Towards New Materiality and Structures

Generating 3D topological interlocking assemblies from 2D Penrose tiling using interactive geometric software

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The term Topological Interlocking Assembly (TIA), proposed by A.V. Dyskin et al., refers to an assembly system whose internal elements are both translationally and rotationally locked. Such systems allow masonry construction to form stable assemblies more sustainably, relying not on mortar or special connectors but by the geometric arrangement of bespoke blocks. Prior studies in TIA focused mainly on platonic geometries, and a few have considered curved structures, derived primarily from regular 2D tessellations. In contrast to existing methods, this paper explored nonplatonic mortarless vaults with TIA generated from aperiodic Penrose tiling for both structural function and aperiodic aesthetic in architecture. The patterns formed by Penrose tiles, invented by Sir Roger Penrose in the 1970s, are aperiodic, i.e. they do not repeat regularly. This research used Rhino Grasshopper to enable real-time control and visualization of design parameters and variations in 2D tiles and 3D aggregation methods. Using geometric software as an interface enables architectural form-finding through material behavior and structural principles. The TIA approach considered is shown to offer ample design space for nonplatonic geometries and breaks masonry architecture's dependence on conventional construction methods. This computer-aided design outcome allows us to formulate new understandings of materiality and structures.

Keywords: *Masonry Architecture, Topological Interlocking Assembly, Penrose Tiling, Mortarless Structure, Aperiodic Tiling, Double-curved Surface.*

INTRODUCTION

Built construction comprises 45% of total UK carbon emissions (*IGPP*, 2021). Masonry, the most ubiquitous building craft, combines basic components such as bricks or stones into architectural geometries in various arrangements, colors, and textures. Binding masonry blocks into a stable structure conventionally requires mortar or special connectors. Brickwork, for example, is typically constructed using cement mortar, which accounts for 17.5% of the total volume when laid in stretcher bond (Bustillo Revuelta, 2021). In addition, cement production accounts for around 7% of global carbon emissions (Supriya *et al.*, 2023), so cement-based mortar contributes substantially to the emissions behind masonry architecture. As a result, mortarless masonry construction emerges as a more sustainable method to reduce construction waste, onsite assembly duration and labor.

In 1699, Joseph Abeille invented a renowned flat vault that covered space, without mortar, by assembling identical convex ashlars (Weizmann, Amir and Grobman, 2016). Abeille's Flat Vault, defined as a material system, is the origin of the Topological Interlocking Assembly (TIA) concept. TIA refers to an assembly system whose internal elements are both translationally and rotationally locked (Dyskin *et al.*, 2003). Architectural innovations based on traditional materials and structures have been made by introducing digital tools into the design and fabrication process. Such tools reduces technical limitations, but formal limitations remains and still constrain masonry structures. Numerous studies have attempted to transfer the complexity from masonry bonds onto individual block modules in order to create more complex geometries. With the integration of geometric software into contemporary masonry architecture, pre-fabricated non-standard interlocking blocks promise potential to be employed in constructing vaults, whether flat or curved.

This paper discusses precedent studies on ancient Flat Vaults and contemporary Topological Interlocking Assembly, analyzing their basic principles, block types, assembly logic, and examining their advantages and limitations. It seeks to address the following questions:

- Can topological interlocking methods be extended from planar assemblies to double-curved vaults, with planar or nonplanar contacting surfaces between blocks?
- Can 2D aperiodic tiling patterns, like Penrose tilings, form 3D topological interlocking assemblies and create curved vaults?

To answer these questions, Rhinoceros Grasshopper is employed as interactive geometric software to design and simulate TIA on both flat and curved surfaces. A digital workflow is developed to facilitate real-time control and visualization of design parameters and variations in 2D tile patterns and 3D aggregation methods. The Parakeet plugin (Mottaghi and KhalilBeigi, 2018) is used to generate 2D Penrose Rhombuses in an easy-to-use approach. Additionally, this research will explore the potentials and limitations of employing these patterns in TIA for curved vaults.

