Lost in the Libor Transition

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Abstract

In the face of the transition away from LIBOR-type term rate benchmarks in key jurisdictions such as the US and the UK, towards interest rate benchmarks based solely on overnight rates, this paper considers the information on the market's view of "roll-over" or "refinancing" risk, which is contained in term rate benchmarks, but lost in the transition to overnight benchmarks such as SONIA and SOFR. Considering Eurozone and, to a lesser extent, US data, we show that a model of this risk, when fitted to existing term rates for a subset of tenor frequencies, performs quite well in recovering the omitted tenors. This type of "out-of-sample-performance" clearly demonstrates that term rate benchmarks such as LIBOR have substantial informational value above their (overnight) replacements, providing a rigorous underpinning to practitioners' reservations about the benchmark transition. In particular, in jurisdictions such as the Eurozone, which have not yet committed to eliminating the term rate benchmarks, our findings may contribute to the ongoing debate.

Keywords: LIBOR benchmark reform, term rates, term risk and term premium, roll-over risk, term-based information differential, overnight interest rate benchmarks, SOFR, SONIA, OIS, interest rate swaps, basis risk.

1 Introduction

When the phase-out of the London Interbank Offer Rate (LIBOR) and similar IBOR-type benchmarks was announced, first and foremost in the United Kingdom and the United States of America, it was expected that this transition would be completed in these jurisdictions in a few years and that others (for example the Eurozone) would quickly follow. Several years on, this project has turned out to be more complex than its main proponents anticipated. The timeline has slipped, though at least the UK and USA have almost fully transitioned to the new risk-free rate (RFR) benchmarks, to SONIA (Sterling Overnight Index Average) and SOFR (Secured Overnight Financing Rate), respectively. For other jurisdictions, this is less clear: In the Eurozone it is not a given that EURIBOR will disappear even if the EUR RFR, €STR, becomes the benchmark of choice. Thus it is worthwhile to consider the wider implications of continuing to pursue the elimination of IBOR-type benchmarks from the market.

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In this paper, we look at a particular aspect of this issue, namely the information about lending and borrowing at term embodied in IBOR-type term rates, which is lost to the market when these benchmarks are eliminated. IBORs are true term rates in the sense that at least some market participants are able to borrow for a term (e.g., three months or six months) at these rates, while this is not true of synthetic "term" rates constructed from the RFR benchmarks, which are invariably based on overnight rates. The reason for this lies in the "multicurve phenomenon", which has been an established feature of money markets since the financial crises of 2007/8: Term (or "tenor") matters, which is why the market adds a spread to the shorter tenor leg of a floating-for-floating interest rate swap (a.k.a. "basis swap"). This phenomenon was first noted in the finance literature by Henrard (2007), and subsequent studies attributed it to credit risk (see, e.g., Morini (2009) and Bianchetti (2010)) or, both, liquidity risk and credit risk (see in particular Filipović & Trolle (2013), as well as Acharya et al. (2011), Crépey & Douady (2013) and Gallitschke et al. (2017)). For an introduction to the multi-curve phenomenon and an extensive description of the impacted interest rate derivatives, we refer to the textbooks Grbac & Runggaldier (2015) and Henrard (2014). Under pre-2007 "single-curve" interest rate theory, the spread on a basis swap represents an arbitrage opportunity, but (as explained in detail in Alfeus et al. (2020)) taking advantage of this "arbitrage" would involve lending at the longer tenor frequency (say, six-monthly) and borrowing at the shorter tenor frequency, thus exposing the arbitrageur to the risk of not being able to refinance their borrowing at the market rate. Backwell et al. (2023) built a model making this *roll-over risk* explicit in a way amenable to econometric estimation and found that in addition to credit risk (in the form of the risk of an increase in a borrower's credit spread to the market rate), funding liquidity risk plays a substantial role in explaining the basis spreads between rates of different tenors. They show that even if credit risk is eliminated through secured (e.g., repo) borrowing, substantial basis spreads should persist. For a more comprehensive description of the complex relationships between interest rate, funding and liquidity risks in the context of term structure models we refer to Backwell et al. (2023) and Filipović & Trolle (2013). In a rigorous theoretical work, Fontana et al. (2023) show that in the presence of funding liquidity risk, basis spreads can be represented as value functions of suitable stochastic optimal control problems, and can arise endogenously from the risk preferences of a representative investor.

Since eliminating IBOR benchmarks means eliminating benchmarks for all tenors except the overnight one, this eliminates all benchmark information on roll-over risk and the market pricing of this risk (in the form of basis swap spreads). Although sometimes conflated with the pricing of interbank credit risk, this issue is increasingly being recognised by market practitioners. In a recent article in the widely read, premier industry publication *Risk*, Goyder (2023) argues that the end of LIBOR-based forward rate agreements (FRAs) and the associated demise of basis swaps between LIBOR and rates implied by OIS (overnight index swaps) make it difficult to hedge bank funding risk. For example, this article quotes a "treasurer at a European dealer in New York", who states, "You've got your fundamental rate risk that you can swap out with SOFR, but the actual systemic credit and liquidity spreads, you really have no way to hedge it[...]" Goyder goes on to report the lament of a "rates head at a second large dealer", that "such a barometer [i.e., FRA-OIS spreads] to measure stress in the banking system no longer exists and could even be contributing to the record levels of market volatility seen in March [2023]."

Rather than developing a new modelling methodology, the aim of our present study is empirical, to demonstrate the extent of the information which is lost, in particular the information which would be lost to markets in the Eurozone, were it to follow the example of the UK and the USA and eliminate EURIBOR. Thus, we consider the extent to which information about roll-over risk can be extracted from the still existing IBOR-type benchmarks. If roll-over risk is indeed the driver of basis swaps spreads, then one should be able to correctly price intermediate tenor frequencies given a model fitted to other tenor frequencies (i.e., interpolate basis swap spreads based on a model of roll-over risk), as well as extrapolate basis spreads to lower or higher tenor frequencies. *Ex ante*, one would expect the model to perform better on the former application than on the latter, for even good models often struggle with extrapolation. However, the extrapolation case is particularly interesting for future applications of the model to markets (e.g., in South Africa) which lack a discount curve based on an overnight tenor (in more developed markets, this curve is given by OIS, the overnight index swaps). The OIS curve is especially important, as it is the appropriate discount curve for collateralized derivative transactions (see for example Piterbarg (2012)). These questions, while clearly important in informing the LIBOR transition debate, have hitherto not been explored empirically. In the literature cited above, Filipović

& Trolle (2013) come closest by considering what they call "the term structure of interbank risk," but their approach pre-dates the advent of the new RFR benchmarks and the explicit modelling of roll–over risk as the driver of single–currency basis swap spreads.

The paper is organized as follows: Section 2 describes the modeling approach, Section 3 explains our econometric method, Section 4 shows our results, and Section 5 concludes. Appendices A to D contain supplementary material.

2 Model

We consider a single, risk-free short-rate process $r(t)_{t\geq 0}$, defined on the filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F})_{t\geq 0}, \mathbb{Q})$. The risk-neutral measure $\mathbb{Q}$ is defined in terms of the risk-free, continuous, savings account numeraire with value process $B(t) = B(0)\mathrm{e}^{\int_0^t r(u)du}$. We first define the collateral rate process $r^c(t)_{t\geq 0}$, which we assume has the following relation to the risk-free rate:

$$r^c(t) := r(t) + \Lambda(t),$$

where $\Lambda(t)_{t\geq 0}$ is a spread factor process. Pricing a collateralized derivative thus amounts to discounting with $r^c(t)_{t\geq 0}$ as opposed to the risk-free rate $r(t)_{t\geq 0}$, see Piterbarg (2012). In this paper, the price process of the collaterized zero-coupon bond is denoted $P(t,T)_{0\leq t\leq T}$, and its dynamics are given by

$$P(t,T) := \mathbb{E}\left[\exp\left(-\int_t^T r^c(u)du\right)\middle|\mathcal{F}_t\right], \tag{1}$$

where $0 \leq t \leq T$. Here, $\mathbb{E}[\cdot]$ denotes an expectation under $\mathbb{Q}$. We then proceed to construct the benchmark term rate or xIBOR. We follow the so-called roll-over risk approach of Backwell et al. (2023), which we summarize briefly for completeness. The overall assumption in the roll-over risk approach is that there exists an entity representative of the panel which determines xIBOR at a given time $t \geq 0$. At time t, this entity can borrow at the rate $r^c(t)$ on an infinitesimal basis and at the prevailing xIBOR when borrowing at term. However, as time progresses there is a risk that the entity will no longer be representative of the xIBOR panel, and thus it becomes unable to fund itself at the prevailing rate. The cost paid by the entity that is representative at time t to fund itself on a running basis, that is at time $u \geq t$, is the unsecured funding rate $r_t^f(u)_{u\geq t}$. We define this rate by

$$r_t^f(u) := r^c(u) + \gamma_t(u), \quad \gamma_t(t) = 0. \tag{2}$$

The spread factor process $\gamma_t(u)_{u\geq t\geq 0}$, we call it the roll-over risk spread. For simplicity, we shall assume that $\gamma_t(u)$ be independent of the rate r^c. It has an initial condition of $\gamma_t(t) = 0$ to reflect the fact that the entity in question can indeed borrow at the infinitesimal rate at the time $u = t$ when the entity is representative of the panel. But a roll-over risk event may happen at any time $u > t$, and thus the entity will have to pay an additional spread $\gamma_t(u)$. We generally assume that the roll-over risk event $\{\gamma_t(u) > 0\}$ is fairly rare, and is induced by either a decrease in credit quality or a non-credit related increase in the cost of funding. We model these two causes of roll-over risk by assuming that the roll-over risk component $\gamma_t(u)_{u\geq t\geq 0}$ can be additively decomposed as

$$\gamma_t(u) = \lambda_t(u) + \phi_t(u). \tag{3}$$

We refer to $\lambda_t(u)_{u\geq t}$ as the credit risk component and $\phi_t(u)_{u\geq t}$ the funding risk component, and we assume initial conditions $\lambda_t(t) = 0$ and $\phi_t(t) = 0$. Following Backwell et al. (2023), Section 3.2, we can derive the term rate xIBOR as

$$L(t,T) = \frac{1}{T-t}\left(\frac{\mathbb{E}\left[\mathrm{e}^{\int_t^T \phi_t(u)du}|\mathcal{F}_t\right]}{P(t,T)\mathbb{E}\left[\mathrm{e}^{-\int_t^T \lambda_t(u)du}|\mathcal{F}_t\right]} - 1\right). \tag{4}$$

