



45 literature review on the history of developments are provided. It should be noted that the  
46 details of each of the presented models is beyond the scope of this paper and the reader may  
47 refer the corresponding literature cited. Through the course of this review, several numerical  
48 techniques pertaining to multi-scale modelling shall be covered. However, we refrain from  
49 making any best practices recommendations as these strongly depend on the problem at hand  
50 and, thus, could be highly subjective. Rather, the aim of this review is to provide the reader  
51 with a comprehensive listing of the available methods. This listing has been developed based  
52 on the authors' prior experience with multi-scale modelling as well as through a  
53 comprehensive review of the state-of-the-art. In the following sections, these mathematical  
54 models are discussed with their governing equations to handle physical problems,  
55 assumptions, implementation strategies and adopted numerical methods along with their  
56 applications. The existing numerical efforts carried out worldwide are provided along with a  
57 detailed discussion on numerical model development actualized by the authors' research  
58 group that has been supported in-part by the Newton International Fellowship. The remainder  
59 of the paper is structured as follows: the spatio-temporal scales associated with various  
60 physical processes in ocean engineering along with application-specific levels of  
61 approximation necessary in a given model are discussed in §2, the depth-resolving Navier-  
62 Stokes models along with numerical strategies for solution, wave/current generation and  
63 absorption, free-surface tracking as well as turbulence modelling have been discussed in  
64 detail in §3; potential flow models are introduced in §3.2, the depth-averaged Boussinesq-  
65 type models are discussed in §4, the state-of-the-art in global and regional-scale ocean  
66 science multi-scale modelling is presented in detail in §5, multi-scale modelling achieved  
67 through coupling of different models is discussed in detail from the standpoint of both  
68 domain as well as functional decomposition strategies in §6, the past and present effort of  
69 benchmarking numerical models through comparative studies is reviewed in §7 and finally  
70 the future of multi-scale modelling in WSI is discussed from the standpoint of AI/ML  
71 techniques as well as the development of hybrid models for floating renewables in §8. The  
72 reader will appreciate that significant effort has been made to cover a broad range of  
73 modelling techniques in ocean engineering in general and WSI in particular. However, this  
74 review is not all-inclusive and hence some fields of research such as hydroelasticity,  
75 metocean analysis, wind-wave interaction and phase-averaged wave action modelling could  
76 not be included.

77

## 78 **2 Different Levels of Approximations**

79 A single numerical tool to address all class of problems in ocean engineering is ideal.  
80 However such a model is not possible due to the following reasons: (a) a large sea area,  
81 having a large range of spatial and time scales, (b) highly nonlinear wave-structure  
82 interaction process (here not only fluid, sometimes the structure can also behave nonlinearly  
83 such as vegetation or fenders or hydro-elasticity), (c) waves co-exist with nonlinear currents  
84 of various levels, sediment transport and others, (d) viscosity, surface tension and turbulence,  
85 (e) two phase (air-sea) or multiphase processes (air-sea-oil or air-sea-sediment), (f) violent  
86 wave impacts (during cyclonic storm surges, flooding) and aeration on rubble mound  
87 structures, green water shipping and slamming. For these above phenomena, one needs to  
88 model large spatial/time scale to capture wave propagation phenomenon as well as resolve

89 small spatial/time scale to understand the wave-structure(-soil) interactions processes. A  
 90 single mathematical model may not always be a solution for this complex problem. Hence,  
 91 the researchers have developed various levels of approximations in the mathematical  
 92 modelling.

93 The level of approximations in the mathematical modelling are decided based on two guiding  
 94 principles: (a) which physical process is governing the problem at hand and (b) strive to  
 95 minimize the computational effort in the resulting numerical algorithm for industrial/practical  
 96 application by balancing computational efficiency and fidelity. In context to the first  
 97 principle, the requirement of modelling a physical process is mapped with respect to various  
 98 applications in Table 1. Table 1 lists various applications that require either large domain or  
 99 local/small domain modelling.

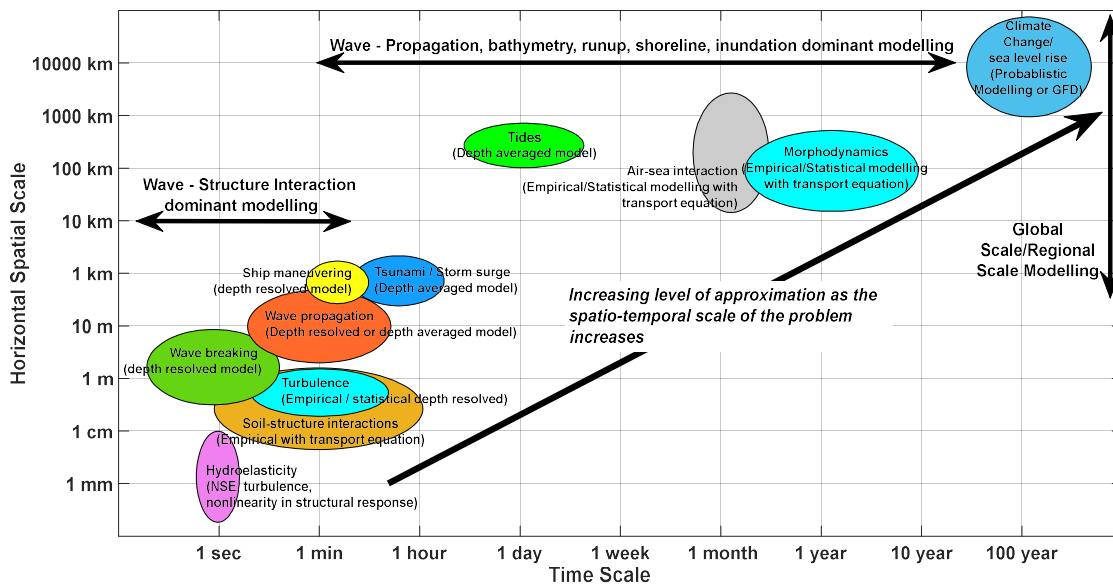
100 *Table 1. A summarization of the various physical processes and the requirement for them to*  
 101 *be modelled for various large and small-scale ocean engineering applications (the*  
 102 *information is based on the authors' experience with multi-scale modelling).*

		PHYSICAL PROCESSES					
		Surface tension	Viscous effects and/or turbulence modelling	Nonlinearity in fluid	Nonlinearity in structure	Modelling the air-phase	
<b>APPLICATIONS</b>	<b>LARGE-SCALE</b>	<b>Current/flow-structure interaction</b>	No	It depends	Yes	No	No
		<b>Wave propagation and interaction</b>	No	It depends	It depends	No	No
		<b>Seakeeping/motion of marine structures</b>	No	It depends	Yes	It depends	No
		<b>Geophysical flows</b>	No	It depends	It depends	No	It depends
		<b>Sediment transport</b>	No	Yes	Yes	No	No
		<b>Wave-breaking</b>	No	Yes	Yes	Yes	It depends
		<b>Aeration dynamics</b>	It depends	Yes	Yes	Yes	Yes
		<b>Wind-wave interaction</b>	It depends	Yes	Yes	No	Yes
	<b>LOCAL-/SMALL-SCALE</b>	<b>Steep wave and rigid structure</b>	Yes	It depends	Yes	No	No
		<b>Extreme waves and rigid structure</b>	It depends	It depends	Yes	No	Yes
		<b>Wave-structure-soil interaction</b>	It depends	It depends	Yes	No	No
		<b>Wave-deformable structure-interaction</b>	It depends	It depends	Yes	Yes	Yes
		<b>Current/flow-structure interaction</b>	No	Yes	Yes	It depends	No
		<b>Sediment transport</b>	No	Yes	Yes	No	No
	<b>Wave-breaking</b>	It depends	Yes	Yes	It depends	Yes	
	<b>Aeration dynamics</b>	Yes	Yes	Yes	It depends	Yes	
	<b>Wind-wave interaction</b>	It depends	Yes	Yes	No	Yes	

103  
 104 A typical example for large domain modelling in coastal engineering is wave propagation  
 105 from offshore to near shore and its interactions with a harbour structure to understand its  
 106 tranquillity, run-up or inundations. Similar examples from naval architecture and offshore

107 engineering would be ship maneuvering under the action of waves and an offshore wind  
 108 turbine farm interacting with waves or offshore platform interactions with waves,  
 109 respectively. For these applications, the physical processes such as surface tension and  
 110 nonlinearities in structural response are not important. Further, complete physics in modelling  
 111 the air-sea process is also not required; some empirical treatment would be deemed sufficient.  
 112 Thus, the full continuity and momentum equations can be simplified based on these  
 113 approximations.

114 Similarly, consider an application of small domain modelling, wherein one is interested in  
 115 quantifying the forces experienced by structures (such as ships, semi-submersible platforms,  
 116 seawalls, scour around monopiles and jackets etc.) against operating or extreme sea state  
 117 conditions. In this scenario, normally researchers would carry out the physical model studies  
 118 in an experimental wave tank. A similar study can be done using numerical modelling based  
 119 on so-called numerical wave tanks. A numerical wave tank is a numerical tool that could  
 120 reproduce the experimental facility with a high degree of fidelity. Thus, a detailed physical  
 121 flow process is realized by solving the continuity and momentum equations, only slightly  
 122 reducing the physical approximations, nonetheless reproducing the dominating forces as  
 123 close to reality as possible. For instance, in coastal engineering, surface tension, nonlinearity  
 124 in structure, sediment to sediment interactions or rigid body interactions (say in a rubble-  
 125 mound breakwater) can be relaxed without greatly compromising the fidelity of the numerical  
 126 approximation. Thus, for large scale problems, one can employ various levels of  
 127 approximations based on the wave characteristics and its applications thus leading to savings  
 128 in the computational cost. This aspect of modelling is further emphasised by means of a  
 129 bubble plot in Figure 1 wherein the various environmental aspects in spatial and time scale  
 130 for mathematical modelling of wave-structure interaction as well as wave-propagation are  
 131 illustrated.



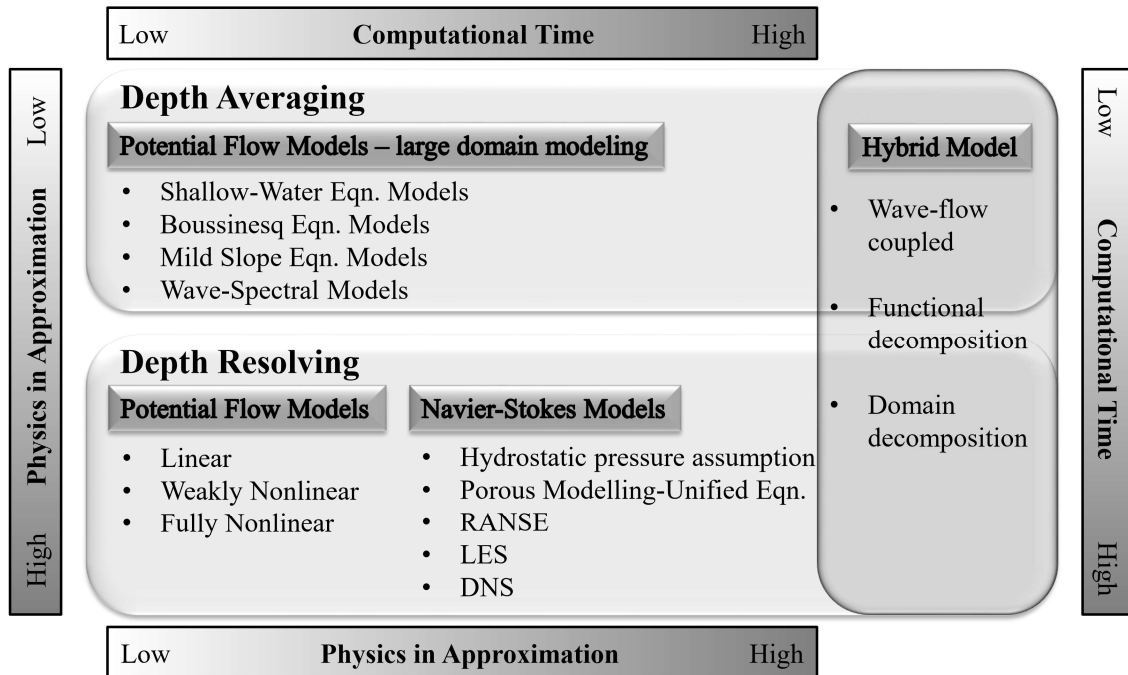
132  
 133 *Figure 1. Bubble plot variation to represent the spatio-temporal scales of various physical*  
 134 *processes in ocean engineering.*

135 Figure 1 showcase various processes ranging from climate change, sea-level rise,  
136 morphodynamics, tides, tsunami/storm surges, wave propagation over varying bathymetry  
137 from offshore to their interactions with structures. Each of these cases has a different  
138 horizontal spatial scale from 1mm to more than 10,000 km and time scale from less than 1s to  
139 100 years. Further, which type of modelling is dominant and ought to be carried out is also  
140 represented (in brackets) in Figure 1, along with global scale or regional scale modelling. The  
141 mathematical modelling approximations based on depth averaging can be seen as  
142 predominant for increasing large scale problems. This is based on the assumptions of the  
143 vertical flow structure. When the vertical flow motion is considered weak or insignificant,  
144 then depth averaged horizontal velocities can be adopted. Such classifications of  
145 mathematical models are called as depth averaged models and depending upon the  
146 approximations adopted in the horizontal velocities different models are available. This will  
147 be discussed in the later part of this paper. When the time scale and horizontal spatial scale  
148 are small, then the wave-structure interaction becomes dominant; in such cases depth  
149 resolving models are normally adopted. This is solved based on Navier-Stokes equations  
150 (NSE) with various simplified approximations. Depending upon the application (such as  
151 porous-structure, vegetation interactions, hydroelasticity or sediment transport) either  
152 microscopic or macroscopic modelling can be adopted within the NS framework to model the  
153 structure interaction process. On the other hand, the physical process involved in the ship  
154 manoeuvring is of the order of kilometers and minutes at the prototype-scale, however for  
155 numerical modelling the same would normally be carried out at a reduced scale using depth  
156 resolving models. Hence, for some applications, although the physical process is at a large  
157 scale, the numerical simulations are normally carried out at a smaller scale due to  
158 computational limitations.

159 In the past decades, one of the major reasons for resorting to the different physical  
160 approximations to model the different scales of the problem was to reduce the computational  
161 time. However, this leads to a compromise on the physics of the problem. Figure 2 shows  
162 three different broader classifications namely depth averaging, depth resolving and hybrid  
163 models. In this broader classification, different governing equations for modelling based on  
164 approximations are available, which are currently in practise within the numerical modelling  
165 community. The basic modelling task in each case is to solve the continuity and momentum  
166 equations for the fluid dynamics problem. However, the modelling complexity increases  
167 based on the physical problem being addressed, type of structure (coastal, offshore or marine)  
168 and type of sea-state under consideration.

169 In coastal engineering, the majority of the structures (e.g. breakwaters, sea-walls and pile  
170 structures) are fixed or stationary. Then the physical problem to represent is the wave  
171 transformation process (i.e., wave shoaling, diffraction, refraction, reflection, wave-  
172 overtopping and wave-breaking) and its interaction with the structures. In case of offshore  
173 engineering, the structures may be fixed (e.g. offshore wind turbine foundations in < 50 m  
174 deep water) or floating (e.g. oil production platforms, floating offshore wind turbines and  
175 floating solar arrays). In the latter case, the modelling complexity increases because the fluid  
176 flow and structure motion(s) are coupled and thus need to be solved in conjunction; failure to  
177 do so would over-predict the hydrodynamic loads. Nonetheless, the overall excursion of an  
178 offshore structure is small when compared to marine structures such as a ship or submarine.

179 For a marine structure, the numerical modelling needs to account for large displacements  
 180 (e.g. ship maneuvering in waves) thus necessitating large domains and, if the sea-state is  
 181 violent, also hydroelasticity plays a role (e.g. hull-slamming in violent sea-states).  
 182 Nonetheless, these scenarios may not always necessitate the NS equations; potential-flow  
 183 models are a viable alternative as long as the hydrodynamic loads and resulting body motions  
 184 are properly accounted for. For instance, models based on the Boussinesq equations are quite  
 185 popular for modelling wave tranquillity and recently, for ship-generated waves.



186  
 187 *Figure 2. Different modelling strategies characterized by the level of physics approximation*  
 188 *and the resulting computational cost.*

189 Various numerical methods are currently in practice to solve a given mathematical model.  
 190 The numerical methods are broadly classified into strong and weak forms. The traditional  
 191 methods such as the Finite Difference Method (FDM), the Finite Element Method (FEM), the  
 192 Boundary Element Method (BEM) and the Finite Volume Method (FVM) as well as modern  
 193 techniques such as particle/mesh-free methods are being employed in the ocean engineering  
 194 problems. Mostly, the choice of the numerical methods depends upon the developers and one  
 195 is not superior to the others as one might expect. Each of these numerical methods has their  
 196 own advantages and disadvantages, and the overall goal is to reduce or minimize the  
 197 disadvantages using numerical treatments/algorithms/schemes.

### 198 3 Depth Resolving Mathematical Models

199 In the present section, we review the depth-resolving models. These models are mostly used  
 200 for the wave-structure interaction problems to estimate the wave loads, wave damping  
 201 characteristics and motion/structural responses. The problems that are based on small spatial  
 202 and time scale are normally handled by the depth-resolving approach which models the

203 physical process using a high (spatio-temporal) resolution thus leading to high computational  
 204 costs.

