

# Stochastic Resonance-Based Power Amplifier Design for Orthogonal Frequency-Division Multiplexing Signals

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**Abstract**—A stochastic resonance-based power amplifier (PA) is proposed for linearly and efficiently amplifying signals with large envelope fluctuations such as those transmitted in orthogonal frequency-division multiplexing (OFDM) systems. The proposed architecture uses a Collins summing network in conjunction with a bandpass filter. The performance of the proposed PA is evaluated through the amplification of OFDM signals in terms of bit-error rate characteristics for both uncoded and coded message symbols. It is demonstrated that the performance improves as the number of parallel paths in the summing network increases, with a significant performance gain being obtained even for a moderate number of parallel paths.

**Index Terms**—Stochastic resonance, power amplifier, delta-sigma modulation, constant envelope, switch-mode power amplifier, OFDM.

## I. INTRODUCTION

**S**TOCHASTIC resonance (SR) is a phenomenon in which perturbations or fluctuations in a nonlinear system enhance the response to a weak input signal. SR was introduced by Benzi *et al.* [1] to explain global climate changes that occur in a  $10^5$ -year cycle [2]. Nowadays, nonlinear effects that improve the output of a system by adding appropriate noises are widely referred to as the SR effect. Recently, Nakashima *et al.* have shown that four-level pulse-amplitude modulation (PAM) signals are detectable using a one-bit comparator output with the aid of noise [3]. Tatematsu *et al.* discussed the use of a few-bit analog-to-digital converter (ADC) for an SR-based receiver [4]. Collins *et al.* discussed waveform reconstruction using a summing network with  $N$  excitable units of a FitzHugh–Nagumo model neuron [5], and it was also applied to the ADCs [6]–[8]. A floating-gate-based neurons were discussed for the summing network [9]. In this paper, we show that Collins summing network, using simplified excitable units of signum functions, is applicable to power amplifier (PA) designs suitable for the transmission of signals with high peak-to-average power ratio (PAPR). Orthogonal frequency-division multiplexing (OFDM) signals are a typical example of high PAPR signals and require linear PA for low-distortion transmission. However, it is known that the basic linear PAs significantly degrade the transmission power efficiency. For example, when the PAPR is 10 dB, the average drain efficiency is 5% for an ideal class-A amplifier and 28% for an ideal class-B amplifier [10]. When more realistic input-output characteristics

of PAs are taken into account, the efficiency becomes even significantly lower [11]. Therefore, there is a long history of improvements in PA design [12]. The Doherty [13], [14] and Chireix outphasing [15]–[17] architectures are popular, and the Kahn envelope elimination and restoration (EER) architecture and envelope tracking technique are also being studied [18], [19]. However, to the best of our knowledge, PA design using the SR effect has not yet been reported.

This paper is organized as follows. Collins summing network is introduced, and the basic concept of the PA design is described in Section II. In Section III, the SR-based PA (SRPA) design for OFDM signals is elaborated, and the bit-error rate (BER) performance of OFDM signals amplified by SRPAs is shown in Section IV. The conclusion is given in Section V.

**Notations:** Continuous-time signals are expressed with  $t$  in parentheses, as in  $f(t)$ , and their discrete-time versions are expressed with the subscript  $i$ , as in  $f_i$ . Vector forms of signals are denoted by boldface letters, as in  $\mathbf{f}$ , whose  $i$ th entry is  $f_i$ . The notations  $(\cdot)^\mathcal{T}$  and  $(\cdot)^\mathcal{H}$  stand for transpose and Hermitian transpose operations, respectively.  $\delta_{i,j}$  is the Kronecker delta defined as  $\delta_{i,j} = \{1(i = j), 0(i \neq j)\}$ , and  $E[\cdot]$  is the expectation operator.  $\Re[\cdot]$  and  $\Im[\cdot]$  are the real and imaginary parts of a complex number, respectively.  $|\cdot|$  is the modulus of a complex number and  $j = \sqrt{-1}$ . The symbol  $*$  denotes the convolution operation and  $f_i \leftrightarrow F_k$  denotes that  $f_i$  and  $F_k$  are the discrete Fourier transform (DFT) pair.

