further adjusting the growth rate α and decreasing rates ϕ^1 fitted on the successful exit companies dataset in Section 3.2. The decreasing jump rate ϕ^1 is set at -0.5, which is chosen from the logarithm of average decrease firm value for deals displaying a decreasing trend. The growth rate is assumed to follow a mixed distribution with the right tail (50% of data) following the growth rate distribution of the successful exit companies dataset and the left tail following the normal distribution with a mean equal to a negative decrease mean. The portfolio is first assumed as diversified and therefore no correlation between firms, and this assumption will be further discussed in Section 4.5.

Table 5 demonstrates the final parameters we chose for the simulated VC portfolio. Note that all parameters can be adjusted following the characteristics of different VC portfolios and a sensitivity analysis will be given in section 4.6 to ensure the stabilizing of this model and the choice of the parameters.

Based on the work of Kettunen and Lejeune (2022), the solution of the model B1 can be numerically found using theorem B.1. It is calculated through the iterative checks of whether a finite set of integer linear inequalities admits a feasible solution. The decomposition algorithm and the lemma for supporting the algorithm are shown in Appendix 2. We choose the number of scenarios as $K = 500$ which proved to be stable in section 4.6.

Table 5. Parameters chosen for the simulated VC portfolio.

Variables	Values
Number of companies	10
Investment amount (% of firm value)	20%
Decrease rate within the firm value model ϕ^1	−50%
Increment percentage within the firm value model ϕ^2	100%
Exit multiple γ	6
Total investment constraint (Millions $)	300

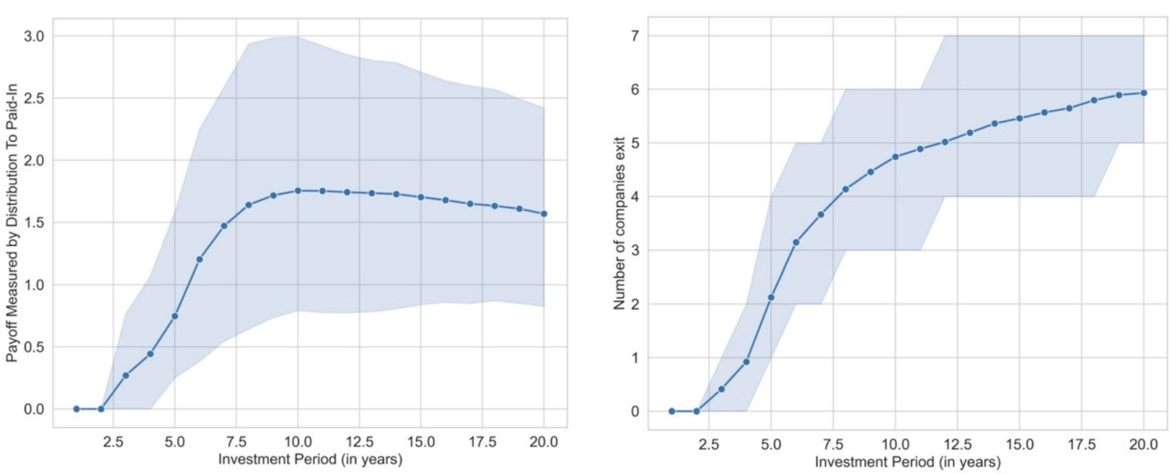
The SIAM model and numerical solving steps are coded in python[7] and all experiments run in Visual Studio Code with a 64-bit computer equipped with an Intel Core i7-1185G7 CPU running at 3.00 GHz. A summary of computational complexity and runtime results is provided in the Appendix 3 to support transparency and reproducibility.

4.2. The minimum investment period

4.2.1. Results on the payoff and investment period length

We first compare how different expected payoffs influence the investment period. The expected DPI payoff is set as $R = 0.5, 1.0, 1.5, 2$ with a probability higher than $1 - p = 0.9$. However, we find that only after an investment period longer than 7 years, the DPI payoff can be higher than 0.5 with 90% probability. Given the high volatility of the VC fund payoff, it is common that the 90% threshold results in a rather low expected payoff. Therefore, to further examine the change in payoff relative to the investment period length, we plot the simulated payoff measures with DPI for different investment period lengths in Figure 4(a) including the mean, 10th and 90th percentile. The solid line denotes the mean of the payoff, and the lower boundary of the shadow represents the 90th percentile of the simulated portfolio payoffs, corresponding to the expected DPI payoff the portfolio can reach under 90% probability. The upper boundary represents the top 10th percentile.

