

Downward water mobility in applied smoldering

Jiahao Wang^{a*}, Marco A. B. Zannoni^a, Tarek L. Rashwan^{b, c}, José L. Torero^c,
Jason I. Gerhard^{a, †}

^a*Department of Civil and Environmental Engineering, The University of Western Ontario, London, Ontario, N6A 5B9, Canada*

^b*School of Engineering & Innovation, The Open University, Milton Keynes, MK7 6AA, UK*

^c*Department of Civil, Environmental and Geomatic Engineering, University College London, London, WC1E 6BT, UK*

Abstract

Applied forward smoldering is used in energy-efficient combustion systems to treat high moisture content waste. However, these systems must be operated in a robust manner far from quenching conditions. Quenching can lead to process failure; therefore, it is critical to accurately resolve water transport in different phases throughout space and time to optimize these smoldering systems. In this work, liquid mobility was integrated into a validated smoldering model that previously only included immobile water. The model was applied to a vertical reactor with an upward propagating forward smoldering reaction. Comparisons between mobility and non-mobility models indicate that the water mobility must be considered to accurately simulate both smoldering ignition and propagation in systems with high water saturations. Water mobility during smoldering can lead to two opposing effects: i) water accumulation at the bottom of the pack, which results in large ignition times or ignition failure; and ii) downward displacement of re-condensed water ahead of the smoldering front, which results in robust propagation with high peak temperatures and front velocities. Finally, a sensitivity analysis revealed the influence of operational parameters on water mobility, which can control the fate of smoldering. Tall fuel pack, high permeability, well-sorted particles, and low capillary-bound saturation were found to significantly accelerate water downward mobility and inhibit ignition. Nevertheless, extending the duration of the initial convective heating process and reducing the packing height favored ignition. These results are highly relevant to current industry applications and address a key knowledge gap in smoldering research.

Keywords: Liquid mobility; Smoldering; Energy balance; Ignition; Extinction

*Corresponding author.

† Deceased

Information for Colloquium Chairs and Cochairs, Editors, and Reviewers

1) Novelty and Significance Statement

Smoldering is proven to be an effective approach towards waste to energy. Key to the successful use of smoldering as a waste destruction technique is the stability of the reaction front. Excess water in high moisture content wastes is a major energy sink that can lead to reaction quenching and process failure. Therefore, water dynamics need to be well-understood to design efficient smoldering systems. The thermal effects of water on smoldering have been studied previously; nevertheless, the fate of smoldering has been demonstrated to be directly related to processes that evolve in space and time – which are poorly understood. Quenching is very sensitive to heat and mass transfer occurring in the porous media far from the reaction. Thus, water mobility is key. This paper is the first effort to address water mobility in the context of smoldering combustion. The conclusions presented here are widely applicable to all smoldering processes including other liquid compounds, e.g., crude oil sludge waste.

2) Author Contributions

- J. Wang: conceptualization, funding acquisition, methodology, formal analysis, writing – original draft.
- M. A. B. Zanoni: conceptualization, supervision, methodology, writing – review & editing.
- T. L. Rashwan: funding acquisition, supervision, methodology, writing – review & editing.
- J. L. Torero: supervision, methodology, writing – review & editing.
- J. I. Gerhard: funding acquisition, resources.

3) Authors' Preference and Justification for Mode of Presentation at the Symposium

The authors prefer **OPP** presentation at the Symposium for the following reasons:

- Focusing on managing wet wastes, where waste to energy is well-aligned with the theme of energy transition.
- Highlighting the impact of water mobility on smoldering is best presented through visual animations in an oral format.
- The impact of spatial heat exchange on smoldering is poorly understood; thus, is a subject of strong academic discussion.

Nomenclature

Latin Letters

$A_{s,sp}$	Surface area of sphere, m^2
A_{cs}	Cross-section area, m^2
D_g	Gas diffusion coefficient, $m^2 s^{-1}$
$\dot{E}$	Energy rate, $J s^{-1}$
h_{sg}	Interfacial heat transfer coefficient, $W m^{-2} K^{-1}$
k	Thermal conductivity, $W m^{-1} K^{-1}$
k_p	Intrinsic permeability, m^2
$k_{p,r}$	relative permeability, m^2
$\dot{m}_w'''$	Evaporation rate, $kg m^{-3} s^{-1}$
P	Pressure, Pa
P_c	Capillary pressure, Pa
R_{GAC}	Reaction rate, s^{-1}
S_b	Capillary-bound saturation, -
S_r	Residual saturation, -
S_w	Water saturation, -
T	Temperature, K
u	Velocity, $m s^{-1}$
U	Global heat loss coefficient, $W m^{-2} K^{-1}$
ν_{O_2}	Stoichiometric coefficient, $kg.O_2 kg.fuel^{-1}$

Greek Symbols

ρ	Density, $kg m^{-3}$
ϕ	Porosity, -
μ	Dynamic viscosity, Pa.s
δ, γ	Empirical constant, -
λ	Pore size distribution index, -

