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Power Minimization for Half-Duplex Relay in Fluid Antenna System

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***Abstract*—The exploration of Sixth Generation (6G) networks has led to a paradigm shift, emphasizing the synergy between next-generation reconfigurable antennas and wireless communications. Notably, Fluid Antenna System (FAS) stands out as a promising solution for advancing 6G networks. One of the unique features of FAS is its ability to instantly reconfigure the antenna's position over a given area, enabling extreme spatial diversity with just one antenna and one Radio Frequency (RF)-chain. This spatial diversity can be harnessed to create an energy-efficient system. While FAS has been explored in various scenarios, its performance as a half-duplex relay remains unclear. In this letter, we investigate the application of fluid antennas in half-duplex relays, considering both Amplify-and-Forward (AF) and Decode-and-Forward (DF) schemes. Moreover, we jointly minimize the total transmit power of the Base Station (BS) and relay through optimal port and transmit power for both schemes, while ensuring the user rate requirement is met. We demonstrate that optimal port and transmit power can be determined using a one-dimensional search for AF and a closed-form expression for DF. Simulation results show that FAS relay is more energy efficient than Traditional Antenna System (TAS) relay with fixed-position antenna.**

***Index Terms*—6G, Fluid Antenna System, Half-Duplex Relay, Amplify-Forward, Decode-Forward, Power Minimization, Energy Efficient**

I. Introduction

The development of Sixth Generation (6G) networks has brought about a paradigm shift, with the potential to combine next-generation reconfigurable antennas with wireless communications to further enhance performance [1]. Nevertheless, network performance improvements will often result in increased power consumption [2]. Therefore, it is essential to explore technologies that can achieve equal or better performance while simultaneously reducing power consumption. One of the promising technologies to realize this is Fluid Antenna System (FAS) [3], [4], [5] and [6]. FAS is a wireless communication system that encompasses any software-controllable fluidic, dielectric or conductive structures, including but not limited to liquid-based antennas, pixel-based antennas, and metasurfaces, that can dynamically reconfigure their shape, size, position, length, orientation, and other radiation characteristics. In its simplest form, a fluid antenna consists of a single Radio-Frequency (RF) chain and multiple preset positions, referred to as ports, distributed within a designated area [7].

Several studies have explored the performance limits of FAS. For example, [8] proposed an efficient channel estimation method for FAS using machine learning techniques, while [9] has investigated online learning framework to address the port selection issue without instantaneous channel state information. In addition, [10] explored the integration of fluid antenna and index modulation techniques to enhance adaptability and wireless communication performance, and [11] analyzed the outage probability. Furthermore, [12] characterized the diversity and multiplexing trade-off for multiple-input multiple-output FAS, while its antenna configurations could be optimized using physics-inspired heuristics such as coherent ising machines and quantum annealing, as discussed in [13]. Overall, existing studies show that FAS is superior to Traditional Antenna System (TAS) with fixed-position antenna. As a result, FAS has also been integrated with other emerging technologies such as full-duplex communications [14], non-orthogonal multiple access [15], and terahertz communications [16], among others to further enhance their performance.

While several scenarios have been investigated, the performance of fluid antennas in half-duplex relay remains elusive. Half-duplex relays are a reliable and cost-effective technology that can improve network coverage without significant power consumption and they have been employed in the fifth generation networks [17]. There are mainly two types of relays: i) Amplify-and-Forward (AF) and ii) Decode-and-Forward (DF) [18]. In the DF scheme, the radio signal is decoded and processed in the digital baseband before being transmitted to the users in an optimized manner. In contrast, the AF relay amplifies the incoming signal and transmits it to the users without baseband processing. Typically, the transmission and reception processes in half-duplex relays occur in different time slots. Full-duplex relays, developed more recently, can simultaneously receive and transmit signals. However, full-duplex relays are prone to self-interference between their transmitting and receiving antennas [19]. As a result, complex digital and analog interference cancellation techniques are