If $\phi_t(u) = \lambda_t(u) = 0$, for all $u \geq 0$ and $t \geq 0$, the above expression collapses to

$$L(t,T) = OIS(t,T) = \frac{1}{T-t}\left(\frac{1}{\mathbb{E}\left[\mathrm{e}^{-\int_t^T r^c(u)du}|\mathcal{F}_t\right]} - 1\right) = \frac{1}{T-t}\left(\frac{1}{P(t,T)} - 1\right). \tag{5}$$

Quantity (5) is recognized as the spot rate derived from a single payment OIS, see Appendix A. Our aim is to construct a model that can be identified from OIS and linear derivatives (i.e., swaps) written on xIBOR rates. From relation (5) it should be clear that the dynamics of $r^c(u)_{u\geq 0}$ can be identified from OIS data, but disentangling the pure risk-free rate $r(u)_{u\geq 0}$ from $r^c(u)_{u\geq 0}$ requires separate data to identify the spread process $\Lambda(u)_{u\geq 0}$. This exercise is not necessary for our analysis, for we only analyze data from collateralized instruments and thus discount directly with $r^c(u)_{u\geq 0}$, see for example Piterbarg (2012). We therefore construct our model for $r^c(u)_{u\geq 0}$ directly in the following manner:

$$dr^c(u) = \kappa(\theta(u) - r^c(u))du + \sigma dW(u), \tag{6}$$

$$d\theta(u) = \kappa^*(\theta^* - \theta(u))du + \sigma^* dW^*(u), \tag{7}$$

where $W(u)_{u\geq 0}$ and $W^*(u)_{u\geq 0}$ are standard Brownian motions under $\mathbb{Q}$, with $dW(u)dW^*(u) = \rho\, du$. Furthermore, we follow Backwell et al. (2023) and, using the same notation as in their paper, model the specific components of roll-over risk by a system of jump processes given by

$$d\lambda_t(u) = -\beta^\lambda \lambda_t(u)\, du + dJ^\lambda(u),\ \lambda_t(t) = 0, \tag{8}$$

$$d\phi_t(u) = -\beta^\phi \phi_t(u)\, du + dJ^\phi(u),\ \phi_t(t) = 0, \tag{9}$$

where $J^\lambda(u)_{u\geq 0}$ and $J^\phi(u)_{u\geq 0}$ are Poisson jump processes, and $\lambda_t(u)_{u\geq t\geq 0}$ and $\phi_t(u)_{u\geq t\geq 0}$ both define a continuum (or random field) of jump processes indexed by $t \geq 0$. While they can jump independent of each other, we assume for simplicity that $\beta^\lambda = \beta^\phi = \beta$, and that $J^\lambda(u)_{u\geq 0}$ and $J^\phi(u)_{u\geq 0}$ share the same stochastic intensity process $x(u)_{u\geq 0}$, given by

$$dx(u) = y(u)\, du + \sigma^x \sqrt{x(u)} dW^x(u), \quad x(0) = x_0 > 0, \tag{10}$$

$$dy(u) = \sigma^y \sqrt{y(u)} dW^y(u), \quad y(0) = y_0 > 0, \tag{11}$$

where $W^y(u)_{u\geq 0}$ and $W^x(u)_{u\geq 0}$ are independent standard Brownian motions under $\mathbb{Q}$. Conditional on the value of $x(u)$ at time u, jump events and jump sizes are independent, with the jump size being exponentially distributed with a mean of 2% (again following Backwell et al. (2023)). The state vector of our model can thus be written as

$$X_t(u) = [r^c(u), \theta(u), x(u), y(u), \phi_t(u), \lambda_t(u)],$$

which defines a six-dimensional system of stochastic processes. In Appendix B, we demonstrate that for each fixed time t, $X_t(u)$ is an affine jump diffusion in the sense of Duffie et al. (2000), and thus it lends itself to tractable pricing of xIBOR and OIS derivatives. In particular, the value of a discounted xIBOR payment is given by an exponential-affine function of the state vector

$$\mathbb{E}[e^{-\int_s^T r^c(u)du} L(t,T)|\mathcal{F}_s] = \frac{1}{T-t}\left[\mathrm{e}^{\alpha^c(T-t)+\alpha^L(T-t)+\alpha^D(t-s)+\beta^D(t-s)\cdot X_s(s)} - \mathrm{e}^{\alpha^c(T-t)+\beta^c(T-t)\cdot X_s(s)}\right], \tag{12}$$

where the coefficients are given as the numerical solution to the Riccati equations in Appendix B, and where $s \leq t \leq T$. Notice that $X_s(s) = [r^c(s), \theta(s), x(s), y(s), 0, 0]$ because $\phi_s(s) = \lambda_s(s) = 0$. This means that the fundamental pricing equation is only a function of the reduced state vector $[r^c(s), \theta(s), x(s), y(s)]$. The common intensity of jumps $x(u)_{u\geq 0}$ for $\lambda_t(u)_{u\geq t\geq 0}$ and $\phi_t(u)_{u\geq t\geq 0}$, should be interpreted as the state-variable driving roll-over risk as a whole. In Backwell et al. (2023), $\lambda_t(u)_{u\geq t\geq 0}$ and $\phi_t(u)_{u\geq t\geq 0}$, have different intensities of jumps since their purpose is to decompose the LIBOR-OIS spread into a credit and funding component. Our purpose here is different since we aim to construct a simpler model that can be identified using only OIS and IRS data without the use of CDS data. As a consequence we remain agnostic about the decomposition of roll-over risk into its funding and credit risk components.

2.1 OIS and IRS

In Appendix A, we consider the detailed pricing of interest rate swaps (IRS) and overnight-index swaps (OIS).

An OIS swaps a fixed rate for a floating rate determined by an overnight reference rate. Let $\mathcal{T} = \{T_1, \ldots, T_n\}$ with corresponding daycount fraction $\{\delta\}_{i=1}^{n}$. For maturities greater than or equal to one year, the fair fixed rate (paid annually) for an OIS with $T = T_n$ years to maturity, is given by

$$OIS(t; \mathcal{T}) = \frac{1 - P(t, T)}{\sum_{i=1}^{n} \delta_i P(t, T_i)}, \tag{13}$$

see Appendix A.3 for more details.
Similarly an IRS with a tenor x and a payment schedule $\mathcal{T}^x$, corresponding daycount fraction $\{\delta_{x,i}\}_{i=1}^{n^x}$ has fair rate given by

$$S(t; \mathcal{T}^x) = \frac{\sum_{i=1}^{n^x} \mathbb{E}[e^{-\int_{T_{i-1}^x}^{T_i^x} r^c(s)ds} \delta_i^x L\left(T_{i-1}^x, T_i^x\right))|\mathcal{F}_t]}{\sum_{i=1}^{T} \delta_i P(T_{i-1}, T_i)},$$

There is typically a standard tenor in a particular market, but, in Appendix A, we show how synthetic swaps of various tenors can be constructed from basis swap spreads in a model-free manner. (Basis swaps involve the swapping of xIBOR payments of different tenors along with a spread commanded by the market.)

3 Econometric method

Section 3.1 describes our data and Section 3.2 explains our calibration approach.

3.1 Data description and curve construction

We collect data from the Refinitiv database accessed on 15 February 2022. We have data on OIS and IRS dating back to the year 2000, but basis swap spread data is generally not available before 2008 in the Eurozone and 2012 in the US, respectively. We have daily data for EONIA and €STR, Federal Funds Rate, and SOFR based overnight indexed swaps (OIS). Furthermore we have data on interest rate swaps and basis swaps referencing EURIBOR and LIBOR. Beyond the ten-year and below the one-year maturities, the data can be quite sparse and for that reason we focus exclusively on maturities from 1-10 years.

We use piecewise cubic Hermite interpolating polynomials using Matlab's PCHIP function as described below. We avoid any extrapolation. Missing values are also found by interpolation, but we avoid interpolating between points that are more than 2 years apart for maturities less than 10 years, and points that are more than 5 years apart for maturities above 10 years.

In the Eurozone, the major exchanges switched from remunerating collateral at EONIA to €STR on Monday 27 July 2020. We will therefore similarly switch to €STR discounting after that date. We emphasise that the EONIA index has been calculated as €STR plus 8.5 basis points since 1 October 2019, but we do not explicitly model this switch—we merely take the OIS curve as given.

In the USA, the major exchanges switched from remunerating collateral at the federal funds rate to the SOFR on 19 October 2020, but we only have data on SOFR-based OIS from 5 April 2021. We therefore switch from federal funds rates to SOFR-discounting from that date, instead.

We use basis swap spreads to construct IRS rates of various tenors. In the Eurozone case, this is a model-free construction. In the USA case, one needs to solve for synthetic zero-coupon bond prices. This is due to the payment frequency of US basis swaps, see Appendix A for details.

3.2 Calibration and recovery approach

We take weekly steps—with five-business-day gaps—through the data described in Section 3.1. If a particular cross section lacks data, we use the data on the following or previous day instead, if possible. We calibrate the model with two main goals in mind. Firstly, we look to recover the one-month curve.[1] We calibrate the model to the OIS curve, as well as to the curves of three-month,

[1]The one-month curve refers to the cross-section of swap rates linked to the one-month xIBOR, not to the zero rates bootstrapped from this curve (similarly for other tenors). Likewise, the OIS curve is simply the cross-section of OIS rates.