The figures generally demonstrate an 'S'-shaped relationship between the portfolio's DPI payoff and the investment period length. The lower boundary shows a steady increase in payoff from year 4 to year 10, converging to 0.75 afterwards. The same trend is observed in the mean and the upper boundary of the payoff, with a decline in payoff after year 10. This value corresponds to the average successful exit companies we retrieved from PitchBook, where the investment period is around 8–9 years. Note that this result is slightly higher than



(a) Payoff Measures by DPI (b) Number of Companies Successful Exit

Figure 4. The investment payoff and number of companies exit result for the simulated VC portfolio.

the average investment period of 5–7 years as shown by Cumming and Johan (2010), Sethuram, Taussig, and Gaur (2021), which is due to we focus on multiple rounds of financing modeling in this study.

We also show the number of companies that exit from the portfolio shown in Figure 4 (b). Only around 4–5 companies can successfully exit before year 10 and after year 10, 1–2 companies exit but the DPI payoff of the portfolio remains unchanged or declined. Therefore, we conclude that for a VC portfolio with the market average assumptions, it is crucial to consider terminating the VC portfolio before year 10 and letting go of approximately 50% of the companies.

4.2.2. Implication on the 'S'-shape relationship

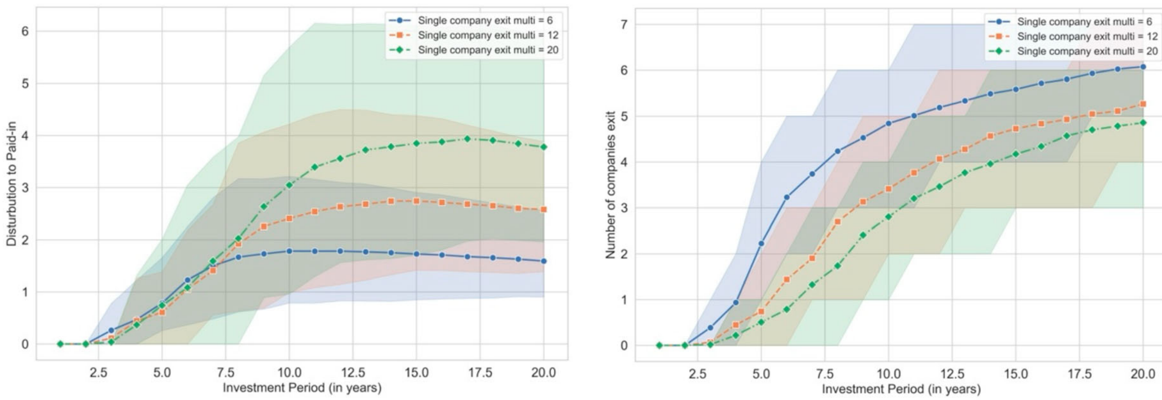
This 'S'-shaped relationship has important implications for investors, as it reflects distinct phases of portfolio performance over time. In the early years (years 1–4), the payoff remains low as investments are in their early stages, and companies are yet to achieve satisfying growth. Investors need to focus on monitoring progress and providing financial investments to portfolio companies to prepare for the growth phase. During the growth phase (years 4–10), payoffs increase significantly as companies mature and achieve successful exits, such as increasing investments in high-potential companies or facilitating strategic exits. Beyond year 10, the curve flattens, and payoffs may even decline, suggesting diminishing returns from holding investments for too long. Investors should consider exiting or restructuring their portfolios during this phase to lock in gains and reallocate resources to new opportunities.

Understanding the 'S'-shaped relationship between payoff and investment period is critical for both founders and VC fund managers. For venture founders, this insight helps in setting realistic expectations about the timeline required to achieve expected returns, which can inform fundraising strategies and exit planning. For VC fund managers, this relationship provides a framework that helps in three aspects. First, it guides the timing of investment exits by suggesting that extending the investment period beyond 10 years may not yield significant additional returns and could even lead to a decline in payoff. Second, it supports active portfolio management by highlighting the importance of focusing on the growth phase (years 4–10) when payoffs increase. Third, it assists in fund lifecycle planning by enabling VC portfolio managers to adjust investment period length in response to their expected payoff goal.

In addition to informing VC founders and portfolio managers of the investment and exiting timing, the 'S'-shaped relationship also provides clear guidance on investment strategies that maximize returns. The model estimation indicates that the optimal payoff is achieved when the VC fund is terminated around the 10-year period. Consequently, an effective investment strategy involves actively capitalizing on the growth phase by prioritizing follow-on investments in high-performing companies while planning exits during 4–10 year time points. This strategy allows investors to lock in returns during the most productive phase, while also freeing resources to pursue new opportunities. Furthermore, by continuously assessing payoff trajectories based on the model's insights, fund managers can make more informed, data-driven decisions regarding the reallocation of capital, ensuring the portfolio remains aligned with its growth potential.

The S-shaped investment curve provides a systematic basis for the timing of exit for optimal purposes within standard VC investment strategy. However, as noted in the literature, VC funds may sometimes deviate from their initial investment strategy – a phenomenon of 'style drift' – due to past performance or competitive pressure. This drift has been shown to affect their overall performance (see, for example, Cumming, Fleming, and Schwienbacher 2009). Style drift can happen on more than one dimension, for example, the stage of investments in startups, geographic location, and sector specialization. For instance, a fund with a focus on early-stage investments can shift its interest towards later-stage investments.