FLAT VAULTS OF INTERLOCKING BLOCKS

Whether working with natural or man-made materials, the term "bonding" refers to the regular arrangement of building blocks into a whole. Simply stacking one block on top of another results in instability, so mortar complements the bonding with binding, adhering the blocks into a stable masonry structure. It has long been believed that, in the absence of binding, any geometry made up of convex blocks can be dismantled by removing blocks one by one. While true in two dimensions, it is not always true in three dimensions (Dyskin *et al.*, 2003). The innovative stereotomy by Abeille and Truchet around the end of the 17th century serves as compelling evidence.

Abeille's flat vault

In the seventeenth century, adopting form to forces through arches and vaults was a prevalent method of construction. Abeille's Flat Vault was published in *Machines et inventions approuvées par l'Académie Royale des Sciences* in 1699 (Fallacara, 2006). In this patent, identical truncated tetrahedron-shaped stones were arranged in a 2D woven pattern that spanned the vault with neither a keystone nor mortar (Tessmann and Becker, 2013). The flat vault is a type of planar assembly that can withstand structural instability when perpendicular loads are applied to the planar surface. The essential characteristics of such assemblies are that each block is held in place by geometrical restrictions, and the assembly itself is bound at the periphery (Dyskin *et al.*, 2003). As a result, there is no need to bind the blocks together.

Visually, each identical polyhedron in this system presents isosceles trapezium-shaped axial sections (Figure 1a). The unique geometry of the ashlar blocks, along with the rotation of neighboring blocks by ninety degrees, enables a simple method of arranging them together. To be specific, each ashlar is supported by two adjacent ones via its protruding cuts, while also supporting two other ashlars via its sloped cuts (Vella and Kotnik, 2016). Interlocking stones leave small, square voids on the intrados of the two visible surfaces, which can be filled either with mortar or with stones of a different color for aesthetic purpose. However, the presence of pyramidal holes, while serving as decoration, may also be seen as a disadvantage of Abeille's approach.

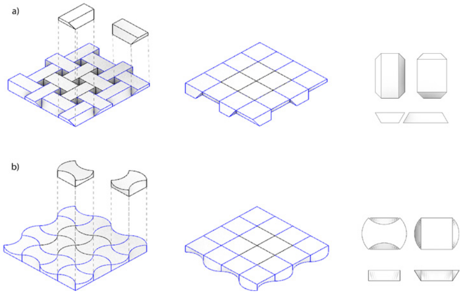

Figure 1
Flat Vault
a) Abeille's flat vault
b) Truchet's flat vault with boundary conditions highlighted in blue

Truchet's flat vault

Father Truchet developed his design in 1704 as an enhancement of Abeille's vault, building upon a more intricate version with a similar organizational pattern. His interlocking vault features no voids on both sides of the ceiling. The distinction lies in the shape of four intersecting surfaces (Figure 1b): Abeille's blocks consist of four intersecting flat surfaces, whereas Truchet's blocks feature concave and convex surfaces resembling saddles to ensure contact at all points (Borhani and Kalantar, 2017). On one hand, this property of contacting surfaces prevents the formation of voids, which is truly ingenious. On the other hand, its complexity poses challenges in fabrication.

It is significant that these flat vaults designed by the two Frenchmen are optimized to utilize only one shape of stone. Moreover, their structures bear loads and control the displacement of blocks, making them capable of withstanding both orthogonal and transverse forces (Akleman *et al.*, 2020).

However, flat vaults are rarely used successfully as an architectural construction method in contemporary building because they function only after the entire assembling process is accomplished. Furthermore, the assembly's large horizontal forces necessitate a strong boundary to pin the structure in place, such as buttresses or substantial walls. Historically, the stone cutting involved in masonry architecture required skilled craftsmen proficient in structural and geometric analysis of architectural shapes. As a result, using basic identical bricks and mortar to compensate for tolerances proved to be more appropriate for the construction sites and unskilled workers during that period.

