Sparse Bayesian Learning-Based Channel Estimation for Fluid Antenna Systems

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***Abstract*—A fluid antenna system (FAS) has been demonstrated to achieve comparable performance to a conventional MIMO system with much fewer RF chains. The performance of the FAS depends on port selection, which in turn relies on the accurate acquisition of channel state information (CSI). Therefore, it is essential to achieve accurate channel estimation in the FAS. In this letter, we propose a sparse Bayesian learning approach (SBL) to estimate the channel and select the best ports. Simulation results demonstrate that our proposed algorithm outperforms competitive benchmarks in terms of accuracy for channel estimation and outage probability for port selection.**

***Index Terms*—Fluid antenna system, sparse Bayesian learning, channel estimation, port selection.**

I. Introduction

MULTIPLE-input multiple-output (MIMO) technology is one of the most important wireless communication technologies due to its exploitation of new spatial freedom. Conventional massive MIMO systems are expensive due to the requirement of a large number of RF chains. To reduce the number of RF chains, a fluid antenna system (FAS) was proposed. In such a system, a few antennas are flexibly switched to different positions (referred to as 'ports') in linear space, and then the favorable ports are selected to adapt to the dynamic wireless channels [1]. The FAS has been proven to achieve high diversity and multiplexing gains, thus being extensively studied by many scholars [2]–[5].

The performance of the FAS is influenced by port selection, which itself depends on the accurate acquisition of channel state information (CSI). Consequently, the channel estimation problem is crucial for optimal performance in the FAS. Moreover, the number of ports in the FAS can be very large, which results in high computational complexity for channel estimation. Therefore, it is essential to develop an efficient channel estimation technique for the FAS. A successive Bayesian reconstructor was proposed in [6] to estimate the FAS channel. This technique successively eliminated the uncertainty of the stochastic channel by employing kernel-based sampling and regression techniques. Several fast port selection algorithms were proposed in [7] utilizing a combination of machine learning methods.

However, existing channel estimation and port selection algorithms for the FAS have several limitations. In [6], the performance of the successive Bayesian reconstructor heavily depends on the experiential covariance kernel, which is unattainable in many dynamic scenarios. In [7], deep learning-based channel estimation methods require a large amount of training data, which is impractical to obtain in many real-world scenarios.

To efficiently solve the channel estimation problem for the FAS, we propose a sparse Bayesian learning approach (SBL). The channel to be estimated is the posterior mean of the beamspace channel vector, derived by alternate evaluation using the SBL algorithm. Simulation results demonstrate that our proposed channel estimation technique exhibits superior accuracy compared to benchmarks while also achieving lower outage probability for port selection.

The rest of this paper is organized as follows. In Section II, the system model and the channel model of the FAS are introduced. In Section III, we transform the channel estimation problem into a basis selection problem. Additionally, we propose the SBL algorithm and analyze its performance. Numerical results are presented in Section IV. Finally, conclusions are provided in Section V.

II. System and Channel Model

A. System Model

We consider a point-to-point system, where a receiver is equipped with an N-port FAS which consists of M RF chains ($M \ll N$), and a transmitter is equipped with a fixed-position antenna. As shown in Fig. 1, at the receiver, N ports are evenly distributed along a linear dimension of length, $W\lambda$, where λ denotes the signal wavelength and W is the normalized size of the FAS. The M RF chains can be randomly switched to M different locations of N available port locations.

Let $\mathbf{h} \in \mathbb{C}^N$ denote the channel of N ports and L denote the number of transmit pilots within a coherence-time frame. As illustrated in Fig. 2, at each of the first L subframes, the M RF chains can randomly switch their positions to receive pilots. Similar to [6], to characterize the locations of M antennas at time slot l, we introduce a binary indicator matrix $\mathbf{S}_l \in \{0,1\}^{M\times N}$ (referred to as a switch matrix), which denotes a configuration of M antennas at time slot l. The (m,n)-th entry being 1 (or 0) means that the m-th antenna is (or not) positioned at the n-th port. Since M of N are selected at each time slot, each row of $\mathbf{S}_l$ contains one entry of 1, and all entries of 1 in $\mathbf{S}_l$ are not in the same column, i.e., $\|\mathbf{S}_l(m,:)\| = 1$, $\forall m \in \{1,...,M\}$, $\|\mathbf{S}_l(:,n)\| \in \{0,1\}$, $\forall n \in \{1,...,N\}$, and $\mathbf{S}_l\mathbf{S}_l^H = \mathbf{I}_M$, where $(\cdot)^H$ denotes the conjugate transpose operation.