### 205 3.1 Navier Stokes Equations

206 The Navier-Stokes equations include the equations governing the conservation of mass  
 207 (termed “equation of continuity” (EOC) for incompressible flows which is in turn a  
 208 reasonable assumption for WSI) and conservation of momentum. The term “full” indicates  
 209 the *absence* of simplifying assumptions such as irrotationality, depth-averaging, Reynolds-  
 210 averaging, two-dimensionality, axisymmetry, single-phase nature of the flow (density is  
 211 spatio-temporally constant) etc.

#### 212 3.1.1 Governing Equations

213 The full Navier-Stokes equations (NSE) governing fluid motion are written here in  
 214 differential form for the instantaneous velocity field  $\vec{V}$ :

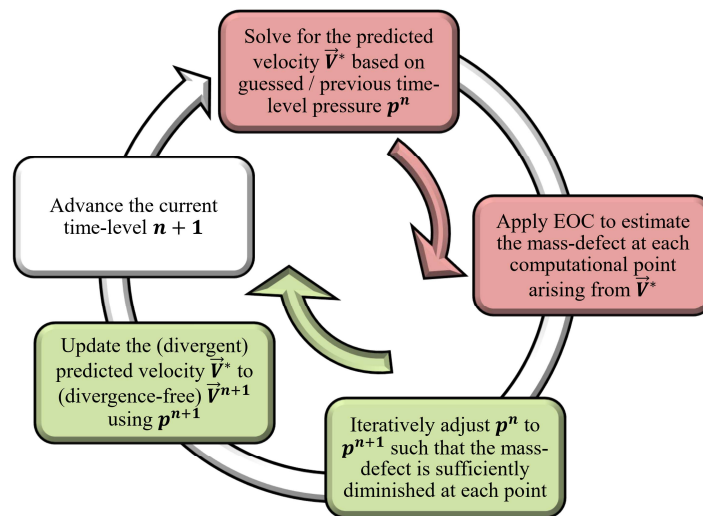
$$\begin{aligned}
 & \underbrace{\frac{\partial \rho}{\partial t} + (\vec{V} \cdot \vec{\nabla})\rho}_{\text{conservation of mass}} = 0 \\
 & \underbrace{\frac{\partial \rho \vec{V}}{\partial t}}_{\text{time}} + \underbrace{\vec{\nabla} \cdot (\rho \vec{V} \otimes \vec{V})}_{\text{advection}} = \underbrace{-\vec{\nabla} p}_{\text{pressure}} + \underbrace{\vec{\nabla} \cdot \left( \mu \left\{ \vec{\nabla} \vec{V} + (\vec{\nabla} \vec{V})^T - \frac{2}{3} (\vec{\nabla} \cdot \vec{V}) \bar{\bar{I}} \right\} \right)}_{\text{diffusion}} + \underbrace{\rho \vec{g}}_{\text{gravity}}
 \end{aligned} \tag{1}$$

215 where,  $p$  is the total pressure,  $\rho$  and  $\mu$  are the density and viscosity respectively,  $\bar{\bar{I}}$  is the  
 216 identity tensor and  $\vec{g}$  is the gravitational acceleration vector. Equation (1) represents the  
 217 compressible Navier-Stokes equations, however, the compressibility of the fluid may only be  
 218 important during violent wave-structure interaction at large-scale. For the remainder of the  
 219 applications, the conservation of mass simplifies to the equation of continuity (EOC):  $\vec{\nabla} \cdot \vec{V} =$   
 220  $0$  which holds for incompressible flow. It is also worth noting that Equation (1) is written for  
 221 the “instantaneous” velocity-field (Anghan *et al.*, 2019) indicating that  $\vec{V}$  is neither time-  
 222 averaged (RANS) nor spatially-filtered (LES). The fluid properties  $\rho$  and  $\mu$  can be replaced  
 223 with the mixture properties  $\rho^*$  and  $\mu^*$  to account for the presence of multiple contiguous  
 224 phases in the domain. Here, advantage is derived from the fact that the phases can be  
 225 considered as being “individually incompressible” (Saincher and Banerjee, 2018) for most  
 226 applications which precludes the necessity of solving (say)  $N$  sets of the NSE for  $N$  phases.  
 227 This results in the so-called “single-fluid formulation” wherein the entire computational  
 228 domain is assumed to be filled with a single, albeit, variable-property fluid (Saincher and  
 229 Sriram, 2022a). It should also be noted that within the single-fluid framework, equation (1) is  
 230 “conservative” (Saincher and Sriram, 2023) meaning  $\rho$  is on the left-hand-side with the time  
 231 and advection terms. On the other hand, the formulation would be termed “non-conservative”  
 232 if  $\rho$  were on the right-hand-side with the pressure and diffusion terms. The positioning of  $\rho$   
 233 in the governing equations is immaterial for a single-phase treatment of the NSE (for instance  
 234 cf. Sriram *et al.*, 2014). The same, however, would have far-reaching consequences for a  
 235 multiphase framework especially for violent flows involving wave-breaking and/or slamming  
 236 loads; a conservative formulation is recommended in these cases (Saincher and Sriram,  
 237 2023). However, an important limitation of the conservative formulation is that it may lead to

238 the formation of unrealistically large velocities at the interface (Tryggvason *et al.*, 2007) and  
 239 thus is deemed unnecessary for more benign wave propagation and WSI scenarios. In context  
 240 to equation (1), it is also worth mentioning that the total pressure  $p$  is comprised of static,  
 241 hydrostatic as well as dynamic contributions;  $p$  is not the true pressure but rather a pseudo  
 242 pressure which satisfies the EOC. The advantage with WSI and ocean engineering problems  
 243 in general is that the simulation begins from quiescent/calm water conditions which allows  
 244 for a very accurate “guess” of the initial pressure field using the hydrostatic law. This results  
 245 in a dynamic pressure field that is very close to the true (say experimentally measured)  
 246 dynamic pressure, once the hydrostatic contribution has been removed (Saincher and Sriram,  
 247 2022a ; 2022b).

### 248 3.1.2 Solving the Navier-Stokes Equations

249 For a given flow problem, the solution variables of interest include the velocity  $\vec{V}$  and  
 250 pressure  $p$ . It is characteristic of the incompressible Navier-Stokes equations to *not* have a  
 251 separate equation for pressure. Owing to this, a majority of incompressible NSE flow solvers  
 252 are based on a predictor-corrector approach which was pioneered by Chorin (1967); the same  
 253 is illustrated in Figure 3.



254  
 255 *Figure 3. A typical predictor-corrector loop characteristic of projection methods pioneered*  
 256 *by Alexandre Chorin in 1967.*

257 At the beginning of the solution, both  $\vec{V}^{n+1}$  and  $p^{n+1}$  at the current time-level are unknown  
 258 and the momentum equations are solved for a predicted velocity field  $\vec{V}^*$  wherein either:

- 259 • the pressure term  $\left(-\frac{1}{\rho^*} \vec{\nabla} p\right)^n$  from the previous time-level is considered (Saincher and  
 260 Banerjee, 2015) or,
- 261 • the pressure term is not considered at all which was the case with Chorin’s original  
 262 method (normally adopted in Meshfree methods ; cf. Sriram and Ma, 2021).



263 At this point, the incompressibility condition  $\vec{\nabla} \cdot \vec{V}^{n+1} = 0$  is invoked at the current time-  
264 level and the same is split into a mass defect  $\vec{\nabla} \cdot \vec{V}^*$  and divergence correction  $\vec{\nabla} \cdot \vec{V}'$   
265 contributions. This marks the end of the “predictor-step” (highlighted in red in Figure 3).  
266 Following this, the property  $\vec{V} = -\frac{\Delta t}{\rho} \vec{\nabla} p$  is invoked to establish a relationship between either  
267  $\vec{\nabla} \cdot \vec{V}^*$  and  $p^{n+1}$  (Sriram and Ma, 2021) or between  $\vec{\nabla} \cdot \vec{V}^*$  and the pressure correction  $p'$   
268 (Saincher and Banerjee, 2015). In either case, one ends up with a Pressure Poisson Equation  
269 (PPE) which needs to be iteratively solved for  $p^{n+1}$  (or an Equation Of Pressure Correction  
270 (EOPC) which needs to be iteratively solved for  $p'$ ). This is oftentimes the most  
271 computationally-intensive step in a flow solver. Following solution of the Poisson equation,  
272  $\vec{V}^{n+1}$  can be obtained using  $\vec{V} = -\frac{\Delta t}{\rho} \vec{\nabla} p$  which marks the end of the “corrector-step”  
273 (highlighted in green in Figure 3). The splitting of the solution into predictor and corrector  
274 steps is also known as the “projection method” since the pressure is used to project  $\vec{V}^*$  onto a  
275 space of divergence-free velocity-field which is essentially the Helmholtz decomposition.

276 Various flow solvers (or so-called “pressure-velocity coupling” schemes) such as SIMPLE,  
277 PISO, PIMPLE essentially have the same predictor-corrector constitution but differ with  
278 regards to how  $\vec{V}^*$  is calculated as well as the number of predictor-corrector cycles per time-  
279 step. In fact, regardless of whether  $\vec{V}^*$  is computed fully-explicitly or semi-implicitly  
280 (because a fully-implicit treatment of the advection term is not possible), the solver still  
281 belongs to the SIMPLE class of algorithms (Ferziger *et al.*, 2020). However, some authors  
282 also call the fully-explicit category of algorithms “semi-explicit” (Dave *et al.*, 2018 ; Sharma,  
283 2022) owing to the implicit nature of solution of the PPE. It is important to note that, for a  
284 given order of time-discretization, the solutions obtained from a fully-explicit or semi-  
285 implicit predictor step should be identical. Nonetheless, the semi-implicit treatment would  
286 accord further stability to the solution.

287 In context to WSI, a forward Euler time-discretization and fully-explicit evaluation of  $\vec{V}^*$  has  
288 been extensively used by the authors (Saincher and Sriram, 2022a ; 2022b ; 2023). From the  
289 authors’ experience, explicit (forward Euler) time discretization is recommended for waves  
290 owing to the hyperbolic nature of solution propagation and a fully-explicit evaluation of  $\vec{V}^*$   
291 was found to be sufficient for relatively benign WSI scenarios especially ones that did not  
292 involve slamming loads. In fact, it is demonstrated in Saincher *et al.* (2023a ; 2023b) that a  
293 fully-explicit evaluation of  $\vec{V}^*$  works even in slamming conditions for modest mesh  
294 resolutions. Thus, the CFD user ought to make an informed decision whilst selecting the  
295 pressure-velocity coupling scheme keeping in mind the trade-off between numerical stability  
296 (better for semi-implicit treatment) and computational efficiency (better for fully-explicit  
297 treatment). Unfortunately, users of commercial CFD solvers seldom have fully-explicit  
298 pressure-velocity coupling available to them and thus alternatively opt for (say) the PISO  
299 solver with a Non-Iterative Time Advancement (NITA) option available in ANSYS®  
300 FLUENT.

301 When the predictor and corrector steps are considered in conjunction, say for a 3D flow  
 302 problem, a single time-step would have one iterative solution loop (for  $p$ ) in case of a fully-  
 303 explicit solver and four (for  $U, V, W, p$ ) in case of a semi-implicit solver. However, our  
 304 experience suggests that the computational effort required for solving  $p$  may sometimes be  
 305 greater than the three velocity components  $U, V, W$  combined. This is primarily because of  
 306 differences in the rate of convergence which is in turn dependent on the type of boundary  
 307 conditions involved. The boundary conditions are predominantly Dirichlet in case of  
 308 velocities which results in predominantly Neumann conditions for the pressure thus leading  
 309 to an increase in the computational effort for solving the PPE.

### 310 3.1.3 Boundary conditions – Wave/Current Generation and Absorption

311 A prerequisite to accurate WSI simulations in ocean engineering applications is high fidelity  
 312 wave generation as well as reflection-free absorption of waves/currents in the computational  
 313 domain. The task of absorption is generally more challenging for WSI simulations involving  
 314 regular and irregular waves when compared to focusing waves primarily due to the larger  
 315 number of wave cycles/periods involved in the former case. The task of absorption also  
 316 becomes complex if currents co-exist with waves. The various methods of wave/current  
 317 generation and absorption in NSE-based NWTs are mapped against their numerical  
 318 characteristics in Table 2.

319 *Table 2. Type of wave/current generation and absorption strategies in NSE-based NWTs.*

<b>Wavemaker</b>		<b><math>U</math></b>	<b><math>V</math></b>	<b><math>W</math></b>	<b><math>p</math></b>	<b><math>\eta</math></b>
Inflow-boundary		Dirichlet	Dirichlet	Dirichlet	Dirichlet	Dirichlet
Mass-source function		--	--	--	--	Source-term in EOC
Momentum-source function		Source-term in momentum equation			--	--
Internal inlet		--	--	Dirichlet	--	--
Relaxation zone		Dirichlet	Dirichlet	Dirichlet	Dirichlet	Dirichlet
Moving wall	Flap / Piston type	Prescribed motion	--	Prescribed motion	--	--
	Segmented type					
<b>Wave-absorber</b>		<b><math>U</math></b>	<b><math>V</math></b>	<b><math>W</math></b>	<b><math>p</math></b>	<b><math>\eta</math></b>
Outflow boundary		Orlanski / Continuity / Sommerfeld radiation boundary condition				
Sponge-layer		Sink terms in momentum equation			--	--
Relaxation zone		Solution gradually ramped from/to wave theory to/from numerical model				
Moving wall (active absorption)		Prescribed motion	--	Prescribed motion	--	--
Adaptive passive absorption		Adaptively predicted using on-board elevation		Neumann	Neumann	--

320 With reference to Table 2, the development of “numerical wavemakers” for NSE models was  
 321 pioneered by Lin and Liu (1998; 1999) wherein the inflow-boundary and mass-source  
 322 function techniques were proposed. As seen from Table 2, the inflow technique involves a  
 323 Dirichlet prescription of the wave-induced orbital velocities (predicted from a suitable wave  
 324 theory) as well as the free-surface elevation at the domain boundary. The present research  
 325 group has proposed a modified inflow technique to improve the volume-conservation

326 properties of inflow-boundaries, particularly for scenarios involving strong Stokes drift such  
327 as steep wave generation in near-shallow water (Saincher and Banerjee, 2017a).

328 In conjunction with inflow boundaries, the mass-source function technique was also  
329 developed which involved the modification of the EOC through the inclusion of a time-  
330 varying source term that is in turn proportional to the wave elevation. Wave generation is  
331 achieved through periodic ejection/ingestion of water-volume from/into the source region and  
332 this offers some advantages over inflow-boundaries. For instance, the only wave  
333 characteristic to be input is the time-varying free-surface elevation  $\eta(t)$  and thus wave-  
334 records from the field could be reproduced. Also, waves reflected from the domain  
335 boundaries would not interfere with the wave generation. Nonetheless, the source region  
336 itself has several design variables requiring parameterization and, in this context, the authors  
337 have proposed guidelines to decide the geometry, placement and strength of the source region  
338 based on the relative depth and wave-steepness (Saincher and Banerjee, 2017b). As listed in  
339 Table 2, other similar methods have also been proposed such as the internal inlet (Hafsia *et al.*,  
340 2009) and momentum-source function (Choi and Yoon, 2009) techniques. Some  
341 researchers have also attempted to directly model piston/flap-type wave-paddle motions into  
342 their NWTs using embedded boundary treatment for the solid (cf. fast-fictitious-domain  
343 (FFD) based modelling of wave-paddles in Anbarsooz *et al.* (2013)).

344 However, currently, the most popular technique of numerical wave generation is the so-called  
345 “relaxation zones” developed by Jacobsen *et al.* (2012) for OpenFoam<sup>®</sup>. Here, the solution is  
346 spatio-temporally “ramped-up” from wave-theory to NSE before the structure/region of  
347 interest and again “ramped-down” from NSE to calm-water conditions after the  
348 structure/region of interest. Thus, relaxation zones not only prevent upstream reflection of  
349 waves from the far-end of the NWT but also downstream re-reflection of waves reflected off  
350 the structure. It is also worth mentioning that relaxation zones in and of itself is a more  
351 general concept that has been implemented in hybrid potential theory-NSE models (Agarwal  
352 *et al.*, 2022b) as well as in hybrid spectral theory-NSE models (Aliyar *et al.*, 2022).

353 Apart from relaxation zones, other methods of wave absorption have also been implemented  
354 for NSE-based NWTs. For instance, Lin and Liu (1999) employed a radiation/outflow  
355 boundary condition for wave absorption at the far-end of the NWT. Outflow boundaries  
356 generally implement the Sommerfeld condition (Dave *et al.*, 2018):  $\frac{\partial \phi}{\partial t} + C \frac{\partial \phi}{\partial n} = 0$  where  $\phi$   
357 is the property to be effluxed from the boundary,  $t$  is time,  $C$  is the phase velocity and  $n$   
358 points normal to the boundary. The prescription of  $C$  is relatively straightforward for “flow  
359 problems” making outflow boundaries suitable for tsunamis, tidal flows, scour etc. which  
360 involve a dominant current component. Sommerfeld conditions are also suitable for  
361 absorbing small-amplitude waves. However, these pose a challenge for absorbing steep  
362 waves particularly because  $C$  is spatio-temporally variable along the boundary. It has been  
363 shown in Dave *et al.* (2018) that improper prescription of  $C$  leads to severe (inward)  
364 reflections even for free-shear flows.

365 Self-adaptive wavemaker theory has also been used popularly in both the physical and  
366 numerical wave tanks. This method utilizes wavemaker (moving wall) whose motion is  
367 specified to generate both the incident waves and an additional wave to cancel the  
368 undesirable wave (e.g. the reflected wave from somewhere within the tank). More details can  
369 be found in Yan *et al.* (2016). In addition, the same concept of “adaptive absorber” was also  
370 used in our recent work on developing a passive wave absorber (Yan *et al.*, 2020). This  
371 boundary behaves similarly to the inflow boundary, however the fluid velocity condition is  
372 specified by considering its relation with the wave elevation recorded at the boundary. This  
373 method does not require the use of the relaxation zone for wave absorption and thus results in  
374 a considerable improvement of the computational efficiency. A recent application of the same  
375 can be found in Xiao *et al.* (2024).

