



Adopting a Framework for Investigating Mathematics Teachers' Technology-integrated Classroom Teaching Practice: Structuring Features of Classroom Practice

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Abstract

In recent years, there has been a growing effort to deepen our understanding of the complexities and mechanisms involved in integrating technology into mathematics education. This pursuit has led to the emergence of various theoretical frameworks, among which the Structuring Features of Classroom Practice (SFCP) (Ruthven, 2009) stands out. This paper presents a thorough review of the SFCP framework and its fundamental components, with a particular emphasis on its utilisation in examining teachers' domain-specific classroom practices involving digital technology. Drawing upon data from a recent multiple case study, this paper aims to illustrate the adoption and operationalisation of the SFCP in analysing how secondary mathematics teachers integrate dynamic digital tools into their practices as they teach the mathematical domain of geometric similarity. By contributing to the testing and refinement of the SFCP, this paper advances our comprehension of this innovative yet promising framework. Additionally, it provides a demonstration of its practical application and offers a critical reflection on its utility in exploring teachers' everyday classroom practices involving technology for teaching specific mathematical concepts.

Keywords Technology-integrated Classroom Practice · Mathematics Teacher · Theoretical Framework · Structuring Features of Classroom Practice · Geometric Similarity

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Introduction

Research in the field of mathematics education has undergone a substantial shift in focus and approach towards educational technologies (Hwang et al., 2023). Initially, research studies centred primarily on investigating how students' use of technology could enhance their understanding of mathematical concepts (e.g. Borba & Confrey, 1996; Heid, 1988). However, starting in the early 2000s, research attention has gradually shifted towards understanding of the role of the teacher in integrating technology, along with of their beliefs regarding and attitudes towards the use of technology (Clark-Wilson et al., 2014b, 2023; Hoyles & Lagrange, 2010; Mariotti, 2002; Thurm & Barzel, 2021). Recent research has pivoted its focus towards elucidating the features that characterise technology-integrated teaching practices, while also providing support for teachers in integrating these technologies (Bozkurt & Ruthven, 2017; Clark-Wilson et al., 2014b, 2023; Drijvers & Sinclair, 2023; Monaghan, 2004; Ruthven & Hennessy, 2002; Simsek, 2021; Vahey et al., 2020). This shift in focus and approach has led to a clearer distinction between research studies solely examining technology itself and those investigating its practical implementation within classroom settings (Drijvers et al., 2023; Sinclair et al., 2023).

Theoretical frameworks play a fundamental role in guiding research inquiries and understanding complex phenomena within the field of mathematics education (Bikner-Ahsbals & Prediger, 2014; Drijvers & Sinclair, 2023; Ruthven, 2014; Sinclair et al., 2023). They provide researchers with valuable tools and perspectives for formulating precise research questions, selecting appropriate methodologies, and communicating findings effectively. Drijvers and Sinclair (2023) argue that theories serve as a means for researchers to explore the intricacies of "how, why, and when" by directing their attention towards specific phenomena while excluding others. In this way, they function as a tool for elucidating certain aspects of teaching and learning (with technology). Therefore, in parallel with the above-mentioned shift in research focus and approach, the landscape of research in mathematics education has witnessed the emergence of two primary theoretical trends over the past few decades. Initially, student-centred theories, such as situated abstraction (Noss & Hoyles, 1996), dominated the field, emphasising the role of digital tools in enhancing students' learning of mathematics. However, in the mid-2000s, there was a notable transition towards teacher-specific theories, which prioritise the role of teacher, their integration of technology, the expertise necessary for effective implementation, their learning and professional development (Clark-Wilson et al., 2014b, 2023).

Despite the well-documented potential of digital technologies in the broader literature (Ball et al., 2018; Drijvers, 2015), there remains a notable gap between this potential and its successful implementation by teachers in their everyday classroom teaching practices (Bray & Tangney, 2017; Bretscher, 2014). To bridge this identified gap, a comprehensive exploration of teachers' integration of technologies is imperative, encompassing an in-depth analysis of the opportunities and challenges inherent in this process, as well as the requisite expertise for its

successful integration. In pursuit of this goal, several emerging theoretical frameworks have provided valuable lenses for examining teachers' classroom practices and their expertise in educational technology within mathematics education (Ruthven, 2014; Sinclair et al., 2023). Notable among these contemporary frameworks are the Technological Pedagogical Content Knowledge (TPACK) (Koehler & Mishra, 2009), Instrumental Approach (Guin & Trouche, 1998; Guin et al., 2005; Haspekian, 2005; Rabardel, 1995; Rabardel & Bourmaud, 2003), Instrumental Orchestration (Drijvers et al., 2013; Trouche, 2004), Documentational Approach to Didactics (DAD) (Gueudet, 2019; Gueudet & Trouche, 2009), Double Instrumental Genesis (Haspekian, 2011, 2014), [Mathematical] Pedagogical Technology Knowledge (MPTK) (Clark-Wilson & Hoyles, 2017; Thomas & Hong, 2013; Thomas & Palmer, 2014), and Structuring Features of Classroom Practice (SFCP) (Ruthven, 2009). It is worth noting that Sinclair and colleagues (2023), through their examination of research articles in seven leading mathematics education journals from 2014 to 2020, find that recent research predominantly focuses on teaching mathematics with technology employing the Instrumental Approach, with the TPACK framework closely following as the second most prominent approach.

More recently, researchers in mathematics education have increasingly recognised the importance of interconnectedness of theories. They have sought to adopt and combine diverse theoretical frameworks to foster a holistic understanding of the intricate integration of technologies in a manner that is both more inclusive and reliable (Bikner-Ahsbahs & Prediger, 2014; Drijvers & Sinclair, 2023; Haspekian et al., 2023; Ruthven, 2014; Sinclair et al., 2023). For example, Haspekian et al. (2023) state that "Theories are never isolated from each other, nor are they isolated from methodologies" (pp. 5–6). Likewise, Drijvers and Sinclair (2023) argue that there is reciprocity among theory, methodology, and design concerning digital resources, meaning that each aspect influences and is influenced by others in the context of using technologies for mathematics education. Researchers have collaborated to establish networking of existing and/or emerging theories within mathematics education (Bikner-Ahsbahs & Prediger, 2014; Haspekian et al., 2023). For example, several researchers have networked a variety of theories and theoretical constructs (e.g. the Instrumental Approach), resulting in the development of the DAD framework (Gueudet, 2019; Haspekian et al., 2023). Furthermore, researchers emphasise the mutual relationship between theory and research, noting that the application of theories in research studies often leads to the evolution of theories themselves and enhances their potential for practical application (Ruthven, 2014). Existing theories, particularly the Instrumental Approach, have undergone notable evolution over time (Sinclair et al., 2023). For instance, the Instrumental Orchestration framework (Drijvers et al., 2010, 2013; Trouche, 2004) was developed through an extension of the Instrumental Approach (Rabardel, 1995) to focus on the teacher and their technology-rich activities in the classroom. Likewise, the DAD framework (Gueudet, 2019; Gueudet & Trouche, 2009) was built based on the distinction between artifact and instrument introduced by the Instrumental Approach. In this regard, Sinclair et al. (2023) highlight the dynamic nature of the research field by offering a comprehensive overview of the key influences, assumptions, and developments within this

theoretical landscape. Overall, the evolution of theory development and its application in technology-enhanced mathematics education is driven by both theoretical advancements and technological innovation. This dynamic relationship continues to shape research endeavours aimed at promoting successful integration of digital technologies in mathematics education worldwide, ultimately leading to improved learning outcomes for students.