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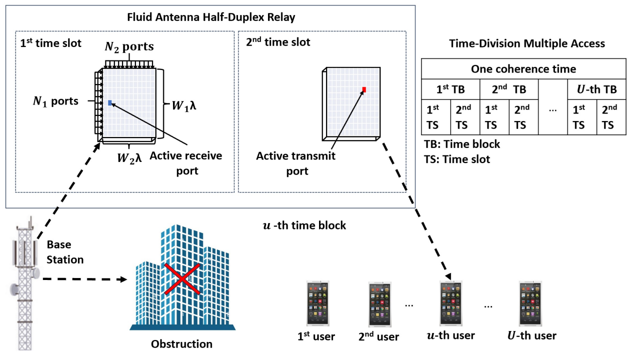

Figure 1. A BS serving multiple users through half-duplex relay with 2D fluid antenna.

required. To mitigate this, virtual full-duplex relays, consisting of at least two half-duplex relays, have been proposed in [19]. As a preliminary investigation into the advantages of fluid antenna, we focus on half-duplex relays in this letter.

Motivated by the aforementioned research gaps, this letter explores the use of half-duplex relay in FAS. Specifically, we investigate the minimum total transmit power required by the base station (BS) and relay to meet the users' minimum rate requirements for both AF and DF schemes. The main contributions of this letter are summarized as follows:

- We design an energy-efficient system by formulating optimization problems that minimize the total transmit power of the BS and relay, subject to the user's minimum rate requirement. For both AF and DF schemes, we seek the optimal port selection for the fluid antenna relay and the optimal transmit power for the BS and relay.
- While the optimization problem is generally non-convex, we exploit its structure and show that the optimal port and power allocation can be determined through a one-dimensional search for AF and a closed-form expression for DF.
- Simulation results are provided to illustrate the impact of the number of ports, the surface area of the fluid antenna, and the location of the FAS relay. We demonstrate that FAS relay is more energy-efficient than TAS relay with fixed-position antenna in all scenarios. Moreover, our discussions offer fundamental insights into the performance limitations of FAS relays.

The remainder of the letter is structured as follows: Section II presents the system model, Section III analytically solves the power minimization problem, Section IV provides numerical results and discussion, and Section V concludes the letter.

II. System Model

As shown in Fig. 1, we consider a scenario with a single BS, a relay, and U multiple users. The direct channels between the BS and the users are assumed to be obstructed and strongly attenuated. Thus, communication can only be made through the relay. To avoid multi-user interference, we consider a time-division multiple access (TDMA). Specifically, the transmission time, which is assumed to be within the coherence time, is divided into U time blocks. Each time block is used to serve a single instance of particular user. Without loss of generality, let us focus on a single instance of a time block.

We assume that the BS and the user are equipped with a fixed-position antenna, while the relay is equipped with a two-dimensional (2D) fluid antenna. Specifically, the fluid antenna has a surface area of $W = W_1\lambda \times W_2\lambda$, where λ is the wavelength. In this surface area, there are $N = N_1 \times N_2$ ports (also known as preset positions) where N_i ports are uniformly distributed along the length of $W_i\lambda$, with $i \in \{1,2\}$. To simplify the notation, we refer the 2D indices of the ports from top-to-bottom and left-to-right, and assign the resulting numbers as the new indices for the ports. For instance, the (n_1, n_2)-th port can be denoted as

$$k\left(n_1, n_2\right) = \left(n_2 - 1\right) N_1 + n_1. \tag{1}$$

The spatial correlation between the (n_1, n_2)-th and $(\tilde{n}_1, \tilde{n}_2)$-th ports can be modeled as [12]

$$\begin{aligned} &J_{k(n_1,n_2),k(\tilde{n}_1,\tilde{n}_2)} \\ =&j_0\left(2\pi\sqrt{\left(\frac{|n_1-\tilde{n}_1|}{N_1-1}W_1\right)^2+\left(\frac{|n_2-\tilde{n}_2|}{N_2-1}W_2\right)^2}\right), \end{aligned} \tag{2}$$