While these changes sometimes indicate deliberate strategic shifts, evidence from Koenig and Burghof (2022) indicates that style drift itself reduces exit rates, potentially impacting the growth phase of investments (typically years 4–10). In light of such concerns, the model framework proposed still holds when adjustments are added to the fitted dataset and threshold for the expected payoff to correct for style drift. Specifically, when an early-stage fund changes direction towards later-stage investments due to market drift, it can be expected to have a lower expected return multiple due to diminished growth opportunities. Portfolio managers can forecast an updated optimal exit duration under these new conditions by taking into account historical data for funds that have exhibited style drift and adjusting the assumptions regarding expected payoff. Consequently, the framework



(a) Payoff Measures by DPI

(b) Number of Companies Successful Exit

Figure 5. Comparing different company exit conditions based on multiples.

is maintained in a flexible form so that investors can examine how various style drift scenarios influence best timing decisions and risk exposure.

4.3. Comparing different company exit conditions

In Section 4.2, the exit of portfolio companies is conditioned on achieving exit multiples greater than or equal to 6. Given the high-risk, high-payoff nature of the VC industry, we aim to investigate whether implementing stricter exit conditions would lead to a higher return in this section.

According to the mean and median values of the exit multiples in the dataset shown in Table 3, we compare the exit multiples γ of 6, 12, and 20 to analyze the DPI payoff behavior under different investment periods.

Figure 5(a) illustrates that for the first 7 years, different exit multiples yield similar investment periods to reach the expected DPI payoff. After year 8, higher γ results in a higher payoff. Specifically, at year 10, the payoff for an exit multiple of 20 almost doubles that of an exit multiple of 6. This difference stabilizes around year 13. Corresponding to the payoff, Figure 5(b) shows the number of successful companies exits, demonstrating that higher exit multiples decrease the number of exits.

Therefore, a relatively high exit multiple does not hamper the payoff at the portfolio's earlier stages; moreover, it results in a higher DPI payoff in the long run. This reflects the nature of the VC industry, where a few high-potential companies, often referred to as unicorns, significantly contribute to the overall high payoff. However, too high exit conditions would result in companies exiting too slowly or not exiting till the end of the investment period.

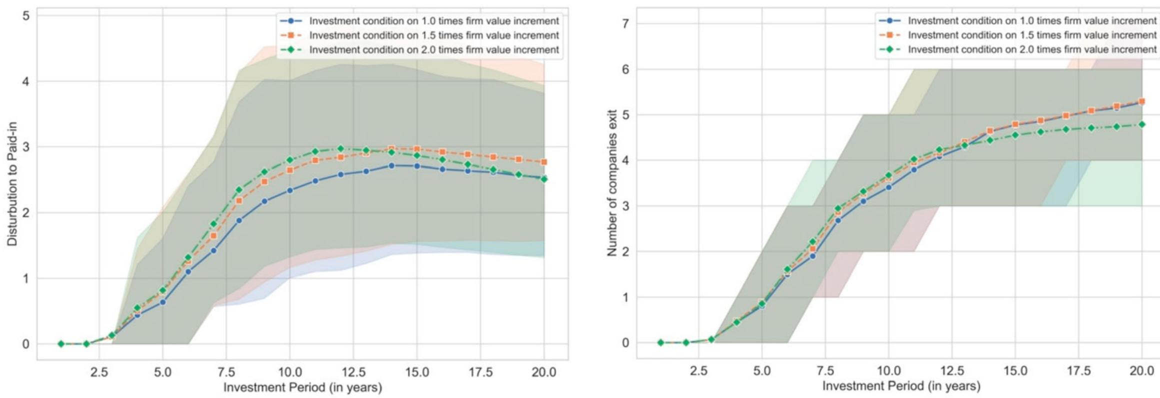
As a consequence, to achieve the maximum payoff, it is necessary to wait for these exceptional companies to reach their full potential by setting relatively high exit multiples. Considering both the payoff and number of exits, we select the exit multiple 12 to use in the later experiment.

4.4. Comparing different investment conditions

In this section, we explore the influence of different investment conditions θ and total investment amount values on the investment period.

Recall that we earlier assume $\theta = 1$, in which the investment is conditioned on the expected firm value increment being higher than one times the investment amount. Now, we select θ values of 1.0, 1.5, and 2.0 as the 'lower', 'medium' and 'higher' investment expectations, respectively.