Subscripts/Superscript

Cl	Cylinder
0	Ambient/Initial
w, v, g	Water, vapor, gas

1

2 1. Introduction

3 Smoldering is a flameless form of combustion
4 driven by the oxidation of a condensed phase fuel [1].
5 Accidental smoldering fires can cause catastrophic
6 losses (e.g., wildfires), while applied smoldering is
7 emerging for soil remediation and waste management
8 purposes. To date, numerous laboratory and pilot tests
9 have demonstrated that high moisture content wastes
10 (HMWs) can be treated via smoldering in a cost-
11 effective manner, such as sewage sludge [2-4], feces
12 [5] and food wastes [6]. Most of these wastes contain
13 moisture content above 50 wt% and exhibit low
14 heating values (~ 20 kJ/g), posing a challenge to
15 conventional thermal methods and energy recovery.
16 Applied smoldering of HMWs is designed to
17 operate as a “self-sustained” process, which requires
18 two distinct stages [7]. The first stage is pre-heating,
19 where heat is provided to a small, localized area
20 (referred to as the ignition zone). The external heater
21 causes a rise in local temperature until the wet wastes
22 dry and then reach the ignition point [8]. The second
23 stage introduces airflow to ignite smoldering. During
24 this phase, the smoldering front propagates in a self-
25 sustained manner without additional external energy
26 input, provided the local energy generation rate

27 overwhelms local heat losses. Local heat losses result
28 from the energy balance in and out of the reaction
29 zone from other areas of the system. Thus, mobility of
30 any liquid compound (e.g., water) affects heat losses.

31 The self-sustained nature of smoldering makes it an
32 energy-efficient technology for processing HMWs.
33 However, in both stages, it is crucial that the energy
34 provided by the heater or generated from the oxidation
35 process is sufficient to evaporate water and sustain
36 smoldering, i.e., achieve a positive energy balance [9].
37 Under this condition, a dry zone appears ahead of the
38 smoldering propagation (i.e., buffer zone) so the
39 combustion reaction only encounters dry fuel [8].

40 Nevertheless, the water saturation (S_w) distribution
41 is non-uniform in smoldering systems, which evolves
42 over time and space within the heated porous medium
43 due to phase change [5, 8, 10] and gravitational
44 drainage [11]. The former aspect has been thoroughly
45 studied. Yermán et al. [5] and Wang et al. [8] showed
46 that water re-condensation can increase the local S_w
47 ahead of smoldering significantly beyond the initial
48 condition (e.g., up to $5\times$) and cause extinction. This
49 water accumulation can slow down drying processes,
50 and thereby suppress the dry buffer between
51 smoldering and wet regions. A minimum dry buffer
52 thickness is required to achieve stable smoldering [8].

53 In addition, water can move downwards due to
54 gravity and accumulate near the ignition zone [2]. The
55 unintended water accumulation might require a
56 prolonged heating time for complete local
57 evaporation, which would require additional energy
58 supply and reduce economic efficiency. However,
59 water mobility in porous media is challenging to
60 predict, where air and water form an interlinked two-
61 phase flow governed by highly non-linear equations
62 [12]. The mobility of each phase relies on the
63 interdependence of capillary pressures and relative
64 permeabilities in the pore space, which is a function
65 of system properties like initial water saturation
66 ($S_{w,0}$), intrinsic permeability (k_p), packing height (H),
67 and capillary-bound saturation (S_b). Note that S_b
68 defines the threshold where capillary forces resist
69 mobility, and water cannot be displaced without a
70 phase change. Therefore, mobility can be neglected
71 when $S_w < S_b$ [12]. While the effect of two-phase
72 flow on smoldering systems has been observed
73 experimentally [2-6], it is not yet well-understood.
74 Thus, a modelling approach is necessary to
75 comprehensively understand the impact of water
76 mobility on smoldering systems, which can ultimately
77 govern treatment success.

78 This study aims to extend the utility of a previously
79 validated one-dimensional wet smoldering model
80 from Wang et al. [8] by adding water mobility
81 physics. A series of experiments were performed to
82 explore water mobility and validate model results. A
83 sensitivity analysis was also conducted to investigate
84 the role of key system parameters on water dynamics
85 and smoldering performance. Altogether, this updated
86 model provides novel insights that are critical in
87 optimizing HMW smoldering systems.

1 Table 1

2 Summary of all model and experimental cases.

Case (-)	Simulated water mobility?	$S_{w,0}$ (%)	C_f^a (%)	$u_{g,in}$ (cm/s)	H (cm)	S_b (%)	λ	k_p ($\times 10^{-10} \text{ m}^2$)	t_g, t_h (s)	SS? ^b
1	-	20	3	5	73	14	2.5	5	6000, 6480	SS
2	-	30	3	5	73	14	2.5	5	6120, 6720	SS
3	Validation	20	3	5	73	14	2.5	5	6000, 6480	SS
4	Validation	30	3	5	73	14	2.5	5	6120, 6720	SS
5	✓	30	3	5	36.5	14	2.5	5	6120, 6720	SS
6	✓	30	3	5	80	14	2.5	5	6120, 6720	SS
7	✓	30	3	5	73	7	2.5	5	6120, 6720	NSS ^c
8	✓	30	3	5	73	28	2.5	5	6120, 6720	SS
9	✓	30	3	5	73	14	1.25	5	6120, 6720	SS
10	✓	30	3	5	73	14	5	5	6120, 6720	SS
11	✓	30	3	5	73	14	2.5	2.5	6120, 6720	SS
12	✓	30	3	5	73	14	2.5	10	6120, 6720	SS
13	✓	30	3	5	73	7	2.5	5	7400, 8000	NSS
14	✓	30	3	5	73	7	2.5	5	6120, 8000	SS
15	✓	30	3	5	36.5	7	2.5	5	6120, 6720	SS

3 ^a Fuel concentration [$\text{kg.O}_2 \text{ kg.fuel}^{-1}$]; ^b Self-sustained?; ^c Non-self-sustained.5 **2. Methodology**

7 The experimental setup in Fig. 1 used water, sand,
8 and granular activated carbon (GAC). Material
9 properties and established experimental procedures
10 are detailed in [7-8]. A radiative heater was used for
11 ignition. When the ignition point was reached, air was
12 injected at t_g to support upward smoldering. Once
13 self-sustaining smoldering was established, the heater
14 was turned off at t_h . The experiments included
15 continuous measurements of axial temperatures via 25
16 thermocouples (TCs) at 3 cm intervals, which were
17 used for the model validation [8]. Refer to
18 Supplementary Material (SM) Section S1 for more
19 details about the experimental setup.