#### 376 3.1.4 Free surface capturing/tracking

377 A majority of ocean engineering problems involve waves and/or other flows such as bores,  
378 hydraulic jumps, etc. which necessitates computing the topology of the free-surface. In  
379 reality, the free-surface marks a discontinuity between two media (say air and water) and thus  
380 acts as an interface. The numerical algorithms for computing the interfacial topology can be  
381 broadly classified into interface-tracking and interface-capturing techniques. In the former  
382 category of algorithms, the free-surface is modelled as a boundary and is tracked by updating  
383 the mesh as the solution progresses. In the latter category, the free-surface evolves spatio-  
384 temporally within a fixed domain wherein the interface is identified by an indicator function.  
385 Interface-capturing algorithms are obviously more advantageous (especially for violent flows  
386 involving complex interfacial deformation such as overturning and aeration) and thus have  
387 been extensively employed in NSE-based flow solvers; the same have been listed in Table 3.

388 As evidenced from Table 3, the interface-capturing algorithms can be further classified based  
389 on the technique used for interface identification (“reconstruction”) and advection. The  
390 interface identification techniques differ based on the type of indicator function used (volume  
391 fraction or level-set function) as well as whether the identification itself is geometric in nature  
392 or not. The level-set method and high-resolution schemes such as the Compressive Interface  
393 Capturing Scheme for Arbitrary Meshes (CICSAM) are algebraic in nature in that they do not  
394 involve explicit geometrical computations of the placement (or advection) of the interface  
395 within the domain. In comparison, geometric methods such as the Piecewise Linear Interface  
396 Calculation-Volume Of Fluid (PLIC-VOF) and Moment Of Fluid (MOF) are higher fidelity  
397 in that the interfacial coordinates are geometrically computed subject to conservation of the  
398 primary phase volume in each cell.

399 Volume conservation is intrinsic for geometric VOF methods and also for single-phase  
400 meshfree methods such as the Improved Meshless Local Petrov-Galerkin method with  
401 Rankine source function (IMLPG\_R). This is not the case for algebraic VOF schemes or the  
402 level-set method where additional numerical treatment is necessary to achieve volume  
403 conservation. This has been comprehensively demonstrated by the present authors (Saincher  
404 and Sriram, 2022a) and others (Anghan *et al.*, 2021 ; Arote *et al.*, 2021) wherein a material  
405 redistribution algorithm originally developed for geometric VOF (Saincher and Banerjee,

406 2015) has been shown to dramatically improve the volume conservation properties of  
 407 algebraic VOF schemes.

408 Similarly, interfacial diffusion is intrinsic for algebraic VOF as well as level-set methods.  
 409 This could be mitigated to some extent using operator-split/direction-split advection as doing  
 410 so would eliminate multi-fluxing errors (Saincher and Sriram, 2022a). Whilst algebraic VOF  
 411 techniques are indeed capable of capturing large-scale interfacial segregation in WSI  
 412 problems (Saincher *et al.*, 2023a), small-scale droplets and bubbles would still diffuse upon  
 413 separation from the parent phase. This diffusion seldom contributes to the hydrodynamics in  
 414 a WSI simulation and, in fact, provides numerical stability to the solution. Conversely,  
 415 droplets/bubbles separating from the parent phase would never dissipate in geometric VOF  
 416 and thus excessive interfacial fragmentation might, in fact, lead to solver instability.

417 *Table 3. Various interface-capturing algorithms developed for NSE-solvers; cf. nomenclature*  
 418 *for the abbreviations.*

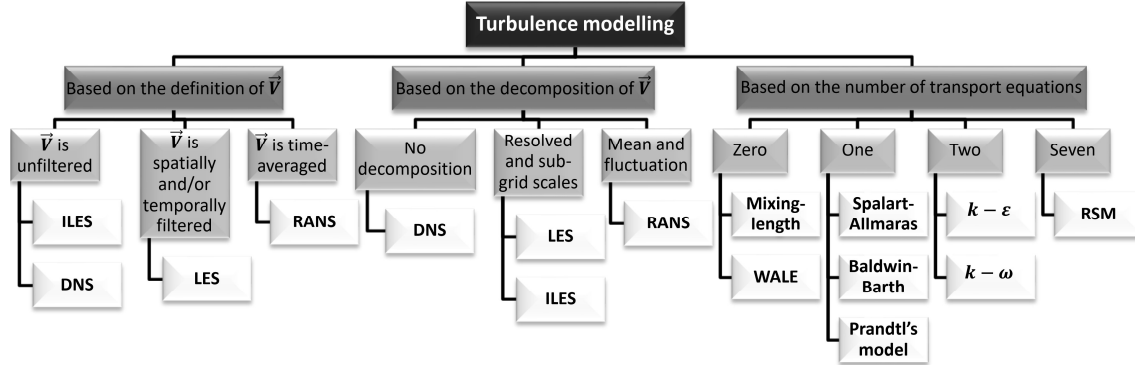
<b>Authors</b>	<b>Algorithm</b>	<b>Interface identification</b>	<b>Interface advection</b>	<b>Interface diffusion</b>	<b>Volume conservation</b>	<b>Mesh</b>
O'Shea <i>et al.</i> (2014)	NFA	Geometric VOF	Unsplit Eulerian	Zero	Intrinsic	Cartesian
Sriram <i>et al.</i> (2014)	IMLPG_R	MPNDAF	Lagrangian	Zero	Intrinsic	--
Saincher and Banerjee (2015)	Redistribution-based PLIC-VOF	Geometric PLIC-VOF	Operator-split Eulerian	Zero	Intrinsic	Cartesian
Bihs <i>et al.</i> (2016)	REEF3D	Level-set	Unsplit Eulerian	Intrinsic	Extrinsic	Cartesian
Zinjala and Banerjee (2016)	LEAS-MOF	Geometric MOF	Lagrangian-Eulerian	Zero	Intrinsic	General Polygonal
Zinjala and Banerjee (2017)	RMOF	Geometric MOF	Lagrangian-Eulerian	Zero	Intrinsic	General Polygonal
Anghan <i>et al.</i> (2021)	MSTACS	Algebraic VOF	Unsplit Eulerian	Intrinsic	Extrinsic	Cartesian
Arote <i>et al.</i> (2021)	SAISH	Algebraic VOF	Unsplit Eulerian	Intrinsic	Extrinsic	Cartesian
Saincher and Sriram (2022a)	OS-CICSAM	Algebraic VOF	Operator-split Eulerian	Intrinsic	Extrinsic	Cartesian

419

### 420 3.1.5 Turbulence Modelling

421 Ocean engineering problems involve flow of sea-water which has a kinematic viscosity of  
 422  $\nu \sim 1e - 06 \text{ m}^2/\text{s}$ . The corresponding Reynolds number  $\text{Re} = \mathcal{V} \cdot \mathcal{L}/\nu$  would typically be  
 423  $O(10^6)$  even if the characteristic velocity ( $\mathcal{V}$ ) and length ( $\mathcal{L}$ ) are  $O(1)$ , that is, at model-  
 424 scale. This is generally the case since the Froude-law is invoked for scaling based on the fact  
 425 that gravity is the dominant restoring force in ocean engineering applications. As a  
 426 consequence, most scenarios being simulated are not laminar and some form of modelling  
 427 may be required to account for the additional viscous effects near the structure. Some of the  
 428 typical applications necessitating turbulence modelling include:

- Wave/tsunami interactions with vegetation: turbulence-induced viscous effects arising from flow separation need to be accounted for to correctly estimate energy attenuation.
- Response of floating bodies: failing to account for viscous effects within the boundary layer may result in over-prediction of the motion response.
- Resistance of marine vessels in waves/calm water: failing to account for viscous effects within the boundary layer may result in under-prediction of resistance.



436  
437 *Figure 4. An illustration of the different means to categorize various strategies to model*  
438 *turbulence in depth-resolving models; cf. nomenclature for the abbreviations.*

439 Several popular methods have been developed for modelling turbulence in NSE-based  
440 solvers; some have been integrated with self-developed codes by the present research group.  
441 There exist different means of classifying turbulence modelling strategies for depth-resolving  
442 methods; the same are depicted in Figure 4. In conjunction with Figure 4, the momentum  
443 equation (1) is also re-written to account for turbulence modelling:

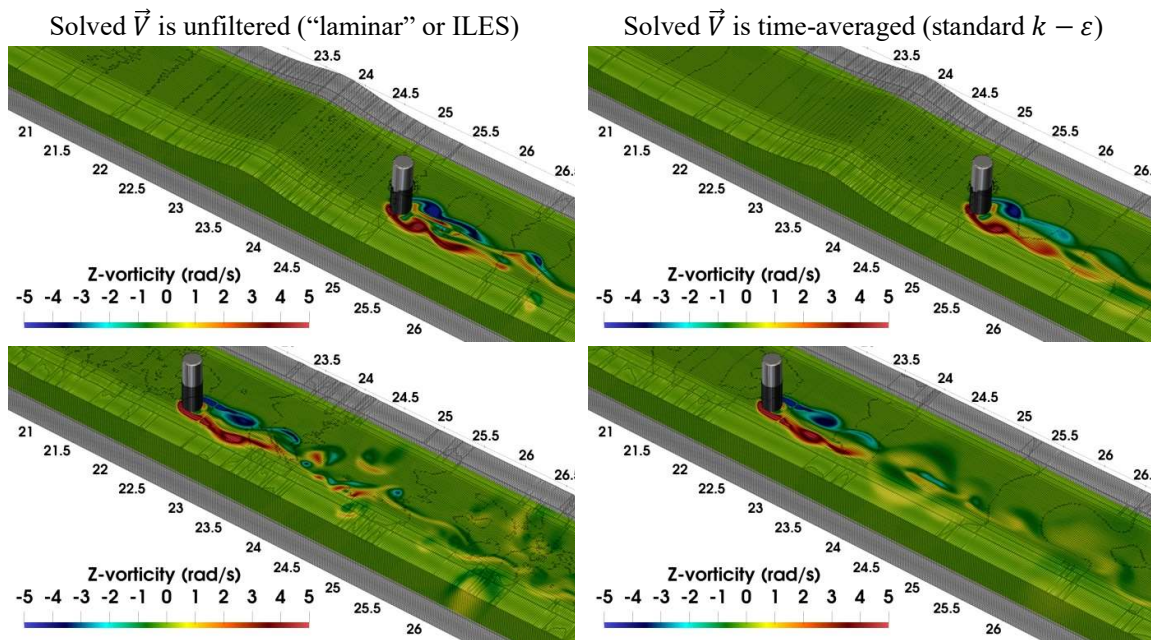
$$\frac{\partial \vec{V}}{\partial t} + \underbrace{(\vec{V} \cdot \nabla) \vec{V}}_{\text{advection}} = - \underbrace{\frac{1}{\rho^*} \nabla p'}_{\text{pressure}} + \underbrace{\frac{1}{\rho^*} \nabla \cdot ((\mu^* + \mu_t) \nabla \vec{V})}_{\text{diffusion}} + \underbrace{\vec{g}}_{\text{gravity}} \quad (2)$$

444 where,  $p'$  is the modified pressure which includes the normal components of the Reynolds or  
445 Sub-Grid-Scale (SGS) stress tensor and  $\mu_t$  is the turbulent viscosity; the terms  
446 modified/introduced by turbulence modelling have been highlighted in bold. In context to  
447 equation (2) and Figure 4,  $\vec{V}$  can be unfiltered, spatio-temporally filtered or time-averaged.  
448 The filtering and time-averaging operations are essentially decompositions of the unfiltered  
449 velocity and thus, once performed, information about the instantaneous velocity field is  
450 invariably lost. For instance, the  $\vec{V}$  field obtained following solution to the RANS equations is  
451 time-averaged and thus, (temporal) fluctuations in  $\vec{V}$  do not represent fluctuations in the  
452 instantaneous field. Only the effect of the true fluctuating field on  $\vec{V}$  is modelled through the  
453 eddy viscosity  $\mu_t$ .

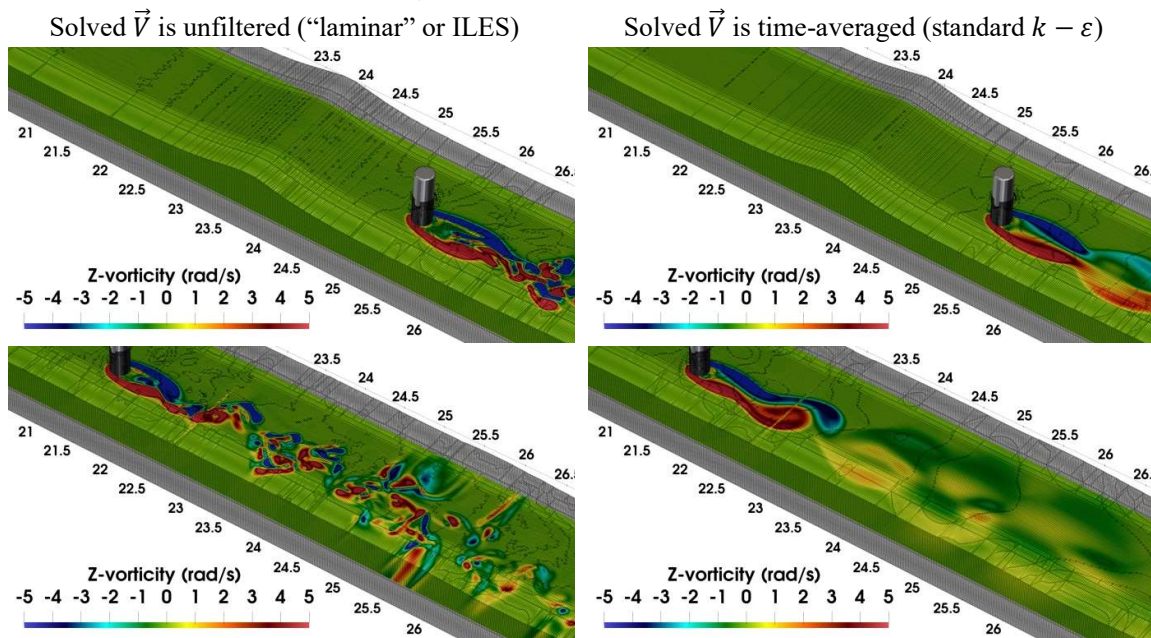
454

455

$U_{\text{cyl}} = 0.34 \text{ m/s}$  (case A25002)

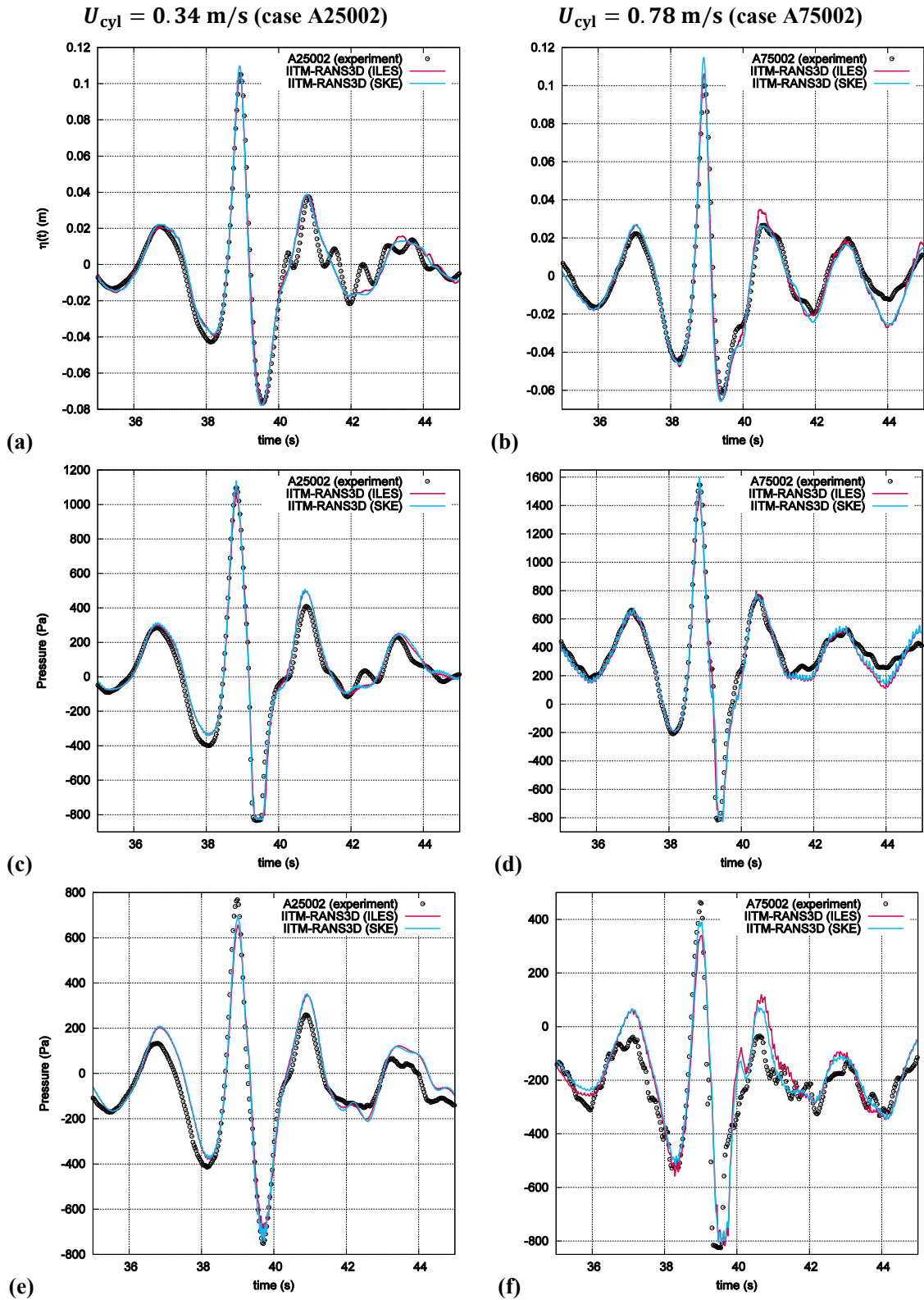


$U_{\text{cyl}} = 0.78 \text{ m/s}$  (case A75002)



456 Figure 5. The vorticity field ( $\vec{\nabla} \times \vec{V}$ ) generated by a moving cylinder interacting with a  
 457 focusing wave (Saincher and Sriram, 2022b); note the change in the nature of the solution  
 458 based on the definition of  $\vec{V}$ . The cylinder moves from bottom-right to top-left.