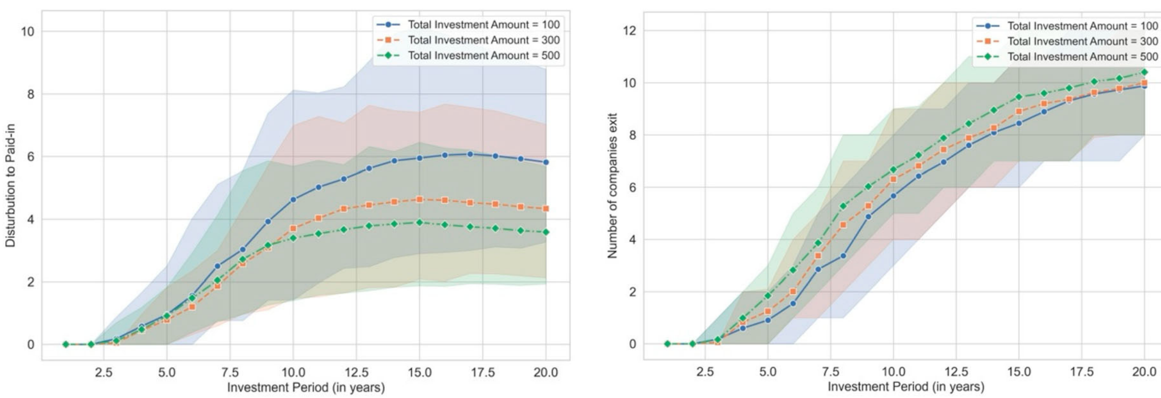
Figure 6(a) illustrates that higher investment conditions, $\theta = 2$, slightly outperform others in the middle of the investment period (years 6–12), but then the payoff all converge to a similar level. Thus, we conclude that there are no significant differences in the payoff across different investment conditions.



(a) Payoff Measures by DPI

(b) Number of Companies Successful Exit

Figure 6. Comparing different investment conditions.



(a) Payoff Measures by DPI

(b) Number of Companies Successful Exit

Figure 7. Comparing different total investment amounts.

Additionally, we compare how different total investment sizes influence the payoff, selecting the portfolio sizes 100, 300, and 500. As Figure 7 DPI_investperiod_invest_amount 7 (a) shows, the payoffs for the first 7 years are similar, while a higher total amount results in lower DPI payoffs in the later investment period. However, we find that the net present value of the payoff for a higher total investment is greater. Therefore, there is a trade-off between the net payoff and the relative payoff measurements, so it depends on the need for the portfolio manager to choose the measure to follow their aim.

4.5. A diversified portfolio or not

In the earlier studies, we assume the portfolio to be uncorrelated. However, many VCs would target specific industries or areas and therefore cannot be diversified. Therefore, we further include the correlation factors within the firm value modeling to model the portfolio with a positive correlation.

Figure 8 demonstrates the portfolio DPI payoffs. With a positive correlation, we find the portfolio shows a significantly higher payoff compared with the uncorrelated one. Also, the number of companies' successful exits is large for the positively correlated portfolio. This corresponds to the specification and success of the venture which has been proved by Gompers, Kovner, and Lerner (2009), Chemmanur, Loutskina, and Tian (2014), Lindgaard Christensen (2007).

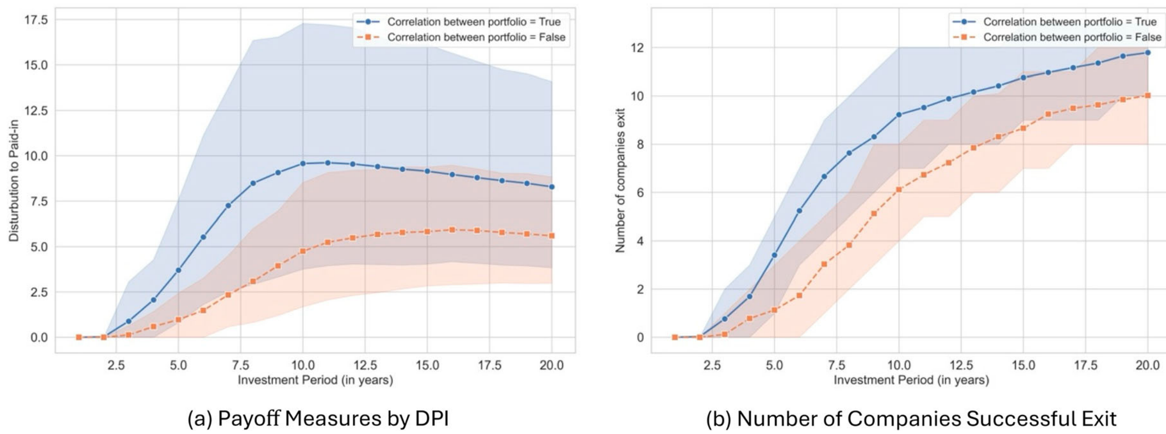


Figure 8. Comparing the correlated and diversified portfolio.