The Present Study

The central focus of this paper revolves around the SFCP framework (Ruthven, 2009), a relatively recent theoretical framework that has garnered attention in the field. Its aim is to provide a comprehensive exposition and review of the SFCP framework along with its constituent components grounded on insights gleaned in a recent multi-case research study in England that applied the SFCP as the primary theoretical framework (Simsek, 2021). This paper also offers a critical reflection on its application and utility when investigating the integration of technology into teaching practices of mathematics in everyday classrooms. Specifically, we explore how this framework can serve as a more effective theoretical tool for gaining a holistic understanding of teachers' domain-specific classroom practices involving dynamic digital technology (DDT). The central research question addressed in this paper is: How does the Structuring Features of Classroom Practice (SFCP) framework serve as a theoretical perspective for examining teachers' (domain-specific) classroom practices that involve (dynamic) digital technology?

In the following sections, we first introduce the SFCP framework, detailing its theoretical underpinnings, components, potential, and limitations. Drawing upon data from recent research (Simsek, 2021), we then demonstrate its application in analysing teachers' domain-specific practices with DDT by operationalising each construct and presenting research findings. Finally, we discuss our findings by contextualising them within existing literature, and we present the broader implications of the paper for educational practice and policy, along with recommendations for further research and acknowledgment of limitations.

Introducing the Structuring Features of Classroom Practice Framework

Advancing a comprehensive understanding of technology integration in mathematics education requires a close examination of the distinct attributes evident in teachers' actual technology-enhanced classroom practices (Bozkurt & Ruthven, 2017; Clark-Wilson et al., 2014b, 2023; Goos, 2014; Trgalová et al., 2018). However, prior research has predominantly focused on exploring the (types of) knowledge, beliefs, and attitudes of teachers, overlooking the holistic experience of their technology use during everyday classroom teaching. Recently, there has been a growing recognition of the importance of observing and examining teachers within

their classroom settings as they make use of technology to enhance students' understanding of mathematical concepts in tangible ways (Bozkurt, 2016; Clark-Wilson et al., 2014a; Mariotti, 2009; Saunders, 2022; Simsek, 2021).

The SFCP framework emerges as a theoretical perspective to assist researchers in examining the characteristic features of teachers' teaching practices and identifying the corresponding expertise required for successful integration of technology into everyday classroom teaching, which evolves with their teaching and learning experience over time. This framework revolves around a system of five key inter-linked constructs fundamental to how teachers incorporate (or fail to incorporate) technology into teaching practice. According to Ruthven (2009), these constructs — *working environment*, *resource system*, *activity format*, *curriculum script*, and *time economy* — have been derived from a rigorous examination within early research endeavours focusing on classroom organisation, interaction, and teacher expertise (e.g. Cohen et al., 2002; Leinhardt et al., 1991; Putnam, 1987; Rivlin & Weinstein, 1984). Additionally, they have been informed by research studies on technology integration into the classroom within, and beyond mathematics (e.g. Deane et al., 2006; Hennessy et al., 2005a, b; Ruthven & Hennessy, 2002). Table 1 exemplifies how the constructs of the SFCP framework are related to the integration of technology into classroom practice. The initial column in Table 1 lists the core structuring components of the framework. The second column presents a set of corresponding defining characteristics aligned with each aforementioned component. The third column provides instances of teacher expertise associated with each construct, underscoring teachers' actions and behaviours within the contextual realm of the first column. The framework therefore provides a structured approach to help us understand the complex interplay between classroom practices, technology integration, and teachers' expertise.

The SFCP framework offers valuable insights into the intricate relationship between teacher expertise, craft knowledge, and technology integration in mathematics education. It illuminates the pivotal role of these elements in shaping teaching practices and provides a structured framework for analysing their interactions. Emphasising that successful technology integration transcends mere tool usage, the SFCP framework underscores the indispensability of teachers' expertise and craft knowledge in successfully incorporating technology into teaching practice within classroom settings (Ruthven, 2009). Within the SFCP framework, teacher expertise refers to a cohesive blend of accumulated knowledge, skills, and teaching experiences that inform classroom teaching practices involving technology (Ruthven, 2014). This expertise, which includes both explicit and craft knowledge acquired mostly through classroom experience and continuous professional development, avoids rigid categorisations into technology-, pedagogy-, or content-focused domains, as observed in the TPACK framework (Ruthven, 2014). Grimmer and Mackinnon (1992) liken teacher expertise to the 'glue' that integrates diverse knowledge bases pertinent to teaching practices. This expertise serves as the foundation upon which craft knowledge is built and refined through classroom-based teaching experience and professional development (Grimmett & Mackinnon, 1992). Craft knowledge provides teachers, for example, with theoretical underpinnings and pedagogical principles essential for developing instructional strategies tailored to

Table 1 The constructs of the structuring features of classroom practice (SFCP) framework (Ruthven, 2014, p. 387)

Structuring feature	Defining characterisation	Examples of associated craft knowledge related to incorporation of digital technologies
Working environment	Physical surroundings where lessons take place, general technical infrastructure available, layout of facilities, and associated organisation of people, tools and materials	Organising, displaying and annotating materials Capturing or converting student productions into suitable digital form Organising and managing student access to, and use of, equipment and other tools and materials Managing new types of transition between lesson stages (including movement of students)
Resource system	Collection of didactical tools and materials in use, and coordination of use towards subject activity and curricular goals	Establishing appropriate techniques and norms for use of new tools to support subject activity Managing the double instrumentation in which old technologies remain in use alongside new Coordinating the use and interpretation of tools
Activity structure	Templates for classroom action and interaction which frame the contributions of teacher and students to particular types of lesson segment	Employing activity templates organised around predict-test-explain sequences to capitalise on the availability of rapid feedback Establishing new structures of interaction involving students, teacher and machine and the appropriate (re)specifications of role
Curriculum script	Loosely ordered model of goals, resources, actions and expectancies for teaching a curricular topic including likely difficulties and alternative paths	Choosing or devising curricular tasks that exploit new tools, and developing ways of staging such tasks and managing patterns of student response Recognising and responding to ways in which technologies may help/hinder specific processes and objectives involved in learning a topic Managing modes of use of tools so as to reduce the time “time cost” of investment in student learning to use them or to increase the “rate of return”
Time economy	Frame within which the time available for class activity is managed so as to convert it into “didactic time” measured in terms of the advance of knowledge	Fine-tuning working environment, resource system, activity structure and curriculum script to optimise the didactic return on time investment

technology like GeoGebra to create technology-embedded real-world problem solving scenarios and facilitate student exploration of mathematical concepts through the affordances of technology. Craft knowledge, in turn, represents the practical application of expertise within the specific context of classroom practice, predominantly acquired and employed, often implicitly, during classroom teaching, thereby substantially influencing teaching practices (Grimmett & Mackinnon, 1992; Ruthven, 2014). However, while the importance of teacher expertise in technology integration is widely recognised (Clark-Wilson et al., 2014b, 2023), there remains a gap in understanding the nuanced forms of expertise crucial for successful technology integration in mathematics education. This gap partly arises from the tendency to overlook this dimension of knowledge in alternative frameworks (Ruthven, 2014). According to the SFCP framework, teachers' use of technology in the classroom that results in improved student comprehension of mathematical concepts, corresponds to the continuous adaptation and refinement of their expertise and craft knowledge through reflective engagement with technology-enhanced classroom teaching practices. This continuous learning process is informed and enriched through encounters with challenges, errors, hiccups, and unexpected contingencies (Clark-Wilson, 2010; Clark-Wilson & Noss, 2015; Coskun et al., 2020; Elbaz, 1981; Rowland et al., 2015).