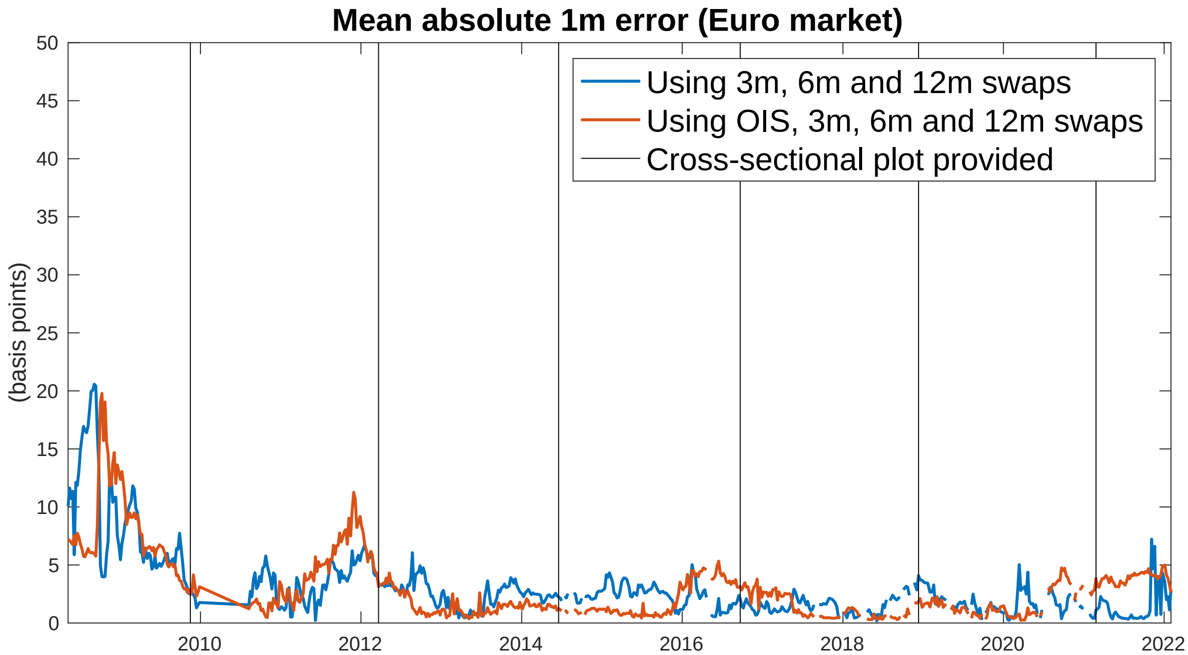

Figure 1: A summary of 647 cross-sectional calibrations to Eurozone data. To measure the success of the estimation, we take the average of the absolute value of the one-month curve estimation errors. The two lines correspond to two methods of calibration: one with the OIS curve, the other without. Certain cross-sections are singled out and indicated, where these are plotted in Figure 3.

six-month and twelve-month IRSs, and then determine the one-month curve implied by the calibrated model. With the one-month curve being in between the OIS curve and three-month curve, we regard this calibration as an interpolation. However, we also consider a variation where we exclude the OIS from the calibration, thus considering the exercise as an extrapolation from higher-tenor curves down to the one-month curve. Secondly, we aim at recovering the OIS curve. We calibrate to the three-month, six-month, and to the twelve-month IRS curves, and then price the OIS curve with the calibrated model. This is an extrapolation from the tenor curves down to the OIS curve. We again consider a variation, where we include the one-month IRS curve. However, if the one-month swap curve is below the OIS curve, the one-month-inclusive calibration is not attempted. The OIS curve is estimated by subtracting the model-estimated IRS-OIS spread from the shortest market IRS curve (one- or three-monthly).

Our calibrations look to minimize the sum of squared differences between market swaps and model-implied swaps, calculated as per Appendix B, by varying $r^c(u)$, $\theta(u)$, θ^*, $x(u)$, $y(u)$ and β. The first three of these variables control the OIS curve (the short, middle and longer parts, respectively), while the final three control the spreads of interest-rate swaps relative to OIS; $x(u)$ and $y(u)$ control the short and long part of the term structure, β controls the relation between different reference tenors. We fix the volatilities, mean-reversion rates and the correlation parameter ρ at moderate values. These quantities are not particularly influential for the swaps curves we consider in the calibration and recovery exercises. They do not correspond to a specific qualitative feature of the model-implied swap curves. We set these parameters at the following values, which we view as typical and moderate, and are based on, e.g., the results of Backwell et al. (2023): $\sigma = 0.005$, $\sigma^* = 0.015$, $\sigma^x = 0.1$, $\sigma^y = 0.1$, $\kappa = 0.25$, $\kappa^* = 0.15$, $\rho = -0.7$.[2]. We plot time-series of selected parameters values and calibrations in Appendix D, see Figure 9.

Once we have calibrated the model, we recover a curve as follows: We take the market curve above the curve in question. That is, if we are attempting to recover the one-month curve, we take the market three-month curve and then subtract the model-implied spread between that curve and the considered

[2]Our results are robust to these parameter settings. For example, if we instead set $\sigma = 0.01$, $\sigma^* = 0.015$, $\sigma^x = 0.15$, $\sigma^y = 0.05$, $\kappa = 0.3$, $\kappa^* = 0.1$, and $\rho = -0.5$, we obtain a very similar fit, with the mean absolute calibration error never changing by more than a basis point. Averaging across all calibration exercises, the mean absolute error changes by less than a tenth of a basis point.

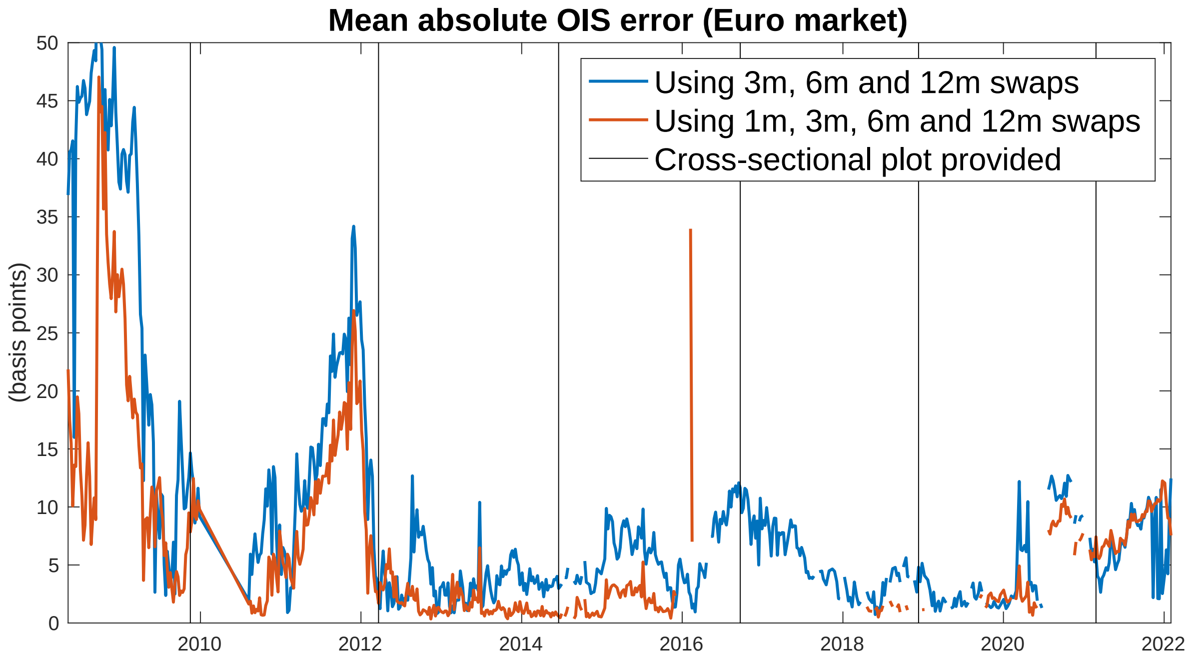

Figure 2: A summary of 647 cross-sectional calibrations to Eurozone data. To measure the success of the estimation, we take the average of the absolute value of the OIS-curve estimation errors. The two lines correspond to two methods of calibration: one with the one-month curve, the other without. (The former is not always possible, because the one-month curve might be below the OIS curve.) Specific cross-sections are indicated and are plotted in Figure 4.

curve below it. This is different from directly defining the recovered curve as the model-implied curve.

4 Results

We consider the Eurozone market in this section, and defer the US market results to Appendix C. In Section 4.1, we consider the interpolation task of recovering the one-month curve given the OIS curve and the higher tenor curves. In Section 4.2, we then look at extrapolating the tenor curves down to the OIS curve.

4.1 Recovering the one-month curve

Figure 1 summarizes our calibrations over the whole sample, where the goal is to recover the one-month curve. We quantify the success of the recovery by taking the average of the absolute differences between the market one-month rates and the model-implied counterparts. With the exception of the market turbulence in 2008, the mean absolute error we obtain is under five basis points. Averaging the mean absolute error over the whole sample, we register 3.1 basis points, which drops to 2.4 if we exclude the year 2008.[3]

In Figure 1, six cross sections are singled out as examples, which are illustrated in Figure 3, in detail. These occur on 16-Nov-2009, 21-Mar-2012, 18-Jun-2014, 22-Sep-2016, 11-Dec-2018 and 25-Feb-2021. For each of these dates, there is a pair of panels in Figure 1; the left-hand side does not use OIS rates (and therefore represents an extrapolation), while the right-hand side uses the OIS curve in the calibration (and effectively seeks to interpolate the one-month curve). In all cases, the one-month curve is plotted in red (with solid lines representing market rates and dashed lines presenting model-implied rates, such that the mean absolute difference between red-solid and red-dashed lines is the quantity considered in Figure 1).

[3]These averages apply to both variations, i.e., both of the lines in Figure 1, where the OIS curve is included/excluded. In other words, the averages turned out to match to the nearest tenth of a basis point.