The received signal $\mathbf{y}_l \in \mathbb{C}^M$ at the receiver at time slot l is

$$\mathbf{y}_l = \mathbf{S}_l\mathbf{h}s_l + \mathbf{z}_l, \tag{1}$$

where s_l is the pilot transmitted by the transmitter and $\mathbf{z}_l \sim \mathcal{CN}\left(\mathbf{0}_M, \sigma^2\mathbf{I}_M\right)$ is the additive white Gaussian noise (AWGN) at M selected ports. Without loss of generality, we

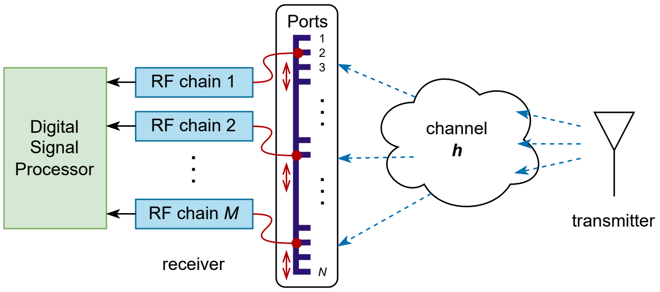

Fig. 1. A point-to-point system with a receiver equipped with a FAS and a transmitter equipped with a fixed-position antenna.

assume that $s_l = 1$ for all $l \in \{1, ..., L\}$. Considering the total L timeslots for pilot transmission, we have

$$\mathbf{y} = \mathbf{S}\mathbf{h} + \mathbf{z}, \tag{2}$$

where $\mathbf{y} = \left[\mathbf{y}_1^T, ..., \mathbf{y}_L^T\right]^T$, $\mathbf{S} = \left[\mathbf{S}_1^T, ..., \mathbf{S}_L^T\right]^T$ and $\mathbf{z} = \left[\mathbf{z}_1^T, ..., \mathbf{z}_L^T\right]^T$. The $(\cdot)^T$ above denotes the transpose operation. At the receiver, the FAS chooses the first M ports with the maximum channel gain magnitude as receiving ports, to which the RF chains are then switched for receiving signals, aiming to achieve the best performance.

B. Channel Model

We consider the far-field spatially sparse channel model with N_c clusters each containing N_r rays. The FAS channel can be approximately modeled as

$$\mathbf{h} = \sqrt{\frac{N}{N_c N_r}} \sum_{i=1}^{N_c} \sum_{j=1}^{N_r} \alpha_{ij} \mathbf{a}\left(\theta_{ij}\right), \tag{3}$$

where α_{ij} and θ_{ij} are the complex path gain and the incident angle associated with the j-th ray in the i-th cluster, respectively. $\mathbf{a}\left(\theta_{ij}\right)$ denotes a steering vector, which is defined as

$$\mathbf{a}\left(\theta_{ij}\right) = \frac{1}{\sqrt{N}} \left[1, e^{-j\frac{2\pi}{\lambda} d \cos\left(\theta_{ij}\right)}, ..., e^{-j\frac{2\pi}{\lambda}(N-1) d \cos\left(\theta_{ij}\right)}\right]^T, \tag{4}$$

where d denotes a port spacing. Our objective is to reconstruct the N-dimensional channel $\mathbf{h}$ from the LM-dimensional noisy observation $\mathbf{y}$ ($LM \ll N$) as described in (2).

III. Proposed Sparse Bayesian Learning-Based Channel Estimator

To estimate the high-dimensional channel according to the low-dimensional received signal as described in (2), we introduce a sparse Bayesian learning algorithm. Initially, we convert this problem into a basis selection problem, which is defined in [8]. Subsequently, we alternately evaluate the posterior covariance, mean and hyperparameter estimates of the beamspace channel vector until the SBL algorithm converges to the optimum. Ultimately, we map the estimated beamspace channel vector back to the desired channel vector $\mathbf{h}$.