Ruthven (2009) states that the SFCP framework situates teachers' technology integration within the broader landscape of their everyday classroom teaching practice. He argues that employing the SFCP is beneficial, particularly in highlighting the disparity between anticipated outcomes and actual classroom realities, as it promises "a system of constructs closer to the lived world of teacher experience and classroom practice" (Ruthven, 2012, p. 100). The SFCP framework therefore offers a systematic approach to understanding and analysing the complex dynamics of teacher knowledge and its relation to classroom practice. Its analytical lenses provide valuable insights into the processes involved in teachers' use of digital technology in classroom settings, shedding light on the multi-faceted dynamics inherent in everyday classroom practice (Ruthven, 2014). By exemplifying and clarifying its components, the SFCP framework contributes to enhanced clarity and accessibility in technology-enhanced classroom teaching contexts. It guides research inquiries within the field of educational technologies in mathematics education, offering valuable perspectives on how teachers design and implement teaching practices enhanced by digital tools. Below, we discuss each of the five core components of the SFCP framework, elucidating their interconnectedness with the integration of emerging technologies in classroom practices within educational settings.

The Components of the SFCP Framework

The integration of (new) technologies into classroom teaching poses various demands on teachers regarding how to shape the *working environment* of the classroom (Ruthven, 2009, 2014). These demands go beyond simple adjustments to room layout and location; they require transformative shifts in classroom organisation and procedural routines. Teachers are expected to manage the use of instructional

materials (both physical and digital), in addition to capturing and converting student work into suitable digital formats, regulating technology access, and navigating transitions between instructional phases. Moreover, the adoption of technologies alongside traditional resources, such as textbooks, necessitates teachers to establish a cohesive and harmonised *resource system* (Ruthven, 2009, 2014). This involves not only selecting, organising, and interpreting didactical tools and materials within instructional settings but also developing suitable pedagogical approaches and norms to integrate both traditional and new tools synergistically.

Furthermore, incorporating technologies into teaching practices demands that teachers adapt established *activity formats* to foster interactive engagement among teachers, students, and technological tools (Ruthven, 2009, 2014). This adaptation entails developing innovative classroom routines and utilising activity templates such as those structured centred on *predict-test-explain* approach (Clark-Wilson & Hoyles, 2017) to maximise the potential for rapid feedback DDT offers. Teachers are also expected to clearly delineate roles and responsibilities for themselves, students, and technological tools to ensure effective implementation of technologies.

Moreover, the incorporation of (new) technologies compels teachers to refine and adjust their *curriculum script*—a vital component of their expertise—guiding instructional design and implementation (Ruthven, 2009, 2014). This script comprises meticulously structured elements, including learning objectives, instructional strategies, outcomes, and actions for teaching a mathematical topic with technology. It also encompasses considerations of potential misconceptions and instructional challenges specific to the mathematical domain, alongside diverse resources, tasks, linguistic elements (both mathematical and technological), thought-provoking questions and alternative pathways. It is important to emphasise that a teacher's curriculum script allows for flexibility and adaptability in response to the dynamic and complex nature of technology-integrated classroom teaching and students' individual needs, interests, and learning progress. Drawing from their classroom teaching experience, teachers can reflect on and revise their script to craft responsive instructional approaches that resonate with diverse learning trajectories. Finally, the adoption of (new) technologies calls for a re-evaluation of the classroom's *time economy*, prompting teachers to formulate strategic approaches for optimising the efficient use of instructional time available (Ruthven, 2009, 2014). This involves devising time-saving strategies aimed at enhancing students' academic engagement and maximising learning outcomes (Wyne & Stuck, 1982).

The Limitations of the SFCP Framework

The SFCP framework is not without its limitations, as documented in the literature (Bozkurt & Ruthven, 2017; Chorney, 2021; Gustafsson, 2017). Ruthven and colleagues formulated this framework mainly by amalgamating various concepts associated with classroom practice from different studies in the broader literature. Importantly, these studies did not exclusively focus on mathematics education, nor were they specific to any particular mathematical subject or artifact. Consequently, the concepts that inform and shape the development of the constructs of the SFCP

possess a degree of generality that extends beyond the boundaries of mathematics education. Despite this limitation, the framework demonstrates potential for research studies exploring technology integration in mathematics education. This is evidenced by a recent study conducted by Bozkurt and Ruthven (2017), which adopts the SFCP as the theoretical backbone for their entire research endeavour, marking the first instance of its application in such a context. Bozkurt and Ruthven emphasise the importance of the SFCP framework in comprehending teachers' integration of technology into classroom practice. However, they also identify a challenge in implementing the framework's components, particularly concerning the curriculum script. This aspect is observed to be abstract and demanded further details on how teachers teach the same subject with technology to various classes to fully grasp the teacher's complete curriculum script. They conclude that additional testing and close examination are necessary to clarify and improve the concept of curriculum script, which might entail conducting deeper analyses of data pertaining to technology-integrated practices. Additionally, other researchers (e.g. Chorney, 2021; Gustafsson, 2017; Simsek, 2021; Skott et al., 2021; Villarreal & Esteley, 2023) also use the SFCP framework in their studies, acknowledging its value as a theoretical tool for examining technology integration in mathematics teaching within the classroom setting.

For example, Gustafsson (2017) investigates the applicability of the SFCP framework in analysing interview-based empirical data from two cases within a design research project in a Swedish lower secondary school. The project aims to develop design principles for Classroom Response System (CRS) tasks and understand teachers' reasoning regarding technology integration in the mathematics classroom. The study finds that the SFCP framework effectively captures a substantial portion of teachers' reasoning, with activity format, curriculum script, and resource system being prominently represented. Gustafsson suggests that these results indicate the usefulness of the SFCP framework in analysing teachers' reasoning about technology integration in mathematics education, providing insights into various aspects of their decision-making processes. Gustafsson argues that the components of the SFCP framework may not effectively capture aspects related to students' attitudes and behaviours, proposing an extension of the framework to address this limitation. However, while it is important to consider students' perspectives in technology-integrated classroom settings, the primary focus of the SFCP framework is on understanding teachers' expertise and practices. Therefore, while the framework may not directly address students' attitudes and behaviours, it provides valuable insights into teachers' decision-making processes and classroom practices.

Another study, conducted by Chorney (2021), focuses on how four high school mathematics teachers integrate Desmos, a connected graphing package (CGP), into their classrooms. Guided by the SFCP framework, it seeks to understand how teachers develop their relevant craft knowledge as they use Desmos, along with the challenges teachers face and the associated strategies they employ in using Desmos effectively. The research findings suggest that teachers develop their craft knowledge through hands-on experience in teaching environments rather than through formal instructional methods. The findings also indicate that the SFCP framework is closely connected to real classroom practice, making it highly relevant for understanding

teachers' actions and responses to various teaching situations. He claims that the broad range of categories of the SFCP framework offers valuable perspectives for capturing the complexity of classroom practice when integrating technology. Chorney also argues that the strength of the SFCP framework lies in its departure from scrutinising isolated events and static knowledge. Rather, it aims to delineate a set of categories that characterise classroom teaching practices involving technology, subsequently engaging with dynamic evolution of teaching practices over time. One limitation related to the framework identified in Chorney's study is the challenge posed by the component of working environment. He asserts that this aspect may not always align smoothly with teachers' classroom realities, impacting their ability to effectively and economically manage time and resources.