20 Two wet smoldering conditions were simulated in
21 COMSOL (see Table 1). Cases #1-2 simulated wet
22 smoldering without mobility. Cases #3-4 considered
23 water mobility and were then validated against
24 experiments. Case #4 served as the base case for the
25 numerical sensitivity analysis, where Cases #5-15
26 were conducted to study the impacts of packing height
27 (H), capillary-bound saturation (S_b), pore size
28 distribution index (λ), and permeability (k_p). The
29 model based on Wang et al. [8] addressed GAC
30 smoldering coupled with water phase change
31 processes, while the water mobility addition followed
32 key methodologies from Gerhard and Kueper [12].

34 **2.1 Governing equations**

35
36 Eqs. 1-7 in Table 2 represent the conservation
37 equations for the solid, water, and gas phases. All
38 symbols are defined in the Nomenclature. The
39 governing equations are briefly discussed here, and

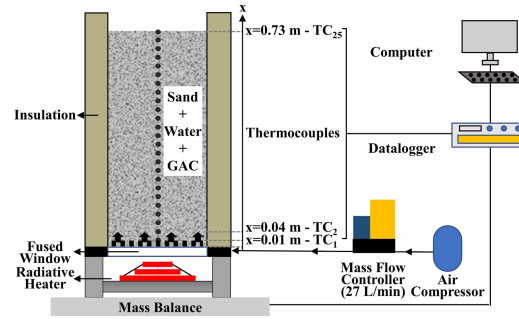


Fig. 1: Schematic of the experimental setup.

40 further details on the model input parameters and
41 boundary conditions can be found in SM Sections S2-
42 4 and [8]. Eqs. 1-4 account for the conservation of
43 mass for GAC, water, vapor, oxygen, and total gas
44 mixture. GAC is a by-product of coal pyrolysis, and
45 its smoldering reaction can be modelled with a 1-step
46 oxidation reaction in Eq. 8 [13]. The water velocity,
47 u_w , was governed by gravity and capillary pressure,
48 while water evaporation was simulated by the water
49 evaporation rate ($\dot{m}_w'''$). Eqs. 6-7 solved local thermal
50 non-equilibrium transport between solid and
51 gas phases; however, the solid and liquid phases were
52 assumed to be in local thermal equilibrium. This
53 assumption is reasonable since water flowed slowly
54 compared to gas ($u_w \ll u_g$) [8, 12]. In this work,
55 when the local S_w was beyond 60%, γ was set to 0.2
56 in the entire domain to consider the effects of fast
57 water phase change; otherwise, γ followed Eq. 10.
58 More details on this methodology can be found in [8].

60 **2.2 Two-phase flow**

1 Table 2

2 Governing equations.

Equations	No.
$\frac{\partial(Y_{GAC})}{\partial t} = -R_{GAC}$	Eq. 1
$\phi \rho_w \frac{\partial(S_w)}{\partial t} = -\rho_w \frac{\partial(u_w)}{\partial x} - \dot{m}_w'''$	Eq. 2
$\phi \frac{\partial(S_g \rho_g Y_v)}{\partial t} + \frac{\partial(\rho_g u_g Y_v)}{\partial x} = \phi \frac{\partial}{\partial x} \left(S_g \rho_g D_{g,wv} \frac{\partial Y_v}{\partial x} \right) + \dot{m}_w'''$	Eq. 3
$\phi \frac{\partial(\rho_g S_g Y_{O_2})}{\partial t} + \frac{\partial(\rho_g u_g Y_{O_2})}{\partial x} = \phi \frac{\partial}{\partial x} \left(\rho_g S_g Y_{O_2} D_{g,O_2} \frac{\partial Y_{O_2}}{\partial x} \right) - \rho_{GAC,b} v_{O_2} R_{GAC}$	Eq. 4
$\phi \frac{\partial(S_g \rho_g)}{\partial t} + \frac{\partial(\rho_g u_g)}{\partial x} = \rho_{GAC,b} R_{GAC} + \dot{m}_w'''$	Eq. 5
$(\rho C_p)_{eff} \frac{\partial T_s}{\partial t} + \rho_w C_{pw} u_w \frac{\partial T_s}{\partial x} = \frac{\partial}{\partial x} \left(k_{eff} \frac{\partial T_s}{\partial x} \right) - U \left(\frac{A_{s,cl}}{V_{cl}} \right) (T_s - T_0) + h_{sg} \left(\frac{A_{s,sp}}{V_{sp}} \right) (T_g - T_s) - \rho_{GAC,b} \Delta H_{GAC} R_{GAC} - \Delta H_{evap} \dot{m}_w'''$	Eq. 6
$\phi S_g (\rho_g C_{pg}) \frac{\partial T_g}{\partial t} + \rho_g C_{pg} u_g \frac{\partial T_g}{\partial x} = \phi S_g \frac{\partial}{\partial x} \left(k_g \frac{\partial T_g}{\partial x} \right) + h_{sg} \left(\frac{A_{s,sp}}{V_{sp}} \right) (T_s - T_g) + \dot{m}_w''' C_{pw} (T_s - T_g)$	Eq. 7
$GAC + v_{O_2} O_2 \xrightarrow{R_{GAC}} GAS, R_{GAC} = A_{GAC} \exp \left(-\frac{E_{GAC}}{RT_s} \right) (Y_{GAC})(Y_{O_2})$	Eq. 8
$k_{eff} = \delta [(1 - \phi)(k_s + k_{rad}) + (\phi S_w)k_w + \phi_{GAC}k_{GAC}], t \leq t_g, \delta = 6; t > t_g, \delta = 1$	Eq. 9
$(\rho C_p)_{eff} = \gamma [(1 - \phi)\rho_s C_{ps} + (\phi S_w)\rho_w C_{pw} + (\phi_{GAC})\rho_{GAC} C_{pGAC}], \gamma = 0.56 (S_{w,0})^{-0.19}$	Eq. 10
$u_g = -\frac{k_{rg} k_p}{\mu_g} \left(\frac{\partial P_g}{\partial x} - \rho_g g \right)$	Eq. 11
$u_w = -\frac{k_{rw} k_p}{\mu_w} \left(\frac{\partial P_w}{\partial x} - \rho_w g \right), S_w > S_{w,b}$	Eq. 12