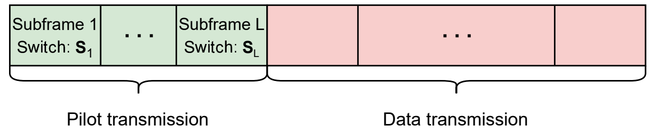

Fig. 2. A coherence-time frame.

A. Problem Transformation

Consider a partition of the feasible angle of arrival (AoA) space with N angular grid points. The set of spatial angles $\Phi = \{\phi_g : \phi_g \in [0, \pi], \forall 1 \leq g \leq N\}$ is chosen according to the following condition [9]:

$$\cos\left(\phi_g\right) = \frac{2}{N}(g-1) - 1, \forall 1 \leq g \leq N. \tag{5}$$

The quantized receive array response matrix $\mathbf{A}(\Phi) = [\mathbf{a}(\phi_1), ..., \mathbf{a}(\phi_N)] \in \mathbb{C}^{N \times N}$. The array response vector $\mathbf{a}(\phi_g)$ is rewritten according to (4). Therefore, $\mathbf{h}$ can be transformed into its beamspace representation $\mathbf{h}_b$. Then the received signal $\mathbf{y}$ can be rewritten as

$$\mathbf{y} = \mathbf{S}\mathbf{A}(\Phi)\mathbf{h}_b + \mathbf{z} = \mathbf{\Psi}\mathbf{h}_b + \mathbf{z}, \tag{6}$$

where $\mathbf{\Psi} = \mathbf{S}\mathbf{A}(\Phi) \in \mathbb{C}^{LM \times N}$ is the overcomplete dictionary matrix, whose columns represent a possibly overcomplete basis. Therefore, (6) is a standard observation equation for basis selection problem [8].

B. Proposed Channel Estimation Algorithm

In this section, we propose the SBL algorithm for the estimation of sparse beamspace channel vector $\mathbf{h}_b$ with the measurement vector $\mathbf{y}$. Let's assume that the noise variance σ^2 is known. We can obtain the Gaussian likelihood model

$$p\left(\mathbf{y}|\mathbf{h}_b\right) = \left(\pi\sigma^2\right)^{-N} \exp\left(-\frac{\|\mathbf{y} - \mathbf{\Psi}\mathbf{h}_b\|_2^2}{\sigma^2}\right), \tag{7}$$

where $p(\cdot)$ denotes the probability density function and $\|\cdot\|_2$ denotes the l_2 norm. The SBL framework assigns a parameterized complex Gaussian prior below to the unknown sparse beamspace channel vector $\mathbf{h}_b \in \mathbb{C}^{N \times 1}$ [9]:

$$p\left(\mathbf{h}_b; \boldsymbol{\gamma}\right) = \prod_{n=1}^{N} \left(\pi\gamma_n\right)^{-1} \exp\left(-\frac{|\mathbf{h}_b(n)|^2}{\gamma_n}\right). \tag{8}$$

In (8), $\mathbf{h}_b(n)$ denotes the n-th component of $\mathbf{h}_b$. The hyperparameter $\gamma_n, \forall 1 \leq n \leq N$ denotes the unknown prior variance corresponding to $\mathbf{h}_b(n)$ and $\gamma = [\gamma_1, ..., \gamma_N]^T$. $|\cdot|$ denotes the magnitude of the complex number. It is evident that as γ_n approaches 0, $\mathbf{h}_b(n)$ tends to 0. Consequently, the estimation of the beamspace channel vector $\mathbf{h}_b$ simplifies to the estimation of the corresponding hyperparameter vector $\boldsymbol{\gamma}$.

Now we start to estimate $\boldsymbol{\gamma}$. we employ the expectation-maximization (EM) algorithm by treating the beamspace channel vector $\mathbf{h}_b$ as hidden variables. Let $\hat{\boldsymbol{\gamma}}^{(k)}$ denote the estimate of the hyperparameter vector $\boldsymbol{\gamma}$ in the k-th iteration. In the E step, we compute the posterior distribution of $\mathbf{h}_b$ according to Bayes' theorem as follows [10]:

$$\begin{aligned} p\left(\mathbf{h}_b|\mathbf{y}; \hat{\boldsymbol{\gamma}}^{(k)}\right) &= \frac{p\left(\mathbf{h}_b; \hat{\boldsymbol{\gamma}}^{(k)}\right) p\left(\mathbf{y}|\mathbf{h}_b\right)}{\int p\left(\mathbf{h}_b; \hat{\boldsymbol{\gamma}}^{(k)}\right) p\left(\mathbf{y}|\mathbf{h}_b\right) d\mathbf{h}_b} \\ &\sim \mathcal{CN}\left(\boldsymbol{\mu}^{(k)}, \mathbf{\Sigma}^{(k)}\right), \end{aligned} \tag{9}$$

Algorithm 1 SBL Channel Estimator

Input: Overcomplete dictionary matrix $\mathbf{\Psi}$, observation $\mathbf{y}$, convergence parameter ϵ and maximum number of iteration $k_{\max}$.