Furthermore, the SFCP framework lacks detailed descriptions for the component of activity format, which hinders the exploration and depiction of interactions among teachers, students, and technologies during classroom teaching (Bozkurt & Ruthven, 2017). To overcome this limitation and capture the nature of complex orchestration of classroom activities driven by technology, it is important to complement the SFCP framework with a suitable framework specifically tailored to the domain of mathematics education. Such a framework would be instrumental in highlighting the various forms of classroom activity organisation, thus providing valuable insights into how mathematics teachers design and structure lessons integrating technology. For example, Bozkurt and Ruthven (2017) integrate the Instrumental Orchestration model (Drijvers et al., 2010) into their data analysis as a complementary framework to the SFCP. Their findings reveal that this combination offers a useful lens for identifying and analysing overarching patterns within teachers' classroom activity structures involving technology.

Despite the aforementioned limitations, various studies cited above, (including those by Bozkurt and Ruthven (2017) and Simsek (2021)), offer substantial evidence of the SFCP framework's potential to facilitate a comprehensive understanding of what teachers actually do (or do not do) and experience in their classrooms when they integrate technology into their teaching practices.

An Application of the SFCP Framework to Research Teachers' Domain-specific Practice with Dynamic Digital Technology

In this paper, our objective is to examine how Ruthven's (2009, 2014) SFCP framework serves for investigating teachers' actual classroom practices involving technology. Specifically, we aim to demonstrate how the SFCP can be applied and operationalised by drawing on data from recent multiple case study research conducted by the principal author of this paper (the lead researcher in the study) (Simsek, 2021). This multiple case study investigates how secondary school mathematics teachers integrate DDT into their classroom teaching practices, focusing particularly on the mathematical domain of geometric similarity (GS). When referring to DDT in this research, we adopt the definition provided by Clark-Wilson and Hoyles (2017), which describes DDT as "technology offering various mathematical representations (such as geometric shapes, graphs, tables and algebraic expressions) that teachers

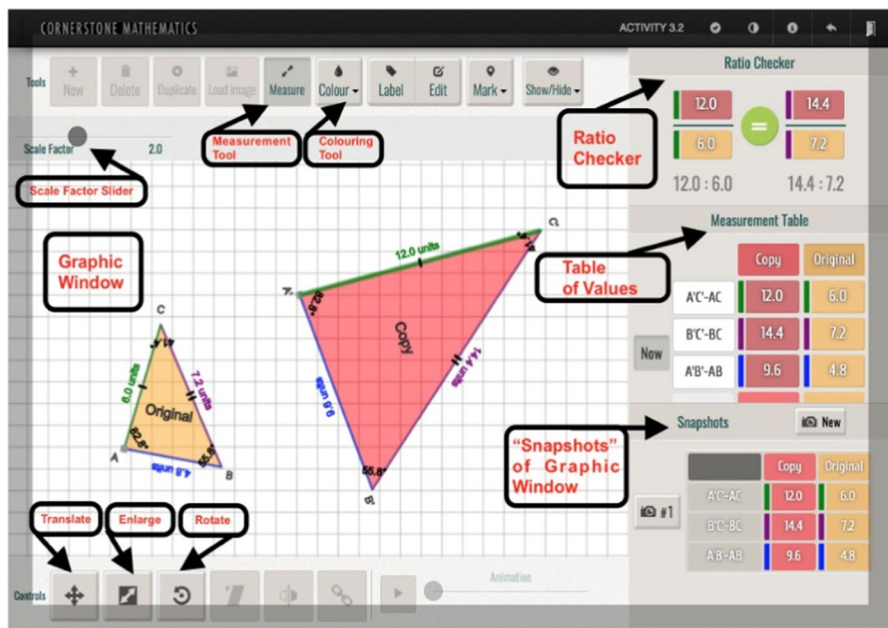


Fig. 1 Selected affordances (indicated in red text) and mathematical representations (e.g. shapes, ratio checker, and measurement table) integrated into the learning environment of the software *Cornerstone Maths*

and [students] can manipulate and by doing so, engage with the underlying mathematical concepts and relationship” (p. 13). This study represents one of the early examples of research studies in the literature that used the SFCP framework as the primary theoretical foundation guiding the design and implementation of the entire research process, including data collection and analysis. In the research, the SFCP was complemented by the Instrumental Orchestration model, with a particular focus on the identified orchestration types such as the *discuss-the-screen* (Drijvers et al., 2010).

The participants in the study were three mathematics teachers who had taken part in the Cornerstone Maths (CM) project¹ (details of which can be found in Clark-Wilson and Hoyles (2017)). The case study teachers (Jack, Alex, and Lara, pseudonyms) worked in lower secondary public schools in London, England at the time of data collection in 2018.

Unlike Alex and Lara, who had similar levels of experience and confidence in the use of digital technology to teach mathematics, Jack demonstrated notably greater proficiency and confidence in employing digital tools in the classroom. To enhance students’ understanding of three selected CM curriculum units, including the GS unit, the participating teachers committed to integrating DDT-enhanced CM

¹ <https://www.ucl.ac.uk/ioe/research/projects/cornerstone-maths>

resources into their classroom practices at their respective schools. These resources comprised a well-established DDT (the software *Cornerstone Maths*; hereafter referred to as ‘the DDT’) and associated supplementary materials such as student workbooks and teacher guides (Clark-Wilson & Hoyles, 2017). It is worth emphasising that the software *Cornerstone Maths* is a web-based DDT accessible globally, providing a dynamic and interactive learning environment spanning three challenging mathematical concepts, such as GS, aimed at lower secondary mathematics (see Fig. 1). It boasts a multitude of features, including dynamic investigations and sub-tasks carefully crafted to captivate students with visually linked multiple representations, encompassing geometric shapes and measurement tables. By exploiting affordances such as dragging, translating, enlarging, and rotating, students can explore variant and invariant properties within geometrically similar shapes. Consequently, the software enables students to formulate conjectures, test them, and explore any differences between their expectations and actual results.

The data collection methods were comprehensive and included: video-recorded observations of classroom sessions (totaling 23 lessons), audio-recorded interviews with teachers following lessons (totaling 20 interviews, each lasting approximately 35–45 min), and a scrutiny of both teacher-created resources and the resulting student work. Data analysis was conducted utilising NVivo 12, employing a combination of *within-case* and *cross-case* analyses guided by the SFCP framework to ensure thorough exploration and interpretation of findings. The research findings unveiled substantial differences and some commonalities among the teachers, shedding light on the fundamental aspects of teaching practices when using DDT to teach GS in the classroom. In this paper, the reported study intends to offer an illustrative example of its implementation within a research context focusing on integrating technology into teachers’ domain-specific everyday classroom practices.

As previously mentioned, the SFCP framework is not specifically tailored to any particular mathematical domain, including the mathematical domain of GS; rather, it serves as a broader framework for investigating classroom practices involving technology. Therefore, in the reported research, we operationalised the SFCP by integrating insights from a wide literature base related to GS, refining it to suit the specific context of our study. Below, we present our operationalisation of each construct within the SFCP framework, along with corresponding findings derived from our reported research. It is important to note from the outset that our research revealed notable differences in the importance and central focus among the five fundamental constructs comprising the SFCP framework. Specifically, our findings indicated that the constructs of curriculum script, resource system, and activity structure differed notably from the remaining two constructs, namely working environment and time economy. We will discuss this divergence in detail later in the discussion section. Consequently, departing from the initial sequence of constructs outlined by Ruthven (2009, 2014), as mentioned earlier, we have opted to begin by presenting our findings pertaining to the constructs of curriculum script, resource system, and activity structure. This will be followed by a discussion of the findings associated with the remaining two constructs, namely working environment and time economy.