In earlier studies, we assumed the portfolio to be uncorrelated. However, many VCs are specialized in certain fields and they will then target firms in specific industries or areas, resulting in portfolios that cannot be fully diversified. Therefore, we include correlation factors within the firm value modeling to represent portfolios with correlations.

We compare the DPI payoff between a positively correlated portfolio, in which the firm values between portfolio companies are positively correlated with $\rho = 0.2$. Specifically, the Wiener process in Equation (10) are updated with the correlated normal random variables Z that are obtained by:

$$Z = L \cdot W$$

where L is the Cholesky decomposition of the correlation matrix.

Figure 8(a) demonstrates the portfolio DPI payoffs under these conditions. With a positive correlation, we observe that the portfolio exhibits significantly higher payoffs compared to an uncorrelated portfolio. Additionally, the number of successful company exits is greater for the positively correlated portfolio as shown in Figure 8(b). As evidenced by studies such as Gompers, Kovner, and Lerner (2009), Chemmanur, Loutskina, and Tian (2014), Lindgaard Christensen (2007), specialization aids in the success of ventures. For example, the specialized knowledge and expertise that VCs develop can lead to a better selection of investments and value-added strategies, and companies within the portfolio can share knowledge and experience, which allows companies to achieve higher returns and improved overall performance.

4.6. Sensitivity and stability test for SIAM

This part shows that the model is robust to different choices of initial firm value and investment amount (percentage of the firm value) and stable across the different simulation samples.

We first test the stability of the SIAM by repeatedly calculating the minimum investment period under the setting in Section 4.1. It's found that for all the repeated tests, the mean of DPI payoffs generally converge at 1.8 or slightly decrease after year 10. This result corresponds to what we demonstrated in Section 4.2.

In the meantime, we also test the SIAM for different initial firm values and investment amounts (set by different percentages of the firm value) for values in Table 6. Results show that for different firm values and investment

Table 6. Sensitivity test parameters.

Parameters	Testing Range
Initial firm value $\xi_{i,0}$	[20.0, 80.0]
Investment amount (% of firm value) In	[0.3, 0.6]

amounts, the same conclusion found in Section 4.2,4.3, and 4.4 can be validated which demonstrates the robustness of the model. As a result, the SIAM model is proved to be robust for VC fund portfolio investment period projection and the insights found by the model are valid.

5. Conclusion

This paper introduces a new model, Sequential Investment Allocation Model (SIAM), for modeling the VC staged financing portfolio investment period. Our analysis revealed several key findings that contribute to the understanding of the investment duration within VC funds.

Firstly, the observed 'S'-shaped relationship between the payoff and investment period indicates that most of the portfolio's payoffs reach their peak around 8–10 years. In the meantime, setting a high exit multiple increases the payoff, while excessive selectivity results in too few companies achieving successful exits, which negatively impacts the maximum payoff and the likelihood of achieving the expected return. A recommended exit multiple of around 12 balances a satisfactory payoff with a reasonable number of exits.

Additionally, our study reveals that positively correlated portfolios exhibit significantly higher DPI payoffs compared to uncorrelated diversified funds. This finding supports the VC strategy that specialized expertise in a specific industry allows companies within the portfolio to exchange knowledge and experience, thereby helping each other achieve higher returns and overall better performance.

In the meantime, the assumptions and the modeling results of the model are validated by empirical data retrieved from PitchBook. This strengthens the robustness of our findings and ensures the applicability of the model in further analysis of investment strategies in VC fund portfolios.

However, several boundary conditions could be further enhanced. Firstly, the model uses firm value as the financial representation of VC portfolio companies. In practice, additional factors such as other financial metrics, the terms between entrepreneurs and investors, and the industry's overall potential need to be considered. This requires the availability and quantification of those data. Additionally, the assumptions used in firm value modeling can be strengthened with more comprehensive data. Specifically, if data on unsuccessfully exited firms and how their firm values changed over time were available, the model's parameters could be better calibrated.

Furthermore, while our model incorporates a discount rate based on the average interest rate to reflect market conditions, market valuation trends throughout different time periods are not explicitly included. Specifically, market-wide valuation shifts could influence investment and exit strategies by affecting the fund availability or exit opportunities. Future research could extend the model by integrating macroeconomic factors, such as public market performance or industry-specific valuation trends with time factors, to further refine investment timing decisions modeling.

Finally, an analysis of the strategic, non-monetary determinants of VC fund returns provides a fertile ground for theoretical model development concerning investment timing and financial mechanisms of VC.

As a result, the implications of our research extend to both academics and industry practitioners. Our model offers valuable insights for VC fund managers seeking to optimize their investment decisions and effectively navigate the complexities of the investment period. By understanding the relationship between payoff and investment duration, fund managers can develop more informed strategies to achieve their desired financial outcomes. Additionally, our findings emphasize the importance of considering company performance in investment decisions, highlighting the potential benefits of a rigorous and selective investment approach.