Output: The estimate of beamspace channel vector $\hat{\mathbf{h}}_b$.

Initialization: $\hat{\mathbf{\Gamma}}^{(0)} = \mathbf{I}_N$, $\hat{\mathbf{\Gamma}}^{(-1)} = \mathbf{0}_N$ and $k = 0$.

```
while ‖γ̂^(k) − γ̂^(k−1)‖_2 > ε and k < k_max do
    Calculate the intermediate variable Σ_y by (13)
    Calculate the covariance Σ^(k) by (11)
    Calculate the mean μ^(k) by (10)
    for n = 1, 2, ···, N do
        Calculate the hyperparameter γ̂_n^(k+1) by (16)
    end for
    Update k to k + 1
end while
return ĥ_b = μ^(k)
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in which the mean and covariance are given by

$$\boldsymbol{\mu}^{(k)} = \hat{\mathbf{\Gamma}}^{(k)} \mathbf{\Psi}^H \mathbf{\Sigma}_y^{-1} \mathbf{y}, \tag{10}$$

$$\mathbf{\Sigma}^{(k)} = \hat{\mathbf{\Gamma}}^{(k)} - \hat{\mathbf{\Gamma}}^{(k)} \mathbf{\Psi}^H \mathbf{\Sigma}_y^{-1} \mathbf{\Psi} \hat{\mathbf{\Gamma}}^{(k)}, \tag{11}$$

where

$$\hat{\mathbf{\Gamma}}^{(k)} = \operatorname{diag}\left(\hat{\boldsymbol{\gamma}}^{(k)}\right), \tag{12}$$

$$\mathbf{\Sigma}_y = \sigma^2 \mathbf{I} + \mathbf{\Psi} \hat{\mathbf{\Gamma}}^{(k)} \mathbf{\Psi}^H. \tag{13}$$

In (12), diag $(\cdot)$ creates a diagonal matrix with the elements of a vector on its main diagonal. To speed up matrix operations, (11) have been transformed by Woodbury matrix identity. We can evaluate the expected complete-data log-likelihood in the k-th iteration given by [9]

$$\mathcal{L}\left(\boldsymbol{\gamma}|\hat{\boldsymbol{\gamma}}^{(k)}\right) = \mathbb{E}_{\mathbf{h}_b|\mathbf{y};\hat{\boldsymbol{\gamma}}^{(k)}} \left[\log p\left(\mathbf{y}, \mathbf{h}_b; \boldsymbol{\gamma}\right)\right] \tag{14a}$$

$$= \mathbb{E}_{\mathbf{h}_b|\mathbf{y};\hat{\boldsymbol{\gamma}}^{(k)}} \left[\log p\left(\mathbf{y}|\mathbf{h}_b\right) + \log p\left(\mathbf{h}_b; \boldsymbol{\gamma}\right)\right], \tag{14b}$$

where $\mathbb{E}[\cdot]$ is an expectation operator.

In the *M* step, we determine the estimate $\hat{\boldsymbol{\gamma}}^{(k+1)}$ by maximizing the expected log likelihood mentioned above. As observed, the first term in (14b) is uncorrelated with $\boldsymbol{\gamma}$ and can therefore be ignored. Then we have

$$\hat{\boldsymbol{\gamma}}^{(k+1)} = \arg\max_{\boldsymbol{\gamma}} \mathbb{E}_{\mathbf{h}_b|\mathbf{y};\hat{\boldsymbol{\gamma}}^{(k)}} \left[\log p\left(\mathbf{h}_b; \boldsymbol{\gamma}\right)\right]. \tag{15}$$

Therefore, it can be solved to acquire the estimates for $\hat{\gamma}_n^{(k+1)}$ in the k-th iteration of the EM algorithm as [9]

$$\hat{\gamma}_n^{(k+1)} = \mathbb{E}_{\mathbf{h}_b|\mathbf{y};\hat{\boldsymbol{\gamma}}^{(k)}} \left[\left|\mathbf{h}_b\left(n\right)\right|^2\right] = \mathbf{\Sigma}^{(k)}\left(n, n\right) + \left|\boldsymbol{\mu}^{(k)}\left(n\right)\right|^2, \tag{16}$$

where $\mathbf{\Sigma}^{(k)}\left(n, n\right)$ and $\boldsymbol{\mu}^{(k)}\left(n\right)$ denote the (n, n)-th element in $\mathbf{\Sigma}$ and n-th element in $\boldsymbol{\mu}$ in k-th iteration respectively. When the iteration converges, the SBL algorithm yields the estimate of sparse beamspace channel vector employing $\hat{\mathbf{h}}_b = \boldsymbol{\mu}^{(k)}$. Algorithm 1 presents a concise overview of the sequential procedures involved in our proposed SBL technique for sparse channel estimation. In practical applications, when any hyperparameter becomes sufficiently small, it gets pruned from the model along with the corresponding column of the dictionary matrix $\mathbf{\Psi}$. $\mathbf{h}$ can be obtained by $\mathbf{h} = \mathbf{A}\left(\Phi\right)\mathbf{h}_b$.