Operationalising Curriculum Script in the Study

A teacher's curriculum script comprises a multitude of elements, including, but not limited to, mathematical concepts and ideas aimed at deepening students' understanding through the use of DDT (Ruthven, 2014). It also incorporates an array of mathematical and technological vocabulary carefully chosen to facilitate connections between mathematical concepts and technological aspects of DDT-enhanced tasks. This strategic selection of vocabulary helps students articulate precise language in their oral and written explanations and justifications, thereby enriching classroom discourse and promoting their evolving understanding. Additionally, the curriculum script encompasses questions referring both to mathematical and technological aspects that teachers plan to ask students to probe and promote their mathematical thinking and justifications. The curriculum script also outlines strategies for addressing any misconceptions students may have or encounter through the use of technology.

In our reported research context, it is anticipated that teachers would establish and delineate appropriate teaching objectives (supported by the DDT) for guiding students in understanding the concepts and underlying ideas of GS within their curriculum scripts (Simsek, 2021). For example, they may aim to formulate a learning objective for students to learn the concept of one-to-one correspondence between sides and angles of geometrically similar polygons. This involves ensuring congruence of corresponding angles and equality of ratios of corresponding side lengths *within* and *between* shapes, while also understanding the multiplicative scale factor relationship between polygons.

In developing their curriculum script, teachers may find it crucial to thoughtfully select various vocabularies to foster connections between mathematical and technological elements in tasks involving DDT. To ensure clarity and precision in communication, teachers could focus on the accurate and precise usage of mathematical and technological language, both for themselves and their students, to support explanations and justifications. For instance, by incorporating terms like 'scale factor slider' from technology and 'congruence' from mathematics, teachers can prompt students to recognise and explore the effect(s) of adjusting the scale factor slider to 1 (see Fig. 1). They can also encourage students to consider the defining properties of congruent shapes, where the scale factor corresponds to the ratios between corresponding sides of congruent shapes.

Moreover, in their curriculum script, it is essential for teachers to prioritise mathematical concepts and underlying ideas over the technical features of technology. This underscores the role of DDT as an artifact to enhance students' grasping of GS, rather than solely highlighting its technical aspects in a lesson. For example, after guiding students in using the *Controls* for transformation, enlargement or translation, as well as the *Tools* for measuring sides and angles, in the interactive and dynamic environment of the DDT, teachers may direct their focus to the side and angle properties of a set of geometrically similar shapes. This enables students to explore what changes and what remains constant for two, three, or more similar shapes. Furthermore, teachers might formulate open-ended questions covering both mathematical and technical dimensions of DDT-involved tasks, posing these questions to students

throughout different phases of a lesson. They might employ ‘think-pair-share’ routines to provide students with opportunities to ponder the questions and collaboratively develop and share their responses. For instance, teachers could create questions like: “Place the quadrilaterals on top of each other, aligned at one corner. Does this help you understand which quadrilaterals are geometrically similar? If so, how?” and “What is the numerical relationship between measurements of corresponding sides in the two triangles, and how can you determine whether two triangles are geometrically similar by considering angles alone?”. Finally, when devising their curriculum scripts, teachers could consider and address potential misconceptions that students could confront while grappling with mathematical concepts. Teachers are expected to anticipate and prepare for these misconceptions in the classroom, utilising the affordances of the DDT to help students overcome them. For example, teachers may anticipate a common misunderstanding related to congruency by assuming that similar shapes cannot be congruent. To address this, teachers are expected to plan how students can use the affordances of the DDT, such as the scale factor, to confront and reflect on this misconception.

Key Findings of the Research related to Curriculum Script

- a. **Diversity of Teaching Goals:** The three case study teachers adjusted and enriched their curriculum scripts in response to using the DDT for teaching GS. As a more experienced and confident teacher in the use of digital technology, Jack demonstrated greater diversity in formulating teaching goals related to GS. For instance, he introduced decimal (less than 1 but greater than 0) and integer (including 1) scale factors when creating geometrically similar shapes. He also discussed congruency as a special case of similarity and applied a transformations-based approach to GS using the DDT. In contrast, Alex and Lara primarily utilised integer scale factors and provided verbal explanations without directly incorporating the use of the DDT into their teaching. Unlike them, Jack manipulated the scale factor slider to dynamically change side lengths, encouraging students to explore the properties of resulting shapes.
- b. **Mathematical and Technological Discourse:** While all three teachers demonstrated some degree of emphasis on precise mathematical and technological language, Jack’s utilisation of the DDT fostered a deeper and more enriched discourse on both mathematical and technological concepts during the observed lessons. He engaged students more extensively and effectively, using the DDT to demonstrate the underlying mathematical concepts and connections. In contrast, Alex and Lara had briefer interactions and utilised the DDT less dynamically. As a result, Jack created and capitalised on more opportunities to employ and highlight a broad range of mathematical and technological language. For example, he frequently referenced terms like ‘scale factor’, ‘scale factor slider’, and ‘ratio checker’ when describing and explaining his own (and his students’) actions on the DDT and the associated learning outcomes.
- c. **Questioning Strategies:** Jack demonstrated proficiency in posing open-ended questions during both whole-class discussions and student independent work with the DDT at computers, effectively stimulating students’ mathematical think-

ing through the dynamic capabilities of the DDT. His questions often addressed both mathematical and technological aspects, guiding students' exploration. In contrast, Alex leaned more towards technology-focused open-ended questions, such as "What functions do we have here which are slightly different from the last Activity [Task] which we carried out?". However, he later transitioned to more mathematical questions. On the other hand, Lara primarily prioritised mathematical aspects in her questions, with minimal reference to the use of the DDT.

- d. **Addressing Misconceptions:** Jack demonstrated adeptness in identifying and addressing (a total of six) misconceptions using the DDT during his observed lessons. He actively engaged students in investigating and addressing misconceptions, such as the misuse of additive strategies within tasks involving geometrical reasoning GS, through dynamic utilisation of the DDT. In contrast, Alex and Lara recognised fewer misconceptions (three and two, respectively) and did not necessarily employ the DDT in a dynamic mode to address them. The analysis highlighted more pronounced differences between the teachers concerning spontaneous identification of misconceptions, such as the misconception that scaling the lengths of a shape by a factor of k results in scaling the area of the shape by the same factor, k , and utilising the potential of DDT to address them. Jack, in particular, showed a tendency to involve the entire class or a group(s) of students in exploring and identifying the underlying reasons behind these misconceptions using the DDT in a dynamic mode.

Operationalising Resource System in the Study

Moving on to consider resource system in a general sense, teachers are expected to establish a coherent resource system that incorporates technology-enhanced tasks and employ the system in a coherent and complementary manner in the classroom (Ruthven, 2014). This may involve creating or adopting additional teaching resources, such as paper-and-pencil tasks that enhance technology-related tasks, reinforce students' understanding acquired through their engagement with technology. These supplementary resources serve various purposes, including assisting students in formulating conjectures in a traditional setting and then validating them using technology, or reinforcing insights obtained from their interactions with technology.