where $j_0\left(\cdot\right)$ is the zero-order spherical Bessel function (or the Sinc function). Consequently, the correlation matrix can be rewritten as [12]

$$\boldsymbol{J} = \begin{bmatrix} J_{1,1} & \cdots & J_{1,k(\tilde{n}_1,\tilde{n}_2)} & \cdots & J_{1,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ J_{k(n_1,n_2),1} & \cdots & J_{k(n_1,n_2),k(\tilde{n}_1,\tilde{n}_2)} & \cdots & J_{k(n_1,n_2),N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ J_{N,1} & \cdots & J_{N,k(\tilde{n}_1,\tilde{n}_2)} & \cdots & J_{N,N} \end{bmatrix}. \tag{3}$$

Since $\boldsymbol{J}$ is a symmetric matrix, we can rewrite it as $\boldsymbol{J} = \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{Q}^H$ using the eigenvalue decomposition. The complex channel between the BS and the relay can be modeled as [12]

$$\boldsymbol{h}_{b,r} = \sqrt{\frac{1}{d_{b,r}^{\alpha}}}\boldsymbol{Q}\boldsymbol{\Lambda}^{1/2}\boldsymbol{z}_{b,}, \tag{4}$$

where $d_{b,r}$ is the distance from the BS to relay, α is the path loss exponent, and $\boldsymbol{z}_b \in \mathbb{C}^{N\times 1}$ is an independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random vector with zero mean and variance of $\boldsymbol{I}_N$. Likewise, the complex channel between the relay and u-th user can be modeled as [12]

$$\boldsymbol{h}_{r,u} = \sqrt{\frac{1}{d_{r,u}^{\alpha}}}\boldsymbol{Q}\boldsymbol{\Lambda}^{1/2}\boldsymbol{z}_r, \tag{5}$$

where $d_{r,u}$ is the distance from the relay to the u-th user, and $\boldsymbol{z}_r \in \mathbb{C}^{1\times N}$ is an i.i.d. circularly symmetric complex Gaussian random vector with zero mean and variance of $\boldsymbol{I}_N$.[1]

[1]For consistency, we represent the complex channel between the relay and UE as a column vector instead of a row vector.

In this letter, we investigate the two common half-duplex relays, namely: (i) AF and (ii) DF schemes. Note that a half-duplex relay requires two time slots to complete a transmission. Thus, each time block is further separated into two time slots to complete the communication. In AF scheme, the received signal at the relay during the first time-slot of the u-th time block is

$$y_r[1,u] = \left(\boldsymbol{a}_{r,u}^H \boldsymbol{h}_{b,r}\right) x_{b,u} + \sigma_r, \tag{6}$$

where $\boldsymbol{a}_{r,u} \in \{\boldsymbol{e}_1, \ldots, \boldsymbol{e}_N\}$ is the activated port for reception, $\boldsymbol{e}_n$ represents an all-zero column vector except the n-th entry being unity, $x_{b,u}$ is the information signal for the u-th user with $\mathbb{E}\left\{|x_{b.u}|^2\right\} = P_{b,u}$, where $P_{b,u}$ denotes the transmit power of the BS for the u-th user, and σ_r is the additive white Gaussian noise with a noise level of N_0. During the second time slot of the u-th time block, the received signal of the u-th user is

$$\begin{aligned} y_u^{\rm AF}[2,u] &= \left(\boldsymbol{a}_{t,u}^H \boldsymbol{h}_{r,u}\right)\left(\beta y_r[1,u]\right) + \sigma_u, &(7)\\ &= \left(\boldsymbol{a}_{t,u}^H \boldsymbol{h}_{r,u}\right)\left(\beta\left(\left(\boldsymbol{a}_{r,u}^H \boldsymbol{h}_{b,r}\right)x + \sigma_r\right)\right) + \sigma_u, &(8)\end{aligned}$$