TOPOLOGICAL INTERLOCKING ASSEMBLY (TIA)

Digital craft connects humans and materials through the intervention of computers, expanding the possibilities of block making and block aggregation beyond traditional techniques. Integrating flat vault systems into contemporary digital environments could offer economic and environmental benefits, as all the modules could be easily disassembled and reused.

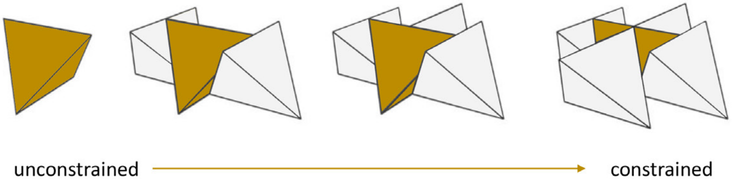

Figure 2
Repetitive Tetrahedra based on Dyskin's definition

Designers have proposed two classes of TIA systems inspired by Abeille's and Truchet's Flat Vaults. These structures comprise a repeating pattern of interlocked tetrahedra and osteomorphic blocks. Their distinguishing element is the contact surface between modules. TIA systems are neither scale- or material-dependent, as their properties derived from geometric configurational conditions (Vella and Kotnik, 2016). More generally, TIA systems are defined by A.V. Dyskin et al. as assemblies in which an element is considered translationally locked if it cannot be removed by translation, and its neighboring elements are fixed (Figure 2). Once fully interlocked, an assembly is linked both translationally and rotationally, rendering it typically ten times more impact-resistant than its monolithic form (Mirkhalaf, Zhou and Barthelat, 2018).

Abeille-based repetitive tetrahedra

In 1970, Michael Glickman first expanded Abeille's system by introducing regular tetrahedrons (Glickman, 1984). Considering a single regular tetrahedron, all of its degrees of freedom can be locked through simple contact with nearby modules. As Dyskin's team defined the term TIA in 2001, they researched various aspects such as stiffness, deformation, bearing capacity, and even sound absorption of planar configurations, that were comparable to Abeille's fundamental concept (Borhani and Kalantar, 2017). In 2006, Giuseppe Fallacara exploited the topological interlocking principles of the Abeille vault to apply the construction method to non-planar vaulted structures and domes. His focus was on the woven aesthetics inherent in the construction principle (Fallacara, 2006). Oliver Tessmann further developed the concept of interlocking tetrahedra. He began exploring the potential for variation within the ashlar itself, examining its generative impact on the shape of the assembly, as well as its potential for creating architectural differentiation in surface qualities (Vella and Kotnik, 2016). If this interlocking structure is constrained at the boundary, the neighboring blocks will hinder the mobility of each platonic-shaped block, and there is no way to remove a block from the assembly. However, the surface of these structures is uneven, and the structure's thickness varies. This allows it to be used for the outer shell of some small constructions but makes it unideal for designed objects requiring high shell performance.

Figure 3
Osteomorphic clay bricks

Truchet-based osteomorphic blocks

The second type is osteomorphic blocks by Estrin et al (2011). These blocks interlock when the convexities on the surface of one block match convexities on the other (Figure 3). Here, non-planar surfaces are adequately smooth to avoid stress concentrations when blocks are interlocked. To implement this block type in actual masonry construction projects, the blocks need to meet planar bottom and corner requirements. Additionally, one block can be translatable by one half-length to lock into another. For corner constructions, blocks need to be rotatable by 90 degrees. Osteomorphic modules have the potential for use in the contemporary building industry. While the non-planar surface is relatively complex to manufacture and requires additional post-tensioning to complete the structure, it presents significant advantages and is worth further research for masonry architecture.

Existing TIA research mainly focused on platonic floor geometry formed primarily from regular 2D tessellations. This paper explores nonplatonic mortarless vaults with TIA generated from aperiodic Penrose tiling for both structural function and aperiodic aesthetic in architecture.