To exemplify within the context of our research, when incorporating the DDT into teaching GS in the classroom, teachers might design a paper-and-pencil task that prompts students to conjecture conditions for congruency, then validate these conjectures in a dynamic learning environment that the DDT provides (Simsek, 2021). Moreover, teachers might be also anticipated to grasp the potential affordances of DDT and exploit them to enhance students' comprehension of (dynamic) concepts related to GS. An illustration of this understanding would be their recognition and exploration of the dynamic angle slider's role in realising a crucial insight for maintaining mathematical similarity that the size of corresponding angles remains equal while moving the angle slider that leads to shapes staying geometrically similar.

Additionally, in a DDT-enriched classroom, teachers might be expected to make sense of multiple mathematical representations available within DDT (e.g., geometric shapes, sliders, ratio checker, measurement table). Teachers may encourage students to recognise and understand (dynamic) mathematical connections among these representations. Additionally, teachers could be expected to elucidate the impact of dynamic values on these representations and enable students to explore their relevance to fundamental concepts and relationships about GS. For example, it might be desirable for teachers to understand the purpose of dragging angle slider and make it clear to the class that changing the value of angle slider does not result in a change in the value on scale factor slider; since while the size of corresponding angles increase or decrease at the same time, the ratio of pairs of corresponding sides remains to be the same. Furthermore, teachers might be expected to be mindful of opportune moments and strategies to exploit the potential of DDT during different phases of a lesson. For instance, whole-class teaching may involve the use of DDT to elaborate on and extend GS-related concepts, to elucidate counterexamples that debunk misconceptions, or confirm or refute students' mathematical conjectures. Finally, teachers could also be expected to facilitate student familiarity with the capabilities of DDT, allowing them to explore underlying mathematical ideas and relationships. This might involve conducting a technical demonstration of the features of DDT in a dynamic mode, such as demonstrating how to drag and rotate shapes on top of one another, before students engage in a related DDT-enriched task.

Key Findings of the Research related to Resource System

- a. **Appreciation of potential of DDT:** All three teachers acknowledged the potential of the DDT in offering a meaningful support to students' learning experience and, ultimately, their exploration and grasping of (especially dynamic elements of) GS. In their post-lesson reflections, they highlighted that the DDT provided a learning environment conducive to students' exploration of mathematical concepts and relationships related to GS. However, there were differences in the emphasis they placed on its dynamic nature and potential. While Jack predominantly underlined the dynamic affordances of the DDT, such as its ability to drag and manipulate shapes, Alex and Lara placed more stress on the potential of its static visualisations.
- b. **Dynamic use of DDT:** The case study teachers' use of the DDT throughout different phases of a lesson displayed differences in mode, level, and intent. Jack consistently used the DDT in a dynamic mode across whole-class discussions and students' independent work at computers. He often took advantage of the dynamic nature of the DDT, which involved dragging, manipulating, and linking various mathematical tools and representations to support students' engagement with the mathematical concepts and relationships. In contrast, Alex and Lara used the DDT intermittently in a dynamic mode in the way Jack used.
- c. **Bridging DDT-enriched and non-DDT tasks:** All the teachers supplemented the DDT-enriched CM resources on GS with additional materials (e.g. paper-and-pencil tasks, traditional exam-style questions). During their reflections, they all highlighted the goal of encouraging students to apply concepts beyond the digital

context. However, differences emerged among the teachers regarding both (i) bridging and establishing connections between the DDT-enriched CM resources and other non-DDT-based supplementary resources and (ii) their awareness of its importance. Jack showed an awareness of the value of promoting bridging or transfer to tasks outside the DDT in consolidating students' mathematical thinking beyond their digital experience. In the class, Jack either adopted or developed additional resources, including those created by him using the context of CM resources. For instance, while students were engaged in supplementary tasks in a traditional paper-and-pencil setting (such as filling in the blanks in the statements using appropriate words/expressions based on geometric shapes provided in that environment), Jack motivated them to make use of the knowledge gained from their interaction with the DDT to complete the tasks. In contrast, Alex and Lara only occasionally prompted students to apply insights they gained using the DMT to solve national exam-style questions on a few occasions.

Operationalising Activity Format in the Study

In terms of activity format, teachers are expected to create and employ (new) activity formats to orchestrate technology-enriched activities when integrating technology into the classroom teaching of a mathematical domain (Ruthven, 2014). They may need to employ various types of orchestrations, wherein technology is used in a dynamic mode, to organise classroom activities across various lesson phases. Within the context of the presented research, teachers could be supposed to use, for example, the *work-and-walk-by* orchestration to circulate among students engaged in the DDT-related tasks to offer them support and guidance (Simsek, 2021). When using the different types of orchestrations, teachers might also be anticipated to adopt a 'predict-test-explain' pedagogic approach to offer students a meaningful experience with the DDT and to balance whole-class discussions with individual and group work incorporating the use of DDT. For example, teachers could be expected to encourage students to: predict what happens to corresponding angles and sides when setting the scale factor slider to the value of 1; and check their mathematical conjectures using the DDT; and then explain any differences between what they predict and what they explore along with their justifications.

Key Findings of the Research related to Activity Format

- a. **Employing various orchestration types:** The analysis identified the orchestration types employed by the participant teachers when using the DDT in their teaching of GS in their observed classrooms. Disparities were noted in how the teachers employed the same types of orchestrations, including the *explain-the-screen*, *guide-and-explain*, *work-and-walk-by*, *board-instruction*, both in terms of frequency and approach. For instance, Jack exploited the individual *discuss-the-screen* orchestration more predominantly than Alex and Lara. While employing this orchestration, noticeable differences arose in the respective patterns of interaction between students, technology and mathematics at stake. Whenever

students worked on the DDT-enriched tasks at computers in the observed lessons, through his exploration of this particular orchestration type, Jack tended to circulate among students, interact with them working in pair on the assigned tasks using the DDT. He frequently engaged in discussions about mathematical phenomena presented statically and dynamically on the screen, often using the DDT in a dynamic mode to enhance students' understanding of mathematical concepts and ideas.

Furthermore, Jack commonly used three orchestration types—*discuss-the-screen*, *predict-and-test*, and *spot-and-show*—that were rarely, or never, used by Alex and Lara. Specifically, Jack employed the orchestration of the discuss-the-screen during his main whole-class discussions involving his (and his students') use of the DDT on the IWB (or from the teacher computer) in each of his observed lessons. His aim with this orchestration was to discuss the potential effect(s) of manipulating and linking several mathematical representations within the DDT, such as the measurement table and ratio checker. In contrast, Alex and Lara primarily employed more teacher-led orchestration types in which the dynamic use of the DDT was not necessarily required, such as the explain-the-(static) screen.

Operationalising Working Environment in the Study

When it comes to the construct of working environment, teachers incorporating technology into their classroom practices need to adapt the working environment of their lessons (Ruthven, 2014). This may entail changes in room arrangement, physical layout, and class organisation. Such adjustments may prompt teachers to re-consider and modify their established classroom routines to ensure smooth and effective lesson implementation in a new environment. For example, teachers might need to configure student seating to suit various classroom activities around the use of DDT (Simsek, 2021). They may also need to prepare workstations and resources to facilitate the seamless integration of DDT. Additionally, developing or modifying new routines may be crucial in fostering a working environment that enables students to concentrate on their tasks, participate in discussions, formulate, and evaluate conjectures, and thus explore mathematical concepts while working independently or in pairs with DDT. To illustrate, teachers could consider using IWBs in conjunction with the DDT. This could involve displaying content from a teacher computer via a digital projector, interacting with the DDT in a dynamic mode on the IWB, and subsequently initiating a whole-class discussion centred around the actions taken on the DDT and the corresponding learning outcomes.