where $\boldsymbol{a}_{t,u} \in \{\boldsymbol{e}_1, \ldots, \boldsymbol{e}_N\}$ is the activated port for transmission, $\beta = \sqrt{\frac{P_{r,u}}{P_{b,u}|\boldsymbol{a}_{r,u}^H \boldsymbol{h}_{b,r}|^2 + N_0}}$ is the amplifying gain, $P_{r,u}$ is the transmit power of the relay for u-th user, and σ_u is the additive white Gaussian noise with a noise level of N_0. Thus, the signal-to-noise ratio (SNR) of the u-th user is given as

$$\mathrm{SNR}_{\mathrm{HD},u}^{\mathrm{AF}} = \frac{\mathit{\Gamma}_{b,u}\left|\boldsymbol{a}_{r,u}^H \boldsymbol{h}_{b,r}\right|^2 \mathit{\Gamma}_{r,u}\left|\boldsymbol{a}_{t,u}^H \boldsymbol{h}_{r,u}\right|^2}{\mathit{\Gamma}_{b,u}\left|\boldsymbol{a}_{r,u}^H \boldsymbol{h}_{b,r}\right|^2 + \mathit{\Gamma}_{r,u}\left|\boldsymbol{a}_{t,u}^H \boldsymbol{h}_{r,u}\right|^2 + 1}, \tag{9}$$

where $\mathit{\Gamma}_{b,u} = \frac{P_{b,u}}{N_0}$ and $\mathit{\Gamma}_{r,u} = \frac{P_{r,u}}{N_0}$ represent the transmit SNRs, which correspond to the scaled versions of the transmit power of the BS and relay, respectively, for u-th user. Since two time slots are required in half-duplex relay, the rate of the u-th user is given as

$$R_{\mathrm{HD},u}^{\mathrm{AF}} = \frac{1}{2}\log\left(1 + \mathrm{SNR}_{\mathrm{HD}}^{\mathrm{AF}}\right). \tag{10}$$

In DF scheme, the received signal at the relay during the first time slot of the u-th time block follows (6). The relay then decodes and forwards the signal $x_{b,u}$ to the u-th user.[2] Therefore, the received signal of the u-th user during the second time slot of the u-th time block is

$$y_u^{\mathrm{DF}}[2,u] = \left(\boldsymbol{a}_{t,u}^H \boldsymbol{h}_{r,u}\right) x_{b,u} + \sigma_u, \tag{11}$$

and the SNR of the u-th user is given as

$$\mathrm{SNR}_{\mathrm{HD},u}^{\mathrm{DF}} = \min\left(\mathit{\Gamma}_{b,u}\left|\boldsymbol{a}_{r,u}^H \boldsymbol{h}_{b,r}\right|^2, \mathit{\Gamma}_{r,u}\left|\boldsymbol{a}_{t,u}^H \boldsymbol{h}_{r,u}\right|^2\right). \tag{12}$$

Consequently, the rate of the u-th user is

$$R_{\mathrm{HD},u}^{\mathrm{DF}} = \frac{1}{2}\log\left(1 + \mathrm{SNR}_{\mathrm{HD}}^{\mathrm{DF}}\right). \tag{13}$$

It is worth noting that the spectral efficiency of half-duplex relays can be further improved by employing full-duplex relays or the concept of virtual full-duplex relays. However, extending the investigation to these approaches is left for future work, as more in-depth discussions are required.

[2]Similar to [18], we assume the signal is successfully decoded without error.

III. Optimal Port and Power Minimization

In this section, we aim to minimize the total transmit SNRs of the BS and relay subject to the minimum rate requirement via optimal port and transmit power. It is important to recall that the transmit SNRs are scaled versions of the transmit power.