## II. SUMMING NETWORK AND WAVEFORM RECONSTRUCTION

We begin by considering waveform reconstruction using a summing network of  $N$  excitable units with the aid of noises [5], as shown in Fig. 1. In this study, the signum function  $\text{sgn}(x) = \{+1 (x > 0), -1 (x < 0)\}$  ( $x \in \mathbb{R}$ ) is used for excitable units as shown in Fig. 1. Signals  $s_n(t)$  ( $n = 1, 2, \dots, N$ ) are the sums of a common source signal

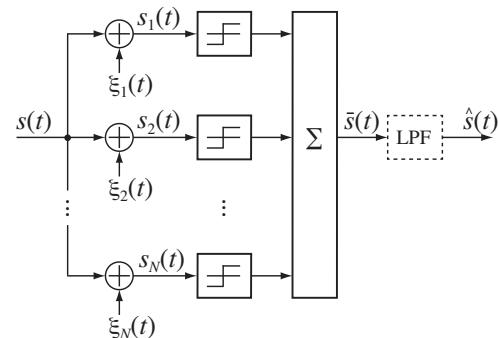


Fig. 1. Summing network of  $N$  excitable units of signum functions for waveform reconstruction with the aid of noises.

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$s(t)$  and independent noises  $\xi_n(t)$ , and are summed together through the signum functions  $\text{sgn}(x)$  to produce a reconstructed signal  $\bar{s}(t)$ . We assume that the source signal  $s(t)$  is a band-limited zero-mean Gaussian random process with a unit variance of  $E[s^2(t)] = 1$  and the noises  $\xi_n(t)$  are zero-mean Gaussian random processes with an identical variance of  $E[\xi_n^2(t)] = \sigma^2$ . Therefore, the signal-to-noise ratio SNR of  $s(t)$  and  $\xi_n(t)$  is represented as  $\text{SNR} = E[s^2(t)]/E[\xi_n^2(t)] = 1/\sigma^2$  for all  $n$ .

Let  $s_i$  ( $i = 1, 2, \dots, M_s$ ) be a discrete-time representation of the source signal  $s(t)$  sampled at time  $t = (i-1)\Delta_t$ , where  $\Delta_t = T_N/R_{\text{os}}$  is the sampling time interval,  $T_N = (2B)^{-1}$  is the Nyquist interval,  $B$  is the bandwidth of  $s(t)$ , and  $R_{\text{os}}$  is the oversampling ratio. Again, let  $\xi_{n,i}$  be the discrete-time signals of the noises  $\xi_n(t)$  sampled at time  $t = (i-1)\Delta_t$ . We assume that  $E[\xi_{n,i}\xi_{m,j}] = \delta_{n,m}\delta_{i,j}\sigma^2$ .

Next, to assess the accuracy of reconstruction, we study the cross-correlation between the source and reconstructed signals.

#### A. Average Cross-Correlation Between Source and Reconstructed Signals

Let  $s \in \mathbb{R}^{M_s \times 1}$  be a source signal vector and  $\bar{s} \in \mathbb{R}^{M_s \times 1}$  be a reconstructed signal vector whose  $i$ th entry is  $\bar{s}_i = \sum_{n=1}^N \text{sgn}(s_i + \xi_{n,i})$ . Using  $s$  and  $\bar{s}$ , we obtain the cross-correlation value between the source signal  $s(t)$  and the reconstructed signal  $\bar{s}(t)$  such that  $s^T \bar{s} / (\sqrt{s^T s} \sqrt{\bar{s}^T \bar{s}})$ .

Figure 2(a) shows the cross-correlation values, averaged over 500 Monte Carlo simulation trials, as a function of SNR, for  $N = 1, 10, 100$ , and  $1000$ , where  $R_{\text{os}} = 8$  and the observation period for each trial was chosen to be  $0 \leq t < 100T_N$  (i.e.,  $M_s = 800$ ).<sup>1</sup>

It is observed from Fig. 2(a) that the average cross-correlation value for  $N = 100$  or  $1000$  reaches more than 0.95 if SNR is appropriately chosen. In other words, appropriate values of the noise variance  $\sigma^2$  improve the quality of waveform reconstruction. It should be noted that for all values of  $N$ , the cross-correlation values converge to a specific value of 0.8 as the SNR becomes large. This is due to the fact that all  $N$  outputs  $\text{sgn}[s_n(t)]$  from the signum functions in Fig. 1 become an identical waveform  $\text{sgn}[s(t)]$  if  $\text{SNR} \rightarrow \infty$ . Consequently, the summed signal  $\bar{s}(t)$  of these waveforms becomes  $\bar{s}(t) = N \cdot \text{sgn}[s(t)]$ , which is still binary in  $\{+N, -N\}$ . Thus,  $N$  is irrelevant for the cross-correlation value if  $\text{SNR} \rightarrow \infty$ .