459 This important aspect is illustrated in Figure 5 wherein vortices shed by a moving cylinder  
 460 interacting with focusing waves are shown (adapted from Saincher and Sriram (2022b)). The  
 461 same problem has been simulated first using unfiltered NSE (a “laminar” solver) and then  
 462 using time-averaged NSE (a RANS solver based on standard  $k - \epsilon$  (SKE)). The  
 463 aforementioned loss of information regarding the true fluctuating velocity field is readily



464 Figure 6. Time-histories of free-surface elevation ( $\eta(t)$ ) and pressure ( $p(t)$ ) corresponding  
 465 to a moving cylinder interacting with focusing waves (Saincher and Sriram (2022b)): (a,b)  
 466  $\eta(t)$  variation in-line with the center of the moving cylinder and  $p(t)$  variation just below the  
 467 SWL at the (c,d) forward and (e,f) rear stagnation points.



468 apparent from Figure 5; the vorticity field is “instantaneous” in both cases. It should also be  
469 noted that the so-called “laminar” solver is a misnomer as it simply refers to solving the NSE  
470 without any turbulence modelling. In this regard, the laminar approach is not a Direct  
471 Numerical Simulation or DNS (since no attempt is made to resolve the Kolmogorov scales)  
472 but rather a form of Implicit Large Eddy Simulation or ILES (wherein the discretization  
473 errors would mimic SGS modelling (Rodi *et al.*, 2013)). Having said that, figure 5 indicates  
474 that the laminar solver captures more “turbulence” than the actual turbulence model!  
475 In addition to the above qualitative assessment, it is also important to quantify the impact of  
476 turbulence modelling (or lack thereof) on quantities of engineering importance. In order to do  
477 this, the time-variation of the free-surface elevation measured in the vicinity as well as  
478 pressure measured on the surface of the moving cylinder is reported in Figure 6 for both ILES  
479 and SKE simulations. It can be seen that ILES and SKE results are practically identical for  
480 the lower towing speed. This is corroborated by the vorticity fields (for  $U_{cyl} = 0.34$  m/s)  
481 reported in Figure 5. For the higher towing speed, the SKE results show a closer agreement  
482 with experiments in terms of both  $\eta(t)$  variation as well as pressure at the rear stagnation  
483 point. However, the improvement gained from turbulence modelling in this case is not  
484 dramatic (even though the computed vorticity fields are dramatically different). The findings  
485 are in line with conclusions drawn from the ISOPE 2020 comparative study which was based  
486 on the same experimental dataset (Agarwal *et al.*, 2021a).

487 The necessity and nature of turbulence modelling depends on the nature of the problem itself  
488 and oftentimes the fidelity of the solution/simulation (against experiments) depends on the  
489 expertise of the CFD practitioner (this is later discussed at length in §7 on comparative  
490 numerical studies). One is not only required to assess the need of a turbulence model but also  
491 the impact of a particular model on the solution. Considering a wave-floating structure  
492 interaction problem as an example, a need for turbulence modelling may arise due to an over-  
493 prediction of the angular acceleration of the body by a laminar model. If RANS-based  
494 turbulence modelling is introduced to supplement the viscous damping in the near-field of the  
495 body, the same may also negatively impact the simulation through unwanted damping of the  
496 incident waves. In such a case, the unwanted damping could be mitigated by:

- 497 • Stabilizing the unbounded growth of  $\mu_t$  using limiters (Larsen and Fuhrman, 2018).
- 498 • Increasing advection using higher-order upwind schemes (Saincher and Sriram,  
499 2022b).
- 500 • Increasing advection using conservative NSE formulations (Saincher and Sriram,  
501 2023).
- 502 • Switching to a less empirical model such as WALE (zero-equation model with a  
503 single model constant) for computing  $\mu_t$  (Rodi *et al.*, 2013).

504 The above discussion indicates that there exist multiple solutions to a given problem and  
505 there is a general consensus that the simplest models also prove to be the most robust. Taking  
506 into account the strongly empirical nature of turbulence modelling in general (RANS in  
507 particular), a modestly accurate albeit robust model applicable to several problems should be  
508 preferred over a heavily calibrated model that works perfectly albeit only for a single

509 problem. Further, turbulence model should only be employed for the practical problems in  
510 need and not for all scenarios.

511

### 512 *3.1.6 Numerical Methods*

513 In addition to the algorithms used for pressure-velocity coupling, interface capturing and  
514 turbulence modelling, the flow solver is also comprised of spatio-temporal discretization  
515 schemes as well as linear equation systems solvers. Both categories of algorithms directly  
516 impact the accuracy and stability of the flow solver.

517 Some of the popular discretization schemes that have been widely implemented for ocean  
518 engineering problems are now discussed in context to the momentum equation (1) and Table  
519 4. Discretization of the time-term can be carried out either using Linear Multi-step Methods  
520 (LMMs) or Runge-Kutta (RK) methods. These two categories of methods can be further  
521 classified into explicit and implicit schemes. Explicit LMMs are also known as the Adams-  
522 Bashforth Methods (ABMs) whilst implicit LMMs are known as Adams-Moulton Methods  
523 (AMMs). As the name suggests, LMMs build accuracy by storing the flow solution across  
524 multiple time-levels such that a first-order LMM would require an existing flow-field  
525 solution from one time-level, a second-order LMM would necessitate solutions from two  
526 time-levels and so on. Owing to the requirement of an existing flow-field solution, LMMs  $>$   
527  $\mathcal{O}(1)$  are not “self-starting” and some complexities exist in implementing these methods for  
528 variable time-steps. Moreover, the region of stability of LMMs shrinks with increasing order  
529 of accuracy (Drikakis and Rider, 2005). Nonetheless, a key advantage of LMMs is that the  
530 per-time-step computation effort does not increase with increasing order of accuracy; only the  
531 storage requirements increase.

532 On the other hand, RK methods divide a single time-step into a number of intermediate steps  
533 with all intermediate velocity fields made divergence-free; only the most recently known  
534 velocity field is necessary for a given intermediate step. Given this characteristic, RK  
535 methods are self-starting and automatically account for variable time-steps. However, the fact  
536 that the predictor-corrector loop (cf. Figure 3) is executed multiple times within a time-step  
537 introduces a unique set of merits and shortcomings. The chief merit is the numerical stability  
538 which, unlike LMMs, increases with increasing order of the method. Another merit over  
539 LMMs is that storage requirements do not increase with increasing order. The chief  
540 shortcoming associated with RK methods is that each intermediate step entails a  
541 computationally expensive solution of the elliptic PPE or EOPC; per-time-step computation  
542 effort thus increases with increasing order. Referring to Table 4, it is seen that a number of  
543 NSE algorithms implement explicit time-integration (ABM or Total Variation Diminishing-  
544 RK (TVD-RK)) which is suitable given the hyperbolic nature of wave propagation. In cases  
545 where a greater amount of numerical stability is desired, say conservative NSE formulations  
546 for violent WSI (Benoit *et al.*, 2023) authors opt for AMM rather than TVD-RK. This is  
547 probably because the additional linear equation systems encountered for AMM (one system  
548 for each component of  $\vec{V}$ ) is parabolic and less expensive to solve than the elliptic PPE/EOPC  
549 encountered multiple times within a time-step in the case of TVD-RK. It is also possible that  
550 very high-order AMMs might lead to dispersive (phase) errors in wave-propagation.

551 *Table 4. A summary of the various discretization methods and linear equation system solvers*  
 552 *implemented for NSE algorithms applied to ocean engineering problems reported in the*  
 553 *literature; cf. nomenclature for abbreviations. (X: data unavailable)*

Authors	Time	Momentum Advection		Pressure	Diffusion	PPE/EOPC
		Scheme	Treatment			
Sriram <i>et al.</i> (2014)	ABM1	Lagrangian	--	SFDI	SFDI	GMRES
Bihs <i>et al.</i> (2016)	TVD-RK3	WENO	Non-conservative	X	X	BiCGStab
Xie and Stoesser (2020)	AMM1	Second-order TVD	Conservative	CD2	CD2	ADI / BiCGStab
Agarwal <i>et al.</i> (2021b)	ABM1	Lagrangian	--	SFDI	SFDI	BiCGStab
Anghan <i>et al.</i> (2022)	ABM2	Blended FOU-FiOU	Non-conservative	CD2	CD4	GSSOR
Saincher and Sriram (2022b)	ABM1	Blended FOU-FiOU	Non-conservative	CD2	CD2	GSSOR
Benoit <i>et al.</i> (2023)	AMM1	Slope-limited SOU	Conservative	CD2 with third-order numerical smoothing		GMRES
Saincher and Sriram (2023)	ABM1	Blended FOU-FiOU	Conservative	CD2	CD2	GSSOR

554

555 In addition to time-integration, the numerical schemes chosen for momentum advection,  
 556 pressure and diffusion terms as well as the linear systems solver chosen for solving the  
 557 pressure field also play a key role in deciding the robustness and accuracy of NSE solvers. In  
 558 context to the discretization of the pressure and diffusion terms, second-order central  
 559 differencing (CD2) suffices for most scenarios and is thus the most widely used (cf. Table 4).  
 560 However, recent studies involving DNS of marine outfalls have instead implemented fourth-  
 561 order central differencing (CD4) for higher resolution treatment of the diffusion term (cf.  
 562 Anghan *et al.*, 2022). It should also be noted that CD4 treatment of the pressure gradient does  
 563 not dramatically improve the accuracy of a solver and should rather be avoided to save  
 564 computational effort (Tafti, 1996).

565 Numerical formulations of the NSE inherently contain some form of numerical diffusion. In  
 566 context to ocean engineering applications, this diffusion gets manifested as a gradual  
 567 reduction in wave-height (Saincher and Banerjee, 2017). Whilst the numerical diffusion can  
 568 be arrested through mesh refinement, a more computationally efficient way to do this  
 569 (especially for mesh-based Eulerian solvers) is by increasing the order of advection  
 570 discretization. However, computational efficiency does not translate to a straightforward  
 571 implementation, especially for multiphase solvers. Implementation of a high order advection  
 572 scheme in its “pure form” leads to severe dispersion errors in regions of sharp velocity  
 573 gradients which, in case of waves, prevail at the air-water interface; the consequence is  
 574 unphysical deformation of the generated waves. This can be corrected by either using  
 575 inherently bounded schemes such as WENO (Bihs *et al.*, 2016) or blended schemes where  
 576 (say) only 50% of the advected momentum is estimated using the high-order scheme, the rest  
 577 being estimated using FOU (Saincher and Sriram, 2022b). For more violent scenarios  
 578 involving wave-breaking and/or wave-slamming, a higher order treatment of advection may  
 579 not be sufficient and rather the correct amount of advection being attributed to each fluid-

580 phase needs to be ensured. This is where conservative NSE formulations come into picture  
581 wherein  $\rho^*$  is shifted to the left-hand-side of equation (1) with the time and advection terms.  
582 It has been recently demonstrated by the authors that conservative NSE solvers are necessary  
583 for correctly capturing the topology of waves overturning over a long distance; such as  
584 solitary waves breaking over a beach/shallow water (Saincher and Sriram, 2023). It is worth  
585 mentioning that conservative NSE formulations strongly and consistently couple mass and  
586 momentum transport (cf. the discussion on mass inconsistency in Saincher and Sriram  
587 (2023)) and thus momentum advection is more strongly governed by material transport  
588 (owing to the 1:800 density ratio between air and water) rather than the momentum  
589 advection scheme itself (Bussmann *et al.*, 2002). This makes conservative NSE a suitable  
590 alternative to high-order advection schemes for arresting wave-damping in non-  
591 violent/moderately violent WSI scenarios.

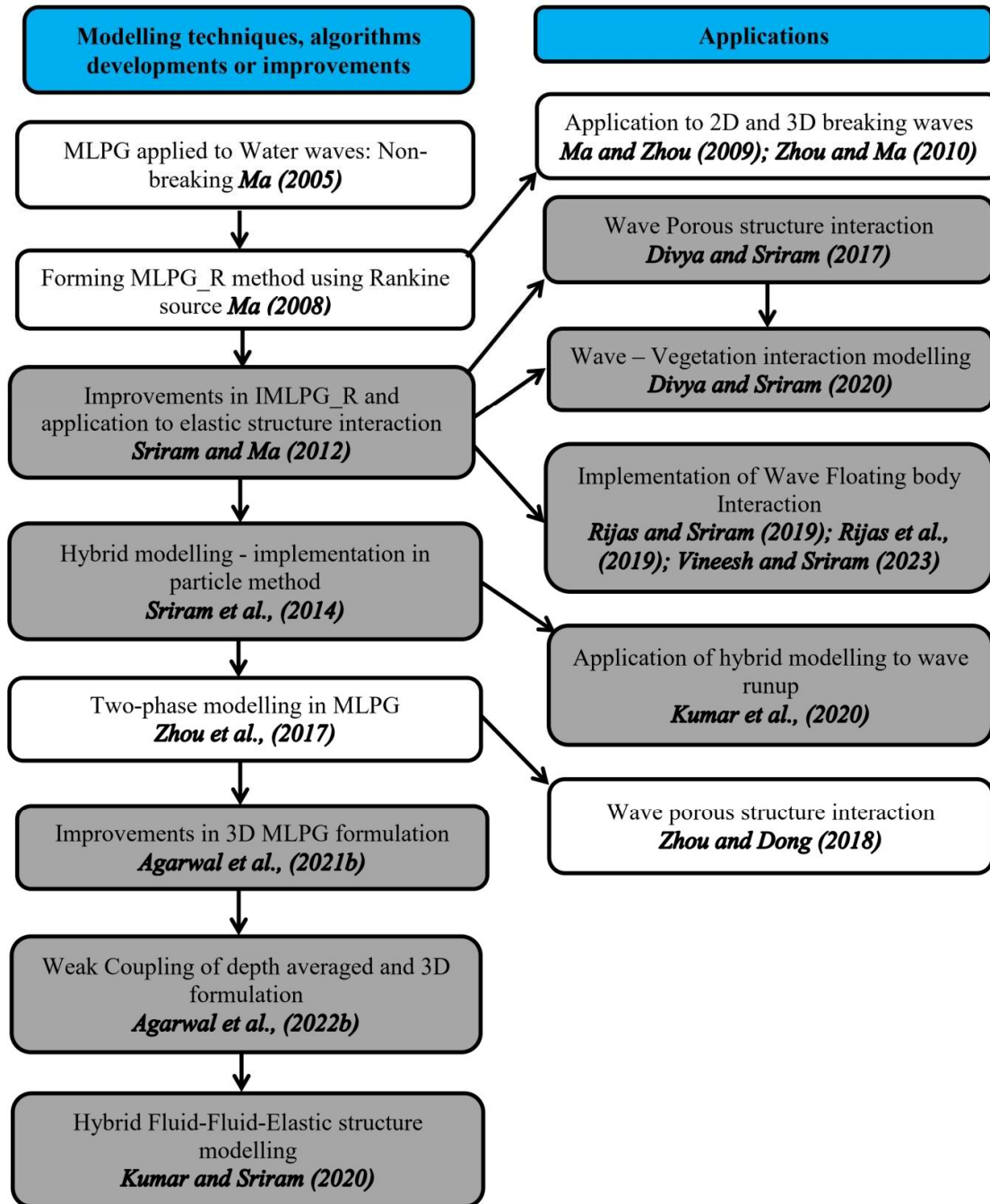
592 For rigid structures, one can incorporate this in the computational domain and solve the  
593 interaction problems as shown in Figure 5. For elastic and floating structures, a separate  
594 equation of motion will be solved to understand the fluid-structure interaction process, see,  
595 Sriram and Ma (2012), Rijas *et al.* (2019) and Vineesh and Sriram (2022). In the case of  
596 modelling porous/vegetation structure interactions with waves, one can adopt microscopic or  
597 macroscopic approaches. The macroscopic approach is commonly adopted due to the  
598 computational advantages as well as in terms of requirement for physical process (see, Divya  
599 and Sriram (2020)). For modelling the porous/vegetation structure interaction with waves,  
600 additional resistance terms such as the: (a) linear drag coefficient representing the laminar  
601 flow, (b) non-linear drag coefficient representing the turbulent flow, (c) coefficient for the  
602 transitional flow and (d) virtual mass coefficient for inertia terms were incorporated in the  
603 governing equations. The numerical studies on the wave porous structure can be carried out  
604 in two different ways:

605 (i) Coupling of pure fluid and porous flow equations, in which the fluid flow is solved using  
606 the NSE and porous flow with different porous flow model, following which the interface  
607 was coupled by matching the flow properties. Such coupling can be explicit, implicit or  
608 iterative in nature.