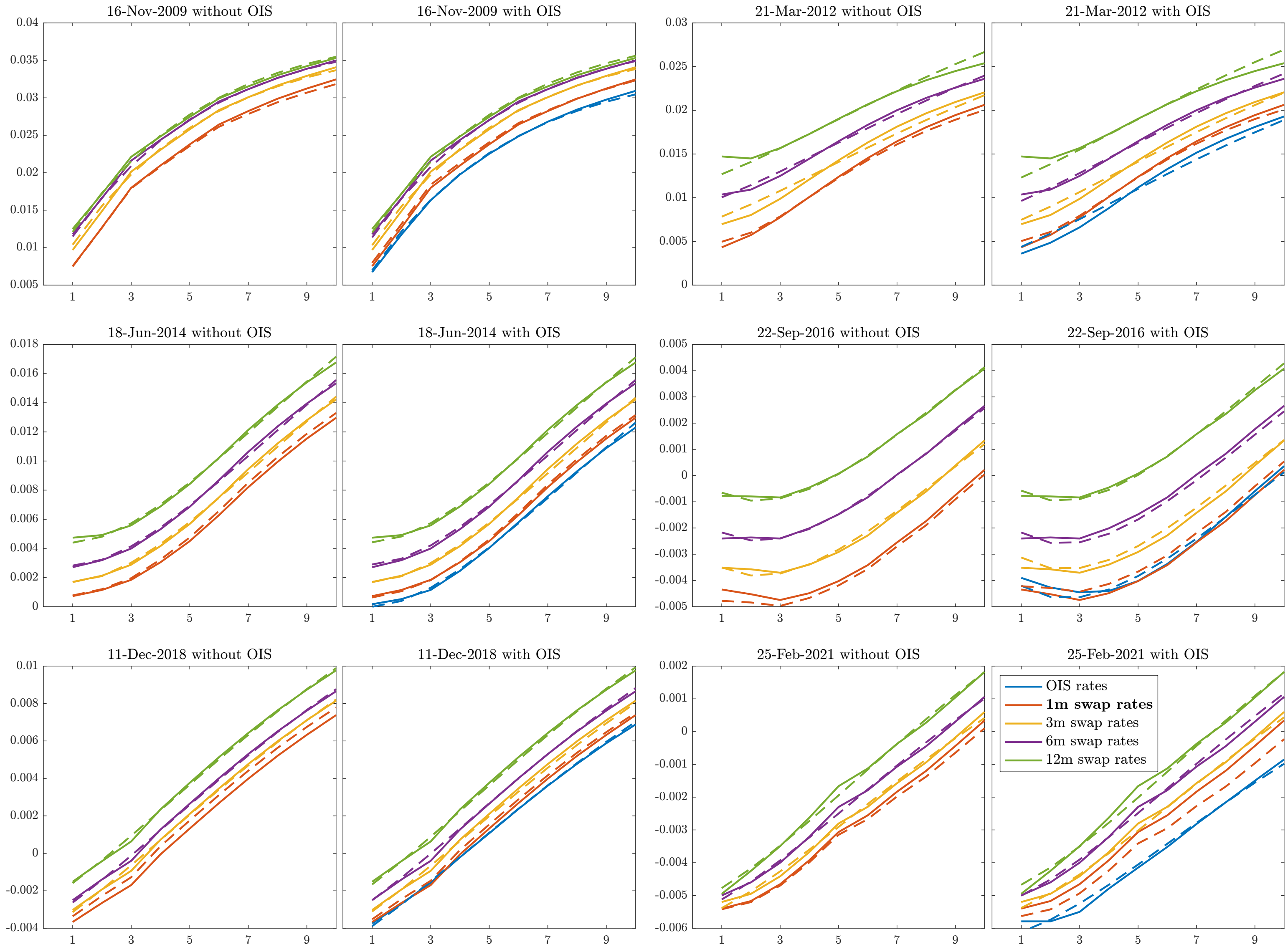

Figure 3: Cross-sectional calibration to Euro-market swaps. As emphasized in the legend, the goal is to estimate/recover the one-month curve, with the calibrated model. The left (right) panel corresponds to a calibration where the OIS curve is excluded (included). Market rates are plotted with solid lines, and model-fitted rates are plotted with dashed lines.

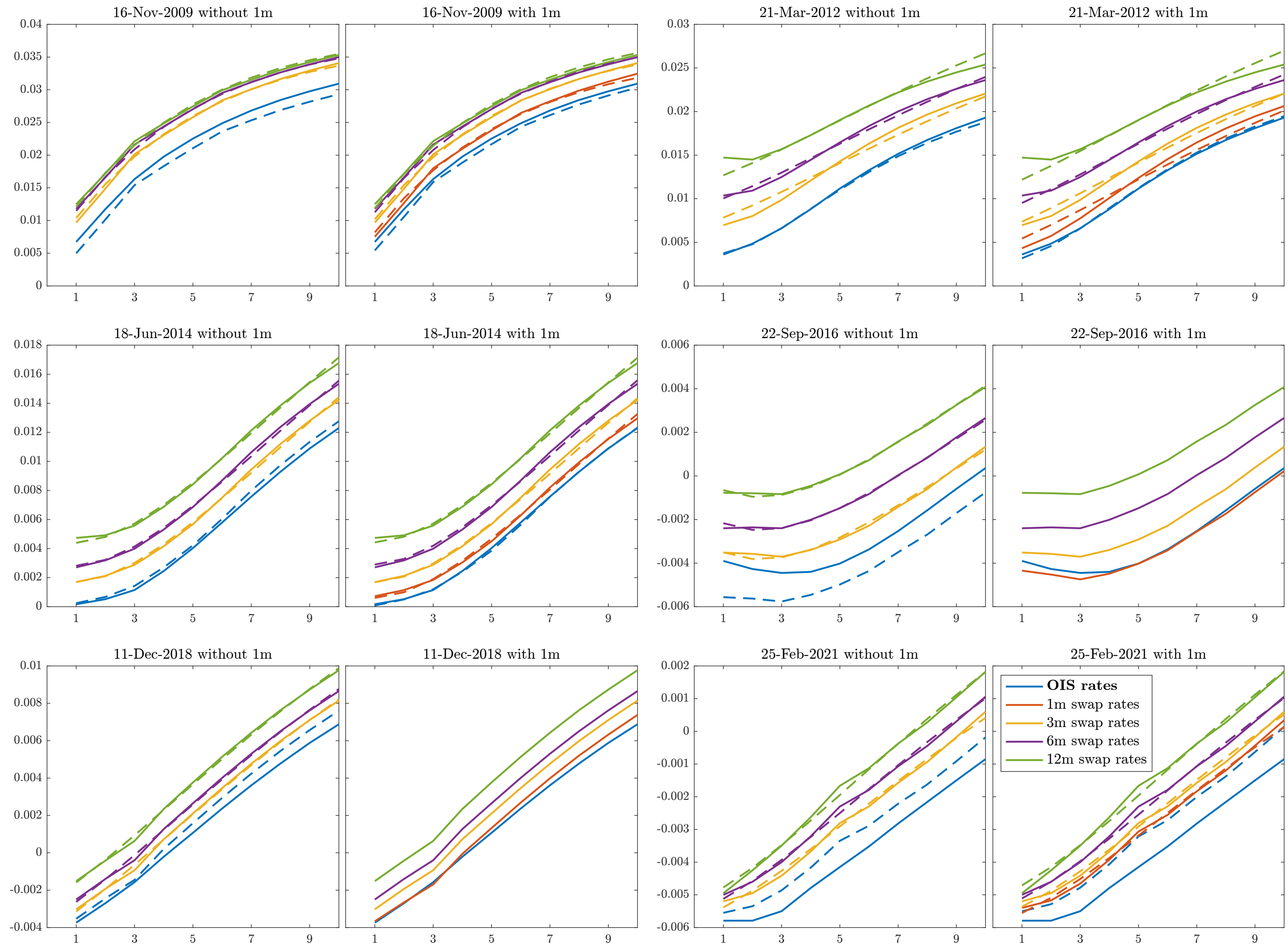

Figure 4: Cross-sectional calibration to Euro-market swaps. As emphasized in the legend, the goal is to estimate/recover the OIS curve, with the calibrated model. The left (right) panel corresponds to a calibration where the one-month curve is excluded (included). Market rates are plotted with solid lines; model-fitted rates are plotted with dashed lines. Model-fitting is at times so accurate that the dashed lines coincide with the solid lines and thus cannot be visually discerned.

4.2 Recovering the OIS curve

Figure 2 summarizes calibrations over the whole sample, where the goal is to recover the OIS curve. We again show the mean absolute difference, in this case between the market OIS rates and the model-fitted ones. As expected, recovery is less accurate in this extrapolation relative to the interpolation in Section 4.1. The average of the mean absolute recovery error over the sample is 9.6 basis points without the one-month curve, which improves to 6.5 when the one-month swaps are included. When 2008 is excluded, these improve to 6.8 and 4.8 basis points, respectively. As argued for example in Alfeus et al. (2020), basis spreads are a result of the market recognising roll–over risk after the turmoul of 2007/2008, so one could justify ignoring the first few years of our data sample as years in which the market was still learning to price this risk, and certainly both interpolation and extrapolation of tenors works much better in the later years of our sample than in the earlier years.

Nevertheless, in Figure 2 there are extensive periods throughout the sample where the one-month tenor rates were lower than the corresponding OIS rates. This is not possible in the context of our model, so this corresponds to times in Figure 2 where the blue curve is plotted, but the red curve is absent. Arguably, OIS is a different market from vanilla and basis interest rate swaps, so perhaps it not surprising that the spread between the latter rates and OIS can not be fully explained by the roll–over risk reflected in our model.

The cross sections indicated in Figure 2 are illustrated in Figure 4. The dates of the cross sections are the same as in Figure 1, and there are again two versions on each date: the left-hand side excludes the one-month curve, while the right-hand includes it. Recall that if the red point is missing and the corresponding blue point is present in Figure 2, it means that some of the one-month swap rates were less than the corresponding OIS rates, and the one-month-inclusive calibration was not attempted (two of these instances can be seen in Figure 4, on 22-Sep-2016 and 11-Dec-2018).