A. Amplify-Forward Scheme

For AF scheme, the optimization problem can be formulated as

$$\begin{aligned} \min_{\mathit{\Gamma}_{b,u}, \mathit{\Gamma}_{r,u} \boldsymbol{a}_{r,u}, \boldsymbol{a}_{t,u}, \forall u,} \quad & \sum_{\forall u} \mathit{\Gamma}_{b,u} + \mathit{\Gamma}_{r,u} &(14a)\\ \text{s.t.} \quad & R_{\mathrm{HD},u}^{\mathrm{AF}} \geq R_{\min}, \forall u, &(14b)\\ & \boldsymbol{a}_{r,u} \in \{\boldsymbol{e}_1, \ldots, \boldsymbol{e}_N\}, \forall u, &(14c)\\ & \boldsymbol{a}_{t,u} \in \{\boldsymbol{e}_1, \ldots, \boldsymbol{e}_N\}, \forall u, &(14d)\end{aligned}$$

where (14a) minimizes the total transmit SNRs of the BS and relay, (14b) ensures that each user rate satisfies the minimum rate requirement, while (14c) and (14d) ensure that only one active port is activated at a time. Note that (14) is generally a non-convex optimization problem due to (14b), (14c), and (14d).

To address (14c) and (14d), we can exploit the fact that $\frac{c_1 x c_2 y}{c_1 x + c_2 y + 1}$ is monotonically increasing w.r.t. to x for $c_1, c_2, x, y, > 0$. Therefore, we can conclude that the optimal $\boldsymbol{a}_{r,u}^*$ (or $\boldsymbol{a}_{t,u}^*$) is $\boldsymbol{e}_{n^*}$ where the n^*-th entry of $|\boldsymbol{h}_{b,r}|^2$(or $|\boldsymbol{h}_{r,u}|^2$) is the largest. For ease of exposition, let us denote $\boldsymbol{a}_{r,u}^{*H} \boldsymbol{h}_{b,r} = h_{b,r}^{\mathrm{FAS}}$ and $\boldsymbol{a}_{t,u}^{*H} \boldsymbol{h}_{r,u} = h_{r,u}^{\mathrm{FAS}}$. Furthermore, we can decouple (14) into U subproblems due to the independence of u. As a result, we can simplify (14) as follows

$$\begin{aligned} \min_{\mathit{\Gamma}_{b,u}, \mathit{\Gamma}_{r,u}} \quad & \mathit{\Gamma}_{b,u} + \mathit{\Gamma}_{r,u} &(15a)\\ \text{s.t.} \quad & \frac{\mathit{\Gamma}_{b,u}\left|h_{b,r}^{\mathrm{FAS}}\right|^2 \mathit{\Gamma}_{r,u}\left|h_{r,u}^{\mathrm{FAS}}\right|^2}{\mathit{\Gamma}_{b,u}\left|h_{b,r}^{\mathrm{FAS}}\right|^2 + \mathit{\Gamma}_{r,u}\left|h_{r,u}^{\mathrm{FAS}}\right|^2 + 1} \geq \gamma, &(15b)\end{aligned}$$

where $\gamma = 2^{2R_{\min}} - 1$. Unfortunately, due to (15b), (15) remains a non-convex optimization problem. However, we can exploit the following lemma.

Lemma 1. *In* (15)*, the optimal variables must satisfy (15b) with equality for* $\gamma > 0$. *Otherwise, the total power of the BS and relay can be further minimized. If* $\gamma = 0$, *the optimal variables are zeros.*

Let us prove this by contradiction. Suppose $\mathit{\Gamma}_{b,u}^*$ and $\mathit{\Gamma}_{r,u}^*$ are the optimal variables such that

$$\frac{\mathit{\Gamma}_{b,u}^*\left|h_{b,r}^{\mathrm{FAS}}\right|^2 \mathit{\Gamma}_{r,u}^*\left|h_{r,u}^{\mathrm{FAS}}\right|^2}{\mathit{\Gamma}_{b,u}^*\left|h_{b,r}^{\mathrm{FAS}}\right|^2 + \mathit{\Gamma}_{r,u}^*\left|h_{r,u}^{\mathrm{FAS}}\right|^2 + 1} > \gamma, \tag{16}$$

then we can always find some $\Delta > 0$ such that

$$\frac{\left(\mathit{\Gamma}_{b,u}^* - \Delta\right)\left|h_{b,r}^{\mathrm{FAS}}\right|^2 \mathit{\Gamma}_{r,u}^*\left|h_{r,u}^{\mathrm{FAS}}\right|^2}{\left(\mathit{\Gamma}_{b,u}^* - \Delta\right)\left|h_{b,r}^{\mathrm{FAS}}\right|^2 + \mathit{\Gamma}_{r,u}^*\left|h_{r,u}^{\mathrm{FAS}}\right|^2 + 1} = \gamma, \tag{17}$$