METHODOLOGY

Designers can interact with digital materials through CAD software for deeper insight into their limitations and possibilities (Piker, 2013). Thus, using geometric software as an interface can link the stages of design and fabrication, demonstrating robust interaction capabilities that transition seamlessly from 2D to 3D in computer graphics (Menges, 2015). Based on prior studies on TIA, this research employs Rhino Grasshopper as interactive geometric software. The software workflow can be divided into five steps:

1. reference curved surface form-finding,
2. inputting 2D pattern data,
3. generating 3D topological interlocking structure with surface approximation,
4. refining the modules,
5. and simulating the assembly sequence.

Steps one and two would differ slightly between constructing TIA from square tiles and from Penrose Rhombuses.

PRE-DESIGN: REGULAR RECTANGLE-BASED TILING

The goal of this pre-design is to enhance understanding of the flat vaults designs by Abeille and Truchet, as well as present-day TIA methods, before proceeding to creating TIA based on more complex aperiodic Penrose rhombuses. This pre-design will investigate the generation of both planar and nonplanar contacting surfaces, drawing inspiration from both Abeille- and Truchet-based TIA methods. The goal is to create double-curved geometries.

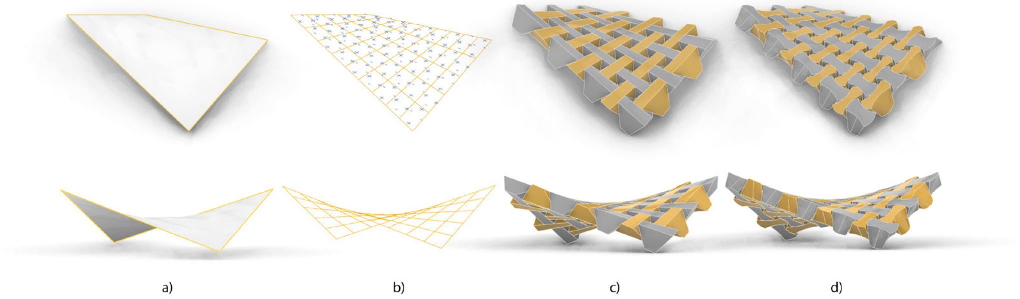

Figure 4
Generating Abeille based TIA
a) reference double-curved surface, b) tessellation,
c) Abeille-based TIA, d) Truchet-based TIA

Step one and step two: inputting the reference curved surface and generating a rectangle-based tessellation

In this step, the input is a NURBS surface, which can be conceptualized as a deformation of a flat rectangle (Figure 4a). Moreover, it offers bidimensional domains that can be divided into segments (Figure 4b), with flexibility to adjust the segment density as needed. Using a rectangular grid applied on the reference surface, users can interactively modify the scale of the tessellation. Then, surface segments are dispatched at intervals and divided into two groups. The aim here is to facilitate the creation of two groups of blocks oriented in different directions, as the overall shape of an assembly depends on the orientation of blocks and their four contacting surfaces.

Step three: generating a 3D topological interlocking structure

Before generating a 3D TIA on a double-curved surface, it is necessary to explore the geometric construction method for creating planar TIAs.

Following the approach developed by Estrin et al., the process starts with constructing planes perpendicular to the flat base surface. Subsequently, these planes are tilted either towards or outwards from the center of the shape by a specific angle. Ultimately, 3D modules are formed through the intersections of every set of planes (Figure 5). When this method is applied to curved surfaces, it becomes more complicated due to the varying orientations of points on the surface.

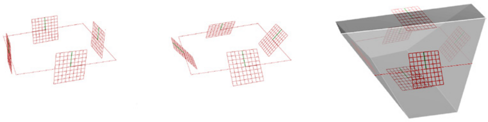

Figure 5
Generating truncated tetrahedra from left to right: perpendicular plan, rotate plan, truncated tetrahedra

In steps one and two, the reference curved surface has been segmented, which is necessary to determine the medial sections of final blocks. Then, middle points of the four edges of each medial section are evaluated; these determine the origins of four perpendicular planes. An angle α is selected; one of two perpendicular planes rotates around its respective edge by +α, and the adjacent one rotates by −α (Figure 4b). As a result, the intersections of all four planes define a tetrahedron within an interlocked grid. To ensure more even top and bottom surfaces, the tetrahedra are truncated using two offset surfaces set at an equal distance from both sides of the reference surface. In this case, increasing the angle α promotes interlocking but also reduces the contact area between modules. In general, this process generates both tetrahedra and truncated tetrahedra (Figure 4c), whose geometries are determined by three parameters: the size of medial sections, the rotation angle α, and the offset distance from the curved reference surfaces.