3

4 In this study, it was assumed that when S_w was
5 lower than the capillary-bound saturation, S_b (14%),
6 water was immobile [12]. Consequently, the two-
7 phase flow mobility feature was only activated when
8 $S_w \geq 14\%$, where u_g and u_w were calculated from
9 Eqs. 11-12. They were coupled through the capillary
10 pressure-saturation relationship $P_c = P_g - P_w$, which
11 followed Brooks-Corey Capillary pressure curve and
12 associated relative permeability in SM Section S3.

13

14 2.3 Global Energy Balance

15

16 A global energy balance was applied:

17

$$18 \quad \dot{E}_{net} = \dot{E}_{in} + \dot{E}_{evap} + \dot{E}_{oxid} + \dot{E}_{loss} + \dot{E}_{out} \quad (13)$$

19

20 where $\dot{E}_{net}$ represents the global net energy rate
21 stored in the porous media, which is the sum of five
22 energy rates: heater input energy ($\dot{E}_{in}$), energy
23 consumed by evaporation ($\dot{E}_{evap}$), oxidation energy
24 generated ($\dot{E}_{oxid}$), system heat losses ($\dot{E}_{loss}$), and
25 energy out through the effluent gas ($\dot{E}_{out}$). Each term
26 was defined in [8, 14] and detailed in SM Table S3. A
27 positive $\dot{E}_{net}$ is needed for self-sustained smoldering.

28

29 3. Results and Discussion

30

31 3.1 Model Validation

32

33 Fig. 2 shows the simulated temperature and S_w
34 computed with and without water mobility compared
35 to the experimental results (i.e., Cases #1-4). Note that
36 the non-mobility model was validated with low
37 saturation experiments in [8]. After air injection,
38 smoldering was initiated and propagated in a self-
39 sustained manner at similar smoldering front
40 velocities with and without mobility. For example, the
41 case of $S_w = 30\%$ had average smoldering velocities
42 of $0.619 \pm 0.048 \text{ cm.min}^{-1}$ (two experimental repeats),
43 $0.662 \text{ cm.min}^{-1}$ (mobility model), and $0.600 \text{ cm.min}^{-1}$
44 (non-mobility model); and peak temperatures of 847
45 $\pm 44 \text{ }^\circ\text{C}$ (two experimental repeats), $783 \text{ }^\circ\text{C}$ (mobility
46 model) and $765 \text{ }^\circ\text{C}$ (non-mobility model). Moreover,
47 both models captured the $\sim 50 \text{ }^\circ\text{C}$ plateau, which
48 resulted from the water re-condensation ahead of
49 smoldering [8]. These similar results suggest that the
50 direct impact of water mobility on the “smoldering
51 propagation” stage was relatively small.

52 Nevertheless, the “pre-heating” stage (i.e., with air
53 off) can only be simulated with the mobility model.
54 The water in the bottom of the pack started to boil at
55 $100 \text{ }^\circ\text{C}$ due to radiative heating. Then the produced
56 vapor moved upwards due to buoyancy, which
57 condensed in the upper cold regions, releasing the
58 latent heat and increasing temperatures until boiling
59 [10]. The succession of crossing temperature profiles
60 at $100 \text{ }^\circ\text{C}$ represents the propagation of the
61 condensation front ahead of water boiling (Fig. 2b).
62 However, in the mobility model, water in the upper
63 region above S_b could move downwards and
64 accumulate at the bottom of the pack, as shown by a

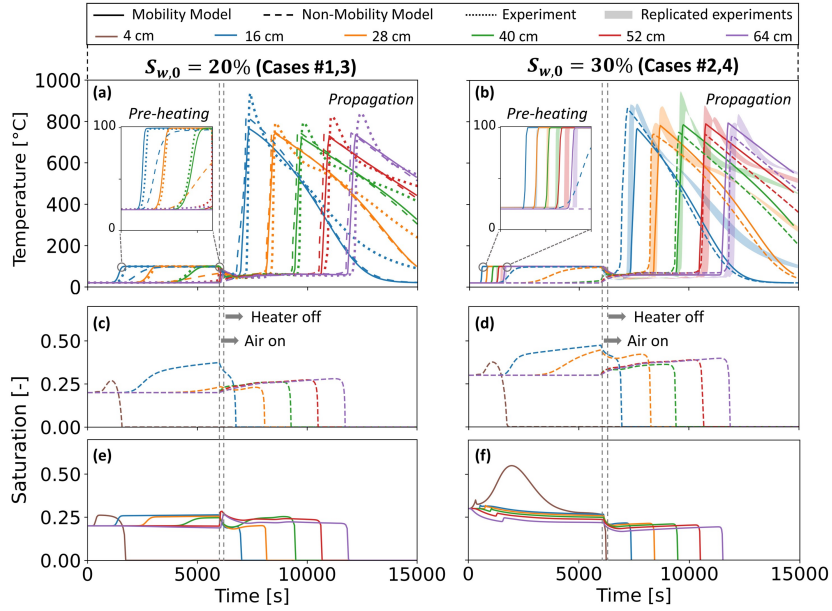


Fig. 2: Comparisons between the mobility model (solid lines), non-mobility model (dashed lines), and experiments (dotted lines), and shadings showing the ranges for two experimental repeats) for Cases #1-4. (a-b) Temperature and (c-f) water evolutions versus time. Colors describe the measurement positions of temperatures and water saturations at 0.12 m intervals.