Notes

1. Disjunctive normal form is a logical formula consisting of a disjunction of conjunctions. It can also be described as the composite logical operation made of an OR of ANDs (Hilbert and Ackermann 2022).
2. This is equivalent, as the main objectives for both levels are to maximize the portfolio companies' final enterprise value or profitability. By considering management fees, carried interest, and contract terms, the SIAM model will retain the same structure and can be considered from the perspective of LPs (Robinson and Sensoy 2013).
3. One challenge in grouping the VC deal on the company is that not all financing rounds disclose pre-money valuation information. Unlike publicly traded companies, private firms are not required to report financial details consistently. However, this is a well-recognized characteristic of VC datasets rather than a fundamental limitation of our study. PitchBook is one of the most

comprehensive and widely used data sources for VC investment analysis (Gornall and Strebulaev 2021). Previous studies have demonstrated that even with some missing valuation data, such datasets reliably capture investment patterns and firm valuation dynamics (Gompers 2022). While future research could explore complementary sources, such as financing deal records from news reports, integrating such data would require extensive validation and is beyond the scope of this study. Given the robustness of PitchBook as a research dataset and its established use in the literature, we believe our validation of the SIAM model remains well-supported within the constraints of available data.

4. We also find the model Gornall and Strebulaev (2021) proposed, between the future rounds investment amount to firm value and the earlier rounds holds, with an R-Square equal 0.3. However, we still adopt the investment amount as a fixed percentage of the firm value for all rounds as it fits with the data and works more efficiently.
5. The risk-free rate is set as 0.03, the volatility σ is set as 0.5, and the decrease Poisson jump is set as 5 years following Brandon, Hoesli, and Shan (2022) and Gornall and Strebulaev (2021).
6. There are some companies with extreme investment periods. To prevent that from influencing the result, we assume the maximum period as 15 years as more than 90% companies exit within 10 years in our dataset.
7. Code is available at https://github.com/GMD67/Later_Stage_VC_Portfolio

Disclosure statement

No potential conflict of interest was reported by the author(s).

Data availability

The data used in this study is provided by PitchBook. Access to the data is available upon request from PitchBook.

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Appendices

Appendix 1. Comparing the SIAM with Decision Tree for growth rate and investment period prediction accuracy

A1.1. Single company level

As mentioned in the main sections, we have the information for each company such as initial firm value, total investment amount, final exit multiples, investment period length and have assumed the rest parameters like the parameters controls the jumps. Therefore, the only unknown parameter that needs to be fitted is the growth rate. Therefore, we calibrate the data based on the data and incorporate the use of Decision Tree model to assist in the prediction accuracy calculation for SIAM.

A Decision Tree model is trained on the training data (with features including initial firm value, total investment amount, exit multiples, etc. and the target is the investment period length) and used to predict the investment period for the testing set. Then we input the predicted investment period to SIAM and find the predicted growth rate. Finally, we can calculate the square root of the mean squared error (RMSE) between the predicted growth rate and the actual growth rate.

Additionally, the data is input into a Decision Tree model to predict the growth rate directly. Cross-validation is applied to further ensure the effectiveness of this validation process.

A1.2. Portfolio level

Companies are grouped into portfolios according to their initial investment years, spanning from 2002 to 2016. Then, we consider the test set to be composed of portfolios from 2010 to 2016, and the training set includes portfolios from the eight years leading up to the respective testing year. We apply a similar methodology as described earlier, where we initially train and predict the growth rate using a Decision Tree model.

Subsequently, for each company within the test set, we employ SIAM to determine their investment period, assuming the longest investment period among them to represent the portfolio's investment period. This value is then compared to the actual longest investment period observed within the test portfolio.

Appendix 2. Solving the SIAM

After defining the time to exit parameter d and investment amount In for portfolio companies, the only decision variable left is τ, therefore, we can rewrite SIAM as

$$\min \quad \tau$$

$$\text{s.t.} \quad \tau \in \bigvee_{t\in\{0,\cdots,\tau\}} \mathcal{G}_t$$

$$\tau \in \mathbf{T} \subset \mathbb{Z}_+$$

where

$$\mathcal{G}_\tau := \left\{ \mathbf{d}, \boldsymbol{\xi}, \mathbf{In} : \mathbb{P}\left(\frac{\sum_{t'=1}^{t} \sum_{i\in C_{t'}(\mathbf{d})} \frac{\xi_{i,t'}}{(1+r)^{t'}}}{\sum_{t'=0}^{t-1} \sum_{i\in I_{t'}(\mathbf{d})} \frac{In_{i,t'}}{(1+r)^{t'}}} \geq R \right) \geq 1-p \right\}$$

Based on the work of Kettunen and Lejeune (2022), the solution of the model can be numerically found using Theorem A2.1. It is calculated through the iterative checks of whether a finite set of integer linear inequalities admits a feasible solution. Two lemmas are used to prove Theorem A1: Lemma A2.2 allows the check of the feasibility of SIAM and bound the optimal solution of SIAM. Lemma A2.3 proves the minimum solution of the below equation provides the optimal solution for SIAM. We refer the reader for detailed proof.