5 Conclusion

Because benchmarks such as LIBOR and EURIBOR, at least in principle, reflect rates at which (some, e.g., major banks) market participants can borrow at term, they contain market information which is not reflected in overnight rates such as SONIA, SOFR or €STR. Synthetic "term rates" such as "SOFR compounded" fail to address this shortcoming and are no suitable replacement for IBOR-contingent term rates. By fitting an interest rate term structure model incorporating roll-over risk into empirical term rate dynamics, we see that such a model can largely recover omitted tenors. This demonstrates that the pricing of roll-over risk in these markets is internally consistent over time. The model performs particularly well in the case of tenor interpolation (i.e., when the model is fitted to tenors both shorter and longer than the one omitted), but still works surprisingly well for extrapolation, i.e., when fitting the model to longer tenors, only, and extracting an implied overnight tenor term structure (e.g., OIS rates). This does not yield perfect results: There are some market features, which are not captured by our model. This observation is in particular confirmed by the fact that there seems to be a structural break between the overnight rate markets and the longer tenor rates. This is especially true in the US markets, where EFFR can be seen as a more direct reflection of a monetary policy rate than, say, EONIA or €STR in the Eurozone. Nevertheless, given a set of term rate benchmarks, one can largely recover overnight rate dynamics, while the converse cannot even be attempted. This is so because in the absence of term rates, any information about the market's view of roll-over risk is lacking. Therefore, in jurisdictions that have not yet locked in the demise of their IBOR-type benchmarks, making these benchmarks more robust and sustainable (rather eliminating them outright) is a matter worth considering in earnest.

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Appendix

A Swap instruments

The appendix contains background material on swap and basis swap contracts. Furthermore, one finds pricing formulae for swaps in an affine jump-diffusion setting. The appendix is concluded with results pertaining term-based curve information in the US market.

A.1 EUR interest rate swaps and basis swaps

A EURIBOR-linked IRS exchanges six-month EURIBOR against a fixed rate, provided the swap maturity is above one year (the one-year swap is linked to three-month EURIBOR instead). The fixed rate is paid on an annual frequency, while the floating rates are paid at the frequency of their respective tenors. Let $\mathcal{T}^x = \{T_i^x\}_{i=1}^{n^x}$ be the payment schedule for the floating rate in an IRS of tenor x and let $\delta_{i,x}$ be the corresponding accrual factor. We denote and value of the floating leg by

$$IRS_{\text{float}}(t;\mathcal{T}^x) := \sum_{i=1}^{n^x} \mathbb{E}[\mathrm{e}^{-\int_{T_{i-1}^x}^{T_i^x} r^c(s)ds} \delta_{i,x} L\left(T_{i-1}^x, T_i^x\right)) | \mathcal{F}_t]. \tag{14}$$

We let $\mathcal{T} = \{T_i\}_{i=1}^n$ be the payment schedule with an annual frequency and δ_i the corresponding accrual factors. Using (1), the fixed leg paying a rate of K is given by

$$\begin{aligned} IRS_{\text{fixed}}(t;\mathcal{T},K) &:= \sum_{i=1}^{n} \mathbb{E}[\mathrm{e}^{-\int_{T_{i-1}}^{T_i} r^c(s)ds} \delta_i K | \mathcal{F}_t] \\ &= \sum_{i=1}^{n} P(T_{i-1},T_i)\delta_i K, \end{aligned} \tag{15}$$

Setting

$$IRS_{\text{float}}(t;\mathcal{T}^x) = IRS_{\text{fixed}}(t;\mathcal{T},K), \tag{16}$$

and solving for K, we can define the swap rate,

$$S(t;\mathcal{T}^x) := K = \frac{IRS_{\text{float}}(t,\mathcal{T})}{IRS_{\text{fixed}}(t;\mathcal{T},1)}. \tag{17}$$

EURIBOR-linked basis swaps exchange payment streams of three-month EURIBOR against six-month EURIBOR, six-month against twelve-month, or one-month against three-month. The basis swaps function in practice as entering two regular fixed-for-floating swaps of the same period. A payers swap on the longer tenor EURIBOR rate and a receivers swap on the shorter tenor EURIBOR rate. Since the fixed leg of the two swaps pay at the same frequency the basis swap is quoted at the value of the spread, $\bar{K}$ between the fixed rate paid on the longer vs. the shorter tenor swap. We can therefore write the value of a basis swap of tenor x_1 against tenor x_2, where $x_1 < x_2$, as

$$P_{BS}(t;\mathcal{T}^{x_1},\mathcal{T}^{x_2},\bar{K}) = IRS_{\text{float}}(t;\mathcal{T}^{x_2}) - IRS_{\text{float}}(t;\mathcal{T}^{x_1}) - IRS_{\text{fixed}}(t;\mathcal{T},\bar{K}).$$

Setting this value to zero, and solving for the fair basis-swap spread $\bar{K}$, one obtains the definition

$$BS(t;\mathcal{T}^{x_1},\mathcal{T}^{x_2}) := \bar{K} = \frac{IRS_{\text{float}}(t;\mathcal{T}^{x_2}) - IRS_{\text{float}}(t;\mathcal{T}^{x_1})}{IRS_{\text{fixed}}(t;\mathcal{T},1)}. \tag{18}$$

We use basis-swap spreads to identify the one-month, three-month and twelve-month swap rates (for swap maturities above one year, i.e., $T > 1$). In particular, from (16) and (18), one sees that

$$S(t;\mathcal{T}^{x_2}) = S(t;\mathcal{T}^{x_1}) + BS(t;\mathcal{T}^{x_1},\mathcal{T}^{x_2}).$$

The above formula allows us to construct synthetic swap rates for swaps with 1M, 3M, 6M, and 12M EURIBOR as the floating leg. For maturities more than one year we observe 6M IRS quotes directly, and use 3M vs 6M and 6M vs 12M basis swap quotes to calculate 3M and 12M swap rates respectively. We then use the synthetic 3M swap rates along with the 1M vs 3M basis swap quotes to get synthetic 1M swap rates. For the 1 year maturity the 3M swap rate is quoted directly. We therefore use 3M vs 6M and 1M vs 3M basis swaps quotes to calculate synthetic 1M and 6M swap rates respectively. Finally, the synthetic 6M swap rate along with the 6M vs 12M basis swap is used to calculate the 12M swap rate.

A.2 US interest rate swaps and basis swaps

A USD-LIBOR-linked IRS exchanges quarterly payments of three-month LIBOR against an annual fixed rate. This is captured by (17). In a USD-LIBOR-linked basis swap you have two floating legs. One pays the longer tenor (USD)-LIBOR rate and the other pays the shorter tenor (USD)-LIBOR plus a fixed spread $\bar{K}$. The spread is paid at the frequency of the corresponding shorter tenor. This is unlike the Eurozone, where the spread is always paid annually. We therefore denote and compute the value of a US-style , x_1 against tenor x_2 basis swap, where $x_1 < x_2$, as

$$\overline{P}_{BS}(t; \mathcal{T}^{x_1}, \mathcal{T}^{x_2}, \bar{K}) = IRS_{\text{float}}(t; \mathcal{T}^{x_2}) - IRS_{\text{float}}(t; \mathcal{T}^{x_1}) - IRS_{fixed}(t, \mathcal{T}^{x_1}, \bar{K}) \tag{19}$$

As before, the fair (US-style) basis-swap spread $\bar{K}$ is found by setting the the above quantity to zero, and we get

$$BS(t; \mathcal{T}^{x_1}, \mathcal{T}^{x_2}) := \bar{K} = \frac{IRS_{\text{float}}(t; \mathcal{T}^{x_2}) - IRS_{\text{float}}(t; \mathcal{T}^{x_1})}{IRS_{\text{fixed}}(t; \mathcal{T}^{x_1}, 1)}. \tag{20}$$

We get 3M swap rates quoted directly in the market. In order to calculate synthetic 1M, 6M and 12M (USD)-LIBOR rates, we first determine the value of the corresponding IRS floating legs. We obtain $IRS_{\text{float}}(t; \mathcal{T}^{3M})$ from (16). Using the market quote for the 1M vs 3M basis swap, we use (19) and (15) to get

$$IRS_{\text{float}}(t; \mathcal{T}^{1M}) = IRS_{\text{float}}(t; \mathcal{T}^{3M}) - IRS_{\text{fixed}}(t, \mathcal{T}^{1M}, BS(t; \mathcal{T}^{1M}, \mathcal{T}^{3M})) \tag{21}$$

We proceed similarly for $x_2 = 6M, 12M$ to get:

$$IRS_{\text{float}}(t; \mathcal{T}^{x_2}) = IRS_{\text{float}}(t; \mathcal{T}^{3M},) + IRS_{\text{fixed}}(t, \mathcal{T}^{3M}, BS(t; \mathcal{T}^{3M}, \mathcal{T}^{x_2})) \tag{22}$$

The floating values can then be used to calculate the synthetic swap rates using (17).