1 significant increase in S_w at $x = 4$ cm (e.g., 60% vs.
2 30% in Fig. 2f and 2d, respectively). This
3 accumulation produced more vapor that transported
4 upwards and further enhanced the release of the water
5 latent heat. As a result, water mobility caused the
6 condensation front to propagate faster, and thus the
7 entire column was rapidly heated to 100 °C, compared
8 to the case without mobility (Fig. 2a, b). In addition,
9 the condensation front propagated quicker in Case #4
10 ($S_{w,0}=30\%$) than Case #3 ($S_{w,0}=20\%$); again, due to
11 water mobility. Case #4 had a $S_{w,0}$ well-above S_b
12 (30% vs. 14%), i.e., it caused more water
13 accumulation in the bottom and faster vapor
14 production than Case #3. Besides, the maximum S_w
15 at $x = 4$ cm was doubled in the mobility model than
16 the non-mobility model for $S_{w,0}=30\%$, while the case
17 of $S_{w,0}=20\%$ had a similar bottom S_w in both models.
18 These results demonstrate that water mobility is
19 important at high saturations and must be simulated at
20 these conditions to predict key smoldering dynamics.
21 Although the water boiling rate increased due to
22 water mobility, the time to dry the bottom lengthened
23 with mobility. The drying time at $x = 4$ cm for the
24 $S_{w,0}=30\%$ condition was predicted to be 6000 s
25 (mobility, Case #4) and 1700 s (non-mobility, Case
26 #2). This increased drying time aligns well with
27 common experimental trends observed in HMWs
28 smoldering [2-6], revealing the water migration rate is
29 much faster than the increased boiling rate. Moreover,
30 this water accumulation can create challenging
31 ignition conditions. For instance, it took a longer time
32 for $x = 16$ cm (the blue TC in Fig. 2b) to ignite after
33 considering mobility, suggesting weaker ignition.

34 In contrast, the downward water migration
35 decreased S_w across space away from the bottom ($x >$
36 4 cm in Fig. 2d, f), which favored smoldering during
37 the propagation stage. A higher peak temperature and
38 faster smoldering front velocity were observed in the
39 mobility model than the non-mobility model (783 vs.
40 765 °C and 0.662 vs. 0.600 cm.min⁻¹, respectively,
41 Fig. 2b). This finding suggests that the major obstacle
42 in smoldering HMWs might be the long drying period
43 during pre-heating. Once it is overcome, smoldering
44 propagation is successful as less S_w remains in the
45 system due to initial mobility.

47 3.2 Water and vapor movement

48
49 Fig. 3 illustrates the migration of water and vapor
50 during the wet smoldering process for the base case
51 (#4). At $t = 0$ s, the entire domain is at $S_{w,0}$ (30%) and
52 there was no water vapor in the gas phase. Then, water
53 migrated downwards where local $S_w > S_b$ (14%).
54 This resulted in a non-uniform water distribution,
55 reducing S_w at the top while increasing S_w at the
56 bottom (e.g., 75% in Fig. 3d). With the heater turned
57 on, accumulated water in the bottom started to boil
58 and produce vapor, leading to a fully saturated vapor
59 (i.e., $Y_v = 100\%$). Due to the vapor pressure gradient,
60 vapor moved upwards and condensed in the upper
61 cold region away from the heater. At the condensation
62 front, Y_v immediately reduced from 100% to 0%
63 while the local S_w increased, corresponding to a small
64 peak in the S_w profile (Fig. 3d) and the temperature
65 rose to 100 °C (Fig. 3e). At $t = 2000$ s, the
66 condensation front reached the top of the pack, and

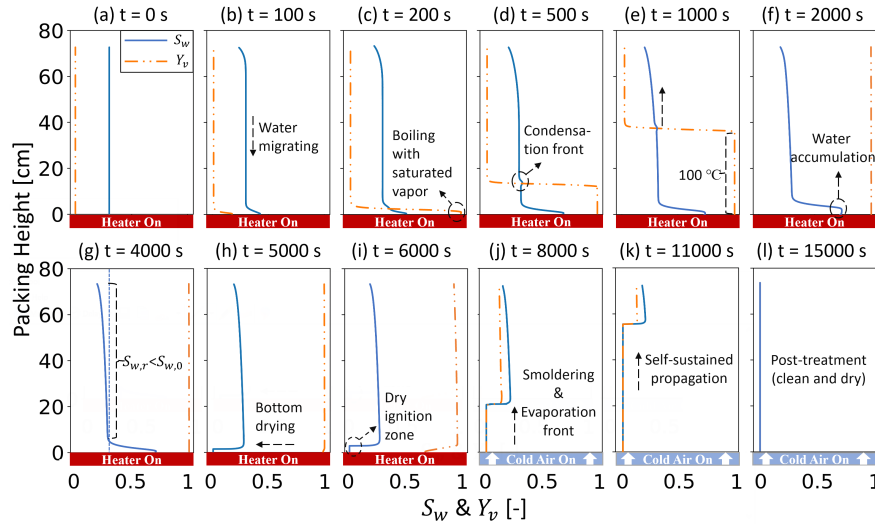


Fig. 3: The predicted water and vapor saturation distribution for base case (#4) considering water mobility. The blue solid lines and orange dotted-dashed lines represent the local water saturation (S_w) and vapor saturation (Y_v), respectively. The dotted blue line is $S_{w,0}$.

the entire pack was boiling at 100 °C, with vapor leaving the top outlet. However, the water velocity was faster than the evaporation rate, which led to water accumulation in the bottom (as discussed above in Section 3.1 and shown in Fig. 2f).