609 (ii) Based on unified or single governing equations to model both porous structure and fluid  
610 flow. In the microscopic approach, the aim is to capture detailed flow physics, directly  
611 resolved by the NSE (see Xie and Stoesser (2023)).

612 Apart from mesh based approach in solving the NS, mesh-free or particle methods are quite  
613 popular and further developments are actively being carried out. These developments have  
614 been the topic of many recent review papers such as Luo *et al.* (2021), Sriram and Ma (2021),  
615 Lind *et al.* (2020) and references therein. However, the acceptability of the mesh-free  
616 methods or particle methods for industry and practical applications in the projects are not  
617 matured compared to mesh-based methods. The consolidation of the work carried out by the  
618 authors with regards to the mesh-free method based on Meshless Local Petrov Galerkin  
619 Method (MLPG) has been reviewed in detail in Sriram and Ma (2021) and shall not be  
620 repeated here for the sake of brevity. Further, an important relation between the widely

621 popular Smoothed Particle Hydrodynamics (SPH), Moving Particle Semi-Implicit Method  
 622 (MPS) and MLPG was established. However, as this special issue concerns the Newton  
 623 fellowships, a flow chart of development has been reproduced for completeness as shown in  
 624 Figure 7.



625  
 626 *Figure 7. Summary of the history of the development of the Meshless Local Petrov Galerkin*  
 627 *method (MLPG) and its application in Ocean Engineering (revised and updated from Sriram*  
 628 *and Ma, 2021). Grey shaded boxes are development with partial or full support from the*  
 629 *Newton fellowship.*

630

### 631 **3.2 Potential Flow Theory**

632 The fully nonlinear potential flow theory (FNPT) has significantly matured in today's context  
633 and is being extensively used by both researchers as well as industry. The methodology was  
634 pioneered by Longuet-Higgins and Cokelet (1976) using a mixed Eulerian and Lagrangian  
635 approach. The simulation of nonlinear waves using FNPT can be carried out either by fully  
636 discretising the domain and then solving the Laplace equation using numerical approaches  
637 (like FEM, BEM and so on) or by obtaining the solution of the Laplace equations using  
638 spectral, Eigen function or Fourier methods. In the former case, the computational effort  
639 would be quite significant when one extends the method to 3D, however, the advantage is  
640 that one can simulate waves interacting with any arbitrarily complex structure. In the latter  
641 approach, the computational effort is lesser in comparison and such methods are largely  
642 employed for simulating the fully nonlinear waves. Dommermuth and Yue (1987) and West  
643 *et al.* (1987) proposed an attractive fast convergence, high accuracy and fast resolution  
644 properties-based higher order spectral (HOS) method. These fast methods of computation are  
645 very useful for calculating the long-time evolution of nonlinear waves and can be used as an  
646 input for the numerical models based on the NS equations. A detailed review of these models  
647 can be found in Kim *et al.* (1999) and in Ma (2008) and references therein. Normally, the  
648 FNPT-based models are quite effective in reproducing the extreme steep non-breaking waves,  
649 however, once the wave begins to overturn (the crest becomes vertical), the simulation  
650 crashes (Mohanlal, 2023). For some models, the crash may be delayed up to the point when  
651 the overturning crest hits the free-surface (Grilli *et al.*, 2001). Naturally, the conventional  
652 FNPT models cannot handle wave-trains in which multiple breaking events occur in  
653 succession (over a period of time). In order to overcome these effects and carry out the  
654 simulations of overturning waves for a longer duration, researchers employ empirical  
655 treatment such as eddy viscosity models to incorporate breaking effects (Tian *et al.* (2010),  
656 Barthelemy *et al.* (2018), Sieffert and Ducrozet (2018), Hasan *et al.* (2019)). Very recently,  
657 Mohanlal (2023) has developed a FNPT model to handle multiple depth-limited two-  
658 dimensional breaking events in irregular sea-states, steepness-limited breaking of two-  
659 dimensional focusing waves as well as depth-limited breaking of regular waves over three-  
660 dimensional bathymetry. In their model, incipient breaking is detected based on whether the  
661 ratio of the orbital velocity to the wave-celerity exceeds a given threshold. Following  
662 detection, the energy of the over-turning wave is dissipated through an absorbing/damping  
663 surface pressure term introduced into the dynamic free-surface boundary condition (Grilli and  
664 Horrillo, 1997). It is worth mentioning that the wave generation techniques discussed in  
665 Table 2 in context to the NS models can also been incorporated in the FNPT models, mostly  
666 using moving wall, relaxation zone and/or prescribing inlet wave characteristics.

667

### 668 **4 Depth-averaged Mathematical Models**

669 The depth-averaged models are governed by the Boussinesq Equations (BSNQE), the Green-  
670 Naghdi equations, the Korteweg-De Vries (KdV) equation or the Shallow-Water Equations  
671 (SWE). These models are based on an assumption that the horizontal velocities (in the  
672 shoreward and longshore directions) are uniform or varying over the water column. A  
673 summary of the depth-averaged mathematical models is provided in Table 5; it can be  
674 appreciated that the models are being actively developed since the 1960s. In some of the

675 models, the approach is based on the notion of a uniform horizontal velocity, which is a  
 676 characteristic of long-wave-induced orbital kinematics, the baseline models are limited to  
 677 shallow-water. It is evidenced from Table 5 that the research effort has been driven by the  
 678 need to expand the applicability of these models to deep-water ( $kd \geq \pi$ ). This was achieved  
 679 through various means such as:

- 680 • replacing the depth-averaged velocity by velocity defined at an arbitrary depth to act  
 681 as the velocity variable (Nwogu, 1993),
- 682 • improving the dispersion characteristics through modification of the governing  
 683 equations (Beji and Nadaoka, 1996) and
- 684 • piecewise integration of the momentum equations over multiple layers yielding  
 685 separate velocity profiles within each layer (Lynett and Liu, 2004a).

686 *Table 5. A summary of various depth-averaged mathematical models developed for wave-*  
 687 *propagation reported in the literature; cf. nomenclature for abbreviations.*

<b>Authors</b>	<b>Model</b>	<b>Numerics</b>	<b>Waves</b>	<b>Breaking</b>	<b>Multilayer</b>
Peregrine (1967)	Nonlinear BSNQE for varying depth	FDM	Solitary	No	No
Green and Naghdi (1976)	Wave propagation in variable depth (rotational)	Analytical	--	No	No
Madsen <i>et al.</i> (1991)	Linear dispersive BSNQE for deep water	FDM	Regular, Bichromatic ( $1.95 \leq kd \leq 2.72$ )	No	No
Nwogu (1993)	Nonlinear BSNQE with velocity at arbitrary depth	FDM	Regular, Irregular ( $0.44 \leq kd \leq 3.13$ )	No	No
Wei <i>et al.</i> (1995)	Fully nonlinear BSNQE for varying depth	FDM	Solitary, Undular bore	No	No
Beji and Nadaoka (1996)	Improved BSNQE for varying depth	FDM	Regular ( $0.47 \leq kd \leq 1.91$ )	No	No
Lynett <i>et al.</i> (2002)	Fully nonlinear BSNQE	FDM	Regular, Solitary ( $kd = 0.14$ )	Yes	No
Madsen <i>et al.</i> (2002)	Fully nonlinear BSNQE	FDM	Solitary, Regular ( $0.65 \leq kd \leq 2\pi$ )	No	No
Lynett and Liu (2004a)	Multi-layer BSNQE	Analytical	Regular ( $\pi \leq kd \leq 8\pi$ )	No	Yes
Lynett and Liu (2004b)	Multi-layer BSNQE	FDM	Regular, Solitary, Landslide ( $0.70 \leq kd \leq 9.00$ )	No	Yes
Sitanggang and Lynett (2005)	Fully nonlinear BSNQE	FDM	Gaussian hump, Solitary, Regular ( $kd = 1.27$ )	No	No
Shi <i>et al.</i> (2012)	Fully nonlinear BSNQE	FDM + FVM	Regular, Irregular, Solitary ( $0.36 \leq kd \leq 0.78$ )	Yes	No
Yang and Liu (2020)	Wave-current model for varying depth based on Euler equations	Galerkin, Subdomain methods	Regular, Focusing + sheared current ( $1.00 \leq kd \leq 22.0$ )	No	No
Agarwal <i>et al.</i> (2022a)	Fully nonlinear BSNQE	FEM	Regular, Solitary, Ship-generated ( $0.73 \leq kd \leq 1.92$ )	No	No

688 These models have been employed to address larger-scale spatial ( $\sim$ km) and temporal  
689 ( $\sim$ min) processes in ocean engineering such as ship-generated waves in bays and inland  
690 waterways (cf. Agarwal *et al.*, 2022a). The depth-averaged models are widely used in the  
691 industry for waves and current hydrodynamics with models capable of handling interactions  
692 between waves and sheared currents being recently proposed by Yang and Liu (2020). Whilst  
693 the BSNQE-based models are based on irrotational and inviscid assumptions, turbulence and  
694 wave breaking have also been treated using the empirical approaches (Lynett *et al.*, 2002 ;  
695 Shi *et al.*, 2012). The BSNQE-based models have also been used for coupling with NS  
696 equations-based models to minimize the computational time (cf. Agarwal *et al.*, 2022b and  
697 references therein). It is worth mentioning that the different approaches of wave-  
698 generation/absorption discussed in Table 2 in context to the NS models are also applicable to  
699 depth-averaged models except the moving wall approach. It is also worth noting that the  
700 present review only aims at providing a brief overview of the development of BSNQE-type  
701 models for the sake of completeness in context to multi-scale modelling and is by no means  
702 all-inclusive. The reader is referred to Brocchini (2013) for a detailed review.

703

## 704 **5 Regional and global-scale modelling in Ocean Sciences**

705 If one refers back to the spatio-temporal scale classification of physical processes and models  
706 in Figure 1, it is seen that the BSNQE-type depth-averaged models belong in the  $\sim$ 1 km/1 h  
707 category. Whilst this is considerably “large-scale” compared to the depth-resolved models  
708 ( $\sim$ 1 m/1 min), the depth-averaged models are also considerably “small-scale” to regional  
709 and global-scale models ( $\sim$ 10<sup>3</sup> km/1 ka) that are typically applied in ocean sciences. Both  
710 regional and global-scale simulations in ocean sciences have been traditionally and  
711 fundamentally based on the concept of multi-scale modelling. Having said that, popular  
712 ocean-science models are generally based on RANS-type momentum equations; multi-scale  
713 modelling is seldom achieved through the inclusion of potential theory or Laplace equations.  
714 This is primarily because such models belong to the class of Ocean General Circulation  
715 Models (OGCMs) wherein the assumptions of zero viscosity and vorticity need to be relaxed.  
716 A summarization of the state of the art in the development of regional and global-scale ocean  
717 models is provided in Table 6.

718

719 *Table 6. A summary of various regional and global ocean models developed for various*  
720 *ocean science applications reported in the literature; cf. nomenclature for abbreviations.*

Authors	Model	Mathematical framework	Scales		Application
			Spatial	Temporal	
Bryan (1968)	--	NSE in geodetic coordinates (Hydrostatic, Boussinesq approximation, turbulent viscosity)	Indian Ocean	30 years	Circulation of the World Ocean
Bleck and Boudra (1981)	--	Euler equations in hybrid coordinates (isopycnic + non- isopycnic) (thermodynamic coupling of density and pressure, Coriolis and wind- forcing)	2400 km × 1200 km	5 years	Formation and evolution of ocean gyres



Blumberg and Mellor (1987)	POM	RANS in $\sigma$ -coordinates (hydrostatic, Coriolis forcing, Boussinesq approximation, equation of state, MY turbulence closure)	65 km $\times$ 700 km	2.5 days	Coastal trapped waves, upwelling, Ekman transport
Chen <i>et al.</i> (2003)	FVCOM	3D RANS equations in $\sigma$ -coordinates (MY level 2.5 and Smagorinsky schemes for vertical and horizontal turbulence closures respectively)	Bohai sea (450 km $\times$ 500 km $\times$ 20 m)	10 days	Tidal amplitudes, residual currents and temperature variation
			Satilla river (40 km $\times$ 30 km $\times$ 4 m)	--	Tidal amplitudes, currents and residual currents
Shchepetkin and McWilliams (2005)	ROMS	Coupled barotropic (depth-averaged) and baroclinic (residual) momentum equations in hybrid $z - \sigma$ topography-following coordinates	--	--	--
Barron <i>et al.</i> (2006)	NCOM	RANS in surface-topography-following hybrid $\sigma - z$ coordinates (hydrostatic, Coriolis, Boussinesq, equation of state, MY turbulence closure, curvilinear surface mesh)	Global	~6 years	Sea-surface and depthward temperature variations
Chassignet <i>et al.</i> (2007)	HYCOM	Euler equations in hybrid coordinates (isopycnal + hydrostatic + $\sigma$ ) Open ocean to mixed-layer to coastal regions	Denmark Strait	--	Undersea overflow
			Gulf of Cadiz	--	Undersea overflow
			Gulf of Mexico	--	Sea surface height in the Gulf Stream
				--	Chlorophyll concentration in the Gulf Stream
			North Atlantic	--	Sea-surface temperature
				--	Salinity and temperature gradients
Barth <i>et al.</i> (2008)	ROMS + HYCOM	RANS equations in topography-following coordinates + Euler equation in hybrid coordinates (ROMS fully nested in HYCOM)	West Florida Shelf	~1 year	Effect of deep-ocean currents on shelf circulation
Haidvogel <i>et al.</i> (2008)	ROMS	RANS equations in topography-following coordinates (Hydrostatic, Boussinesq approximation, $k - \epsilon$ and $k - \omega$ turbulence closures, Ecological sub-routines, Rheology and Thermodynamics for sea-ice)	Hudson river estuary	50 days	Tidal dynamics, salt transport
			NENA continental shelf	3 years	Nitrogen cycling
			North Pacific Basin	6 years	Basin-scale climate modelling
			Barents Sea	12 years	Sea-ice distribution and dynamics
Bomminayuni <i>et al.</i> (2012)	FVCOM	3D RANS equations in $\sigma$ -coordinates (MY level 2.5 and Smagorinsky	Rose Dhu Island (40 km $\times$	32 days	Identification of hydrokinetic energy hotspots

		schemes for vertical and horizontal turbulence closures respectively)	40 km)		from tidal streams
Delandmeter <i>et al.</i> (2018)	SLIM 3D	3D Hydrostatic Boussinesq equations in ALE formulation (Equation of state to correlate density, temperature and salinity, Coriolis forcing, Smagorinsky and $k - \epsilon$ closures for horizontal and vertical turbulence respectively)	Lake Tanganyika (650 km × 50 km × 0.57 km)	~3 years	Dynamics of thermocline oscillations in a lake
Adcroft <i>et al.</i> (2019)	MOM6 + OM4	3D Hydrostatic Boussinesq equations in generalized orthogonal curvilinear coordinates with vertical Lagrangian remap (Equation of state to correlate density, temperature and salinity; Coriolis forcing; EPBL for planetary boundary layer; MLE for baroclinic eddies; parameterizations for shear instabilities, internal breaking waves, BBL, lateral friction and mesoscale eddies; SIS2.0 model for sea-ice and icebergs)	World Ocean	300 years	Ocean-surface climate in terms of sea-surface temperature
					Seasonal cycling of mixed layer depths (MLDs)
					Ocean ventilation
					Temperature and currents in the upper ocean
					Arctic and Antarctic sea-ice
Hanert <i>et al.</i> (2023)	SLIM 2D	Nonlinear SWE (Horizontal baroclinity, surface wind-stress, turbulent diffusion, bottom-drag)	Persian / Arabian Gulf (gulf-scale to coastal-structures-scale)	~1 month	Multi-scale regional circulation patterns

721

722 Referring to Table 6, the popular ocean models include POM, FVCOM, ROMS, HYCOM  
723 and MOM6. The horizontal variable arrangement in these models is based on various classes  
724 of the finite-difference Arakawa grids, namely:

- 725 • **A-grid** in which scalars (eg. temperature and salinity) and vectors (eg. velocity) are  
726 defined at the same point,
- 727 • **B-grid** in which scalars are staggered from the velocity by half a grid dimension,  
728 however both velocity components are defined at the same point and
- 729 • **C-grid** in which both velocity components as well as scalars are staggered by half a  
730 grid dimension from one another.

731 In the vertical direction, the following coordinate system(s) are implemented:

- 732 •  $z$ -coordinates which follow the vertical direction and are suitable for resolving free-  
733 surface flow features,
- 734 •  $\sigma$ -coordinates which follow the bottom topography/bathymetry and are suitable for  
735 resolving the bottom boundary layer and
- 736 • isopycnal or isopycnic coordinates which follow the density contours and are suitable  
737 for resolving tracer (temperature, salinity etc.) transport in the open ocean.

738 As evidenced from Table 6, most ocean models employ some combination of the above  
739 coordinates which accords the capability for multi-scale simulations. A hybridization of

740 coordinates is necessary because free-surface features, topographical features and density  
741 stratification all occur at different vertical scales and moreover the individual scales vary as  
742 one move from the open ocean to coastal regions (Chassignet *et al.*, 2007). Another important  
743 aspect which differentiates OGCMs from FNPT and BSNQE approached is the need for  
744 modelling the feedback from the subgrid-scales to the inertial-scales in terms of both  
745 momentum as well as scalar transport (turbulent diffusion). Referring to Table 6, this is  
746 achieved through various turbulence closure models. It is also interesting to note that,  
747 because the horizontal scale in such problems is significantly greater than the vertical scale  
748 (cf. select domain sizes in Table 6), the horizontal and vertical directions employ different  
749 turbulence closures. Typically, the Smagorinsky model (zero-equation turbulence model  
750 wherein the evaluation of  $\mu_t$  is conceptually similar to Prandtl's mixing length model (Rodi  
751 *et al.*, 2013)) can be applied in the horizontal direction whilst a more comprehensive two-  
752 equation model (the MY-2.5 model is a popular choice; cf. Mellor and Yamada (1982) and  
753 Chen *et al.*, (2003) for details) can be applied in the vertical direction. This unique numerical  
754 constitution makes ocean models ideally suited for multi-scale modelling across a wide range  
755 of applications which is evidenced from Table 6. It is worth mentioning that the present  
756 review only aims at providing a brief overview of the development of OGCMs for the sake of  
757 completeness in context to multi-scale modelling and is by no means comprehensive. The  
758 reader should refer to the literature listed in Table 6 for further details.