This waveform reconstruction can be expressed as follows: For a certain normalization constant  $\beta \in \mathbb{R}$ ,  $\beta \bar{s}(t)$  asymptotically approaches  $s(t)$  for  $N' \rightarrow \infty$  with respect to (wrt) the cross-correlation criterion if SNR is appropriate, where

$$\bar{s}(t) = \sum_{n=1}^{N'} \text{sgn}[s(t) + \xi_n(t)]. \quad (1)$$

The symbol  $N'$  was used instead of  $N$  in (1) for some reason as will be apparent shortly.

<sup>1</sup>The source signal  $s(t)$  was produced with a sum of  $L$  sinusoidal waves  $\sqrt{\frac{2}{L}} \cos(2\pi f_l t + \theta_l)$  ( $l = 1, 2, \dots, L$ ), where  $\theta_l$  are independent random numbers with a uniform distribution in  $(0, 2\pi)$ . To prevent  $s(t)$  from being periodic, the frequencies  $f_l$  are also randomly chosen with uniform distributions in unoverlapped frequency bins  $((l-1)\frac{B}{L}, l\frac{B}{L})$ . For each trial, independent random numbers are generated and altered.  $L = 30$  was chosen.

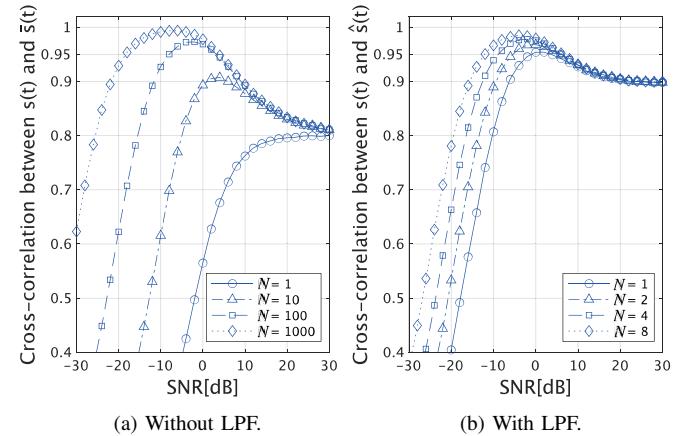


Fig. 2. Average cross-correlation between the source signal  $s(t)$  and the reconstructed signals: (a) for  $\bar{s}(t)$  (without LPF) and (b) for  $\hat{s}(t)$  (with LPF).

#### B. Average Cross-Correlation Between Source and Lowpass-Filtered, Reconstructed Signals

To improve the smoothness of the reconstructed signal  $\bar{s}(t)$ , we employ a lowpass filter (LPF) as shown in Fig. 1. We assume that the LPF is linear and has an impulse response  $h(t)$ . In this paper, we employ an ideal LPF with a rectangular frequency response, which is realized using the DFT and yields a discrete-time lowpass-filtered, reconstructed signal  $\hat{s}_i$  ( $i = 1, 2, \dots, M_s$ ) such that  $\hat{s}_i \leftrightarrow \hat{S}_k$  ( $k = 1, 2, \dots, M_s$ ), where

$$\hat{S}_k = \begin{cases} \bar{S}_k & (1 \leq k \leq M_B + 1, M_s - M_B + 1 \leq k \leq M_s) \\ 0 & (M_B + 2 \leq k \leq M_s - M_B) \end{cases}. \quad (2)$$

Here  $M_B$  is an integer that defines the bandwidth of the ideal LPF and  $\bar{S}_k = \frac{1}{\sqrt{M_s}} \sum_{i=1}^{M_s} \bar{s}_i e^{-j2\pi(i-1)(k-1)/M_s}$ .