The process is not confined to rectangle-based tiling: modules with nonplanar contact surfaces can also be generated in a similar way (Figure 4d). This iteration begins not with inclined planes, but rather by constructing four arcs from the surfaces of medial segments. These arcs are constructed by moving the midpoints of each of four edges outward or inward along the curved reference surface in opposite directions. To define solids, the arcs are moved upward and downward by the same distance from the medial sections, which are then lofted to create the four faces of a block. Finally, these four faces, plus the top and bottom cap surfaces, are joined to form a six-surface module.

Step four: refining the modules

For ease of future fabrication, it is better for the top and bottom surfaces of each module are planar. Since the truncated tetrahedra built in step three have slightly curved surfaces, I approximate each of these surfaces by two planar triangles. Another method for constructing Planar Quadrangular (PQ) meshes of curved surfaces is using the Kangaroo plugin at the beginning of steps one and two, which involves data input. This plugin allows each segment of the reference curved surface to be optimized to have planar faces, which also brings convenience to step three.

Step five: simulating the assembly sequence

In this step, two key points need to be considered: a border condition and a formwork. Topological interlocking does not occur for the boundary modules, which have only three neighboring module and hence do not fulfil the conditions for interlocking to occur. Thus, a fixed boundary is necessary. There are various ways to achieve this:

- a surrounding frame
- external prestressed cable
- gluing the boundary modules together

Moreover, a formwork could prove beneficial in providing support for incomplete structures during the assembly process.

FURTHER DESIGN: APERIODIC PENROSE RHOMBUS-BASED TILING

Generating 2D patterns is a common task in various computational tools. However, numerous algorithmic methods depend on predetermined pattern configurations, typically derived from a single polygon or a restricted set of polygon types. Previous studies on TIA focused primarily on periodic tiling, in other words when there is an organized replication and translation of a specific set of finite tiles (Cocks, Nguyen and Peters, 2023). Penrose tilings are a family of aperiodic tilings of the plane, initially introduced by Roger Penrose (1974). Whereas periodic tiling exhibits translational symmetry but may lack rotational or reflection symmetry, the Penrose patterns offer a distinct modularity and variability. They offer a restricted number of module types while maintaining design diversity. As they do not depend on regular patterns, Penrose tilings offer more flexibility in creating novel outcomes. From the pre-design phase, a clear design workflow has been developed and will be followed here. The next challenges involve implementing complex Penrose rhombuses on double-curved surfaces and devising rules for further developing interlocking modules.

Step one and step two: generating a Penrose pattern and inputting a reference curved surface

The generative method in this phase differs slightly from the previous design. It begins with generating a Penrose pattern, which is then projected onto a double-curved surface. Given a base plan, the size of grid shells and number of iterations, an initial set of planar rhombuses are created. The algorithm here is based on the Parakeet plugin developed by Esmaeil Mottaghi and Arman KhalilBeigi. This plugin helps create a Penrose tiling comprising two types of rhombuses: a thin rhombus with corner angles of 36, 144, 36, and 144 degrees, and a thick rhombus with corner angles of 72, 108, 72, and 108 degrees.

In order to proceed to step three, I found the midpoints of every one of the two sets of opposite sides in a rhombus and connected them as references (Figure 6a). Note that the vertices of the reference lines within adjacent rhombuses do not intersect. This ensures the correct direction for module generation. Both the rhombuses and their reference lines are projected onto the rotation-symmetric reference surface. This method can also be applied to other double-curved surfaces.