At $t = 4000$ - 5000 s, the water accumulation height decreased as the water mobility became slower than the evaporation. By this time, a large amount of water was evaporated out, and the system exhibited a relatively low residual saturation relative to the initial ($S_r < S_{w,0}$, Fig. 3g). S_r represents the maximum S_w that the porous material can retain against gravity drainage, which is greater than S_b (see SM Section S6). Below S_r , water can cling to the solid due to capillary forces, and therefore water is immobile [15]. This finding is novel for both research and applications, as it quantifies how $S_{w,0}$ might not limit smoldering propagation because water mobility enhances the evaporation and decreases overall S_w to S_r before the ignition.

After 6000 s of heating, a 4 cm dry ignition zone appeared, and smoldering ignition was achieved when airflow was injected (Fig. 3i). The water evaporation front was driven by the smoldering front, accelerating the drying process in the entire system. At $t = 15000$ s, the reaction front reached the top of the pack, leaving behind dry and fuel-free porous media.

3.3 Sensitivity Analysis

Previous studies have identified packing height (H), capillary-bound saturation (S_b), pore size distribution index (λ), and intrinsic permeability (k_p) as key factors influencing water mobility within porous media [12, 16]. These parameters are not only crucial to understanding water dynamics but also allow engineers to fine-tune wet smoldering systems.

Consequently, these properties, using practical values [1-10, 12, 13-14, 16], were chosen for a sensitivity analysis of water mobility across eight cases. Each parameter was varied by half or double the base case value (#4), except the packing height, where a high H condition was only increased from 73 to 80 cm to optimize computational time (Fig. 4).

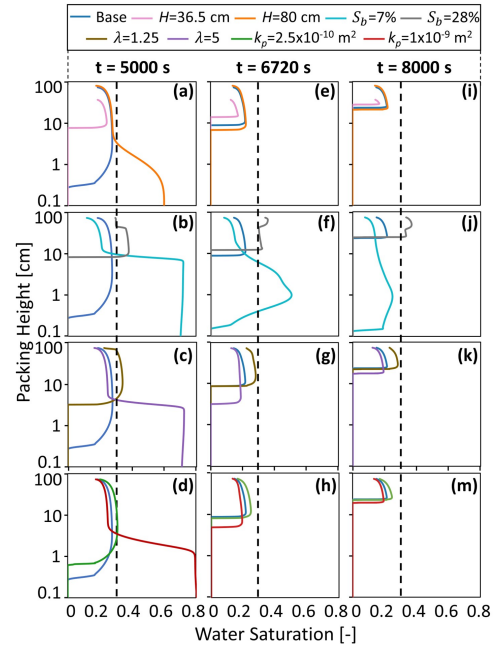


Fig. 4: Spatial water distribution for the sensitivity analyses at 5000, 6720, and 8000 s. The vertical dashed black line indicates $S_{w,0}$.

Fig. 4a-d show S_w distributions at $t = 5000$ s (air off). At this time, the bottom of the pack in the base case (#4) was dried with a 0.2 cm dry zone. However, a small increase of H to 80 cm led to a substantial rise in the bottom S_w (60%). The extra water height effectively increased the water migration rate, and more water flowed downwards and accumulated in the bottom (Fig. 4a). In contrast, a decrease in H to 36.5 cm reduced water migration, and a 10 cm dry zone was observed after the same pre-heating time. This finding agrees with [16], revealing the importance of packing height in HMW smoldering.

As shown in Fig. 4b-c, low S_b and high λ cases also accelerated the downward water mobility, which accumulated water in 7 and 3 cm thick pools, respectively, with $S_w = 70\%$ (note, $S_{w,0} = 30\%$). A porous medium with a low S_b means that the pores hardly retain water; thus, water can easily move downwards with gravity. A high λ represents a porous medium that is composed of well-sorted particles with a homogeneous pore size, i.e., without tiny pores inhibiting water flow. Therefore, water moved through the well-sorted pores more freely. Nevertheless, even though water pooled at different heights, neither S_b nor λ affected the S_w of the water pool, which was fixed at 70%. This constant pool S_w emerged because water can only invade pores and increase S_w when the water pressure exceeds the displacement pressure, which is limited by the porosity, permeability, and surface tension between water and air [12]. Therefore, as expected, Fig. 4d shows that an increase in the S_w of water pool (80% vs. 70%) resulted from increasing the permeability. Moreover, increased downward water mobility drained water from the upper region, as evidenced by a lower S_w in high H , λ , k_p , and low S_b conditions above $x = 10$ cm (Fig. 4a-d).