A.3 Overnight Index Swaps

An OIS involves the exchange of fixed and floating payments each year for the life of the swap (we do not consider maturities less than one year). Letting $R_d(t)$ denote the prevailing overnight rate over time, the floating payment to be paid at some time T is given by

$$R(T) = \prod_{i=0}^{n_T - 1} (1 + \delta_i R_d(t_i)) - 1,$$

where $\{t_i\}_{i=0}^{n}$ includes all business days in the year preceding T, such that $t_0 = T - 1$ and $t_{n_T} = T$, and where $\delta_{d,i} = t_{i+1} - t_i$ (the year fraction associated with each overnight gap between business days). If the maturity is less than a year then there is only one fixed and floating payment at maturity, the latter determined by setting $t_1 = 0$ in the above formula. The benchmark floating rates $R_d(t)$ are the federal funds rate (until October 2020) and SOFR in the US and EONIA (until October 2019) and ESTR in the eurozone. Let us now consider an OIS with an annual payment schedule $\mathcal{T} = \{T_1, \ldots, T_n\}$, with corresponding daycount fractions $\{\delta_i\}_{i=1}^{n}$ with floating leg $\{R(T_i)\}_{i=1}^{n}$ defined above. Following the same arguments as in Filipović & Trolle (2013) section 2.5(but see also Grbac & Runggaldier (2015)) we can calculate the fair rate on an OIS as

$$OIS(t, \mathcal{T}) := \frac{1 - P(t, T_n)}{\sum_{i=1}^{n} \delta_i P(t, T_i)}$$

The above formula allows us to construct the OIS term structure on an annual frequency. This is sufficient to calculate the value of the fixed leg EUR swaps in (18). Pricing the fixed leg in US swaps in (21) and (22) requires knowing the OIS term structure on at least a monthly frequency. We calculate this term structure by interpolating at the level of yield-to-maturity.

B Instrument Pricing

Let $X_t(u) = [r^c(u), \theta(u), x(u), y(u), \phi_t(u), \lambda_t(u)]^\top$, $0 \le t \le u$. For a fixed t, the process $X_t(u)_{0\le t\le u}$ is an affine jump diffusion in the style of Duffie et al. (2000) that satisfies

$$dX_t(u) = (K_0 + K_1 X_t(u))du + L(X_t(u))dW(u) + dJ(u), \quad u \ge t \ge 0,$$

and where $W(u-t)_{u\ge t}$ is a four-dimensional Brownian motion[4]. Moreover,

$$K_0 = \text{diag}[0, -\kappa^*\theta^*, 0, 0, 0, 0, 0],$$

$$K_1 = \begin{bmatrix} -\kappa & \kappa & 0 & 0 & 0 & 0 \\ 0 & -\kappa^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\beta & 0 \\ 0 & 0 & 0 & 0 & 0 & -\beta \end{bmatrix},$$

$$L(x) = \begin{bmatrix} \sigma & 0 & 0 & 0 \\ \rho\sigma^* & \sqrt{1-\rho^2}\sigma^* & 0 & 0 \\ 0 & 0 & \sigma^x\sqrt{x_3} & 0 \\ 0 & 0 & 0 & \sigma^y\sqrt{x_4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$J(u) = \sum_{i=1}^{N^\lambda(u)} Y_i^\lambda + \sum_{i=1}^{N^\phi(u)} Y_i^\phi,$$

with jump size vectors $Y_i^\lambda = [0, 0, 0, 0, Z_i, 0]$ and $Y_i^\phi = [0, 0, 0, 0, 0, Z_i]$. The jumps Z_i are independent, identical, and exponentially distributed where $N^\lambda(u)_{u\ge0}$ and $N^\phi(u)_{u\ge0}$ are (conditionally) independent Poisson processes with stochastic intensity given by $x(u)_{u\ge0}$. Following Duffie et al. (2000), we have

$$f(X_t(t), t, T) = \mathbb{E}\left[\mathrm{e}^{-\int_t^T(\rho_0+\rho_1 X_t(u))du + wX_t(T)} \,\middle|\, \mathcal{F}_t\right] = e^{\alpha(T-t)+\beta(T-t)\cdot X_t(t)}, \tag{23}$$

where

$$\begin{aligned}
\beta_1'(s) &= -\kappa\beta_1(s) - \rho_{1,1}, \\
\beta_2'(s) &= -\kappa^*\beta_2(s) - \rho_{2,1}, \\
\beta_3'(s) &= \frac{1}{2}(\sigma^x)^2\beta_3^2(s) + \frac{0.02}{1-0.02\beta_4} + \frac{0.02}{1-0.02\beta_5} - 2 - \rho_{3,1}, \\
\beta_4'(s) &= \beta_3(s) + \frac{1}{2}(\sigma^y)^2\beta_4^2(s) - \rho_{4,1}, \\
\beta_5'(s) &= -\beta\beta_5(s) - \rho_{5,1}, \\
\beta_6'(s) &= -\beta\beta_6(s) - \rho_{6,1}, \\
\alpha'(s) &= -\kappa^*\beta_2(s) + \frac{1}{2}\beta_1(s)^2 + \frac{1}{2}\beta_1(s)\beta_2(s)\rho\sigma\sigma^* - \rho_0,
\end{aligned}$$

with initial conditions $\alpha(0) = 0$ and $\beta(0) = w$. The $\beta_3'(s)$ term includes terms associated with the jump size distributions; recall that we set the mean of the exponentially distributed jumps to 2%, so that $\mathbb{E}[\mathrm{e}^{wZ_i}] = 0.02/(1-0.02w)$. The linear ODE dimensions of the form: $\beta'(t) = -\kappa\beta(t) + \rho, \beta(0) = w$ have the solution

$$\beta(t) = e^{-t\kappa}(w - \rho/\kappa) + \rho/\kappa$$

[4]The components of this Brownian motion are independent, unlike the one-dimensional Brownian motions in (6) and (7). In particular, $W^*(t)$ in (7) is given by $\rho W^1(t) + \sqrt{1-\rho^2}W^2(t)$, where $W^i(t)$ are the scalar components of the four-dimensional $W(t)$.

while the remaing $\beta_3(t), \beta_4(t)$ are solved numerically using the Runge-Kutta method. Setting $\rho_0 = 0$, $\rho_1 = [1, 0, 0, 0, 0, 0]$ and $w = 0$ in (23), one can price the bond in (1) and determine fair OIS rates via (13). In this case, we denote the obtained affine coefficient functions as follows:

$$P(t,T) = \mathbb{E}\left[\mathrm{e}^{-\int_t^T r^c(u)du}|\mathcal{F}_t\right] = \mathrm{e}^{\alpha^c(T-t)+\beta^c(T-t)\cdot X_t(t)}.$$

In the case that downgrade risk is included (in other words, for $\rho_1 = [1, 0, 0, 0, 0, 1]$, but leave $\rho_0 = 0$ and $w = 0$), we have

$$\mathbb{E}\left[\mathrm{e}^{-\int_t^T (r^c(u)+\lambda_t(u))du} \mid \mathcal{F}_t\right] = \mathrm{e}^{\alpha^\lambda(T-t)+\beta^\lambda(T-t)\cdot X_t(t)}.$$

For $\rho_1 = [0, 0, 0, 0, -1, 0]$, we obtain

$$\mathbb{E}\left[\mathrm{e}^{\int_t^T \phi_t(u)du} \mid \mathcal{F}_t\right] = \mathrm{e}^{\alpha^A(T-t)+\beta^A(T-t)\cdot X_t(t)}.$$

The xIBOR rate in (4) can then be given as,

$$L(t,T) = \frac{1}{T-t}\left(\mathrm{e}^{\alpha^L(T-t)+\beta^L(T-t)\cdot X_t(t)} - 1\right)$$

where $\alpha^L(u) = \alpha^A(u) - \alpha^\lambda(u)$ and $\beta^L(u) = \beta^A(u) - \beta^\lambda(u)$. Finally, we consider setting $\rho_0 = 0$, $\rho_1 = [1, 0, 0, 0, 0, 0]$ and $w = \beta^L(T-t) + \beta^c(T-t)$ in (23), and obtain

$$\mathbb{E}\left[\mathrm{e}^{-\int_t^T r^c(u)du+(\beta^L(T-t)+\beta^c(T-t))X_t(T)}|\mathcal{F}_t\right] = \mathrm{e}^{\alpha^D(T-t)+\beta^D(T-t)\cdot X_t(t)},$$

which can be used to price the discounted value of a future xIBOR payment as follows:

$$\begin{aligned}
\mathbb{E}\left[\mathrm{e}^{-\int_s^T r^c(u)du}L(t,T) \mid \mathcal{F}_s\right] =& \mathbb{E}\left[\mathbb{E}\left[\mathrm{e}^{-\int_t^T r^c(u)du}|\mathcal{F}_t\right]\mathrm{e}^{-\int_s^t r^c(u)du}L(t,T) \mid \mathcal{F}_s\right] \\
=& \frac{1}{T-t}\mathbb{E}\left[\mathrm{e}^{-\int_s^t r^c(u)du}e^{\alpha^c(T-t)+\alpha^L(T-t)+(\beta^L(T-t)+\beta^c(T-t))\cdot X_t(t)} \mid \mathcal{F}_s\right] \\
&- \frac{1}{T-t}\mathbb{E}\left[\mathrm{e}^{-\int_s^t r^c(u)du}e^{\alpha^c(T-t)+\beta^c(T-t)\cdot X_t(t)} \mid \mathcal{F}_s\right] \\
=& \frac{\mathrm{e}^{\alpha^c(T-t)+\alpha^L(T-t)+\alpha^D(t-s)+\beta^D(t-s)\cdot X_s(s)} - \mathrm{e}^{\alpha^c(T-t)+\beta^c(T-t)\cdot X_s(s)}}{T-t}.
\end{aligned}$$

Thus, we are able to evaluate (14) and then the swap rate (17).

C US Market Results

C.1 Recovering the one-month curve

Figure 5 shows attempted recovery of the one-month curve in the context of the US market, in the same format as Figure 1 in Section 4.1, with Figure 6 again showing the indicated cross sections. The average mean absolute recovery error over the sample is, to the nearest tenth of a basis point, 4.1 basis points, both with and without the OIS curve.