By defining a vector  $\hat{s} \in \mathbb{R}^{M_s \times 1}$ , we can calculate the cross-correlation value between the source signal  $s(t)$  and the lowpass-filtered, reconstructed signal  $\hat{s}(t)$  such that  $s^T \hat{s} / (\sqrt{s^T s} \sqrt{\hat{s}^T \hat{s}})$ . The cross-correlation values between  $s(t)$  and  $\hat{s}(t)$ , which are averaged over 500 trials, vs SNR are shown in Fig. 2(b), where  $M_B = 50$  was chosen for the pass bandwidth. Unlike in Fig. 2(a),  $N$  is chosen to be small such that  $N = 1, 2, 4$ , and  $8$  in Fig. 2(b), showing that with the LPF, the small  $N$  obtains significantly large cross-correlation values. Surprisingly, even for  $N = 1$ , the average cross-correlation value reaches 0.95 if SNR is appropriately chosen. This waveform reconstruction can be summarized as follows: For a certain normalization constant  $\gamma \in \mathbb{R}$ ,  $\gamma \hat{s}(t) \approx s(t)$  for  $1 \leq N \ll N'$  wrt the cross-correlation criterion if SNR is appropriate, where

$$\hat{s}(t) = h(t) * \left( \sum_{n=1}^N \text{sgn}[s(t) + \xi_n(t)] \right). \quad (3)$$

Equation (3) is the basis for the design of highly efficient SRPAs for signals with large envelope fluctuations. In (3), the term  $\text{sgn}[s(t) + \xi_n(t)]$  takes binary values in  $\{+1, -1\}$ , which can be amplified using switching-mode (or saturation-mode) PAs (SMPAs) that achieve the maximum PA efficiency. Suppose that we use  $N$  SMPAs that have a common gain of  $K$ . In this case, the source signal  $s(t)$  is efficiently amplified

and we obtain a  $K$ -fold amplified, reconstructed signal  $K\hat{s}(t)$  such that

$$h(t) * \left( \sum_{n=1}^N K \cdot \text{sgn}[s(t) + \xi_n(t)] \right) = K\hat{s}(t). \quad (4)$$

### III. SR-BASED PA DESIGN FOR OFDM SIGNALS

#### A. OFDM Signals

The complex envelope  $z(t)$  of OFDM signals is given by

$$z(t) = \frac{1}{\sqrt{N_c}} \sum_{l=1}^{N_c} b_l e^{j2\pi(l-1)t/T_0}, \quad 0 \leq t < T_0, \quad (5)$$

where  $N_c$  is the number of complex subcarriers,  $T_0$  is the duration of OFDM signals, and  $b_l = b_l^{(1)} + j b_l^{(2)}$  is the complex message symbol that modulates the corresponding  $l$ th complex subcarrier  $e^{j2\pi(l-1)t/T_0}$  ( $l = 1, 2, \dots, N_c$ ).

Without loss of generality, we assume 2PAM symbols for both  $b_l^{(1)}$  and  $b_l^{(2)}$ , that is,  $b_l^{(1)}, b_l^{(2)} \in \{+1, -1\}$ . This corresponds to 4QAM or quaternary phase-shift keying (QPSK) for the  $l$ th complex subcarrier.

Using  $z(t)$ , the OFDM signals  $f(t)$  are given by

$$f(t) = \Re[z(t)e^{j2\pi f_c t}] \quad (6)$$

$$= x(t) \cos \omega_c t - y(t) \sin \omega_c t, \quad \omega_c = 2\pi f_c, \quad (7)$$

where  $x(t) = \Re[z(t)]$ ,  $y(t) = \Im[z(t)]$ , and  $f_c$  is the frequency for the first subcarrier ( $l = 1$ ).

In the following, we assume that the discrete-time signals are sampled at time  $t = (i-1)\Delta_t$  ( $i = 1, 2, \dots, N_c R_{\text{os}}$ ), where  $\Delta_t = T_0/(N_c R_{\text{os}})$ .

#### B. SR-Based PA Design

Using (4) and (7), we can readily obtain the structure of SRPAs as shown in Fig. 3, where  $u_n(t)$  and  $v_n(t)$  ( $n = 1, 2, \dots, N$ ) are independent zero-mean Gaussian noises, which cause the SR effect, with a variance of  $E[u_n^2(t)] = E[v_n^2(t)] = \sigma^2$ . Since  $E[x^2(t)] = E[y^2(t)] = 1$ ,  $\text{SNR} = E[x^2(t)]/E[u_n^2(t)] = E[y^2(t)]/E[v_n^2(t)] = 1/\sigma^2$  for all  $n$ . As for the discrete-time signals  $u_{n,i}$  and  $v_{n,i}$ ,  $E[u_{m,i}u_{n,j}] = E[v_{m,i}v_{n,j}] = \delta_{m,n}\delta_{i,j}\sigma^2$  and  $E[u_{m,i}v_{n,j}] = 0$ .