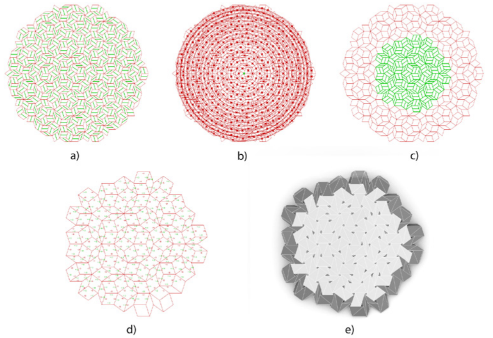

Figure 6
Generating Penrose rhombus-based tiling
a) reference lines,
b) reference circles,
c) projection boundary determined, d) rotation reference map, e) final 3D TIA

Step three: generating a 3D topological interlocking structure

The third step essentially automates the procedure used in the pre-design phase of the research to generate TIAs, but determining the direction of plane rotation is more complicated. First, the centroids of all rhombuses and the distances from these centroids to the centroid of the overall Penrose pattern are calculated. Second, the distance values are sorted, and centroids are grouped by distance and connected by circles drawn from the center of the overall pattern (Figure 6b). Third, these circles and groups determine the projection boundary of the Penrose pattern, which can be adjusted in Grasshopper (Figure 6c). Fourth, reference lines determine the rotation direction of perpendicular planes on each edge (Figure 6d). The remainder of the procedure is almost the same as in the pre-design phase (Figure 6e).

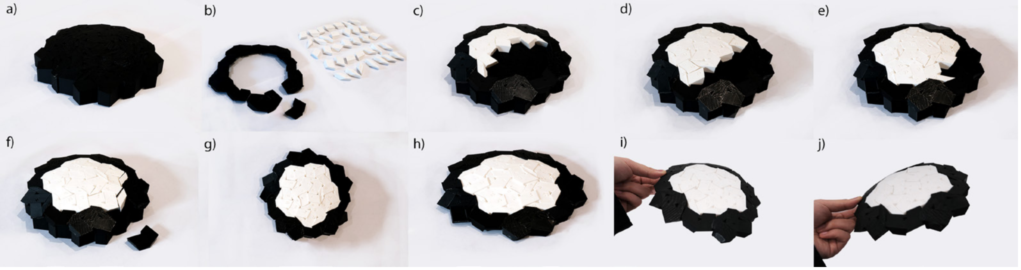

Figure 7
Assembly sequence of Penrose rhombus-based tiling
a) formwork
b) boundary components and 3D-printed blocks
c)-g) assembly sequence
h) removal of formwork
i)-j) varying tilting directions

Step four and step five: refining the modules and simulating the assembly sequence

Step four automates the previous processes. In step five, a dome prototype is 3D printed using PLA, with a formwork employed to support blocks during assembly (Figure 7). The outer boundary is not topologically interlocked, as there are only three neighbors per block. One solution is to construct a monolithic boundary. However, when assembling the final block to achieve the interlock, there is limited moving space available. To address this issue, the boundary is divided and printed as two separate components: one large and one small. Since the assembly sequence of TIA relies on placing each block in its precise location, the blocks are numbered in Grasshopper prior to 3D printing. Starting with the large boundary component, all blocks are inserted into their positions, leaving only the small boundary component to be placed. The entire TIA is then completed by gluing the remaining component to the large part, forming a monolithic boundary. The final assembly step involves removing the formwork, after which the dome exhibits stability under varying tilting directions.

DISCUSSION AND FUTURE WORK

The design process underscores the benefits of interactive geometric software in swiftly and automatically generating blocks with bespoke shapes. A variety of surfaces can be inputs for the digital workflow, and the curvatures of these surfaces directly impacts upon the results of module generation. In particular, surfaces with excessively steep or variable curvature may lead to workflow errors. To address this issue, one solution is to increase the surface discretization to create smaller-sized modules. During the pre-design phase, this is accomplished by adjusting the segment density of the 2D domain. In later design phases, the same objective can be achieved by increasing the number of iterations of Penrose rhombuses before projecting onto the reference surfaces. Within this workflow, it is easy to adjust the plane rotation angle, the module thickness, and the voids between modules, leading to a variety of visual outcomes (Figure 8).