C. Performance analysis of the SBL algorithm

In terms of convergence, the increase in the value of the log-likelihood function in each iteration ensures the convergence of the EM algorithm. Similarly, in the SBL algorithm, each iteration is guaranteed to enhance the log-likelihood function until reaching a fixed point, thereby establishing global convergence [8].

Considering computational complexity, given the condition $LM \ll N$, each iteration of the SBL algorithm has a complexity of $O\left(L^2 M^2 N\right)$, thereby achieving linear complexity with respect to N [10]. When the number of ports is significantly larger than the number of RF chains and pilots, the SBL iteration becomes highly attractive.

IV. Simulation Results

In this section, numerical simulation results are provided to validate the performance of our proposed algorithm. In the simulation, the FAS at the receiver is configured with $N = 256$ ports and $M = 4$ antennas. The normalized size of FAS is set to $W = 10$. Therefore, the port spacing is $d = \frac{W\lambda}{N-1}$. Additionally, the number of pilots is set to $L = 10$.

To verify the effectiveness of our proposed SBL channel estimator, the following benchmark schemes are considered.

1) **OMP**: the orthogonal matching pursuit (OMP) algorithm is utilized under the assumption that the number of clusters N_c and the number of rays N_r are known.[1]

2) **ML**: the maximum likelihood (ML) algorithm that maximizes the likelihood function $p\left(\mathbf{y}|\boldsymbol{\alpha}, \boldsymbol{\theta}\right)$ is adopted, where $\boldsymbol{\alpha}$ and $\boldsymbol{\theta}$ are defined in [6].

3) **Interpolation**: the nearest neighbour interpolation is employed by sampling LM ports at a uniform interval.

4) **SBL**: our proposed SBL algorithm is employed with the convergence parameter $\epsilon = 10^{-5}$ and the maximum iteration number $k_{\max} = 100$.

In Fig. 3, we show the results of the normalized mean square error (NMSE) of all ports versus the average signal-to-noise ratio (SNR). The NMSE of all ports is defined as

$$\mathrm{NMSE_{all}} = \mathbb{E}\left[\frac{\left\|\mathbf{h} - \hat{\mathbf{h}}\right\|_2^2}{\left\|\mathbf{h}\right\|_2^2}\right], \tag{17}$$

where $\hat{\mathbf{h}}$ denotes the estimate of $\mathbf{h}$. As shown in Fig. 3, our proposed algorithm achieves a notably lower NMSE of all ports compared to the other three schemes across all SNR ranges. The improvement is particularly significant under high SNR conditions. This demonstrates the superior capability of our proposed algorithm in accurately estimating the channel for all ports. Our proposed algorithm leverages the Bayesian framework to estimate the CSI by maximizing the posterior probability, which integrates the observed data and prior information to mitigate the influence of noise, thereby delivering more accurate and robust estimation results compared to conventional methods.