In Fig. 3, since all output signals  $\bar{x}_n(t)$  and  $\bar{y}_n(t)$  ( $n = 1, 2, \dots, N$ ) from signum functions always take binary values in  $\{+1, -1\}$ , all input signals  $\bar{x}_n(t) \cos \omega_c t - \bar{y}_n(t) \sin \omega_c t$  to the SMPAs have a constant envelope level of  $\sqrt{\bar{x}_n^2(t) + \bar{y}_n^2(t)} = \sqrt{2}$ . These signals are highly efficiently amplified through the SMPAs, and the subsequent bandpass filter (BPF) improves the quality of waveform reconstruction as in the LPF in Fig. 1. Thus, as the SMPA evolves, the PA efficiency of the SRPA improves.<sup>2</sup>

<sup>2</sup>For example, it is known that the maximum efficiency of a class-F amplifier increases with the number of harmonics, reaching 0.9045 when the number of harmonics is 5, and can theoretically approach 1 when the number of harmonics is increased [10].

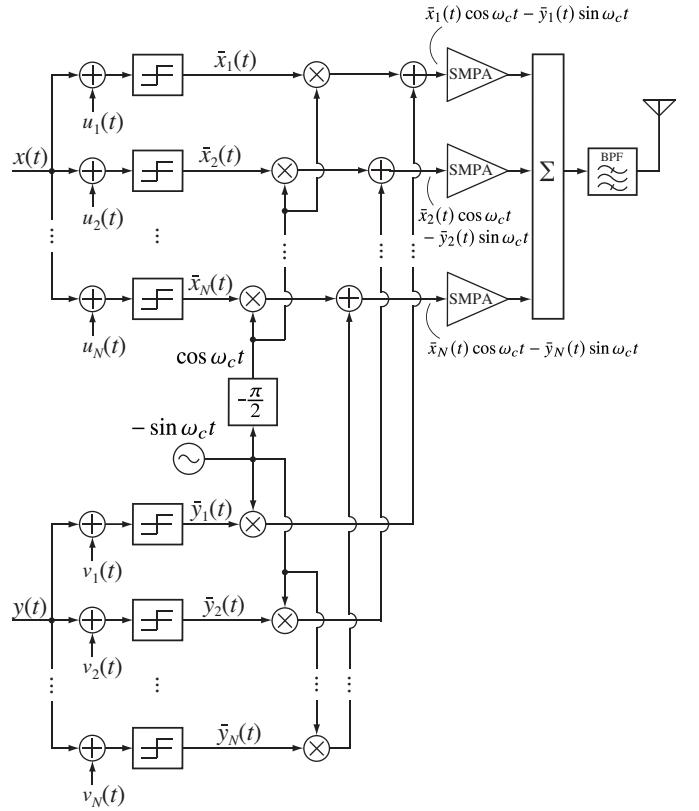


Fig. 3. Design of SR-based PAs.

#### C. Equivalent Lowpass Representation of SR-Based PAs

In order to simplify the analysis of communication systems, equivalent lowpass system representations are employed. Let  $\eta_n(t)$  ( $n = 1, 2, \dots, N$ ) be complex noises defined using  $u_n(t)$  and  $v_n(t)$  as  $\eta_n(t) = u_n(t) + j v_n(t)$ . If we define a complex signum function  $\text{csgn}(z)$  as  $\text{csgn}(z) = \text{sgn}(\Re[z]) + j \text{sgn}(\Im[z])$  ( $z \in \mathbb{C}$ ), an equivalent lowpass system representation of the SRPAs is obtained as shown in Fig. 4, where  $K$ s are intended as the gains of the SMPAs in Fig. 3 and the LPF is the equivalent lowpass system representation of the BPF in Fig. 3. This has an identical form to the summing network shown in Fig. 1 except for the difference in the real or complex system if  $K = 1$ . Therefore,  $K\hat{z}(t)$  is obtained as the lowpass-filtered version of  $K\bar{z}(t)$  as in Fig. 1, where  $\bar{z}(t) = \sum_{n=1}^N \bar{z}_n(t)$  and  $\bar{z}_n(t) = \text{csgn}[z(t) + \eta_n(t)]$  as shown in Fig. 4.