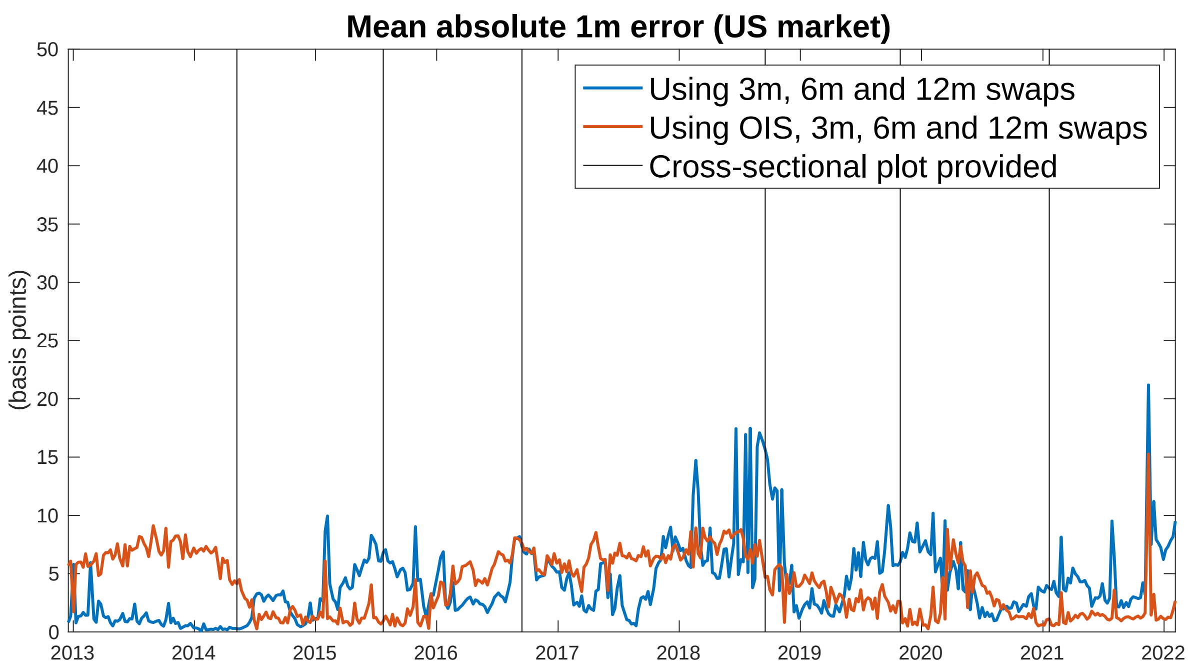

Figure 5: A summary of 452 cross-sectional calibrations to US data. To measure the success of the estimation, we take the average of the absolute value of the one-month curve estimation errors. The two lines correspond to two methods of calibration: one with the one-month curve, one without (the former is not always possible, as the one-month curve might be below the OIS curve). Certain cross sections are indicated; these are plotted in Figures 6.

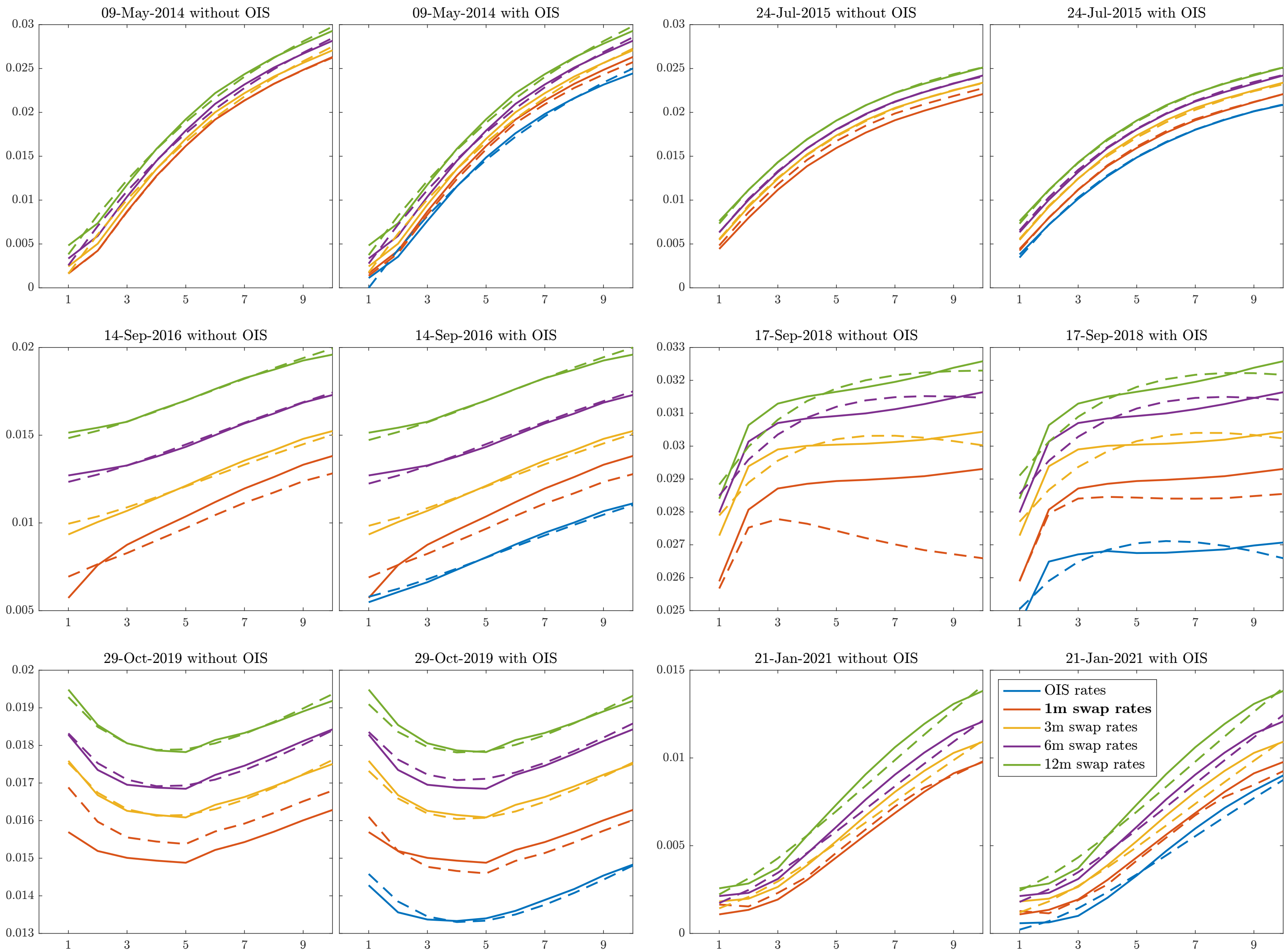

Figure 6: Cross-sectional calibration to US-market swaps. As emphasized in the legend, the goal is to estimate/recover the one-month curve, with the calibrated model. The left (right) panel corresponds to a calibration where the OIS curve is excluded (included). Market rates are plotted with solid lines, the model-fitted rates are plotted with dashed lines.

C.2 Recovering the OIS curve

Figure 7 shows our attempted recovery of the OIS curve in the context of the US market, in the same format as Figure 2 in Section 4.2, with Figure 8 again showing the indicated cross sections. Averaging over the sample, we obtain an average mean absolute recovery error of 11.1 basis points; this drops to 8.7 basis points when the one-month curve is included.

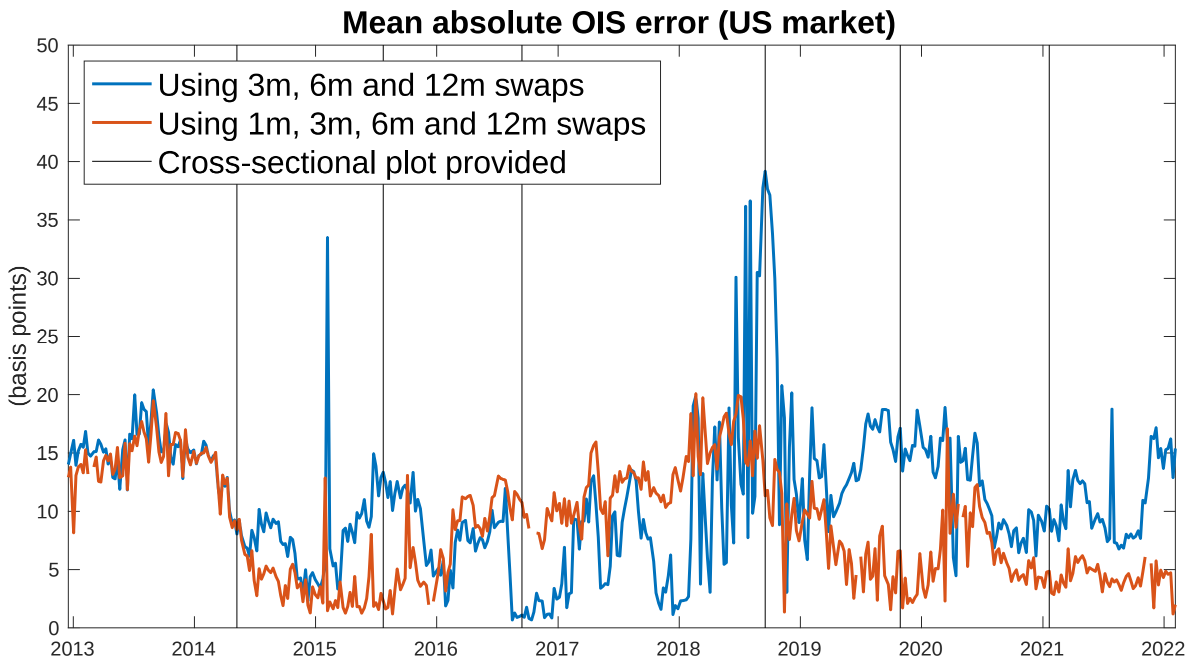

Figure 7: A summary of 452 cross-sectional calibrations to US data. To measure the success of the estimation, we take the average of the absolute value of the OIS-curve estimation errors. The two lines correspond to two methods of calibration: one with the one-month curve, one without (the former is not always possible, as the one-month curve might be beneath the OIS curve). Certain cross sections are indicated; these are plotted in Figures 8.