Theorem A.1: *Consider $l \in \{0, \cdots, T\}$. The return level R in SIAM can be reached by time l if and only if the system of integer linear inequalities* FA_l

$$FA_l : \begin{cases} \sum_{t'=0}^{t} \frac{\sum_{t'=1}^{t+1} \sum_{i\in C_{t'}(\mathbf{d})} \frac{\xi_{i,t'}}{(1+r)^{t'}}}{\sum_{t'=1}^{t} \sum_{i\in I_{t'}(\mathbf{d})} \frac{In_{i,t'}}{(1+r)^{t'}}} \geq R\beta_t^k \\ \qquad +(1-\beta_t^k) \sum_{t'=0}^{t} M_{t'}^k & k \in \mathbf{K}, t \in \tilde{\mathbf{T}} \\ \sum_{k\in\mathbf{K}} q^k \beta_r^k \geq p \\ \beta_t^k \in \{0,1\} & k \in \mathbf{K} \end{cases}$$

admits a feasible solution.

Lemma A2.2*: If $(\bar{y}, \overline{beta})$ is a feasible solution for the equation, then*

- *$(\bar{\beta}, \bar{\gamma})$ is feasible for A-RM and $(\tau = r)$ is feasible for A-M*
- *$(\bar{\beta}, \bar{\gamma})$ with $\bar{\gamma}_t = 1$ if $t = r$ otherwise $\bar{\gamma}_t = 0$ is feasible for A-RM and $(\tau = r)$ is feasible for A-M*

Lemma A2.3*: Let $l^* \in \{0, \cdots, T-1\}$ and τ^* be the optimal value of AM a*

- *If the feasible solution set for equation $l = l^*$ is $\emptyset$, then $\tau^* > l^*$*
- *If the feasible solution set for equation $l = l^*$ is not $\emptyset$, then $\tau^* \leq l$*
- *If the feasible solution set for equation $l = l^*$ is $\emptyset$, and when $l = l^* + 1$ is not $\emptyset$, then $\tau^* = l^* + 1$.*

Therefore, based on the above Theorems, a solution for SIAM can be calculated following algorithm:

Appendix 3. Computational complexity and runtime summary

As introduced above, the decomposition algorithm solves the SIAM by iteratively increasing the investment period length and checking feasibility at each step. The algorithm requires at most T iterations, where T is the maximum investment horizon.

At each iteration, the algorithm performs two main steps: (1) Simulating scenarios for each company firm value paths, (2) solving a feasibility-checking mixed-integer program (denoted as *FA*). Therefore, the total computational complexity is at most

$$\left(\sum_{l=1}^{T} \mathrm{Cost}_{FA_l}\right)$$

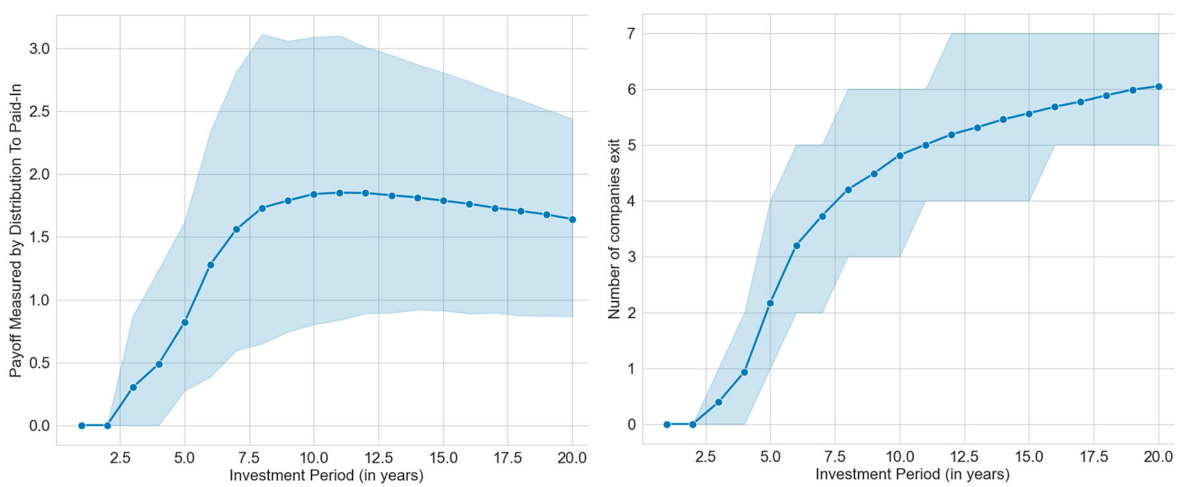
where Cost_{FA_l} is the computational cost of solving the feasibility problem FA. This cost depends on three main factors: the number of companies (E), the time period length (l), and the number of decision variables and constraints in the model. To improve transparency and reproducibility, the table below reports key computational statistics from selected experiments.