In this paper, we employ an ideal BPF with a rectangular frequency response in Fig. 3. Therefore, the discrete-time signal  $\hat{z}_i$  ( $i = 1, 2, \dots, N_c R_{\text{os}}$ ) of  $\hat{z}(t)$  is obtained such that

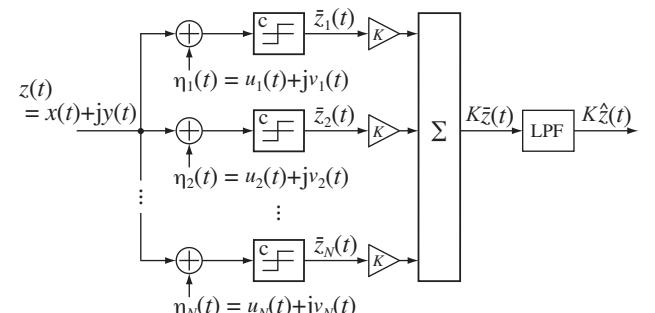


Fig. 4. Equivalent lowpass system representation of SR-based PAs.

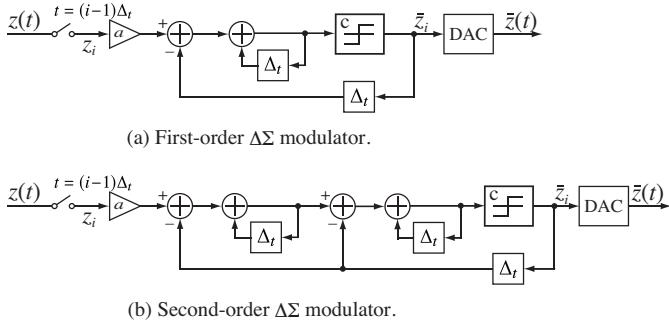


Fig. 5. First- and second-order  $\Delta\Sigma$  modulators models.

$$\hat{z}_i \leftrightarrow \hat{Z}_k \quad (k = 1, 2, \dots, N_c R_{\text{os}}), \text{ where}$$

$$\hat{Z}_k = \begin{cases} \bar{Z}_k & (1 \leq k \leq N_c) \\ 0 & (N_c + 1 \leq k \leq N_c R_{\text{os}}) \end{cases} \quad (8)$$

$$\text{and } \bar{Z}_k = \frac{1}{\sqrt{N_c R_{\text{os}}}} \sum_{i=1}^{N_c R_{\text{os}}} \bar{z}_i e^{-j2\pi(i-1)(k-1)/(N_c R_{\text{os}})}.$$

Without loss of generality, we assume that  $K = 1$  in the following discussions.

#### IV. PERFORMANCE EVALUATION

##### A. Coding Efficiency

As in Section II, the quality of waveform reconstruction in terms of cross-correlation values is improved by using the LPF; however, the energy contained in  $\hat{z}(t)$  becomes smaller than that contained in  $\bar{z}(t)$  owing to the LPF that discards the out-of-band spectral components contained in  $\bar{z}(t)$ . This energy loss caused by the LPF, equivalently, the energy loss caused by the BPF in Fig. 3, is called *coding efficiency* *CE* introduced by Johnson and Stapleton [20], which is given by  $\text{CE} = \mathbb{E}[|\hat{z}(t)|^2]/\mathbb{E}[|\bar{z}(t)|^2] = \mathbb{E}[\hat{\mathbf{z}}^H \hat{\mathbf{z}}]/\mathbb{E}[\bar{\mathbf{z}}^H \bar{\mathbf{z}}]$ .

##### B. $\Delta\Sigma$ Modulations

The CE has been used in the context of  $\Delta\Sigma$  modulation-based PA ( $\Delta\Sigma$ PA) designs (see [20]–[23] and the references therein). In this paper, we consider a basic configuration of the first- and second-order  $\Delta\Sigma$  modulators respectively shown in Figs. 5(a) and (b) for performance comparison, where each has an adjustable input gain  $a$ . The initial values stored in two memories in Fig. 5(a) and three memories in Fig. 5(b) for the time delay of  $\Delta_t$  were all set to zero. Since the output signals  $\bar{z}(t)$  (and also  $\bar{z}_i$ ) from both  $\Delta\Sigma$  modulators are obtained through the signum functions, their amplitudes become binary and are thus efficiently amplified by the SMPAs.