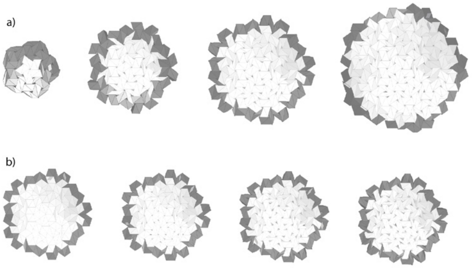

Figure 8
Various visual outcomes
a) different sizes of Penrose tiling,
b) changing angles of rotation

Exploring the application of TIAs to design the unique shapes for building blocks and the corresponding assembly topologies paves the way for large-scale mortarless construction. In contrast to monolithic structures, TIA systems rely on kinematic constraints imposed by their special shape and mutual arrangement of modules to maintain their structural integrity. Thus, the complexity arises primarily from the absence of rigid connections between modules: it is challenging to predict the behavior of such interlocking systems under load. While the proposed design phases showcase visually appealing aggregations of interlocking modules, the absence of structural analysis in the workflow prevents their validation. Conducting such analysis, along with additional analytical skills, is necessary to further explore the translation from a digital model into physical construction and, hence, to ensure the feasibility of the design.

In addition to structure analysis, future work could explore three other directions. Firstly, the investigation of more complex 2D patterns. While this research mainly focuses on quadrilateral polygon-based tilings, hexagon-based tilings could also be considered. Moreover, although this study primarily examines the Penrose P3 rhombus tiling, further exploration of the other two types, such as the original P1 pentagonal Penrose tiling and the P2 kite-and-dart tiling, could be valuable. Secondly, there is a need to explore the control of module variability. The intricate surfaces in this design produce a multitude of unique blocks due to the varying curvature. It would be beneficial to minimize the number of unique module types while guaranteeing both potential and structural stability. Thirdly, the fabrication process plays a crucial role in assessing the global behavior of TIAs. Using 3D-printed blocks could be an effective approach for assembling small-scale prototypes. Additionally, while formwork is often employed during assembly, efforts should be made to minimize its usage. Before advancing to a full-scale prototype, careful consideration must be given to material selection and experimentation. Material properties significantly determine the results of interlocking systems. For example, when dealing with clay bricks, it is crucial to allow for shrinkage during both drying and firing. Other fabrication processes such as casting and additive manufacturing are also worth exploring.

CONCLUSION

The research revolves around the development of the TIA in masonry architecture throughout history. In contrast to many traditional masonry construction methods, TIAs offer the possibility of mortar-free assembly sequences, making it easier for unskilled people to construct complex structures. This thesis details a digital workflow for generating TIAs for architectural design through two design phases. It begins with inputting 2D patterns and 3D reference surfaces, followed by the automated generation of 3D interlocking blocks. The regular rectangular-based tiling serves as a fundamental element for exploring the geometric possibilities of TIAs. The digital workflow expands TIA applications from planar surfaces to double-curved vaults. Additionally, it introduces novel findings regarding the generation of TIAs from Penrose tiling, achieving both structural functionality and aperiodic aesthetic qualities for architecture. In general, this study illustrates how the geometric principles underlying TIA facilitate a more comprehensive understanding of the interactions among blocks, thereby expanding perspectives on masonry architecture to embrace new materiality and structures.

REFERENCES

Akleman, E. *et al.* (2020) 'Generalized abeille tiles: Topologically interlocked space-filling shapes generated based on fabric symmetries', *Computers & Graphics*, 89, pp. 156–166. Available at: https://doi.org/10.1016/J.CAG.2020.05.016.

Borhani, A. and Kalantar, N. (2017) 'APART but TOGETHER - The Interplay of Geometric Relationships in Aggregated Interlocking Systems', *Volume 1*, 1, pp. 639–648. Available at: https://doi.org/10.52842/CONF.ECAADE.2017.1.639.