Key Findings of the Research related to Working Environment

- a. **Organising the working environment for DDT-integrated lessons:** In the observed lessons of Alex and Lara, who were colleagues at the same school, their instructional settings diverged into two distinct types: a computer room with row-aligned tables and facing students, and a conventional classroom with clus-

tered tables. In the traditional classroom, students collaborated in pairs or trios using iPads, while the computer room witnessed predominantly individual use of computer desktops by students. Despite their differences, both spaces offered common resources such as the IWB, an ordinary whiteboard, a teacher-accessible computer with a data projector, and desktops/iPads for students. Notably, in their observed lessons, the seating arrangements were not pre-arranged.

In Jack's classroom, he took the initiative to arrange cluster desks in a traditional classroom layout specifically designated for his regular classes. His intention was to foster the DDT-integrated individual or group work utilising laptop computers. Jack's classroom was specifically designed to support the laptop-based DDT tasks, with a seating plan prominently displayed on the IWB.

- b. **Coordinating physical materials and resources:** The case study teachers demonstrated a variety of strategies in making use of shared classroom materials and resources. Jack used the IWB effectively for the use of DDT in a dynamic mode, particularly in whole-class discussions. Alex and Lara, however, used it mainly in a static mode for the use of DDT or projected static images from PowerPoint. In their observed lessons, Jack and Lara employed the ordinary write-on whiteboard to note students' responses or illustrate mathematical concepts. Conversely, Alex did not use the ordinary write-on whiteboard during his observed lessons.

All teachers accessed the classroom computer assigned for teacher use to display their DDT-enriched contents, yet unlike Jack and Alex, Lara did not operate the DDT in a dynamic mode from the teacher computer. Jack also personalised Smart Notebook slides with student work captured using his smartphone during students' independent work, enhancing whole-class interaction— an approach not observed in Alex's and Lara's lessons.

Operationalising Time Economy in the Study

Regarding the construct of time economy, teachers need to strategically plan how to allocate lesson time available to ensure effective integration of technology, thereby maximising students' learning opportunities (Ruthven, 2014). Incorporating technology into classroom practice can pose challenges related to time management, prompting teachers to develop and implement various strategies to optimise lesson delivery. For example, teachers may circulate among students during independent work with the DDT at computers, providing necessary support and guidance (Simsek, 2021). This practice may enhance students' academic productivity during activity time, ultimately prolonging their engagement and learning experiences with the technology. Additionally, teachers could make use of applications on the IWB to set designated time frames for specific student tasks on computers, allowing students to monitor the remaining time for completing a particular activity with technology.

Key Findings of the Research related to Time Economy

- a. **Starting lessons smoothly and efficiently:** Jack's familiarity with his classroom provided him with several advantages, enabling him to use the allocated lesson time more efficiently and economically by minimising set up time for available technological resources like laptop computers and the IWB. During breaks between his observed lessons, since he did not need to move to a different classroom, he could organise the classroom environment for the next lesson, for example, by displaying his presentation on the IWB, logging into his classroom desktop for use during the lesson, or distributing laptops to students' desks. Conversely, Lara and Alex, teaching in different classrooms across observed lessons, encountered difficulty becoming familiar with their classroom environments and utilising them effectively for teaching. At the beginning of their lessons, it took them some time to get their lessons start smoothly, negatively impacting their use of the allocated lesson time available. This delay stemmed from the need to distribute iPads to students' desks or prepare desktop computers for student use during the lessons.
- b. **Giving clear and concise instructions about the use of DDT:** With the aid of slides displayed on the IWB, Jack consistently provided clear instructions to students regarding the tasks they were to undertake during independent work with the DDT at computers. This approach in his lessons facilitated students' understanding of Jack's expectations, resulting in more effective use of instructional time. However, Alex and Lara did not adopt this practice. Instead, they preferred to quickly read the instructions provided in the CM student workbook for each activity in their observed classrooms. This approach hindered their ability to utilise lesson time efficiently and economically. Students seemed to take longer to complete DDT-enriched tasks due to their limited familiarity with what was expected of them by their teacher during independent work with the DDT at computers.
- c. **Setting time spans between assigned tasks:** In Jack's case, he utilised the Smart Notebook Timer Tool to set time limits for students' independent tasks involving the DDT. This allowed Jack to keep his students focused, on track, and punctual. However, Alex and Lara did not consistently inform students of the allotted time for tasks or employ any Timer Tool to set time limits. Instead, they occasionally verbally notified students of the time allocated for independent work with the DDT at computers. Instead, they occasionally raised their voices to communicate the time allotted for students' independent work with the DDT at computers or the remaining time to complete their DMT-embedded tasks.

Discussion and Conclusion

The exploration of the intricate and dynamic nature of technology integration in mathematics education has given recently rise to various theoretical frameworks (Clark-Wilson et al., 2014a; Sinclair et al., 2023). Among these, the Structuring Features of Classroom Practice (SFCP) framework, the central focus of this paper,

emerges as a novel but promising tool for examining teachers' technology-integrated classroom practices. Through its five key constructs—*working environment*, *resource system*, *activity structure*, *curriculum script*, and *time economy*—the SFCP framework provides valuable insights into the mechanisms and complexities of integrating technology into the classroom. This paper offers an extensive review of the SFCP framework, with a critical reflection on its application in a research study concentrating on the incorporation of dynamic digital technology (DDT) into the teaching and learning of geometric similarity (GS). The research presented in this paper serves as an exemplary reference case, illustrating how the SFCP framework can be effectively applied in a real research context. Through critical examination and testing, we shed light on the framework's utility in analysing technology integration into teachers' domain-specific classroom practices, particularly within the mathematical domain of GS. Thus, this paper reinforces further the 'tentative' position of the SFCP framework as a useful instrument (Ruthven, 2009, p. 134).

With respect to the five foundational components that constitute the SFCP framework, our reported research findings shed light on notable differences within the framework, particularly regarding the importance and focus of three key components—curriculum script, resource system, and activity structure—in comparison to the other two, namely working environment and time economy. Our analysis highlights a distinct alignment of curriculum script, resource system, and activity structure with mathematical and technological aspects, contrasting with the relatively limited incorporation of mathematical considerations in working environment and time economy components. For example, curriculum script component provides a window into the mathematical dimensions of the study, underlining the importance of teachers' developed scripts in teaching specific mathematical topics with technology integration. Our findings illustrate how teachers, particularly exemplified by Jack, adapted and enhanced their curriculum scripts in response to using DDT for teaching GS. Jack's diverse teaching goals, ranging from decimal and integer scale factors to congruency discussions and a transformations-based approach to GS, highlight the richness of integrating DDT into the curriculum script.

However, when considering the latter two components, namely working environment and time economy, they do not function as effective theoretical lenses compared to the other three components in addressing our research questions concerning teachers' domain-specific classroom practices involving DDT. We argue that, fundamentally, working environment and time economy, while important, emphasise general considerations that teachers need to think and address prior to engaging deeply with mathematics they teach using technology. These considerations encompass factors such as ensuring the availability of computers, arranging the classroom environment, organising student seating, and efficiently managing time allocated. These aspects highlight prerequisites (e.g. technological facilities) that need to be established for teachers to progress toward comprehensive considerations of teaching specific mathematical domains using technology. We propose that, in research on teachers' technology-integrated classroom practices, especially domain-specific ones, the components of *working environment* and *time economy* could serve better as contextualising lenses, providing a backdrop for case study participants and their contexts. This approach could contribute to a clearer understanding of crucial

findings related to the core structuring constructs: *curriculum script*, *resource system*, and *activity structure*.