759

## 760 **6 Coupled models**

761 In the previous sections, we presented several different models that are available to treat the  
762 problem at hand. However, rather than different models, it would be ideal to have one  
763 particular model to handle a wide range of problems spanning various spatio-temporal scales.  
764 One way of achieving this, as discussed in context to ocean sciences in §5, is to employ  
765 hybrid coordinate systems. Another way of achieving this is coupling different modelling  
766 tools that are developed over the period of years leading to multi-scale modelling in ocean  
767 engineering. Such coupled models are discussed in the following subsections.

768

### 769 **6.1 Overview of coupling/decomposition strategies**

770 With regards to coupling models, there are two approaches; one is domain decomposition and  
771 the other functional decomposition. The domain decomposition (DD) strategy divides the  
772 computational domain into parts and applies different mathematical models in each part. This  
773 is ideally done to avoid computationally expensive (and energy dissipative) NS simulations in  
774 the entire domain. Thus, multi-scale modelling is achieved by gaining the ability to model  
775 larger computational domains than would normally be allowed for a pure NS model. The  
776 functional decomposition (FD) strategy was pioneered by Dommermuth (1993) who  
777 simulated the formation of striations and scars on a free-surface due to impingement by a pair  
778 of vortex tubes shed from the tips of a submerged delta wing. In the case of FD, rather than  
779 physically decomposing the domain, the instantaneous velocity and pressure fields are  
780 decomposed into irrotational and vortical components. Another key characteristic of FD is  
781 the prescription of a constraint that the normal component of the vortical velocity is zero at  
782 the free-surface. This yields a transport equation for the free-surface elevation  $\eta$  in terms  $\eta$   
783 and the velocity potential  $\phi$  which accords an FNPT-like framework for the solution of

784  $\eta(x, y, t)$ . Such a numerical treatment means that FD yields “natural and exact transition”  
785 from viscous vortical flow to inviscid vortical flow to potential flow (Dommermuth, 1993);  
786 this is not possible in a conventional primitive-variable formulation of the NSE.

787

## 788 **6.2 Navier-Stokes coupled with potential-flow models (Domain Decomposition)**

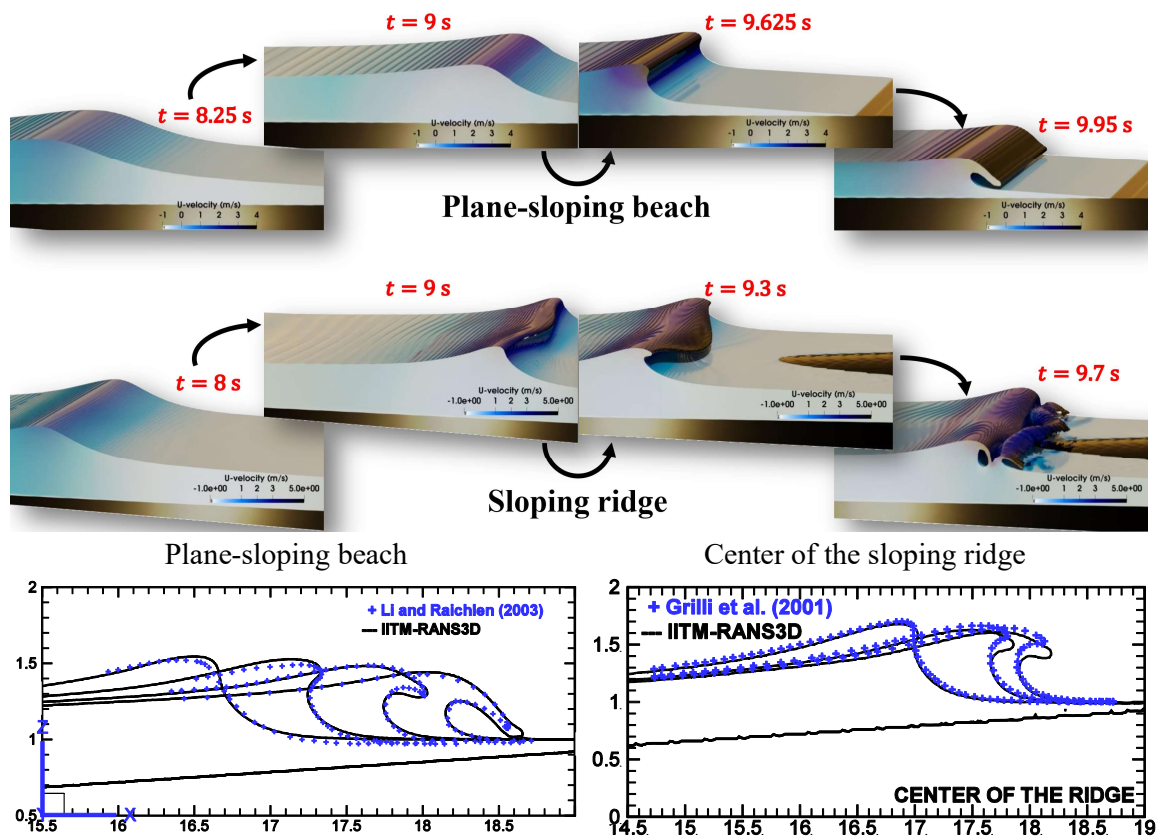
789 In most DD-based problems, the computational domain is decomposed into a viscous inner  
790 sub-domain and a potential outer sub-domain. The information (velocity, pressure and  
791 surface elevation) will be transferred through either relaxation zones or a sharp interface.  
792 Also, based on how the information between the solvers is being transferred, it can be either  
793 one-way coupling (weak coupling) or two-way coupling (strong coupling). In one way  
794 coupling, information is transferred only from the potential solver into the viscous solver, but  
795 in two-way coupling, the information is transferred in both ways, from depth-averaged or  
796 depth-resolving irrotational models to full NS and vice-versa. The two-way coupling is  
797 advantageous since it allows for a significantly smaller computational region for the viscous  
798 solver. However, it necessitates an iterative process or an implicit approach between the two  
799 models on a shared interface, which might increase the computational costs (Sriram *et al.*,  
800 2014). The advantage of one-way coupling is that no such iterations are needed, but it needs a  
801 longer viscous domain to avoid the reflection from outer boundaries. This method is suitable,  
802 wherein, one needs to analyse the kinematics of the breaking waves in deep water or depth-  
803 induced breaking in the shallow water region (Saincher and Sriram, 2023).

804 An early implementation of the concept of weakly-coupled hybrid modelling can be seen in  
805 the work of Fujima *et al.* (2002). They spatially nested a three-dimensional Navier-Stokes  
806 model within a two-dimensional nonlinear long-wave model to simulate tsunami-breakwater  
807 interaction at 1: 200 scale. An obvious shortcoming of the model was that spanwise vortices  
808 generated in the depth-resolving model that could not be transferred back to the depth-  
809 averaged model due to the latter’s reduced dimensionality. Grilli and co-workers (1999,  
810 2003, 2004) coupled the 2D HOBEM-FNPT with NS model based on SL-VOF. Extension of  
811 the FEM code with the NS model has also been carried out by Clauss and co-workers (2004,  
812 2005). For the NS model they have tested with the commercial softwares such as FLUENT,  
813 CFX and COMET. They tested their coupling approach by studying the deep water wave  
814 breaking (breaking of freak waves) and comparing with experimental measurements. Yan and  
815 Ma (2009) coupled the QALE-FEM with the commercial software STAR-CD to study the  
816 wind effects on breaking waves. Hildebrandt *et al.* (2013) coupled the FEM with the  
817 commercial software ANSYS to model the wave impacts with tripod structure.  
818 Narayanaswamy *et al.* (2010) and Kassiotis *et al.* (2011) used one way coupling of the  
819 Boussinesq model with the SPH method for solitary wave simulations. Without feedback  
820 from the SPH to the Boussinesq model, a fixed overlapping zone was considered to transfer  
821 the information. Recently, this was improved by Agarwal *et al.*, (2022b) by coupling a  
822 Boussinesq model with the MLPG (Meshless Local Petrov Galerkin) method.

823 However, if one needs to analyse the wave structure interactions in the presence of floating  
824 bodies or fixed structure, then strong coupling of the two models is required, wherein the  
825 radiated waves will propagate from NS model to depth-averaged or depth-resolved  
826 irrotational models. In strong coupling, the computational domain is divided into two parts, in  
827 one part the generation and propagation of waves is being considered and in the other part

828 structure/breaking region will be present. The modelling of first part of the domain will be  
 829 carried out using depth-averaged or depth-resolved irrotational models and then the boundary  
 830 conditions (velocity and pressure) are fed into the NS model at the same time steps, to study  
 831 the remaining part of the domain. Then the velocity from NS model is again feed back to the  
 832 depth-averaged or depth-resolved irrotational model domain for the next time step. Thus, in  
 833 general, the strong coupling needs to couple the models both in space and time domains. For  
 834 the coupling in space domain, the following four methods have been found to be employed,  
 835 as pointed out by Sriram *et al.* (2014): (a) fixed boundary interface, (b) moving boundary  
 836 interface, (c) fixed overlapping zone and (d) moving overlapping zone.

837

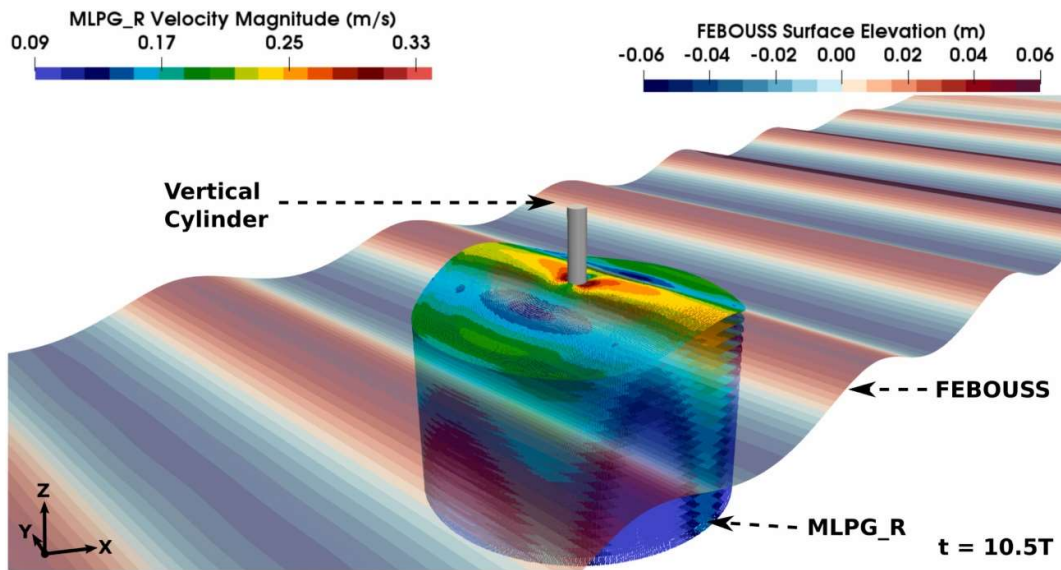


838 *Figure 8. Simulation of solitary wave-breaking over a 1: 15 plane-sloping beach and sloping*  
 839 *ridge using weakly coupled IITM-FNPT2D and IITM-RANS3D: (top) topology of the over-*  
 840 *turning wave visualized using iso-volumes of VOF and coloured using streamwise velocity,*  
 841 *(bottom) validation of the breaking topology against literature (Saincher and Sriram, 2023).*

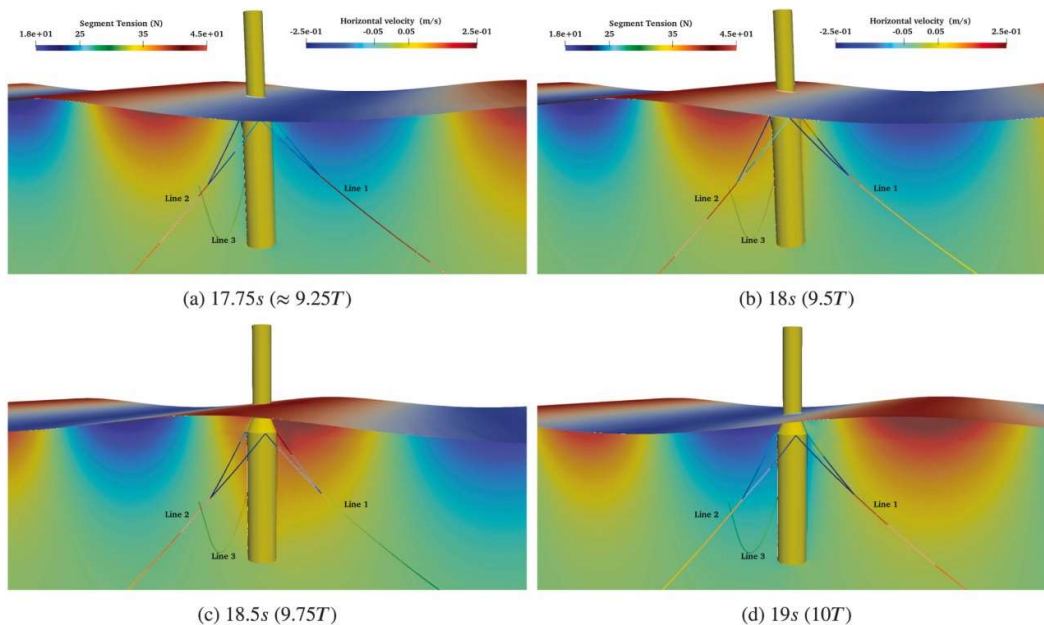
842

843 One of the pioneering works in this regard was carried out by Grilli and co-workers (2005,  
 844 2010) wherein, they extended the model from weak coupling to strong coupling for studying  
 845 the 3D breaking waves by coupling 3D HOBEM-VOF. Later, Grilli and co-workers (2007,  
 846 2008, 2009) coupled the NWT and NS based on Large Eddy Simulation (LES) to study the  
 847 forced sediment transport simulations. Later studies from Greco (2001), Colicchio *et al.*  
 848 (2006), Greco *et al.* (2007) and Sitanggang and Lynett (2010) further established the

849 feasibility of the DD strategy. Following these studies, a ground-breaking contribution was  
 850 made by Sriram *et al.*, (2014) wherein the full capability of coupled DD modelling was  
 851 explored in detail.  
 852



853  
 854 *Figure 9. Simulation of directional regular waves (aligned at 30° to the x-axis) interacting*  
 855 *with a fixed cylinder using weakly coupled FEBOUSS and MLPG\_R using 3D cylindrical*  
 856 *coupling interfaces (Agarwal et al., 2022b).*  
 857



858  
 859 *Figure 10. Simulation of regular waves interacting with a moored floating spar using a*  
 860 *coupled model employing HOS-NWT, foamStar and MoorDyn (Aliyar et al., 2022).*

861 However, until now mostly these strong coupling are realised only in the 2D problems, and,  
 862 to the best of the authors' knowledge, the strong coupling in 3D is yet to be attempted.  
 863 Typical examples of simulations performed based on one-way coupling using the codes

864 developed by the authors and their co-workers: IITM-RANS3D, HOS-NWT-foamstar (using  
865 depth-resolved potential and viscous models) and FEBOUSS-MLPG (using depth-averaged  
866 potential and viscous models) are reported in Figures 8-10.

867 One more popular hybrid model is qaleFOAM, which has been developed based on the  
868 experience from QALE-FEM. This model adopts the domain decomposition approach, which  
869 combines a two-phase Navier–Stokes (NS) model with a model based on the fully nonlinear  
870 potential theory (FNPT). In a region around the structures and/or the breaking waves (NS  
871 domain), the open-source NS solver OpenFOAM/interDyMFOam is applied. In the rest of the  
872 computational domain (FNPT domain), the FNPT-based quasi arbitrary Lagrangian–Eulerian  
873 finite element method (QALE-FEM) is adopted. The qaleFOAM was originally developed  
874 for modelling the turbulent flow near offshore structures subjected to extreme waves (Li *et*  
875 *al.*, 2018). It has now been extended and applied to model a wide range of wave-structure  
876 interaction problems, such as the wave resistance (e.g. Gong *et al.*, 2020), violent wave  
877 impact on sea walls (Li *et al.*, 2023), survivability and performance of floating wind turbines  
878 (Yu *et al.*, 2023; Yuan *et al.*, 2023) and wave energy converters (Yan *et al.*, 2020) as well as  
879 wave-driven drift of floating objects (Xiao *et al.*, 2024). Recently blind tests and numerical  
880 comparative studies have confirmed its superiority over single-model methods including the  
881 potential theory and the NS solvers. The details will be discussed below. However, one of the  
882 theoretical issues in these DD coupling is that the researchers coupled irrotational flow model  
883 with the rotational flow models. Particularly for strong coupling, there is a mathematical  
884 discontinuity in the velocity field and they overcome this with numerical approaches (Sriram  
885 and Ma, 2021). This needs to be overcome in the future modelling efforts, see Yang and Liu  
886 (2022) for the development of the multi-layer model based on rotational flow.