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or

$$\frac{\mathit{\Gamma}_{b,u}^{*}\left|h_{b,r}^{\mathrm{FAS}}\right|^{2}\left(\mathit{\Gamma}_{r,u}^{*}-\Delta\right)\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}}{\mathit{\Gamma}_{b,u}^{*}\left|h_{b,r}^{\mathrm{FAS}}\right|^{2}+\left(\mathit{\Gamma}_{r,u}^{*}-\Delta\right)\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}+1}=\gamma. \quad (18)$$

The existence of Δ can be easily verified by simplifying the inequalities: left of side of (16) $>$ (17) or (18). Overall, this implies that the optimal value can be reduced by Δ while satisfying the (15b). This contradicts that $\mathit{\Gamma}_{b,u}^{*}$ and $\mathit{\Gamma}_{r,u}^{*}$ are the optimal variables. Furthermore, if $\gamma=0$, it can be easily verified that the optimal values of the variables are $\mathit{\Gamma}_{b,u}^{*}=\mathit{\Gamma}_{r,u}^{*}=0$ since the optimal variables must be non-negative.

Since (15b) must be satisfied with equality for $\gamma>0$, we know that $\left(\mathit{\Gamma}_{b,u}^{*},\mathit{\Gamma}_{r,u}^{*}\right)$ must lie in the set

$$\mathcal{C}\triangleq\left\{\left(\tilde{\mathit{\Gamma}}_{b,u},\tilde{\mathit{\Gamma}}_{r,u}\right)\mid\frac{\tilde{\mathit{\Gamma}}_{b,u}\left|h_{b,r}^{\mathrm{FAS}}\right|^{2}\tilde{\mathit{\Gamma}}_{r,u}\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}}{\tilde{\mathit{\Gamma}}_{b,u}\left|h_{b,r}^{\mathrm{FAS}}\right|^{2}+\tilde{\mathit{\Gamma}}_{r,u}\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}+1}=\gamma\right\}. \quad (19)$$

For this reason, for a given $\tilde{\mathit{\Gamma}}_{r,u}$, we can obtain the corresponding $\tilde{\mathit{\Gamma}}_{b,u}$ as follows:

$$\tilde{\mathit{\Gamma}}_{b,u}=\frac{\gamma\left(\tilde{\mathit{\Gamma}}_{r,u}\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}+1\right)}{\left|h_{b,r}^{\mathrm{FAS}}\right|^{2}\left(\tilde{\mathit{\Gamma}}_{r,u}\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}-\gamma\right)}. \quad (20)$$

This suggests that we can solve (15) using an one-dimensional searching method on $\mathit{\Gamma}_{r,u}^{*}$ within the interval $\left(\frac{\gamma}{\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}},\infty\right)$. For example, (9) is a unimodal function and thus we can employ golden section search between the interval $\left[\frac{\gamma+\epsilon}{\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}},\mathit{\Gamma}_{\max}\right]$, where ϵ and $\mathit{\Gamma}_{\max}$ are arbitrarily small and large numerical values, respectively. The time-complexity of golden section search is $\mathcal{O}\left(\frac{\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}\mathit{\Gamma}_{\max}-\gamma+\epsilon}{\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}0.618}\right)$ per user, where 0.618 is known as the golden ratio.