In this regard, in categorising the five constructs of the framework, we propose that curriculum script, resource system, and activity structure are more appropriately regarded as ‘necessary and sufficient conditions’ (essential components) for integrating technology into teaching specific mathematical domains in the classroom. Conversely, we classify working environment and time economy as ‘necessary but not sufficient conditions’ (important but not solely impactful). It is important to note that this categorisation emerged from our data analysis, although it had not been initially our intention, as earlier research tends to assign equal importance across all five components (Bozkurt & Ruthven, 2017; Chorney, 2021; Ruthven, 2009; Ruthven et al., 2009). Therefore, while this categorisation is tentative and requires further evidence from additional research studies, our study suggests a departure from previous research by highlighting the potential to treat these constructs differently, rather than equally, when employing the SFCP framework as a theoretical lens, emphasising their varying impacts on technology-enriched, domain-specific classroom practices. By categorising the SFCP constructs based on their relative impact on technology-integrated practices, we provide insights into how researchers and practitioners can prioritise certain components for more focused investigations. This categorisation offers a more nuanced understanding of the applicability of SFCP framework in technology-integrated classroom teaching practices in mathematics education.

Furthermore, we argue that, as the SFCP framework was initially formulated at a broader pedagogical level (in Science, English, and Mathematics) (Ruthven & Hennessy, 2002), its lenses are not exclusively tailored to mathematics or any specific mathematical domains, including the concept of GS and any associated artifacts. While Ruthven (2009, 2014) demonstrates its applicability in mathematics teaching in a broad sense, a challenge that has persisted is the extent to which it can be used to understand and explain teachers’ technology-integrated practices within specific mathematics domains. In this regard, we demonstrate, through our fine-grained data analysis, how to specify the SFCP framework in a particular mathematical context (i.e. GS) with DDT (i.e. the software *Cornerstone Maths*), by operationalising it based on wider research literature pertinent to the mathematical domain focused on in this paper (see the full operationalisation of all five components of the framework in Simsek (2021)). Our study points to that operationalising the SFCP framework for teaching GS with DDT in the classroom is beneficial in obtaining more meaningful findings through the finer-grained analysis. Similar to Chorney (2021), we assert that the operationalised framework has the potential to render the craft knowledge of teachers more observable and accessible, making it not only apparent to researchers but also accessible to practitioners and thus bridging the gap between research and practice in mathematics education.

Moreover, the present paper contributes to advancing the understanding of theoretical frameworks in technology-related research within mathematics education. As outlined in the introduction, the evolution of research from an initial focus on students’ use of technology to a nuanced examination of teachers’ roles, their practices and associated expertise underlines the dynamic nature of the

field. Theoretical frameworks, such as the SFCP framework and the Instrumental Orchestration model, are pivotal in guiding research inquiries and understanding complex phenomena within this evolving landscape. However, probing more deeply by analysing the actual role of theories in technology-related research studies is important for gaining a better, holistic understanding of complex phenomena and for driving forward the evolution of theories themselves towards practical application. By combining the SFCP framework and Instrumental Orchestration model, this study provides additional evidence of the complementary value of using two theories in investigating and analysing teachers' classroom practices involving technology. The adoption of these frameworks provides useful constructs for the reported research, as both frameworks focus on teachers' technology-mediated classroom practices. The Instrumental Orchestration framework aids particularly in focusing attention on how the teachers managed activities involving DDT in the classroom, thereby offering a valuable lens for a finer-grained analysis of the construct of activity structure within the broader SFCP framework. This finding is consistent with extant literature recognising the complementary nature of the SFCP framework and Instrumental Orchestration, suggesting their effectiveness as complementary theoretical lenses for elucidating teachers' professional knowledge (Bozkurt & Ruthven, 2017; Drijvers et al., 2010; Ruthven, 2014). Moreover, the combination of the SFCP framework and Instrumental Orchestration in the present study serves as an exemplar of the dynamic interplay between theory and practice in analysing teachers' technology-integrated classroom practices. This collaborative approach reflects the networking of theories and their role in shaping research studies (Bikner-Ahsbals & Preidiger, 2014).

While this paper has primarily focussed on the analysis and application of the SFCP framework, acknowledging its networking with complementary frameworks enriches our understanding and enhances the flexibility in its applicability. For example, networking the SFCP and DAD frameworks has the potential to deepen our comprehension of how teachers use resources interacts with the structuring elements outlined in the SFCP framework. The 'resource system' construct within the SFCP framework aligns with the emphasis of the DAD framework (Gueudet, 2019; Gueudet & Trouche, 2009) on the diverse range of resources accessible to teachers, encompassing both digital and traditional resources. Understanding how teachers manage and coordinate these resources in a coherent and harmonious manner can provide valuable insights into their pedagogical decision-making processes (Sinclair et al., 2023). What is more, Skott et al. (2021) conducted a study employing both the SFCP framework and Patterns-of-Participation (PoP) (Skott, 2013) to analyse a specific episode of classroom lessons taught by a single teacher. Their findings suggest that researchers could adopt a variety of frameworks and theoretical approaches, including networking and dual analyses, to develop comprehensive insights into the integration of digital technologies in mathematics classrooms. Emphasising the potential of the SFCP framework and its networking with other frameworks, this paper informs and advances technology-related research in mathematics education by bridging the gap between research and classroom practice. While the SFCP framework offers valuable insights into technology-integrated teaching practices, it

may not fully capture the complexity of all dynamics and complexities of technology-integrated classroom practices. Future research could explore the ways of the networking of the SFCP framework and other frameworks to address additional aspects of classroom practices involving technology.

Ruthven (2014) has previously advocated for researchers to employ the SFCP framework throughout the entire research process, from data collection to analysis, to thoroughly test, elaborate on, and refine this innovative yet tentative framework. Responding to this call, the study reported in this paper adopted the SFCP framework as the primary theoretical lens for designing and guiding the entire research study, including data collection and analysis. By doing so, this study contributes to the testing, elaboration, and refinement of the framework, providing an important example of its operationalisation and application in exploring teachers' everyday classroom practices involving technology for teaching specific mathematical domains. Addressing our research question stated in the introduction section, the present paper concludes that the (operationalised) SFCP framework serves well as a theoretical perspective for examining teachers' (domain-specific) classroom practices that involve (dynamic) digital technology. As noted by Sinclair et al. (2023) and Ruthven (2014), theories evolve through their application in research endeavours, thereby enhancing their practical application. By emphasising the reciprocal relationship between theory and research, this study underscores the potential for theoretical frameworks to inform and advance educational practices in mathematics. Future research endeavours can further explore how diverse subject areas within mathematics can adapt to and benefit from the SFCP framework in various educational contexts. This could involve examining its effectiveness in different curriculum settings and exploring its use in addressing specific pedagogical challenges. This contributes to an improved and more comprehensive understanding of technology integration across curricula worldwide. Lastly, despite the valuable insights gained from this study, the focus on a specific mathematical domain (GS) and DDT tool (the software *Cornerstone Maths*) may limit the generalisability of the findings to other contexts. Future research might aim to explore the applicability of the SFCP framework across diverse mathematical domains and technological tools to ensure broader relevance.

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Data Availability The datasets produced and examined in the present study are not accessible to the public due to privacy and ethical concerns. However, they can be obtained from the corresponding author upon reasonable request and contingent upon obtaining the required approvals.

Declarations The authors declare they have no financial interests.

Ethics Statement The research study reported in this paper received official approval from University College London (UCL) at the time of data collection.

Conflict of Interest The authors declare no conflicts of interest.

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