887

### 888 **6.3 Navier-Stokes coupled with potential-flow models (Functional Decomposition)**

889 As described earlier in §6.1, the fundamental concept for the functional decomposition (FD)  
890 is to use the Helmholtz decomposition to separate the velocity field into the rotational and  
891 irrotational parts to investigate the free surface flow (Dommermuth, 1993). The FD approach  
892 has also been adopted to simulate ocean engineering scenarios. In context to WSI, there are  
893 two categories under this decomposition, based on whether the structure is considered in the  
894 potential solver or not.

#### 895 *6.3.1 First category: structure handled by both potential and viscous solvers*

896 In the first category, the WSI problem is split into a potential component and a viscous part.  
897 The complete problem is initially solved by a potential solver, and then rectified by adding  
898 the viscous correction (Kim *et al.*, 2005; Edmund *et al.*, 2013; Rosemurgy *et al.*, 2016;  
899 Robaux and Benoit, 2021). One drawback of this strategy is that the potential solver must  
900 first solve the entire problem before applying the viscosity correction. As a result, challenges  
901 such as higher-order waves, stability issues in the steep waves and breaking induced by  
902 presence of structure with complex interactions are still constraints in this classification.  
903 Recently, Robaux (2020) published a thorough description of nonlinear waves' interactions  
904 with a horizontal cylinder with a rectangular cross section employing potential solver, CFD  
905 solver, and HPC-OpenFOAM coupled DD and FD based solvers. In comparison to the full  
906 CFD simulation, both coupling approaches, in particular the FD-based approach, need a

907 minimal amount of computational time while providing an accurate representation of the  
908 loads and associated hydrodynamic coefficients.

### 909 *6.3.2 Second category: structure only handled by the viscous solver*

910 In the second category, the total unknown is decomposed into the incident part and the  
911 complementary part. Only the incident flow is modelled in the incident part (wave only),  
912 leaving all the interaction with structure calculated by the viscous solver as the  
913 complementary part. The common name among researchers for this classification is  
914 SWENSE (Spectral Wave Explicit Navier Stokes Equations), proposed by (Ferrant *et al.*,  
915 2002) and actively developed by (Gentaz, 2004; Li *et al.*, 2018; Kim, 2021). The NS equation  
916 modified into the SWENSE is solved to yield the complementary fields. The advantage of  
917 this method is that the wave models directly provide incident wave solutions, minimising the  
918 problem's complexity and cost. For a detailed derivation of single-phase and two-phase  
919 SWENSE, refer to Luquet *et al.* (2007) and Li *et al.* (2018) respectively. The applications in  
920 single-phase SWENSE over the years can be read in (Luquet *et al.*, 2007; Monroy *et al.*,  
921 2010). Recently, the two-phase SWENSE method (Li *et al.*, 2021) has been implemented on  
922 top of *foamStar* and is called as *foamStarSWENSE*, and the only difference is that in this  
923 solver, the NS equations in *foamStar* are replaced by SWENSE. Recent developments of  
924 *foamStarSWENSE* such as efficient regular and irregular wave generation in the solver and  
925 higher-order forces estimation on a vertical cylinder, buoy and floating spar can be referred to  
926 in Choi (2019), Kim (2021), Li *et al.* (2018) as well as in Aliyar *et al.* (2022).

927

## 928 **6.4 Navier-Stokes coupled with geophysical fluid dynamics / ocean-science models**

929 Recently, there has been a research effort to implement the domain decomposition (DD)  
930 strategy to (strongly) couple geophysical fluid dynamics (GFD) / ocean-sciences models with  
931 Navier-Stokes solvers. A couple of such hybrid GFD-CFD models have been listed in Table  
932 7 wherein FVCOM (cf. Table 6 for details) has been coupled to either the overset mesh-based  
933 single-phase NS solver SIFOM or the unstructured mesh-based two-phase NS solver SIFUM.  
934 It is worth mentioning that in addition to the NS solver being based on overset meshes  
935 (SIFOM), the overset grids have also been employed to nest the SIFOM/SIFUM domain  
936 within the FVCOM domain. Referring to the domain sizes in Table 7, it should be noted that  
937 in some cases, the SIFUM domain is not necessarily entirely nested within the FVCOM  
938 domain along the vertical ( $z$ ) direction. This is attributable to the ability to model the air-  
939 phase within the SIFUM framework which is necessary for violent WSI scenarios.

940 Such hybrid GFD-CFD modelling provides the ability to perform multi-scale environmental,  
941 geological as well as FSI/WSI simulations over domains spanning several hundred or even  
942 several thousands of square kilometres. However, it is worth mentioning that some of the  
943 problems listed in Table 7 are comparatively “small-scale” and can indeed be tackled by  
944 more conventional FNPT-RANS models. For instance, Saincher and Sriram (2022b) have  
945 applied IITM-RANS3D to a  $0.045 \times 0.0022 \times 0.002 \text{ km}^3$  domain to simulate the  
946 interaction between focusing waves and a moving cylinder. In another study Saincher *et al.*  
947 (2021) had applied IITM-RANS3D to a  $0.3 \times 0.005 \times 0.007 \text{ km}^3$  domain to study the run-  
948 up characteristics of violently breaking long and high waves that could be generated by an



949 extreme coastal event. Thus, the lower-limit of applicability of the hybrid GFD-CFD models  
 950 can also be tackled by hybrid FNPT-RANS models.

951

952 *Table 7. A brief overview of geophysical fluid dynamics / ocean science models recently*  
 953 *hybridized with Navier-Stokes equations models reported in the literature; cf. nomenclature*  
 954 *for abbreviations. (✗: data unavailable)*

Authors	Models		Maximum domain extents for each model		Coupling strategy	Application
	Large-scale	Small-scale	Large (km <sup>3</sup> ) (x × y × z)	Small (km <sup>3</sup> ) (x × y × z)		
Tang <i>et al.</i> (2014)	FVCOM (weakly 3D)	SIFOM (fully 3D, single-phase)	1.8 × 0.6 × 0.1	0.4 × 0.4 × 0.1	DD (strong coupling)	Flow over flat plate
			3.5 × 0.4 × 0.15	~2.0 × 0.3 × 0.1		Transient sill flow
			3 × 0.3 × 0.009	~0.05 × 0.02 × 0.009		Bridge pier (lab-scale)
			3 × 0.6 × ✗	~0.8 × 0.4 × ✗		Thermal effluent
			~70 × 140 × 0.013	✗		Bridge pier (river-scale)
			~170 × 170 × 0.05	~0.7 × 0.7 × 0.05		Flow past seamount
Qu <i>et al.</i> (2016)	FVCOM (weakly 3D)	SIFOM (fully 3D, single-phase)	3 × 0.6 × 0.012	0.075 × 0.03 × 0.012	DD (strong coupling)	Thermal effluent
			~20 × 15 × 0.004	✗		Lagrangian tracking of estuary flows
			~250 × 1 × 0.01	✗		Storm surge impact on river bridge pier
Qu <i>et al.</i> (2019a)	FVCOM (weakly 3D)	SIFUM (fully 3D, two-phase)	14 × 0.1 × 0.2	~2 × 0.1 × 0.05	DD (strong coupling)	Tsunami wave runup Tsunami wave impacting coastal highway bridge
Qu <i>et al.</i> (2019b)	FVCOM (weakly 3D)	SIFUM (fully 3D, two-phase)	3.5 × 0.4 × 0.15	~1 × 0.3 × 0.225	DD (strong coupling)	Transient sill flow
			0.2 × 0.2 × 0.01	~0.1 × 0.12 × 0.01		3D dam-break flow
			0.04 × 0.001 × 0.0004	0.004 × 0.001 × 0.0005		Long-wave impingement on a vertical cylinder
			0.04 × 0.03 × 0.002	0.015 × 0.03 × 0.002		Hydraulic jump
			~60 × 60 × 0.025	~0.1 × 0.1 × ✗		Coastal flood impacting beachfront house

955

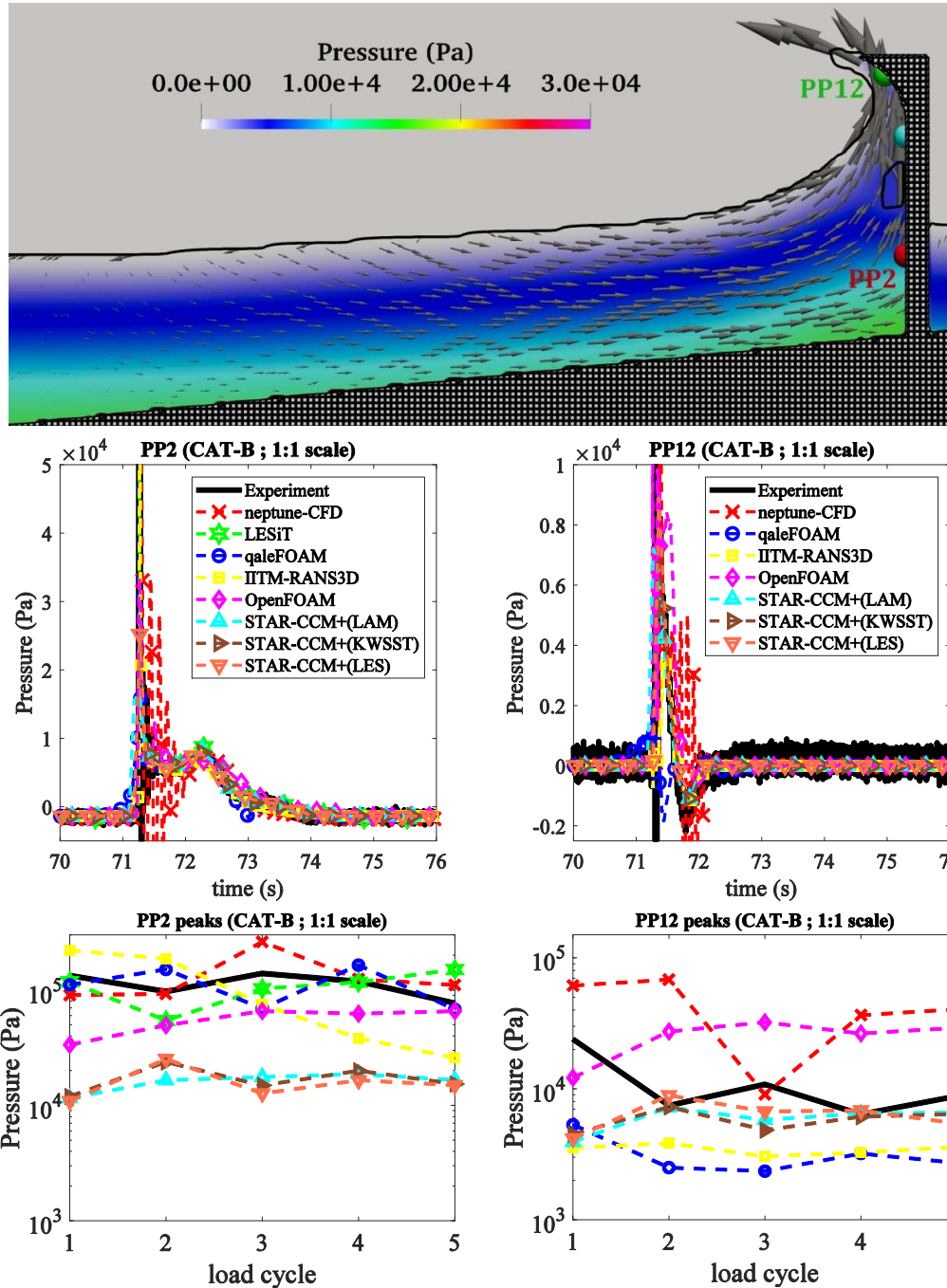
## 956 7 Benchmarking the Numerical Models through Comparative Studies

957 In the past researchers developed numerical models and validated with their own  
 958 experimental simulations, the data sharing and comparison between different numerical  
 959 models, its accuracy and performance in terms of computational efficiency are not attempted.  
 960 In the field of ocean engineering, when the concept of numerical wave tank was developed  
 961 inline with the numerical wind tunnels that are quite popular in those times, Clément (1999)  
 962 and Tanizawa and Clément (2000) carried out such exercise for fully nonlinear potential flow  
 963 theory. Recently, major initiatives were undertaken by Ransley *et al.* (2019, 2020), Sriram *et al.*  
 964 (2021), Agarwal *et al.* (2021a) and Saincher *et al.* (2023a). These studies highlighted  
 965 some of the commonly adopted guidelines by the researchers pertaining to WSI simulations:

- 966 • The fidelity of regular/focusing wave generation deteriorates away from the  
 967 wavemaker irrespective of the nature of the numerical model and no single wave-  
 968 generation method may be regarded as superior over others. Far from the wavemaker,

969 the models generally deviate by 5 – 10% in terms of primary energy content which  
970 is acceptable. However, the deviation across models may be as high as 50% in terms  
971 of the sub- and super-harmonic wave components.

- 972 • The performance of a solver should be judged based on the peak values of the  
973 surface-elevation/hydrodynamic pressures/loads as well as the phase agreement  
974 captured by the model. Phase disagreement is acceptable for WSI scenarios involving  
975 regular waves (Saincher *et al.*, 2023a) as long as the phase-shift remains constant  
976 over several wave cycles. However, for WSI scenarios involving transient waves  
977 such as focused and/or overturning waves, the phase agreement is critical as it  
978 determines the shape of the impacting wave as well as the time-varying load profile  
979 on the structure (Sriram *et al.*, 2021 ; Agrawal *et al.*, 2021a).
- 980 • The inclusion of turbulence modeling does not necessarily improve the accuracy of a  
981 simulation. This rather depends on the problem at hand. Further, for the same  
982 problem, different turbulence models may lead to the same/similar results. The  
983 expertise of the user should also be factored-in whilst using turbulence models,  
984 especially RANS-based models which are strongly empirical. These statements are  
985 substantiated through Figure 11 wherein results from the ISOPE-2022 comparative  
986 study on WSI are reported. In this study, one of the participating institutions had  
987 employed STAR-CCM+ for the simulations and had assessed the effect of different  
988 turbulence modeling strategies (implicit LES (“laminar”), RANS and explicit LES)  
989 on the solution. It can be observed from Figure 11 that changing the modeling  
990 strategy hardly affects the pressure time-history or the value of maximum impact  
991 pressure across multiple loading cycles. This could be interpreted from two  
992 perspectives: (a) the physics of the problem under consideration is independent of  
993 turbulence modeling or (b) the employed numerical framework is independent of  
994 turbulence modeling. The first statement is (obviously) incorrect. The second  
995 interpretation, however, holds merit from the standpoint of the relation between LES  
996 and RANS (Rodi *et al.*, 2013). It is worth mentioning that the STAR-CCM+  
997 simulations were two-dimensional and a rather coarse resolution of  $L/300$  and  $H/50$   
998 (where  $L$  is the wavelength and  $H$  is the wave-height) was chosen for the horizontal  
999 and vertical directions respectively. This corresponds to a mesh-size of  $\Delta x \sim 12$  cm  
1000 and  $\Delta z = 1.4$  cm respectively which is comparable to the  $\Delta x = \Delta z = 5$  cm chosen  
1001 for the RANS model IITM-RANS3D in the same study (Saincher *et al.*, 2023a). The  
1002 numerical framework exhibits “independence” from turbulence modeling because a  
1003 RANS resolution was applied to LES and ILES. Since the transport equations for the  
1004 mean-flow are the same between unsteady RANS and LES (Rodi *et al.*, 2013),  
1005 STAR-CCM+ apparently resolved the same mean-flow in all three cases. The minor  
1006 differences in peak impact pressure observed in Figure 11 stem from the unresolved  
1007 scales that are modeled differently across RANS, ILES and LES (Rodi *et al.*, 2013).



1008 *Figure 11. Results from the ISOPE-2022 comparative study on breaking waves*  
 1009 *impacting a seawall with a recurved parapet (Saincher et al., 2023a): (top) seaward*  
 1010 *deflection of the breaking wave, (center) time-history of the hydrodynamic pressure*  
 1011 *over the vertical wall (PP2) as well as the parapet (PP12) and (bottom) variation of*  
 1012 *peak impact pressure over five loading cycles.*

1013  
 1014 It is also worth noting that had the opposite been done wherein an LES grid was  
 1015 applied to RANS, dramatically different results would have been obtained. This is  
 1016 because RANS would have become grid-independent on the scale of the LES grid

1017 whilst LES itself would only become grid-independent at the Kolmogorov scale  
 1018 (Rodi *et al.*, 2013). Thus, one might argue that this instance constitutes a case where  
 1019 turbulence modeling was attempted but eventually proved to be unnecessary.

- 1020 • Hybrid modeling invariably improves the computational efficiency of the solver and  
 1021 should be adopted for large-scale WSI problems (Agarwal *et al.*, 2021a ; Saincher *et*  
 1022 *al.*, 2023a).
- 1023 • The state of the art in modelling large domain problems for transient waves appeared  
 1024 to be based on hybrid numerical modelling using weakly coupled algorithms (or one-  
 1025 way coupling); this strategy was adopted by most of the participants.
- 1026 • In simulating the same WSI problem at different scales, no general correlation could  
 1027 be obtained between computational effort and the scale of the problem. For instance,  
 1028 amongst the ten models compared for breaking waves interacting with a recurved  
 1029 seawall, the hybrid codes qaleFOAM and IITM-RANS3D were simultaneously the  
 1030 fastest and slowest at two different scales of the problem (Saincher *et al.*, 2023a).