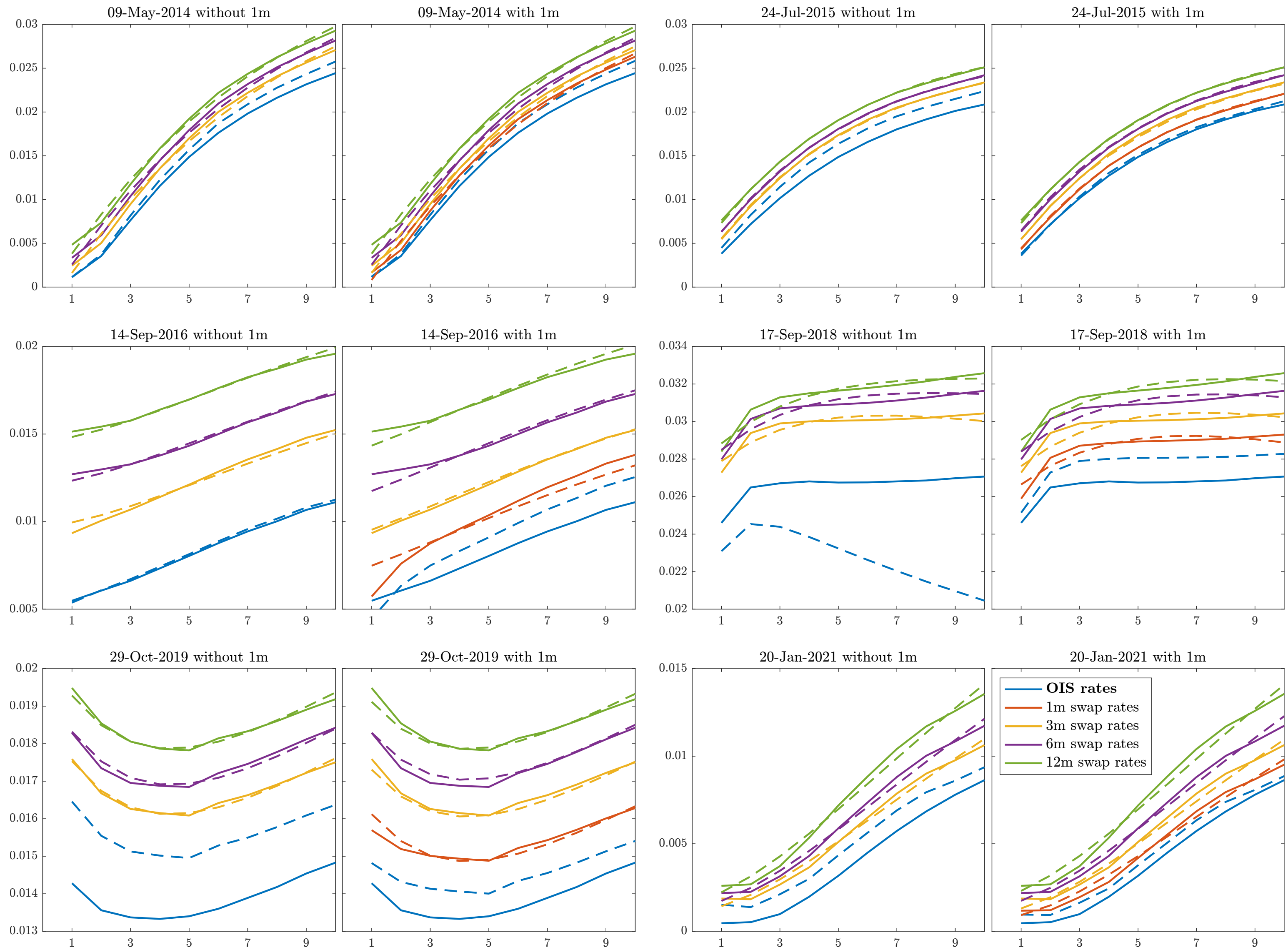

Figure 8: Cross-sectional calibration to US-market swaps. As emphasized in the legend, the goal is to estimate/recover the OIS curve, with the calibrated model. The left (right) panel corresponds to a calibration where the one-month curve is excluded (included). Market rates are plotted with solid lines; the model-fitted rates are plotted with dashed lines.

D Time-Series of Calibrated Parameters

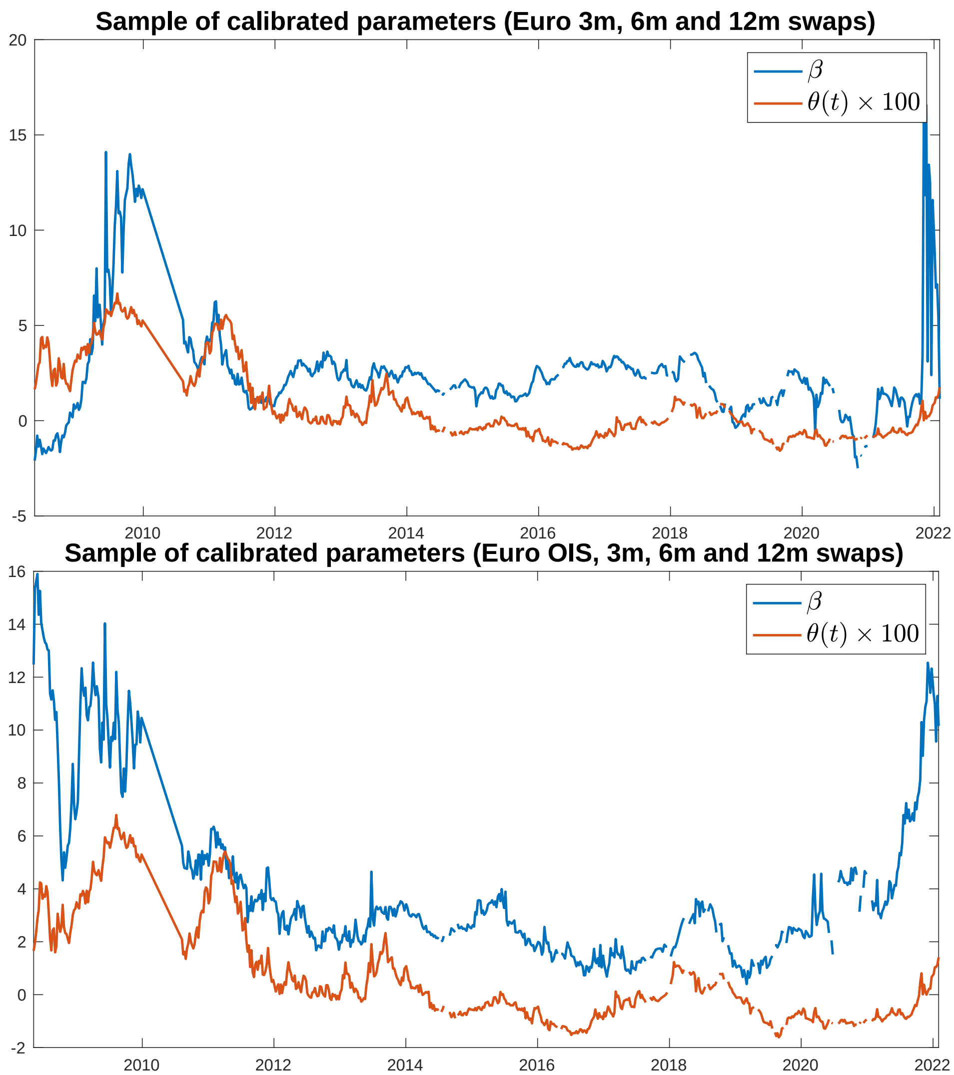

Figure 9: The above show the time-series of the calibrated parameters to the EUR market swaps. The top and panel shows the calibration with and without the OIS included. The parameters show stable behaviour throughout the sample except for the most volatility periods in the beginning and the end of the sample period

Disclosure of interest

There are no conflicting interests to declare.

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References

Acharya, V., Gale, D. & Yorulmazer, T. (2011), 'Rollover risk and market freezes', Journal of Finance **66**(4), 1177–1209.

Alfeus, M., Grasselli, M. & Schlögl, E. (2020), 'A consistent stochastic model of the term structure of interest rates for multiple tenors', *Journal of Economic Dynamics and Control* **114**.

Backwell, A., Macrina, A., Schlögl, E. & Skovmand, D. (2023), 'Term rates, multicurve term structures and overnight rate benchmarks: A roll-over risk approach', *Frontiers of Mathematical Finance* **2**(3).

Bianchetti, M. (2010), 'Two Curves, One Price', *Cutting Edge Interest Rate Derivatives, Risk (August 2010)* .

Crépey, S. & Douady, R. (2013), 'LOIS: Credit and Liquidity', *Risk* pp. 82–86.

Duffie, D., Pan, J. & Singleton, K. (2000), 'Transform analysis and asset pricing for affine jump-diffusions', *Econometrica* **68**(6), 1343–1376.

Filipović, D. & Trolle, A. B. (2013), 'The Term Structure of Interbank Risk', *Journal of Financial Economics* **109**, 707–733.

Fontana, C., Pavarana, S. & Runggaldier, W. J. (2023), 'A stochastic control perspective on term structure models with roll-over risk'.
URL: *https://arxiv.org/abs/2304.04453*

Gallitschke, J., Seifried, S. & Seifried, F. T. (2017), 'Interbank interest rates: Funding liquidity risk and XIBOR basis spreads', *Journal of Banking and Finance* **78**, 142–152.

Goyder, B. (2023), 'FRA–OIS demise leaves hole in bank treasury risk management', *Risk.net* .

Grbac, Z. & Runggaldier, W. J. (2015), *Interest Rate Modeling: Post–Crisis Challenges and Approaches*, Springer Briefs in Quantitative Finance, Springer.

Henrard, M. (2007), 'The irony in the derivatives discounting', Wilmott Magazine **3**, 92–98.

Henrard, M. (2014), *Interest Rate Modelling in the Multi-Curve Framework*, Palgrave Macmillan, UK.

Morini, M. (2009), 'Solving the puzzle in the interest rate market', SSRN:1506046 .

Piterbarg, V. (2012), 'Cooking with Collateral', *Cutting Edge Derivatives Pricing, Risk (Aug. 2012)* .