At $t = 6720$ s, the heater was turned off (airflow was turned on at 6120 s) and smoldering propagated in a self-sustained condition, where $\dot{E}_{net}$ was positive (Eq. 13). That is, the energy rate generated by oxidation exceeded the energy consumption rate by water evaporation and system heat losses. Here, the magnitude of $\dot{E}_{net}$ represents the robustness of smoldering ignition. In Fig. 5, base case (#4) had a $\dot{E}_{net}$ around 640 J/s. However, the enhanced downward water mobility decreased $\dot{E}_{net}$ and weakened ignition. For example, in the case of $S_b = 7\%$, the bottom ignition zone was not completely dried (Fig. 4f) and thus smoldering ignition was not successful, with a $\dot{E}_{net}$ of -270 J/s. On the other hand, stronger ignition occurred with higher $\dot{E}_{net}$ when the water mobility and accumulation were reduced (e.g., with low H and high λ).

Fig. 5 also shows $\dot{E}_{net}$ at 8000 s when smoldering reached the end of the pack. This figure shows the impact of mobility on $\dot{E}_{net}$ at this time was diminished in most cases due to the similarities in remaining S_w profiles (see Fig. 4i-m). This remaining S_w was dominated by the water mobility and water re-

enhanced water mobility could mitigate water re-condensation and lead to a low remaining S_w and more robust smoldering. For instance, Case #10 with well-sorted particles ($\lambda = 5$) had substantial downward water migration, and a large fraction of the water was evaporated during pre-heating. These dynamics led to a much lower remaining S_w than Case #8 with $S_b = 28\%$, i.e., 10% vs. 40%, respectively (Fig. 4j-k). The lower S_w thereby fostered more robust smoldering in Case #10 than #8 (650 J/s vs. 350 J/s, respectively, in Fig. 5). Similar behavior was found for the other cases. The peak temperatures and front velocities of all the cases 75 analyzed are available in SM Table S4.

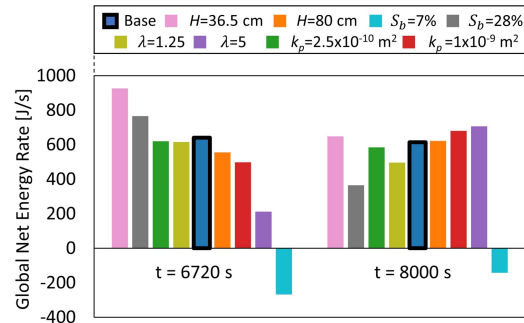


Fig. 5: Global net energy rates from the sensitivity analyses at 6720 s and 8000 s.

Altogether, downward water mobility accumulates more water initially in the base that must be evaporated during pre-heating by an external heater, which lowers the remaining S_w to hinder subsequent smoldering. Therefore, if ignition is successful, initial downward water mobility can promote smoldering propagation and increase system robustness with a high $\dot{E}_{net}$. This is a key finding of this study.

3.4 Re-ignition

Three additional cases were conducted to investigate extinction of Case #7: air-on time (t_g), heater-off time (t_h), and packing height (H).

Case #13 extended both airflow and heating times by ~ 21 minutes ($t_g = 7400$ s, $t_h = 8000$ s). Fig. 6b shows that a longer pre-heating time was not sufficient to dry the bottom of the pack; therefore, ignition was unsuccessful. Nevertheless, Case #14 increased t_h (8000 s) but kept t_g (6120 s) fixed – like in the Case #7 – resulting in successful ignition (Fig. 6c). This approach extended the convective heating time (i.e., the period when air and heater were both on), which enhanced water evaporation under the forced airflow. This condition accelerated the drying processes and enabled ignition before the heater was turned off. The comparison between Cases #13 and #14 is important for industrial applications as it suggests that when ignition failures occur due to water

1 quenching, convective heaters should be used for re-
 2 ignition. Besides, this observation confirmed the
 3 conclusion of Section 3.3, i.e., once ignition is
 4 successful, smoldering should be sufficiently robust
 5 to propagate through wet fuel regions.

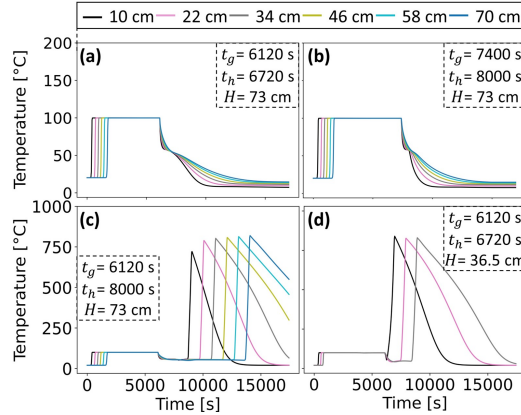


Fig. 6: Different strategies to re-ignite Case #7 (a): (b) increasing pre-heating stage; (c) increasing convective heating stage; and (d) halving the packing height.

6
 7 Finally, when the packing height was decreased by
 8 half (Case #15, Fig. 6d), smoldering ignition was also
 9 successful because less water accumulated at the pack
 10 bottom. This finding is important in designing
 11 smoldering reactors to best treat HMWs.

12 Conclusion

13
 14 In this work, water mobility was integrated into a
 15 smoldering numerical model with immobile water
 16 dynamics. This new model was successfully validated
 17 with experimental data. The simulations showed that
 18 water mobility must be considered to accurately
 19 simulate smoldering ignition and propagation under
 20 high water saturations. Water mobility was shown to
 21 lead to i) severe water accumulation near the heater,
 22 which can inhibit ignition; and ii) reduced water
 23 content ahead of the smoldering front, which can
 24 favor subsequent smoldering propagation. These
 25 observations indicate that water mobility
 26 predominantly occurs in the initial phase of ignition
 27 and significantly influences ignition success. The
 28 fundamental finding should be applicable across a
 29 variety of fuels and smoldering systems, particularly
 30 in upward configurations.