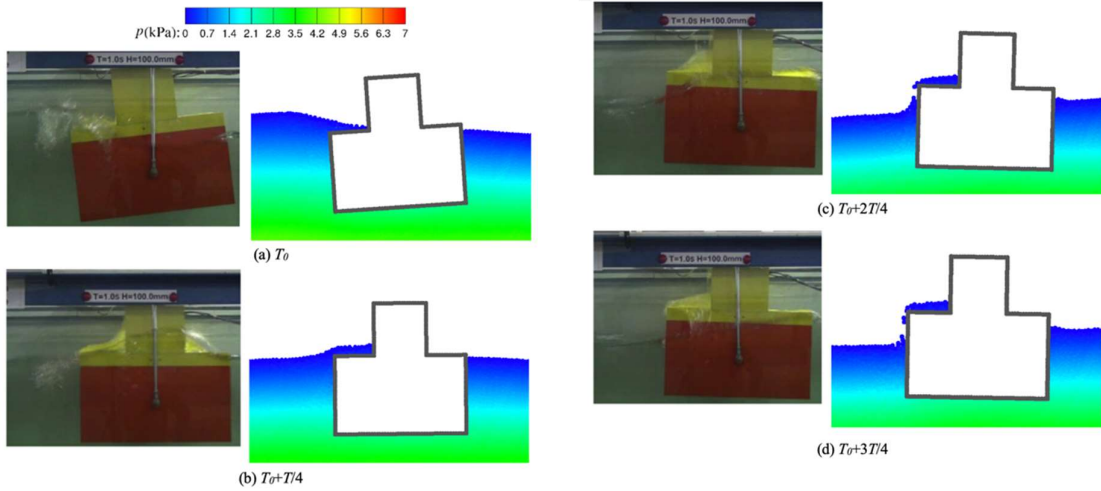
1031 In these studies it was also noted that the experimental error/uncertainty should be taken into  
 1032 consideration during validation. The inclusion of the experimental uncertainty would make  
 1033 the above guidelines less stringent. However, a conservative approach is beneficial in order to  
 1034 maintain a reduced error margin when adopting the said guidelines in practice.

1035

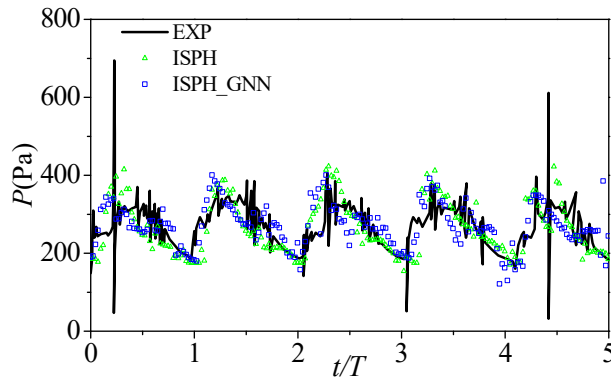
## 1036 **8 The Future**

### 1037 **8.1 Application of machine learning algorithms**

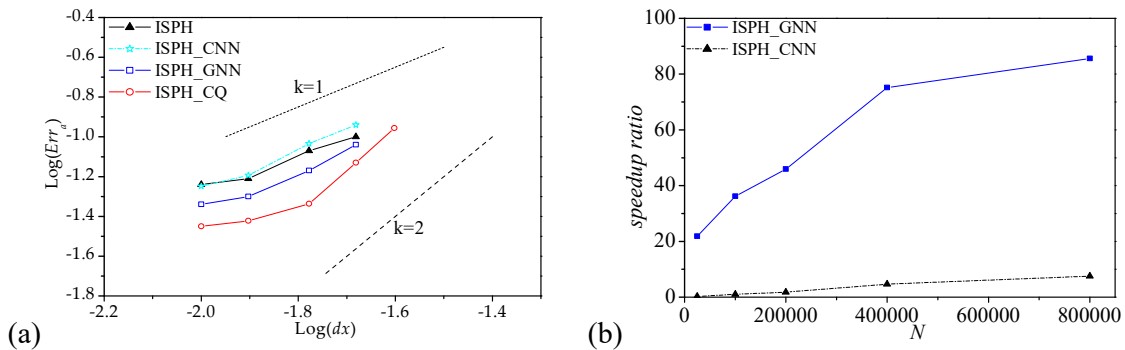
1038 The machine learning (ML) techniques is becoming popular in assisting the fluid simulation,  
 1039 e.g. to reconstruct the fluid field from data (Raissi *et al.*, 2020), to predict the turbulence  
 1040 related parameters (Ling *et al.*, 2016; Zhang *et al.*, 2015; Kutz, 2017), and to approximate  
 1041 time-independent flow field governed by NS models, such as the projection-based Pressure  
 1042 Poisson Equation (PPE, e.g. Yang *et al.*, 2016; Xiao *et al.*, 2018; Tompson *et al.*, 2017; Dong  
 1043 *et al.*, 2019; Ladicky *et al.*, 2015; Wessels *et al.*, 2020, Li *et al.*, 2022). Recently, both the  
 1044 convolution neural network (CNN, Zhang *et al.*, 2023) and graphic neural network (GNN,  
 1045 Zhang *et al.*, 2024a, 2024b) have been coupled with the incompressible smoothed particle  
 1046 hydrodynamics (ISPH) model to accelerate the numerical simulations. In these work, high-  
 1047 fidelity time-domain numerical results are produced using stand-alone ISPH simulation on  
 1048 wave propagation and impact on fixed structure. The CNN or GNN are used to train a  
 1049 machine learning algorithm to predict the pressure in the future step based on the numerical  
 1050 results at the current time step including the velocity, velocity divergence and pressure. After  
 1051 the algorithm is trained, it will be used to replace the PPE solver in the classic ISPH. Both the  
 1052 CNN-supported and GNN-supported ISPH models have been applied to modelling wave  
 1053 propagation, impact on seawall and interaction with other structures. Figure 12 and Figure 13  
 1054 illustrate some numerical results from the GNN-supported ISPH, which does not only show  
 1055 the capacity of the ML-supported ISPH but also demonstrate its promising accuracy. Further  
 1056 evidence on numerical accuracy and CPU speeding-up can be demonstrated in Figure 14 for  
 1057 the cases with solitary wave propagation. In this figure, the error is defined by the L2-norm of  
 1058 the time history of the wave crest; ISPH and ISPH-CQ adopt the linear and 2<sup>nd</sup>-order PPE  
 1059 solvers, respectively.



1060 *Figure 12. Comparisons of the floater movement progress during green water impact*  
 1061 *between laboratory photos (Zheng et al., 2016) (left) and ISPH\_GNN simulations (right) at*  
 1062 *different instants (duplicated from Zhang et al., 2024b).*  
 1063



1064  
 1065 *Figure 13. Time histories of the impact pressure on deck at P1 (duplicated from Zhang et al.,*  
 1066 *2024b).*  
 1067



1068 *Figure 14. Averaged errors of numerical results corresponding to different particle spacing*  
 1069 *in the solitary wave propagation (a) and the CPU speeding ratio (b) against solving PPE*  
 1070 *directly (solitary wave height = 0.28\*water depth ; duplicated from Zhang et al., 2024a).*

1071 As shown in Figure 14(a), both the convergence and accuracy of the ISPH-GNN are bounded  
1072 by the corresponding values of the ISPH and ISPH-CQ, implying a promising computational  
1073 accuracy. Figure 14(b) illustrates excellent CPU time speeding-up ratios against directly  
1074 solving the PPE using the 2<sup>nd</sup> order solver. For the solitary wave propagation using 80k  
1075 particles, the GNN can speed up the simulation by 80 times.

1076 The existing work related to AI and ML may be quantified as hybrid model combing a CFD  
1077 solver with the ML algorithms, e.g. Zhang *et al.* (2024a) combining ISPH with graph neural  
1078 network for simulating free surface flows. Data are needed to train the ML algorithms.  
1079 Recently, researchers started solving the fluid mechanics and fluid-structure interaction  
1080 problems using the AI library for discretising the required partial differential equation  
1081 (AI4PDE, see, e.g. Chen *et al.*, 2024). This work does not need to train the neural network  
1082 but directly modifying the filters of the neural network. Limited benchmarking rest has  
1083 demonstrated its promising computational accuracy and efficiency. The applications of the  
1084 AI/ML to existing hybrid model, such as the qaleFOAM, have yet found to the best of our  
1085 knowledge. Based on our preliminary work on CNN/GNN supported ISPH, its feasibility to  
1086 the hybrid modelling is confirmed.

1087 The challenges in the hybrid modelling can be fully or partially solved by the AI/ML  
1088 technologies. These include: (1) replacing the NS solver by the ML-supported version; (2)  
1089 intelligently decomposing the computational domain in an adaptive way, i.e. to minimising  
1090 the NS domain in the run-time depends on the development of the viscosity/turbulence effect  
1091 and breaking wave occurrence; (3) intelligently choosing the appropriate models, such as  
1092 RANS or LES; (4) in the function-decomposition approach, using the ML algorithms for  
1093 solving the compromised equations instead of solving them directly; (5) dynamic load  
1094 balancing in the cases with parallel computing.

1095

## 1096 **8.2 Hybrid FNPT-RANS-LES models for floating renewables**

1097 Another important aspect in the blue economy theme is the renewable energy. The  
1098 development of offshore wind farms based on Floating Offshore Wind Turbine (FOWT)  
1099 arrays is one of the popular, potential and realizable area. In order to reduce the CAPEX and  
1100 installation costs, shared mooring systems have been proposed for FOWT arrays where  
1101 anchors and a part of the mooring line are shared between turbines. This introduces  
1102 challenges that manifest differently in shallow and deep water. The deep-water mooring  
1103 system is susceptible to motions of the FOWT platform being amplified leading to large  
1104 displacements in the mooring line and peak anchor loads. Chain catenary moorings in  
1105 shallow water experience snap loads due to their susceptibility to violent wave-current-  
1106 structure interactions during extreme events and individual loads superimposing nonlinearly  
1107 with the structural response. In order to develop a comprehensive understanding of the  
1108 mechanisms leading to snap loads and peak anchor forces in shared mooring systems of a  
1109 FOWT farm, high fidelity multi-scale solver is required. To achieve this, the existing FNPT  
1110 (Fully Nonlinear Potential Theory), RANS (Reynolds Averaged Navier-Stokes) and Large  
1111 Eddy Simulation (LES) codes can be coupled via a zonal approach to yield a high-fidelity  
1112 multi-scale solver for wave-current-structure interaction. A FEM-based structural solver will

1113 be integrated to accurately predict the coupled fluid-structure interaction of several mooring  
1114 lines and to facilitate the modelling of elastic materials. A critical aspect of the model  
1115 development would be scaling-up the code for prototype-scale FOWT arrays whilst retaining  
1116 computational efficiency and accuracy. This could be achieved using AI and ML-based  
1117 prediction of turbulence-generation near the floating platforms, as this is expected to be the  
1118 most computationally intensive aspect of the modelling (traditionally handled using hybrid  
1119 RANS-LES). Thus, a continuous research efforts in the field of computational  
1120 hydrodynamics is required. This is in fact supported by the Newton Fellowship (recently  
1121 awarded to the second author from the authors research group in 2023) wherein the existing  
1122 understanding in hybrid modelling as well as AI/ML-based prediction of turbulence shall be  
1123 carried forward.

1124

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1132

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1751 **NOMENCLATURE**

1752 **Roman symbols**

$\vec{g}$	Gravitational acceleration vector
$H$	Wave-height
$k$	Turbulence kinetic energy
$kd$	Relative depth
$L$	Wavelength
$\mathcal{L}$	Characteristic length scale
$p$	Pressure in instantaneous NSE
$p'$	Modified pressure in time-averaged or filtered NSE
$Re$	Reynolds number
$U ; V ; W$	Components of $\vec{V}$ along the $x$ , $y$ and $z$ directions respectively
$U_{cyl}$	Speed of the moving cylinder
$\mathcal{V}$	Characteristic velocity scale
$\vec{V}$	Instantaneous or Reynolds averaged velocity vector
$t$	Time
$x ; y ; z$	Cartesian coordinate directions

1753

1754 **Greek symbols**

$\Delta x ; \Delta y ; \Delta z$	Cell-sizes along the $x, y$ and $z$ directions respectively
$\varepsilon$	Turbulence dissipation rate
$\eta$	Free-surface elevation
$\mu^*$	Mixture dynamic viscosity
$\mu_t$	Turbulent or eddy viscosity
$\nu$	Kinematic viscosity
$\rho^*$	Mixture density
$\sigma$	Topography-following vertical coordinate
$\phi$	Velocity potential
$\omega$	Specific dissipation rate

1755 **Superscripts**

$Q^*$	Predicted value of $Q$ in a projection method
$Q'$	Corrected value of $Q$ in a projection method
$Q^n$	Previous time-level value of $Q$
$Q^{n+1}$	Current time-level value of $Q$

1756 **Abbreviations**

<b>ABM</b>	Adams-Bashforth Method
<b>ADI</b>	Alternating Direction Implicit
<b>AMM</b>	Adams-Moulton Method
<b>BBL</b>	Bottom-Boundary Layer
<b>BEM</b>	Boundary Element Method
<b>BiCGStab</b>	Bi Conjugate Gradient Stabilized
<b>BSNQE</b>	Boussinesq Equations
<b>CD</b>	Central Difference
<b>CFD</b>	Computational Fluid Dynamics
<b>CICSAM</b>	Compressive Interface Capturing Scheme for Arbitrary Meshes
<b>CNN</b>	Convolution Neural Network
<b>DD</b>	Domain Decomposition
<b>DNS</b>	Direct Numerical Simulation
<b>EOC</b>	Equation Of Continuity
<b>EOPC</b>	Equation of Pressure Correction
<b>EPBL</b>	Energetically constrained Parameterization of the surface Boundary Layer
<b>FD</b>	Functional Decomposition
<b>FDM</b>	Finite Difference Method
<b>FEBOUSS</b>	Finite Element model for BOUSSinesq equations
<b>FEM</b>	Finite Element Method
<b>FFD</b>	Fast-Fictitious Domain
<b>FiOU</b>	Fifth Order Upwind
<b>FNPT</b>	Fully-Nonlinear Potential Theory
<b>FOU</b>	First Order Upwind
<b>FOWT</b>	Floating Offshore Wind Turbine
<b>FVCOM</b>	Finite Volume Coastal Ocean Model
<b>FVM</b>	Finite Volume Method
<b>GFD</b>	Geophysical Fluid Dynamics
<b>GMRES</b>	Generalized Minimal RESidual method

<b>GNN</b>	Graph Neural Network
<b>GSSOR</b>	Gauss-Seidel Successive Over-Relaxation
<b>HOS</b>	High-Order Spectral
<b>HYCOM</b>	HYbrid Coordinate Ocean Model
<b>IITM-RANS3D</b>	IIT Madras-RANS3D
<b>IMLPG R</b>	Improved MLPG with Rankine source function
<b>ILES</b>	Implicit LES
<b>ISPH</b>	Incompressible SPH
<b>KdV</b>	Korteweg-De Vries
<b>LBM</b>	Lattice-Boltzmann Method
<b>LES</b>	Large-Eddy Simulation
<b>LEAS-MOF</b>	Lagrangian-Eulerian Advection Scheme-MOF
<b>LMM</b>	Linear Multi-step Method
<b>MLD</b>	Mixed Layer Depth
<b>MLE</b>	Mixed Layer Eddies
<b>MLPG</b>	Meshless Local Petrov-Galerkin
<b>MOF</b>	Moment Of Fluid
<b>MOM</b>	Modular Ocean Model
<b>MPNDAF</b>	Mixed Particle Number Density and Auxiliary Function
<b>MPS</b>	Moving Particle Semi-implicit
<b>MSTACS</b>	Modified Switching Technique for Advection and Capturing of Surfaces
<b>NCOM</b>	Navy Coastal Ocean Model
<b>NITA</b>	Non-Iterative Time Advancement
<b>NFA</b>	Numerical Flow Analysis
<b>NS</b>	Navier-Stokes
<b>NSE</b>	NS Equations
<b>NWT</b>	Numerical Wave Tank
<b>OGCM</b>	Ocean General Circulation Model
<b>OS-CICSAM</b>	Operator-Split CICSAM
<b>PIMPLE</b>	PISO-SIMPLE
<b>PISO</b>	Pressure-Implicit with Splitting of Operators
<b>PLIC</b>	Piecewise Linear Interface Calculation
<b>POM</b>	Princeton Ocean Model
<b>PPE</b>	Pressure Poisson Equation
<b>QALE-FEM</b>	Quasi-Arbitrary Lagrangian–Eulerian Finite Element Method
<b>RANS</b>	Reynolds-Averaged Navier Stokes
<b>RANSE</b>	RANS Equations
<b>RK</b>	Runge-Kutta
<b>RMOF</b>	Refined MOF
<b>ROMS</b>	Regional Oceanic Modeling System
<b>RSM</b>	Reynolds Stress Model
<b>SAISH</b>	Smoothly Adapting Interfacial Scheme based on Hybridization
<b>SFDI</b>	Simplified Finite Difference Interpolation
<b>SGS</b>	Sub-Grid-Scale
<b>SIFOM</b>	Solver for Incompressible Flow on Overset Meshes
<b>SIFUM</b>	Solver for Incompressible Flow on Unstructured Mesh
<b>SIMPLE</b>	Semi-Implicit Method for Pressure Linked Equations
<b>SIS</b>	Sea-Ice Simulator

<b>SLIM</b>	Second-generation Louvain-la-Neuve Ice-ocean Model
<b>SOU</b>	Second Order Upwind
<b>SPH</b>	Smoothed Particle Hydrodynamics
<b>SWE</b>	Shallow-Water Equations
<b>SWENSE</b>	Spectral Wave Explicit Navier Stokes Equations
<b>TVD-RK</b>	Total Variation Diminishing-RK
<b>VOF</b>	Volume Of Fluid
<b>WALE</b>	Wall-Adapting Local Eddy-viscosity
<b>WENO</b>	Weighted Essentially Non-Oscillatory
<b>WSI</b>	Wave-Structure Interaction

1757