[1] This method is a modified version of the one proposed in [11].

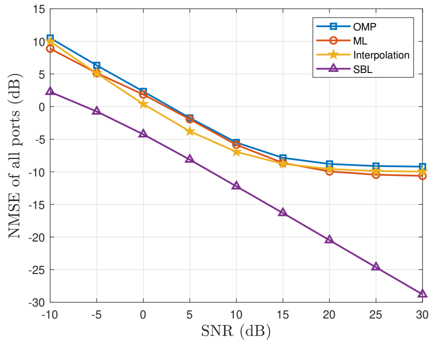

Fig. 3. NMSE of all ports versus SNR in the FAS.

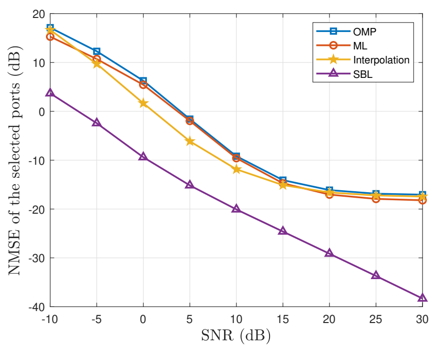

Fig. 4. NMSE of the selected ports versus SNR in the FAS.

In Fig. 4, we show the results of the NMSE of selected ports versus the average SNR. The NMSE of the selected ports is defined as

$$\mathrm{NMSE}_{\mathrm{selected}} = \mathbb{E}\left[\frac{\left\|\mathbf{h}\left(\Omega\right) - \hat{\mathbf{h}}\left(\Omega\right)\right\|_2^2}{\left\|\mathbf{h}\left(\Omega\right)\right\|_2^2}\right], \tag{18}$$

where Ω denotes a set of selected ports. Such a performance metric better demonstrates the accuracy of the channel estimation for the selected ports. It is crucial because once the M RF chains have selected M ports, the performance of the subsequent digital signal processing operations depends on the accuracy of the channel estimation for these selected ports rather than all ports. The $\mathrm{NMSE}_{\mathrm{selected}}$ using our proposed algorithm is significantly lower than that achieved by benchmark algorithms. This indicates that our proposed algorithm not only provides accurate overall channel estimation but also ensures high accuracy in the estimation of the channels for the selected ports.

In Fig. 5, we plot the outage probability as a function of the average SNR. The outage probability is defined as

$$p_{\mathrm{out}} = \Pr\left(\rho < \rho_{\mathrm{th}}\right), \tag{19}$$

where $\Pr\left(\cdot\right)$ is the probability of an event $\left(\cdot\right)$, $\rho = \frac{|\mathbf{h}(n)|^2}{\sigma^2}$ is the SNR of the n-th port and ρ_{th} is the SNR threshold meeting the requirement $\frac{\rho_{\mathrm{th}}}{\rho} = 1\mathrm{dB}$. Our proposed algorithm achieves a lower outage probability compared to the baseline methods across the entire range of SNR values, demonstrating a more precise port selection. The outage probability of our proposed algorithm decreases rapidly with increasing SNR, even reaching as low as 2.5×10^{-6} at an average SNR of 30 dB. These results indicate that our proposed algorithm can significantly reduce communication failures and enhance the reliability of communication performance.

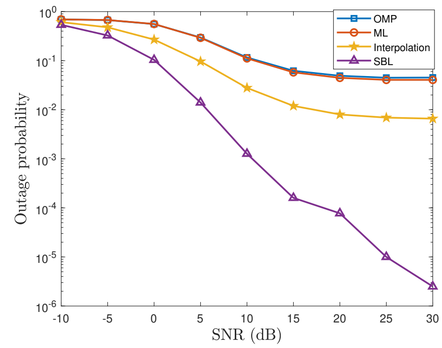

Fig. 5. Outage probability versus SNR in the FAS.

V. Conclusion

In this letter, we proposed an SBL-based approach for channel estimation in FAS by converting this issue into a basis selection problem that leverages the sparse beamspace representation of the channel responses. By alternately updating the posterior covariance, mean and hyperparameter of the beamspace channel vector, the estimated beamspace channel vector was obtained as the posterior mean after iteration and convergence. Simulation results demonstrated that our proposed channel estimation algorithm achieves superior accuracy compared to competitive benchmarks, while also exhibiting a lower outage probability for port selection.

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