##### C. BER Characteristics

Figure 6 shows a transmission model that contains the output portion of the PA (transmitter) shown in Fig. 4 ( $K = 1$ ) and the receiver including the OFDM demodulator in the presence of additive white Gaussian noise (AWGN) with two-sided power spectral density of  $N_0/2$ . Usually, the characteristics of BER are evaluated using the received energy per bit,  $E_b$ , defined using the received signal  $\hat{z}(t)$ . In this paper, the energy per bit,  $\bar{E}_b$ , defined using  $\bar{z}(t)$  such that  $\bar{E}_b = T_0 \mathbb{E}[|\bar{z}(t)|^2]/(2N_c)$ , is used. Consequently, the BER characteristics encompass both the impact of waveform distortion, as indicated by the cross-correlation, and the influence of energy loss, as quantified by the CE.

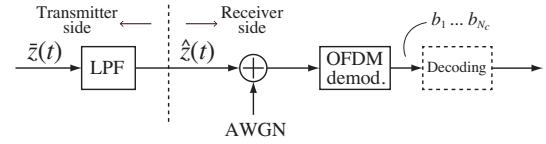


Fig. 6. Transmission model.

TABLE I  
CES FOR THE UNCODED AND CODED CASES.

	$\Delta\Sigma$ PAs		SRPAs			
	1st-order	2nd-order	$N=1$	$N=2$	$N=4$	$N=8$
Uncoded case	0.269	0.401	0.460	0.564	0.662	0.761
Coded case	0.451	0.717	0.736	0.740	0.786	0.837

In the following,  $N_c = 128$ ,  $R_{\text{os}} = 32$ , and 4QAM (QPSK) message symbols for  $b_l$  are commonly considered. The design parameter  $\text{SNR}^3$  for the SRPAs and the input gain  $a$  for the  $\Delta\Sigma$  PAs are set to minimize the required value of  $\bar{E}_b/N_0$  that achieves  $\text{BER} \approx 10^{-3}$ .<sup>4</sup>

1) *Uncoded Case*: Figure 7(a) shows the BER characteristics of OFDM signals transmitted using SRPAs for  $N = 1, 2, 4$ , and 8 as a function of  $\bar{E}_b/N_0$ . Here, we assume that the uncoded message symbols are transmitted as  $b_l$  ( $l = 1, 2, \dots, N_c$ ). An ideal curve, which is the case of  $\hat{z}(t) = z(t)$  (distortionless transmission and no energy loss of  $\text{CE} = 1$ ), is also plotted. The values of SNR and  $a$  were identified through simulations as given in Fig. 7(a), and the resultant CEs are summarized in Table I. It is verified from Fig. 7(a) that the BER performance improves as  $N$  increases, and the chosen SNR value becomes small as  $N$  increases. In other words, the required contribution of noises added for the SRPAs increases with  $N$ .

2) *Coded Case*: For practical system illustration, we use the standardized second-generation specification of the digital video broadcast standard (DVB-S2) [24] signal format, which uses low-density parity check (LDPC) coding.<sup>5</sup> The BER characteristics are shown in Fig. 7(b) for the coding rate of  $R_c = 1/3$ . CEs are summarized in Table I. It is observed from Fig. 7(b) that the SRPA of  $N = 1$  and the second-order  $\Delta\Sigma$ PA have almost identical BER characteristics. It is seen that the SNR chosen for SRPA of  $N = 1$  was  $\text{SNR} \rightarrow \infty$ , which means that the case without any noise (i.e.,  $\eta_1(t) = 0$ ) was the best. This is a valid conclusion only for  $N = 1$  with powerful error correction such as  $R_c = 1/3$ .

The key advantage of using the SR approach over conventional  $\Delta\Sigma$ -OFDM amplifying methods is clearly shown

<sup>3</sup>In a realistic scenario of analog circuit development, independent Gaussian noises can be introduced through amplifiers that generate thermal and shot noises with white power spectra, such as those observed over AWGN channels. The SNR value can be adjusted by attenuating the input signal power of the amplifier. The random variation of the comparator offset was employed as noise in [7].