Appendix 4. Further validation on different choices of data

As mentioned in Section 3.1, we conducted two additional validation tests using two datasets. One includes all rounds of financing and successful exit deals, while the other consists only of successful exit deals. Neither dataset was trimmed. After removing deals

Algorithm: Decomposition Algorithm

Let β denote a feasible solution for equation B2, $l \in 1, \ldots, T$. Let τ^+ be the optimal value of SIAM.
Step 1: Initialization and Fix Variables
$l = 0$;
$\beta_{tk} = 0$ for $t \in 2, \ldots, T, k \in K$
Step 2: Simulate Scenarios
Simulate ξ_{ikl} for $i \in I, k \in K$
Step 3: Feasibility Check
Check emptiness of equation FAI
Step 4: Stopping Criterion
If FAI is feasible then:
    Optimal value of SIAM is r: $\tau^+ = l$
    Stop.
Else:
    If FAI is not empty and $r < T$, then:
        Increment l: $l := l + 1$
      Go to Step 2
    If FAI is empty and $l = T$, then:
      SIAM is infeasible
      Stop.



(a) Payoff Measures by DPI

(b) Number of Companies Successful Exit

Figure A1. Comparison test for the successful exit data without trimming.

without investment dates or amounts (essential information for the later analyze), we obtained 756 successful exits and 1903 rounds of financing. We then repeated the procedures from Section 3.2 to validate the firm value model and assess the fitting results.

Our findings show similar results to the main analysis. The RMSE decreases as the jump rate increases, but an excessively high jump rate leads to many deals being unfittable. Based on this, we validated the choice of the same jump rate for the firm value model. Regarding the growth rate, its mean and standard deviations are slightly higher for the full dataset, as shown in Table A1. By incorporating this adjusted growth rate (μ) into the model, we find that the maximum payoff is achieved around years 10, following a similar trend. Therefore, our conclusion that a 10-year investment horizon is optimal for general VC funds remains valid.

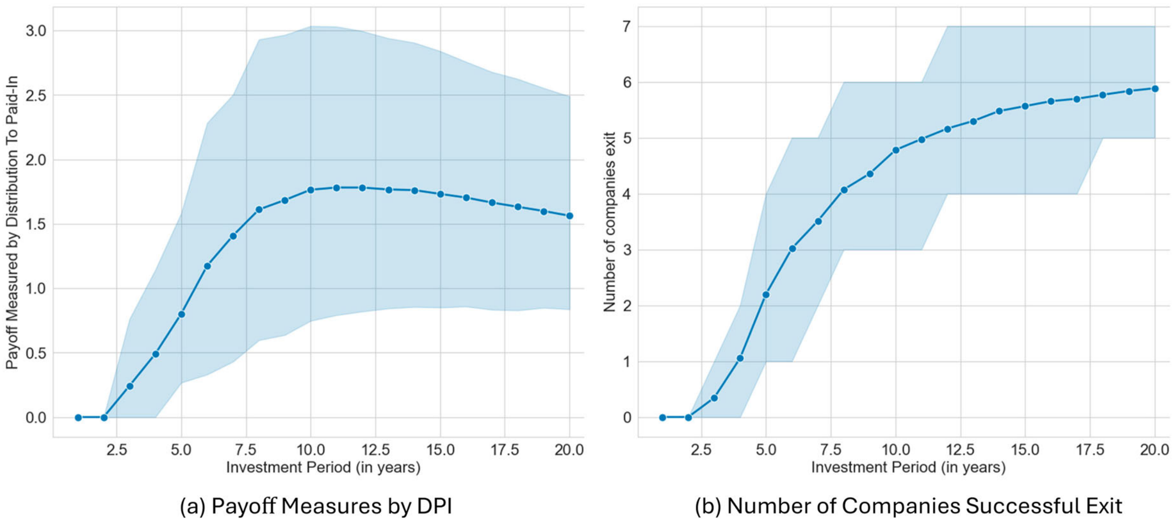


Figure A2. Comparison test for all rounds of financing data, both exited and non-exited.

Table A1. Comparison of fitted growth rate for different dataset

Dataset	Number of Companies within this dataset	Mean of Fitted Growth Rate
All deals with deal date and amounts	2, 659	3.4
Successfully Exits	756	3.1
Successfully Exits with trimmed	710	3.0

Table A2. Computational statistics and runtime summary for selected experiments

	Maximum investment period	Scenarios Number	Avg Computational Time (seconds)
1	10	500	2.6
2	10	1000	6.1
3	10	2000	9.8
4	20	500	5.3
5	20	1000	10.2
6	20	2000	20.0