Decode-Forward Scheme

For DF scheme, the optimization problem can be formulated as

$$\begin{aligned}
\min_{\mathit{\Gamma}_{b,u},\mathit{\Gamma}_{r,u},\boldsymbol{a}_{r,u},\boldsymbol{a}_{t,u},\forall u}\quad & \sum_{u}\mathit{\Gamma}_{b,u}+\mathit{\Gamma}_{r,u} && (21a)\\
\text{s.t.}\quad & R_{\mathrm{HD},u}^{\mathrm{DF}}\geq R_{\min},\forall u, && (21b)\\
& \boldsymbol{a}_{r,u}\in\left\{\boldsymbol{e}_{1},\ldots,\boldsymbol{e}_{N}\right\}, && (21c)\\
& \boldsymbol{a}_{t,u}\in\left\{\boldsymbol{e}_{1},\ldots,\boldsymbol{e}_{N}\right\}. && (21d)
\end{aligned}$$

Similarly, (21) is a non-convex optimization problem. However, by exploiting the monotonic increasing and independent properties, we can reformulate (21) as

$$\begin{aligned}
\min_{\mathit{\Gamma}_{b,u},\mathit{\Gamma}_{r,u}}\quad & \mathit{\Gamma}_{b,u}+\mathit{\Gamma}_{r,u} && (22a)\\
\text{s.t.}\quad & \mathit{\Gamma}_{b,u}\left|h_{b,r}^{\mathrm{FAS}}\right|^{2}\geq\gamma, && (22b)\\
& \mathit{\Gamma}_{r,u}\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}\geq\gamma, && (22c)
\end{aligned}$$

which is a linear optimization problem. Clearly, the optimal variables must satisfy (22b) and (22c) with equality for $\gamma>0$. Otherwise, the total power can be further minimized. Therefore, it can be easily verified that the optimal solutions are $\mathit{\Gamma}_{b,u}^{*}=\frac{\gamma}{\left|h_{b,r}^{\mathrm{FAS}}\right|^{2}}$ and $\mathit{\Gamma}_{r,u}^{*}=\frac{\gamma}{\left|h_{r,u}^{\mathrm{FAS}}\right|^{2}}$. If $\gamma=0$, it can be easily verified that the optimal values of the variables are $\mathit{\Gamma}_{b,u}^{*}=\mathit{\Gamma}_{r,u}^{*}=0$. Due to closed-form expression, the time-complexity is $\mathcal{O}\left(1\right)$ per user.

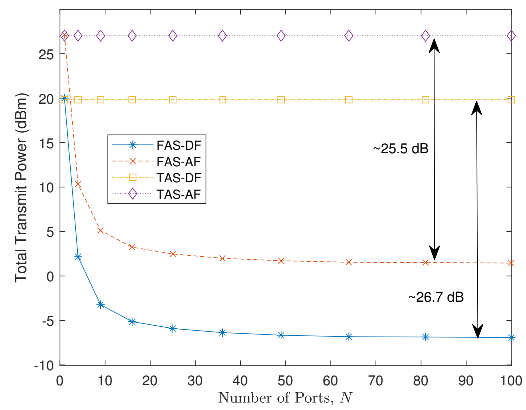

Figure 2. Total transmit power vs. number of ports, N.

IV. Simulation Results

In this section, we investigate the minimum total transmit power of the BS and relay in FAS. For benchmarking, we consider fixed-position antenna, referred to as TAS.[3] Furthermore, we consider both AF and DF schemes. Unless stated otherwise, the simulation parameters are set as follows: $U=3$, $d_{b,r}=400$m, $d_{r,1}=400$m, $d_{r,2}=500$m, $d_{r,3}=600$m, $\alpha=3$, $N_1=N_2=10$, $W_1=W_2=1$, $N_0=-80$dBm, $\mathit{\Gamma}_{\max}=500$dB, $\epsilon=1\times10^{-12}$, and $R_{\min}=5$ bps/Hz.