31 Furthermore, the model revealed the important role
 32 of operating properties on water mobility and thus the
 33 fate of smoldering performance. A porous material
 34 characterized by high permeability, homogeneity, and
 35 low capillary-bound saturation could accelerate water
 36 mobility and create challenging ignition conditions.
 37 However, extending the convective heating time and
 38 decreasing the packing height are two viable
 39 strategies to promote smoldering ignition.
 40

41 Overall, this work provides unique insights into the
 42 critical role of water mobility in applied smoldering
 43 systems. The observations presented in this work will
 44 help researchers and engineers better understand and
 45 optimize these systems. Moreover, the conclusions
 46 presented here are widely applicable to smoldering in
 47 other contexts (e.g., wildfires with wet fuels).
 48 However, more complex fuels with different water
 49 dynamics, such as bonded water, require further
 50 investigation.

51 Declaration of competing interest

52
 53 The authors declare that they have no known
 54 competing financial interests or personal relationships
 55 that could have appeared to influence the work
 56 reported in this paper.

57 Acknowledgements

58
 59 Funding was provided by the Natural Sciences, the
 60 Engineering Research Council of Canada, the Royal
 61 Society (RG/R2/232528), and The Open University
 62 through Engineering & Innovation Research Funding,
 63 Higher Education Innovation Funding Knowledge
 64 Transfer Vouchers, and the Open Societal Challenges
 65 Programme in support of the SPLICE challenge, and
 66 Mitacs Globalink. We appreciate support from Chris
 67 Power, Zia Miry, Angelos Almpanis, Joshua Brown,
 68 Taryn Fournie, Shuyang Wang, and Huiyi Yang.

69 Supplementary material

70
 71 Supplementary material can be found online.

72 References

- 73 [1] J.L. Torero, J.I. Gerhard, M.F. Martins, M.A.B. Zanoni,
 74 T.L. Rashwan, J.K. Brown, Processes defining smoldering
 75 combustion: integrated review and synthesis, Prog. Energy
 76 Combust. Sci. 81 (2020) 100869.
- 77 [2] T.L. Rashwan, J.I. Gerhard, G.P. Grant, Application of
 78 self-sustaining smoldering combustion for the destruction
 79 of wastewater biosolids, Waste Manage. 50 (2016) 201-212.
- 80 [3] T.L. Rashwan, T. Fournie, J.L. Torero, G.P. Grant, J.I.
 81 Gerhard, Scaling up self-sustained smoldering of sewage
 82 sludge for waste-to-energy, Waste Manage. 135 (2021) 298-
 83 308.
- 84 [4] T.L. Rashwan, T. Fournie, M. Green, A.L. Duchesne,
 85 J.K. Brown, G.P. Grant, J.L. Torero, J.I. Gerhard, Applied
 86 smoldering for co-waste management: Benefits and trade-
 87 offs, Fuel Process. Technol. 240 (2023) 107542.
- 88 [5] L. Yermán, R.M. Hadden, J. Carrascal, I. Fabris, D.
 89 Cormier, J.L. Torero, J.I. Gerhard, M. Krajcovic, P. Pironi,
 90 Y.-L. Cheng, Smoldering combustion as a treatment
 91 technology for faeces: exploring the parameter space, Fuel
 92 147 (2015) 108-116.
- 93 [6] Z. Song, T. He, M. Li, D. Wu, F. You, Self-sustaining
 94 smoldering as a novel disposal approach for food waste with
 95 high moisture content, Fuel Process. Technol. 228 (2022)
 96 107144.

1 [7] J. Wang, G.P. Grant, J.I. Gerhard, The influence of
2 porous media heterogeneity on smouldering remediation, J.
3 Contam. Hydrol. 237 (2021) 103756.

4 [8] J. Wang, M. A. B. Zanoni, J. L. Torero, J. I. Gerhard,
5 Understanding the role of water evaporation and
6 condensation in applied smoldering systems, Combust.
7 Flame 253 (2023) 112780.

8 [9] M.A.B. Zanoni, J.L. Torero, J.I. Gerhard, Experimental
9 and numerical investigation of weak, self-sustained
10 conditions in engineered smouldering combustion, Combust.
11 Flame 222 (2020) 27-35.

12 [10] M.A.B. Zanoni, J. Wang, J.L. Torero, J.I. Gerhard,
13 Multiphase modelling of water evaporation and
14 condensation in an air-heated porous medium, Appl. Therm.
15 Eng. 212 (2022) 118516.

16 [11] M. G. Letts, N. T. Roulet, N. T. Comer, M. R. Skarupa,
17 D. L. Verseghy, Parametrization of Peatland Hydraulic
18 Properties for the Canadian Land Surface Scheme, ATMOS.
19 OCEAN, 38 (2000).

20 [12] J. I. Gerhard, B. H. Kueper, Relative permeability
21 characteristics necessary for simulating DNAPL infiltration,
22 redistribution, and immobilization in saturated porous
23 media, Water Resour. Res. 39 (2003).

24 [13] M.A.B. Zanoni, J. Wang, J.I. Gerhard, Understanding
25 pressure changes in smouldering thermal porous media
26 reactors, J. Chem. Eng. 412 (2021) 128642.

27 [14] M.A.B. Zanoni, J.L. Torero, J.I. Gerhard, Determining
28 the conditions that lead to self-sustained smouldering
29 combustion by means of numerical modelling, Proc.
30 Combust. Inst. 37 (2019) 4043-4051.

31 [15] A. I. Johnson, Specific yield-compilation of specific
32 yields for various materials, Geological survey water supply
33 paper 1662-D, (1967)

34 [16] L. Kinsman, J.L. Torero, J.I. Gerhard, Organic Liquid
35 Mobility induced by smoldering remediation, J. Hazard.
36 Mater. 325 (2017) 101-112.