<sup>4</sup>Thorough simulations were used to determine the optimal values of SNR and  $a$ .

<sup>5</sup>In addition, for the decoding, soft decisions are made using the approximated log-likelihood ratio, and the receiver has prior knowledge of signal power and channel noise variance. The maximum number of iterations was chosen to be 50. One packet for DVB-S2 consists of 64800 bits of a coded message, whereas one 4QAM-OFDM signal with  $N_c = 128$  can carry only 256 bits. Therefore, the OFDM signals are transmitted 254 times repeatedly for the transmission of one packet, in which 224 dummy bits were included and were not used for decoding in our simulations.

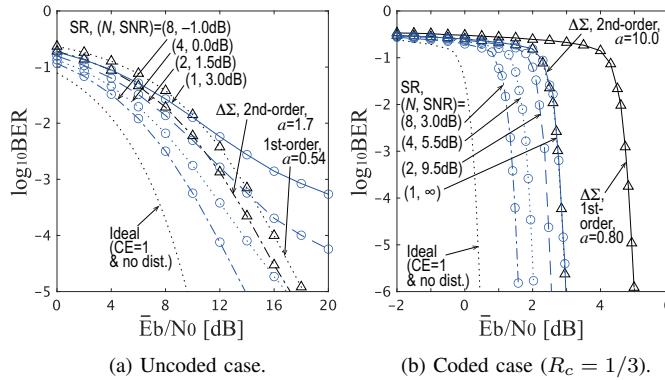


Fig. 7. BER characteristics of 4QAM(QPSK)-modulated OFDM signals transmitted using the SR- and  $\Delta\Sigma$  modulation-based PAs ( $N_c = 128$  and  $R_{\text{os}} = 32$ ).

in both parts of Fig. 7; (a) for the uncoded case and (b) for the coded case. In the uncoded case, our proposed design has an advantage over the conventional second-order methods of at least 1.6 dB for  $N = 4$  improving to 3.1 dB for  $N = 8$  at  $\text{BER} = 10^{-3}$ . For the coded case, with practical DVB-S2 signals, the advantage is more apparent for all values of  $N$ . This narrows the gap, relative to the ideal case<sup>6</sup>, from 2.5 dB for the conventional second-order  $\Delta\Sigma$ PA to 1.1 dB for the new design SRPA of  $N = 8$  at  $\text{BER} = 10^{-3}$ .

## V. CONCLUSIONS

This paper has explored the use of the stochastic resonance concept for the transmission of OFDM signals. The cross-correlation studies of the  $N$ -path noise-aided summing network with lowpass filtering have shown that accurate signal reconstruction may be achieved for reasonable SNR values, even for small  $N$ . The concept has then been applied to a transmitter structure with power amplifiers and bandpass filtering. Detailed modelling of the signal transmission link in AWGN channel, with various values of  $N$  and appropriate SNR of the summing network, has shown advantages over  $\Delta\Sigma$  modulation-based OFDM power amplification, in terms of BER performance improvement, even for the moderate value of  $N = 8$ , with only 1.1 dB energy consumption penalty relative to the ideal performance limit.<sup>7</sup> The results may open a new way of overcoming the power amplifier limitations of multicarrier systems, at the expense of a small energy consumption penalty and some increase in system complexity. Since all the optimal values of SNR depend on the target BER and  $N$ , system optimization will be required especially when fading channels are considered, and this is part of our ongoing work, as well as studies of appropriate predistortion techniques for higher order modulations, including optimal subcarrier power allocation.

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<sup>6</sup>In this paper, the ideal case is defined as distortionless transmission and  $\text{CE} = 1$ . This gives the optimal results in the context of our discussion, achieving quasi error free, defined as a packet error rate of  $10^{-7}$ , at  $\bar{E}_b/N_0 = 0.59$  dB for  $R_c = 1/3$  [24].

<sup>7</sup>As for the coded 16QAM with Gray encoding, the energy consumption penalty was approximately 1.3 dB for  $N = 8$ , while for the uncoded 16QAM,  $N \geq 10$  was required to attain  $\text{BER} \approx 10^{-3}$ ; it was obtained at  $\bar{E}_b/N_0 = 26.0$  dB and 17.1 dB for  $N = 10$  and 16, respectively.

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