Fig. 2 illustrates the total transmit power for different numbers of ports, N. As shown, the total transmit power decreases as the number of ports increases. For instance, FAS-DF reduces the total transmit power by 26.7 dB compared to TAS-DF, while FAS-AF achieves a 25.5 dB reduction compared to TAS-AF. This indicates that FAS can enhance the performance of the DF scheme more effectively than the AF scheme. However, as the number of ports increases, the reduction saturates due to limited spatial diversity within a fixed surface area, W. To further improve performance, increasing the surface area can be considered.

Fig. 3 presents the total transmit power for varying surface areas, W. As it is seen, the total transmit power decreases as the surface area increases. However, this reduction eventually saturates — not due to limited spatial diversity, but because the maximum channel gain cannot be increased indefinitely by merely expanding the surface area. Nonetheless, when W is sufficiently large, FAS-DF and FAS-AF achieve maximum transmit power reductions of 32 dB and 30.6 dB, respectively.

Fig. 4 illustrates the impact of varying the relay's location. Specifically, the distances are adjusted as follows: $d_{b,r}=(400+D)$m, $d_{r,1}=(400-D)$m, $d_{r,2}=(500-D)$m, and $d_{r,3}=(600-D)$m, where D spans the range$(-400,400)$,

[3]In this paper, we fix the active port of the FAS throughout the duration to represent a TAS with a fixed-position antenna ($a_{t,u}=\boldsymbol{e}_1$and $a_{r,u}=\boldsymbol{e}_1$).

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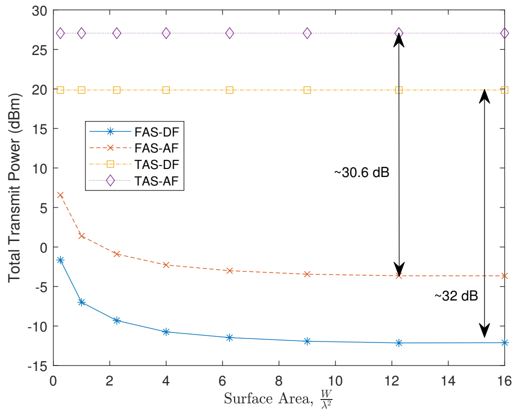

Figure 3. Total transmit power vs. surface area, W.

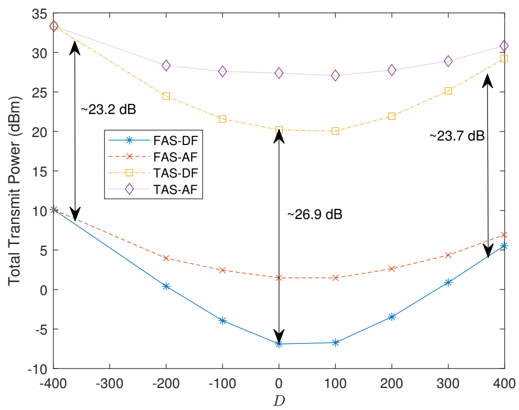

Figure 4. Total transmit power vs. varying distance, D.

thereby mimicking the effect of relocating the relay. As shown in the figure, placing the relay midway between the BS and the users is the most effective strategy for minimizing the transmit power of both the BS and the relay. At this optimal location, FAS-DF also achieves an additional 3.7 dB gain, while FAS-AF provides an additional 1.7 dB gain compared to locations closer to the BS or the users. This result highlights the superior performance of FAS, particularly when the relay is placed approximately equidistantly between the BS and the users.

V. Conclusion

In this letter, we investigated half-duplex relay in FAS. Specifically, we considered AF- and DF-based fluid antenna relays. We minimized the total transmit SNRs of both the BS and the relay, subject to a minimum rate requirement, by optimizing the port selection and transmit power. The resulting optimization problems are generally non-convex. To address this, we leverage monotonicity and independence properties. Consequently, we show that the AF scheme's optimization problem can be solved using a one-dimensional search method, while the DF scheme's problem can be solved in closed-form. Our results demonstrate that FAS can significantly reduce the total transmit power compared to TAS with fixed-position antenna due to its extreme spatial diversity. Furthermore, we observed that the FAS relay performs best when positioned midway between the BS and the users.

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