Bustillo Revuelta, M. (2021) 'Mortars', pp. 307–337. Available at:

https://doi.org/10.1007/978-3-030-65207-4_11.

Cocks, R., Nguyen, J. and Peters, B. (2023) 'Wieringa Surface: The Implementation of Aperiodicity into Architectural Acoustics', *Communications in Computer and Information Science*, 1819 CCIS, pp. 190–203. Available at: https://doi.org/10.1007/978-3-031-37189-9_13/COVER.

Dyskin, A. V. *et al.* (2003) 'Topological interlocking of platonic solids: A way to new materials and structures', *Philosophical Magazine Letters*, 83(3), pp. 197–203. Available at: https://doi.org/10.1080/0950083031000065226.

Estrin, Y., Dyskin, A. V. and Pasternak, E. (2011) 'Topological interlocking as a material design concept', *Materials Science and Engineering: C*, 31(6), pp. 1189–1194. Available at: https://doi.org/10.1016/J.MSEC.2010.11.011.

Evans, R. (1995) *The projective cast : architecture and its three geometries*. Cambridge, Mass ; MIT Press.

Fallacara, G. (2006) 'Digital Stereotomy and Topological Transformations: Reasoning about Shape Building'.

Glickman, M. (1984) 'THE G-BLOCK SYSTEM OF VERTICALLY INTERLOCKING PAVING', in *Second International Conference on Concrete Block Paving*, pp. 10–12.

IGPP (2021). Available at: https://igpp.org.uk/blog/article/sustainability-greatest-challenge-of-construction-industry (Accessed: 31 March 2024).

McCullough, Malcolm. (1996) *Abstracting craft : the practiced digital hand*. Cambridge, Mass. ; MIT Press.

Menges, A. (2015) 'Material Synthesis. Fusing the Phisical and the Computational', *Angewandte Chemie International Edition, 6(11), 951–952.*, pp. 5–24.

Mirkhalaf, M., Zhou, T. and Barthelat, F. (2018) 'Simultaneous improvements of strength and toughness in topologically interlocked ceramics', *Proceedings of the National Academy of Sciences of the United States of America*, 115(37), pp. 9128–9133. Available at: https://doi.org/10.1073/PNAS.1807272115/SUPPL_FILE/PNAS.1807272115.SAPP.PDF.

Mottaghi, E. and KhalilBeigi, A. (2018) *Parakeet | Food4Rhino*. Available at: https://www.food4rhino.com/en/app/parakeet (Accessed: 31 March 2024).

Piker, D. (2013) 'Kangaroo: Form Finding with Computational Physics', *Architectural Design*, 83(2), pp. 136–137. Available at: https://doi.org/10.1002/AD.1569.

Supriya *et al.* (2023) 'Low-CO2 emission strategies to achieve net zero target in cement sector', *Journal of Cleaner Production*, 417, p. 137466. Available at: https://doi.org/10.1016/J.JCLEPRO.2023.137466.

Tessmann, O. and Becker, M. (2013) 'Extremely Heavy and Incredibly Light – Performative Assemblies in Dynamic Environments', *Open Systems: Proceedings of the 18th International Conference on Computer-Aided Architectural Design Research in Asia (CAADRIA 2013) / Singapore 15-18 May 2013, pp. 469-478*, pp. 469–478. Available at: https://doi.org/10.52842/CONF.CAADRIA.2013.469.

Vella, I.M. and Kotnik, T. (2016) 'Geometric Versatility of Abeille Vault A Stereotomic Topological Interlocking Assembly', *Proceedings of the International Conference on Education and Research in Computer Aided Architectural Design in Europe*, 2, pp. 391–397. Available at: https://doi.org/10.52842/CONF.ECAADE.2016.2.391.

Weizmann, M., Amir, O. and Grobman, Y.J. (2016) 'Topological interlocking in buildings: A case for the design and construction of floors', *Automation in Construction*, 72, pp. 18–25.